# Energy efficient hopping with Hill-type muscle properties on segmented legs 

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#### Abstract

The intrinsic muscular properties of biological muscles are the main source of stabilization during locomotion, and superior biological performance is obtained with low energy costs. Man-made actuators struggle to reach the same energy efficiency seen in biological muscles. Here, we compare muscle properties within a one-dimensional and a two-segmented hopping leg. Different force-length-velocity relations (constant, linear, and Hill) were adopted for these two proposed models, and the stable maximum hopping heights from both cases were used to estimate the Cost of Hopping. We then performed a fine-grained analysis during landing and takeoff of the best performing cases, and concluded that the force-velocity Hill-type model is, at maximum hopping height, the most efficient for both Linear and Segmented models. While hopping at the same height the force-velocity Hill-type relation outperformed the Linear relation as well. Finally, knee angles between $60^{\circ}$ and $90^{\circ}$ presented a lower energy expenditure than other morphologies for both Hill-type and Constant relations during maximum hopping height. This work compares different muscular properties in terms of energy efficiency within different geometries, and these results can be applied to decrease energy costs of current actuators and robots during locomotion.


## 1. Introduction

Animals move in a plethora of gaits with different energy consumptions. Among these gaits, the hopping gait is the first choice of many animals, and can also be considered as a widely accepted simplification of the running gait [1]. The importance of understanding how leg and body behave during hopping has prompted scientists to conduct biological experiments focusing on this gait. One-legged hopping experiments with humans pointed to the existence of a preferred hopping frequency which maximizes the effects of the stretch reflex [2], while other experiments with hopping and jumping subjects suggested that the suppression of the H-reflex changes the behavior of human muscles from springs to dampers [3].

Whenever biological experiments are limited by biological factors, simulations can, to some extent, explain locomotory phenomena and provide new insights on how animals move. Hopping simulations considering a two-segmented leg with a positive force feedback suggested that this proposed stretch reflex can stabilize running behavior and act as a replacement for central motor commands [4]. From a biomimetic perspective
[5], experiments with artificial muscles within a musculoskeletal biped proved that the presence of stretch reflex helps stabilization on the frontal plane [6]. The integration of such feedback properties with feed-forward control within different muscle architectures can be seen in [7]. There, force-length and force-velocity relations are simulated with a one-dimensional linear hopping muscle as constant, linearly increasing, or Hill-type behavior, and they proved that the Hill-type muscle recovered from disturbances faster than other architectures. A similar work from the same author [8] focused on the intrinsic properties of muscles with the same one-dimensional hopping muscle, and the author concluded that force-length relation has a very low contribution to stability, while linear and Hill-type force-velocity relations increased stability.

Beyond locomotory stability, energy expenditure is another remarkable feature observed in animals, which allows longer periods without food and thus a higher chance of survival. The manifold implications of a lower energy consumption is such that it can also be used to trace the human transition from quadrupedal to bipedal locomotion [9]. While energy efficiency has been studied from a biological perspective [10], only a few works recreate locomotion to understand how energy is used. From passive walkers [11] to regenerating electric quadrupeds [12], roboticists aim to reduce energy consumption to increase autonomy, with special attention to the works of [13] and [14]. While the former presents a beam hopper which explores the natural frequency of the system to perform vertical hops, the latter adopts a parallel elastic actuator with an openloop sinusoidal control to horizontally displace the hopping robot. Nonetheless, studies regarding the energy efficiency of muscular properties during locomotion are, to the best of the author's knowledge, nonexistent.

In this work we propose a vertical hopping simulation to better understand the influence of muscular properties on energy efficiency. We adopted two hopping models - A one-dimensional linear model and a two-segmented leg model - and used three different simplifications for the force-length and force-velocity behaviors - Constant, Linear, and Hill-type. With a total of nine cases per model, we simulate stable hops with the proposed 18 cases, and estimate the Cost of Hopping ( CoH ) for each one of them. Additionally, we compare their CoH while hopping at the same height. Beyond previous works in the area [8], we aim to show that the introduction of non-linearity on locomotion, inherent to segmented limbs, alters the hopping behavior of the model. We also aim to understand the impact of force-length and force-velocity relations on the energy expenditure, going beyond an understanding of hopping stability. Our conclusions suggest that biological muscles and geometries are optimized for efficiency, and better actuators should follow similar cues to improve autonomy in robots.

