

# Cambridge Working Paper Economics

Cambridge Working Paper Economics: 1625

## BARGAINING WITH RENEGOTIATION IN MODELS WITH ON-THE-JOB SEARCH

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This version June 28, 2017.

First version May 31, 2017

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# Bargaining with renegotiation in models with on-the-job search\*

Latest version

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## Abstract

This paper studies a model with wage bargaining, random on-the-job search and renegotiation. Wages are determined by a bargaining process in which the firm and the worker alternate in making offers and there is a probability that this process breaks down. In the model, a given wage contract ends at a Poisson rate, and then the firm and the worker bargain over a new wage. In this setting, a higher wage decreases the firm's markup, but this effect is partly offset by lowering turnover, which increases match surplus. This increase in match surplus enables the worker to capture a higher share of the surplus. This positive effect of a higher wage on match surplus diminishes when they renegotiate more frequently. Eventually, as the contract length tends to zero, the equilibrium of the model converges to the equilibrium payoffs discussed by [Pissarides \(1994\)](#). My model thereby justifies using the Nash bargaining solution with perfectly transferable values in models with on-the-job search. In contrast, when the Poisson rate goes to zero, the equilibrium in the model, which I show to be unique, converges to one of the equilibria found by [Shimer \(2006\)](#).

**JEL classification:** C78, J31, J41, J64

**Keywords:** on-the-job search, bargaining, renegotiation, wage contracts

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\*I would like to thank Jake Bradley, Mike Elsby, Nils Gottfries, Marcus Hagedorn, Ryoko Ito, Hannes Malmberg and Coen Teulings for comments and suggestions and the Economic and Social Research Council and the Tom Hedelius Foundation for financial support.

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# 1 Introduction

In many models featuring on-the-job search (OJS), whether a worker quits depends on the wage. The effect of a higher wage on profits is twofold. First, the markup decreases, and second, turnover is lower. These two effects move in opposite directions. The value function of the worker increases with the wage only via the direct effect on flow income. Adding the profit function of the firm and the worker's value function gives the total match surplus, which is not a constant function of the wage. With wage contracts the worker and firm values are then not perfectly transferable. In some cases, profits can be locally increasing in the wage as the lowered turnover can more than offset the cost of higher wages. Then both the profit function and the worker's value function are increasing in the wage. Under these conditions, it is not possible to apply a Nash bargaining solution (Nash, 1950) because the bargaining set is not convex (Shimer, 2006).

The literature has proposed different approaches to resolve this issue. Pissarides (1994) assumes that turnover is fixed, which allows him to apply the Nash bargaining solution to a bargaining problem with a convex bargaining set.<sup>1</sup> Shimer (2006) takes the interactions between wage and turnover into account. Instead of Nash bargaining, he uses an alternating offer bargaining game in the spirit of Binmore et al. (1986). In this setting, there is an equilibrium despite the potential nonconvexity of the bargaining set. However, the equilibrium in his model is not unique. When the productivity of firms is homogeneous, there are two features to note. First, there exists a continuum of equilibrium wage distributions. Second, given a wage distribution, the bargaining solution within a match is not unique. These indeterminacies present a complication for applied work.

My model features random on-the-job search, bargaining and renegotiation. The model extends the work by Shimer (2006) to incorporate renegotiation and different bargaining powers. Wages are determined by bargaining with alternating offers by the firm and the worker. Between offers, there is an exogenous probability that this bargaining process breaks down. I consider the solution of this model as the breakdown probability goes to zero. In the event of a breakdown, the worker becomes unemployed and the firm's payoff is zero. If an agreement is reached, the wage remains

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<sup>1</sup>Krause and Lubik (2007) analyze a model in discrete time where wages are set in the beginning of the each period, the timing restriction means that this is an equilibrium result. Similarly, in Moscarini (2005) when a worker meets another firm the two firm engage in a auction for the worker. There is an arbitrary small cost to participate in the auction and the equilibrium outcome is that the auction never occurs and the payoffs correspond to the Nash bargaining solution treating turnover as fixed.

fixed until the end of the wage contract when they bargain over a new wage. The end of the contract arrives at a Poisson rate. In this model, the value function of the worker depends on both the current wage and the firm's type, and their relative importance depends on the frequency of renegotiation. If there is no renegotiation, the worker's value function depends only on the wage. This compares with [Burdett and Mortensen \(1998\)](#) and [Shimer \(2006\)](#). When the worker and the firm renegotiate more frequently, and hence the contract length goes to zero, only the firm type determines the worker's value function. For an intermediate frequency of renegotiation, both the firm type and the wage determine the value function of the worker. I show that, in this model, the aforementioned indeterminacies in [Shimer's \(2006\)](#) solution are avoided. The equilibrium wage distribution is unique in my model, and given this equilibrium wage distribution, the bargaining solution within a match is also unique.

The key aspect in my model is that the worker has two state variables, the wage and the firm's type. The frequency of renegotiation (determined by the Poisson rate) plays a key role in my model: it governs the relative importance of the state variables to the worker, and the sensitivity of turnover and match surplus to the wage. For instance, if the worker and the firm never renegotiate, the length of the contract goes to infinity and turnover depends only on the wage. Instead, if the worker and the firm renegotiate sufficiently frequently, the length of the contract goes to zero and wages have no effect on turnover. If turnover is highly responsive to the wage, so is the match surplus – then the worker captures a greater share of the surplus.

