Music Learning and Mathematics Achievement: A Real-World Study in English Primary Schools

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Abstract

This study examines the potential for music education to enhance children's mathematical achievement and understanding. Psychological and neuroscientific research on the relationship between music and mathematics has grown considerably in recent years. Much of this, however, has been laboratory-based, short-term or small-scale research. The present study contributes to the literature by focusing on specific musical and mathematical elements, working principally through the medium of singing and setting the study in five primary schools over a full school year.

Nearly 200 children aged seven to eight years, in six school classes, experienced structured weekly music lessons, congruent with English National Curriculum objectives for music but with specific foci. The quasi-experimental design employed two independent variable categories: musical focus (form, pitch relationships or rhythm) and mathematical teaching emphasis (implicit or explicit). In all other respects, lesson content was kept as constant as possible. Pretests and posttests in standardised behavioural measures of musical, spatial and mathematical thinking were administered to all children. Statistical analyses (two-way mixed ANOVAs) of student scores in these tests reveal positive significant gains in most comparisons over normative progress in mathematics for all musical emphases and both pedagogical conditions with slightly greater effects in the mathematically explicit lessons.

This investigation addresses concerns that UK and US governments' quests for higher standards in mathematics typically result in impoverished curricula with limited access to the arts. In showing that active musical engagement over time can improve mathematical achievement, as hypothesised, this work adds to a growing body of research suggesting that policy-makers and educationalists should reconsider curriculum balance.

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I have deepest gratitude for my best friend and husband, Eddie Sanders. He encourages me to go for my dreams, including this one. I love him and thank him for his forbearance, love and continual emotional support.

This thesis is dedicated in memoriam to my mother and father, who had similar dreams but were unable to fulfil them, and to my daughter Sky Marie Sanders, who is always in my heart.

Statement of Authorship and Authenticity

This thesis is the result of my own work and includes nothing which is the outcome of work done in collaboration except as declared in the preface and specified in the text. It is not substantially the same as any that I have submitted, or, is being concurrently submitted for a degree or diploma or other qualification at the University of Cambridge or any other university or similar institution except as declared in the preface and specified in the text. I further state that no substantial part of my thesis has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the University of Cambridge or any other university or similar institution.

I hereby declare that the sources of which I have availed myself have been stated in the body of the report and in the bibliography and that the rest of the work is my own. This following report, exclusive of the bibliography, does not exceed 80,000 words in length. Specifically, with exclusions, it is: 78,396.

Preface

The inspiration for this research has autobiographical and professional origins. These are summarised here to explain the motivations that drive this work and to confirm my qualifications to have carried out the intervention. I believe that music in the schools is valuable for its own sake, yet also for its potential to enhance one's life as well as other abilities such as mathematical achievement.

I also believe in the value of scientific enquiry. I have appreciated science since childhood and won a "young scientist" award at age 12 after monitoring and analysing the effects of differing environmental vibrations on plant growth. Yet perhaps we are all scientists in a sense, as we create and revise our own theories throughout life.

In secondary school, I became absorbed in mathematics and represented my school in contests across the region. At the same time, I was playing Chopin and singing Mozart. I had always wondered whether my musical training helped to bring out mathematical abilities or did I just happen to love both? What is the connection between the two disciplines and cognitive domains? My goal in this study was to answer those questions. Both disciplines fully engaged me and I frequently seemed to have experienced what Csikszentmihalyi (1988) calls "flow," the mental state of immersion in which an optimum balance of challenge and enjoyment is present. Therefore, I began my undergraduate studies double-majoring in music and mathematics.

The pattern-making and creative problem-solving elements I enjoyed in musical composition exist in mathematics as well. Therefore, I found a similar satisfaction in my work as a composer that I felt in my work as a mathematician. I believe that every melody, harmony and rhythm has spatial relevance and numerical value. All of the pieces in my honours thesis opera at Columbia University were composed with a mathematical foundation in combination with creativity and heart; it later won the ASCAP Classical Music Award.

At the same time, I became fascinated with the function of the brain after doing well in a neuroscience-based psychology class at Columbia University and continued studies in that area as well. I had been teaching composition, piano and voice while working on my baccalaureate and was continually interested in what was going on in my students' brains as they composed or learned technique and repertoire.

While teaching young children (aged four-six) music notation in New York, I noticed that they could learn the counterintuitive method of writing rhythms (less is of a longer duration, e.g., take away a stem then it is doubled) more effectively through expressing it with their bodies. Therefore, I created different moves for each type of rhythm note and called it the rhythm dance. While saying "quarter, quarter, quarter, quarter" we would take one step per quarter note, then while stretching out the steps to a duration twice as long spatially and temporally, we would say "half note, half note," then we would drop to the floor on the whole note, and shout "whole 2 3 4" while sitting the full duration. The mathematics pedagogy of Milan Hejný and colleagues (2009) at the Faculty of Education at Charles University is reminiscent of this method of physicalizing knowledge through acting out the concept. For example, the understanding of order numbering, addition and subtraction could be enhanced with five-six-year-olds by taking steps forwards and backwards with his "Walk Environment" methodology.

I also briefly taught mathematics in a primary school. This occurred by chance, while I was a teacher's assistant over 15 years ago. Each day, the students played kickball during a break. The teacher had a rule whereby students who were "in trouble" for any reason, including talking out of turn, would have to stay in the classroom and do homework instead. It was my task to watch them during this time. I offered to help them with their homework and the students quickly realized that I explained the mysterious workings of mathematics for them very clearly. They were learning long division and most had difficulty understanding the teacher, who went rather quickly when putting equations on the blackboard, only briefly noting the steps. I helped each of them individually, as each had a different way of understanding (or misunderstanding) the concepts and processes. Quite soon thereafter, the word must have spread, because students started to "misbehave" on purpose so that they could stay inside and discover the joy of long division and other mathematical adventures with Mrs Sanders.

This experience convinced me that I wanted to pursue an education degree at the post-graduate level, so I entered a master's program in education at Columbia University. I soon became a research associate at the Center for Arts Education Research at CU Teachers College and continued while working on both of my master's degrees there. Details of a longitudinal study I co-conducted at the centre will be noted within this thesis. In Chapter 1, I further illustrate my motivation and intent for this study when I discuss that research.

Professor Mallucci at the University of Cambridge Medical Research Council (MRC), notes that research is a creative job and is "all about ideas and thinking – and being prepared to change your mind" (2010). Indeed, this doctorate has been a long process of creative exploration, exciting discovery and convergent synthesis that I hope will help others experience similar processes in a fulfilling way, whether through music, mathematics or research.

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Chapter 1
Introduction, Justification & Overview

I would give the children music, physics and philosophy, but the most important is music, for in the patterns of the arts are the keys to all learning.
Plato (c. 428 – 347 BC)

The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful.
Poincaré (1854-1912)

Motivation & Intent

The inspiration for this doctoral research was my role as researcher in the New Jersey String Orchestra and Columbia University's Center for Arts Education Research longitudinal project with elementary (primary) school children in the economically deprived city of Newark, New Jersey (Abeles & Sanders, 2007). It revealed to me the potential for music education to help students in numerous ways, from the academic to the personal. This project was introduced in Chapter 2, but I am returning to it here to provide a backdrop to the conduct and findings of the present study.

As noted in the literature review, we had been hired to assess the academic transfer effects of early musical experiences on children aged seven to nine. Children in this Newark Early Strings Program were taught how to play the violin and had opportunities to perform in school as well as with the orchestra and even at the city's concert hall. The measures used to track the children's progress through this five-year study included standardised tests of attainment, attitudes about school, themselves, and their career aspirations, as well as through interviews and age-appropriate questionnaires.

The results revealed statistically significant improvements in Language Arts and mathematics after two years of violin study and performance, compared with students in the same school who did not have music lessons, and most markedly in comparison to students across different schools within the same city.

The children's powerful experiences were revealed in their interviews, including comments about learning and performing music such as: "It helps you learn a lot of stuff. It makes you feel good," "It changed our lives. Before we had nothing to do and now we come to school and we have something to do and "it's fun," and "Playing the violin is really a good way to take away all your hard feelings. It soothes you a lot" (Abeles & Sanders, 2007, p. 10).

The enhanced quality of life, new outlook on school and the improved scholastic results shown by the children motivated me to investigate this connection further, but also to encourage policymakers to include more music in the schools, particularly in areas where pupils would be less likely to have opportunities to learn music formally outside of school. As improvement in mathematics achievement is among the priorities of most school districts in the United States, England and many other countries, I reasoned that perhaps music in the schools can be preserved or even expanded if it could be shown that a fresh, enjoyable addition to the curriculum, such as music education, could be effective in improving numeracy understanding and attainment. Indeed, anecdotal evidence of impact of the Newark study followed my presentation of the preliminary results of the Newark study at a conference in 2006. A teacher's school board decided to retain its threatened music programs, after seeing the results of the study.

In seeing the benefits for the participating children on so many levels, my motivation was, and still is, ultimately for the sake of children's wellbeing and happiness. Along with motivation, however, can come bias. Thus, I realised that it was even more important to carefully design and analyse research to answer questions in an objective and robust way. This influenced my decision to focus primarily on quantitative data and to double mark all tests, which required hundreds of hours and a financial investment as well due to the need to hire markers.

Observations of the children's learning process and the interviews with children, teachers and principals (head teachers) revealed numerous potential reasons, beyond the music learning itself, why their musical experiences may have helped raise their academic achievements, both in school and in state-run standardised assessments. For example, music learning can develop and improve attentional ability, self-esteem, courage, enjoyment of school, social and cooperative abilities, creativity and self-discipline (Hallam, 2010; Posner, Rothbart, Sheese & Kieras, 2008). These competences can *indirectly* improve academic achievement.

Yet I wanted to test for the possibility of *direct* effects specifically related to musical development on cognitive development in other domains, particularly mathematics, therefore this study was undertaken.

1.1 Background

The relationship between music and mathematics has been pondered for millennia. Yet the realms of music and numerical cognition often continue to be perceived as very different cognitive domains. Perhaps they are not as dissimilar as one might think. Over the past four decades, enquiries have emerged in cognitive science and educational research literature regarding the potential connection between the two. Why should such seemingly different domains be linked? What aspects of both are connected? Can music education and learning improve mathematical ability? The aim of this study is to add to the body of literature that attempts to answer these questions. This study, therefore, examined how a course of musical training focused primarily on singing, lasting nine months and taught as part of the regular curriculum in school, might affect seven- to eight-year-olds' mathematical development. A key question underpinning the research design was whether a pedagogical focus on pre-specified musical elements (form, pitch relationship – referred to in the thesis as melody – and rhythm) would influence potential cognitive correlates between musical and mathematical components and therefore would lead to improvements in the related aspects of mathematics as the main hypothesis states: *Music education via the voice as the primary instrument can enhance mathematical achievement*.

1.2 The Case for More Music Education

This research addresses concerns that governments' quests for higher standards in mathematics may result in impoverished curricula with limited access to the arts (Berliner, 2011). Shortly before the launching of this study, governments across the globe had become concerned about their countries' rankings in the international league tables such as PISA (Programme for International Student Assessment). For example, in England, the fall in the ranking of eight-year- old children from 8th to 28th in the 2009 PISA league table (Department for Education, 2010) was described by Michael Gove, then shadow Secretary of State for Education, as "plummeting down" the international achievement tables.

If it is shown that musical learning can benefit mathematical thinking, as hypothesised, it would add to a growing body of research suggesting that policy-makers and educationalists reconsider curriculum balance. As long as each school provides the recommended mathematics education foundation, which includes a certain number of qualified mathematics teachers as well as the appropriate number of mathematics instruction hours appropriate for the age level of the pupils, then rather than potentially overwhelming struggling students with more mathematics lessons, instead, adding music lessons to the curriculum could foster mathematical learning from a fresh and often more motivating angle. In other words, student attention and retention levels might be increased if numerical skills were fostered via an additional modality.

Additionally, community or school-based musical experiences in general have been shown to increase health, wellbeing (Welch & Ockelford in Hallam & Creech, 2010, pp.36-52; Ashley 2002; Clift & Hancox, 2001), social connectivity and enjoyment in school (Abeles & Sanders, 2007). Additionally, enhanced listening skills and attentional control (Posner, 2008) and can extend to executive control abilities such as planning, memory, cognitive task switching and inhibitory development (Posner, Rothbart, Sheese & Kieras, 2008). Cognitive task switching is one of five main executive function skills, indicating flexible thinking. Executive function and self-regulation skills are the mental processes that enable us to focus, plan, remember things and handle multiple tasks.

Even though improvements in aforementioned areas such as cooperation, social support, wellbeing, focus and general cognition can indirectly lead to improvements in school, this study focuses on the direct connection that potentially exists between musical and mathematical cognitive development. The theoretical and research-based arguments for this relationship are presented in the literature review in detail (Chapters 2 and 3).

1.3 Practicality & Significance of the Study

This study included a number of features that were designed to take account of both practical feasibility in classrooms, and the limitations of many existing studies. As singing is the primary teaching medium in this study, and given the popularity of the "Sing Up!" initiative, its inherent accessibility and low cost may increase the relevance of its results for consideration in schools

Though theories of brain plasticity and neural transfer are fundamental to this study, a neuroscientific study was not practicable in this project for numerous reasons. First, the relatively large sample could not be accommodated in a neuroscientific study due to limitations of time, equipment and cost. Second, a large sample of behavioural evidence to justify a neuroscientific investigation is needed. Hence, this could be considered as the first stage of a two-stage study, as the conceptual basis for the study lies in the workings of the brain.

In terms of research design, I am aware of limitations within the existing body of research, which includes many small-scale, short-term, correlational or laboratory-based studies, this study is

(i) medium-scale, (ii) longitudinal (a full school year, i.e., nine months), (iii) quasi-experimental study with behavioural evidence and (iv) conducted in a real-world classroom setting as part of pupils' every day school lives, with all the potential threats to validity that implies. Furthermore, this appears to be the first study that sought to (v) align specific components of music, such as rhythm and melody with corresponding cognitive correlates related both to musical and mathematical processing due to experiential music learning.

In addition to filling in gaps in the research, my contribution to knowledge in the field lies in highlighting the connection between the two domains and in specifying detailed musical and mathematical relationships that may inspire new thinking about research itself, curriculum design, influence effective pedagogical practice, and have a positive and productive impact on the educational process.

Behavioural and neuroscientific research on the relationship between music and mathematics has grown considerably in recent years. Though a reasonable amount of research has been conducted regarding the relationship between musical experiences and mathematically-related development (e.g., Rauscher, Shaw & Ky 1994; Sarnthein, von Stein, Rappelsberger, Petsch, Rauscher & Shaw,1997; Graziano, Peterson, & Shaw, 1999; Rauscher & Zupan, 2000; Schlaug, et al.,[1] 1994, 1995, 2001, 2004, 2008; Schmithorst & Holland, 2004; Spelke, 2008), there is no published study of this type that focuses solely on singing and cognitive transference to the mathematical domain.

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2 Schlaug works with multiple and varying authors, who will be named in the references. Only his name is noted here for the sake of space and readability.

2.1 Considerations Regarding the Research Questions & Design

This study examined if and why music education could enhance children's mathematical thinking and therefore achievement, with recognition of the differing components within both domains.

Therefore, the research questions are as follows:

2.1.1 Research Question 1

How far, if at all, can school-based musical learning primarily via the voice improve mathematical achievement?

2.1.2 Research Question 2

Does focusing on specific musical elements while teaching enhance understandings of possible corresponding mathematical concepts?

2.1.3 Research Question 3

Does teaching music with brief yet explicit references to hypothesised mathematical correlates enhance children's corresponding mathematical skills, and if so, does it do so more than teaching music without these references?

Chapter 4 comprises a detailed discussion of each research question.

Pragmatically speaking, to test any causal relationship as suggested by the research questions, an experimental or quasi-experimental pretest-intervention-posttest design is required, in which the independent variables of different types of musical training are introduced and the dependent variables indicating potential cognitive changes in numeracy are examined. To the extent possible, given a normal school setting, influencing factors were restricted to just one: particular modes of musical experiential education. Of course, the context of the students' lives must be considered. Each student is living, learning and growing in the naturalistic settings of school, home and elsewhere, moved by the constant yet ever-changing laws of human nature and the environment. All of this was considered, so that a realistic and holistic assessment could be made.

Education itself is a search for ways of knowing; hence Crotty's (1998) recognition that there are many epistemological positions and theoretical perspectives is appreciated. Post-positivism as research undertaken in a scientific, yet non-absolutist way for the sake of learning and knowledge, indeed does not need to conflict with interpretive understandings (Carr & Kemmis, 1986). Through gathering evidence that rigorously tested the hypotheses of this study, it is recognised that what has been discovered will be part of a larger process of enquiry, rather than a final and absolute truth.

2.2 Overview of the Thesis

This thesis presents and scrutinises the research in eight chapters. Chapter 1 includes the background, justification for, practicality and significance of the study, brief considerations and introduction of the research questions, an overview of the thesis and closing remarks.

Chapter 2 is part one of the two-part literature review with historical, theoretical and educational perspectives reinforced by research in numerical, mathematical and spatial understandings. Then Chapter 3 provides part two of the literature review, presenting the theoretical framework in detail as well as examining further theoretical support in neuroscientific and educational research.

Chapter 4 restates the research questions and main hypothesis, presents the sub-hypotheses, discusses potential approaches to this investigation, puts forth the research design as well as the assessment and monitoring tools used for the study, while justifying and defining each in detail.

Chapter 5 describes this research intervention fully, describing the school settings and details of the content and teaching of the music programmes. Chapter 6 presents the analyses and results in detail.

Chapter 7 provides the interpretation of the results and discussion of the findings. Finally, Chapter 8 closes this thesis with a critical reflection on the research, implications of the study and conclusion. This is followed by the bibliography and appendices.

2.3 Closing Remarks

It is possible that mathematical understandings can enhance musical development as well, particularly in recognising the architecture of music from a theoretical perspective. Mathematical ability is useful for serious improvisational and compositional work as well as advanced theoretical analysis. Anecdotally, while I was teaching music improvisation and composition to a mathematics postgraduate at the University of Cambridge, at one point when analysing a Liszt work harmonically, the student exclaimed, “This is simply mathematics.” During that term, she learned advanced music theory at an impressive rate, perhaps not by coincidence.

It is hoped that the following in-depth investigation of relationships between music education and children's mathematical skills will help answer questions that are relevant yet unique to the field and will contribute valuable knowledge for the benefit of children, families, schools and ultimately society.

Chapter 2
Music, Number, Time & Space:
Links Between Music & Mathematics

The beginning of this chapter provides a historical backdrop of the link between music and mathematics dating back to the Classical period in ancient Greece. Then the existence of this connection is argued via a progression of concepts beginning with the physical phenomena within music itself. Therefore, foundational ideas concerning the physics of music are presented, starting with the tones themselves, which inherently contain numerical relationships. Bamberger and Disessa refer to the inherent numerical nature of music as embodied mathematics (2003). Since Pythagoras plays a large role in ancient history citing the link between music, physics and number, the physics of music is discussed shortly afterwards.

Following this, a quick return to history will demonstrate the origin of number systems. This discussion includes prominent mathematical concepts such as the binary numeral system (used in computers) and the Fibonacci sequence, which were borne from ancient Indian calculations of rhythm patterns (Hall, 2005). The two-toned drumming of ancient Africa also employed a sophisticated binary numeral system for communication (Finnegan, 2012) and will therefore be described in more detail.

Next, the terms – music and mathematics – are briefly defined for the purposes of this thesis. This is followed with a concise mention of the history of mathematics education noting Freudenthal's (1905-1990) lament regarding the lack thereof.

Then, concepts of spatial sense, including relationships to both musical and mathematical understandings are explored using logic as well as behavioural evidence from numerous studies, followed by a summary at the end of the chapter.

2.1 Music & Mathematics: Historical Perspectives

A concrete argument for a fundamental link between mathematics and music perhaps began with the noted mathematician, Pythagoras (569-475 BC), who is often referred to as the "father of numbers." He could also be considered the "father of harmony," given that his alleged elucidation of the overtone series and analyses of musical acoustics in terms of ratios have served as the foundation of harmony in western hemispheric music composition ever since.

In ancient Greece, the four liberal arts of the *Quadrivium* all relate to number (Lundy, Sutton, Ashton & Martineau, 2010). *Arithmetic* directly concerns number, *Geometry* could be defined as number in space, *Music* is often understood as number in time, but as will be discussed further, it involves number in space *and* time, as does *Astronomy*. Pythagorean, *Quadrivial,* and Platonic classifications of mathematics were based on hierarchical dimensions, starting with arithmetic, then geometry, astronomy and finally music. Crane (1995, p. 1) refers to the early belief that music follows astronomy because the laws of harmony govern both, and the flow of celestial bodies was considered harmonious to the eye as music is to the ear.

2.2 The Physics of Music

Reportedly, Pythagoras experimented with tones produced when plucking strings of different lengths. He found that playing string lengths of specific ratios created pleasing harmonies when combined and others did not. Based on careful observations Pythagoras identified intervals, or distances between notes, that form the primary harmonic system still used today (Parker, 2009, pp. 3-5).

Ratios in terms of sound wave frequencies and the corresponding intervals that he observed include 4:3 (or 1.33 x, known to musicians as the interval of a perfect fourth, or two pitches that are 5 *semitones* apart from each other) and 3:2 (or 1.5 x, a perfect fifth, 7 semitones apart). A semitone is the distance (or interval) between two tones that equals 1/12 of an octave on a log frequency scale. It is also known as a half step and is the smallest step used in Western musical tradition.

The most prominent interval that he noted highlights the universality and fundamentally physical properties of his findings. It is composed of the ratio 2:1 (2 x) and is known as the octave (12 semitones apart). Pitch is the frequency or rate of vibration of a physical source such as a string. When the frequency of a tone is twice the rate of another it is said to be an octave higher than the second tone. Interestingly, the tones are often perceived as being the same or almost the same as each other.

For example, a woman's voice may fluctuate around 220 Hertz and a man's around 110 Hertz, approximately half the frequency of the woman's. One could assume they are in unison while singing the same melody together, even though they are actually an octave apart. This frequency relationship of 2:1 is so elemental to what humans consider music, that even though there are great differences among musical cultures around the world, the octave is the basis of all musical systems that have been documented. Even nonhuman animal species such as monkeys and cats recognise this association (Wright, Rivera, Hulse, Shyan & Neiworth, 2000; Levitin, 2006, p. 31).

When vibrating molecules or sound waves reach the *pinnae* (the outer part of the ear that is visible) and the *tympanic membranes* (the eardrums), the latter respond by oscillating at the same frequency as the sound waves. Sound itself is a wave, as is water. With use of a microphone and an oscilloscope, one can literally see the sound by producing a visual representation of the specific vibratory pattern. This should not be confused with a symbolic system, such as musical notation. The cochlea portion of the inner ear converts the vibrations within the inner ear fluid into patterns of electrical nerve impulses, which are normally transmitted to the brain. This process leads to a mental representation perceived as pitch (Levitin, 2008; Parker, 2009).

We have seen that tones and therefore music involve numerical and spatial relationships. Furthermore, the mathematical structure of pitched sound deepens to yet another level. A single naturally occurring tone contains within it a series of additional frequencies above its fundamental frequency, also called partials, of which we are normally unaware on a conscious level. This harmonic series, or *overtone series* typically occurs in a pattern in which the nth partial is n times the frequency of the fundamental (Randel, 2003, p. 8). In other words, there exists a mathematical relationship among the frequencies as specific integer multiples of each other. For instance, if the slowest frequency, the fundamental, were 100 Hz then the partials would be 2 x 100 (200 Hz), 3 x 100 (300 Hz) and so forth. In *overtone series* terminology, the overtones are designated by numbers with the first overtone being the first vibrational frequency above the fundamental, the second overtone the second vibrational frequency above the fundamental, and so forth.

For clarification, the same overtone phenomenon is also described by some physicists in a different way. In this nomenclature, the overtone series is referred to as *harmonics*. This differing terminology can be a source of confusion due to the different way of naming the fundamental tone and its related partials. In *harmonics* terminology, the first harmonic is the fundamental frequency and the second harmonic is the same as the first overtone, and so forth.

Another way of describing this phenomenon is that a *harmonic partial* is any of the sine waves by which a complex tone is described. Further, any set of partials are whole number multiples of a common fundamental frequency. This set includes the *fundamental (*or *first harmonic in harmonics nomenclature),* which is a whole number multiple of itself (1 times itself). Overtone does not imply harmonicity or inharmonicity and has no other special meaning other than to exclude the fundamental (Roederer, 1995; Levitin, 2008).

See Figure 1 and Table 1 below for two visual depictions of this phenomenon. The first shows harmonics in terms of a vibrating string and the second shows the first six harmonics via a table.

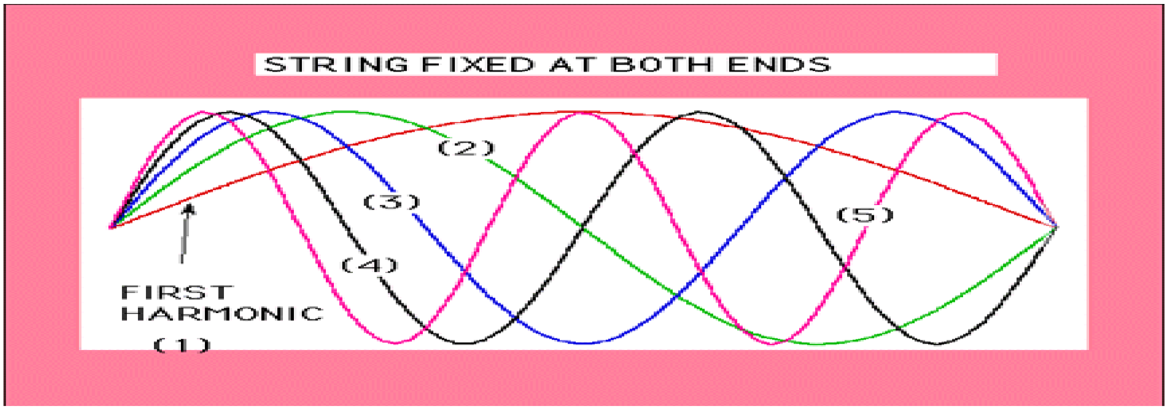


Figure 1 Illustration of harmonics from Educational Materials @ www.pppst.com in partnership with Phillip Martin (www.phillipmartin.com, n.d.)

Harmonic	**Freq. Hz**	**Note**	**Comments**
1	131	C3	Fundamental
2	262	C4	1 Octave Higher
3	393	G4	A Fifth above C4
4	524	C5	2 Octaves above fundamental and a fourth above G4
5	655	E5	A Third above C5
6	786	G5	A Fifth above C5 **Harmonics 4, 5 & 6 form a major chord**

Table 1 The first six harmonics showing frequencies measured in hertz as well as the musical terminology for intervallic relationships. This is adapted from the Physics Department website, Michigan Technological University, (Suits, 1998).

Of particular interest is that the auditory cortex normally responds to each of the components of these natural, harmonic sounds with synchronous neural firings, therefore establishing a neural foundation for the coherence of these sounds (Levitin, 2008, p. 46). Therefore, since all the partials of a tone are mathematically aligned as in nature, if the fundamental frequency is removed electronically, the brain will automatically restore the missing fundamental. This phenomenon was illustrated by the experiment described below.

Janata (1993) conducted this experiment with a barn owl by placing electrodes in the inferior colliculus part of the auditory system. He then played "The Blue Danube Waltz" with all the fundamentals removed. His hypothesis was that the owl's brain would restore the missing fundamental if that process occurs at the early levels of auditory processing. He proposed that neurons in the owl's inferior colliculus should fire at the rate of the missing fundamental. This Gestaltian reaction is indeed what happened. Additionally, since the electrodes emit a small electrical signal corresponding with the frequency of firing, Janata could play back the sound of the owl's neurons through a loudspeaker. The firing rates were identical to the frequency of the missing fundamental, therefore the firing rates of the neurons could be heard back, that is, the melody of the "Blue Danube Waltz." Indeed, the brain is so "attuned to the overtone series that if we encounter a sound that has all of the components except the fundamental, the brain fills it in for us in a phenomenon called restoration of the missing fundamental" (Levitin, 2008, p. 46).

The above elaboration of certain elements of the physics of music could be stated as a sub-hypothesis as such: *If there exist mathematical relationships within the many layers of music from a tone itself to a fully composed piece of music, and the brain responds in a correspondent manner to these patterns and proportions, musical learning over time could enhance mathematical capabilities.*

2.3 Numbers, Rhythms & Sequences

Music is the pleasure the human mind experiences from counting without being aware that it is counting. ***Gottfried Leibniz – mathematician and philosopher***

2.3.1 Origins of Numerical Systems

If certain relationships between and within tones (as in harmony and overtones) could be described as vertically spatial, perhaps the relationships between sound and silence durations (rhythm) could be described as horizontally spatial, with corresponding numbers attached based on specific temporality. It may not be surprising then, that the history of mathematics reveals key relationships with music. Rather than delving into the depths of this history, only critical historical developments relevant to the main hypothesis will be discussed.

The connection between mathematics and music in ancient India is now widely recognised by scholars (van Nooten, 1993; du Sautoy, in Maw, BBC Radio 4, 29 November, 2007). Indeed, certain essentials of mathematics developed in India. The modern numerical system, which uses zero both as a number and as a place value were key innovations. Additionally, the use of binary numbers and a noted numerical sequence found in nature and prominent in art are said to have origins in India (van Nooten, 1993, pp. 31-50; Pogliani, Randic & Trinajstic, 1998, pp. 729-744; Kaplan, 1999; Hall, 2005, pp. 1-4; du Sautoy, BBC Radio 4, 2007; Brown, 2008, pp. 1-15).

The latter two will be discussed briefly. The binary numeral system (base-2 numeral system) is the foundation for computer science. Yet, in the earliest known Sanskrit treatise on prosody, Pingala first used it in the analysis of musical and poetic metres in his Chandahsutra (Science of Metres) text (c. 200 BC, Pingala in Bag, 1966, pp. 68–74). Metres were arranged in differing patterns of long and short, with the temporal ratio of 2:1. Pingala's study of metre led him to describe the addition rules defining rows of the triangle later known as Pascal Triangle, widely used in the mathematical branches of geometry, algebra and probability.

Ancient African bush tribes communicated via intricate messages using two-toned drums (Shectman, 2003; Finnegan, 2012). They used a form of binary system that included an error-correcting code. Retransmission messages were sent to neighbouring villages, possibly with protocol similar to what was used hundreds of years later by Ethernet. This would have ensured a clear channel for retransmission (Hellman, 2013).

Use of long-short binary patterns identifies the temporal organisation of dance music in various cultures. For example, *clave* rhythmic cycles in Salsa music, which most likely originated in sub-Saharan Africa before moving to Cuba and other regions, utilises multiple variations of 2 and 1 (Hall, 2005, p. 2). Argentine tango, influenced both by African rhythms and music from Europe (Davis, 1995), is another case of mathematical patterns utilised within dance structure, often revealing mathematical sequences such as the Fibonacci sequence, to be discussed below. Demonstrating potential layers of complexity in dance, a mathematician at the University of Colorado created geometrical models to mathematically illustrate rotating spatial configurations within classic tango choreography (Farsi, 2010).

In 12th century India, Hemacandra also studied metre, as did his countryman many centuries earlier. After creating a matrix of all of the rhythmic possibilities of a binary pattern with a ratio of 2:1 he discovered a sequence in which each subsequent number following the first two is created by adding the sum of the two preceding numbers (Koshy, 2001). This sequence later became known as the Fibonacci sequence, after the Italian mathematician who introduced it to Europe in the 13th century. Fibonacci had been educated in North Africa and travelled widely with his father for business. After visiting India, he spoke of the advantages of the Hindu-Arabic system of mathematics and introduced it to Europe. He subsequently became prominent for the now-famous numerical series even though he acknowledged the origins in India (Brown, 2008). Note the incidence of this ratio (2:1) in harmonic and overtone structures within musical compositions and tones as well.

This sequence is related closely with the Golden Ratio. The Golden Ratio, also known as the Golden Section or Golden Mean, has been described as a universal law found in natural phenomena. It parallels the cycle of growth and decline, and is reflected in the microcosmic spirals of a seashell and a pinecone as well as the macrocosmic spirals of a hurricane and the solar system. It has also been considered the essence of beauty and is found in the proportions of the Parthenon and the paintings of Leonardo da Vinci (Brown, 2008).

Numerous composers have employed this sequence or its related Golden Ratio, *whether consciously or unconsciously*. Beethoven utilised the Golden Ratio at key points in his Fifth Symphony and use of this ratio also appears in works of Bach, Bartok, Debussy, Satie and Schubert (Knott, 2010). These examples further demonstrate a link between mathematics and music. Indeed, the very existence of the *Journal of Mathematics and Music: Mathematical and Computational Approaches to Music Theory, Analysis, Composition and Performance* with titles such as "Music, mind and mathematics: theory, reality and formality" indicates that there is a growing recognition of the link between the aforementioned physical phenomena and therefore between the corresponding cognitive domains required to perceive and process said input as well.

Before moving forward from this preparatory discussion of the link between music and mathematics, each topic will be briefly defined in order to establish working definitions. The definition of mathematics will be expanded to include subsets such as spatial reasoning in order to focus on particular types of mathematical conceptualisations believed to correspond most closely with music.

2.4 Defining Terms and Exploring Concepts

2.4.1 Music

As concepts and availability of music continue to expand, it is increasingly difficult to define. Social contexts and technological advances have changed not only perception of music, but music itself in some cases (Hargreaves & North, 2010).

Entire books and numerous essays have been written about what music is (Brendel, 2000; Cook, 1998). Yet the purpose of my research is not to add to this field. Therefore, straightforward and well-accepted definitions of music will be used here.

The composer Edgard Varèse described music as “organized sound” (Clayson, 2002). The Oxford English Dictionary defines it as “the art of combining vocal or instrumental sounds together in a pleasing way,” “the sound so produced” or “the written or printed signs representing such sound” (Soanes, Ed., 2003, p. 742).

The argument that it is organised is basic to this thesis, for the patterns and structures inherent in music indeed relate to mathematics. Whether it is music of the West or of the East, patterns and relationships exist that can be numerically mapped. It could be argued that in order for there to be, using terminology from the Oxford English Dictionary, “pleasing” sounds from the Western perspective at least, certain patterns and structures would be mathematically organised, whether consciously generated or not. In addition, sounds recognised as music in some cultures that might seem "unpleasant" to the Western ear, such as microtones, may be pleasant to the Eastern listener due to acculturation and vice versa. Nonetheless, sounds that are organised in patterns that are familiar within the cultures these structures are used would still be defined as music.

Though there are differences among varying cultural perceptions of harmony and consonance, this discussion will focus on elemental aspects of music and perception that are based on currently recognised laws of physics and biology. According to Trainor and colleagues, certain sounds that many define as “consonant” appear to be universally and innately preferred over dissonant sounds as has been revealed by infants (Schellenberg & Trainor, 1996; Trainor & Heinmiller, 1998, pp. 77-88).

