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Numerical simulations of dam-break floods with MPM

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Abstract

Due to the ability to model large deformations and the ease of dealing with boundary conditions in a Lagrangian framework, the material point method is getting more popularity in geotechnical engineering applications. In this paper, the material point is tentatively applied to model problems of hydrodynamics as an introduction to this field. Dam-break flows with different initial aspect ratios are simulated in both the material point method and an extensively verified shallow water equations model. In order to test the accuracy and stability of the material point method, simulations of dam-break flows are carried out and the results are in good agreement with other validated numerical methods and experimental data. From the comparisons between the simulations in both the material point method and shallow water equations model, a critical aspect ratio value for the applicability of the shallow water equations is found to be 1. The material point method shows its potential to tackle hydrodynamics related problems.

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Keywords: hydraulics; hydrodynamics; material point method; shallow water equations.

1. Introduction

The material point method (MPM) is an extension of the fluid implicit particle (FLIP) method which was developed from particle in cell (PIC) to alleviate many of its inherent numerical problems in the early

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implementations. The FLIP method uses fully Lagrangian particles and thus it eliminates the convective transport which is the major numerical source of error in PIC [1]. And the FLIP method has been widely used for modelling highly distorted flows and materials with history-dependent behaviour. The extension of the FLIP method to MPM was originally motivated by the problems of solid mechanics with history-dependent variables. The key difference between the two methods is that in MPM, the governing equations are presented in the format of a weak formulation contrary to the FLIP method. This feature enables the MPM to be consistent with the finite element method (FEM) and therefore, the MPM can be presented within an FEM framework. Also, the constitutive equations are invoked at the material points in MPM whereas, in FLIP method, they are solved at the grid nodes [2].

MPM has many attractive advantages over traditional numerical methods [3]. First, it is convenient to use with history-dependent constitutive models because information such as strain, stress, and history-dependent variables can be carried by material points, which enables the spatial and temporal tracking of history-dependent variables. Furthermore, the use of a background mesh allows for the implementation of boundary conditions in a manner similar to that in FEM. Compared with other mesh-free methods, MPM has computationally efficient features. In addition, it also avoids the tensile instability that is evident in Smoothed Particle Hydrodynamics (SPH) [4], [5].

On the other hand, in the field of hydraulic engineering, free surface hydrodynamic flows are of significant industrial and environmental importance but are difficult to simulate because the surface boundary conditions are specified on an arbitrarily moving surface [6]. Due to the fact that the MPM combines both the ease of dealing with boundary conditions of mesh-based methods and the ability of modelling large deformations of mesh-free methods, it is natural to employ the MPM to tackle the mentioned difficulty in free surface hydrodynamic flows.

The purpose of this study is to establish a MPM model for tentatively solving hydrodynamic problems and apply to dam-break flow modelling. For the purpose of model validation, the MPM simulations of dam-break flows are compared with available experimental data [7] and results of other numerical methods, like SPH [4], volume of fluid (VOF) [8]. The results show a good agreement. Then, the parametric study of 2D dam-break flow is conducted using the MPM and traditional SWEs solver. By comparing the simulation results, it is found that with the increase of the aspect ratio, the SWEs solver tends to overestimate the propagation speed of the dam-break flow, and a critical aspect ratio for the applicability of the SWEs is found to be 1. And through the evaluated cases, the MPM can be viewed as a useful tool for study hydrodynamic problems.

2. MPM Model

2.1. Governing equations

Let the material domain Ω be represented by N_p number of material points (Figure 1) and let X_p denote the position of a material point in the current configuration (i.e. time t). The same material point in the initial configuration (time t=0) is denoted as X_p^0.

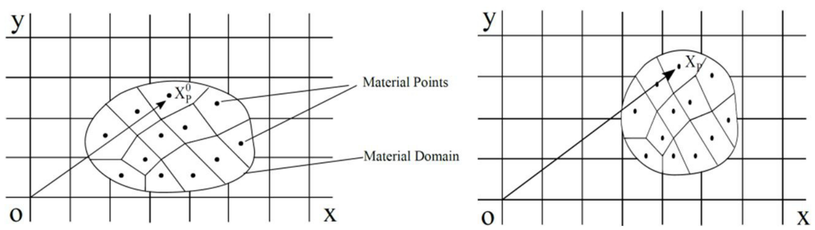


Fig. 1. Sketch of typical computational grid and material points: Initial configuration(left) and deformed configuration (right).