One of the equilibria in [Shimer \(2006\)](#) corresponds to a special case of my model in which the worker and the firm never renegotiate and the bargaining powers are symmetric. On the other hand, the payoff function in the model of [Pissarides \(1994\)](#) corresponds to another special case of my model in which the worker and the firm renegotiate continuously and the length of the contract goes to zero. Then an important implication of the present paper is that, in models with continuous renegotiation (also see [Moscarini \(2005\)](#)), the equilibrium can be described by the Nash bargaining solution with transferable utilities. Therefore, this paper justifies the use of the Nash bargaining solution with transferable utilities in the presence of OJS.<sup>2</sup>

In my model, the only commitment possible by the agents is to the wage during the length of the contract. The main difference between this model and other models of wage determination

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<sup>2</sup>In [Coles and Mortensen's \(2016\)](#) model, firms have all the bargaining power and can continuously adjust the wage, but the worker cannot observe the firm's type. This is similar to [Burdett and Mortensen \(1998\)](#) and [Shimer \(2006\)](#) in that the worker's value function only depends on the wage, unlike my model.

with OJS is the type of commitment available to the agents. Wage posting models typically assume that firms have all the bargaining power, and that the wages are set only at the time of vacancy posting (i.e. no renegotiation).<sup>3</sup> See, for instance, [Burdett and Mortensen \(1998\)](#). The wage is chosen so that the marginal gain from hiring and retaining exactly matches the increased wage cost. After hiring a worker, the firm has an incentive to change the agreed wage as it no longer affect the hiring of the worker. Then the wages are not time consistent, as pointed out by [Coles \(2001\)](#). [Postel-Vinay and Robin \(2002\)](#) also consider a case in which the firm has all the bargaining power. But the firm is able to observe outside offers and make counter-offers. When the worker receives an offer, the firm employing the worker and the other firm making the offer engage in Bertrand competition over the worker. The equilibrium entails the worker moving to the most productive firm and to receive a wage that leaves the worker indifferent to working at the less productive firm for a wage equals its productivity. The wage thereby increases within a match as counter-offers arrive. The model requires the firm to commit to the wage when the counter-offer expires.<sup>4</sup>

Section 2 defines the general model and expands on the contributions of the paper discussed above. Section 3 provides a closed form solution in the case of homogeneous productivities. Section 4 concludes.

## 2 Model

There is a frictional labor market with a continuum of two types of risk neutral and infinitely lived agents, firms and workers. Time is continuous and discounted at a rate  $\rho$ . Firms differ in their type  $F$ , drawn from the standard uniform distribution. A firm matched with a worker produces a flow output of  $x(F)$ , where  $x(\cdot)$  is a differentiable and weakly increasing function. The flow profit is given by production  $x(F)$  minus the agreed wage. Workers are homogeneous but differ in their employment state (unemployed or employed), the wage  $w$  and the type of the firm  $F$ . An unemployed worker receives a flow benefit  $b$  and job offers at rate  $\lambda_u$ . An employed worker receives a wage  $w$  and job offers at rate  $\lambda$ . The job gets destroyed at rate  $\delta$  in which case the worker becomes

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<sup>3</sup>[Gautier et al. \(2010\)](#), like [Shimer \(2006\)](#), consider a model where wages are set after the match is formed. The increased retention is then the only reason to increase pay. [Coles and Mortensen \(2016\)](#) consider model where hiring cost are independent of the wage the increased retention is then the only reason for increased pay irrespective of the timing of wage setting.

<sup>4</sup>[Dey and Flinn \(2005\)](#) and [Cahuc et al. \(2006\)](#) extend this model to include bargaining where the threat point is given by the best outside offer.

unemployed and the firm gets no payoff. The wage in a match is renegotiated at a Poisson rate  $\gamma(F)$  until which time it remains fixed, where  $\gamma(F)$  is a weakly decreasing differentiable function. At the time of renegotiation, a new wage is determined by bargaining with alternating offers between the worker and the firm. We restrict our attention to Markov strategies in the bargaining game and to equilibria in which the wage is weakly increasing in the firm type.  $w(F)$  denotes the wage that the worker and the firm type  $F$  expect from renegotiation. An equilibrium requires that, for all  $F \in [0, 1]$ ,  $w(F)$  is a bargaining outcome. Lastly, we assume that workers move if the worker is indifferent between the offer and the current job.

## 2.1 Value functions

Given the wage function  $w(\cdot)$  that the firm and the worker expect, the value function for an employed worker,  $W(F, w)$ , is given by the expression

$$(\delta + \rho + \gamma(F))W(F, w) = w + \lambda \int_0^1 \max \left\{ E(\tilde{F}, w(\tilde{F})) - E(F, w), 0 \right\} d\tilde{F} + \delta U + \gamma(F)E(F, w(F)).$$

The value function for an unemployed worker,  $U$ , is given by

$$\rho U = b + \lambda_u \int_0^1 \max \left\{ W(\tilde{F}, w(\tilde{F})) - U, 0 \right\} d\tilde{F}.$$

Define the excess value of a job to a worker as

$$V(F, w) = W(F, w) - U,$$

and the normalized outside value for the worker as

$$\bar{b} = b + (\lambda_u - \lambda) \int_0^1 \max \left\{ W(\tilde{F}, w(\tilde{F})) - U, 0 \right\} d\tilde{F}.$$

The value function of the firm,  $\Pi(F, w)$ , is given by

$$(\delta + \rho + \gamma(F) + \lambda(1 - G(V(F, w))))\Pi(F, w) = x(F) - w + \gamma(F)\Pi(F, w(F))$$

where  $G(V(F, w)) = \int_{\{\tilde{F}: V(F, w) > V(\tilde{F}, w(\tilde{F}))\}} d\tilde{F}$  is the fraction of firm types with an implied value that is strictly less than  $V(F, w)$ . The measure  $F$  is normalized to only include productivity levels that lead to matches. That is, the productivity satisfies  $x(0) > \bar{b}$ .

## 2.2 Bargaining game

Here I describe the bargaining game and the payoff relating to the bargaining stage. Bargaining occurs in discrete artificial time as in [Shimer \(2006\)](#).<sup>5</sup> The players alternate in making offers. After

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<sup>5</sup>For a treatment of bargaining in real time in a non-stationary search model see [Coles and Muthoo \(2003\)](#).

the proposer has made an offer, the responder chooses to accept or reject the offer. If the offer is accepted, the agents gets the payoff associated with the agreed wage. If the offer is rejected, we move to the next period and there is a probability that this bargaining process breaks down. If this process breaks down, the parties get their outside option. The probability that there is no breakdown is  $(1 - \Delta)^\beta$  after the worker makes an offer and  $(1 - \Delta)^{1-\beta}$  after an offer by the firm.  $\beta$  reflects the relative bargaining power of the worker. We define  $w_\Delta(F)$  to be the bargaining outcome treating the wage distribution as fixed when the friction is  $\Delta$ . I define  $w(F)$  to be a bargaining outcome if, for all  $\epsilon > 0$ , there exist a  $\bar{\Delta}$  such that the difference  $|w(F) - w_\Delta(F)| < \epsilon$  for all  $\Delta < \bar{\Delta}$ .