As defined in the music theory and well as the music cognition literature, consonance is the harmonious combination of notes due to the relationship between frequencies. “Harmonious” might be seen as a subjective term that varies from culture to culture and even within a culture, depending on personal experience and taste. Yet certain proportional relationships or intervals such as $4^{ths,}$ 5^{ths} and 8^{ths} (octaves), which appear in the harmonic series for example, are considered to be universally consonant and are preferred even by human infants (Trainor & Heinmiller, 1998) and day-old chicks for example (Chiandetti & Vallortigara, 2011).

On the other hand, dissonance exists when two tones with non-identical harmonics that are separated by less than a critical bandwidth (slightly less than 3 semitones) occur simultaneously (Kameoka & Kuriyagawa, 1969; Plomp & Levelt, 1965). The critical bandwidth corresponds to the width of the auditory filter within the basilar membrane of the inner ear (Greenwood, 1991). Therefore, two simultaneous tones separated by less than a critical band are not fully resolved by the ear and the beating (amplitude fluctuations) that arises from their interaction is perceived as coarseness or dissonance (Trainor & Heinmiller, 1998).

Certain aspects of perceived consonance and dissonance seem to play a role in virtually every musical system observed (Schellenberg & Trehub, 1994). From another perspective, however, familiarity can shift the meaning of what some might consider absolute phenomena, bringing into play the possibility of environmental influence.

Melodic (successive tones) and *rhythmic* (of duration, stress or time) structures may consist of contours and patterns that repeat and vary according to the style and distinctiveness of the piece of work. In could be seen that there are three levels of *harmonic*[5] (simultaneous tones) structures within most musical works: 1) the harmonics of the tone itself, as discussed previously, in which specific instrumentation or voice determines timbre, 2) the distances between the vertical, simultaneously sounded tones that create particular intervals such as a 4^{th}, 5^{th} or an 8^{th}, and 3) the progression from one harmony to another, usually analysed in terms of chord structure in relation to the tonality or key of the piece (harmonic progression). Harmonic structure can also be referred to in terms of harmonies changing in time, as in harmonic rhythm.

To provide more detail regarding the last level, a musical chord is normally defined as three or more pitches sounded simultaneously. Two pitches, however, can suggest the chord structure and may be referenced in that way for the sake of the harmonic analysis of a musical work (harmonic progression). For example, the tonic (I chord) to the subdominant (IV chord) to the dominant (V chord) and back to the tonic is a classic chord progression utilised in much of Western music ranging from folk to jazz to classical genres.

2.4.2 Mathematics

The Oxford English Dictionary defines mathematics as “the branch of science concerned with number, quantity, and space either as abstract ideas (pure mathematics) or as applied to physics, engineering, and other subjects (applied mathematics)” (Soanes, 2003, p. 697).

The fields of mathematics education research, cognitive psychology and neuroscience investigate how the mind processes the differing elements of mathematical thinking. These elements will be discussed from all three viewpoints, and parallels with musical thinking will be made.

In conjunction with the Mathematics Education and Neurosciences (MENS) project at the Freudenthal Institute for Science and Mathematics Education in the Netherlands, van Nes and de Lange (2007) present a convincing argument concerning the developmental trajectory of mathematical skills in young children. The association between children's awareness of spatial structures and the subsequent development of spatial and number sense is a focus of their research and is incorporated within the institute's pedagogical philosophy. Freudenthal (1984, p. 48) emphasises the importance of spatial sense when defining it as the capacity to "grasp the external world."

2.5 Realistic Mathematics Education

"Rarely, if ever, do researchers ask, let alone investigate, whether and to which degree 'errors' are due to education, and what educational developers and teachers can learn from them, although only tackling these questions could justify this kind of research as properly educational." Hans Freudenthal -- mathematician and educator

Fortunately, since Freudenthal wrote this statement, many researchers have asked these questions and investigated ways to improve mathematics teaching. For example, for nearly two decades Rowland has sought to improve the teaching of mathematics, often using grounded theory to analyse the teaching of novice teachers, such as the use of examples in their teaching of elementary mathematics (2008) with a goal of aiding instructor meta-cognition for more effective lessons or developing theories with colleagues such as "the Knowledge Quartet as an organising framework for developing and deepening teachers' mathematics knowledge" (Turner & Rowland, 2011, p. 1; Rowland, 2013).

Yet preceding the relatively recent increase of research in this field, Dutch mathematician and educator Hans Freudenthal sought to improve mathematics education through research and theory. Consequently, based primarily upon his theories regarding mathematics education, Realistic Mathematics Education (RME) was developed. Rather than viewing mathematics merely as subject matter that must be transmitted, he emphasised the idea of mathematics as a human activity (Freudenthal, 1977). Freudenthal believed that mathematics should be accessible to children, connected to reality and relevant to society in order to be of value. "Reality" within this conceptual framework, however, can be misunderstood to mean only "real-word" problems. Yet it also includes problems in the minds of students as well. For instance, a child may be able to mentally solve problems just as effectively while imagining a fantasy character in challenging situations.
Even imagining abstract ideas as in formal mathematics can be a context appropriate for a particular

pupil, as long as the problem is real in the pupil's mind. The verb "to imagine" is "zich REALISEren" in Dutch. The emphasis on making something real in one's mind gave RME its name (van den Heuvel-Panhuizen, 2001, p. 3).

The importance of structures and spatial sense in the development of numerical understandings was emphasised by Freudenthal. Work that progressed from this area of his teachings on mathematics education is particularly relevant for this thesis.

2.6 Spatial Sense

From the perspectives of van Nes and de Lange and as seen in certain mathematics curricula, spatial reasoning or spatial sense contains three components: geometry, spatial visualisation and spatial orientation (Mathematics in Context, 1998; van Nes & de Lange, 2007). They assert that all should be included in young children's learning environments.

2.6.1 Geometry

"I have made an attempt to construct it like a Bruckner symphony, with crescendos and climaxes, little foretastes of pleasure to come, and abundant cross-references. The geometric, algebraic and group-theoretic aspects of the subject are interwoven like different sections of an orchestra" said the geometer Coxeter, about his book *Regular Complex Polytopes* (p. ix). From Euclid to Coxeter, geometers have been connected with music and mathematics for centuries. Geometry raises awareness of shapes and figures, including geometrical patterns and structures such as dot configurations on dominoes (Clements & Sarama, 2007). Jones and Mooney (2003, pp. 1-15) make a case for a rethinking of the National Numeracy Strategy (NNS) in England, which devotes little time (10%) to geometry. Yet geometry is undergoing a renaissance due to a renewed, research-based recognition of the need and importance for the development of spatial reasoning skills, of which geometry is a subset. James and Mooney note that technological advances in areas such as cinema animation, robotics and architecture as well as the new technologies of global positioning and MRI (magnetic resonance imaging) require advanced skills in geometry (2003, p. 4).

2.6.2 Spatial Visualisation

Spatial visualisation refers to the ability to mentally manipulate objects in space. This is applied regularly in a child's daily life for basic activities such as imagining where one's snack may be in the kitchen before getting it, eventually leading to higher mathematical reasoning skills in addition to other cognitive abilities (van Nes & de Lange, 2007).

2.6.3 Spatial Orientation

Spatial orientation is an individual's awareness of the space around him or her in terms of distance, surrounding forms, direction and relative positioning. When children learn to orient themselves as such, they are able to take different perspectives, describe routes and understand relationships between objects (van den Heuvel-Panhuizen & Buys, 2005; Lee & Spelke, 2008, pp. 743-749).

Spatial-temporal reasoning is not included as a component of spatial sense by van Nes & de Lange (2007) yet it is particularly relevant for this discussion. Much of recent research in fields such as computer science are emphasising the importance of spatial-temporal reasoning. It can be defined as the facility to visualise spatial patterns and manoeuvre them over a time-ordered sequence of spatial transformations. As with spatial-visual reasoning, it is becoming generally accepted that this ability may extend to finding, creating and conceptualising solutions to multi-step problems existing in areas such as science, engineering and mathematics (Casasanto & Boroditsky, 2003; Renz & Nebel, 2007). Furthermore, this link between space and time is now discussed in research involving additional, wide-ranging topics such as geographical information systems (GIS), artificial intelligence (AI), aeronautics, evolutionary biology and cognitive linguistics (Bennett, Cohn, Wolter & Zakharyaschev, 2002, pp. 239-251; Casasanto & Boroditsky, 2003; Lin, Huang, Jiang & Chen, 2009, pp. 1-5).

How are these two elements – space and time – linked cognitively? Gentner (2000, pp. 203-222) examines this question at length. She notes that languages across cultures often use spatial terminology to describe time. For example, we may say that we look *forward* to tomorrow and forget the troubles *behind* us while listening to music all *through* the night. Gentner suggests that spatial mappings may influence the cognitive processing of temporal sequences and go beyond mere analogical metaphor. These space-time metaphors facilitate reasoning in three ways: they use ordered space to represent elements (events) and their relations (sequential ordering) while using spatial dimensions (a single linear dimension placed in correspondence with time's single dimension) providing non-arbitrary analogs for abstract concepts.

Dehaene and colleagues had devised a methodology to test whether number magnitude is spatially represented in the mind via a task in which responses were recorded based upon hand speed. They found that participants respond faster with the left hand when the answer involves smaller numbers and faster with the right hand with larger numbers later, referring to this phenomenon as the SNARC Effect, or Spatial-Numerical Association of Response Codes (Dehaene, Bossini & Giraux, 1993, p. 394). This experiment, as well as the experiment by Santens and Gevers (2008) using the close-far dimension, both suggest the existence of a spatial representation of numbers in the brain and further the argument for the hypothesis mentioned above concerning the use spatial cognition in reasoning, though the latter authors argue that the SNARC effect alone does not prove its existence definitively (p. 269).

2.7 Spatial Sense and Musical Processing

Numerous studies have suggested a connection between music and spatial-temporal reasoning (Rauscher, Shaw, Levine, Ky & Wright, 1994; Graziano, Peterson, & Shaw, 1999, both to be discussed in the following section). Since music is sound existing in space and usually moving in time, it seems logical that training in this domain could facilitate spatial-temporal reasoning. Therefore, it is reasonable to ask the question: Does spatial-temporal reasoning in particular seem to improve with exposure to and active engagement in music and if so, why?

2.7.1 Correlative and Infant Studies

A study by Spelke (2008) offered promising clues to help answer this. She had found in her extensive research on mathematical cognition and development that mathematical ability is not confined to one system in the brain and was intrigued by the longstanding idea that there is a special tie between music and mathematics. Therefore, when challenged to see how the arts relate to the organisation of cognitive systems, she and members of her lab began a correlative study among three school-age groups to see if those with music training had an associated advantage in any area of mathematical aptitude. The three studies Spelke conducted (2008) looked at levels of training from low intensity to high. Of three core mathematical abilities, the children who received moderate or intensive music training performed significantly higher on geometrical and spatial tasks. There was also an associated advantage for the music group in using number lines and maps, which utilise spatial skills as well, even when controlling for elements such as reading IQ and motivation.

Therefore, in order to discover the source of these correlations, Spelke led an infant experiment, patterned after one conducted by her colleagues (Carey & Srinivasan, 2008). Carey and Srinivasan had worked with note durations and visual objects (worms) of different lengths to see whether there might be an inherent relationship between the perception of musical space and the perception of time. They had auditorially presented infants with short and long note durations accompanied by corresponding short and long worms. The infants readily learned to connect the relationship. To test whether this was arbitrary, they presented another group of babies with the same sets of tones and worms, yet the sets were reverse-paired. Those infants did not learn the connection, suggesting a cognitive relationship between auditory duration and visual length that could reveal a foundational link between the perception of sound and the representation of space.

Spelke's experiment (2008) looked at pitch contour in relation to space, in this case height. Ascending or descending sequences of tones were matched with corresponding heights of objects, and then reverse pairs were shown to a different group of babies. Again, the infants learned the relationship with the congruent pairs but did not learn to match the incongruent pairs, showing a connection between melodic contours and positions in space. This study clearly suggests that an inherent relationship exists between musical and spatial processing; both may serve as a foundation for an emergence of the positive relationship between music and mathematics.

In another rare study that investigated potential effects of a specific mode of music, in this case rhythm, Parsons, Martinez, Delosh, Halpern & Thaut (1999) found that concentrated rhythmic training yielded higher levels of spatial-temporal reasoning than did focusing on other musical elements.

2.7.2 Piagetian-Inspired Studies and Vygotskian Responses

Though the studies discussed in this section were conducted decades before, it seems worth looking at this work due to the relationship between foundational cognitive concepts first explored by Piaget and later by Pflederer and their relationship with ideas concerning the musical-spatial-mathematical connection that are explored in this thesis. In 1964, Pflederer designed "musical conservation" experiments to parallel Piaget's well-known conservation studies. Though much of Piaget's work evaluating the cognitive abilities of children has been challenged, his idea of conservation is generally accepted. Piaget pioneered the field of "genetic epistemology" with his theory of children's biologically guided developmental stages and designed experiments that tested for logical reasoning. He claimed that there are age-related differences marking one's understandings of physical phenomena. For example, in a series of cognitive tests he termed "conservation," he looked at children's responses to manipulated materials. Piaget (1964) defines conservation as "the capacity to grasp that despite certain changes in an object, there are particular properties that remain unchanged" (p. 19). For example, after changing the shape of something, he would then ask the child if the second item was the same amount as the first one. Those in the younger age group (~ 4-7) who would have been in what he called the "pre-operational stage," tended to think the quantity of the material was different because it had changed shape, whereas children of approximately the age of seven and up would usually recognise both as the same quantity, denoting a revolution in the child's physical knowledge, therefore entering what he termed the "concrete operational stage" (Lovell & Ogilvie, 1960).

Several elements of mathematical and scientific reasoning are involved in conjunction with "conservation" abilities, including number, classification, space, distance, time, speed and volume. These parallel elements of music. Though more abstract than time, for instance, even volume could be comparable to the texture of a piece (how many instrumental layers), the resonance of the tones or the strata of the harmony. Pflederer, a music educator and researcher, indeed saw that certain spatial as well as numerical elements related to ideas of Piaget's conservation tasks could correspond well with musical reasoning and designed a study testing what she called "music conservation" (1964).

She used nine tasks, each utilising two musical examples, and followed the basic design of Piaget's experiment. In general, the results of her pilot aligned with Piaget's, in which the 8-year- old subjects understood the conservation of certain elements even though other elements such as pitch, rhythm or metre had changed. Therefore, after considering various flaws in her original test such as small sample size and confusing terminology, she conducted another study with modifications (Pfederer & Sechrest, 1968) yielding similar results, which supported the Piagetian theory of conservation. The study does not claim that a child must go through biologically- prescribed stages one at a time before cognitively moving on but notes cognitive realisations in scientific thought that parallel musical thought, the latter which could be argued is also scientific. Others have followed Pfederer's line of reasoning: Tunks (1980) produced similar results showing that music conservation improves with age, with a slight plateau at around age nine. Though Foley (1975) found that music conservation abilities could be trained at the pre-operational age, contrary to a follow-up study of Pflederer-Zimmerman and Sechrest (1970 in Hargreaves, 1986), the empirical evidence from the research in this area generally supports Piaget's theory (Hargreaves, 1986). This work implies underlying connections between musical, logical and spatial reasoning, all of which can contribute to mathematical reasoning.

As suggested before, many researchers, however, have refuted elements of Piaget's theories. Bruner, Olver and Greenfield (1966) echo Vygotsky (1930-1934/1978) in their emphasis on the role the environment plays in influencing thought. After conducting a cross-cultural study, evidence of stages was present in children of non-Western countries, though with less delineation. Others have also challenged Piaget's conservation tests noting that misleading verbal or perceptual elements of the tasks could affect the outcome, rather than the child's cognitive ability at the time (Bruner, et al., 1966; Freudenthal, 1984, 1991). This questioning led to the concept of "contextual understandings" and an area of theory and research known as "social cognition" (Bryant, 1974).

It is hoped that this doctoral study will build on and contribute research supporting the idea that the environment indeed does play a role in development, hence advocating education that may enhance cognitive abilities, in this case, musical training for the enhancement of mathematical skills. The child's understanding of "conservation," does appear to be influenced by environmental factors such as teaching clarity and repetition. Therefore, having a good teacher as well as practising should increase musical comprehension; in turn, enhanced musical understandings may indeed enhance cognitive understandings in other domains. This research recognises the existence of nature in cognition and yet calls for the maximisation of nurture to support natural development.

In another reference to Piaget's work, Gardner (1973), of the Multiple Intelligences theory, takes the stance that a child does not need to have concrete operational thought to participate in the "artistic process." He proposes that there is no need to be at any specific developmental stage for the arts since "the groupings, groups and operations described by Piaget do not seem essential for mastery or understanding of human language, music, or plastic arts" (p. 45). Grossly oversimplifying the cognitive processes involved in music comprehension in certain examples he gives, Gardner nonetheless admits that the lack of fluency in motor skills by young children can inhibit high levels of musical competency (p. 197). Yet perhaps fluency of mind is needed as well as that of motor skills in order to reach the realisations to which Gardner refers.

2.7.3 The "Mozart Effect"

Starting a conversation among researchers, teachers and parents anxious to boost their children's IQ scores, is the well-publicised experiment that Rauscher and colleagues (Rauscher, Shaw & Ky, 1993). This was conducted with university students in which ~1/3rd of the matched groups listened to Mozart's Sonata K. 448 in D Major for 10 minutes and then took spatial reasoning tests using the Stanford-Binet Intelligence Scale. Another group listened to relaxation instructions and the other, nothing. Those who had listened to Mozart (N=36) scored 8-9 IQ points higher than the other groups; the effect lasted approximately 10 minutes. Hence was born the phrase, "The Mozart Effect," which implies that listening to Mozart leads to higher intelligence.

The researchers in their findings claimed a causal relationship between Mozart's music and spatial-temporal reasoning, as the students were specifically tested for spatial-temporal reasoning, not general intelligence. Yet this study should be noted with caution as the findings were frequently exaggerated, particularly in the media, sparking a commercial frenzy with claims that Mozart's music could create "Baby Einsteins."

Numerous replications of the study were later conducted. For example, Chabris analysed 15 of such studies (1999) and concluded that the findings were not statistically significant. In response, Rauscher criticised his report, noting that certain studies should not have been included in his analysis since they had tested for general intelligence, not spatial intelligence. Others found minimal support of the study (Pietschnig, Voracek & Formann, 2010; Vaughn, 2000), though in Vaughn's meta-analysis, for example, only six studies were included, therefore weakening the validity of the analysis.

Shortly after the initial experiment of note, Rauscher and colleagues responded to challenges to go beyond listening by conducting two studies that looked at the effects of training with preschool children. Both yielded statistically significant increases in spatial-temporal reasoning ability after months of consistent training. Using controls, one (N=33) included daily group singing lessons and weekly private keyboard lessons taught by professionals (Rauscher, Shaw, Levine, Ky & Wright, 1994) and another involved private keyboard lessons and computer lessons (Rauscher, Shaw, Levine, Wright, Dennis, & Newcomb, 1997). Responses to this research include replications, clarifications (Rauscher, Shaw & Ky, 1995), enquiries investigating why the effect occurs (Rideout & Laubach, 1996) and attempts to generalise to other cognitive abilities (Wilson & Brown, 1997).

Hetland (2000a) conducted a meta-analysis of 36 experiments with 2,469 subjects and compared tasks that qualified as spatial-temporal (31 of 36) to other types of spatial measures (5 of 36). Contrast analysis showed that the average effect size of the experiments utilising spatial-temporal measures alone is $r = .20$. Experiments employing only nonspatial-temporal measures produced a small effect size of $r = .04$, and experiments that employed a combination of spatial-temporal and nonspatial-temporal measures showed an intermediate effect size ($r = .15$). Hetland's analyses substantiate the idea that the consequence, though temporary, of listening to music at least similar to Mozart's music may therefore be specific to spatial-temporal, more than general spatial thinking and even more than general intelligence.

Hetland conducted another meta-analysis (2000b), this time of 19 experiments involving musical training in a variety of instruments with children ranging in age from 3-12 that met certain criteria, such as having at least one control. She implemented three meta-analyses based on outcome measurements. In line with evidence from the listening studies, categorisations were based on which spatial reasoning tests were employed in the studies. The first, which included 15 (N = 704) studies that used spatial-temporal tests, yielded a large average effect size by meta- analysis standards (r = .37, d =.79). The second meta-analysis that included 5 (N = 694) studies employing nonspatial-temporal measures yielded a small average effect size of r = .08, d = .16.

The third meta-analysis included 9 studies (N = 655) that utilised a variety of spatial tests (including spatial- temporal). The average effect was moderate but still relatively strong (r = .26, d =.55), indicating that spatial reasoning skills in general are enhanced via musical training but not as significantly as spatial-temporal ones are (r = .37, d = .79).

Hetland's meta-analyses substantiate the idea that effects of exposure to music are more specific to spatial-temporal than general spatial thinking and much more than to general intelligence. An experiment conducted in light of the prevailing assumption that spatial reasoning skills enhance complex mathematics skills employed keyboard lessons coupled with a mathematics computer game (Graziano, Peterson, & Shaw, 1999) and explored whether skills that are particularly difficult to teach verbally, such as proportional mathematics and fractions, could be enhanced with these two interventions. The experimental portion of the 237 Grade 2 children[*] were given keyboard lessons and showed statistically significant improvements in proportional mathematics and fractions compared with the control groups who were given control training along with the mathematics video game, therefore supporting the hypothesis presented here. (*Grade 2 in the United States corresponds with Year 3 in the United Kingdom.)

The specificity of numerous findings mentioned above regarding spatial-temporal reasoning could weaken an alternate hypothesis that arousal and mood were the causes of the enhanced spatial reasoning abilities demonstrated (Husain, Thompson & Schellenberg, 2002; Husain, Thompson & Schellenberg, 2002). Also, seeming to rule out the arousal and mood suggestion were studies that used other forms of music such as Philip Glass music and repetitive trance dance music in comparison to the Mozart sonata (Rauscher, Shaw & Ky, 1995). These forms of music had a significantly lower effect on spatial-temporal reasoning ability, yet this may not be surprising, as one may also assume that arousal would not have been as high with the particular types of non-Mozart music used in this experiment.

In the case of numerous other studies attempting to show a Mozart Effect, including Rauscher et al.'s, Mozart's music was contrasted with silence, speech or monotonous music. Therefore, in those cases Mozart's music could have simply represented *most* non-monotonous music. Therefore, in those studies, Mozart's music had a stronger effect than other groups yet it should be questioned whether the results of the studies showed the efficacy of Mozart's music specifically. On the other hand, in most studies that used Mozart's music, after subjects had been asked to focus on the music, their spatial reasoning scores were higher, suggesting that the music itself perhaps did affect the outcome.

Schellenberg and Hallam (2006) conducted a robust study that reanalysed a large data set that initially had been produced in response to Rauscher et al.'s seminal study (1993); this replication by Hallam (1996) did not show evidence of a "Mozart Effect." This reanalysis, however, investigated the possibility that the "Mozart Effect" that had been shown in numerous studies could be the result of listening to music that activates optimum arousal levels and mood states, which in turn enhances cognition (Schellenberg and Hallam, 2006).

In Hallam's original study (1996, later published in 2000), over 8,000 children, aged 10-11 took part. This examined the spatial reasoning effects of three different groups after each group listened to 10 minutes of one of the three types of auditory stimuli: Mozart's music, popular music such as a song by Blur or a spoken explanation of the study itself. Of the two spatial tests administered right after listening, the students who had listened to the popular music scored notably higher than the other two groups on the paper folding test, yet at an equal level as the other two groups on the square-completion task. This evidence supports the arousal and mood hypothesis, since the popular music was most likely the most enjoyable music for children at this age (Schellenberg & Hallam, 2006, p. 202-209).

That testing order could have played a role was noted in the article on the study (Schellenberg & Hallam, 2006, p. 207). The paper folding task was consistently the second task completed during this study, which was more difficult for the students than the square-completion task. It would be interesting to see what the outcome might be if the order of the test-taking were to be reversed. As the alleged "Mozart Effect" lasts approximately 10 minutes, the effect may have "worn off" by the time the more difficult test was taken and therefore the children who had listened to the popular music and therefore heightened arousal and mood levels may have had an advantage due to a potential for the "Blur Effect" to last longer than the "Mozart Effect."

Additionally, it would be interesting to see the results of a study in which children enjoyed Mozart's music to an equal degree as popular selections similar to music used in the original Hallam study conducted in 1996. (The original study is described in Schellenberg & Hallam, 2006, p. 203.) This comes to mind when reflecting upon research undertaken with underprivileged children in Newark, New Jersey (Abeles & Sanders, 2007). The majority of the participants had no previous experience with classical music prior to the music intervention. In the intervention, Mozart's music was learned on an elementary level through a modified Suzuki pedagogical approach and further exposure to classical music was experienced via participation in concerts with a professional orchestra. In interviews with the children, their enjoyment of the music was expressed, therefore suggesting that experience with classical music had expanded their range of preferences.

In addition to the interviews, participants took a vocational survey (Vocational Choice Scale, Cuiettia, 1995 in Abeles & Sanders, 2007)) before and after the intervention to monitor whether the children's interest in music would increase due to the exposure and active engagement with music, in this case classical music. The survey was modified in an attempt to eliminate gender and ethnic bias and showed different images of musicians, including that of a violinist, a profession to which children in the region where the study took place may not have been exposed. There was a statistically significant difference (Chi-square = 7.47, $p < .01$) between the frequency of the choice of Musician for students participating in the strings program (31%) compared to students who did not participate in the program (14%). This vocational preference shift could be assumed to correlate with a potential musical preference shift. This suggests that a Mozart Effect study using different types of music in addition to Mozart's music may reveal higher numbers of subjects who prefer or equally like classical music, thereby potentially altering results regarding levels of arousal, mood enhancement and cognitive priming.

If there are indeed aspects about Mozart's music that are conducive to particular types of cognitive enhancement such as spatial-temporal or proportional reasoning, then participants who also enjoy this music could potentially strengthen or perhaps even double their advantage in certain cognitive tasks through meeting both discussed conditions, 1) enjoyment leading to arousal and mood enhancement as well as 2) mental priming for pattern, structural and proportional recognition.

2.7.4 Elements in Music Encouraging Organised Thinking

What are possible musical elements in Mozart's music or in similar music that may prime the brain to perform better in spatial-temporal or proportional tasks that therefore may encourage processes strengthening mathematical understandings? The intricate, highly organised and sequential nature of much of Mozart's music may explain part of this effect. As is often found in Bach's music, there is normally continual motion and repetition of logical patterns that exist within the course of subtle yet multiple melodic and harmonic modulations and unexpected rhythmic and textural shifts within a symmetrical design. Mozart often plays with symmetry in reversing musical phrases, both horizontally and vertically, thereby creating mirrored music, not unlike rotating tasks found in spatial-temporal reasoning tests, which include questions such as imagining what shapes one would see if inside a shop window, with each question containing varying shapes (Smith & Lord, 2002). Retrograde inversion (when a musical theme is repeated in full reverse order, that is, backwards and upside down) could also be compared to geometric transformations such as rotation, in which a figure is turned about a given point that can be inside the figure or outside of it.

Yet Mozart's music is only a representative body of work that holds certain musical elements potentially conducive to priming of mathematical cognition. Listening to and performing other forms of music such as jazz or "world" music could presumably have similar cognitive outcomes to classical music of this nature if containing similar properties and balance of logic, repetition and complexity. Indeed, listening to a style one prefers has been shown to be most beneficial (Schellenberg & Hallam, 2006), though one might conjecture that the cognitive results might be even more powerful when the preferred music contains the said elements as well.

Therefore, recognising and widening the educational repertoire to include non-classical and non-Western styles of music should be considered. The interlocking rhythms and melodies of African-American Jazz, Indonesian, Indian, Turkish, Latin or African music are frequently comprised of multiple layers that can be highly logical, intricate and variant within phrases of patterned repetitions. It would be interesting to explore possible cognitive changes after experiencing one or more of these additional choices of music that also include the use of multi-layered rhythmic, melodic and/or harmonic elements of different cultures (Sanders, 2012).

2.8 Spatial Sense & Numerical Sense: A Convergence

Different threads of descriptions and categorisations exist in the current literature on numerical sense, or number sense. In cognitive psychology and neuroscience, three primary categories are recognised within this larger category. First, is *spatial sense*. Second, immediate identification of small amounts of numbers (1-4), labelled *subitization* (Butterworth, 1999), exists as early as infancy and among nonhuman animals (Xu & Spelke, 2000; Xu, Spelke & Goddard, 2005; Uller & Lewis, 2009). This elementary number system includes quantity discrimination among groups of objects that may be recognised regardless of mode of delivery (for example, visual or auditory) or language ability (Dehaene, Dehaene-Lambertz & Cohen 1998; Piazza & Dehaene, 2004). Third, *numerical magnitude representation* involves the estimation of larger quantities and leads to symbolic mathematics (Whalen, Gallistel & Gelman, 1999; Piazza & Dehaene, 2004).

These three systems – spatial reasoning, small numerical quantity recognition and magnitude judgment, begin to converge in our culture between the ages of three and six (Spelke, 2008). The latter two systems join to create representations of exact number, or natural number concepts including counting. The combination of spatial sense and number sense connects points on a line as well as positions in space and time with numbers. However, there is evidence suggesting that early intuitions about these relationships exist in infancy. In preschool, early symbolic abilities linking space with objects enables a child to connect geometrical points on a page to a real, three-dimensional environment (Lipton & Spelke, 2005; Xu, Spelke & Goddard, 2005).

2.9 Spatial Skills & Numerical Skills: Comparisons with Musical Thinking

Children continually attempt to organise their world by finding patterns and creating structures (Gopnik, 2004). Mathematics is an activity of organisation, of problem solving. Organising subject matter within reality must be accomplished according to mathematical patterns in order to find solutions in life (Freudenthal, 1991). Music-making also requires the organisation of material, and like a mathematician, the musician seeks patterns, creates structures and solves problems (Pogonowski, 1987; du Sautoy, in Maw, BBC Radio 4, 2007).

In agreement with van Nes and de Lange (2007), one could define a pattern as a numerical or spatial regularity and the relationship between the elements of a pattern as an object's or phenomenon's structure. The researchers above give examples of spatial structures that young children would normally be familiar with such as the dot configurations on dice, beads on a necklace and block constructions. If sound is considered in terms of space, a musical piece is a spatial structure made up of patterns of sound. Additionally, its notation is in fact a visually spatial structure containing patterns of lines, curves and dots.

Van Nes and de Lange (p. 217) suggest that the intertwinement of the three components of early spatial sense may contribute to the development of children’s number sense, the discernment of quantities and relationships between numbers. In parallel, since every element of music is spatial in some form, whether it is rhythmic, melodic, harmonic or tonal, each can be attached to a number. Additionally, all of these musical components are in specific, measurable relationships to the others within their own categories as well as between categories. An understanding of these spatial-temporal and numerical elements within a musical piece may at least contribute to an implicit understanding of the patterns that make up the structure of a musical composition. Therefore, these spatial-numerical connections within music may help explain the potential link between musical and mathematical understandings.

Once children can imagine a spatial or temporal structure (whether visually or aurally) of a certain number of objects or sounds that are to be manoeuvred, the emerging number sense, which includes knowledge of quantities as well as counting, should be largely simplified. Below is an adaptation of the hypothesised relationship between early spatial sense and emerging number sense as conceived by van Nes and de Lange. Following this is an extension of these concepts to the domain of music.

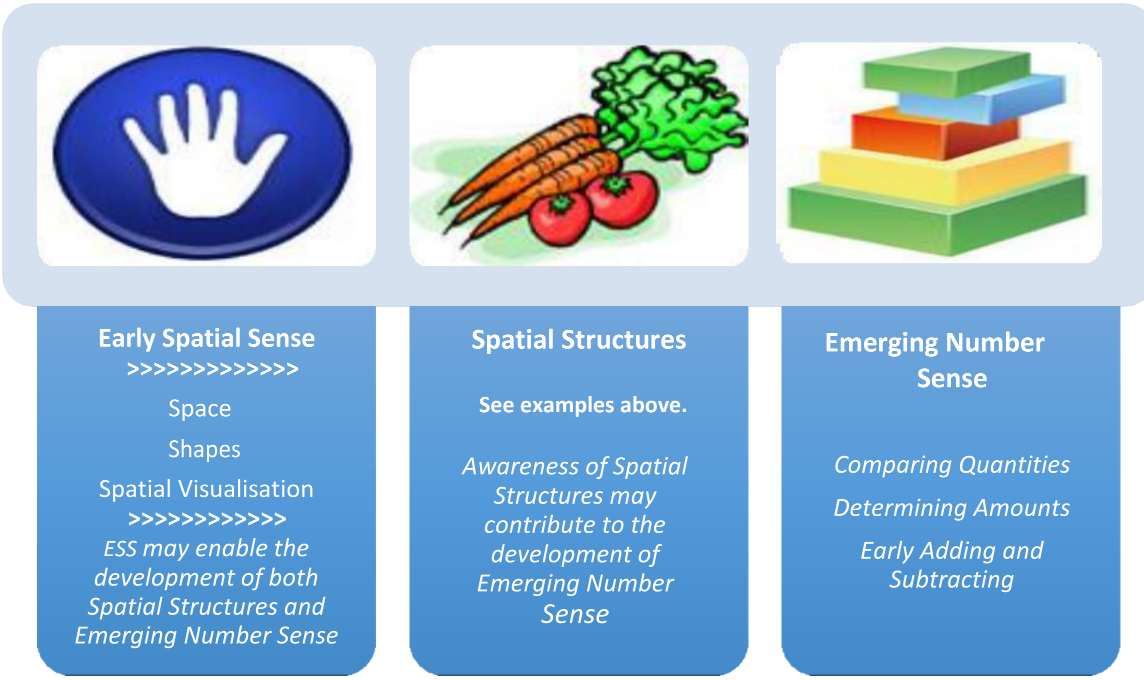


Figure 2 Early spatial sense. This leads to awareness of spatial structures and numerical concepts, which may support the development of emerging number sense. Adapted from van Nes and de Lange (2007).