In section 2, we introduce the hopping models and the parameters adopted during simulation. In section 3, we present the results from our hopping experiments, and in section 4 , we discuss these results and conclude this paper.


Figure 1. Proposed hopping experiment to assess the influence of the intrinsic muscle properties to the energy efficiency. Above, a human hopper lands, compresses until reaching a height of $0.9 l_{0}$ and takes off to reach a maximum hopping height $\left(y_{1}\right)$. We adopted $l_{0}=1$ for all experiments. Below, the proposed Linear and Segmented models with dampers. Future changes on the leg geometry can reproduce similar results if parameters (e.g. $m, d, k$ ) are changed accordingly, which generalizes our results for different animals.

## 2. Methods

Hopping can be considered as a horizontal [14, 15] or a vertical [16] motion, and within this work we will focus on the vertical aspects of this movement. While simplifying this vertical motion as a mass attached to one leg might deprive our studies from intricate details from the motion observed in animals, this simplification guides us to the essence of the movement, which leaves the aforementioned details to be subject of different studies.

Our simulations adopted two different models: The first, known as the Linear Model, is a one-dimensional muscle which expands when actuated, and the second model, known as Segmented Model, is a two segmented leg with a monoarticular kneeextending muscle, similar to a vastus lateralis. The reason for adopting two models instead of one is to compare the behavior of a linear and a non-linear morphology to the influence of different muscle properties. In stark difference from previous works $[8,4]$, we compare the influence of these two morphologies to the energy efficiency of the system and systematically change the resting angle to understand the observed differences. In Fig. 1 both models are compared to a hopping human, and a damping
element was introduced to force the muscle to restitute the energy losses. We measure energy efficiency by accounting the energy spent in this corrective measure.

### 2.1. Linear model

As shown in Fig. 1, we idealized the leg as a single expanding massless muscle with the body mass on top of it. The model falls from a predefined height $y_{0}$, touches the floor, compresses the muscle until it reaches the mid-stance with a leg compression of $0.1 m$, and propels itself upwards to reach the final height of $y_{1}$. Our simulations considered the hopping as a hybrid dynamic system, with a flight and a stance phase, and the equation of motion is defined as

$$
m \ddot{y}=-m g+ \begin{cases}0 & \text { if } y>l_{0}  \tag{1}\\ F_{m}+d \dot{y} & \text { otherwise }\end{cases}
$$

where $m$ is the total mass of the system, $y$ is the vertical position of the center of mass, $d$ is the damping coefficient during stance, $l_{0}$ is the rest length of the muscle, and $F_{m}$ is the total force applied by the muscle against the floor. $F_{m}$ is a combination of the activation, force-length-velocity relations and maximum force, and is described by the following equation:

$$
\begin{equation*}
F_{m}=A(t) F_{l} F_{v} F_{\max } \tag{2}
\end{equation*}
$$

This model is, for the sake of comparison, largely based on a similar model presented in [8] with the addition of damping losses. The force-length and force-velocity relations also follow similar comparison, expressed by

$$
\begin{align*}
& F_{l}= \begin{cases}1 & \text { Constant } \\
k\left(l_{0}-l\right) & \text { Linear } \\
\exp \left[c\left|\frac{l-l_{\text {opt }}}{l_{\text {opt }} w}\right|^{3}\right] & \text { Hill, and }\end{cases}  \tag{3}\\
& F_{v}= \begin{cases}1 & \text { Constant } \\
1-\mu v & \text { Linear } \\
\frac{v_{\max }+v}{v_{\max }-K v} & \text { Hill }(v>0) \\
N+(N-1) \frac{v_{\max }-v}{-7.56 K v-v_{\max }} & \text { Hill }(v \leq 0)\end{cases} \tag{4}
\end{align*}
$$

where $k$ is the spring coefficient for the linear behavior, $l$ is the leg length, $c$ is the curvature of the bell-shaped force-length relation approximation for Hill-type behavior, $w$ is the width of this curvature, and $l_{\text {opt }}$ is the optimal length for maximum force output. Within the force-velocity relation $\mu$ is the angular coefficient of the linear behavior, $v_{\max }$ is the maximum velocity output, $K$ is the curvature constant, $N$ is the dimensionless force $F_{m} / F_{\max }$ at maximum velocity. Fig. 2 depicts the force-length and force-velocity behaviors during muscular activation.