During the bargaining process, the firm and the worker treat future wage negotiations as fixed, which means that the functions  $V$ ,  $U$  and  $\Pi$  are fixed. The bargaining game consists of two players: a firm with payoff function  $\Pi$  and a worker with (excess) payoff function  $V$ . The action set is  $\mathbb{R}_+$  for the proposer and  $\{\text{Accept}, \text{Reject}\}$  for the responder. A Markov strategy is such that the offer and acceptance rules only depend on the type and not on the previous history. For  $i \in \{w, f\}$ , where  $w$  and  $f$  refers to the workers and firms respectively,  $w_{i,\Delta}(F)$  denotes the wage offer by agent  $i$  in the bargaining game with friction  $\Delta$  and a firm type of  $F$ . We require that  $w(F)$  is an outcome of a sub-game perfect equilibrium (SPE) in the bargaining game as the friction goes to zero for every  $F \in [0, 1]$ . In an equilibrium, the firm and the worker make offers such that the value function of the responder evaluated at the offer is equal to their continuation value. A wage offer associated with a value less than the continuation value is not accepted, and, given the costly delay, such an offer is not optimal. Similarly, an offer higher than the continuation value is accepted, but results in a smaller payoff for the proposer. The following theorem summarizes these results. The proof is in Appendix A.

**Theorem 1** *There exists a unique equilibrium in which the two offers, for all  $F \in [0, 1]$ , solve*

$$V(F, w_{f,\Delta}(F)) = (1 - \Delta)^{(1-\beta)} V(F, w_{w,\Delta}(F)) \quad (1)$$

$$\Pi(F, w_{w,\Delta}(F)) = (1 - \Delta)^\beta \Pi(F, w_{f,\Delta}(F)), \quad (2)$$

and  $\lim_{\Delta \rightarrow 0} w_{f,\Delta}(F) = \lim_{\Delta \rightarrow 0} w_{w,\Delta}(F) = w(F)$ . As the cost of delay goes to zero ( $\Delta \rightarrow 0$ ), the two equations yield the differential equation

$$\beta \Pi(F, w(F)) \frac{\partial V(F, w)}{\partial w} \Big|_{w=w(F)} + (1 - \beta) V(F, w(F)) \frac{\partial \Pi(F, w)}{\partial w} \Big|_{w=w(F)} = 0, \quad (3)$$

with the initial condition

$$\beta\Pi(0, w(0)) = (1 - \beta)V(0, w(0)). \quad (4)$$

The Nash product is defined by

$$\Pi(F, w)^{1-\beta}V(F, w)^\beta. \quad (5)$$

In order to see why the bargaining outcome for a given match is unique it is useful to combine (1) and (2). This shows that the Nash product is the same when evaluated at either the firm's or the worker's offer. (3) implies that the bargaining outcome is such that, the derivative of the Nash product with respect to the wage is zero. Worker turnover is less for firms of a higher type as the worker expects a higher wage offer during the renegotiation. The total surplus of a match is therefore increasing in the firm type, and, for higher types, the contracts last (weakly) longer. Combining both of these insights implies that if the derivative of the Nash product is zero for one type, it cannot be zero for a different type (for further discussion see Appendix A). Thus we see that the inclusion of renegotiation in the model of [Shimer \(2006\)](#) implies that the bargaining outcome within a match is unique.

Turning to the uniqueness of the distribution. The unique equilibrium in this model is such that the wage function solves the differential equation in Theorem 1. Where the initial condition for the differential equation is given by the bargaining outcome that arise if turnover is treated as fixed. To prove that there is a unique equilibrium, the bargaining game for the lowest firm is examined. On the interior of the support, the Nash product is a strict local maximum. To have two offers around the lowest wage, the Nash product must decrease as the wage is lowered below the support. Outside the support, the Nash product is a concave function. The maximum is at the wage that corresponds to the bargaining outcome in which turnover is treated as fixed. The Nash product is increasing in wage outside the support, if the wage is less than this wage. But with bargaining with OJS, the bargaining outcome is greater than or equal to the bargaining outcome treating turnover as fixed. There is therefore a unique wage, consistent with bargaining, for which the Nash product decreases as the wage is lowered.

In [Shimer \(2006\)](#), the Nash product is constant on the support of the wage function. For a (sufficiently) small probability of breakdown, an offer by the firm and the worker can then be found on the interior of the support in which the Nash product is the same. The initial value of the distribution is therefore not determined as both offers are on the support of  $w(\cdot)$ .<sup>6</sup> We have a

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<sup>6</sup>This is similar to the [Burdett and Mortensen \(1998\)](#) model where the firms optimal choice of the wage implies a



unique equilibrium for any positive probability of renegotiation. One might therefore worry about any discontinuity in the number of equilibria in the limit. Letting the worker make the initial offer, in [Shimer \(2006\)](#), implies that the offer by the firm will then be outside the support of wages. This results in a unique equilibrium, for any friction  $\Delta$ .<sup>7</sup>

We can now turn to analyzing the properties of the solution. First we can define the match surplus in the usual way as

$$S(F, w(F)) = \Pi(F, w(F)) + V(F, w(F)). \quad (6)$$

Using the derivative of the value functions we can rewrite the bargaining equation as<sup>8</sup>

$$\Pi(F, w(F)) = \frac{(1 - \beta) \left[ 1 - \frac{\lambda}{w'(F)} \frac{\delta + \rho + \lambda(1 - F)}{\delta + \rho + \gamma(F) + \lambda(1 - F)} \Pi(F, w(F)) \right]}{1 - (1 - \beta) \frac{\lambda}{w'(F)} \frac{\delta + \rho + \lambda(1 - F)}{\delta + \rho + \gamma(F) + \lambda(1 - F)} \Pi(F, w(F))} S(F, w(F)) \quad (7)$$

$$V(F, w(F)) = \frac{\beta}{1 - (1 - \beta) \frac{\lambda}{w'(F)} \frac{\delta + \rho + \lambda(1 - F)}{\delta + \rho + \gamma(F) + \lambda(1 - F)} \Pi(F, w(F))} S(F, w(F)). \quad (8)$$