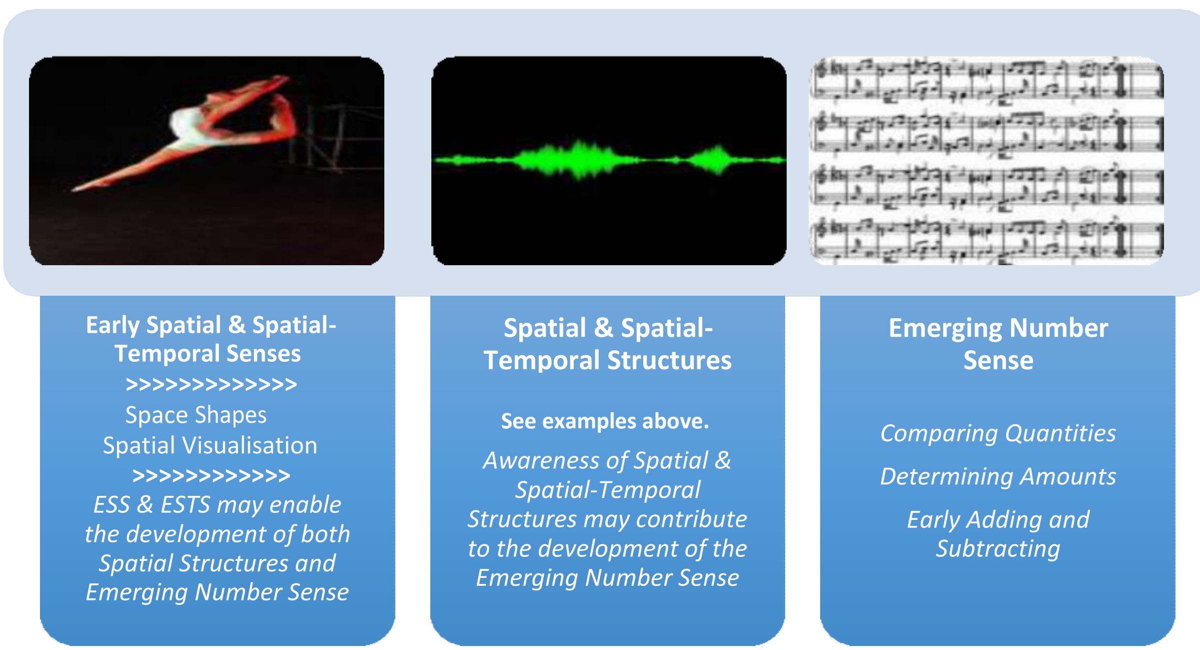


Figure 3 Early spatial and spatial-temporal senses. These lead to awareness of spatial and temporal structures as well as numerical concepts, which may support the development of the emerging number sense. Musical experiences including dancing, singing, playing an instrument and reading notation can encourage further growth and understanding of spatial-temporal concepts

2.10 Spatial Structures within Mathematical Problems

In continuing the argument for spatial reasoning and its relationship to both mathematical and musical problem solving, it should be helpful to consider Gestalt theory. Original proponents of this theory, Wertheimer (1922), Koffka (1922) and Köhler (1929), elucidated the tendency for humans and nonhumans to group stimuli together and to perceive the whole versus the parts.

This "grouping" predisposition causes one to structure and interpret visual, auditory or conceptual phenomena based on proximity, similarity, closure and or simplicity according to symmetry, regularity and smoothness. These factors were referenced as the laws of organisation and explained in the context of perception and problem solving. Wertheimer (1922, 1959) was particularly interested in the problem-solving processes of geniuses such as Einstein as well as children; he saw the essence of successful problem-solving behaviour as the ability to see the overall structure of a problem. Luchins & Luchins were two of many Gestalt psychologists who applied Gestalt theory to mathematical pedagogy (Wertheimer, 1922, 1959; Luchins & Luchins, 1985, 1988, 1997; Arcavi, 2003). Additionally, this theory may explain why one normally hears a musical tone as a whole rather than in terms of separate overtone fragments; this allows insight regarding suggested writing rules and cognitive results of musical composition as well (Lerdahl & Jackendoff, 1983; Levitin, 2006, pp. 76-82).

Richardson (2004) illustrated the existence of Gestalt theory in an illustration of children's use of spatial logic in her preschool classroom. Originally, she had asked her students to count the number of dots on cards (similar to dot arrangements on dice). Yet instead of counting, they made shapes to match what they had seen, such as an "X" shape to show the five-dot configuration and a square shape to show the nine-dot configuration. This example supports the argument by van Nes & de Lange (2007, p. 219) of the propensity of spatial logic to exist prior to and in support of numeracy.

As an illustration of how music can enhance spatial reasoning and therefore logical thought, a case study of a blind learner will be summarised. Clearly, a case study of one person should not be generalised, yet it may provide insight into the potential that active and evaluative listening may hold in encouraging logical thinking. This student, in his early 20s, exhibited certain cognitive difficulties apparently deriving from an episodic rather than holistic, Gestaltian perception of reality. His spatial reasoning and ability to make connections and distinguish relevancies was lacking possibly due to the scarcity of stable systems of reference by which to organise space. Based on the cognitive intervention programme, Instrumental Enrichment (Feuerstein, Rand, Hoffman & Miller, 1980), an intervention was undertaken, which utilised tactile and analytical musical methodologies. This was developed by Gouzman (2000) in order to address cognitive deficiencies that are relatively common for the blind learner.

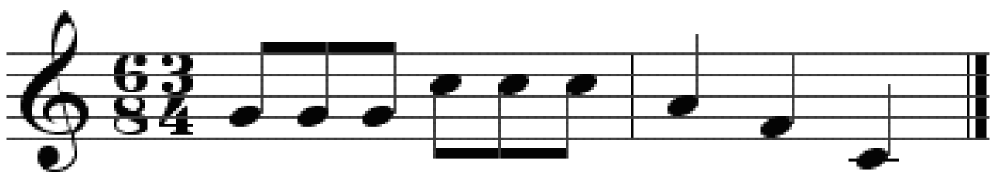
Before listening to numerous musical selections, three compositional techniques of musical form were discussed: *repetition*, used to create a sense of familiarity, balance, unity and symmetry, *contrast* and *variation,* the former of the two also used for symmetry and balance and both for novelty as well as to keep some elements of a musical thought while changing others. Aided by this vocabulary, the student was then guided through numerous listening sessions that encouraged him to analyse the structure of each piece, including how differing elements make up the whole. In order to activate abstract and yet focused thinking, he was asked to aurally follow the pathways of the music and to locate goals and points of articulation throughout. Listening to and locating specific aspects of the music is analogous to visually or kinesthetically locating multiple points and angles on a geometric shape or orienting oneself within a space.

After only three sessions, cognitive skills such as strategic focusing, comparative and hypothetical thinking, communication of mental processes and Gestaltian perception, aided by identification of musical points of reference and overall organisation had been stimulated in this learner, as revealed by the transformed quality of his discussions. The first session had been primarily analytical, the second, which included improvisation and composition, encouraged creative problem solving and syntactical reasoning, building a whole from parts. The third session revealed an emerging ability to hierarchically identify, classify and evaluate several sources of information. This case study suggests the potential that active listening and analysis of music holds for the development of organised and abstract reasoning (Portowitz, 2001). Again, this is a case study of solely one person and the results should not be generalised to an entire population; it is nonetheless reasonable to acknowledge that certain modes of thought could arise from such analytical work. My own experience helping children and adult learners to clarify their thoughts and develop coherent reasoning via listening, evaluating, performing and discussing musical principles resonates with this example.

Additionally, music compositional and improvisational skills encourage both divergent thinking (associated with creativity) and convergent thinking (usually associated with problem solving). As with composition and improvisation, higher-order mathematical problems and calculations have been found to benefit from both forms of thinking also (Dunn, 1975; Haylock, 1987; Toshihiro, 2000; Cropley, 2006).

Indeed, for children, simpler yet similar listening activities and questions can be applied. Van

Nes and de Lange (2007) point out the advantage of applying spatial structures to mathematical problems such as comparing numbers of objects, continuing patterns, building block constructions and identifying quantity using structure and grouping, for example, by seeing six as groups of three and three. Certain musical pieces are particularly effective for encouraging spatial-temporal reasoning as well as counting, accurate quantity discernment, categorisation, pattern recognition and even cognitive task switching[8]. The song *America*, for instance (Bernstein & Sondheim, 1957) is in a mixed metre, with alternating 6/8 and 3/4 time signatures or rhythm patterns. It provides a good opportunity to teach numerous skills such as those mentioned above and to alert students to the role that rhythm can play in structure. See Figure 4 below.



1	2	3	**4**	5	6	**1**	&	**2**	&	**3**	&

1	2	3	**1**	2	3	**1**	2	**1**	2	**1**	2

Figure 4 Different methods of notation for the rhythm in "America" (Bernstein & Sondheim, 1957).

Beneath the standard method of notation shown at the top of Figure 4 above are two other ways to count the song "America" from the musical *West Side Story* (Bernstein & Sondheim, 1957). Though the underlying tempo and pulse remain constant, the accents (in bold) fall on different beats as reflected by the alternating time signature of 6/8 3/4 and are therefore grouped and counted accordingly. The top row is a "proper" way to count in line with the underlying eighth note (or quaver) pulses within this alternating time signature. The first measure (bar) could also be counted as two large beats, each containing three small beats within (compound duple metre, or compound time) followed by the different emphases in the second measure, in which three large beats are each divided into halves. If considered in that way, one could count, "**1** ee uh, **2** ee uh, **1**&, **2**&, **3**&." The bottom row shows an alternate way to count, which points out the accents and subsequent grouping structure and can be readily understood by learners whether or not they have had notational training. This musical piece provides an effective opportunity to teach multi-layered skills including counting, the retention of a steady beat while accentuating alternating ones, recognition of rhythmic patterns and structures as well as the flexibility of cognitive task switching. The top method of counting is most appropriate for students who are learning more complex standard rhythmic notations.

Mathematical abilities such as ordering, comparing, generalising and classifying are supported by an ability to grasp spatial structure (National Council of Teachers of Mathematics, 2000; Waters, 2004). More formal, complex operations such as addition, subtraction, multiplication and use of algebraic variables also benefit from a solid foundation in spatial reasoning (Kieran, 2004; van Nes & de Lange, 2007). Research has shown that children with serious mathematical difficulties tend to use minimal levels of structure if at all (Mulligan, Mitchelmore & Prescott, 2005). Therefore, it seems clear that improving spatial and spatial-temporal reasoning is important for later mathematical development.

As shown and discussed in multiple ways, music education may provide a method for assisting growth in that area. This suggests possibilities for a comprehensive education that includes an inter-disciplinary approach for the enhancement of spatial, structural and comprehensive mathematical skills. It would not be appropriate to simply substitute music training for spatial and structural awareness guidance in the classroom, but it appears to be potentially helpful as a supplement.

2.11 Fostering Mathematical Thinking in Children

When referring to mathematical thinking, Haylock & Cockburn (2003, pp. 296-300) note how important it is for teachers to be aware of its nature. They emphasise helping children learn the "reasoning and language associated with making generalizations" (p. 296-297). Questions such as "What is the pattern you see here?" and "Does that work for all of them?" (p. 299) foster this type of reasoning. Mathematical thinking can be encouraged when teaching music as well. Questions such as, "Do you see a pattern here (melodic or rhythmic)? After going up, do the pitches come down every phrase? If so, why do you think they always come down eventually? Do you see a sequence in this pattern (melodic or rhythmic)? Does the first note in each of these three phrases start higher or lower but keep the same pattern? Or, does every measure (bar) that you just composed total the equivalent of four quarter notes (crotchets)? Do you notice which part of the measure is usually louder than the other parts (the downbeat)? Does that always happen?

The mathematical thinking that is encouraged when teaching about harmonic movement in music analysis and composition, requires syntactic knowledge of the elegant patterns inherent within the circle of fifths found in music. The need to know the rules and processes of numerical patterns used in harmonic movement and modulation parallel the need to know processes found in mathematics as well. Rowland, Turner, Thwaites and Huckstep (2009, p. 21) define syntactic knowledge in mathematics as that of knowing how mathematical processes work.

In the context of exploring pedagogical skills teaching syntax in mathematics, Rowland, et al. (2009, p. 32-33) give an example of how a teacher might respond to a child's misunderstanding of the proper process for a fractional operation. This child was asked to give a fraction between ½ and ¾. Though her answer was correct, her process was not (p. 32). The rhythm matrix comes to mind as a potential aid to help children understand more complex fractional operations in addition to basic ones, the latter which will be discussed in Chapter 5. The rhythm matrix is a set of boxes indicating on which beat the rhythms should sound) used in the intervention and is described more fully in Chapter 5. To reinforce understanding of this fractional operation in an experiential way, once students discover how to transform the denominators of the fractions in the problem so that they are equal through exploring equivalencies, the denominator of 8 should be reached, so three matrices of 8 boxes each could be put on the board. The one with 4 counts is on the left, the one in the middle has empty boxes and the one with 6 counts on the right. Then students could see how many belong in the middle matrix (5 of 8 boxes). The reward as well as reinforcement is given when they actually play the rhythms. Three smaller groups could be formed, with each playing the rhythms in one of the designated boxes. There is no need to have drums, a pair of hands and a desk top will do. The embodiment (Hejný. & Kuřina, 2009) of concepts can be a powerful pedagogical tool.

2.12 Music Education in the Schools

Extensive evidence, including the meta-analyses by Hetland discussed in this chapter, confirms the assumption that students improve more in both near and far transfer domains through individual lessons and when learning standard notation, yet improvement under all conditions are large enough to encourage music education regardless of the level of privacy or reading skill taught (Hetland, 2000b). Sloboda (2005) emphasises that there are now a number of studies (Costa-Giomi, 1999; Schellenberg, 2006) which show clear effects of traditional instrumental instruction on intelligence and that this research is being undertaken against a backdrop of cutbacks in school music provision in a number of countries. Some of this research appears to be a part of a wider "advocacy" movement (Hallam, 2002) to save music education from further cuts in public spending and to argue for a restoration, or even an increase, in the availability of music education within populations (Sloboda, 2005).

Certainly, this advocacy appears to be justified, and is a motivator for this study, which will perhaps encourage the enhancement of both music education and mathematics education using the proposed interdisciplinary approach.

Schellenberg (2001) has argued that robust evidence in support of the “Mozart Effect” can only be assured if such faciliatory effects on performance can be reliably replicated using within-participants experimental manipulations, therefore further justifying this research.

2.13 Summary and Implications for Further Research

In this chapter, a case has been presented that supports the link between music and mathematics using historical and logical argument as well as empirical evidence while underscoring fundamental perceptual and physical features of music and mathematics as well. Phenomena in the sciences of physics and mathematics ranging from ancient to contemporary thought and research reveal connections between the two. Research and experience in the fields of cognitive psychology, music education and mathematics education also highlight and support the connection.

The idea that spatial sense may be a potential cognitive bridge between musical and mathematical domains is featured in this chapter. Yet even the studies that support the hypothesis that music exposure enhances spatial-temporal reasoning still do not adequately answer the question as to *why* this effect may occur, though they provide enough evidence to encourage pursuit of this question. Additionally, most of the extant research supporting my hypothesis is short-term and laboratory based. What is lacking is research that shows whether or not there is transfer between modalities during the course of everyday life. Hence, it is valuable to know what will happen in a long-term, real-world study such as this.

Though this study provides behavioural data rather than neuroscientific data due to practical considerations such as cost and technical feasibility, as well as the recognition of the value of and need for substantial behavioural evidence to support neural theory, the theoretical foundation nonetheless is based on how learning as well as transfer between cognitive domains occurs at the neural level. Therefore, the following chapter explores the neuroscientific literature that is most relevant to the core of this thesis’ primary research question: *How far, if at all, can school-based musical learning primarily via the voice improve mathematical achievement?*

Chapter 3
Neuroscience, Neuroeducation & Interdisciplinary Education: Transfer Across Musical & Mathematical Cognitive Domains

What happens in the brain while listening to or creating music? What happens in the brain while processing spatial structures and numerical information or while solving mathematical problems? Are there overlapping processes for musical and numerical domains and are certain areas of the brain accessed for both?

The questions inspired this research. This chapter reflects the initial attempt to answer these enquiries. It begins with explanations of inter-related theories that serve as the theoretical foundation for this thesis: *neural plasticity*, *functional connectivity* and *distributive processing* and how they relate to the hypothesis and design of the study.

Next, the emerging field of *neuroeducation* is discussed, with concise definitions of the supporting theories of *information processing theory* and *connectionism* included.

In introducing several EEG studies that use Mozart's music (Hughes, 2001, 2002; Ho, Mason & Spence, 2007), an analysis of elements within Mozart's music follows, in an effort to pinpoint potential reasons for physiological and cognitive effects that often appear in both behavioural and neuroscientific studies that use his music. Embedded within the descriptions of studies is an argument for the inclusion in cognitive research of additional music that contains similar components found in Mozart's music (Sanders, 2012) as well as the mention of an evolutionary precedent for the symmetry- and pattern-seeking tendencies that respond so readily to the designated music of such extensive investigation.

Additional neuroscientific studies of relevance that explore musical processing, followed by research showing structural brain changes succeeding consistent musical engagement over time are then discussed. Then neuroscientific studies examining mathematical processing and others illustrating locational and functional resources of the brain that are shared by both discussed cognitive domains follow.

The last section of the chapter includes a discussion regarding interdisciplinary education and music education with a focus on transfer effects, followed by justifications for using the voice as the primary instrument in the intervention for this study as well as evolutionary explanations for the proclivity of our species towards musical and vocal expression. Finally, the need for this doctoral research is highlighted through numerous points and a return to succinct yet pertinent philosophical musings close the chapter.

3.1 Theories of Neural Plasticity, Functional Connectivity & Distributive Processing

The theoretical foundation for this research is located within neuroscientific theory as expounded below. Neuroscientific studies in the fields of music and mathematics education have increased substantially over the last 20 years. Yet, as explained in Chapter 1, real world behavioural evidence to justify the costs and logistics of collecting a large sample of neuroscientific data via measurement tools such as EEG or fMRI (both measures to be defined later in this chapter) had not been provided. Hence, this thesis offers behavioural evidence to support investment in a future neuroscientific study.

Primarily based upon theories of *experience-dependent synaptogenesis* (Greenough, Black & Wallace, 1987), *brain plasticity* (Foscarin, Rossi & Carulli, 2011; Leuner & Gould, 2010; Münte, Altenmüller, & Jäncke, 2002; Pascual-Leone, Amedi, Fregni & Merabet, 2005; Wan & Schlaug, 2010), *functional connectivity* (Fingelkurtsa, Fingelkurtsa & Kähkönenb, 2005; Guye, Bartolomei & Ranjeva, 2008; Jenkins, 2001; Park, Park & Polk, 2013) and *distributive processing* (McIntosh, 2000), this study examines the potential for music education to enhance children's mathematical understanding and hence ultimately their achievement through repeated activation across the brain for related cognitive challenges shared by these two domains – music and mathematics.

The theory of *experience-dependent synaptogenesis* extends the concept of the creation of new synaptic connections or synapses (connections between neurons, or more specifically, neuronal junctions where electrical impulses pass between neurons) beyond what Greenough, et al. (1987) call "experience-expectant information storage" (p. 1) in which development and learning take place from naturally-occurring perceptual responses to environmental stimuli to experience- dependent information storage, in which development and learning occur due to specific stimuli unique to the situation and individual (Greenough, et al., 1987, p. 1). In the case of this doctoral research, the musical experiences of the learners are examples of experience-dependent synaptogenesis, or learning.

The theory of *brain plasticity* postulates that the ability and natural tendency of the human brain to adapt to experiences and changes in the environment is an inherent result of evolution available throughout the lifespan (Pascual-Leone, et al., 2005). This theory further asserts that neural growth and structural change occur due to learning experience (Leuner & Gould, 2010; Foscarin, et al., 2011). Therefore, it predicts that music learning can yield profound changes in the brain.

Functional connectivity is the organisation of activity among different neuronal groups in order to realise a complex cognitive task or perceptual process (Fingelkurtsa, et al., 2005). The related theory of *functional distribution* (also referred as *distributive processing)* states that brain regions are structurally interconnected, and process information in a distributed way (McIntosh, 2000).

Functional distribution is an extension of the concept of functional connectivity and is based upon the general hypothesis that learning and memory are emergent properties of large-scale neural network interactions. A key point in this theory is that a region can play different roles across many functions and that each role is governed by its interactions with anatomically related regions (McIntosh, 2000).

When a brain region is activated and developed to serve a particular cognitive domain, this development can also affect other related cognitive domains, such as musical activity and development affecting mathematical activity and development, due to the interconnected nature of brain activation and functionality. Since networks within different regions can function to serve varying contexts through recruitment of neural networks across the brain, the functional distribution theory supports the idea of *cognitive transfer*, in which knowledge or skills developed through experience in one cognitive domain can be used or transferred to another domain. In other words, synaptic connections and structures may be deployed for a different purpose than that which initially led to their construction.

Functional connectivity can therefore be applied to the thesis that musical learning can enhance mathematical thinking and ultimately therefore achievement, further highlighted by the similarities of cognitive skills potentially developed through both musical and mathematical experiences, such as *pattern, sequential, structural* and *proportional recognition*, as well as *creativity* and *problem solving* due to shared neural network interactions.

Structural proximity of and overlapping brain regions. Returning to the theories of functional connectivity (Fingelkurtsa, et al., 2005) or functional distribution (McIntosh, 2000), structural proximity also can help explain the increase of transfer effects across different cognitive domains that this study and others have shown. Therefore, these theories support the hypothesis put forth in this thesis by signifying that repeated activation of cognitive functions, particularly those occurring in the same or nearby brain regions, can facilitate transfer from one cognitive domain to another. To repeat from Chapter 1, the main hypothesis is that: *Music education via the voice as the primary instrument can enhance mathematical achievement*.

Therefore, for example, it is likely that when multi-modal integration areas in the frontal and parietal regions such as those surrounding the *intraparietal sulcus (IPS)* are repeatedly accessed, cross-modal influence on cognitive and behavioural processes employ neural networks used in both music and mathematics processing.

Indeed, numerous studies in the literature support this proposal by showing that intensive music training can result in intraparietal sulcus (or IPS, the cerebral groove located on the lateral surface of the parietal lobe) modulations (Schlaug, 2001; Lee, Chen & Schlaug 2003; Koelsch, Fritz, Schulze, Alsop & Schlaug, 2005; Schlaug, 2009; Wan & Schlaug, 2011, 2012). This area also is associated with numerical representation and operations (Cohen, Cohen, Kaas, Henik & Goebel, 2007; Dehaene, Dehaene-Lambertz & Cohen, 1998; Piazza, Pinel, Le Bihan & Dehaene, 2007; Pinel, Piazza, Le Bihan & Dehaene, 2004). These shared regions utilise the same neural resources, particularly for the understanding and mental manipulation of symbolic representations, whether musical or mathematical. Therefore, certain sensorimotor and cognitive enhancements beyond music that have been associated with the acquisition of music learning indeed may be due at least in part to functional connectivity (or functional distribution) and plasticity of related neural networks (Fingelkurtsa, Fingelkurtsa & Kähkönenb, 2005; McIntosh, 2000; Pascual-Leone, Amedi, Fregni & Merabet, 2005; Wan & Schlaug, 2010, 2012).

3.1.1 Parietal Cortex, Inter-hemispheric Transmission and Mathematical Performance

Since the corpus callosum aids inter-hemispheric information transfer, the work to be further described later in the chapter by Schlaug and colleagues (Schlaug, et al., 1994, 1995, 2008) that revealed a causal link between music training and corpus callosum growth (aiding in the speed of informational transmission between hemispheres) is relevant for this study, as recent research has shown that there is a positive correlation between inter-hemispheric functional connectivity and mathematical performance (Park, Park & Polk, 2013). The parietal cortex is central to numerical cognition, yet it is also engaged in verbal, spatial and attentional functions that may contribute to calculation (Dehaene, Piazza, Pinel, & Cohen, 2003). Since the right parietal region is mainly involved in basic quantity processing and the left parietal region is involved in precise number processing and numerical operations, Park and colleagues' study confirmed the importance of functional connectivity between the right and left parietal cortex for numerical processing that involves both fundamental number representation and learned numerical operations.

Mathematical results also correlated with the degree of functional connectivity across participants, while activity within each brain region did not. Again, this study emphasises the importance of parietal functional connectivity in numerical processing and suggests that arithmetic processing relies upon communication between hemispheres of the parietal cortex and that this communication plays a role in the degree of numerical competence (Park, Park & Polk, 2013).

Even though the full experience of music is due to activity in widely distributed areas, relevant neural networks are nonetheless dedicated to specific aspects of musical processing. Perception, production and analysis of detailed patterns and structure of rhythm and pitch relationships happen mainly in the left hemisphere, while timbre, melodic contour (Warren, 1999, p. 571) and metrical extraction (Levitin, 2006, p. 169) are processed primarily in the right (Peretz

& Zatorre, 2005). Therefore, the understanding of specific musical patterns and structures could be analogous to the understanding and processing of precise numbers and numerical operations, all of which are processed mainly in the left hemisphere. The processing of the general melodic contour and extracting of the fundamental metrical substrate could be analogous to the mathematical processing of basic quantity (Dehaene, Bossini & Giraux, 1993; Park, Park & Polk, 2013), which are both processed in the right hemisphere.

Bringing this discussion into the real-world classroom, it can be noted that indeed there is a growing recognition among prominent neuroscientists that "classroom interventions can alter neural networks related to cognition in ways that generalize beyond the specific domain of instruction" (Posner & Rothbart, 2005, p. 1). This recognition clearly underlies this thesis's hypotheses.

The theories brought forth here underpin the methodology of this research, which reflects confidence in the importance and power of experiences, of *doing* to ensure lasting learning. Therefore, experiential learning was key to both the design and implementation of the fieldwork intervention. It is believed that through the active learning that music-making entails, cognitive changes can and do occur.

3.1.2 Neuroeducation

Since the brain is the primary organ of learning (Goswami, 2008b), it seems appropriate that concepts from both behavioural and neuroscientific paradigms might be considered in any educational study. There is a call for the fields of neuroscience and education to collaborate successfully, and though it would require both teacher and researcher training, it would be a powerful combination with the potential to help learners profoundly (Ansari & Coch, 2006; Devonshire & Dommett, 2010).

Therefore, neuroeducation is a new field that deserves consideration. Understanding cognitive processes on the neural level has the potential to inform educators regarding the learning process. In addition to the theories described earlier in this chapter (neural plasticity, functional connectivity and distributive processing), other learning theories such as information processing theory and connectionism support concepts inherent within neuroeducation as well.

Classic information processing theory simulates human cognitive skills, including both declarative knowledge as well as procedural knowledge. Similarly, connectionism attempts to model human brain function via computer models, creating artificial neural networks (ANNs) of particular domains (Byrnes, 2009).

Though both theories differ slightly in approach and focus, with information processing theory stressing the importance of goal-directed learning while connectionism the gradual and conventional nature of knowledge change, both emphasise the function of practice and repetition in learning. Practice strengthens associative bonds. Therefore, in any study seeking to monitor the effects of a particular domain within education, consistent lessons in that domain are essential. Therefore, the fieldwork in this study included consistent weekly music lessons for nine months in each of the five participating schools. Details regarding the intervention are given in Chapter 5.

3.2 Explorations in Neuroscience

The exploration of perception, thought processes and learning is expanding as new tools have become available. Though neuroscience does not currently present definitive answers to the mysteries of the mind, its potential to shed light on certain questions about cognition is promising even in its infancy.

3.2.1 Music and the Brain with Potential Links to Mathematics

Following are examples of neuroscientific studies illustrating their contribution to the understanding of musical processing in the brain. This section starts with a seminal study by Janata regarding music cognition, which has influenced studies involving other cognitive domains as well. Next, an EEG study by Rauscher and colleagues will be presented. Additional neuroscientific studies regarding music and the brain close this section. Note that these are only representative examples of relevant studies in the field.

Measuring Perception and Expectations

In 1995, Janata tested cognitive responses to different harmonic sequences that mark the closing of a musical phrase, section or piece, known as cadences. College students (N=23) heard three possible resolutions: best possible (suggesting the tonic or “home” destination of a major key), harmonically plausible (suggesting the tonic destination, yet in a minor mode), or harmonically implausible (not fitting within the harmonic structure of the piece). ERP waveform components revealed both the speed and level of understanding of the subjects, therefore showing also that this measurement technique can investigate the perception and processing of expectations related to the probability of musical events and contexts (p. 153). This seminal experiment has influenced multiple studies seeking to understand musical cognitive processing (Patel, Gibson, Ratner, Besson & Holcomb, 1998; Granot & Donchin, 2002; Trainor, McDonald & Alain, 2002; Fujioka, Trainor, Ross, Kakigi & Pantev, 2004, 2005; Koelsch, Jentschke, Sammler & Mietchen, 2007; Carrion & Bly, 2008). This study has also influenced research involving cognitive processing in other domains such as language processing by noting probability structures in response to various stimuli and illustrating the overlapping nature of cognitive processes in multiple domains (Tueting, Sutton & Zubin, 1971; Squires, Donchin, Herning & McCarthy, 1977; Thatcher, Krause & Hrybyk, 1986; Petsche, 1995).

3.2.2 Neuroscientific Studies Investigating the Mozart Effect

In response to the discourse on the seminal study Rauscher and colleagues conducted, which demonstrated that listening to a ten-minute passage from a Mozart sonata could temporarily improve spatial reasoning (Rauscher, Shaw & Ky, 1993), EEG studies have been implemented in an attempt to reveal on a neural level what cognitive processes Mozart's sonata K448 (of "Mozart Effect" fame) may trigger.

For example, a study by Sarnthein, von Stein, Rappelsberger, Petsch, Rauscher & Shaw (1997) suggested a positive effect on right frontal and left temporoparietal coherence activity after listening to this music. In nearly 50 percent of the subjects, it carried over onto spatial-temporal reasoning tasks. The reported location of activity would have been an estimation, however, due to the low spatial resonance in EEG assessments, particularly at the time of the study.

Another EEG study showed that after listening to Mozart's music for ten minutes, enhanced synchrony of the firing pattern of the right frontal and left temporoparietal areas of the brain persisted for twelve minutes (Rideout & Laubach, 1996). Though these regions have been identified for musical processing via later fMRI studies (Peretz & Zatorre, 2005) the specific localisation identification in these earlier EEG studies is questionable, again due to the spatial limitations in EEG. As uncertain as the claims here for localisation are, it is worth noting that the temporoparietal region of the brain is specified in these studies. In addition to the study by Dehaene and colleagues that was cited at the beginning of this chapter (Dehaene, Piazza, Pinel & Cohen, 2003), additional studies previously indicated that certain aspects of mathematical processing are represented in the related areas of occipito-temporal and intraparietal cortex (Dehaene & Cohen, 1995; Dehaene, Dehaene-Lambertz & Cohen, 1998).

3.2.2.1 Melodic and Form Elements in Mozart's Music

Aspects of Mozart's music that may contribute to spatial-temporal enhancement, pattern recognition and/or proportional understanding after listening and even more so, after learning and performing, include a combination of *extended repetition of patterns* and *logical, intricate structures*. Cognitive mechanisms needed to process patterns and structures existing in the auditory musical mode also may be utilised to recognise and manipulate spatial-temporal structures and patterns in other modes in which the visual sense is key as well as in the mental-procedural mode used to rotate objects in the mind. Mozart's melodic patterns have been studied closely in the Department of Neurology at the University of Illinois Medical Center in Chicago, concerning the stabilising effect that the music has on epileptic patients. Hughes (2001, 2002) theorised that the high incidence of repeated, though varied melodic motifs in Mozart's music might account for the significant decrease in epileptic episodes for patients studied as well as the enhanced spatial reasoning skills exhibited in previous studies due to the regularity of patterns as well as a certain amount of challenge. This is so subtle as to stay within what Vygotsky referred to as the Zone of Proximal Development, in which one can be challenged just beyond one's current capability so that growth is stimulated and achieved within one's reach (Vygotsky, 1930-1934/1978). Hughes refers to the highly-organised music of Mozart paralleling the highly-organised structure of the brain. He notes that repetition and periodic changes occur in the functioning of our brains and bodies as a whole (p. 20) and are mirrored in the repetition and periodic changes of Mozart's music. Indeed, this phenomenon within Mozart's music as well as other music that has similar organisational traits and levels of complexity, whether Western classical or jazz or Eastern classical music of India for example or complex rhythmic or harmonic works of Africa, may also account for the enhanced mathematical pattern, structural and proportional recognition skills that appear to develop further following musical training (Sanders, 2012, p. 3).

Cross-species evidence supports an evolutionary precedent for this pattern-seeking cognitive phenomenon. The highly-structured cortex appears to search for patterns and symmetry in the environment. Therefore, musical structure is a natural attraction for many if not most mammalian neocortices as well as avian dorsal ventricular ridge (DVR) nuclei, the avian equivalent of the mammalian neocortex (Dugas-Ford, Rowell & Ragsdale, 2012; Shanahan, Bingman, Shimizu, Wild & Güntürkün, 2013.)

Below in Figure 5 is a graph resulting from Hughes' research showing the frequency of specified musical properties in the music of seven composers. Mozart's music shows significantly higher values for the repetition of the majority of each of the properties as well as the total of the properties compared with the other composers, designating the high periodicity of his music.

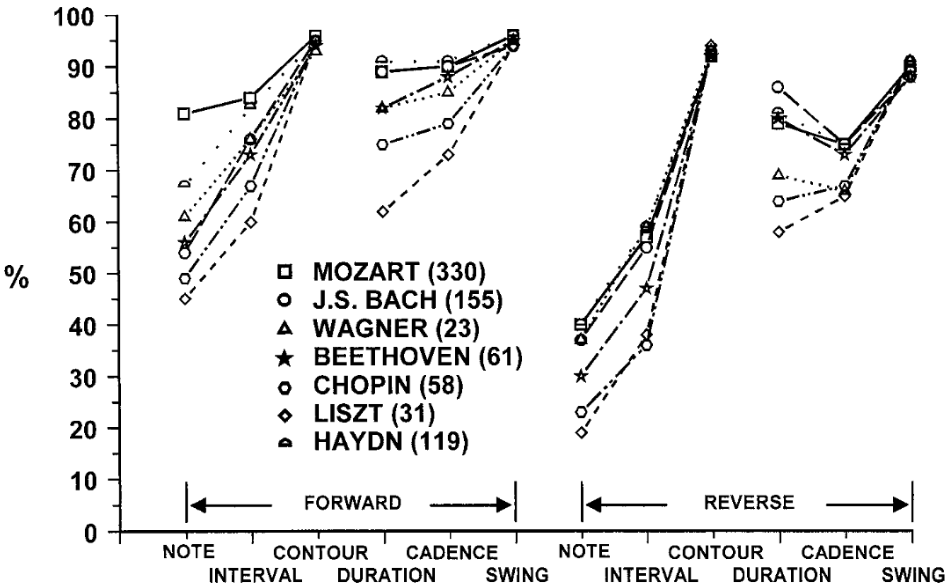


Figure 5 Above are characteristics of a melodic line: sequence length of four notes.
The figure and notes are from "The Mozart Effect, additional data" by Hughes, J.R., 2002, in *Epilepsy & Behavior 3(*2), p. 18.

On the horizontal axis of Figure 5 above are specified (1) note, (2) interval, (3) contour, (4) duration, (5) cadence and (6) swing, followed by the reversal of (1) - (6). On the vertical axis is the percentage of occurrences for each indicated characteristic.

As shown, Mozart's music shows significantly higher values for the repetition of the majority of each of the properties as well as the total of the properties compared with the other composers, designating the high periodicity of his music.

3.2.2.1 Temporal Elements in Mozart's Music

Ho, Mason and Spence (2007) examined whether Mozart's music would influence subjects' temporal attention using a visual attentional blink (AB) task. EEG technology, as mentioned, provides high temporal resolution, therefore this task can provide a reliable measure of the temporal dynamics of visual attention, which can influence performance in spatial-temporal tasks.

Researchers in this study used Mozart's Sonata for Two Pianos in D Major, K.448, as have other studies noted here. This is the same sonata as in the original Rauscher, et al. study.