The governing equations which describe the motion of the continuum body Ω are the standard conservation equations: mass (Equation 1), and momentum (Equation 2).

$$\frac{d\rho}{dt} + \rho\nabla \cdot \mathbf{v} = 0 \quad (1)$$

$$\rho\mathbf{a} = \nabla \cdot \boldsymbol{\sigma} + \rho\mathbf{b} \quad (2)$$

Where $\rho(\mathbf{x}, t)$ is the mass density of the continuum body, acceleration $\mathbf{a}(\mathbf{x}, t)$ is the material derivative of the velocity, $\mathbf{v}(\mathbf{x}, t)$ such that

$$\mathbf{a}(\mathbf{x}, t) = \frac{d\mathbf{v}(\mathbf{x}, t)}{dt} \quad (3)$$

and

$$\mathbf{v}(\mathbf{x}, t) = \frac{d\mathbf{u}(\mathbf{x}, t)}{dt} \quad (4)$$

Where, $\mathbf{u}(\mathbf{x}, t)$ is the displacement.

In the MPM, the convective term is not present in the formulation as a result of the grid which can be interpreted as an updated Lagrangian framework [2]. In fact, a separate convective phase where the material points are advanced through the grid is incorporated in the computation procedure.

2.2. Weak form of the governing equation and numerical implementation

The computational cycle contains three phases: an initialization phase, a Lagrangian phase and a convective phase. The information is mapped from the material points to the grid during the initialization phase to initialize the calculation. Then the balance equations are solved on the grid and the material points are updated with new properties during the Lagrangian phase. Finally, the material points are held fixed while the grid is redefined in the convective phase. The details of the discretization of the governing equations and the numerical implementation are not included here due to the limited space. For interested readers, they can be found in literatures [2], [9].

However, it should be highlighted that, in order to eliminate divisions by nodal masses, the momentum is used instead of velocity as much as possible in the MPM formulation and the velocity gradient used to calculate the strain increments at material points is computed using the updated particle velocities mapped back to the grid nodes [9].

3. Dam-break problem

It has become common practice to solve shallow water equations (SWEs) numerically for fully dynamic models for flood routing. Shocks in the flow can be reproduced in the form of discontinuities in the weak solution. Being a set of hyperbolic equations, the SWEs admit discontinuities in the solution. They can be derived by integrating the three-dimensional (3D) Navier-Stokes equations over depth, therefore, they are much easier to solve compared with Navier-Stokes equations for the analysis dimension is reduced by one and the free surface position is simply obtained from the mass conservation principle. They are favoured in rapid flood simulations, since for nearly horizontal flows the SWEs description is accurate and convenient.

Due to the limited space, the SWEs model is not quoted here. But the discussions of the model are available in many papers and for detailed descriptions, readers can refer to [4,10,11].

However, Liang (2010) questions the accuracy of the results of shock-capturing solver, because he thinks a serious inconsistency behind the shock-capturing schemes to solve the SWEs is overlooked. Close to where the flow changes abruptly, the underlying assumptions behind the SWEs (i.e. hydrostatic pressure distribution, smoothly varied water surface and negligible vertical acceleration) may not hold true. So it may need to evaluate the applicability before solving the SWEs for flood routing. In this section, the MPM model is firstly validated through a dam-break flow problem against the SPH simulation and experimental data. Then, the parametric study of 2D dam-break flow is conducted using the MPM and SWEs. It should be noted that the MPM model is actually a 3D model, and only one layer of elements are employed to save the computational cost.

In this paper, the name of SWEs instead of a specific solver is used because the aim of this paper is to evaluate the SWEs rather a specific SWEs solver. The SWEs here are solved by the TVD-MacCormack model which has been extensively verified and applied to many practical case studies [10], [12], [13]. And given the correctness of different SWEs solvers, their results should be of no big discrepancies because they basically solve the same governing equations. Viscous and turbulent effects are not considered in dam-break floods whose behaviours are dominated by convection process. For the cases in this paper, all the contact surfaces are assumed to be frictionless.