## force-velocity relation $\mathrm{F}_{\mathrm{v}}$



Figure 2. Pictogram with three force-length and three force-velocity relations. As the muscle is activated differences in length and velocity affect the force output from the model, and the Hill-type model is the model that most closely resemble the behavior of a biological muscle. This figure is adapted from Haeufle et al. 2010 [8].

The parameters for this experiment are shown in Table 1, and the rationale for these choices is to mimic observations with human experiments and simulations [4]. Similar observations from the same work will also be the basis for the idealization of the segmented model.

### 2.2. Segmented model

As with the Linear Model, the Segmented Model consists of a body mass attached to the upper part of a massless leg. The main difference between these models lies in the presence of a joint and two links, where a muscle in the upper link forces the leg to extend when activated. While the muscle from the previous model expanded when activated, this model contracts the muscle with activation. The equation of motion for this model follows the same behavior described at Equation (1) without the damping contribution, as the output of the muscle will consider intrinsic damping properties that were embedded to simulate losses during hopping. The leg force is

$$
\begin{equation*}
F_{l e g}=A(t) F_{l} F_{v} F_{\max }+d v_{\text {muscle }}, \tag{5}
\end{equation*}
$$

where $v_{\text {muscle }}$ is the muscle contraction speed, which is negative during takeoff and positive during landing, and $d$ is the muscular damping coefficient. As it is expected, $F_{m}$ will take into account $F_{l e g}$ combined with the mechanical advantage related to the geometry, as shown in

$$
\begin{equation*}
F_{m}=\frac{d_{m a}}{\sqrt{l_{s}^{2}-\left(l_{l e g} / 2\right)^{2}}} F_{l e g} \tag{6}
\end{equation*}
$$

with $d_{m a}$ as the constant moment arm of the muscle, $l_{s}$ the length of both upper and lower links, and $l_{\text {leg }}$ as the distance between the body mass and the floor, as devised by [4]. The instantaneous length of the muscle can be calculated with

$$
\begin{equation*}
l_{\text {muscle }}=l_{\text {ref }}-d_{m a}\left(\beta-\beta_{r e f}\right), \tag{7}
\end{equation*}
$$

where $l_{\text {ref }}$ is the reference length of the muscle, $\beta$ is the joint angle and $\beta_{r e f}$ is the joint angle at which $l_{\text {ref }}$ is reached.

Within our simulations the leg falls from an initial height with an initial resting length $l_{0}=1$, which is defined by

$$
\begin{equation*}
l_{0}=1.44 l_{s} \sqrt{1-\cos \left(\beta_{r e f}\right)}=1 \tag{8}
\end{equation*}
$$

and alters the link length according to the resting knee angle. This results in a link length variation from $l_{s}=0.501$ to $l_{s}=5.737$ with a proportional change in the maximum output force $F_{\max }$.

The definition for force-length and force-velocity relations for the two-segmented case differ in a few aspects from the Linear Model. Initially, the fact that the muscle contracts instead of expanding will likewise change force direction within both relations. Then, with the muscle and the damping element acting on the knee joint the everchanging angle will generate a non-linear relationship between forces and leg length.

Table 1. Hopping model parameters

| Parameter | Linear Model | Segmented Model |
| :--- | :--- | :--- |
| leg rest length $l_{0}$ | 1 m | 1 m |
| body mass $m$ | 80 kg | 80 kg |
| gravitational constant $g$ | $10 \mathrm{~m} \mathrm{~s}^{-2}$ | $10 \mathrm{~m} \mathrm{~s}^{-2}$ |
| maximum muscle force $F_{\text {max }}$ | 2.5 kN | 19.8 kN |
| spring coefficient for linear behavior $k$ | $10 \mathrm{~m}^{-1}$ | $100 \mathrm{~m}^{-1}$ |
| curvature for Hill behavior $c$ | -29.96 | -299.6 |
| width for Hill behavior $w$ | 0.45 | 0.4 |
| optimal length $l_{\text {opt }}$ | 0.9 m | 0.512 m |
| angular coefficient for linear behavior $\mu$ | 0.25 | 0.25 |
| maximum velocity $v_{\text {max }}$ | $-3.5 \mathrm{~m} \mathrm{~s}^{-1}$ | $-15 \mathrm{~m} \mathrm{~s}^{-1}$ |
| curvature constant $K$ | 1.5 | 5 |
| dimensionless force constant | 1.5 | 1.5 |
| damping coefficient $d$ | 0.7 | 3450 |
| segment length $l_{s}$ | 1 m | 0.5773 m |
| resting knee angle $\beta$ |  | $120^{\circ}$ |
| lever arm |  | 0.04 m |