In this attentional blink task, participants attempted to identify two target digits in the correct order of presentation among a stream of distractor letters in three different conditions: 1) while listening to the Mozart sonata played normally, 2) while listening to the same Mozart sonata played in reverse, and 3) while in silence. The results showed that the participants detected the second target significantly more accurately when the Mozart sonata was played normally than in either of the other two conditions. Ho and colleagues demonstrate the potential effectiveness of temporal components in Mozart's music that can aid attentional resources. I suggest that other music could also have the same effect, given that similar elements exist that could be relevant to the cognitive task being studied.

It therefore appears that pattern and structural recognition in addition to attentional levels may be improved through the temporal and other structural components inherent in much of Mozart's music. These abilities also appear in subjects in the in the previously mentioned study, which also utilised Movement I of the much-investigated sonata. Organisational abilities and attentional improvements can be nourished via temporal elements such as a cleanly-organised, rhythmic structure with a symmetrical 4/4 metre and consistent rhythmic patterns, even with subtle variations in note durations as well as changes in dynamics and texture, which often give the illusion of a fundamental rhythmic change. Again, Mozart’s temporal and melodic variations as well as harmonic modulations are usually quite subtle, and therefore can perhaps cognitively lead the listener or musician more effectively as these small changes can function as scaffolds towards each subsequent step.

Alternating metrical emphases of accents including syncopations (accenting unexpected beats) and rubato (expressiveness through slight acceleration or de-acceleration) on occasion add to the complexity within the continuous pulse. The masterful subtlety of Mozart's compositions can seem deceptively simple, though they are complex. The logic and minuteness of the changes enable scaffolding yet can create the illusion of sameness. It should be noted that performances of this sonata can vary, therefore affecting all the properties mentioned above including tempo, though professional recordings normally adhere to the main written components with only slight variations due to differences in interpretation and expressivity.

As previously noted, listening to other forms of music such as jazz or non-western music could have similar outcomes to classical music of a complex yet repetitive nature if the same pieces are similarly complex yet with high periodicity (Sanders, 2012, p. 3). For example, "the interlocking rhythms and/or melodies of Indonesian, Indian, Arabic, Latin or African music are frequently comprised of multiple layers that can be highly intricate" though often repetitive (p. 3).

3.2.3 Additional Neuroscientific Studies Exploring Musical Processing

In another EEG experiment regarding musical processing in the brain, Petshce and colleagues ostensibly showed that imagining music as well as composing differ distinctly from listening to music because these mental activities initiate many more coherence increases in the beta band*.

Additionally, a larger percentage of hemispheric interaction was indicated (Petshce, Richter, von Stein & Etlinger, 1993), further supporting the thesis that active musical processing can support cohesive interhemispheric cognitive processing, which ultimately can create the neural environment to aid mathematical processing. Therefore, as musical processing activates hemispheric interaction (Petshce, et al., 1993), and as stated earlier, repeated musical experiences can thicken the corpus callosum (Schlaug, et al., 1994; 1995; 2008), which aids inter-hemispheric information transfer, musical experiences thereby can impact mathematical processing (Park, Park & Polk, 2013) as noted earlier in this chapter. (* Beta activity refers to a frequency band in the brain (neural oscillation or brainwave with a frequency range between 12.5 and 30 Hz) commonly present in healthy people and associated with normal waking consciousness.)

A final EEG study regarding music and its potential effect on the brain is described here. Musicians in this research showed greater increases in gamma band phase synchronization between the frontal cortex and the right parietal cortex during mental rotation tasks than non-musicians (Bhattacharya, Petsche, Feldmann & Rescher, 2001). Note that mental rotation is a form of spatial- temporal reasoning, therefore this study further supports the link between musical training and spatial- temporal reasoning.

In 2006, a magnetoencephalography (MEG) study by Fujioka and colleagues showed that after one year of musical training, auditory cortical-evoked fields in young children displayed "pronounced morphological change." The researchers noted that music learning had potentially established a neural network associated with sound categorisation as well as involuntary attention (Fujioka, Ross, Kakigi, Pantev & Trainor, 2006, p. 2593).

Though musical processing appears to be directly linked to cognitive improvements such as the development of spatial-temporal reasoning and pattern recognition, it has also been suggested that training in music may improve attention networks (Posner, 2008). Enhanced attention networks have been shown to improve general cognition, which in turn can lead indirectly to better attainment of mathematical skills.

3.2.2.2 Structural Brain Changes Following Consistent Musical Engagement

As noted at the beginning of the chapter, Schlaug and various collaborators in many studies since 1994 have found that musical training can modify general cognition related to mathematical thinking because of the alteration of brain structure as well as the function (Schlaug, et al., 1994, 1995, 2001, 2003, 2008). The individual publications are listed in the bibliography. It is worth revisiting the work of Schlaug and colleagues more extensively, whose numerous investigations look closely at the potential cognitive changes due to musical processing as well as the long-term effects of musical training on brain structure and functionality.

Schlaug and colleagues (Overy, et al., 2004) have performed several in-vivo magnetic resonance morphometries comparing musicians with nonmusicians. Their work showed evidence of larger midsagittal corpus callosum size and degree of myelinisation in musicians, which affects the efficacy of interhemispheric communication (1994). They also found structural brain asymmetry in musicians (1995), leftward asymmetry of the planum temporale in absolute-pitch musicians (2001), grey matter differences between musicians and nonmusicians (2003, p. 514), larger cerebellar volume differences in musicians (2003) and other microstructural differences in motor, auditory and visuospatial brain regions (2003). The researchers point out that early, consistent and intense musical training can account for these differences since there is more structural plasticity in the brain during the preschool and primary years. According to Schlaug, in some structures, functional plasticity is only possible during a critical period of brain development (2001, p. 281) though recent research has found that brain plasticity operates more flexibly later in life than earlier thought. Nonetheless, in general, Schlaug and colleagues found that the earlier the musicians had started and the more intensely they had practised, the more extreme were the brain differences.

In the study that follows, Schlaug and colleagues used “child-friendly” fMRI employing a scanning protocol that uses a sparse temporal sampling technique with clustered volume acquisition in order to test whether children show the same hemispheric sensitivities for melody (right) and rhythm (left) as adults (Overy, et al., 2004, p. 1723). As part of a larger longitudinal study examining the potential effects of music training on cognitive and neural development, children with a mean age of six (N = 33) performed musical discrimination tasks, and images during these tasks were compared to images during silence (baseline). The young children, unlike adults in similar studies, showed similar activation patterns for melody and rhythm, suggesting that hemispheric specialisation for processing the two musical components may develop with age (Overy, et al., 2004). This recognition of delayed cognitive specialisation could have implications for the examination of abilities in other domains as well.

In 2006, Ozdemir, Norton and Schlaug conducted another study to determine whether distinct neural correlates exist for speaking versus musical vocalisation as had been assumed for years. Ozdemir and colleagues compared different conditions of vocalisation, including singing, speaking, humming and production of vowels. The results suggest that a bihemispheric network for vocal production exists regardless of whether the words or phrases were intoned or spoken, yet singing more than speaking showed additional activation in right hemispheric areas. This suggests certain functional hemispheric differences, and may elucidate why patients with non-fluent aphasia due to left hemisphere lesions are able to sing the text of a song even though they are unable to speak the same words (p. 628). These types of illuminations have therapeutic as well as educational applications and lend support for the encouragement of singing. Indeed, as with Janata's EEG work and much of the behavioural work to be discussed in the next chapter, fMRI studies using music may advance knowledge that can be applied in areas beyond music (Zatorre, 2003; Gaab & Schlaug, 2006).

Fundamental to this research is the aforementioned longitudinal study (Schlaug, et al., 2004; Schlaug, 2009) launched in an attempt to answer questions of causality. Schlaug and colleagues have enquired whether there might be pre-existing neural, cognitive, or motoric markers for musical ability. Based on preliminary results, Schlaug suggested that musical training, especially at a young age, could alter these markers. After the three-year mark, Schlaug reported that indeed there were distinct changes in the brains of the musically trained children that did not appear in the matched control group. After the five-year mark, Schlaug reiterated "training-induced brain plasticity" that corresponded with behavioural improvement scores though interestingly, these changes only corresponded slightly in behavioural tests initially. It was speculated at the time that either the tests were not sensitive enough or that changes that appear within the brain may yield external results at a later point, perhaps after foundational changes have been established, which indeed appeared to occur later in time.

Based on the theory that there may be coactivation of certain areas of the brain that are associated with both music and mathematics, or areas that at least have shared resources between these different cognitive tasks, researchers often look specifically for changes occurring in the parietal lobe surrounding the intraparietal sulcus (IPS) region. As noted in the beginning of this chapter, the IPS region has been associated with mathematical processing, and the discussion in the section below will elaborate on this. The surrounding region of the IPS is a multisensory area of the brain that integrates information coming from different domains (Schlaug, 2008). Again, previous studies (Schlaug, et al., 2001, 2003, 2005, 2009; Wan & Schlaug, 2012) have shown a correlation between music training and change in this region. Preliminary results of Schlaug and colleague's longitudinal study have shown changes in the brain that had occurred so far *within* the musically trained (matched) group were in the temporal lobe, frontal lobe and cerebellum. No changes were observed in the control group. Within these areas, the inferior frontal gyrus appears to be more fully preserved, while containing more grey matter in musicians versus non-musicians as they age (Schlaug, 2009). In addition to having a relationship with speaking, it is suggested that the inferior frontal gyrus is also related to mapping auditory sounds and is particularly active when one performs a mental rotation task, a subset of spatial-temporal reasoning. The experiment is ongoing and will be carefully tracked and continually analysed along the way. This may offer further explanations that support the hypotheses presented in this thesis.

3.3 Mathematics and the Brain (with Potential Links to Music and the Brain)

Regarding mathematical reasoning processes, recent understandings that were pointed out at the beginning of the chapter regarding related to cognitive development and anatomy in this domain have been gained by using both EEG and fMRI as well as lesion studies and psychological experiments. This section delineates in more detail three numerical cognition systems as theorised by Dehaene and colleagues, followed by examples of investigations via EEG that are seeking to discern the source of cognitive numerical deficiencies and further suggest musical training as a potential aid.

3.3.1 Three Numerical Systems

Through EEG and fMRI brain imaging techniques, examination of brain lesion patients as well as psychological studies with infants and nonhuman animals, Dehaene and colleagues have answered numerous questions regarding mathematical awareness and thought (Dehaene & Cohen, 1995; Dehaene, 1997; Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999; Feigenson, Dehaene & Spelke, 2004; Barth, La Mont, Lipton, Dehaene, Kanwisher & Spelke 2006). In 1995, Dehaene and Cohen put forth a triple-code model and then updated it in 2003 (Dehaene, Piazza, Pinel & Cohen). They suggest that mathematical ability has an evolutionary precursor by presenting evidence of this ability in infants and nonhuman animals. The three "numerical codes" were originally defined as: the *verbal system* (language that is used later in human development for the verbal processing of numbers), the *parietal quantity system* and the *occipito-temporal system*. The latter two involve deep cortical areas in the intraparietal sulcus (IPS) and in the fusiform gyrus and are of particular interest for this thesis (Dehaene, 1997, pp. 1-25; Dehaene & Cohen, 1995).

The more developed version (2002) of the three “numerical codes,” each accompanied by the primary location in the brain associated with these behaviours indicates: 1) verbal processing of numbers, processed in the angular gyrus of the left hemisphere, 2) visuospatial processing in the posterior superior parietal lobule, and 3) numerical quantity representations, processed in the horizontal segment of the intraparietal sulcus (Piazza & Dehaene, 2004, pp. 1-27). See page 77 for an illustration of Dehaene and Cohen’s original mapping of brain regions for specific aspects of mathematical processing (Dehaene, 1997).

Recall the study that found mathematical results to correlate with the degree of functional connectivity across participants (Park, Park & Polk, 2013). As mentioned, this research emphasises the importance of parietal functional connectivity in numerical processing and suggests that arithmetic processing relies upon communication between hemispheres of the parietal cortex, pointing out that this communication plays a role in the degree of mathematical competence.

Therefore, as noted earlier, since the corpus callosum assists inter-hemispheric information transfer, the work by Schlaug and colleagues (Schlaug, et al., 1994, 1995, 2003, 2005, 2007, 2008, 2009) as well as others (Oztürk, Tascioglu, Aktekin, Kurtoglu, & Erden, 2002; Hyde, Lerch, Norton, Forgeard, Winner & Evans, 2009) is highly relevant to this thesis because it indicates a potential causal link between music training, corpus callosum development and mathematical processing.

3.3.2 EEG Studies and Numerical Magnitude Representation

Szücs and colleagues have conducted numerous investigations utilising ERP (event related potential) components regarding numerical processing and arithmetic performance, many in order to understand the differences between people with normal arithmetic ability and those with developmental dyscalculia (DD). Though still insufficiently understood, dyscalculia is defined as a cognitive disorder that affects arithmetic learning ability, even though other abilities may be intact (Soltész & Szücs, 2009). In a reanalysis of an earlier study (Soltész, Szücs, Dékany, Markus & Csépe, 2007) the researchers identified and compared cognitive processes in adolescents with developmental dyscalculia and in matched control subjects in one-digit number comparison tasks.

Though it is theorised that those with dyscalculia may have a deficiency in magnitude representation*, which is shown to exist in the horizontal intraparietal sulci (HIPS) of the brain (Dehaene, Piazza, Pinel & Cohen, 2003), the automatic processing of numerical information occurred with similar speed in both groups of participants. The ERP correlate of the numerical distance effect, which is the most frequently used marker of the magnitude representation (Dehaene, et al. 2003), was shown to be intact in the developmental dyscalculia (DD) subjects during all processing stages. Therefore, Soltész & Szücs (2009) demonstrated that those with DD do not necessarily have a dysfunction of magnitude representation. Other evidence emerging from this study pointed towards possible monitoring function differences in subjects with DD versus the control group. (*Numerical magnitude representation in humans is shared with non-human animals. This is a mental system for representing number and magnitude that is language-independent and emerges early in development.)

Though not seeming to be relevant to the link between music and mathematics, the results of this and other similar studies open the way for further research that may benefit from using music education to investigate attentional, executive and self-monitoring processes as well as specific cognitive or emotional responses to numerical stimuli that may be deficient or sensitive in individuals with developmental dyscalculia (DD) (pp. 473-485).

Indeed, based upon theories and evidence presented in this thesis thus far, it could be argued that, in addition to directly influencing numerical and spatial cognition due to its very structure containing pattern, shape, proportion and magnitude, music may therefore constitute a treatment for certain cognitive deficiencies due to its potential to assist development in executive functioning capabilities, such as working memory, focus, self-control, planning and mental flexibility (Harvard University Center on the Developing Child, 2012).

As understanding of both musical and mathematical processing grows, comparisons can continue to be made between the two domains in order to better define their relationship.

3.4 Shared Brain Regions and Functions for Musical and Mathematical Cognition

Furthering the discussion regarding shared cortical regions for both discussed cognitive domains, there have been recent innovative studies that have looked at musical and arithmetic cognition as well as attainment, with enquiries similar to Research Question Two in this thesis, which asks: What cognitive mechanisms do both musical and mathematical domains share?

Below are several studies that directly explore and expand the question of regions shared by both musical and mathematical domains.

3.4.1 Structural and Temporal Integration Resources Shared for Both Musical and Arithmetic Processing

Hoch and Tillmann (2012) argue that both musical and arithmetic processing share structural and temporal integration resources. They note that, similar to language, both are combinatorial systems that are structurally organised by rules that integrate events into mental representations (Friedrich & Friederici, 2009; Hoch & Tillmann, 2012, p. 231).

The researchers used a cross-modal paradigm to investigate interactive influences between simultaneous musical and arithmetic processing that had been previously used to investigate influences between music and linguistic processing (Hoch, Poulin-Charronnat & Tillmann, 2011). The methodology was based upon work by Dehaene and colleagues, who investigated the possibility of spatially represented numbers in cognition when judging number magnitude by creating a task whereby responses were recorded according to hand speed. It was found that subjects who are asked whether a number is odd or even generally respond faster with the left hand corresponds to smaller numbers and with the right hand to larger numbers (Dehaene, Bossini & Giraux, 1993, p. 394), therefore suggesting the existence of a mental number line. The researchers (Dehaene, et al., 1993) called this the SNARC Effect based upon the original identification of this phenomenon as the Spatial-Numerical Association of Response Codes.

Santens and Gevers later challenged the rigidity of the left-right cognitive number line paradigm by conducting a similar test using close-far responses whereby smaller numbers corresponded with close orientation and larger numbers with far. This suggests an additional spatial conceptualisation dimension for magnitude (Santens & Gevers, 2008), but does not rule out the tendency for the cognition of a small-large trajectory to correspond with the left-right spatial orientation.

Hoch & Tillmann (2012) combined this with an analogous paradigm developed by Beecham and colleagues (Beecham, Reeve & Wilson, 2009) identified as the Spatial-Musical Association of Response Codes task (SMARC) to explore emerging evidence that numerous abstract concepts are represented cognitively in a spatial layout (Beecham, et al, 2009). Their study investigated the hypothesis regarding shared resources between music and arithmetic structure processing. They hypothesised that if music and arithmetic processing share structural integration resources, as suggested by previous research in each domain, then interactive influences between the simultaneous music and arithmetic structure processing would be found, as was indeed the case.

These interactions were therefore interpreted as reflecting shared structural integration resources. Similarly, Hochs and Tillmann (2012) speculated that the interactive pattern between music and arithmetic processing observed may reveal shared structural and temporal integration resources. These are required for integration of incoming (musical or arithmetic) events based upon preceding events into coherent (musical or arithmetic) structured representations (Hoch & Tillman, p. 234). These findings further support the hypothesis brought forth in this thesis regarding links among musical, spatial and mathematical cognition.

3.4.1 Functional Magnetic Resonance Imaging (fMRI) Studies

Functional magnetic resonance imaging (fMRI) studies investigating the effect of musical training on musical processing and on the neural correlates of mathematics processing are presented in this section.

Schmithorst and Holland (2003) were motivated by previous studies that had exhibited changes in neuronal activity in trained musicians relative to controls while performing assorted music processing behaviours. In one study, these researchers investigated neural correlates of the effect of music training on processing two aspects of music, melody and harmony, via functional magnetic resonance imaging (fMRI). Approximately half of the subjects (n = 7) had experienced continuous musical training from early childhood to adulthood and the other half (n = 8) had not had musical training. The most anterior part of the superior temporal gyrus was activated during melodic processing for both musicians and non-musicians, while different visual association areas were activated for musicians versus non-musicians. Only in musicians were the inferior parietal lobules recruited for both tasks. Therefore, the authors concluded that musical training activates different neural networks for the observed features of music processing.

It is notable that melodic and harmonic processing recruited the parietal lobules (bordered above by the intraparietal sulcus) only in musicians, a region which is also recruited in mathematical processing, particularly for numerical quantity representations (Piazza & Dehaene, 2004; Ansari, Garcia, Lucas, Hamon & Dhital, 2005) and also for visuospatial working memory (Garavan, Kelley, Rosen, Rao & Stein, 2000; Gillebert, Dyrholm,Vangkilde, Kyllingsbæk, Peeters & Vandenberghe, 2012).

Mindful musical experiences over time can improve recognition of numerical and proportional structures inherent within melodies and harmonies. Therefore, the activation of this region only in musicians suggests that numerical properties are indeed recognised in the musical tasks.

Schmithorst and Holland conducted another fMRI study the following year (2004) investigating the neural correlates of the hypothesised link between "formal musical training and mathematics performance" (p. 193). Similar to the earlier experiment, approximately half (n = 7) of the normal adults had experienced musical training since early childhood and the other half (n = 8) had not. FMRI was conducted while the subjects mentally added and subtracted fractions.

Subjects with musical training had increased activation in the left fusiform gyrus and prefrontal cortex yet had decreased activation in visual association areas and the left inferior parietal obule during the mathematical task. Therefore, the researchers hypothesised that the correlation between musical training and math proficiency may be related to enhanced working memory functioning and an improved abstract representation of numerical quantities (Schmithorst & Holland, 2004). See Figure 6 below.

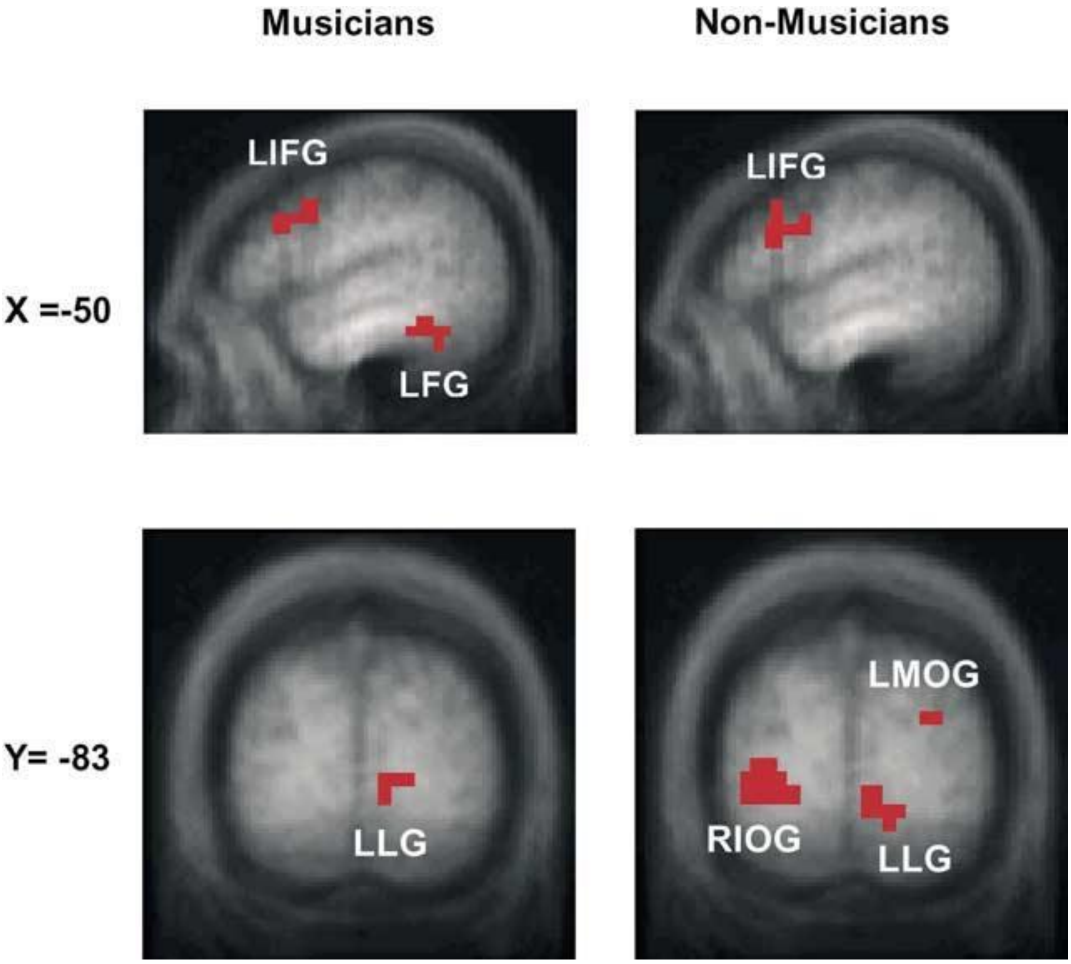


Figure 6 Schmithorst and Holland (2004). fMRI study investigating the neural correlates of the link between formal musical training and mathematics performance.

Figure 6 above shows composite activation maps for seven musicians (left) and eight non- musicians (right) performing the mental addition and subtraction of fractions. Slices selected for display are sagittal at X 1⁄4 2 50 and coronal at Y 1⁄4 283 (Talairach coordinates). Activated pixels have P, 0:001 (corrected). (Labels: LIFG 1⁄4 left inferior frontal gyrus; LFG 1⁄4 left fusi-form gyrus; LLG 1⁄4 left lingual gyrus; LMOG 1⁄4 left medial occipital gyrus; RIOG 1⁄4 right inferior occipital gyrus). *Note.* This figure is from "The effect of musical training on the neural correlates of math processing: A functional magnetic resonance imaging study in humans" by V.J. Schmithorst & S.K. Holland, 2004, *Neuroscience Letters*, 354**,** p. 194. Notes are from the same source.

The studies described in this literature review thus far show only a fraction of the growing body of research on music and neuroscience. Specific studies relevant to the research questions in this thesis were chosen to demonstrate points made and to further the argument regarding the link between musical and mathematical processing. To close the neuroscientific-focused section of the literature review, illustrations are included below that show general brain regions used for musical processing (Levitin, 2006, pp. 263-265), as well as brain regions used for mathematical processing (Dehaene & Cohen, 1995).

Levitin (2006) emphasises that music processing is distributed throughout the brain (p. 263). The first two illustrations are by Tramo and updated by Levitin (Tramo, 2001; Levitin, 2006) and indicate the brain's primary musical processing centres from two views.

Following these illustrations is a general brain map of musical perception in opera singers (Kleber, Veit, Birbaumer, Gruzelier & Lotze, 2010, p. 1152) and the final illustration is a general brain map of mathematical processing (Dehaene & Cohen, 1995).

The Brain in Relation to Music 1

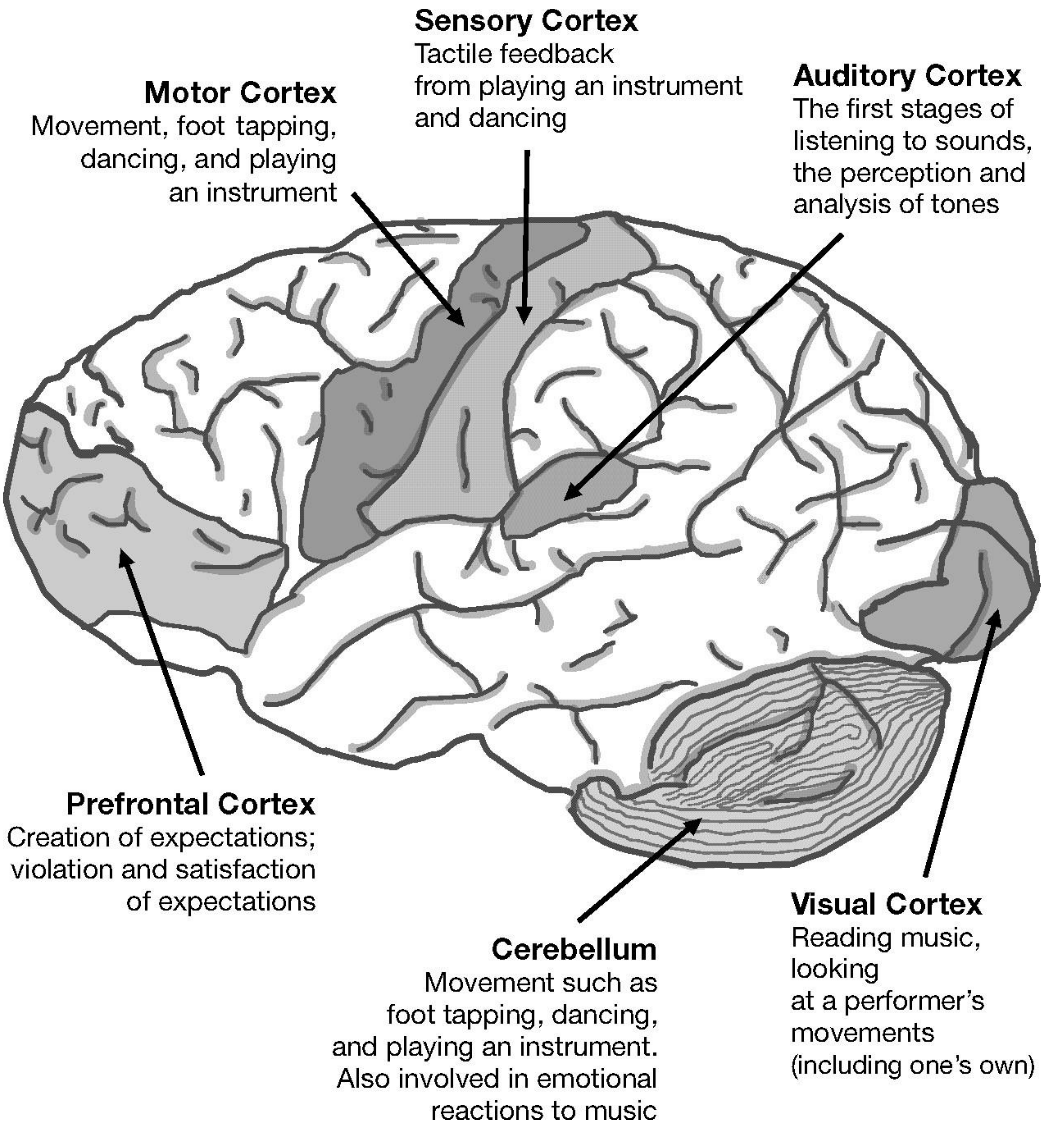


Figure 7 General brain regions used in musical processing from a side view of the brain, with the front of the brain to the left (Tramo, 2001 in Levitin, 2006, p. 264).

The Brain in Relation to Music 2

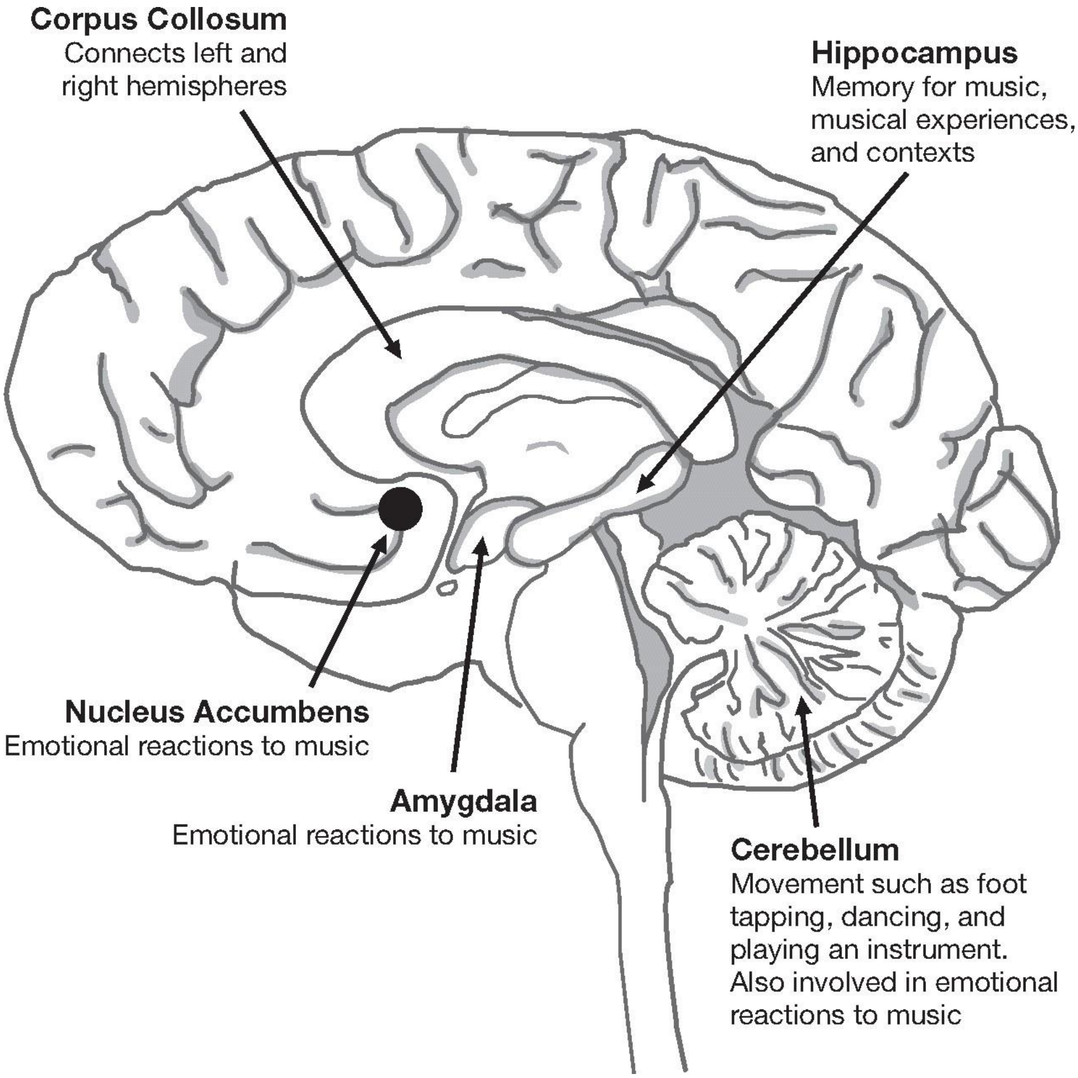


Figure 8 General brain regions used in musical processing showing the inside of the brain from the same point of view as the first illustration (Tramo, 2001 in Levitin, 2006, p. 265).

The Brain in Relation to Opera Singing

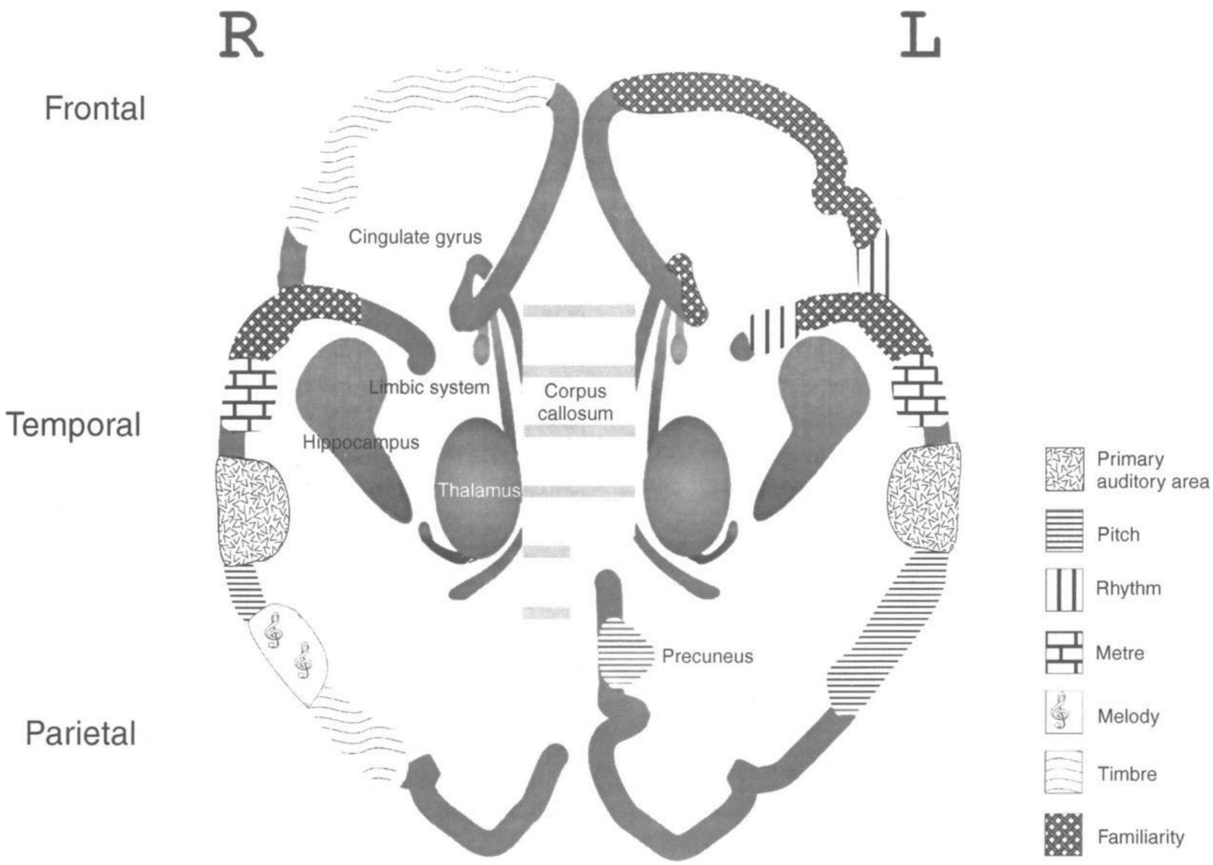


Figure 9 Illustration from article in Cerebral Cortex entitled, "The Brain of Opera Singers: Experience-Dependent Changes in Functional Activation" (Kleber, Veit, Birbaumer, Gruzelier & Lotze, 2010).