Compared to bargaining without OJS, there is an extra term coming from the fact that the wage endogenously affects turnover. This results in the worker receiving a higher wage. The lower surplus results in a higher fraction of the surplus going to the worker. The term  $(1 - \beta) \frac{\lambda}{w'(F)} \frac{(\delta + \rho + \lambda(1 - F))}{\delta + \rho + \gamma(F) + \lambda(1 - F)} \Pi(F, w(F))$  captures this effect. The term comprises of the terms capturing the change in the surplus. If the worker leaves for a marginally better firm then the profits are lost. The turnover is therefore not bilaterally efficient. Thus, by reducing turnover the surplus increases by the change in turnover multiplied by the level of profits. The change in turnover is

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differential equation for the wage function. In order to get the initial condition for the wage function we consider a deviation below the support of the wage distribution. Such an offer must not be accepted by the worker and it must therefore be that the worker is indifferent between the lowest wage and unemployment.

<sup>7</sup>An alternative refinement to [Shimer \(2006\)](#) would be to consider equilibria that are the limit from an arbitrary large initial friction. With a sufficiently large (initial) friction in [Shimer \(2006\)](#) there is also a unique equilibrium. Consider a friction such that the offer by the firm falls below the wage that solves (4). The bargaining game implies that the Nash product evaluated at the offer by the firm's and the worker's offer must be the same. Letting the cost of delay go to zero the offer by the firm and the worker converge to a maximum. The set of wages on the interior of the support corresponds to this maximum only in case (4) holds, otherwise the bargaining outcome converges to a point outside the support. Thus an alternative refinement to the model of [Shimer \(2006\)](#) to get a unique equilibrium would be to consider equilibria where, for each type, the bargaining outcome must converge to the wage function for any initial friction as the friction disappears. A global maximum of the Nash product solves (4). If the lowest wage is higher than the wage that solves (4) then the Nash product is lower on the interior of the support than at the wage which solves (4).

<sup>8</sup>Where we use that  $G'(V(F, w(F))) = (\delta + \rho + \lambda(1 - F))/w'(F)$  and  $\frac{\partial V(F, w)}{\partial w}|_{w=w(F)} = (\delta + \rho + \gamma(F) + \lambda(1 - F))^{-1}$

given by the density of incoming wage offers  $\lambda/w'(F)$  multiplied by the relative length the wage remains fixed for  $\frac{(\delta+\rho+\lambda(1-F))}{\delta+\rho+\gamma(F)+\lambda(1-F)}$ . As the length of the contract decreases, the wage becomes less important, compared to the firm type, for the worker. Thus, as the length of the contract decreases, turnover becomes less responsive to the wage and workers capture a smaller share of the surplus. A longer length of the contract thus increases the payoff to the worker. In the limit, as the contract length goes to zero ( $\gamma(F) \rightarrow \infty$ ), the Nash product becomes unresponsive. The wage then solves the standard Nash bargaining solution with perfectly transferable utilities, given by (9) and (10) below.

$$\Pi(F, w(F)) = (1 - \beta)S(F, w(F)) \quad (9)$$

$$V(F, w(F)) = \beta S(F, w(F)). \quad (10)$$

The model thus provides a justification, based on continuous renegotiation, for using the Nash bargaining solution in [Pissarides \(1994\)](#). Intuitively, it is only future wages that reduce turnover and as renegotiation becomes frequent, the current wage becomes irrelevant for future wages. Similarly as the length of the contract goes to infinity ( $\gamma(F) \rightarrow 0$ ) and the bargaining powers are symmetric, the differential equation corresponds to that in [Shimer \(2006\)](#). In the limit, wages are never renegotiated, and the worker only cares about the wage and not the firm type.

### 3 Homogeneous productivities

In this section, I impose some restrictions on the general model. Firstly, I assume homogeneous productivity of the firm (i.e.  $x(F) = x > \bar{b}$ ). Secondly, the discounted duration of a wage contract is a fixed fraction of the expected duration of the job. That is,

$$\gamma(F) = \frac{1 - \theta}{\theta}(\delta + \rho + \lambda(1 - F)). \quad (11)$$

The parameter  $\theta$  measures the fraction of the discounted duration for which the wage remains fixed. When  $\theta$  is one, the model corresponds to the case of no renegotiation, as in [Shimer \(2006\)](#), and the limit as  $\theta$  goes to zero corresponds to continuous renegotiation. For this specific model, we have a closed-form analytical solution. Theorem 2 presents the distribution function, the inverse of the wage function, and the value functions. The proof is presented in Appendix B.

**Theorem 2** *The wage offer distribution is given by*

$$F(w) = \frac{(\delta + \rho + \lambda)}{\lambda} \left( 1 - \left[ \frac{x - w}{x - \underline{w}} \right]^{1/\theta} \left[ 1 - \frac{(x - \underline{w})^{1/\theta}}{\underline{w} - \bar{b}} \left( \frac{1 - \beta + \beta/\theta}{1 - \beta} \right) \frac{(x - w)^{1-1/\theta} - (x - \underline{w})^{1-1/\theta}}{1 - 1/\theta} \right]^{\frac{\beta}{\theta(1-\beta)+\beta}} \right) \quad (12)$$

and the value functions are given by

$$\begin{aligned}\Pi(F(w), w) &= \frac{x - w}{(\delta + \rho + \lambda) \left[ \frac{x-w}{x-\underline{w}} \right]^{1/\theta} \left[ 1 - \frac{(x-w)^{1/\theta}}{\underline{w}-\bar{b}} \frac{1-\beta(1-1/\theta)}{(1-\beta)} \frac{(x-w)^{1-1/\theta} - (x-w)^{1-1/\theta}}{(1-1/\theta)} \right]^{\frac{\beta}{\theta(1-\beta)+\beta}}} \\ V(F(w), w) &= (\underline{w} - \bar{b}) \frac{\left[ 1 - \frac{(x-w)^{1/\theta}}{\underline{w}-\bar{b}} \left( \frac{1-\beta+\beta/\theta}{1-\beta} \right) \frac{(x-w)^{1-1/\theta} - (x-w)^{1-1/\theta}}{1-1/\theta} \right]^{1-\frac{\beta}{\theta(1-\beta)+\beta}}}{(\delta + \rho + \lambda)},\end{aligned}$$

where  $\underline{w} = \beta(x - \bar{b}) + \bar{b}$ .