Both relations are described as follows

$$
\begin{align*}
& F_{l}= \begin{cases}1 & \text { Constant } \\
k\left(l-l_{0}\right) & \text { Linear } \\
\exp \left[-c\left|\frac{l-l_{\text {opt }}}{l_{\text {opt }} w}\right|^{3}\right] & \text { Hill }\end{cases}  \tag{9}\\
& F_{v}= \begin{cases}1 & \text { Constant } \\
1+\mu v & \text { Linear } \\
\frac{v_{\max }-v}{v_{\max }+K v} & \text { Hill }(v>0) \\
N+(N-1) \frac{v_{\max }+v}{7.56 K v-v_{\max }} & \text { Hill }(v \leq 0)\end{cases} \tag{10}
\end{align*}
$$

where the meaning of each parameter is similar to the ones adopted at the Linear Model.
The parameters for this experiment are shown in Table 1, and the rationale for these choices is to mimic observations with human experiments and simulations [4].

### 2.3. Muscle properties and Activation

As demonstrated with equations (2), (5) and (6), the total force output from the model against the floor depends on the interplay between activation, force-length, force-velocity and maximum force. While Fig. 2 depicted the force-length and force-velocity relations, in Fig. 3 these parameters are combined with the activation and the maximum force to produce the output force.

The activation of our model is divided in two parts: Before and after mid-stance. Initially, the activation has to enforce a maximal deformation of 0.1 m , where the midstance takes place. For this purpose, the simulation chooses one constant value for $A(t)$ until the deformation condition is met. Upon reaching mid-stance $A(t)$ is set to 1 , the maximum value, and is kept in this setting until takeoff. Works such as [8] used a genetic algorithm to find an optimum activation pattern to fit the hopping period within 500 ms, and we decided to adopt a different method to 1 . simplify the parameters during hopping and to 2 . provide a new perspective and comparison between previous works and our contribution.

### 2.4. Hopping height and CoH

Our simulations sought to compare hopping heights for different force-length-velocity relations and, later, perform similar comparison with their CoH . An inter-model comparison was not the focus of this work, as linear muscles have very low energy requirements and thus such comparison would be unfair. An intra-model comparison, though, can be strong enough to show trends from intrinsic properties when it comes to performance or efficiency.


Figure 3. Example of the combined infuence of force-length, force-velocity and activation upon the maximum force, resulting in the output force. The adopted forcelength was of constant behavior, while the force-velocity was of linear behavior. The activation pattern is smaller until mid-stance, and maximum after it to ensure maximal hopping height.

A stable hopping height was chosen by iteratively starting simulations from hopping height $y_{0}$ and registering their final hopping height $y_{1}$, and then adopting $y_{0}{ }^{\prime}=y_{1}$ as the new initial condition until the initial and final hopping height are equivalent ( $y_{0}=y_{1}$ ).

In previous works $[13,17]$ the idea of a CoH is proposed, as opposed to a Cost of Transport (CoT). While CoT focuses on horizontal energy expenditure, and grades robots and simulations by their planar displacement per consumed energy, the CoH compares the total energy input during hopping to the maximum potential energy (height) achieved. The formula used for such is

$$
\begin{equation*}
\mathrm{CoH}=\frac{E_{d}}{m g y_{1}} \tag{11}
\end{equation*}
$$

where $E_{d}$ is the energy loss due to damping. The addition of damping to the system allowed the system to lose energy during landing and takeoff, and if this loss was neglected, the total work of the system would have been conserved. As the damping dissipates energy, the muscle has to input energy to achieve the same height as the initial height. This energy input has a non-linear nature, as it depends on complex force-length and force-velocity relations, but can be easily extrapolated when we consider that initial and final height will incur in the same potential energy, as follows:

$$
\begin{equation*}
E_{p}(\text { initial })=E_{p}(\text { final })+E_{\text {input }}-E_{d} \rightarrow E_{\text {input }}-E_{d}=0 \tag{12}
\end{equation*}
$$

and the implications of damping as the sole source of energy loss are discussed at Study Limitations.