3.1. Validation of MPM

In order to validate the MPM algorithm for simulating dam-break flows, the numerical results obtained for the dam-break flow with an aspect ratio of 2 (an aspect ratio is defines as the value of the initial height over the initial width of the water column and denoted by α) were compared with the validated numerical results by SPH presented in [4].

Liang (2010) models the collapse of a water column in a tank using SPH. As sketched in Figure 2, a rectangular water column is initially confined between the left wall of the tank and a dam inside the tank. The aspect ratio of the water column is 2.0 with L_0=0.1 m wide and H_0=0.2 m high. At the beginning of the computation, the dam is instantaneously removed and the water is allowed to collapse onto the dry horizontal bed.

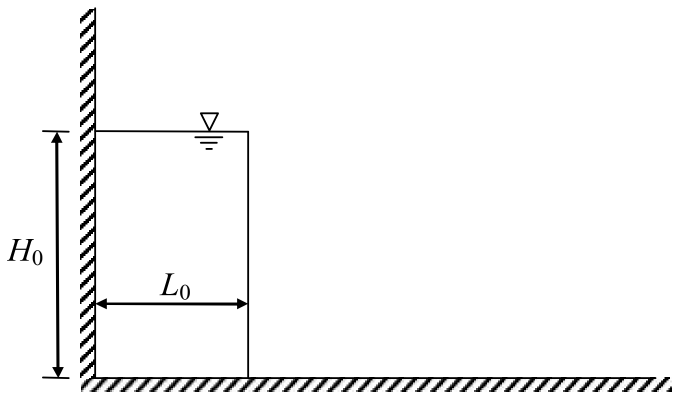
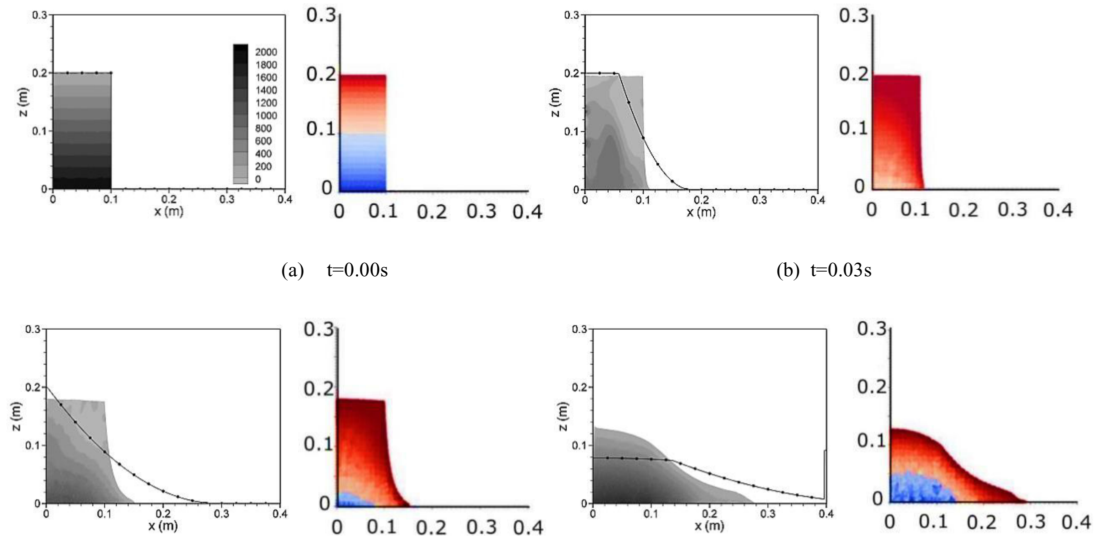


Fig. 2. Sketch of dam-break problem in a tank (Liang 2010).

The propagation of the flood obtained from the MPM simulation at different time steps are compared with the simulation results of SPH and SWEs presented in [4] in Figure 3.



(a) t=0.00s (b) t=0.03s

(c) t=0.07s (d) t=0.15s

Fig. 3. Comparison of water column collapse using SPH (left, contours), SWEs (left, solid lines with circles) (Liang 2010) and MPM (right).

This comparison shows a close agreement between the MPM and SPH simulations.