Theorem 2 extends the solution in [Shimer \(2006\)](#) to incorporate renegotiation and different bargaining powers. I pick parameter values that are typically used for modelling the US economy in the literature (see Table 1). Furthermore,  $x$  and  $\bar{b}$  are normalized to one and zero, respectively. I illustrate the importance of the frequency of renegotiation by varying the frequency of renegotiation. In Figure 1, I plot the wage function and the value function for different values of the frequency of renegotiation.

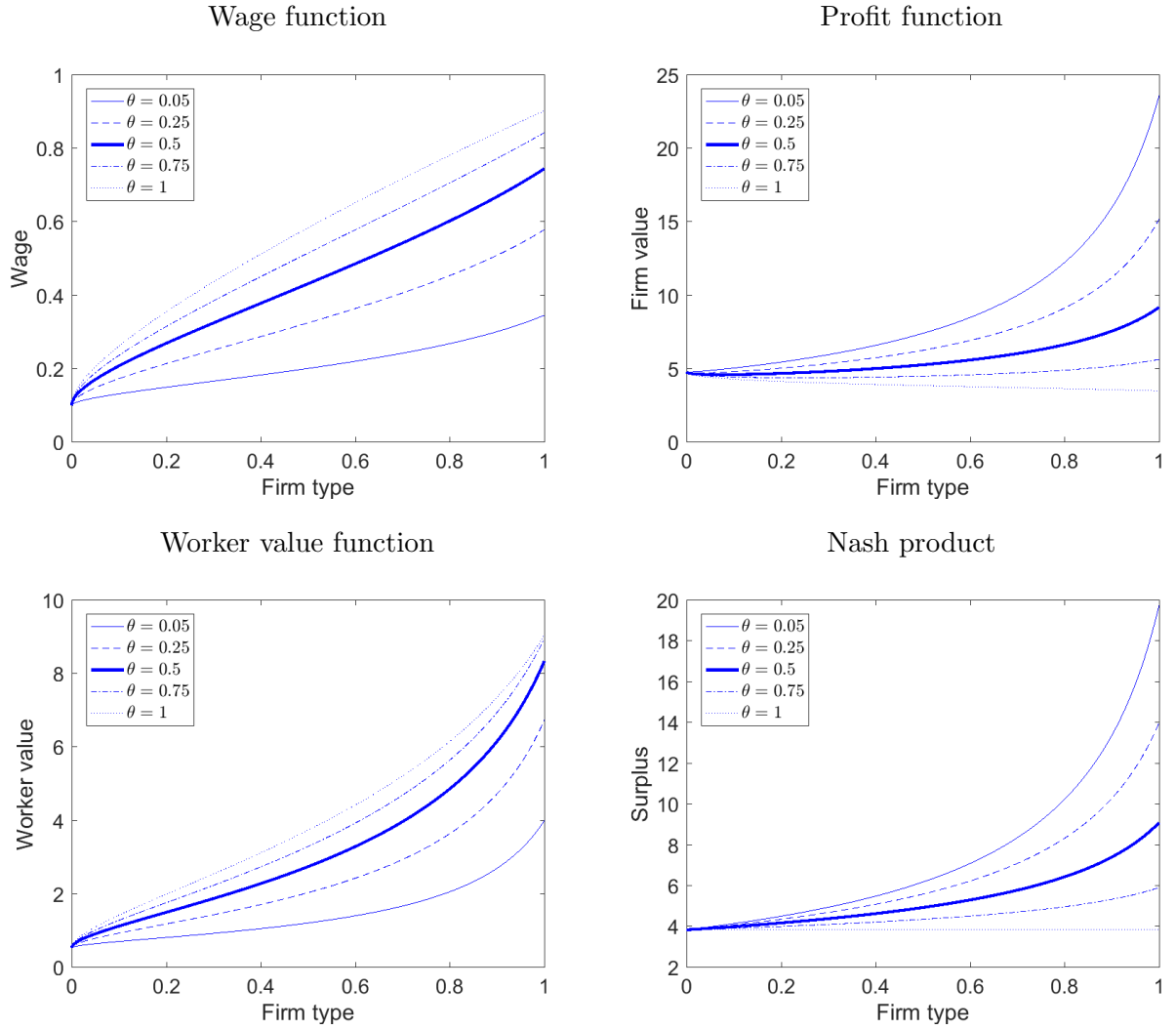
Table 1: Parameters and moments

	Value	Moment	Reference
$\lambda_u$	0.45	45% Monthly job finding rate	<a href="#">Shimer (2012)</a>
$\lambda$	0.16	3.2% Monthly job-to-job transition rate	<a href="#">Moscarini and Thomsson (2007)</a>
$\delta$	0.0237	5% unemployment rate	—
$\rho$	0.004	5% annual discount rate	—

**Note:** Average job finding rate refers to the period 1948Q1-2007Q1.

The more infrequent the renegotiation, the more the wage function increases with firm type. The worker's value function exhibits the same behavior. Worker turnover does not depend on the frequency of renegotiation but wages decrease if renegotiation becomes more frequent. The profits are then increasing in the frequency of renegotiation and particularly so for firms of a higher type. In particular, for sufficiently infrequent renegotiation, profits increase in firm type. If renegotiation is sufficiently infrequent, profits decrease with firm type. Similarly, if renegotiation becomes more frequent, the Nash product increases more with the type of the firm. In the case of no renegotiation, the Nash product is constant ([Shimer, 2006](#)). A central question must then be to find an empirically relevant range of the frequency of renegotiation.

Figure 1: Functions for  $\beta = 0.1$



## 4 Conclusion

This paper generalizes Shimer's (2006) model with OJS and bargaining to include renegotiation and different bargaining powers. Allowing for renegotiation breaks the indeterminacy found by Shimer (2006). Furthermore, the model nests both Pissarides (1994) and Shimer (2006) as special cases of continuous renegotiation and no renegotiation, respectively. This paper further provides a justification for using the Nash bargaining solution with transferable utilities in models with OJS.