It is important to note that the damping coefficient $d$ was artificially chosen for these experiments, and arbitrarily changing this parameter will result in different energy costs. In this vein, a comparison with biological data may be skewed by the choice of this parameter combined with total mass, leg length and other properties of the muscle. Moreover, as the damping element for the segmented case is acting upon the knee joint, a comparison between the results for Linear and Segmented models is not possible. For the sake of brevity, the cases will be called by an acronymn with the combination of the first letters of each relation (e.g. Constant force-length and Hill-type force-velocity will be called CH ).

## 3. Results

Our initial experiments with the Linear Model aimed to create the groundwork for a comparison between this work and previous works from the field [8]. While adopting similar parameters, we added a damping force and a simplified activation pattern. As a result, a few differences between both results can be observed in the upper part of Fig. 4. The results show that the cases CC and HC obtained the highest hopping height, with 1.203 m and 1.204 m , respectively. the cases CL and HL tied as the third best case, and the case LH could not reach a stable hopping condition.

Analyzing the energy lost due to damping within our linear experiments (Fig. 4b), the lowest CoH was found with the case LC , followed by CH and HH . With the exception of $\mathrm{CC}, \mathrm{HC}$ and LH , all cases had similar CoH and there was a correlation between hopping height and CoH . A correlation between the activation pattern during landing, depicted as $A_{l}$, and the CoH was not possible, as the lowest CoH (LC) demonstrated one of the highest $A_{l}(0.915)$. Nonetheless, cases with a constant force-velocity relation $(\mathrm{xC})$ demonstrated the best hopping height, while cases with a Hill-type force-velocity relation ( xH ) showed the best energy expenditure. A second linear experiment (Fig. 4c) considered the same hopping height observed at the HH case for all the proposed cases, and a high similarity of results is easily observed for all successful cases, with CoH values between 0.000168 and 0.000172 . Although the LL and LH cases couldn't normally reach this hopping height, the LC case could when adopting a take-off activation pattern of $A=1.1$.

Experiments with the Segmented Model revealed that the introduction of a joint on the model drastically altered the outcome of our simulations. As seen in Fig. 5, the highest hopping height was observed on the constant force-velocity relations ( xC ), with the cases CC and HC demonstrating the highest stable hops. The presence of a higher $A_{l}$ did not necessarily lead to a higher hopping height or a worse CoH .

The CoH for the Segmented Model followed a similar trend to the Linear Model, where the Hill-type force-velocity cases ( xH ) had superior energy efficiency. The lowest CoH during maximum hopping was found with the LH case, closely followed by the CH and HH case. While hopping at the same hopping height, the Hx cases demonstrated the best CoH , followed by the Cx cases, and the Lx cases were the least energy efficient.


Figure 4. Results for the Linear Model. Darker colors indicate good results, while the worst results are in white. While xC cases excel at hopping height, xH cases demonstrate a better energy efficiency at maximum hopping height. All cases behaved similarly at the same hopping height. $A_{l}$ indicates the activation during landing, while $y_{1}$ is the final hopping height, where $y_{0}=y_{1}$ (stable). Same height experiments adopted $y_{0}=1.066$.


Figure 5. Results for the Segmented Model. Darker colors indicate good results, while the worst results are in white. Unlike the previous results, $x C$ cases excelled at the hopping height criteria. As with the Linear Model, the xH cases are the most energy efficient at maximum hopping height, while Cx and Hx cases excel when hopping at the same height. Same height experiments adopted $y_{0}=1.075$.


Figure 6. Different values for $\beta$ result in different energy efficiencies for the cases xC and xH . As the knee resting angle increases $\left(\beta>90^{\circ}\right)$ the differences between Cx and Hx can also be seen. The condition $l_{0}=1$ was kept for all experiments, and the link geometries obeyed the Eq. (8).

The polarization of results among constant and Hill-type force-length-velocity relations prompted a fine-grained analysis of the cases $\mathrm{CC}, \mathrm{CH}, \mathrm{HC}$ and HH . Would the results be a consequence of the adopted resting knee angle $\beta=120^{\circ}$ ? We probe the problem by hopping at maximum hopping height with different values for $\beta$, as shown in Fig. 6. These new experiments utilise a range of hopping angles from $\beta=10^{\circ}$ to $\beta=170^{\circ}$, and all cases adopted $l_{0}=1$. This condition results in legs in varying lengths (from 1.02 m to 11.4 m ) and, consequently, the force output for each case varied accordingly (from 8.15 kN to 350.5 kN ). These parameters were equally adopted for all four cases.