Some quantitative comparisons of the flood front travelling speed are given in Figure 4, where the experimental data [7] and the numerical results obtained using VOF [8] and SPH [4] are included.

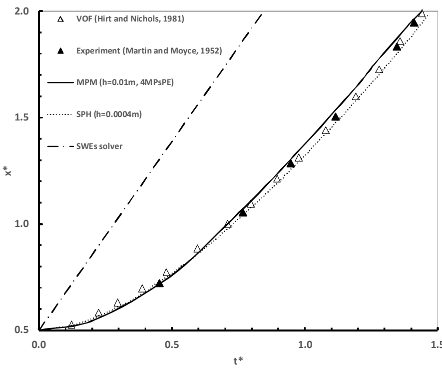


Fig. 4. Variance of the flood front position for the dam-break flow in a tank.

Length and time have been normalized according to the Froude scaling law, in order to compare the experimental data of different dimensions.

$$t^* = \frac{t}{\sqrt{H_1 / \mathbf{g}}}, x^* = \frac{x}{H_1} \tag{5}$$

Where t^* is normalized time, x^* is normalized flow front position, t is time, x is flow front position, H_1 height of water column and $\mathbf{g}$ is gravitational acceleration.

Generally speaking, the MPM simulations compare well with the experimental data and the numerical results of VOF and SPH.

3.2. Effect of mesh size and particle density

The sensitivity of the dam-break flow simulation to the computational mesh size and particle density (the number of material points per element) is investigated through the simulations of dam-break flow with the aspect ratio of 1. Two mesh sizes are employed: coarse (0.1m) and fine (0.05m). Three different particle densities are used: 4, 8 and 10. And the flow front positions at different time are shown in Table 1.

Table 1. Flow front positions at different time.

Condition	Mesh Size (m)	NO. of MPs per Element	Front positions at Different Time (m) 0.0s	0.1s	0.2s	0.5s	0.8s	1.0s	1.2s	1.5s
1	0.1	4	1.00	1.10	1.40	2.86	4.55	5.80	7.05	8.92
2	0.1	8	1.00	1.10	1.40	2.86	4.62	5.83	7.10	9.00
3	0.1	10	1.00	1.10	1.40	2.82	4.58	5.81	7.05	8.92
4	0.05	4	1.00	1.10	1.41	2.85	4.58	5.80	7.05	8.98

From the table above, it is found that the mesh size and the particle density do not play an important role in the results of dam-break flow simulations. And considering about saving the computational effort, comparatively coarse meshes are chosen, i.e. for the simulations with aspect ratios of 0.2, 0.25, 0.5, mesh sizes are chosen to be 0.05m while for other simulations, mesh sizes are chosen to be 0.1m. The number of material points per element in all simulations is chosen as 4.

3.3. Parametric study

Simulations of dam-break flows using the MPM and SWEs are conducted, with different aspect ratios: 0.2, 0.25, 0.5, 1, 2, 3 and 4. The width of the water columns is fixed to be 1.0 m for all of the simulations. And the representative simulations of aspect ratios of 0.2 (shallow column) and 4 (high column) are shown in Figure 5 for comparison. The contours represent the MPM simulations while the solid lines represent the SWEs calculation results. From Figure 5, it can be seen that when the aspect ratio is 0.2, the MPM and SWEs simulations agree well, but when aspect ratio is 4, the simulations show a big difference.