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## A Theorem 1

I prove Theorem 1 using a number of lemmas. Define the equilibrium value function as

$$\bar{V}(F) = V(F, w(F)) \quad \text{and} \quad \bar{\Pi}(F) = \Pi(F, w(F))$$

It is useful to define the wage  $w(F, V)$  which gives the worker excess value  $V$  when matched with a firm of type  $F$ . The derivative of the workers value function with respect to the wage when matched with a type  $F$  firm is strictly positive and equals

$$\left. \frac{\partial V(F, w)}{\partial w} \right|_{w=w(F, V)} = \frac{1}{(\delta + \rho + \gamma(F) + \lambda(1 - G(V)))}.$$

Let  $\bar{x}(F)$  denote the wage that gives the firm zero profits and  $\bar{b}(F)$  denote the wage that makes the worker indifferent between unemployment and being matched to a type  $F$ .

**Lemma 1** *Neither  $\bar{x}(F)$  nor  $\bar{b}(F)$  is a bargaining outcome.*

**Proof.** The highest possible continuation value for the worker matched to a type  $F$  firm is the value function associated with the wage of  $\bar{x}(F)$ . The value function for the worker is differentiable in the wage. This means that, for any  $\Delta$ , there exists an  $\epsilon$  such that an offer of  $\bar{x}(F) - \epsilon$  is strictly preferred by the worker compared to waiting one period to get  $\bar{x}(F)$ . Markov strategies imply that any offer that results in a higher value than the continuation payoff will be accepted. Such an offer must therefore be accepted and yield a strictly positive payoff for the firm. Then, an offer  $\bar{x}(F)$  cannot be accepted by the firm, as the firm would get a strictly positive payoff by rejecting and then offering  $\bar{x}(F) - \epsilon$ . The same argument applies to  $\bar{b}(F)$ . ■

**Lemma 2** *In equilibrium, all offers are accepted. Furthermore, the worker’s and the firm’s offers must converge as  $\Delta \rightarrow 0$ .*

**Proof.** First, I exclude the possibility that no agreement is ever reached. If no agreement is reached, then for all  $\Delta$ , there exist some  $\epsilon > 0$  such that an offer of  $\bar{b}(F) + \epsilon$  by the firm must be accepted. (Note that  $\bar{b}(F) = \bar{b}$  is the outside value of the worker). The firm then has a profitable deviation, as such an offer would yield positive profits to the firm (given the assumption  $x(0) > \bar{b}$ ). Thus, an agreement must be reached for all  $F$ .

Suppose that the equilibrium features agreement at a wage  $w$ . Then, the proposer when agreement is reached would accept  $w$  as the continuation value is lower given the cost of delay. This means that it cannot be optimal for the responder to offer an unacceptable wage, since this would be dominated by offering  $w$ . Thus, all offers are accepted. The worker's value function is strictly increasing in the wage, which implies that, if the two offers do not converge as the probability of breakdown disappears, the worker would reject the lower offer. Declining the lower offer would imply that there are no agreement after both offers. ■

**Lemma 3** *In the equilibrium, the profit function is strictly decreasing in the wage offered by the worker and weakly decreasing in the wage offered by the firm.*

**Proof.** First, we note that, at both the firm's and the worker's offer, the profit function is weakly decreasing in the wage. If the profit function would be strictly increasing at an offer, both the firm and the worker would gain from a higher wage. A higher wage would must then be accepted and is preferred by either party. This also rules out any mass points on the distribution, as a mass point means that the turnover effect dominates the decreased markup.

Thus it only remains to show that the profit function is strictly decreasing in worker's offers. I show this by contradiction. Suppose that the derivative with respect to wage of the profit function is zero at the offer made by the firm. We know that the profit function is eventually decreasing (as the turnover effect disappears outside the support). Thus, the worker offers a higher wage that yields  $\Pi - \epsilon$  (where  $\Pi$  denotes the continuation value).<sup>9</sup> For  $\epsilon$  sufficiently small, such an offer will be acceptable. The offer by the worker must therefore always be at a point where the profit function is strictly decreasing in the wage. ■

Lemma 3 implies that, at the bargaining outcome, the profit function is strictly (weakly) decreasing in the region above (below) the bargaining outcome.

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<sup>9</sup>Since there is no mass point on the distribution the profit function is continuous.

**Lemma 4** *If the profit function is locally strictly decreasing in the wage at the offers, then the offers satisfy*

$$V(F, w_{f,\Delta}(F)) = (1 - \Delta)^{(1-\beta)} V(F, w_{w,\Delta}(F)) \quad (13)$$

$$\Pi(F, w_{w,\Delta}(F)) = (1 - \Delta)^\beta \Pi(F, w_{f,\Delta}(F)). \quad (14)$$

As the probability of breakdown goes to zero, (13) and (14) yield the differential equation

$$\beta \Pi(F, w(F)) \frac{\partial V(F, w)}{\partial w} \Big|_{w=w(F)} + (1 - \beta) V(F, w(F)) \frac{\partial \Pi(F, w)}{\partial w} \Big|_{w=w(F)} = 0. \quad (15)$$

**Proof.** Since the worker's value function is globally strictly increasing in the wage, the worker will offer the highest wage such that the firm is indifferent. Because the profit function is continuous and strictly decreasing, such a wage exist. Because the profit function is strictly decreasing, the wage is locally unique. This gives the necessary conditions

$$V(F, w_{f,\Delta}(F)) = (1 - \Delta)^{(1-\beta)} V(F, w_{w,\Delta}(F)) \quad (16)$$

$$\Pi(F, w_{w,\Delta}(F)) = (1 - \Delta)^\beta \Pi(F, w_{f,\Delta}(F)). \quad (17)$$

If there were an inequality then an offer that is  $\epsilon$  higher (lower) by the worker (firm) would be accepted since the worker's value function and firm's profit function are continuous. As the worker's (firm's) value function is increasing (decreasing), such an offer is preferred. Taking the limit as the probability of breakdown goes to zero ( $\Delta \rightarrow 0$ ), we obtain (15). ■