Figure 9 above is a "schematic axial slice of the human cerebral hemispheres, showing structures involved in music perception. Key areas involved in processing each component of the musical stimulus are coded at the right of the figure. These areas form interdependent neural networks, rather than 'centres' where particular functions are localized" (Kleber, Veit, Birbaumer, Gruzelier & Lotze, 2010, p. 1152).

Concerning mathematical representations in the brain, as noted before, Dehaene originally defined the three "numerical codes" as: the verbal system, the parietal quantity system and the occipito-temporal system. See page 67 for a detailed description of these systems.

Figure 10 on the next page shows this generalised conceptualisation of number systems in the brain by Dehaene and Cohen (1995).

The Brain in Relation to Mathematics

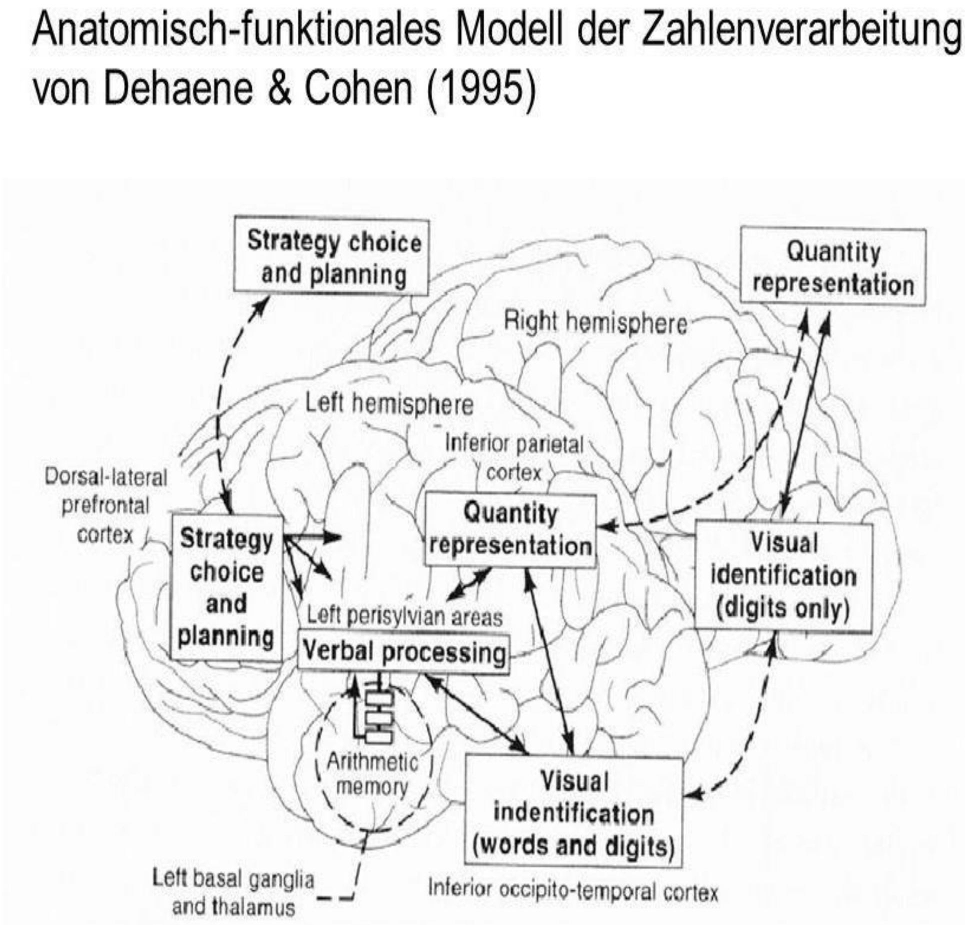


Figure 10 Triple-code model of mathematical processing (Dehaene & Cohen, 1995).

3.5. Educational Perspectives: Interdisciplinary Education & Music Education

3.5.1 Transfer Effects: Support for Interdisciplinary Education

Transfer may be described as a two-part process: transfer *during* learning and transfer *of* learning. The former refers to the way that past learning influences the processing and acquisition of new learning. The latter refers to the application of the new type of learning in future situations (Sousa, 2006, p. 136). Further, within the category of transfer of learning, Mestre (2002) describes two types, *near* and *far. Near transfer* refers to the transfer of learning from one context to another that is closely related and *far transfer* refers to the transfer of learning from one context to one not closely related. Thorndike (1932) formulated the "Identical Elements Theory of the Transfer of Training," in which he postulated that the amount of transfer between the familiar situation and the unfamiliar one is determined by the number of elements that the two situations have in common.

Regarding possible transfer effects due to musical experiences, examples of near transfer would be the development of the muscular control and motor skills used while creating music as well as the development of auditory discrimination abilities. Specific examples related to singing would be an increase in the strength and control of the intercostal muscles, which are used for breathing and controlled during exhalation for lengthened phrasing and the ability to discriminate among pitches and then match them vocally.

Again, this study proposes that enhanced spatial reasoning, pattern recognition and proportional awareness may occur as a result of musical training, therefore improving mathematical achievement. The enhancement of mathematical achievement would be considered far transfer, that is, the transfer of learning from one domain, music, to another domain, mathematics. Therefore, intermediary abilities such as the three noted above could be potential variables leading towards the development of improved mathematical skills.

Skills that are utilised in one domain, which in turn improve ability in another domain that also uses these skills, could be called *medium transfer*. Perhaps spatial reasoning, pattern recognition, proportional understanding and even problem-solving ability and creativity could fit into this category, since they are often activated while processing both musical and mathematical thought. Likewise, cognitive facilities such as the ability to focus (attention) and working memory, both exercised during musical production and skill acquisition could also be improved through training and perhaps could be transferred to mathematical processing. These arguments have led to the research reported in this thesis.

Sousa (2006) offers suggestions for teachers to aid in the process of transfer. He refers to this as "teaching for transfer via bridging," and includes examples for brainstorming with students regarding different ways in which this new learning might be applied to the new situation. For instance, *analogies* can be used to examine the similarities and differences between one topic or skill and another. Encouraging *metacognition,* or *discussing strategies* for learning or problem solving through relating one area of learning to another, are other "teaching for transfer" techniques that are suggested. These three techniques can be used as bridges between musical and mathematical learning; similar strategies were only employed minimally with the explicit groups in this doctoral study since a key aspect of the enquiry here regards whether and how much implicit transfer can occur during and after musical learning and whether even brief references to the connection between the two domains could be enough to make a difference in the pupils' learning.

3.5.2 Academic Transfer Aligned with Arts Partnerships in Schools

Partly in recognition of the potential benefits of interdisciplinary education, arts partnerships are now starting to be implemented more often in certain countries such as the United States and in England (for example, Music Education Hubs, Arts Council UK, Cultural Education Partnerships, founded 2012). Schools often become involved in partnerships with local arts organisations to foster interdisciplinary experiences for students. Abeles & Sanders (2007) at the Columbia University Arts Education Research Center, for example, assessed possible academic transfer effects of over 500 children receiving early strings training in five relatively deprived New Jersey public schools over a period of six years. In this partnership, The New Jersey Symphony Orchestra partnered with the city of Newark's Public Schools.

The researchers evaluated general academic success as shown in state standardised tests and school achievement, as well as personal and social development based on interviews with students, teachers and principals (head teachers). These interviews revealed improvements in school interest and performance as well as increased levels of creativity, self-esteem and general well-being. Each year, the standardised test scores for the Third Grade (equivalent to Year 4 in the UK), gathered from both strings students and control students of the same grade level, consistently showed statistically significant differences between the strings students and non-strings students in the same schools and across schools in the city (Newark, New Jersey), with the strings students out-scoring the non-strings students in both mathematics and literacy. Notable as well is that even though the students in the programme generally were considered underprivileged, often in single-parent homes and many with English as their second language, they eventually were compared to the entire state of New Jersey, which contains a broad socio-economic spectrum. Therefore, the broader comparison included children of high socio-economic status in contrast to the low socio-economic status of the city where the arts partnership programme took place.

As one of the researchers, my determination to further explore this link while looking at potential causality on the level of cognition became strong after seeing the initial results of the study as well as the positive and powerful differences the musical experiences gave the children.

In any school-based study, active musical engagement could contribute to the acquisition of general attributes such as cooperative skills and increased motivation due to simply participating in activities beyond the regular school curriculum, presumably then contributing to academic success indirectly.

Nevertheless, any increases in academic achievement that could be related to higher parental involvement or income were unlikely in this case since Newark is not only one of the most economically-deprived cities in New Jersey, but at least at the time of the study, it was one of the most crime-ridden cities as well. Therefore, the opposite trajectory, one of low parental involvement and economic support could have been expected. Hence, it was surprising that on average, strings students even out-performed other students in the state slightly in literacy as well as in mathematics even though for a majority, English was their second language. Additional research looking at the correlation between musical training and language skills is needed, in addition to research on the link between musical training and mathematics skills, though this thesis focuses on musical learning and mathematical attainment.

3.5.3 Transfer in Schools of Problem-Solving in Music and Mathematics

Bahr and Christensen (2000) investigated the essence of the transfer of problem- solving skills between two domains which they describe as "very dissimilar at a surface level but which overlap at a deep structural level in specific areas" (p.187). The researchers specify these domains as "formal musical skill" and mathematics. They specifically enquired whether this transfer might occur without explicit instruction to encourage transfer. The frame of reference used for the study was Structural Learning Theory (Scandura, 2001) as it elucidates the existence of deep structural similarities between music and mathematics. Structural Learning Theory focuses on the idea that working memory holds both rules and data, therefore Scandura posits that the process of learning and problem solving benefit from use of higher level rules.

Secondary school students (n = 85; mean age 15.5 years) enrolled in an extension math course took an initial mathematics test. One week later, students completed the Musicianship Rating Scale to measure "trained musical knowledge" (p. 187). The results showed that pupils who had training in music achieved at higher levels than those with no musical training in mathematical areas of structural overlap. Yet they did not perform better in areas without overlap. The author concluded that "transfer occurs as a result of deep-structural similarity of domains and that this transfer can occur spontaneously without explicit instruction designed to facilitate transfer" (Scandura, 2001, p. 187).

3.5.4 Inter-domain Transfer of Musical and Algebraic Achievement

A large study (n = 6,026) conducted by Helmrich (2010), (who acquired her PhD after teaching algebra and mathematics in secondary schools for 30 years), revealed significant differences in standardised measures of algebraic achievement by students with sustained instruction (3 years) in instrumental music (3.34, $p < .00$) as well as those with sustained instruction (3 years) in choral music (3.82, $p < .00$), compared to those without formal music instruction. Findings also indicated the greatest gains in achievement were obtained by African American students (instrumental group, 9.39, $p < .00$; choral group, 8.87, $p < .00$). Performance in this subgroup was examined to see whether music instruction might provide a means to narrow the current achievement gap between White and African American students. The results clearly suggest this possibility. This is one of the few studies looking at the potential singing may hold in supporting cognitive development in mathematics (Helmrich, 2010).

3.6. Why Singing?

Full school instrumental programmes do not normally begin until the fourth grade (Year 5) in the United States, if at all. Whether due to a deficiency in funding or lack of recognition of the value of a thorough and consistent early-years music programme, gaps in musical exposure in schools could be prudently diminished by a well-constructed curriculum using a readily accessible instrument, the voice. An extra expense that could be justified is the cost of an expert vocal instructor; this need exists particularly because of the delicate and changing nature of the young voices being taught as well as the positive potential that effective teaching may hold. In addition to expertise, the instructor should be able to provide "an enjoyable and rewarding" learning experience for students (Hallam, 2010, p. 1).

There are multiple advantages to having continuous singing programmes in schools. These instruments are free. The instrument is an inherent part of the child. Singing utilises the entire body and thoroughly trains the ear. The singer ideally must regularly create the necessary type and amount of breath for a good tone and carry oneself with an aligned posture, or with a constructive "use of self" (Alexander, 1932) in order to produce a healthy and "pleasing" sound from one's own instrument. Emotional, social and health benefits are not within the scope of this study, but have been well-researched and discussed elsewhere (Clift, Hancox, Staricoff & Whitmore, 2008; Welch et al., 2008).

In addition to the potential for active engagement that full singing holds, the voice is perhaps the most intrinsic of the instruments as well. As suggested, the instrument is one's own body itself. Infants respond more intensely and positively to the musical form of speech, "motherese," found across all cultures than to regular speech patterns and significantly longer to singing than to any form of speech (Papousek, 1992; Trehub et al., 1997; Nakata & Trehub, 2004). Melodic contour awareness is noted in infants long before they can speak or sing (Trehub, Bull & Thorpe, 1984), accurate pitch and rhythm matching has been found in infants as young as six months (Ostwald, 1973) and according to Dowling, children begin to sing as soon as they begin to speak (Dowling, 1984). I would suggest melodious forms of vocalisation often occur even sooner than speech.

Dowling also notes that a melody is "an integrated whole, a Gestalt" (p. 186). Numerous studies recognise the detailed and yet able processing children create and exhibit through song and the subsequent relationship to development (Dowling, 1978, 1984; Bartlett & Dowling, 1988; Davidson & Scripp, 1988a; Dowling, 1994).

Sloboda (2005, p. 176) alludes to the complexity of music-making when he speaks of four elements needed for musical production (his emphases follow): 1. The relevant dimensions of sound need to be *attended to, 2.* The sounds need to be *coded* or *categorised*, 3. The sounds need to be held together into a *structure* or *pattern*, and 4. All must be translated into a *response*. He notes that if one is not singing "in tune," usually the last element is deficient. This suggests that the process of singing, though natural, necessitates full engagement for example while aligning one's "ear" with the tone, therefore completing the musical experience.

A seven-month study (N=96) employing the Kodaly (1974) curriculum of singing and art, which emphasises sequential skill development, yielded higher standardised test scores in mathematics for the arts students than for the control students. The authors noted that learning arts skills causes mental "stretching" that is useful to other areas of learning. Gardiner and colleagues therefore suggest that the mathematics learning advantage in their data may exhibit the development of mental skills such as ordering, and other elements of thinking on which mathematical learning at this age also depends (Gardiner, Fox, Knowles & Jeffrey, 1996, p. 284).

Schellenberg, 2006, also tested the assumption that training in music "stretches" the mind.

He included four randomly assigned matched groups of students (N=144) in which two experimental groups took either voice or piano lessons and the control groups took drama or no lessons. IQ was measured prior to the experiment. After one year, the children who received music lessons had improved their IQ scores more than those in the control groups (although interestingly but not surprisingly the children in the drama group improved their social skills to a significantly larger degree than the other groups). The increase was generalised across IQ subtests, index scores and a standardised measure of academic achievement, with the voice students' mean increases in full-scale IQ slightly higher than the keyboard students.

A more recent study found that at-risk children who received two years of individual keyboard instruction as well as children who received individual singing instruction scored higher on a standardised arithmetic test than children in control groups, including a group that received computer instruction to rule out a possible Hawthorn effect (Rauscher & LeMieux, 2003). Interestingly, children who received instruction on rhythm instruments performed best on a mathematical reasoning task. Perhaps this should not be surprising as rhythm inherently involves patterns and proportion. Additionally, when teaching rhythm, one could vary the level of complexity to a large degree; perhaps this case is an example of the influence of the teacher and method of teaching as well as the content itself. The next chapter will discuss rhythm further and will illustrate elements of numerical, computer and mathematical systems that have derived from rhythm analyses by ancient musicians.

As emphasised before, the quality of the teaching is important, in terms of both technical and motivating abilities of the instructor. This can affect the level of music being made as well as the degree of mastery and enjoyment experienced by the students. For example, if the music that the students can manifest is more complex, the patterns and proportions absorbed in their minds likely will be more complex. If they enjoy it, the level of commitment and understanding will likely be stronger. Yet it is a concern that most of the studies looking at the potential effects of music education do not specify the level and expertise of the instructor.

Nonetheless, these studies provide evidence supporting the hypothesis that vocal training may aid numerical learning and may even hold certain advantages as mentioned before. As seen in studies with infants and children, singing is not only inherent, but extended experience with it can foster cognitive development as well. Following is a look at evolutionary research in the literature that continues to support the previous argument.

3.6.1 Singing and Synapses

Besson and colleagues pose the question, “Why is vocal music the oldest and still the most popular form of music?” They point out that vocal music involves an intimate combination of speech and music, which are specific, “high-level skills of human beings” (Besson, Faita, Peretz, Bonnel & Requin, 1998, p. 494). Although it should be noted that many if not most bird species also have intricate vocalisation systems often with similar functions such as communication and self-expression (Dugas-Ford, Rowell & Ragsdale, 2012; Shanahan, Bingman, Shimizu, Wild & Güntürkün, 2013). Numerous studies have shown that bird song holds linguistic properties such as grammatical order, which affect the meaning interpreted by the birds (Abe & Watanabe, 2011).

Using event related potentials (ERPs), Besson et al. found that humans treat linguistic and musical elements independently while listening to vocal music. Though their study supports a modular theory of language-music organisation, evidence as detailed before does show overlapping aspects, such as the left hemispheric location of broader rhythmic perception and analysis as well as language usage (Warren, 1999, p. 571). Schön and Magne collaborated with Besson in a later work (2004) that studied the changes in the brain’s electrical activity associated with certain elements of prosodic processing. They then compared the results with those obtained for the melodic processing of short musical phrases, (p. 1201) which confirmed Besson’s earlier theory regarding independence between modes.

Another potential advantage of singing is the cross-modal use of the brain due to its inclusion of language. This research focuses primarily on one sensory mode of learning – auditory – in order to look at the effects of music itself and therefore to avoid possible confounds added by other factors such as reading music or keyboard training (which provides visuospatial information). Yet since it is recognised that use of language can be an additional factor beyond the music itself that may aid general cognitive development, the inclusion of language could be a potential confound as well. Variables have been limited as much as possible, however, in order to reach clearer conclusions.

Other possible aids in cognitive transfer, which also could be seen as potential confounds, can be viewed through the theoretical lenses of behaviourist and constructivist theories of transfer of learning. For groups in which brief references to the link between music and mathematics were made, the contiguity of presenting these two domains in a positively reinforcing environment is one behaviourist factor that might enhance transfer, or the constructivist process of children being influenced by the thinking of a teacher they like could be explained by Bandura’s social learning theory (1977). Therefore, interviews with children might not reveal actual understanding, but rather, might reflect their desire to mimic the instructor’s ideas of the connection between the two domains.

On the other hand, post-intervention tests in mathematics might reveal whether certain numerical or proportional ideas had become embodied by the children, even if they could not express their conceptual understandings in words.

3.6.2 Evolutionary Viewpoints

We sing when we are very young and, according to anthropologists, we sang when our species was very young as well. Evolutionary explanations for the musical propensity of our species have been linked to our linguistic abilities. Scientists, musicologists and linguists have searched for sources of and parallels between both. One belief is that music and language evolved together and therefore, in part, share brain functionality and location. Patel (2008) says these “define us as human” and that these traits appear in every human society no matter what other facets of culture are absent. Across different studies, researchers such as Patel and Peretz have worked on defining music modules in the brain. Their brain-imaging studies suggest music and language share the same neural networks though lesion studies show they can be partially or entirely disassociated (Peretz & Coltheart, 2003; Patel, 2003; Mithen, 2006). Pinker (1997) surmised that music was a by-product of our capacity for language, yet was refuted by Tolbert (2001), Dunbar (2004) and Cross (1999), the latter noting that it is not only deeply rooted in human biology but also critical to the cognitive development of the child. Cross (1999) responds to Mithen’s (1996) earlier work when he suggests that proto-musical behaviours may play a functional role in general development and by implication, in cognitive evolution.

The theory that music and language systems initially evolved together, using the same portions of the brain due to shared communicative functions, but later parted, suggests that once language had stabilised into a system conveying specific content, music then fulfilled other functions such as emotional regulation and expression, cooperation and empathy (Wray, 1998, 2000; Mithen, 2006). McMullen and Saffron (2004) offer an elegant microcosm of this macrocosm in a study that examines infants’ innate ability to similarly categorise auditory events within language and music. They allow that some anatomical distinction between music and language may well exist in adults but note that it remains unclear whether we begin life with this neural specialisation or whether it emerges out of experience, with the two domains becoming distinct from one another (pp. 290-291). Both systems emerge from finite sets of sounds, the former consisting of phonemes and the latter, tones or notes. Humans can shift to a distinction of categories within their own culture by age one in language (Werker & Lalonde, 1988; Kuhl, Williams, Lacerda, Stevens & Lindblom, 1992) and by age four in music (Trehub, Cohen, Thorpe & Morrongiello, 1986; Schellenberg & Trehub, 1999).

Perhaps within the life span of a human, the two modes begin as open reservoirs of possibility, shadowing the evolutionary life span of humanity itself.

Musical expression and mathematical expression both have intrinsic value even when no transfer effects to other domains are found. This research focuses on singing as the musical instrument of choice, due to its accessibility for schools and for the children themselves as well as its direct link to musical development, with no external instruments such as keyboards, which inherently contain spatial information on their own, to confound the design. Singing is an organic part of human heritage (Papousek, 1992; Trehub, 2009) that is frequently overlooked in educational systems (Sloboda, 2005) and may deserve serious consideration. Additionally, if it is recognised that the findings here establish the hypothesised link further, therefore showing the potential that increased musical experiences hold to enhance skills in a normal population, treatment for those who experience mathematical difficulty should be considered as well.

For example, those born with Williams syndrome, a genetic condition characterised by certain medical and developmental challenges (Williams, Barratt-Boyes & Lowe, 1961) have a tendency to be outgoing and to enjoy and be moved by music, particularly singing, yet they struggle with basic skills such as mathematics (Hopyan, Dennis, Weksberg & Cytrynbaum, 2001; Levitin, 2006). These often occur side by side with striking verbal abilities, highly social personalities and an affinity for music. Therefore, singing used as a tool to reach and to work with children with Williams syndrome via a more accessible avenue, can be an enjoyable and effective way to support the learning of the very basics of mathematics, such as adding and subtracting for use in daily life. Personal experience teaching music and basic concepts to a child with Williams syndrome aligned with this suggestion.

3.7. The Need for This Particular Study

Though numerous experiments have been conducted that look for causality (Schellenberg, 2006; Schlaug et al., 2003, 2004, 2005, 2008, 2009), certain controlled elements are lacking or confounds are existing in many of them. For example, participants in one of the Schellenberg studies (2004) were recruited based upon whether they had a piano with at least 4 octaves of full- sized keys at home. Therefore, there was already a difference in the population studied compared to the general population in that a supportive home environment for learning music appeared to already exist. This study, however, which included a singing group and a keyboard group, showed that the former group had the largest enhancement of IQ. This is unlike the results of other similar studies thus far such as the Rauscher & LeMieux study (2003), which indicated a stronger effect for the piano group. Yet the piano group also had singing lessons, which created uneven group interventions, therefore skewing the measurements.

Second, further confirmation of emerging hypotheses is needed. Third, as mentioned, no published studies focus solely on the academic effects of learning music via the voice. Fourth, no studies seek to link specific elements of music with their specifically related cognitive constructs. Fifth, very few studies exist that were conducted in a natural primary comprehensive (state) school setting throughout the course of a full academic school year.

In light of the preceding review, what is lacking in the literature is research that goes beyond looking at the results of listening, but rather, the results of making music in a real-world, classroom setting.

This research also attempts to fill a gap in the existing research, which is comprised of studies that are primarily lab-based, small or short-term, by asking *why* there may be a connection between musical, spatial, structural and subsequently, mathematical reasoning by exploring specific, underlying cognitive mechanisms that may be related and therefore utilised by all discussed forms of reasoning.

Ultimately, through research to better understand children's learning processes, it is hoped that educational opportunities will be provided that can serve children most effectively in terms of intellectual, academic as well as emotional development. This hope provided the motivation for my doctoral research.

In closing this literature review, I will return to the culture of Plato, whose words opened this thesis. The ancients spoke of music in scientific terms; mathematicians of the 20^{th} century spoke of the bond between mathematics and art in poetic terms. Hardy (1940) suggests that a mathematician is like a painter or a poet who is a maker of patterns that “must be *beautiful;* the ideas, like the colours or the words, must fit together in a harmonious way” (pp. 24-25). Storr (1992) illuminates the source of the music-mathematics relationship from the cognitive perspective and concurs with the assertions put forth in the earlier discussion. He notes that both domains are concerned with linking together abstractions and with making patterns of ideas. He speaks of the need for humans to create integrated wholes and of the satisfaction gained from “solving problems, perceiving connections, understanding structure, learning new techniques…. ‘Eureka’is our cry of pleasure at a new Gestalt, even if it has no immediate mundane application” (Storr, 1992, pp. 175- 177). This perhaps explains the pull of the researcher as well, the need to find answers.

Chapter 4
Research Questions, Design & Assessment Tools

4.1. Introduction

The previous chapters provided historical, theoretical and research-based arguments regarding links between music and numeracy, as well as the connection between musical learning and mathematical thinking. This chapter begins with the justification for the type of design used, followed by the assumptions of the study. Then the research questions for this doctoral study are presented and discussed. Following this is the research design in detail, and then the chapter closes with explicit descriptions of the standardised measures and monitoring tools used in this research.

As noted in Chapter 1, this study examined if and why music education could enhance children's mathematical thinking and therefore achievement, with recognition of the differing components within both domains.

4.2. Justification for the Design

Though recognising the value of a case study or of identifying correlative relationships through interviews or questionnaires, it is important to say that these types of research strategies do not try to prove causality. To do that, one needs to take measurements in controlled conditions to compare the effects of those conditions, or in this case as controlled as possible given the real-life circumstances within school settings. For example, one could do a case study of one group, but that would be the weakest design in this instance because there would be no comparison group or opportunity for standardisation, therefore, other factors such as maturation could explain any findings (Campbell & Stanley, 1963).

At the time this study began, Hallam's comprehensive review of studies regarding music transfer had not yet been published, but it is worth noting her point that even though much of the existing research does not establish causality, "all of the research has the potential to make a contribution to our developing understanding of the nature of transfer of musical expertise to other domains and skills, albeit in different ways" (Hallam, 2015, p. 10). Nonetheless, the aim of this research is to contribute a bit more to the body of evidence that at least attempts to show causality and to provide further explanations regarding why the phenomenon of transfer might occur.

4.3. Assumptions

This study included the following assumptions: 1) Since this experiment was conducted in naturalistic school settings, it is therefore a quasi-experiment because all factors could not be systematically controlled to the same degree as one could do so in a laboratory and it lacks true randomization when assigning groups to conditions. That is, one "lacks the full control over the scheduling of experimental stimuli…and the ability to randomize exposures" (Campbell & Stanley, 1963, p. 34). 2)

The pretests and posttests used in this study are age-standardised and have been tested for validity and reliability. Therefore, they measure what they are intended to measure with consistent results among a similar population of children in the same age range. 3) With only a few exceptions, the children understood the vocabulary and concepts within the tests, and 4) the analyses of the quantitative data were accurate, and 5) the interpretations of both quantitative and qualitative data were appropriate.

4.4. Research Questions and Hypotheses

Through investigating the literature and pondering the primary enquiry, four interrelated questions have emerged. Due to the depth of the challenge put forth as well as the limited scope of resources for this study one may expect to answer only a fraction of these questions. Perhaps, however, this project might inform future research and provide a clearer and more thorough understanding of the frequently postulated and anecdotally observed music-mathematics link. It is hoped that research in this area may help guide school curriculum decisions towards education of optimum quality and balance for all students regardless of background and resources.

Below are the research questions, with a discussion of each to follow:

4.4.1 Research Question 1

How far, if at all, can school-based musical learning primarily via the voice improve mathematical achievement?

4.4.2 Research Question 2

Does focusing on specific musical elements while teaching enhance understandings of possible corresponding mathematical concepts?

4.4.3 Research Question 3

Does teaching music with brief yet explicit references to hypothesised mathematical correlates enhance children's corresponding mathematical skills, and if so, does it do so more than teaching music without these references?

4.5. Primary Research Question

Following is a discussion of the first research question:

How far, if at all, can school-based musical learning primarily via the voice improve mathematical achievement?

Previous investigations of the music-mathematics question have been explored principally using keyboard or violin. Keyboard studies have received the most attention (Costa-Giomi, 1999; Rauscher & Zupan, 2000) with violin (Fujioka, Ross, Kakigi, Pantev & Trainor, 2006; Abeles & Sanders, 2007) following in order of frequency. It appears that voice has been included in only a few studies of this sort (Rauscher et al., 1997; Schellenberg, 2004; Hellmrich, 2010).

Use of the voice as the vehicle for musical study is suggested primarily for its availability for all populations and direct access to internal musicality, as mentioned in the previous chapter, although it has other advantages, including encouraging healthy breathing and individual expressivity. As noted before, the nature of early singing training for children does not usually include supplementary learning tools or visual aids such as patterns on a keyboard or in musical notation. Since these may be associated with improving mathematical skills, and therefore might serve as confounding variables, this study does not include keyboard training, nor does it include musical notation beyond a rudimentary introduction. Additionally, though learning keyboard patterns and standard musical notation can be beneficial for the learner, these learning opportunities are not readily available for many. These skills can also be seen as potential factors improving spatial ability, therefore this researcher wanted to see if spatial ability is *intrinsic* to learning music, whether or not visual aids are present in the learning process. As noted in the literature review, advantageous effects from keyboard training in particular have been reported in other studies (Cheek & Smith, 1999; Costa-Giomi, 1999) yet the possible effects of vocal training without spatial aids have yet to be fully explored. Therefore, examining outcomes due to learning music through one primary mode of input and one primary tool of production, that is, auditory and vocal respectively, could therefore add clarity when seeking answers to the research questions by reducing the number of variables involved.

Educators often ask if musical learning improves mathematical reasoning since the link continues to appear in the literature as well as the media and in daily life experiences. I therefore chose a quasi-experimental design to test this hypothesis:

Music education via the voice as the primary instrument can enhance mathematical achievement.

Observing behavioural evidence both first-hand and in the literature over the years had continued to increase my motivation to look for further evidence of this link and to discover the reasons why. To gain additional confirmation and explore potential causes of the suggested link between music and mathematics, an investigation of the potential effects of specific aspects of musical learning is put forth, therefore leading to the second research question.

4.6. Research Question 2

Does focusing on specific musical elements while teaching enhance understandings of possible corresponding mathematical concepts?

Here the question is restated as a hypothesis:

Teaching music while focusing on specific elements does enhance understandings of corresponding mathematical concepts.

Three sub-questions are proposed, though the last two are reasoned conjecture:

- Does emphasis of music structure (form) comprehension particularly enhance pattern recognition, structural recognition and creative problem-solving abilities?
- Does emphasis of pitch relationships (melody) while teaching music particularly enhance pattern recognition as well as geometrical and spatial-temporal understandings?
- Does rhythmic emphasis while teaching music particularly enhance counting ability and pattern recognition as well as proportional and spatial-temporal understandings?

This study is designed to provide evidence of the link between certain musical skills and potentially corresponding mathematical skills. Although it is believed by this researcher that a combination of components in musical experiences can prime the cognitive system for use of several corresponding components of mathematical thinking such as spatial-temporal reasoning, pattern and structural recognition, creativity and problem solving, sub-hypotheses are proposed to suggest that certain musical components may influence specific mathematical correlates. Therefore, the proposed sub-hypotheses are:

- *Form identification skills are particularly related to recognition of patterns and structures as* *well as problem-solving abilities.*

- *Melodic skills are particularly related to geometrical and spatial-temporal understandings.*
- *Rhythmic skills are particularly related to counting and pattern recognition as well as proportional and spatial-temporal understandings.*

4.6.1 Justifications for and Interrelationships Among Variables

The literature regarding these precise correlations is virtually nonexistent, therefore the sub-questions and corresponding hypotheses arise principally from speculation based on experience and reasoning. The sparse literature found was discussed briefly in the literature review. As noted, Parsons and colleagues (1999) conducted an experiment in which concentrated rhythmic training yielded higher levels of spatial-temporal reasoning than did focusing on other musical elements, and experiments with infants at Harvard's Laboratory for Developmental Studies revealed that melodic and rhythmic components, separately tested, both related to spatial cognition (Carey & Srinivasan, 2008; Spelke, 2008).

4.6.2 Hypothesised Relationships Between Variables with Corresponding Theories

Theories regarding why the hypothesised relationships between variables may exist are revisited below from the first portion of the literature review. Also, additional hypotheses are put forth here and expanded upon using musical examples.

Why might long-term focused learning in each independent musical variable have an effect on its corresponding dependent mathematical variable?

4.6.3 Form/Structural Learning

Learning to recognise structures within a musical piece supports analytical thinking beyond the musical domain, as may be reflected in improved mathematical performance involving organisation and categorisation (Mulligan, Mitchelmore & Prescott, 2005; van Nes & de Lange, 2007). This may also lead to improved problem-solving abilities. The Form groups in this study used their newly found knowledge regarding similar and contrasting parts to compose their own song with three distinct parts while also including similar repeating parts. As noted in the literature review, music compositional and improvisational skills encourage both divergent thinking (associated with creativity) and convergent thinking (usually associated with problem solving). Again, as with composition and improvisation, higher-order mathematical problems and calculations have been found to benefit from both forms of thinking also (Dunn, 1975; Haylock, 1987; Toshihiro, 2000; Cropley, 2006).

Additionally, understanding form implies and therefore may foster structural understandings as well as the recognition of larger patterns comprising structure.