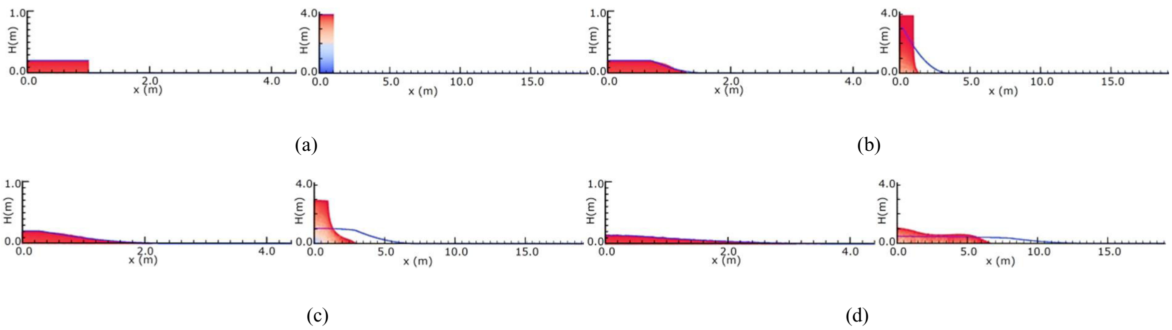


Fig. 5. Flow front simulation (left: $\alpha = 0.2$; right: $\alpha = 4.0$) using MPM (contours) and SWEs (line): (a) t=0.0 s; (b) t=0.2 s; (c) t=0.5 s; (d) t=1.0 s.

The developments of flow front difference Δx with time t of all the simulated conditions are compiled in Figure 6. The flow front difference Δx is defined in Equation 6 as:

$$\Delta x = x_1 - x_2 \tag{6}$$

Where Δx is the flow front difference, x_1 is the calculated front position from SWEs, x_2 is the calculated front position from MPM.

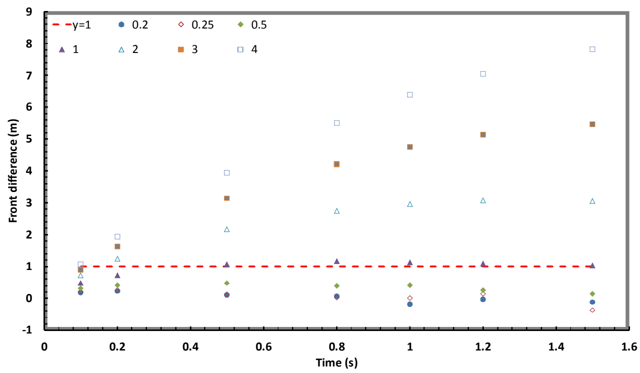


Fig. 6. Front difference between MPM and SWEs with time for different aspect ratio's.

It is worth noticing that an obvious different development trend of the flow front difference between the simulations of larger aspect ratios ($\alpha > 1$) and smaller ones ($\alpha \leq 1$) can be observed. In the case of $\alpha > 1$, the flow front difference gets larger and larger with time, while there is a limit for the difference of about $\pm 1.0m$ when $\alpha \leq 1$. That means when the aspect ratio $\alpha \leq 1$, the simulation results of MPM and SWEs agree well with each other, while this is no longer valid when the aspect ratio exceeds 1.

With regard to the $\pm 1.0m$ deviation, that can be attributed to the fact that for MPM simulations, the flow front position is not defined only as the position of the particle at the most front, but it is defined as the position of the most front particle with continuous following particles. Because of this, the flow front positions are not read directly from the calculation file, but they are read from the visualization of the simulation where some errors can be introduced. Moreover, in this case, the development trend of the flow front difference rather than the value of the difference should be paid more attention to, because the development trend is what we use to evaluate the agreement of the MPM simulations and the results of SWEs. For this reason, $\alpha = 1$ can be regarded as the critical value for the applicability of SWEs.

In addition, for a 2s dam-break flow calculation, a SWEs calculation takes several minutes, while a MPM simulation takes about 2 hours.

4. Conclusions

The performances of the SWEs and MPM models are compared for predicting the simple dam-break flows, using the experimental data and other verified numerical methods (SPH, VOF) as references. And the following conclusions can be drawn:

- Compared with experimental data and other verified numerical methods (SPH and VOF), the MPM simulations can give fairly good predictions of the flow front propagation of dam-break flow.
- The selections of different mesh sizes and numbers of material points per element do not play a significant

role in the dam-break flow modelling.

- For aspect ratio $\alpha \le 1$, the SWEs can accurately capture the front movement of the dam-break flows.
- For $\alpha > 1$, the SWEs can no longer accurately predict the front of the dam-break flows. It tends to overestimate the speed of the propagation speed of the dam-break flows.
- The critical aspect ratio for the applicability of the SWEs is 1.
- With regard to the computational cost, the SWEs is much more efficient than the MPM. In other words, the SWEs still has its advantage in predicting the propagation of dam-break flows with small aspect ratios for it is faster to get a result.

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