**Lemma 5** *The profit function is strictly decreasing in the offer by both players.*

**Proof.** In the proof of Lemma 3, we showed that the profit function is decreasing in the offer by the worker. The profit function must therefore be decreasing to the right at the bargaining outcome. Suppose that there exists a type  $\bar{F}$  such that the profit function is flat as the wage is lowered below  $w(\bar{F})$ . Denote  $V \in (V_u, V(F)]$  to be the interval of the worker's value function where the profit function is constant for type  $\bar{F}$ . The wage  $w(F, V)$  is given by

$$(\delta + \rho + \lambda + \gamma(F))(V - \bar{V}(F)) = w(F, V) - w(F) + \lambda \int_0^1 \left( \max\{V, \bar{V}(\tilde{F})\} - \max\{\bar{V}(F), \bar{V}(\tilde{F})\} \right) d\tilde{F}$$

The functions  $\{\gamma(F), w(F), \bar{V}(F)\}$  are continuous in  $F$  and therefore so is  $w(F, V)$ . The profit function is given by

$$(\delta + \rho + \gamma(F) + \lambda(1 - G(V)))\Pi(F, w(F, V)) = x(F) - w(F, V) + \gamma(F)\Pi(F, w(F))$$



All functions in the expression are continuous in  $F$  and so is the profit function. The profit function is differentiable in the region  $(V_u, V(F))$  with derivative at  $V$  given by

$$\frac{\partial \Pi(F, w)}{\partial w} \Big|_{w=w(F, V)} = -\frac{1}{(\delta + \rho + \gamma(F) + \lambda(1 - G(V)))} + \frac{\lambda G'(V) \Pi(F, w(F, V))}{(\delta + \rho + \gamma(F) + \lambda(1 - G(V)))^2}$$

This derivative is zero for type  $\bar{F}$ . The effect of decreased turnover exactly matches the increased cost for a type  $F$  firm. The profit function  $\Pi(F, w(F))$  is therefore strictly positive. The function  $G(V)$  is differentiable in the region,  $(V_u, V(F))$ , which implies that, for types that offer wages in the interval, the left and the right derivatives of the profit function are the same.

By Lemma 3, the right derivative with respect to the wage has to be negative. The profit function must therefore be strictly decreasing in the wage. By Lemma 4, the bargaining equation must then hold with equality. Denote  $F - \epsilon$  to be a type that offers a wage corresponding to this interval of worker's value function. Note that the derivative of the profit function is continuous in  $F$ . The left limit of the derivative is then equal to the value, which is zero. By the bargaining equation, it has to be that the derivative of the worker's value function  $(\frac{\partial V(F - \epsilon, w)}{\partial w} \Big|_{w=w(F - \epsilon)})$  goes to zero. But the derivative of the worker's value function is bounded below by  $1/(\delta + \rho + \gamma(F - \epsilon) + \lambda)$  and we can therefore reject that the profit function is flat in some part below the wage outcome. ■

I now prove that there is a unique bargaining outcome for each type. The proof is presented in two steps. First I show that the Nash product is increasing and then decreasing on the support of values. I then show that the Nash product is only decreasing outside the support if (4) holds.

**Lemma 6** *The profit function is strictly greater for a higher type evaluated at the equilibrium worker value function for either of the types (see (18)).*

$$\Pi(F, w(F, V)) > \Pi(F', w(F', V)) \text{ if } V = \bar{V}(F') \text{ or } V = \bar{V}(F) \text{ and } F' < F \quad (18)$$

**Proof.** After renegotiation, the worker's value function is strictly greater at  $F$  than  $F'$ . That is,  $\bar{V}(F) > \bar{V}(F')$ . There exists  $\tilde{F}$  such that  $\bar{V}(F') < \bar{V}(\tilde{F}) < \bar{V}(F)$ . The quit rate is therefore strictly less ( $G(\bar{V}(F)) > G(\bar{V}(F'))$ ). For these intermediate quits, the worker's value of remaining in a match at  $F$  is greater than the worker's value from quitting. The profit is also strictly positive at  $F$ . The total surplus,  $V(F, w(F, V)) + \Pi(F, w(F, V))$ , is therefore greater under  $F$  than  $F'$ . Since the worker's value is the same  $V$ , the profits must be greater. ■

**Lemma 7** *No two types pay the same wage and the Nash product is strictly decreasing (increasing) below (above)  $w(F)$  on the interior of the support of values.*

**Proof.** The derivative of the (log) Nash product is

$$A(F, F') \left[ \frac{\beta}{V(F')} + \left[ \frac{\partial V(F, w(F, \bar{V}(F')))}{\partial w} \Big|_{w=w(F, \bar{V}(F'))} \right]^{-1} \frac{1 - \beta}{\Pi(F, w(F, \bar{V}(F')))} \frac{\partial \Pi(F, w(F, \bar{V}(F')))}{\partial w} \Big|_{w=w(F, \bar{V}(F'))} \right],$$

where  $A(F, F') = \left[ \frac{\partial V(F, w(F, \bar{V}(F')))}{\partial w} \Big|_{w=w(F, \bar{V}(F'))} \right]^2$ . Substituting for the derivative of the value functions we get

$$A(F, F') \left[ \frac{\beta}{\bar{V}(F')} - (1 - \beta) \left[ \frac{1}{\Pi(F, w(F, \bar{V}(F')))} - \frac{\lambda(\delta + \rho + \lambda(1 - F'))}{(\delta + \rho + \gamma(F) + \lambda(1 - F'))} \frac{1}{w'(F)} \right] \right].$$

Note that,  $-\Pi(F, w(F, \bar{V}(F')))$  and  $\gamma(F)$  are strict greater (less) at  $F$  than  $F'$  if  $F$  is strictly greater (less) than  $F'$ . The term is therefore smaller for  $F < F'$  and greater for  $F > F'$  than for  $F = F'$ . So if the expression holds for  $F'$  at  $\bar{V}(F')$  it can not hold for  $F \neq F'$ . Furthermore, if it holds at  $\bar{V}(F')$  for  $F'$ , the equation has to be strictly positive (negative) for  $F$  at  $\bar{V}(F')$  if  $F$  is strictly greater (less). This implies that, on the interior of the support, the Nash product is increasing before the bargaining outcome zero at and decreasing thereafter. ■

We know that the two offers must imply the same Nash product. On the interior of the support, for the lowest type, the Nash product decreases as wage is raised. In order to find an offer around the lowest wage we therefore require that the Nash product decreases as the wage is lowered outside the support.