The cases xC and xH are starkingly different when energy efficiency is considered, as the Hill-type force-velocity relation produces less energy losses at maximum hopping height. As the leg geometry straightens and the leg approaches the morphology of a human hopper, the overall energy efficiency worsens and the contributions from the force-length relations are felt. At angles $\beta>90^{\circ}$, Cx cases became remarkably less energy efficient than Hx cases.

We decided to further study the behavior during landing and takeoff of the same four cases at $\beta=150^{\circ}$, analyzing the velocity, damping force and power output during the stance phase, and the results are shown in Fig. 7. Both force output and power output calculations are based on the damping losses. As the touchdown and liftoff speeds have the same absolute values, the total work during stance is zero. The damping energy loss, on the other hand, presents a force that is always opposite to the speed direction


Figure 7. Comparison among the four cases. It is clear that the cases CC and HC require more power for a shorter period, while the cases HH and CH waste less power for longer periods. HH uses approximately the same amount of power that CH uses, but for a shorter period during maximum hopping.
and produces a non-zero value, which represents the amount of energy lost by the muscle during stance.

The resemblances between CC and HC , and also between HH and CH , are clear. An explanation for the superior energy efficiency of the case HH over CH consists in the fact that HH had a shorter stance phase with approximately the same damping force. The xC cases presented a remarkably high CoH , and Fig. 7 leads us to believe that, although shorter in duration, the xC cases always require a higher instantaneous power than the xH cases, which leads to higher amounts of energy loss per time.

A more in-depth analysis of these maximum hopping results shows that the energy efficiency during landing is worse for the HH case than the CH case ( 0.3 J ), while the HH case has a better energy efficiency during takeoff (1.32J), as seen in Fig. 8. Overall, the HH case outperforms the CH case for straight leg morphologies, as previously shown in Fig. 6.

## 4. Discussion

### 4.1. One-dimensional and two-segmented models

In this paper energy efficiency is inversely correlated to the amount of energy lost during hopping, and in this aspect the Linear Model is superior to the Segmented Model (Figs. 4 and 5), and the main explanation lies in the need for the Segmented Model to actuate the joints using a small moment arm, and thus requiring a greater output force. Many


Figure 8. As the four cases hop they lose energy due to damping. The difference between xC and xH cases is remarkable. Although less energy efficient between landing and mid-stance, the case HH is very efficient while taking off and, overall, outperforms the CH cases.
reasons led nature to develop such design, as segmented legs present a morphological advantage over telescopic legs in joint lubrication aspects or when obstacle clearance is required, as discussed by [18].

Previous simulations from [8] showed that, among Linear Models, the case CC obtained the best hopping height, and our results partially agree with their claim, as our best linear hopper adopted the HC case (as seen in Fig. 4). We argue that Hill-type force-length relations take more time to build up speed during takeoff, and, shall the previous work [8] unfreeze the hopping frequency condition ( 2 Hz ), similar results will be obtained. Different activation patterns play a small role on the observed differences, since all simulations considered a full activation after mid-stance. Our findings with the Segmented Model, though, reinforce their claim, and the CC case obtained the highest hopping height, as shown in Fig. 5.

The non-linearity introduced by the Segmented Model was better associated with Constant force-length relations to reach higher hopping heights (Fig. 5), and this is in agreement with the results from the Linear model (Fig. 4), consequently agreeing with the findings from [8].

When sheer performance is the objective, higher hopping heights can be obtained with a Constant force-length-velocity relation with both models, although this kind of relation is not possible with biological actuators. Both models converged for the energy efficiency criteria at maximum hopping height, as the Hill-type force-velocity relation demostrated the best CoH for both, and this demonstrates the higher energy efficiency present in animals during locomotion.

### 4.2. Influence of resting angle on energy efficiency

In [18], it is stated that human-like segmented legs are superior to other designs when energy efficiency and stability are considered. Our simulations with different knee angles showed that for angles above $90^{\circ}$, which are straighter and closer to the human landing angle, the Hill-type force-length-velocity was the most energy efficient, and this strongly agrees with the previous hypothesis from [18]. A clear explanation for this phenomenon is that different landing angles require different leg lengths, which incur in different muscular velocities. Although a low variability was observed between $10^{\circ}$ and $170^{\circ}$, our future works will explore different leg geometries to explore other possible outcomes.