4.6.4 Melodic/Harmonic Learning

The melody/pitch focus includes both melody and harmony because an awareness of both in the context of a musical composition is related to the tonal centre, or the "key." The "2-dimensional" aspect of melody and the "3-dimensional" aspect of harmony, both containing multi-directional elements, may encourage geometrical and spatial understandings. The tonal centre, which relates to both melody and harmony, serves as a continual point of reference. In more complex music, harmonic modulations, or changes in the tonal centre, can occur frequently, therefore "exercising" spatial orientation and structural awareness skills. Mozart, for example, composed virtually continuous modulations that are so subtly embedded within the music that these changes are often imperceptible to the untrained ear.

4.6.5 Rhythmic Learning

In addition to counting and pattern recognition, rhythmic recognition requires structural understanding. Each musical composition contains a certain number of bars or measures (rhythmic groupings). Within each bar are arrangements of differing note durations, with bar count totals aligning with the metre (or metres in some cases) of the piece. Every rhythmic value is a proportion of the whole, that is, it is a specific fraction of the total bar count durations as designated by the metre and the corresponding reference note. For example, in 4/4 time, a breve or whole note receiving four counts is the reference note. Learning music produces at least an implicit understanding of these proportional relationships. (See also Parsons et al., 1999 and Carey & Srinivasan, 2008 from the literature review.)

4.6.6 Overall Musical Learning

Since patterns, structure and proportions exist to some degree in all aspects of music – form, melody and rhythm – it is reasonable to suggest that there would be some overlap in the corresponding mathematical cognitive correlates as measured by tests, and will be dependent upon the quality of those tests. On the other hand, that should not pre-empt the attempt to find out and therefore it is worth an exploration nonetheless.

For example, though rhythm may appear to relate more obviously to proportion, there are specific proportional relationships within melody and harmony as well. The exact distance between pitches defines the specific melody or harmony and therefore is the foundation for melodic and harmonic designations. Famous examples are the notable descending melodic thirds in Beethoven's sequential motif in the opening of his Symphony No. 5 in C minor opus 67 (1808) or the Petrushka chord used by Stravinsky in his ballet of the same name (1911) in which two major triads, which are a tritone (6 semitones) apart, form a distinct, dissonant harmony.

Table 2 below outlines each musical component to be emphasised in the teaching (independent variable) and its possible cognitive correlate in mathematical thinking as indicated via test scores (dependent variable).

Hypothesised Relationships Between Variables

Independent Variable (Music Teaching Focus & Music Learning)	**Primary Dependent Variable (Mathematics Learning)**
Form/Structural Focus, Awareness & Music Learning	1. Pattern recognition 2. Structural awareness and categorisation ability 3. Creative and analytical problem-solving skills
Melodic/Harmonic Focus, Awareness & Music Learning	1. Pattern recognition 2. Geometrical understandings 3. Spatial & spatial-temporal reasoning
Rhythmic Focus, Awareness & Music Learning	1. Pattern recognition 2. Counting 3. Spatial-temporal reasoning ion 4. Proportional mathematics: fractions, division
Overall Musical Learning	**A combination of all abilities listed above**

Table 2 The music teaching focus, awareness and learning independent variables and corresponding mathematics learning dependent variables.

4.7 Three Key Mathematical Concepts Related to Music

Emerging from the literature review as well as from experience and speculation, three key mathematical concepts continue to appear to be particularly related to music: spatial-temporal reasoning (Rauscher, Shaw, Levine, Ky & Wright, 1994; Hetland, 2000a,b), pattern recognition and proportional understanding – yet particularly the latter – are forms of structural awareness, which is an important foundational concepts for mathematical understandings and skills (Friedrich & Friederici, 2009; Hoch and Tillmann, 2012). This will be explored further in Chapters 6 and 7.

4.8 Research Question 3

Does teaching music with brief yet explicit references to hypothesised mathematical correlates enhance children's corresponding mathematical skills, and if so does it do so more than teaching music without these references?

The assumption evoked by logic that teaching related concepts explicitly can increase the appearance of transfer effects in children's learning is also supported by the literature (Thorndike, 1932; Mestre, 2002; Sousa, 2006). This study adds to the literature regarding both music and mathematics education; therefore it may inform future curricula.

In response to the need put forth and emphasised in the literature review, the research presented in this thesis incorporates a carefully designed pretest-treatment-posttest experiment that examines behavioural evidence in the quest to answer the above-stated questions. As defined at the beginning of the chapter, it is a quasi-experiment in that children were not assigned randomly to the various conditions, as it took place in a real-world school setting, where in addition to lack of true randomisation, numerous other factors were beyond my control (Campbell & Stanley, 1963, p. 34). Consideration was given to other possible contributing factors that might indirectly relate to improvements in mathematical skills such as attentional and memory skills, both potentially improved by music training. However, these were not systematically tested; the children were tested only on the hypothesised abilities discussed here, that is, music aptitude, spatial-temporal reasoning and mathematics. Therefore again, the design of this study would strictly be described as quasi-experimental.

In light of the research questions and the research design, which includes curricular and pedagogical aspects, a more specific hypothesis is proposed:

Children (aged 7-8 Year 3) who have had regular weekly music lessons primarily via the voice for nine months will show improvements in mathematical skills beyond normal maturation *rates. The results may differ depending upon the musical foci and will increase with the inclusion of very brief yet explicit references to mathematical correlates during lessons.*

4.8. Research Design

In order to test whether music training would improve mathematical thinking and further, to test whether emphases on specific musical elements would enhance possible mathematical correlates, a multi- factorial design was created. This study used a between-groups, pretest-treatment-posttest 3x2 multivariate design (see Table 3 below). There are two independent variables: *1) music training* with three levels that reflect emphases – Form (structure), Melody (pitch relationships, inherent within melody and harmony) as well as Rhythm, and *2) mathematical content explicitness* (with two levels – Implicit or Explicit).

Three dependent variables as measured by standardised tests are analysed (to be justified and described in detail later in this chapter: *1) music audiation, 2) spatial-temporal reasoning, and 3) mathematical skills* for Year 3, ages 7-8 (for example, addition, fractions, geometry).

The importance of using the standardised mathematics test is critical in relation to the research design. It could be argued that a quasi-experimental design of this kind should have included a set of control groups, one per pair of treatment groups. One set would have the same teacher (the researcher) spending the same amount of time teaching but teaching something other than music, to control for any of the individual teacher effects. In order to eliminate teacher effects, the study would have needed a small group of teachers willing and able to teach the intervention music lessons, or the training of the class teachers to do so. This process would most likely introduce a new set of uncontrolled variables, such as teaching styles and attitudes to and competence in the musical training. Another set of control classes would have their usual class teacher and no musical training, beyond the typically minimal music curriculum, as a basic control group to indicate mathematics progress over the course of the school year. The use of a recently standardised test of mathematics, developed and validated with children who had followed the current mathematics national curriculum in state schools, allows the test norms to be used to represent the normal, "no extra musical training" control sample, therefore obviating the need for separate control groups.

Several additional arguments for not attempting to use control groups were put forward. The first is that of feasibility of training and retaining the teachers in a study of this size. Second was the difficulty of findings schools with at least two parallel classes of approximately 30 each within the target age group. Third was the unlikely and unethical procedure of administering the battery of preand posttests to a large number of children who would not experience the music lessons, and their potential benefits. Fourth, a counterbalanced design to allow an initial control group to experience the music lessons during the second half of the intervention period would reduce the actual length of the intervention by half, therefore undermining the principle that the intervention needed to be long-term for any hypothesised learning and neural development to occur, and to be long enough to exceed the age-standardised norm intervals. Any potential teacher "charisma" effects could still be present, nonetheless. An attempt to assess these factors and lessen any potential confounds was made by creating video and audio recordings of the music lessons, and by observing one mathematics lesson by each class teacher. These were viewed and reflected upon to ensure as consistent treatment of each experimental group as possible, and to identify any major divergence in teaching style.

On the next two pages, the research design in terms of the overall process and then in terms of intervention foci are shown.

Research Design & Overall Process

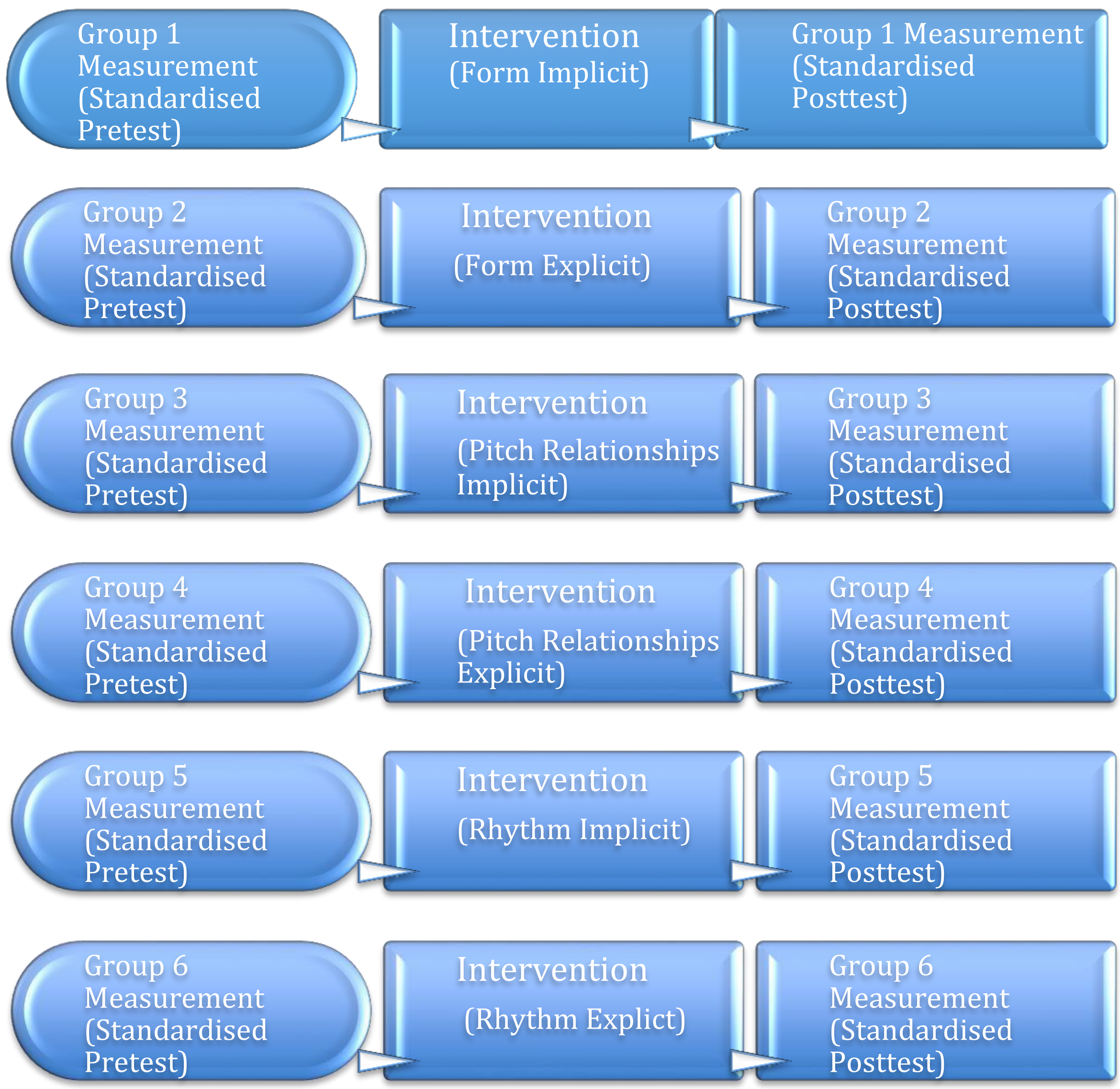


Table 3 The research design in terms of the overall process. This flowchart is adapted from Field and Hole (2008) and illustrates the six-group design in terms of overall pretest-intervention-posttest process.

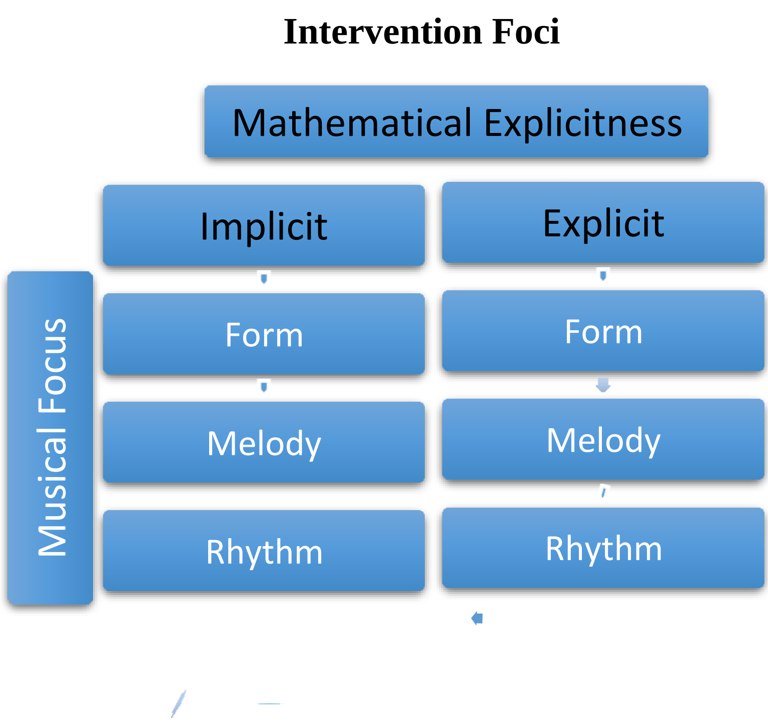


Table 4 The research design in terms of intervention foci. This flowchart is adapted from Field and Hole (2008) and illustrates the 3x2 six-group design.

Note that Table 4 above illustrates the musical focus independent variable category, which has three levels (Form/structure, Melody/Pitch relationship and Rhythm) and the mathematical explicitness independent variable category, which has two levels (Implicit and Explicit).

4.9 Teaching groups according to assigned musical mode

The approach to each lesson in terms of differentiating musical modes in the teaching permeated each lesson. The process started with an awareness and reminder to myself as the teacher in case I needed to bring any special instruments such as percussion instruments, though most classrooms had plenty of Orff-inspired choices. Virtually the moment the class began, the modes were emphasised because the vocal warmups were tailored to each one. For example, the Melody groups would do smooth scalar runs as well as arpeggios, and we would talk about the melodic contour. With the Rhythm groups, we would also do scales and arpeggios, but the rhythms would vary from even to uneven (dotted) for instance. With the Form groups we sometimes would go back to the first exercise and call it Part A.

The piano accompaniments, song arrangements and children's instrumentations would vary according to the musical mode of their group. For example, different ways of playing and singing the

Japanese song “Sakura” according to the mode will be shown in the next chapter, with the Rhythm groups playing percussion and the Melody group playing the glockenspiel for example. Finally, the performance styles took on different styles depending upon the mode, with flowing contoured movements for the Melody groups, rhythmic movements for the Rhythm groups and structural concepts such as dance choreography and original composition for the Form groups.

To summarise, all of the components below were informed by the particular mode involved:

Teacher Preparation before the lesson
Teacher Awareness during the lesson
Warmups
Discussions
Piano Accompaniments
Song Arrangements
Orchestrations
Movements
Performance Styles

4.9.1 Participants

Age

The mean age of students was 7 years 7 months (SD = 3.41) at the start of fieldwork testing in October 2010 (*N* = 180, *M* = 91.54 months, *SD* = 3.41, Minimum age 86 months, Maximum age 97 months). Participants were in Year 3 during the fieldwork timeframe (academic year 2010-2011), which is the first year of Key Stage 2 in England and Wales (The Education Act, 2002). This age group was chosen for several reasons.

First, this is prior to the time when schools in many developed countries start focused music education. In England (Associated Board of the Royal Schools of Music, 2017; Henley, 2011) and the United States (Parsad & Spiegelman, U.S. Department of Education, 2012), most schools do not begin formal instrumental lessons and ensemble work such as orchestra or band until around the age of nine, if at all. Therefore, it was reasoned that there should be little “interference” with this study since the variable of weekly music lessons would be primarily limited to the intervention. In England, there is not strict implementation of music education programmes in primary schools, and therefore many do not have them. Yet in 2011, the Department of Education in England put forth the goal to have music education available to all children aged 5-18 through community hubs over the following few years, though funding would decrease at the same time (Department of Education, England, 2011, p. 9).

Second, if the study were to show a positive correlation between improvements in musical and mathematical abilities, some schools, local authorities or multi-academy trusts (MATs) might be more willing to invest in music programmes and if they exist already, then to consider starting their musical programmes earlier. Third, age seven is an appropriate age to begin a focused yet age-appropriate level of vocal training due to both physical and cognitive factors such as coordination, body awareness and readiness to learn abstract concepts (Fuchs, 2008). Many voice teachers shy away from teaching this age group perhaps in fear of behavioural problems or overworking the child too early in their development. Yet at this point, children are often singing along with the radio and can begin to establish damaging habits (even in group activities such as a choir if the choral director is not trained to work with young voices) that can be difficult to change at a later point. If taught in a cautious, yet enjoyable manner with the health of the voice (and mind) of the child forefront, a healthy foundation can be created. Therefore, that time period in the child's life is an excellent time to start learning music via the voice. Note that in order for children (or anyone) to be able to explore more extensive melodic lines, and therefore more complex music, one needs to be able to sing with a larger range. Therefore, by learning technique enabling one to sing in one's "head" voice (in other words, in a higher range with minimum muscular action), expanding one’s vocal range and therefore repertoire becomes more possible. Additionally, the ability to sing comfortably with a larger range is healthier and more enjoyable. (Doscher, 1994).

Fourth, the mathematics material for Year 3 in England's Primary National Curriculum is excellent for this study since new concepts are being introduced such as fractions in line with many other countries, for example in the United States (English National Curriculum, 2010; The National Council of Teachers of Mathematics, US, 2010).

Some educators and researchers, including Schlaug et al., (2004) may encourage starting at age five instead of seven since research has suggested that learning music at this younger age has a more pronounced effect on the developing brain. Yet the pragmatist perspective influences the design of this research. Therefore, what is practical for implementation in the schools is a very important consideration. If a beneficial effect is shown at the age of seven, school authorities might more readily include more singing training in the schools at least at this level (the beginning of Key Stage 2 in England and Wales, for example). They also might be more open to students starting one or two years earlier if the suggestion or opportunity should arise. Whereas, translation directly into school implementation might be less likely in some cases if the focus were on five-year-olds, since the assumption might be made that one *must* start at this earliest age for the concentrated singing experience to be effective as a method supporting cognitive transfer to other school subjects. If that were the perception, this earlier starting age might seem to policy-makers to require a larger investment of resources and therefore may not even be considered.

Socio-Economic Levels

Two reasons that children of schools serving low-middle socio-economic status populations were selected for this study are, first, to offer opportunities for those who would normally not have them and, second, because the likelihood that certain children may have private music lessons or a mathematics tutor would be lower. Such extra tuition would threaten the validity of the study.

4.9.1 Selecting the Schools

Considerations in the selection process were the children's age, socio-economic levels and similarity of schools (such as state schools with diverse populations) so that participants could be matched as closely as possible. Feasibility of carrying out the fieldwork was also a consideration. For example, the schools needed to be within a certain distance from Cambridge in order that weekly visits could be carried out by bicycle. In locating these schools, Ofsted reports were examined, and then out of the selected schools, the head teachers were asked whether the classes were randomly grouped (therefore of mixed ability) or had been grouped in terms of special needs or talents. Indeed some had been, therefore eliminating them as a potential candidate for the study.

The advantages in working with state schools for this study are that it may be less likely that children have private music tuition and the population is more likely to be diverse, reflecting the general UK population.

4.9.2 Gaining Access

The recruitment process began with an introductory letter on University of Cambridge Faculty of Education letterhead as well as my Curriculum Vitae. These were sent to each school (see Appendix 4) by e-mail. This was followed by a series of phone calls and discussions, usually with the head teachers and occasionally with Year 3 class teachers. The e-mails contained a brief introduction. One interesting reaction is noted here: Approximately the first eight e-mails began with the phrase, "I am a PhD student who would like to…." Two schools replied shortly afterwards to the negative saying they have recently had a university project in their school and they were either too busy or did not want to get approval from the parents so soon for another project. The introductory e- mail was then changed to begin with the sentence, "I am an experienced teacher and have returned to academia for my PhD." Both were true, but the wording clearly made a difference. In contrast to the previous situation, this time several schools responded quickly to the affirmative. Understandably, the shift in the primary identity, experience and abilities of the researcher appears to have been helpful to the process. Over fifteen schools were approached and five were open to and able to participate in the study, therefore giving their consent.

4.9.3 Forming the Groups

Participants were chosen from seven classes in five schools. Out of these, six groups were established for this experiment. Two of the schools contained two different year levels within each classroom (Years 3 and 4 @ St. Lucy's Primary; Years 2 and 3 @ Fremont Primary). Therefore, the Year 3 portions of these two schools were combined (because each was too small by itself for this study) to make one of the six groups for this experiment (Rhythm Implicit Group). Two of the schools each had two Y3 classes of approximately 30 students in each and one had a Y3 class of 25. Therefore, five experimental groups (Form Implicit, Form Explicit, Melody Implicit, Melody Explicit and Rhythm Explicit) comprised self-contained classes and one (Rhythm Implicit), as noted, comprised the two smallest classes. See Table 5 below for group organisation across schools.

Participating Schools Arranged by Experimental Groups

Groups	**School**	**Number of Pupils in Group**
Form Groups		
1. *Implicit*	St. Michael's	**30**
2. *Explicit*	St. Michael's	**30**
Melody Groups		
3. Implicit	Lady Gytha	**30**
4. Explicit	Lady Gytha	**30**
Rhythm Groups		
5a. Implicit	St. Lucy's	**18***
5b. Implicit	**5**b. Fremont	**17***
6. Explicit	**6**. St. Peter's	**25**
Total participants:		**180**

Note: Pseudonyms have been used for all schools.

Table 5 Participating schools arranged by experimental group.

Though class size estimates may be imprecise, nonetheless, the average class size in England according to the Organization for Economic Cooperation and Development (2009) is about 26 but classes are often as many as 30 students, as with this study. Five out of the six groups were randomly assigned to particular music emphasis modes. The one class that was not randomly assigned was the Rhythm Explicit group because the teacher was the only one who had planned to closely follow the UK Primary National Curriculum, which (for Year 3) featured rhythm to a larger degree than other musical elements (English National Curriculum, 2010). The two schools that contained two Year 3 classes were assigned to the same mode (Form for one and Melody for the other) for simplicity, acceptance (avoiding the tendency for students to compare) and ability to group together for the final concert. Each school will be discussed individually below.

4.9.3.1 ***Matching Groups***

An important precondition is that the children in the six groups share a range of abilities that are as similar as possible on a range of characteristics. For validity, levels of aptitude must be controlled at the beginning as much as is possible even within a natural school setting, so that causal relationships may be demonstrated by comparing groups showing similar pretest outcomes with the same groups after the intervention, at posttest. Therefore, as mentioned above, in addition to assuring that schools have not grouped children according to ability, students were tested for aptitudes (musical, spatial and mathematical) relevant to the question at the start in order to ascertain whether ability level is evenly distributed among the groups, to quantify any discrepancies and to take them into account statistically in the pretest-posttest comparisons. Furthermore, this allows specific before-and-after assessments to be made as to the degree of improvement in each area for discernment in the analysis of any potential causal relationships among the independent variables and the dependent variables.

Additionally, the most recent Standard Assessment Task (SATs) scores for the children were collected in order to verify that they were relatively equivalent groups, to control for possible aptitude differences and to verify the construct validity of the MaLT tests. Since this study took place at the start of Year 3, the children had taken these assessments at the end of Year 2 as they were finishing Key Stage 1 (English National Curriculum, 2010). Furthermore, intermediate tests from students who were at the end of Year 3 the prior year (hence being in Year 4 during the intervention when this experimental group was in Year 3) were gathered in order to further assess whether the *schools* were indeed comparable to each other in this way as well as to compare Year 3 *participants* in the study to

Year 3 students of the year before at the same school for additional validity.

Even though it has been verified that the students were matched as closely as possible and, with one exception (the teacher for the Rhythm Explicit group requested rhythm to be in line with the National Curriculum guidelines), the groups were randomly assigned independent variables (musical mode emphasis and mathematical explicitness), this is still a quasi-experimental design since most of the groups were already intact as classrooms. Furthermore, this could – in the strictest sense – be considered a nonequivalent group design, since there were inevitably differences among them, due to the natural setting as well as the relatively small sample size, eliminating true randomness. Of course it would be nearly impossible to conduct this experiment in a laboratory without differences existing among students, even if it were an ethical option.

One possible confounding factor that was taken into account was the classroom teacher, in particular to what degree each engaged the pupils in additional musical activities as well as the quality of mathematics teaching. These were both assessed, as well as other classroom activities and curricular priorities. Though elements of other music and mathematics teaching were similar among all seven teachers, detailed differences will be discussed later in this chapter. Each teacher also completed a questionnaire in order to obtain their evaluation of the intervention and to see if there were any blatant differences in either musical or mathematical levels of content inclusion as well as their attitude in these areas that may not have been apparent through observation. Additionally, students (approximately 30%) in each class were interviewed regarding their musical and mathematical experiences as well as their thoughts on the connection between the two.

4.9.4 Participating Schools

All schools in this study are mixed gender state primary schools in Cambridge. Therefore, they share similar characteristics across multiple areas with only minimal differences. More details of participating schools are shown via Table 6 below, which includes Ofsted results both prior to and after the fieldwork, both timeframes within the nearest time proximity of the study. Schools have been given pseudonyms and will be discussed further, with brief commentaries on teachers and head teachers.

Ofsted normally inspects a school every four years, with some exceptions. Therefore, there is some variation in the timing of school inspection reports noted here. Ofsted inspection grades are 1/outstanding, 2/good, 3/satisfactory and 4/inadequate.

See Table 6 on the following page for the description of schools by experimental group.

Participating Schools Arranged by Experimental Groups

Groups	**School**	**Type & Location in Cambridge**	**No. of Pupils in Group**	**No. of Pupils in School**	**Ofsted Reports**
Form Groups					
1. *Implicit*	St. Michael's	Community/ Centre	**30**	443 463	3 -Satisfactory Feb. 2009 1 - Outstanding Sept. 2011
2. *Explicit*	St. Michael's	Community/ Centre	**30**	443 463	3 - Satisfactory Feb. 2009 1 - Outstanding Sept. 2011
Melody Groups					
3. Implicit	Lady Gytha	Foundation/ South	**30**	411 500	2 - Good June 2008 2 - Good Nov. 2012
4. Explicit	Lady Gytha	Foundation/ South	**30**	411 500	2 - Good June 2008 2 - Good Nov. 2012
Rhythm Groups					
5a. Implicit	St. Lucy's	Community/ North	**18***	181 172	3 - Satisfactory June 2009 3 - Satisfactory Jan. 2012
5b. Implicit	**5**b. Fremont	Community/ South	**17***	227 202	2 - Good March 2007 2 - Good Nov. 2011
6. Explicit	**6**. St. Peter's	Voluntary aided/ Centre	**25**	211 192	2 - Good March 2008 2 - Good July 2012
Total participants:			**180**		

Table 6 shows the participating schools with details and is arranged by experimental groups.

Note that St. Lucy's and Fremont combined to make one experimental group due to the relatively small number of Y3 students in each school. During the academic year in which the fieldwork took place – 2010/2011 – St. Lucy's combined Y3 with Y4 and Fremont combined Y3 with Y2. The only students whose work was analysed were in Y3 at the time of the study. Y3 totals were 18 from St. Lucy's and 17 from Fremont for a combined total of 35. The mean Ofsted report score for both schools, in reports both before and after the fieldwork, is 2.5.

4.9.5 Further Contextual Details of Participating Schools

The comments regarding teachers' preferences or feelings about music and mathematics are taken from the questionnaires, which they completed at the end of the project.

St. Michael's Primary (pseudonym, as with all school names)

This large community primary school is in the centre area of Cambridge. The details below are according to the school's 2011 Ofsted report. Approximately 33 percent of pupils are from a wide range of different minority ethnic groups. The proportion of pupils known to be eligible for free school meals is below average. The proportion of pupils with special educational needs and/or disabilities is below average (School Ofsted Report, 2011).

The school improved to a very large degree from February 2009 to September 2011, according to their Ofsted report, which went from 'Satisfactory' to 'Outstanding' in this short timeframe. The atmosphere was generally positive and seemed to be productive, similar to the other schools. This was the only school, however, where I became aware that there may be some cultural bias in the way certain students were treated in the classroom, particularly by the assistants. Yet also, more worryingly, in that some children's scores appeared to have been changed in the SAT teacher assessments by a teacher of Year 4. When I asked the teacher about this, she told me that she changed some of the scores because, for example, Nigel (names are changed here) normally does better and Mohammed normally does worse. In retrospect, I feel I should have spoken to the head teacher after the fieldwork had been completed.

The teacher for the *Form Implicit* group did not feel competent to include music as part of classroom activities, and was deterred partly by the presence of a few particularly disruptive students. Yet this programme was supported; it inspired the teacher's desire to have students sing rounds and create stories and lyrics to music in the future as he had witnessed their composition process and was surprised at how creative and cooperative his students could be. This teacher said that teaching mathematics and literacy is enjoyable because development in the pupils is visible.

The teacher for the *Form Explicit* group maintained a highly ordered classroom. Having admitted to never doing music with the students due to lack of knowledge, the desire to implement percussion activities using rhythm charts from the intervention in the future was nonetheless expressed. The questionnaire given to the teachers at the end of the study revealed that it is one of four subjects the teacher enjoys teaching the most; in particular, problem solving is a favourite aspect due to the challenge for the students.

The head teacher was supportive and was impressed with the final concert, noting that the children had memorised a great deal of songs.

Lady Gytha Primary

This large foundation primary school is in south Cambridge. The proportion of pupils from minority ethnic backgrounds is well above average; these pupils from a diverse range of backgrounds. Pupils who are known to be eligible for additional funding are proportionally below average. The proportion of disabled pupils and those who have special educational needs is average, yet the proportion supported at "school action plus" or through a statement of special educational needs is well below average (School Ofsted Report, 2012).

The school maintained a good Ofsted score from June 2008 to November 2012. The atmosphere was generally positive and appeared to be productive, similar to the other schools.

The teacher for the *Melody Implicit* group supported the music lessons well, with a sincere desire for the children to participate and benefit fully. Music in the classroom was not normally included due to lack of confidence, but songs from the programme were possibly to be added to future classroom activities. Mathematics teaching was similar to the other classrooms in the study, though the teacher's favourite subject to teach was physical education, especially outdoor activities.

The teacher for the *Melody Explicit* group supported the music lessons in a less active way during the lessons than the ‘Implicit’ group colleague, but in a more active way outside of the lessons, particularly in terms of communication. Before the intervention, music had been included minimally, but the teacher expressed that due to a gain in the confidence to be able to teach singing, songs from the intervention were to be added to the curriculum. Though history and geography are this teacher's favourite subjects to teach, calculation and time are the favourite aspects of mathematics education. PSHE is the least favourite teaching subject for this teacher due to the conviction that it should be taught through example, not enforcement. This instructor did indeed model kind and thoughtful behaviour.

Both teachers from this school gave me a thank you card with flowers and candy at the end of the concert. The head teacher was supportive, but did not attend the final concert.

Fremont Primary

This small community primary school is just to the south of Cambridge proper. The majority of pupils are from a White British background, though a few pupils are from minority ethnic heritages and a few are not native English speakers. The proportion of students who are known to be eligible for free school meals is lower than found nationally. Yet the proportion of pupils with special educational needs and/or disabilities is higher than that found nationally (School Ofsted Report, 2011).

The school maintained a good Ofsted score from March 2007 to November 2011. The atmosphere was generally positive and appeared productive, similar to the other schools.

The teacher for this half of the *Rhythm Implicit group* was warm and supportive. Learning to sing one song with some movement was included occasionally in the classroom (for example, 2-3 weeks before the school Christmas concert). The teacher remarked that in the future, songs from the intervention were to be added to classroom activities. Mathematics teaching was similar to other groups. The teachers' favourite aspect of mathematics to teach is shape and space while the least is fractions because of perceived comprehension difficulty for the children. The favourite subject is art, with science the least, due to lack of confidence in this area.

St. Lucy's Primary

This relatively small community primary school is in north Cambridge. Approximately half of the pupils are White British, while the rest are of a wide range of minority ethnic heritages. Most of the latter speak English as an additional language. The proportion of pupils who are known to be eligible for free school meals is average. In addition, the proportion of disabled pupils and those with special educational needs is slightly below average (Ofsted report, 2012).

The school has kept a satisfactory Ofsted score from June 2009 to January 2012 and is working on improving. The atmosphere was generally positive and appeared to be productive, not unlike the other schools.

The teacher for this other half of the *Rhythm Implicit group* was energetic and often supportive, though away during many of the lessons. This teacher, the only musician among the seven teachers, sometimes included singing in classroom activities while accompanying students on guitar and helped with the annual school musical. The teacher shared that the warm-up exercises from the intervention were to be incorporated into future class activities. Mathematics teaching was slightly less enthusiastic or focused as other classes. Nonetheless, data handling is this teacher's favourite aspect of mathematics to teach due to the real world connection, while division is the least due to children's perceived difficulty in understanding it. Arts and Humanities are the favourite subjects to teach while science is the least favourite for this teacher.

St. Peter's Primary

This relatively small voluntary-aided primary school is in the centre area of Cambridge. Students come from a wide range of minority ethnic backgrounds. An above-average proportion of pupils do not speak English as their first language, though only a few are at the early stages of learning English. The proportion of pupils who are known to be eligible for free school meals is well below average. The proportion of disabled pupils and those who have special educational needs is well below average. A much larger than average proportion of pupils leave or enrol in this school at times other than at the standard times during the year (School Ofsted Report, 2012).

The school maintained an Ofsted score of "good" from March 2008 to July 2012. The atmosphere was particularly positive and seemed productive, similar to the other schools.

The teacher for this *Rhythm Explicit* group was quite pleasant and supportive. The classroom was managed smoothly, with a quiet firmness and cultured manner that included learning how to say proper greetings and acknowledgements, for example. As mentioned before, though not a musician, this is the only teacher who planned to follow the English National Curriculum guidelines for music. Therefore, since rhythm was featured slightly in the QCA Schemes for England at the time, this class was given its designation.

Though this was the only class in which a certain degree of structured music learning might have been included in classroom activities, my intervention took the place of this, as it provided this goal for the teacher. The teacher revealed that plans learned from the intervention to be used in future music classroom teaching include reading and performing "rhythm patterns [written] in boxes" in classroom activities. This instructor's favourite subjects to teach are in the arts and humanities and the least favourite is PE, particularly football, due to lack of expertise. The favourite aspect of mathematics for this teacher are shape, space and "number work" as the former two are "fun" and the latter work is enjoyable when seeing student growth using methods learned in class. The least favourite aspect of mathematics for this teacher are fractions, as this teacher feels the expectations of the National Strategy in this area are too demanding for this age group.

According to the school Ofsted reports as well as my observations of at least one mathematics lesson from each group, the mathematics teaching was similar across all classrooms participating in the study.