**Lemma 8** *The bargaining outcome is weakly greater than the bargaining outcome that treats turnover as fixed.*

**Proof.** The derivative of the Nash product is

$$\frac{\beta}{\bar{V}(F)} - (1 - \beta) \left[ \frac{1}{\Pi(F, w(F, \bar{V}(F)))} - \frac{\lambda(\delta + \rho + \lambda(1 - F))}{(\delta + \rho + \gamma(F) + \lambda(1 - F))} \frac{1}{w'(F)} \right]$$

The effect of OJS is captured in the term

$$\frac{\lambda(\delta + \rho + \lambda(1 - F))}{(\delta + \rho + \gamma(F) + \lambda(1 - F))} \frac{\Pi(F, w(F, \bar{V}(F)))}{w'(F)},$$

which reflects how the joint surplus changes with the wage. The lower turnover associated with a higher wage implies that the derivative of the Nash product is greater. As the derivative of Nash product is zero at the wage outcome, the bargaining outcome is weakly greater than had turnover been treated as fixed. ■

**Lemma 9** *The Nash product decreases as the wage decreases or increases out of the support of wages if and only if*

$$\beta \Pi(0, w(0, \bar{V}(0))) = (1 - \beta) \bar{V}(0) \quad (19)$$

**Proof.** (19) solves for the maximum of the Nash product outside the support. If the wage is higher than this wage, the Nash product increases as the wage is lowered. For the Nash product to be decreasing at the lowest wage, we require (19) to hold. By Lemma 8, we know that the wage outcome is weakly greater than the wage treating turnover as fixed. The Nash product must therefore decrease above the highest wage. ■

The derivative of the Nash product is strictly positive before  $w(F)$ , zero at  $w(F)$ , and strictly negative thereafter. The profits are decreasing in the wage. In a bargaining equilibrium, we therefore require that (1) and (2) are satisfied. For any  $F$ , two offers can be found around  $w(F)$  for which this is the case. The offers converge to  $w(F)$  as the friction goes to zero. If (3) does not hold at a particular wage, that wage is not a bargaining outcome. Similarly, if (4) does not hold, the lowest wage is not a bargaining outcome.

## B Theorem 2

Differentiating with respect to the wage gives

$$\begin{aligned} V_w(F, w)|_{w=w(F)} &= \frac{1}{(\delta + \rho + \gamma(F) + \lambda(1 - F))} \\ (\delta + \rho + \gamma(F) + \lambda(1 - G(V(F, w(F))))\Pi_w(F, w) &= -1 + \lambda G_V(V(F, w))V_w(F, w)\Pi(F, w), \end{aligned}$$

where we use the following

$$G_V(V(F, w(F)))V_w(F, w)|_{w=w(F)} = \frac{\partial V(F, w)}{\partial w}|_{w=w(F)} \left[ \frac{\partial V(F, w(F))}{\partial F} \right]^{-1} = \frac{1}{w'(F)}\theta$$

The differential equation given by the bargaining game is

$$\beta\Pi(F, w(F))V_w(F, w)|_{w=w(F)} + (1 - \beta)\Pi_w(F, w)|_{w=w(F)}V(F, w(F)) = 0 \quad (20)$$

Rewriting it in terms of wages using  $F(\hat{w}) = w^{-1}(\hat{w})$  and solving the differential equation gives the solution.

$$F(w) = \frac{(\delta + \rho + \lambda)}{\lambda} \left( 1 - \left[ \frac{x - w}{x - \underline{w}} \right]^{1/\theta} \left[ 1 - \frac{(x - \underline{w})^{1/\theta}}{\underline{w} - \underline{b}} \left( \frac{1 - \beta + \beta/\theta}{1 - \beta} \right) \frac{(x - w)^{1-1/\theta} - (x - \underline{w})^{1-1/\theta}}{1 - 1/\theta} \right]^{\frac{\beta}{\theta(1-\beta)+\beta}} \right) \quad (21)$$

Using the distribution function, we can solve for the profit function as

$$\Pi(F(w), w) = \frac{x - w}{(\delta + \rho + \lambda) \left[ \frac{x - w}{x - \underline{w}} \right]^{1/\theta} \left[ 1 - \frac{(x - \underline{w})^{1/\theta}}{\underline{w} - \underline{b}} \frac{1 - \beta(1 - 1/\theta)}{(1 - \beta)} \frac{(x - w)^{1-1/\theta} - (x - \underline{w})^{1-1/\theta}}{(1 - 1/\theta)} \right]^{\frac{\beta}{\theta(1-\beta)+\beta}}}$$

The value function of the worker is

$$\begin{aligned}
V(F(w), w) &= \int_{\underline{w}}^w \frac{1}{(\delta + \rho + \lambda) \left[ \frac{x-w}{x-\underline{w}} \right]^{1/\theta} \left[ 1 - \frac{(x-w)^{1/\theta}}{\underline{w}-\bar{b}} \frac{1-\beta(1-1/\theta)}{(1-\beta)} \frac{(x-w)^{1-1/\theta} - (x-w)^{1-1/\theta}}{(1-1/\theta)} \right]^{\frac{\beta}{\theta(1-\beta)+\beta}}} dw + \frac{\underline{w} - \bar{b}}{(\delta + \rho + \lambda)} \\
&= (\underline{w} - \bar{b}) \frac{\left[ 1 - \frac{(x-w)^{1/\theta}}{\underline{w}-\bar{b}} \left( \frac{1-\beta+\beta/\theta}{1-\beta} \right) \frac{(x-w)^{1-1/\theta} - (x-w)^{1-1/\theta}}{1-1/\theta} \right]^{1-\frac{\beta}{\theta(1-\beta)+\beta}}}{(\delta + \rho + \lambda)}
\end{aligned}$$