As the highest energy efficiency (low CoH ) was found with leg geometries between $60^{\circ}$ and $90^{\circ}$, we can infer that differences between HH and CH would not be significant with these geometries (as seen in the colinearity of xH cases in Fig. 6), and the Hilltype force-velocity relation would be the most important trait leading to a higher energy efficiency. This connotes that roboticists can reap the benefits of segmented legs by designing their robots with landing angles between these values, and preferably adopt actuators capable of replicating a biomimetic force-velocity relation, such as [19]. Previous works [8, 4] did not approach the energy efficiency problem nor its interplay with knee angles, and the contribution of this work is unique in these aspects.

Our simulations focused on the Linear and the Segmented models, and our next research steps will focus on different morphologies to fully test the hypothesis from [18] pertaining the optimality of the human segmented morphology and further extend contributions on locomotory efficiency.

### 4.3. Hopping height and CoH trade-off

A very clear trend is visible while analyzing the hopping height for all 18 cases: within every column, the maximum hopping height decreased from xC to xL , and kept this same trend from xL to xH . The same kind of relationship is present within the CoH columns, and it leads us to state that a correlation between these two does exist. The proportionality of the relationship, however, is not clearly seen among hops with the same height, and Figs. 4c and 5c depict an interplay between Constant and Hill-type relations leading to higher efficiencies. Further, Linear force-length relations presented itself as the worst alternative for both hopping height and CoH at the same height and for both models. These results should prompt roboticists to consider which actuation method to avoid, as combustion engines and electrical motors have different relationships between their rotational speed and torque, which could be decisive to determine the energy efficiency and hopping height within a physical experiment, such as [20].

A similar analysis between hopping height, energy efficiency and force-lengthvelocity relations is not present in previous works, but in [8] a correlation between force-velocity and stability is made. Within our simulations for both models, the cases CH and HH demonstrated a remarkable energy efficiency combined with an average hopping height. These findings, combined with the stability association made by [8],
allow us to infer that Hill-type force-velocity relations increase stability while decreasing energy consumption. The generalization for the stability claims from the Linear Model to the Segmented Model still remains to be investigated.

In strong agreement with the results from [8], the simplest muscle models obtained the highest hopping heights, but ironically they were also the ones with the worst energy efficiency. All the xC cases required an activation superior to 0.9 for both Linear and Segmented models, while all the xH cases demonstrated an $A_{l}<0.7$. As explained in [8], "in the more complex models the factors in the force equation of the muscle model mostly reduce the maximum dynamic muscle force", and thus lower hopping heights should be expected in xH cases.

### 4.4. Study limitations and conclusions

As in any experiment in which biological conditions are recreated artificially, our study is limited by the degree of simplification from our system. While musculoskeletal systems are capable of providing more realistic hopping behavior results, the main benefit of a simpler simulation is that unnecessary parameters, noise and disturbances are eliminated, and our results can concentrate on the phenomenon under observation. A few research settings show potential to approach this similar subject with real-world experiments, and this limitation can be addressed properly [20]. Damping losses through the parallel damper were considered as the only source of losses on this simulation, and therefore the energy input from the muscles was the precise amount needed to replenish this loss. In real life experiments many other sources of energy loss exist, and future works in this area should account for additional contributions.

In this work we analyzed 18 different hopping conditions, combined force-length and force-velocity relations to two different hopping models, and we found that the results differ between Linear and Segmented models. The non-linearity introduced by the Segmented Model did not affect the conclusions from [8], as the Constant force-velocity relations also reached higher hopping height for both models. Both models converged for the energy efficiency criteria at maximum hopping height, as the Hill-type forcevelocity relation demostrated the best results for both. This superiority is not obvious when hopping at the same height, as the HC emerged as the best option for Segmented legs, followed by CH and HH . Segmented models with a hopping angle between $60^{\circ}$ and $90^{\circ}$ reached the best energy efficiency, and, independently of the adopted force-length-velocity ratio, this value should be used as a standard for robots with segmented legs.

In the future we will perform new simulations with a higher complexity level by introducing stretch-reflex responses, segment differences, and a third link, and we will assess the contribution from these degrees of complexity to the overall hopping efficiency.

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