4.10. Ethical Considerations

All research was conducted according to the ethical guidelines of the British Educational Research Association (BERA) (2004) as well as the revised ethical guidelines in the University of Cambridge Educational Research Course Reader (2009, pp. 1-13). In order to truly respect the rights of each child and parent involved in this research, all steps were taken to ensure voluntary informed consent, including making sure the participants knew they could withdraw at any time, that their confidentiality was ensured and that no harm, physically or psychologically, would be caused (BERA, 2004, pp. 1-13; British Psychological Society, 2009, pp. 1-6).

Two ethical issues exist with this design:

First, though the research question was not withheld, the group identifying information (Form Implicit, and so forth) was not mentioned or promoted in order to avoid influencing behaviour, though experience and observation may have revealed the emphasis over time. However, all information was available for the school, teacher, as well as any parent or child at the end of the intervention, and it will still be available if they would ever like to know. Fortunately, every child received lessons and of the same duration and amount as all students in the study. Therefore, there was no issue of equal distribution of teaching and learning time, but only of specific content allocation, which was of relatively little difference.

Second, consent for audio recordings was granted by the participating schools and the teachers, since identifying the students would be difficult with this alone. Consent for video recordings was given by the schools, teachers, children and most parents, with varying degrees of use being requested via a letter and permission form for the parents or carers. This letter gave a background of and motivation for the research as well as including a brief description of the children’s activities and outlining ethical considerations.

Positive points were noted to teachers in case there were concerned parents: 1) recording the lessons keeps them open for judgment while the lessons from the research can help 2) the children as well as 3) the community-at-large. There were only a few parents who did not grant one or more of the requests. The one child whose parents did not grant video recording was never included in any videos and the one child whose parents did not allow participation in the recital (on religious grounds) participated in another activity at the school during the recital. The letter and permission slip for parents are in Appendix 4.

In this section, the research questions, followed by the research design, were presented, discussed and described thoroughly as well as ethical considerations of the study. In order to properly implement this design, and further, to contextualise the research, appropriate and thorough assessment measures needed to be employed. Here, all measures are described in detail.

4.11. Assessment & Monitoring Tools

4.11.1 Introduction

The second part of this chapter will put forth and give details about measures used in this study. Additionally, it will draw attention to some of the challenges encountered in using this battery of measures with a large sample in real-world classroom settings. These include quantitative data gleaned from tests of musical audiation, spatial reasoning and mathematics as well as the subset of proportional mathematics. Mention will be made regarding observations, recordings, interviews and questionnaires, which all served as monitors for the study as well as vehicles for recognising the viewpoints of the participants, both students and teachers. Figure 7 below illustrates the overview of assessment and monitoring tools.

Note that Year 3 students were assessed for relevant abilities at the start of the intervention. Posttests were given at the end of the children's academic school year.

Overview of Assessment & Monitoring Tools

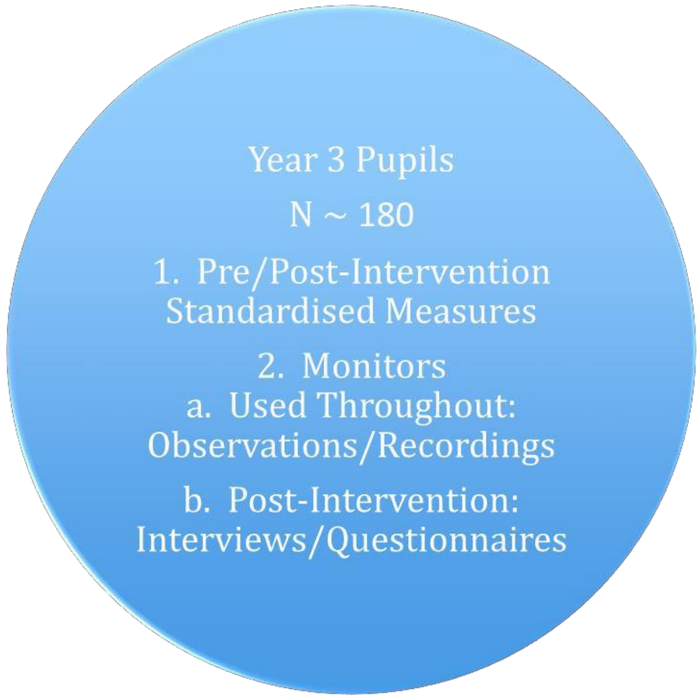


Figure 11 Depicted is an overview of the assessment and monitoring tools used for this study.

4.11.2 Standardised Measures

The pre-intervention and post-intervention tests analysed were: 1) *music audiation* 2) *spatial-temporal reasoning* and 3) *Year 3 mathematics*. The data for these were compared and analysed within- groups and across-groups in order to test the main and sub-hypotheses. Pretests took place after the children, parents, teachers and head teachers granted consent. Children's SAT scores were also collected. Yet due to the inconsistencies of the administration and assessment of these tests, they were not analysed for this study. Below, each measure is described in more detail.

4.11.2.1 Music Audiation

The search for musical aptitude tests was relatively extensive. Reasons for the exclusion of certain tests ranged from obsolescence (Seashore, 1919; Tilson, 1941) to lack of availability (Karma, 1973; 1982; 1993) to protractedness (Webster, 1994). Gordon's *Primary Measures of Music Audiation* (1965, 1986; 2002) was chosen for multiple reasons beyond simply remaining after the process of elimination. This assessment tool is clear and simple to administer, with two subtests: tonal (pitch) and rhythm. Additionally, this test is backed by well-considered theory and development (Gordon, 1965, 1967) and extensive research with tests of reliability and validity (Gordon, 1980, 1986) for standardisation across three regions in the United States.

Gordon defines "audiation" (p.8 in manual) as the perception and finding of meaning in music, whether the sound of the music is externally present. He coined the term because he felt that it is difficult to judge accurately musical "aptitude" until a child is around nine years old and has had a certain amount of musical exposure (Gordon, 1980). This term is now widely used in the fields of music education and music psychology.

Test Development

The test development process started in 1971 (Gordon Main Test Manual, p. 9) and went through several trials using between 200-300 children per grade level each time. After numerous trials in different areas of the country had been conducted – with adjustments made each time to increase reliability – the final testing was completed in 1978, providing norms for each primary grade level from Kindergarten - Grade 3 (Year 1 - Year 4). Grade 2 norms were based upon test results of 280 children in West Irondequoit, New York (of 873 children in total for the final norms referencing procedure). This sample size is relatively small, yet it was reportedly diverse.

Test Statistics, Reliability and Validity

For Grade 2 (Year 3, n = 280), the mean score is 59.7 out of a possible 80. This is standardised to 100 for the mid-range age of 8. The standard deviation is 8.35, the standard error is 2.3 and the reliability (internal consistency) is .92 (Gordon Main Test Manual, pages 87-91). Multiple forms of validity – content, concurrent (criterion-related), congruent, longitudinal predictive – were tested and retested over a four-year period to ensure full test validity, that is, that the test actually does measure what it intends to measure (Gordon Main Test Manual, pages 97-118).

Test Administration and Items

There are 80 items (40 Tonal and 40 Rhythm) and the duration of the test is 40 minutes (20 minutes each subtest), including administration. For each item, children listened to two short musical examples, melodic for the tonal subtest and rhythmic for the rhythm subtest. Then they were instructed to circle two faces that were the same as each other if the musical examples were the same, and two different faces if the examples were different from each other. They were given four practice items before starting the test.

Shortcomings

It would have been preferable if it had been standardised within the UK, but the number of trials and variety of populations used in the development of this test increase the probability that it would apply to a general population of children in the same age range. One key shortcoming is the lack of musicality inherent within this type of test, both in terms of the tone quality of the sounds (created via a Moog synthesiser) as well as the difficulty in assessing "true" musicality with such short listening excerpts.

Webster (2000) addresses this in his test, which involves working with each individual at the piano. The lengthy amount of time this method would take with nearly 200 children, however, made this option unfeasible.

An additional shortcoming is that the standardisation is for a broader age range than is often used by year, rather than by month. Finally, it was surprising to find one clear mistake that somehow passed through all of the testing. Therefore, scores are not entirely accurate. See Appendix 2 for the full test.

4.11.2.2 Spatial-Temporal Reasoning

The *Spatial Reasoning* test by Smith & Lord (2002) was used in this study. For this particular research, it was important to employ tests that include a spatial-temporal component, that is, at least one subtest that requires thinking in time-ordered sequences of spatial transformations rather than general spatial-visual tests that do not require sequential mental manipulation. With spatial-temporal reasoning tests, one must be able to both visually and *sequentially* imagine the solution in *abstract* terms by transforming the object in one's mind, which involves several steps progressing in time. (See below for more details of items.) As noted, spatial-temporal reasoning may relate to musical processing or reasoning, since music is normally sound moving in time. Additionally, current evidence supports this hypothesis as discussed in the literature review.

For example, Hetland's meta-analyses (2000a; 2000b), detailed in Chapter 2, demonstrate the idea that effects of exposure to music are more specific to spatial-temporal than general spatial thinking and much more than to general intelligence.

As stated before, spatial and spatial-temporal tests were originally developed in order to assess nonverbal reasoning. These tests are particularly important cognitive measures for children or adults who may function more optimally or at least at a normal level while thinking (or at least as measured by tests) spatially as compared to verbally, such as those with dyslexia (Smith & Lord, 2002; Mortimore, 2008).

Choosing this test was straightforward, especially since few tests include spatial-temporal reasoning tests. Additionally, it was standardised and tested for reliability and validity in the UK, as well as meeting the criteria for this study.

Test Development

The test development process started in 2000 (Smith & Lord, Main Test Manual, p. 26) and was standardised in 2002 using a national sample from 32 maintained and independent schools proportionally divided across North, Midlands and South of England, Wales and Scotland. Norms for Year 3 were based upon test results of 827 children (413 boys and 414 girls).

Test Statistics, Reliability, Standardisation and Validity

For Year 3, (n = 827), the mean score is 39.0 out of a possible 62 (63%). The standard deviation is 15.07, the standard error is 4.6 and the reliability (internal consistency) is .91. The standardised scale is equated to an average of 100 and each standard deviation to 15 (Manual, p. 19).

Validity of test content was claimed by the authors (Smith & Lord, 2002, pages 4-5), though little detail was provided. Additionally, it was noted (27-28) that other validity measures were not yet possible due to the lack of other spatial tests that were suitable for the comparisons necessary for concurrent validity as well as the lack of the time-frame necessary for predictive validity measurements. One may consider Raven's Matrices for nonverbal reasoning as suitable, yet Hetland (2000b) asserted that these rely more on general logic than spatial-temporal reasoning, and it appears that Smith and Lord concur.

Test Administration and Items

There are 62 items (with 4 subtests: Windows, Hidden Shapes, Jigsaws and Stacks – see below for details) and the duration of the test is 45 minutes, including administration (27 minutes of actual test- taking). Students were given two practice items before starting each subtest. The two subtests that relate most to spatial-temporal reasoning – as they require sequential, step-by-step cognitive processes – are Windows and Jigsaws. Below are brief descriptions of the four subtests.

In the Windows section, students are shown a window with various interconnected shapes juxtaposed within and are told to imagine what the shapes would look like if standing inside a shop, such as a toyshop. These questions assess the capacity to envision shape formations from the opposite spatial perspective. For many of the questions, this requires numerous mental rotation steps due to the relative complexity of the pictures.

In the Hidden Shapes section, or "Hidden Treasures" game, students are instructed to search for a specific shape embedded within a larger, more complex shape. This assesses the ability to hold onto a shape and retain its actual angles and proportions while merging it within the global image. In the Jigsaws section, pupils were told to do jigsaws in their heads by putting the pieces together to make a finished design. This extends the mental process of holding shapes in one's mind required by the previous Hidden Shapes questions to include rotating and combining these images. Therefore, spatial- temporal reasoning is required due to the complex spatial and sequential nature of the task.

In the Stacks section, pupils were instructed to find which shape is on the bottom of a drawing of an imagined stack of shapes. This task assesses an aptitude for envisaging three-dimensions from a two- dimensional image. For a relatively large portion of students in this study, the answers for this section revealed an inverse relationship with the other three subtests. Some of the children who did extremely well on the first three subtests did not do well on this subtest, but many who did not do well on the first three subtests did quite well on this subtest. This suggests that there may be a specific cognitive ability for imagining three-dimensional visual situations that may not be required for the other tasks.

Shortcomings

First, the relatively recent and limited body of research on spatial reasoning necessarily confines conclusions. Second, this specific test contains two mistakes in misnaming two "correct" answers that are actually incorrect. As with the music audiation test, which had one mistake of this sort, I decided to score it as indicated in order to be in line with standardisation comparison measures. Nonetheless, two additional columns in the data file were created with scores based upon the correct answers for future analysis options.

Third, a large portion of one group (Melody Implicit) did not complete the test. This was the only class for which I did not administer the spatial reasoning posttests, due to scheduling. When picking up the tests from the school, I checked to see that they had been done by leafing through at least the first half of most of the tests through to the bottom of the pile. These were completed toward the end of the school year, therefore when I began to mark them two weeks later and found that only two students had completed all four sections of the spatial reasoning tests, it was too late to go back to the school and have the students finish the tests.

All children but two had completed only the first half (two of four sections) of the test for unknown reasons. This illustrates challenges inherent within "real world" research, such as this research conducted in the natural setting of a school. Nevertheless, it seems that the advantages of this methodology still outweigh the disadvantages.

See Appendix 2 for the full test.

4.11.2.2 Mathematics Assessment (MaLT 8)

Group mathematics tests were administered (MaLT 8, Murray, Williams, Wo & Lewis, 2005,

2010) for this research to assess mathematical performance as a measure of mathematical reasoning skills. Items were in line with the curriculum documents for the English National Curriculum and National Numeracy Strategy for England and Wales (English National Curriculum, 2010; YEAR, p. 49) and were based on comparable data from previous research. MaLT (Mathematics Assessment for Learning and Teaching) tests had been and were being used for contemporary studies in the Faculty of Education at the University of Cambridge at that time and were highly recommended. These tests had been developed and updated in the 2000s, and standardised in English state schools. This means that the test norms, as far as possible, were applicable to the present sample and could therefore serve as the basic index for assessment of the relative progress of the children in the treatment groups without the need for a set of control groups who would not have had the musical training. This will be discussed a further in later chapters.

Test Development

Pretests were conducted in 2004 (MaLT Main Test Manual, p. 49) using a representative population from a cross-section of schools and attainment levels. Standardisation took place in 2005 using a nationally representative sample from 120 schools in England and Wales. Norms for Year 3 were based upon test results of 1,358 children (710 boys, 645 girls and 3 of unknown gender).

Test Statistics, Reliability, Standardisation and Validity

For Year 3, (n = 1358), the mean score is 20.8 out of a possible 45 (46.22 %). The standard deviation is 8.50 and the reliability (internal consistency) is .91. The standardised scale is equated to an average of 100 and each standard deviation to 15 (manual, p. 50). Surprisingly, no mention of validity exists in the manual. Yet according to Williams, Wo and Lewis (2005) validation for this test included analyses at the pre-test and main test stages, suggesting that the construct and scale was indeed acceptable (p. 94).

Test Administration and Items

There are 45 questions with six categories interspersed among all questions: 1) counting and understanding number, 2) knowing and using number facts, 3) calculating, 4) understanding shape, 5) measuring and 6) handling data. The duration of the test is 45 minutes, including administration. With the Spatial Reasoning test, all questions were read out loud so that level of reading skill would not be a factor in the results. For the same reason, the MaLT tests, children were encouraged to raise their hands if they would like a particular question to be read to them out loud.

Shortcomings

No apparent shortcomings exist with this test, at least in relation to this research.

To reiterate, the three assessments are age-standardised and have been tested extensively for reliability and validity. General control skills of interest such as attention and memory were not tested due to feasibility in terms of time and money. Furthermore, it seemed most practical to test for skills that might improve as a direct effect of the musical learning. This research is strengthened by the combination of 1) a relatively healthy sample size (n = 180, ~ 150 were tested at both pretest and posttest), 2) age-standardisation of the tests, as well as 3) rigorous statistical analyses, which will be discussed in the following chapter. Below is a summary of the tests showing validity, reliability and limitations.

4.11.2.2 Summary of Standardised Tests

Primary Measures of Music Audiation (Gordon, 1986; 2002)

- Reliability: The standard deviation is 8.35, the standard error is 2.3 and the reliability (internal consistency) is .92 (Gordon Main Test Manual, pages 87-91).
- Validity: content, concurrent (criterion-related), congruent, longitudinal predictive were tested and retested over a four-year period to ensure full test validity (Gordon main test manual, pages 97-118).
- Limitations: 1) Standardised within the US, rather than the UK (though the large number of trials and variety of populations used in the development of this test increased the probability that it would apply to a wider western, English-speaking population of children in the same age range.) 2) Lack of musicality inherent within this type of test, both in terms of the tone quality of the sounds as well as the difficulty in assessing "true" musicality with such short listening excerpts. 3) The standardisation is for a broader age range than is often used, which is by year, rather than by month. 4) There was one clear musical mistake in the publication that had passed through all the testing and proofreading. Therefore, scores are not 100 percent accurate.

Spatial Reasoning test (Smith & Lord, 2002)

- Reliability: The standard deviation is 15.07, the standard error is 4.6 and the reliability is .91 (Smith & Lord test manual, p. 19).Validity: As stated in Chapter 4, validity of test content was claimed by the authors (Smith and Lord, 2002, pp. 4-5), with no specific validity estimate given. In addition, they note (pp. 27-28) that other validity measures were not yet possible due to the lack of other spatial tests that were suitable for the comparisons necessary for concurrent validity as well as the lack of the timeframe necessary for predictive validity measurements.

- Limitations: 1) the relatively recent and limited body of research on spatial reasoning necessarily confines conclusions. 2) This specific test contains two mistakes in misnaming two "correct" answers that are actually incorrect. As with the music audiation test, which had one mistake of this sort, I decided to score it as indicated to be in line with standardisation comparison measures as stated in Chapter **1.** 3) As noted before, a large portion of one group (Melody Implicit) did not complete the test. This was the only class for which I did not administer the spatial reasoning posttests, due to scheduling. Again, this illustrates challenges inherent within "real world" research, such as this research conducted in the natural setting of a school.

Mathematics Assessment (MaLT 8, Murray, Williams, Wo & Lewis, 2005, 2010)

- Reliability: The standard deviation is 8.50 and the reliability is .91 (manual, p. 50). Standard error was not noted.
- Validity: No mention of validity exists in the manual. Yet, as noted in Chapter 4, Williams, Wo and Lewis (2005), stated that validation for this test included analyses at the pre-test and main test stages, suggesting that the construct and scale were acceptable (p. 94).
- Limitations: 1) The lack of a standard error reported for reliability. 2) The lack of a validity report, though validation for this test was discussed in the literature later by external assessors. 3) In relation to this research, the lack of enough items in categories of specific interest for the study, such as pattern recognition and problem solving, which did not allow for appropriate statistical analyses for these categories,
  4) Also in relation to this research and noted in Chapter 4, as well as any testing of logical cognitive processes, questions requiring knowledge of cultural terminology such as “litre” distort assessment.

See Appendix 2 for the full test.

Below Table 7 depicts technical specifications of the three tests analysed, followed by Table 8, which shows test scoring details.

Technical Specifications of Tests Used for the Study.

Name of Test	Author/Year/ Publisher	Norms Samples (Year 3 or equivalent)	Maximum Score/Mean/ Standardised Mean (if age is mid-range, i.e., 8 years)	Standard Deviation/Standard Error	Reliability (Internal Consistency)
1. Primary Measures of Music Audiation	Gordon, 1979/ GIA Publications	280	80/59.7/100	8.35/2.3	0.92
2. Spatial Reasoning 8	Smith & Lord/2002/ GL Assessment	827	62/39.0/100	15.07/4.6	0.91
3. MaLT 8 (Mathematics)	Murray, Williams, Wo & Lewis/2010 new ed./ Hodder & Stoughton	1358	45/20.8/100	8.50/not reported	0.91

Table 7 The table above shows technical specifications of the tests analysed for the study.

Test Scoring Details.

	Music Audiation	Spatial Reasoning	Mathematics
Number of Categories within st	2	4	6
Subscores input for analysis including composite scores	3: a. Rhythm b. Tone c. Composite	5: a. Windows (perspective rotation) b. Hidden Shapes (proportional retention) c. Jigsaws (extension of b with shape rotation and combination) d. Stacks (3-D perception) e. Composite	7: a. Counting and understanding number b. Knowing and using number facts c. Calculating d. Understanding shape e. Measuring f. Handling data g. Composite
Errors detected w/in test itself	1	2	None
For future analysis: Scored versions due to errors in test itself (i.e., two composite scores)	2	2	1

Table 8 The table above shows test scoring details.

4.11.2.2 SATs

The most recent Standard Assessment Tasks (SATs) scores in mathematics for the children were collected in order to verify that the groups were relatively equivalent and to control for possible aptitude differences. Since this study took place at the start of Year 3, the children had taken these assessments at the end of Year 2 as they were finishing Key Stage 1 (English National Curriculum, 2010).

Furthermore, SATs from students who were at the end of Year 2 the prior year (hence being in Year 4 during the intervention when this experimental group was in Year 3) were gathered in order to further assess whether the schools were indeed comparable to each other in this way as well as to compare Year 3 participants in the study to Year 3 students of the year before for additional validity. Yet, for reasons noted below, SATs were not formally analysed.

First, since these tests for Key Stage 1 were administered and assessed by the classroom teachers, there was missing data and there were general inconsistencies, seen primarily in the extremely varied ways of marking, recording and reporting the results of these tests. In many cases, the teachers had a column for their own assessment next to the students' initial marks on the test. These adjusted assessments indicated the mark that the teacher felt the student *should* have received, based upon their expectations or opinions. One teacher openly expressed disdain for the tests, which may have influenced subsequent care in recordkeeping.

As noted before, these irregularities led to a second serious concern beyond the issue of accurate dates, scores and method of assessment. The case in which I noticed that some "final" scores had been raised from the actual performance scores and yet some "final" scores had been lowered, was discussed with the teacher as mentioned after I had become concerned when I saw that the names next to the raised scores were English names and the names next to the lowered scores were Bangladeshi. Therefore, since there was potential evidence of systematic bias in the assessments, the validity of the scores was further undermined. Investigating this concern is beyond the scope and focus of this study, but there is a large body of research that shows consistently widespread bias, whether unconscious or conscious, among teachers as shown through large-scale research and assessments in multiple countries, including UK and US (Tenenbaum & Ruck, 2007; Ouazad, 2008; Burgess & Greaves, 2009).

A third concern regards the apparent low level of reliability and validity of SATs in general. A large body of research questioning the reliability and validity of these tests confirms this suspicion (Harlen, 2004; Baird & Black, 2013). Fourth, because of the use of the more uniformly administered and marked standardised tests (Music Audiation, Spatial Reasoning & Mathematics - MaLT 8) used for this research, there was already a large amount of reliable and valid data for subsequent rigorous analysis.

Therefore again, for the above reasons, SATs were not formally analysed.

4.11.3 Monitors and Vehicles for Voice

Classroom observations, lesson recordings, student interviews and teacher questionnaires provide contextual information and function as monitors to ensure that the research is on track. These activities were not conducted in order to be analysed, as this is a large quantitative study without the resources to analyse qualitative data as well. Rather, this material confirms that the classroom instructors taught mathematics in similar ways, the music teacher taught music (and did not teach mathematics) to all six groups similarly, the children enjoyed the lessons, and the classroom teachers supported the intervention and offered other creative activities during the school year as well. It was helpful to know that they all carry out other creative activities and that they responded equally positively to the music programme provided via the research. Had there been a negative response, this could have confounded the results. In addition, the interviews sought to gain even a bit of insight into how the children might connect the two domains of music and mathematics in their minds. Furthermore, observing the teachers' classroom teaching and hearing about their experience, in addition to the children's voices honours them as an integral part of the study, not in a statistical data sense, but in a holistic sense.

4.11.3.1 Observations

Classroom observations of mathematics lessons were carried out in order to gain a general sense of teaching styles and competencies among the teachers.

4.11.3.2 Recordings

Audio recordings were made during autumn term then video recordings were made in spring and summer terms in order to have evidence of the style, methods, consistency and validity of the teaching in relation to the research question.

4.11.3.3 Interviews

At the end of the learning intervention, in consultation with the teacher, students were selected to take part in interviews. These were conducted with a representative percentage (approximately 30%) of students of both genders with a range of abilities from each class, therefore over fifty in total. Students were asked 1) how they felt about music and mathematics, 2) what they thought about the connection between music and mathematics, 3) their experiences with music and mathematics outside of school and 4) their experiences with music and mathematics in school – including the intervention. Interviews were approximately 10 minutes each. Questions alternated between those asking about music or mathematics first in case there might be a sort effect due to order, such that attitude to one influences attitude to the other. The interview questions and several excerpts from the interviews are in Appendix 3.

4.11.3.4 Questionnaires

Questionnaires were given to the teachers at the end of the intervention. They were asked about any musical or mathematical activities already in the school, their experience of teaching mathematics and other subjects, and my teaching as well as their opinion of the overall programme, including a request for suggestions or comments.

The questionnaire is in Appendix 3. The form given to each teacher had more space between questions than the one presented here. These questionnaires have provided a fuller context and allowed the more in depth description of students, schools and teacher perspectives that was given.

Table 9 below specifies the assessment and monitoring tools used for this study. Pretests and posttests (music, spatial reasoning and mathematics) were administered during the first four and last four weeks of the fieldwork, respectively.

Assessment and Monitoring Tools

Assessment Measures & Monitoring Data Collected for All Six Groups	Name of Test	Assessment Scheduling	Duration	Analysis or Purpose
Primary Data				
1. Music Audiation	Primary Measures of Music Audiation	Pre- & Post-Intervention	40 minutes	MIXED ANOVA, reliability tests
2. Spatial-Temporal Reasoning	Spatial Reasoning	Pre- & Post-Intervention	45 minutes	MIXED ANOVA, reliability tests
3. Mathematics - Year 3	MaLT 8	Pre- & Post-Intervention	45 minutes	MIXED ANOVA, reliability tests
Supporting Data				
4. SAT Scores in Numeracy	SAT (Standardised Assessment Tasks)	Pre-Intervention	45 minutes (flexible timing)	Supporting
Monitors				
5. Observations (Mathematics Lessons)	N/A	During Intervention	40 minutes	Monitoring for validation. Provides contextual details.
6. Recordings (Intervention)	N/A	Throughout Intervention	40-minute lessons	Monitoring for validation. Provides contextual details.
7. Interviews (Students)	N/A	Post-Intervention	10 minutes	Monitoring for validation. Provides contextual details.
8. Questionnaires (Teachers)	N/A	Post-Intervention	Varies with each Teacher	Monitoring for validation. Provides contextual details.

Table 9 The table above depicts the way the data were used: either as primary data for analysis or supporting data for validation.

4.12. Conclusion

The first part of this chapter puts forth the research questions and hypotheses with detail and justifications. It is followed by the research design, including a comprehensive depiction of the schools and teachers, drawing largely from Ofsted reports and teacher questionnaires in addition to observation. This section closes with a discussion of ethics in this study.

The second portion of this chapter presented and described multiple assessment and monitoring tools used in order to rigorously, multi-dimensionally evaluate potential effects on mathematical understandings due to the intervention – an extended period of musical learning within a school.

In summary, the fieldwork in this research included systematic weekly music lessons for nine months in each of the five participating schools with nearly 200 children aged seven to eight years (Year 3). Participating children from all six groups were considered the experimental groups in this study. Since all the tests used were UK standardised tests, all children in the United Kingdom other than study participants were considered the control group.

Lessons emphasised form, melody or rhythm; in half of the classes, the teacher made the musical-mathematical parallels explicit. There were two independent variable categories – musical focus (three types) x mathematical emphasis (two levels) = six groups.

Apart from elements of the specific musical-mathematical foci, the lesson content was kept as constant as possible within primary school settings. Details regarding the intervention will be given in Chapter 5.

Pretests and posttests in behavioural measures of musical, spatial and mathematical thinking were administered. Statistical analyses of student scores in these tests observed change over time while considering differences among the three musical components and two pedagogical conditions for each.

As noted, the intervention was recorded virtually throughout the entire programme and the students and teachers were both asked to evaluate it from their own perspectives. This was for monitoring purposes and to contextualise the study by giving voice to all participants involved. The recordings, interviews and questionnaires were not for analysis, as this study is designed to be solely quantitative research. This is why relatively large numbers were required with as much randomisation as possible and standardised tests were used, with the results analysed. The next chapter will provide full details of this intervention.

Chapter 5
The Intervention: Content and Teaching

5.1 Introduction

This chapter describes the music lessons intervention from the perspective of my role as the teacher. In contrast to the other chapters in this thesis, it is written in the voice of the teacher as opposed to that of researcher. In planning my fieldwork, I asked numerous questions, such as: What is the most practical and enjoyable way to teach music in the schools that is available to me? How can I use music in a way that supports and enhances mathematical thinking in children? How can I awaken in the students their inherent musicality while imparting concepts that are shared by both disciplines such as patterns, symbols, structure, sets and problem solving?

The teaching was founded within a rich theoretical framework that includes philosophical and methodological aspects of Jaques-Dalcroze (2000) and Alexander (1932) in terms of kinaesthetic awareness, Kodály (1974) and Glover (Bennett, 1984) in terms of hand movements and movable-do solfege, Orff (Schumacher, 2013) in terms of instrumentation and improvisation, Suzuki (1969) in terms of creating a caring environment for learning and the importance of high quality music modelling and using early years auditory receptiveness, and a gentle bel canto singing technique.

5.2 Intervention

5.2.1 Curriculum

The curriculum of the intervention lessons was compatible with the English National Curriculum for music in the schools and was appropriate for this age group. It was comprised of music from around the world, although most of it was western-based. This curriculum was presented to the teachers prior to the start of the programme. Since the goal was to learn to actively *make* music while learning *about* music as well, the curriculum consisted of building a repertoire of songs while learning musical concepts, singing, instrumental and analytical skills particularly relevant to the musical mode of the group.

It was good to see that the English National Curriculum guidelines (English National Curriculum, 2010) for Year 3 did not expect the teacher to teach notation, since an additional goal of this research was to explore ways to teach organically, for the sake of the children and the teachers. It was hoped that students would experience the music, beyond listening and factual knowledge.

Additionally, since many primary school teachers do not read musical notation, they are nonetheless expected to teach at least a bit of music, therefore being able to teach them organically, at least in the early years, is important. By organic, I mean using children's natural resources such as their ears, their voices and their bodies (for clapping, tapping and movement).

5.2.2 Timeline and Number of Lessons

The full course ran almost ten months, approximately three weeks short of the full English primary school academic year:

27 September 2010 - 22 July 2011.

Given holidays and teacher training days, all groups (7 classes, each taught separately) had exactly 27 weekly 40-minute group music lessons over this time period. Sessions for taking pretests and posttests as well as concert performances are not counted in this total. Pretests and posttests were started exactly 9 months apart: 27 September and 27 June, respectively. During the four-week pretest period, the schools allowed extra time of approximately one hour. This allowed students to sing for 5-8 minutes at the end of each test as a reward and a preview of what was to come. During the four-week posttest period at the end of the academic school year, full lessons were given at different times than the testing times. Concerts were performed for the most part during the final week, and students were given congratulatory certificates, marking the full gestation of the project.

5.2.3 Music Lessons

I taught and conducted activities with all groups (180 total students) for quality, consistency, reliability and internal validity. The majority of the lessons were audio recorded from September and video recorded from January, with parental permission as well as the children's consent, for monitoring and reference purposes. These group music lessons were based upon singing, with additional musical activities included that corresponded to the particular musical mode of the group (form, melody, rhythm). More specifically, the lessons consisted of rudimentary but essential fundamentals of vocal technique, but primarily focused on learning songs, which naturally comprised ear training. Further into the programme, lessons also consisted of movement and other supplementary musical activities such as playing the glockenspiel for the melody groups, or percussion for the rhythm groups.

Fundamentals of proper vocal technique were taught, not just for the sake of singing beautifully, but in order to develop the tuning, flexibility and range necessary for singing more expansive, varied and complex songs. This complexity enabled the children to learn and experience music of a higher order than that of elementary children's songs, which in turn – according to the hypothesis – activated their minds to a larger degree. Furthermore, good technique, which includes breathing for example, supports wellbeing and health beyond vocal health (Welch, et al., 2008).

The students' ultimate repertoire comprised 11 polished and memorised pieces, which were performed at the final concert held at each school. After the concerts, all of the head teachers were quite pleased with the performances, three out of the five head teachers noted how impressive students' musicality and memorisation were and one commented on the authenticity of the world music arrangements and performances. Parents were also delighted and students were excited and seemed very happy and proud.

See list of songs in Table 10 on the following page, which shows the different approaches to a single song to emphasise a specific mode of music, Form, Melody or Rhythm, as specified in the research design.

Songs for Fieldwork

Form Groups	Melody Groups	Rhythm Groups
1. Hello from All the Children of the World (International)	1. Hello from All the Children of the World	1. Hello from All the Children of the World
2. Freres Jacques (France)	2. Freres Jacques	2. Freres Jacques
3. Scarborough Fair (England)	3. Scarborough Fair	3. Scarborough Fair
4. My Favourite Things (Austria)	4. My Favourite Things	4. My Favourite Things
5. Tafta Hindi (Arabia)	5. Tafta Hindi	5. Tafta Hindi
6. Let's Learn from Each Other (America)	6. Let's Learn from Each Other	6. Let's Learn from Each Other
7. Sakura	7. Sakura (Japan)	7. Sakura
8. Gbodi – The Gazelle (Sudan)	8. Gbodi	8. Gbodi
9. Jamaica Farewell (Jamaica)	9. Jamaica Farewell	9. Jamaica Farewell
10. Woodland Wild (Composed by students)	10. Kookaburra (Australia)	10. Kookaburra
11. Coulter's Candy (Scotland)	11. Swiss Hiking Song (Switzerland)	11. Coulter's Candy

Table 10 Songs for Fieldwork.

The representative country or region of each song is noted in parentheses in Table 10 above. All groups learned the first nine songs listed above, out of the total of eleven songs. An additional song in each group's repertoire was shared by three other groups, and one song was unique to each musical mode group, as it featured the musical component of focus. All songs, however, were at the same level of difficulty and in a similar upbeat style appropriate for children.

More specifically, two musical mode groups (Melody and Rhythm) learned "Kookaburra," and two groups (Form and Rhythm) learned "Coulter's Candy," which features catchy rhythmic patterns and calls for accompanying clapping and bouncing. This coordination was more difficult for them than one might imagine. See reflections in Figure 12 below.

Reflections on Pupils' Rhythm Connections

2 February 2011

As I have now introduced clapping and body movements into the picture, I am seeing some surprising results. The rhythm groups* in particular this week tapped at their tables and clapped along as well, specifically on the final chorus of “Let’s Learn from Each Other.” Even though it is a ballad, it is a bit of a power ballad and seems to inspire them to move with the slow, but strong beat. OK, my keyboard playing gives them a hint as well. Yet generally, they seem to feel compelled at times to move to the music.

Adding clapping with synchronous movement, that is, clapping at the same time as bending one's knees, appears to have raised the difficulty level, even though one might assume that the synchrony of the movement would render the additive element relatively simple to carry out. It was therefore surprising that in each of the five classes that anticipated in rhythmic activities (Form and Rhythm groups) there were children who clearly struggled with being in sync, even with repeated instructions, demonstrations and efforts.

So far children in the Fremont class – comprised of Y2 and Y3 students – have done the best. Perhaps it should not be surprising, since their teacher has done this type of activity with them as shown during the Polish Christmas song they performed for me.

In the future, I would introduce movement a little earlier, perhaps right away for a tiny bit the end of learning a new song starting out with just clapping then adding the bouncing later. I isolated the song learning primarily based on teaching older children and my own process of isolating and mastering one thing at a time. Bel canto training, which only allows singing on vowels for the first seven years, is an extreme example of this.

Yet, just as with language acquisition at a young age, multiple forms of linguistic information come in at once and integrate naturally. Perhaps it is the same with music. It is more natural to move while making music. Activation of the cerebellum and motor cortex, even while only listening to music, suggests that movement and music have evolutionarily been entwined.

*Note: The five classes above include the two Rhythm groups and the two Form groups. Form groups experienced a balance of some rhythm and some melody in their repertoire.

Figure 12 Extract from post-lesson reflections. “Fremont” is the anonymised name for the school.

Form groups composed their own song, "Woodland Wild." They were the only ones to do so, due to the differentiation between groups. It was fitting for the Form groups to create their own work, which contained three distinctive yet cohesive parts, as their focus was on the overall structure of a musical piece. The Melody groups exclusively learned the Swiss Hiking Song as it features melodic leaps for the yodelling parts throughout. As mentioned above, the Rhythm groups learned "Coulter's Candy," due to its rhythmic nature. In terms of additional rhythmic or melodic activities that accompanied the songs, Form groups were given a balance of each of these musical traits so that they were fully participating in the music making, while not leaning too much into either melodic or rhythmic realms for the sake of the study.

5.2.3.1 Rationale for Song Choice

The majority of the songs were folk songs, each representing a different country. The reasoning for the multiculturalism was to be inclusive and to encourage appreciation of other cultures. This does not relate directly to the research itself, though it provides variation and can encourage an openness to explore and learn, musically as well as culturally. Core aspects of the folk songs themselves, however, directly help to answer Research Question 2*, which requires the application of three different musical foci across groups. The simplicity and place in history of these pieces allow for multiple arrangements and modes of performance as these have been open to interpretation for centuries and most do not have one iconic way it must be performed in the minds of others. (*Research Question 2: *Does focusing on specific musical elements while teaching enhance understandings of possible corresponding mathematical concepts?)*

The songs selected were simple enough for children to learn relatively well with a bit of work yet challenging enough to encourage growth as well as the state of "flow" (Csikszentmihalyi, 1988). As noted in the literature review, flow has been described as the mental state that arises when fully immersed in an activity. Custodero, 1998, suggests that it occurs while the balance of enjoyment and challenge is optimum.

Table 11 on the following page provides an example of one song, "Scarborough Fair," showing brief summaries of different treatments for each of the three groups. Though all groups learned this song, different arrangements and pedagogical foci were given to each musical mode group that emphasised different musical components and explicitness of mathematical references. Across all songs, each group had the same amount of additional activities, although the activities themselves varied according to musical mode.

Examples of Different Arrangements for the Same Song to Emphasise Particular Musical Modes

Form/Stricture	**Melody/Pitch**	**Rhythm**
Song: Scarborough Fair	**Song: Scarborough Fair**	**Song: Scarborough Fair**
Changes to song itself: Kept this the same as the original, at a moderate tempo.	**Changes to song itself:** Kept this the same as the original, at a moderate tempo.	**Changes to song itself:** Kept this the same as the original, at a slightly faster tempo.
Origins & Meaning: English folk song about unrequited love, referring to a medieval fair (fayre) in Scarborough, England. Briefly discussed riddles and symbols in lyrics.	**Same**	**Same**
Analysis: Form: AAA or Strophic form (one-part song form) with refrain. Discussed how many refrains there might be and why. (There is more than one correct answer.)	**Analysis:** Melody: Dorian mode (whole half whole whole whole half whole step scale). The song opens with an intervallic leap of a fifth.	**Analysis:** Rhythm: Metre Simple triple (expressed with 3/4 time signature). Pattern Long-short pattern occurs often. Tempo□ Moderato to Allegro.
Accompaniment: Added introduction and transitions between verses. Stately, with clear	**Accompaniment:** Arpeggiated accompaniment; emphasised melismatic	**Accompaniment:** Primarily blocked and slightly blocked chords, rhythmical accompaniment with downbeat emphasised.

transition into each new verse.	singing (more than 1 note per syllable).	
Arrangement/Activity: Movement: Proper English country dance Instruments: both lute-like strings and percussion	**Arrangement/Activity:** Movement: Proper English country dance Instruments: stringed lute-like instruments	**Arrangement/Activity:** Movement: Proper English country dance Instruments: Tambourines, triangles and frame drums
Mathematical Explicitness Group: (About 2-3 minutes) I drew timeline map of form on board of both simple AAA form. Then I asked the class to suggest what form it might be if we included the introduction and transitions between verses. I drew form ideas on a horizontal line as well as other shape possibilities.	**Mathematical Explicitness Group:** (About 2-3 minutes) I drew melodic shapes on board and asked what geometric shapes come to mind. I drew Dorian scale (mode) and the intervallic leap of the fifth on board. We discussed measurement when noting distances between notes in scale and interval.	**Mathematical Explicitness Group:** (About 2-3 minutes) I counted and conducted 3/4 metre. I drew rhythm chart on board for portions of song (showed counts 1-2 as long note and count 3 as short note, so 2 out of the 3 are held = 2/3). I asked them what idea in maths comes to mind (fractions). I asked about which corresponding fractions might match. I then transferred these to a pie chart.

Note that in addition to differing arrangements to accord with a particular musical mode, ideas for bridging into mathematical correlates are shown in Table 11 above. Almost all songs (9 out of 11) were given differing treatments that worked with the song and yet aligned with the emphasised musical mode as well. The other two songs (Coulter's Candy – with strong rhythmic component, and the Swiss Hiking Song – with melodic jumps for yodelling effect) inherently featured one of the three musical modes and therefore were not learned by all groups.

5.3 With Primary School Teachers in Mind

Does a teacher need to be musically trained in order to include music for enlivening the classroom and inspiring numerical ideas? Activities were conceived with the goal of supporting teachers who may not feel confident teaching music due to limited musical training or experience. This section contains fuller examples of lesson activities within a conceptual framework that includes consideration of teachers with a range of musical experiences. The section begins with a broader example of an activity that was carried out with all of the groups as well as a corresponding musical analysis. This is followed by examples of activities with each musical mode group featured in turn. A full circle will have been made with an outline of a typical singing lesson applicable to all groups is put forth.

5.3.1 Analysis and Creative Music Notation

For the first two lessons, the primary activity was listening to and analysing music through illustration, which prepared students for some of the ideas we would be discussing throughout the full course. The children were asked to draw the music they heard in whatever way they would like to do so. There were no rules; they were simply encouraged to make a picture of the sound as it travelled through time and their musical focus was mentioned. For example, to the Melody groups I would simply say something like, "Notice the lovely melody."

Having the freedom to create their own way of illustrating, notating or “tracking” the music heightens awareness of musical proportions and patterns, in addition to being an expressive outlet. Through monitoring compositional structure, melodic contours, intervallic distances and rhythmic patterns within a musical piece, for example, children can learn about relationships among other things in life such as geometrical shapes and proportions of objects, as well as patterns and solutions that one may find in mathematical problems.

Many of the drawings were noteworthy and all were expressed in a variety of styles. Several examples looked like actual sound waves at different frequencies. Some were particularly creative and lovely; some were mixed with standard notation. I showed both classes some examples, including those that looked like sound waves. The first day the students drew with pencils, then the second day they used coloured pencils or crayons, therefore the variety increased even more. With each group, we discussed the drawings in terms of their main musical focus (form, melody or rhythm).

It is fascinating to see how creative and yet relatively accurate children’s illustrations of music can be. In effect, they created their own notational systems, often with inspiring, even impressive results. Importantly as well, they openly expressed their enjoyment and took pride in their "masterpieces."

Table 12 on the next page provides a brief analysis of the musical piece, "Le Cygne" (Saint-Saëns, 1886), in terms of the three featured musical elements (form, melody and rhythm). Following that is a brief example of this work with students in the Form musical mode group. Additionally, for each of the "mathematical explicit" groups within each mode group, I briefly noted the connection between the relevant musical ideas and corresponding mathematical ideas while teaching, whereas in the "mathematical implicit" groups I did not mention mathematics at all, even though mathematical elements such as form, proportion and patterns exist in the music implicitly and *were* discussed.

Three brief Analyses, each emphasising one of three musical modes featured in this study of "Le Cygne"("The Swan") by Camille Saint-Saëns (1886) from *Le Carnaval des Animaux* (Carnival of the Animals)

Form analysis.

Ternary (ABA)

Introduction: 2 bars | A: 16 bars (8 + 8) | B: 16 bars (8 + 8) | A (recapitulation): 8 bars | Transition to closure: 8 bars | Coda: 5 bars

Melody analysis.

The melody has a balanced, elegant shape throughout, played by a solo cello representing the swan. The piece opens with a descending line and is followed by ascending scalar motion. the repeat of the 1st part of the opening melody, a chromatic variation of the Part A melodic theme follows. Part B features a sequential descending-ascending pattern. After the recapitulation of Part A, the transition to closure is once again a series of descending, then ascending lines, yet with each phrase ending with a tonal suspension. These undulations and suspensions seem to reflect the swan moving gently on the water, which is musically represented by the continual piano arpeggiation underneath.

Rhythm analysis.

The rhythm is in 3/4 metre (3 quarter notes per bar) and consists mainly of the stately quarter notes (crotchets) and dotted half notes (dotted minims) of the swan juxtaposed with continuous sixteenth notes (semiquavers) of the water below. In the 2nd portion of the transition to closure, the two entities finally part, each taking its turn. In the coda, the water (piano) returns to its usual flow of sixteenth notes beneath the final suspended tone of the swan (cello).

Table 12 Analysis of "Le Cygne"("The Swan") by Camille Saint-Saëns (1886, pub. 1921).

5.3.2 Creative Notation Activity with Form Groups

After explaining why different parts of a musical composition can be labelled as Part A, Part B, and so forth, I included the concept that when a musical section is repeated with slight changes, then it would be labelled with a "prime" marking such as A' for the slightly altered return of part A. We then listened to "The Swan" with that in mind. I asked them to raise their hands when part A came back, which most of them were able to do. I then drew a map of the parts on the board, similar to a timeline with A-B-A going left to right. With the mathematically explicit group, I compared ABA to shape and asked what shape has three sides, to which most replied, “Triangle.” Then I drew a triangle on the board, labelling B as the bottom side, as it was the one that was different from the other sides.

Similarly, throughout the nine-month programme we discussed musical components of *all* of the pieces students were learning to sing (and also play in some cases), particularly in terms of the musical mode on which they were focusing.

5.3.3 Composing Music with Form Groups

Creating musical compositions develops problem-solving capabilities that can transfer to problem solving in mathematics. Through exploring different possibilities while creating a cohesive musical piece, such as a song, the teacher can then present mathematical problems and show that these too can be solved via exploring different approaches. By facilitating their own explorations, perhaps in small groups, students can discover that solving mathematical problems can be very similar to solving musical problems. It can be presented as similar to putting pieces of a puzzle together. In groups or as a class, pupils can discuss possibilities for solutions, therefore solving problems together, prefaced by or followed with going over the steps taken and ideas used.

The Form groups (2 classes both in the same school) in my research composed a song together called "Woodland Wild." (See Figure 13, p. 145 for lyrics.) I facilitated throughout, and together, we created three distinct parts – A, B and C – as well as a coda. This activity may require that the teacher have a little bit more musical training, depending upon the complexity of the song being created and whether it will be notated or recorded.

5.3.4 Showing Contours with Melody Groups

Again, all groups learned virtually the same songs even though different musical elements were featured during the learning process. Warm ups for the melody groups were similar to Kodály's moveable- do solfège warm up, with the hands showing the vertical place on the scale as each note is sung. During a song, children were sometimes asked to depict pitch changes with their hands, especially when they were still learning the melody. An extension of this activity that reinforces melodic learning is to show the broader melodic shape of a song with one’s arms while singing it.

Standard melodic notation also offers an opportunity to unite auditory and visual depictions of music and can be expressed by the children kinaesthetically as well. Melodic notation evolved to illustrate the contours of sound logically, therefore a person without any training can nonetheless follow the general directional flow and structure of the music as depicted by the notation. Teachers can directly include mathematical ideas by helping pupils discover and illustrate geometric shapes via comparing the shapes of melodies to visual shapes. Additionally, measurement concepts can also be taught when illustrating specific distances between notes and comparing them to the distance between the floor and the table top for example.

Games in which several short melodies are depicted visually on the board (whether via standard melodic notation or general, artistic illustrations of melodic contour) can fully engage students. As each melody is played or sung, children are asked to match the one they are hearing with the appropriate illustration. This again encourages students to pay attention to shapes, patterns and structures, and holds the potential for them to transfer these skills to geometry and architecture. Conversely, pupils can choose a geometric shape first and then choreograph or compose a piece that goes with that shape.

Maintaining the theme of “songs from around the world,” melody groups sang the "Swiss Hiking Song", a folk tune with yodelling. Yodelling is a fun way to feature melodic changes since this style of singing contains dramatic leaps, resulting in the distinctive tonal shift that occurs when jumping from a lower vocal register to a higher vocal register. All groups sang an Arabic folk song, "Tafta Hindi" ("Cloth from India"*)*, containing exotic melodic elements. This was one of the students' favourite pieces.

5.3.5 Counting Development and Using Charts with Rhythm Groups

Rhythm activities that encourage numeracy can feature counting while tapping, clapping, stomping or playing a percussion instrument to a pulse (musical beat or tactus). Counting is essential for young children’s comprehension of foundational mathematical principles and involves higher levels of understanding than one might expect, such as understanding the concept that the final number is the total rather than simply the end of a phrase (Gelman & Gallistel, 1978). In more sophisticated counting, children may use thinking strategies (Cobb & Merkel, 1989 in Anghileri, 2001). For example, if a child knows that 8 + 4 = 12, then he might strategise that 7 + 5 must be 12 (Anghileri, 2001, p. 19).

Through learning music, strategic thinking with arithmetical applications like this example above can be fostered when rhythmic values are “borrowed” within a bar. For example, in 4/4 time, if the child changes the first note of the bar (or measure) from one count to two counts (hence doubling its temporal value), she must then make another note half the temporal value in order to keep the total counts (or beats) within the bar the equivalent of 4 quarter notes (or crotchets).

By using simple charts, the teacher can lead, and anyone can participate whether or not they read standard musical notation. A chart presents a very clear visual depiction of time going by, as monitored by counting. This design contains six connected squares lined up horizontally in a structure I call a "rhythm matrix." (See Figure 11 below.) Each box corresponds with a pulse within the musical phrase. The teacher can then put an "X" in the boxes where the children are to clap, tap, stomp or play their percussion instrument. Empty boxes indicate silence, though time is still passing by. Therefore the counting continues, whether aloud or internally, as students must keep track of time moving forward. Therefore, they are working with patterns. Once students become comfortable with this, they can create their own rhythm patterns by putting "X's" into the boxes of their choice. It is often quite enjoyable for children to hear how their own pattern sounds when classmates are participating in this percussive activity.

Including a second series of boxes below the first series with "O's" in some of the boxes, indicating another rhythmic part, makes an arrangement. Children can be assigned different parts. For example, one side of the room can clap where the "X's" are and the other side can stomp where the "O's" are. Alternatively, children with triangles can play the "X's" and those with Congo drums can play the "O's." Then they can switch parts. Fun patterns can be created and then changed for continual challenge.

1	2	3	4	5	6
X		X		X	
	O		O		

Figure 13 Rhythm matrix used in fieldwork intervention.

Figure 11 above shows an example of a simple rhythm matrix. This contains two layers indicating different parts. A different action or instrument can be assigned for each letter, "X" or "O." Note that the sixth pulse is silent for both groups, which can add a surprise element and keep them counting and focused in a more challenging way. The 6-box chart works well with metres such as 3/4, 3/8, or 6/8.

This activity leads naturally toward learning about fractions, of which I briefly spoke with the "Mathematical Explicit" groups. For example, if three out of the six boxes have "X's" in them, I asked the children to express that ratio as a fraction (3/6). I also asked them what portion of the whole is silent. In this case, silence is in the same proportion as sound. Then I extended this conceptual learning by asking pupils to name what an equivalent fraction might be (e.g., 1/2). If only two boxes are marked, as with the O's, then the fraction would be 2/6, with an equivalent being 1/3, and so forth. The fractions within the matrices can then be transferred to a pie chart for further clarity and additional illustrations of the concept.

Since we were learning songs from around the world as part of my study, the two songs for which we used the 6-box rhythm matrix were a Sudanese children's playing song, "Gbodi" *("*The Gazelle"*),* and a Japanese folk song, "Sakura" ("Cherry Blossoms"*).* For the latter piece, the rhythm groups used a rhythmic introduction with a popular Japanese chant – nam-myoho-renge-kyo – containing six syllables, one for each beat. (The melody groups sang this on a pentatonic scale for their introduction while playing the scale on glockenspiels, with "nam" functioning as a pickup note on the same pitch as "myo.") With their voices and hands, children depicted the rain (and depending on how loud the students played their drums, the storm) that is needed for blossoms to thrive. Use of this imagery, along with variety in accented rhythms, dynamics or the entrance timing (for example, a second group can join in on "myo," which is the 2nd pulse), created an exciting atmosphere, fully engaging the students. Most of the children were concentrating and appeared to be developing cognitive skills without even realising it.

Teachers can use all of the above examples whether or not she or he has had musical training. If the teacher does not play the piano or guitar, if a variety of accompaniments is desired, or more freedom to work with students is wanted, recordings of musical examples can be used and instrumental versions of songs without the vocal guide track can challenge the students further and be used for performances as well.

5.3.6 Singing with All Groups

Throughout the course, the students and I discussed musical components of all of the pieces that they were learning to sing (and also to play in some cases), particularly in terms of the musical mode on which they were focusing.

Below is a skeleton of a typical lesson (based on singing as the instrument):

- Greet and warm up
- Learn a new song
  - Teacher (me) models the song line by line, with children repeating each line back, in a call and response pattern
  - Put lyrics up on the whiteboard
  - Sing the whole song together

- ❖ Work a bit extra on tricky parts
- ❖ Analyse and discuss the song, particularly in terms of the musical mode of the group*
- ❖ Sing the song once or twice again, with awareness of what we just spoke about

♦ Review and sing familiar song(s)

- ❖ Remind students of things to notice, such as
    - Musical and/or vocal parts that need correcting or polishing
    - Expression and meaning in the song
    - Musical elements particularly relating to the musical focus for the group
- ❖ Add additional vocal part, movement or an instrument to the song

♦ Sing, move and/or play entire song for fun finale (either the previous song or another familiar song) followed by applause.

5.3 Conclusion

Through talking about the music they are hearing, yet more importantly, through taking action by drawing it or making the music themselves, students become aware and sensitised, and can appreciate music at a new level, even to the point of spontaneously creating new arrangements and musical works themselves.

As evidence continues to grow regarding the positive link between musical processing and mathematical thinking, it may encourage policy makers to include consistent music lessons in the curriculum. In addition, teachers may be more inspired to use music as a tool and a bridge toward helping their students to understand mathematical concepts with more ease. We have a responsibility to our children to seek creative ways to help them not merely to learn, but to love learning.

In the following chapter, quantitative analyses using mixed ANOVA of the Music Audiation, Spatial Reasoning and Mathematics tests will be described in detail.
Additionally, results of supporting data will be informally described.

Endnote

Below are three figures (Figures 14, 15 and 16). First is a brief description of one day in the life of the fieldwork for this research. This is followed by lyrics of the song, which students co- wrote in the form groups, and then third is a sample of the end-of-year concert programme for the fieldwork.

A Story Behind the Study

16 December 2010

Today was an extremely cold, blustery, icy day – a continuation of the winter freeze that had gripped much of England. I had one last lesson before Christmas break at the furthest school out of the five at which I teach – St. Lucy's. The night before, I typed out lyric sheets, complete with pictures of extra holiday songs that had been requested. I planned to stay an extra 15 minutes in order to play carols for everyone to sing.

It was a dramatic bike-ride down with the treacherous roads, the snow settling on my glasses to combine with the fog already blinding me due to my comparatively hot breath. In addition to the ice, the wind slowed me down as well, so that my 45-minute cycling journey took over an hour.

I would find moments to encourage myself while on South Trumpington Road when admiring the beauty of the icy trees rising from the earth on my left. The expansive Cambridge football fields were white and glistening with snow and ice. I was only allowed millisecond glances, however, if I wanted to avoid sliding into a pole or worse. What really kept me going, especially when the next hill seemed too much or the wind gust almost knocked me over, was envisaging the faces of the smiling children, excitedly greeting me and happily enjoying their music lesson.

I arrived, wet and tired, but glad to be there all in one piece. The bike was locked up, my helmet secured, nose blown and layer upon layer of clothing was removed. One deep breath and a smile; I'm ready to teach. As I entered the gymnasium, which leads to the classroom, I saw and heard many children singing. There was the teacher whose class I teach playing the guitar in the front. Oh, no, I wondered, have I trekked here through the hour plus of blizzard in vain?

Indeed. The teacher had forgotten to tell me that the school had scheduled rehearsal for the winter concert during our usual lesson time.... Onward….

Figure 14 Fieldwork reflection

Woodland Wild
By "St. Michael's" Year 3 & Edel Sanders
Form: AABABCB

Verse 1 (Part A)
Mister squirrel's running with his tiny feet.
Jumping from branch to branch, his wife to meet.
"We must gather food for our family! We must be strong for our sporting spree!"

Verse 2
Pretty bird soars across the splendid sky.
Navigation expert, she is meant to fly!
"I must prepare for the winter nights! I love to plan for my distant flights!"

CHORUS (Part B)
Woodland wild, filled with life – home for lots of friends!
Birds, squirrels and imaginary dragons – what a lovely place to be!!

Verse 3
Magical creature, through the woods he goes.
Sniffing for berries with his giant nose.
"I smell some fruit in the thorny tree. Delicious feast for my friends and me!"

CHORUS
Woodland wild, filled with life – home for lots of friends!
Birds, squirrels and imaginary dragons – what a lovely place to be!!

Bridge (Part C)
The ecosystem goes round and round.
All the clouds in the sky surround!
Rain comes down to the ground, helping all the seeds to grow!

CHORUS

Figure 15 Lyrics of song composed with Form groups

St. Peter's Primary School
Year 3
Presents:

Songs from Around the World!

- Hello to All the Children of the World
- Let's Learn from Each Other! – America
- Sakura (Cherry Blossoms) – Japan
- Gbodi (The Gazelle) – Sudan
- Scarborough Fair – England
- Coulter's Candy – Scotland
- Frère Jacques (Brother John) – France
- Kookaburra – Australia
- Tafta Hindi (Cloth from India) – Arabia
- Jamaica Farewell – Jamaica
- My Favourite Things – Austria

Figure 16 Sample concert program for one of the groups

Chapter 6
Analysis of Quantitative Measures

6.1 Introduction

Chapter 5 describes the intervention in detail. This chapter justifies and presents the statistical analyses of the standardised tests for this study and briefly notes student interviews and teacher questionnaires that were conducted for the study as contextualising and monitoring tools.

In order to test the hypotheses stated in Chapter 4 regarding the potential effects of the intervention, multiple analyses were conducted on the test score data that would indicate any differences in the students' musical, spatial and mathematical achievement over time. All test scores were double marked for validity and inter-rater reliability. Differences between all groups were explored using two-way mixed ANOVAs with post hoc comparisons.

Using the Bonferroni correction, the potential effects of nine months of sustained music learning on the three achievement areas tested were investigated and are illustrated and discussed in detail here. This chapter presents the results with relatively little discussion since the next chapter will offer both a synthesis and a more thorough discussion of the findings.

6.1.1 Analysis of the Standardised Measures

The analyses used for the complete set of student test data from the music aptitude, spatial reasoning and mathematics standardised tests as well as the proportional mathematics subcategory encompassing all six groups were two-way mixed analysis of variance (ANOVA). By taking all of the data into account, the ANOVA tests for inter-group differences and can be an effective test in detecting details within the data. The statistical formula in ANOVA calculates a value to indicate the degree of differences in the means by comparing the within and between group variances using the *F* statistic, therefore yielding an estimated marginal means.

Specifically, the two-way ANOVA is suitable here because there are two independent variables: 1) *time* (pretest and postest times) and 2) *teaching intervention.* Teaching interventions were implemented via three different levels of musical foci (form, melody or rhythm), each with two levels of mathematical emphasis (implicit or explicit). A mixed ANOVA is appropriate because the same participants were used for the first independent variable – *time* (as this is a repeated measures design) and different participants were used for the other independent variable – *teaching intervention* (six different groups) (Field, 2009). In summary, these analyses were employed using age-standardised scores for pretests and posttests in behavioural measures of the children in musical, spatial and mathematical thinking.

6.1.2 Multiple Comparisons and Justification for Using ANOVA

Multiple-comparison procedures on a single set of data risks making Type I errors (false positives) since for each additional hypothesis tested, there is a corresponding increase in the probability of identifying at least one significant result due to chance (Field, 2009; Napierala, 2012).

Yet in this study, changes over time for each of the dependent variables, that is, *test scores* (Music Aptitude, Spatial Reasoning, Mathematics and Proportional Mathematics) are analysed separately. In other words, comparisons of multiple aspects or subsets within each dependent variable were not made in each analysis; one test was examined at a time, therefore reducing the possibility of making Type I errors. Additionally, according to Howell (2007), addressing limited and specific questions prior to performing the analyses (*a priori*), as is the case here, also reduces the potential error rate.

The Bonferroni correction applied to probability values is required when several dependent or independent statistical tests are performed simultaneously on a single data set (Simes, 1986; Napierala, 2012). Therefore, to be cautious, as more than one hypothesis had been put forth, Bonferroni adjustments were made to the ANOVA via SPSS. Furthermore, if significance levels are defined at a more conservative level than perhaps is necessary, .001 rather than .05 for example, then this conservative approach could risk making a Type II error (false negative), and therefore losing statistical power (Field, 2009, p. 56; Howell, 2013). Even though most of the comparisons, particularly in the Music Aptitude tests as well as the primary dependent variable, Mathematics (including Proportional Mathematics) tests were statistically significant even at this stricter level of probability, the significance threshold was set at .05 for all statistics based on convention.

6.1.3 Research Questions

Below are the research questions for reference:

Research Question 1

How far, if at all, can school-based musical learning primarily via the voice improve mathematical achievement?

Research Question 2

Does focusing on specific musical elements while teaching enhance understandings of possible corresponding mathematical concepts?

Research Question 3

Does teaching music with brief yet explicit references to hypothesised mathematical correlates enhance children's corresponding mathematical skills, and if so, does it do so more than teaching music without these references?

6.2 Analysis of the Music Aptitude Tests

This section is dedicated to showing whether children improved in Music Aptitude scores beyond norms, since this is an essential prerequisite for assuming that any gains in mathematics might be associated with the music intervention. Due to the lack of suitable tests available, these scores have been age-standardised to the US population rather than the UK population, which would have been preferred. Standardisation is in the same age range as the students in this study (with 100 being the norm average) in order to determine improvement beyond that which may occur due to normal maturity.

Therefore, any improvement in standardised test scores indicates that students improved to a higher degree than other children did of the same age on average. For example, if a student whose score was 100 (the mean of children's scores of the same age on this test) improves to the same degree that other children of the same age have improved (who function as controls) over the same time period, then the standardised score would go from 100 at pretest and remain 100 at posttest even though the raw score would have risen. On the other hand, if a student's raw score stays the same after nine months, then that student's posttest age-standardised score would decrease since scores are expected to rise with age. One could therefore consider, for this study, that the "control group" has a population score of 100 for the age-standardised pretest and posttest scores, which is the mean standardised score.

If students' mean Music Aptitude scores have indeed improved over time beyond norms or the population mean, based upon the main hypothesis, this would lead one to suspect changes in mathematical skills beyond the norms as well. Without musical development, which is fundamental to this study, changes in mathematics scores would not necessarily show that the hypothesis is true.

6.3 Two-way Mixed ANOVA for the Music Aptitude Test

6.3.1 Pre-testing for Parametric Suitability

Outliers and Homogeneity of Variance

Prior to running an ANOVA, the data needs to be checked for outliers and homogeneity levels. Additionally, reliability tests were run in order to ensure that the test is reliable for the specific sample used, in this case Year 3 children from five schools in Cambridge, UK.

There are no outliers in the data, as illustrated by the boxplot in Appendix 1. Music Aptitude standardised scores are normally distributed for all interventions at both time points, as assessed by the Shapiro-Wilk's test showing normal distribution ($p > .05$). (See the appendix for corresponding table.)

There is not homogeneity of variances for the pretest scores, as assessed by Levene's Test of Homogeneity of Variance, $p < .05$ and Welch's more robust test, $p < .001$. The ANOVA is run nonetheless, as it is fairly robust, while specific significance interpretations are made with caution (Field, 2009, p. 155).

Homogeneity of Co-variance

There is not homogeneity of co-variances, $p < .001$, as shown by Box's test of equality of covariance matrices in Appendix 1. When musical modes are assessed separately, Form groups are the groups that do not have homogeneity of covariance at pretest and therefore have contributed to the overall lack of homogeneity. The primary interest, however, is in improvement levels from pretest to posttest.

It is notable that Music Aptitude tests and Spatial Reasoning tests (to be shown in the next section) do not have homogeneity of variances or co-variances at pretest, yet they do at posttest. Education in these cognitive domains is not formally regulated in schools and therefore it is not surprising that there would be variation at the beginning of the intervention. It is therefore possible that the shift from non-homogeneity at pretest to homogeneity at posttest is due to the musical intervention.

Similar to treatment of the homogeneity of variance test, even without homogeneity of co-variances, ANOVA is often run (Field, 2009, p. 604).

Reliability Analysis

The Music Aptitude pretest subscales achieve very good internal consistency, as shown by Cronbach’s alpha, $\alpha = .88$. Likewise, the Music Aptitude posttest subscales also have good internal consistency, α = .78, though not as high as the pretest subscale reliability score for this sample.

Key: Good = .70 ≤ α ≤ .90. See Appendix 1 for full reliability table (Kline, 2000; George & Mallery, 2003**)**.

6.3.2 Results of the Two-way Mixed ANOVA for the Music Aptitude Test

Results of the two-way mixed ANOVA performed on the Music Aptitude test are given in this section and are accompanied by visual depictions of the findings.

Profile Plots Indicating Gains

Figure 17 below illustrates that all groups improved over time on standardised Music Aptitude scores. In other words, mean scores of all children in the sample increased well above the norms or population scores. The graph indicates a strong *main effect* for Time and Group, with Form Implicit and Melody Explicit being the least dramatic. Additionally, it shows that homogeneity of variance moves from unbalanced to relatively balanced, following nine months of consistent music lessons.

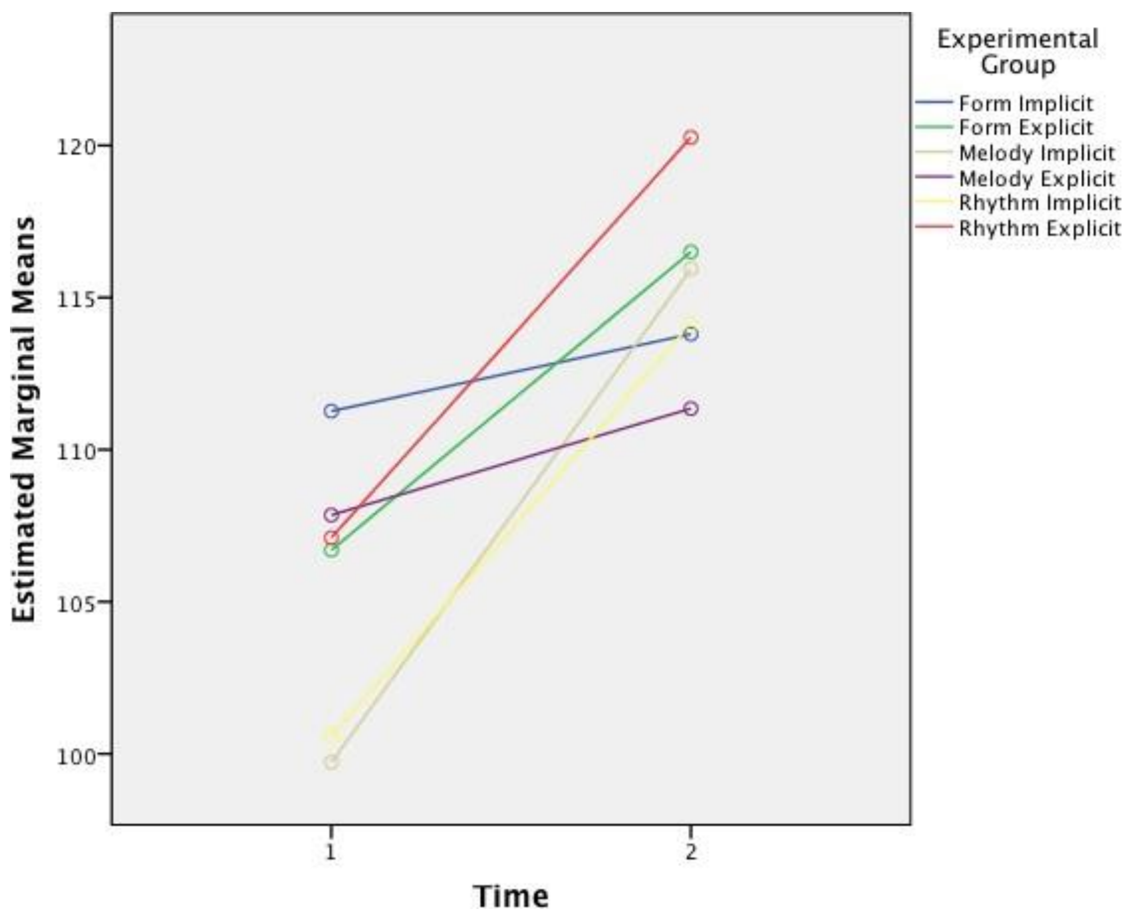


Figure 17 This graph shows change in the treatment groups' Music Aptitude scores over time.

Figure 17 above illustrates that all groups' Music Aptitude test scores improved considerably over time, compared with the test norm.

Children's Music Aptitude scores show a statistically significant estimated marginal mean gain from pretest to posttest (-9.80), $p = .000$ ($p < .05$). Therefore the *main effect of time* shows a statistically significant difference in Music Aptitude test scores from pretest to posttest, $F(1, 108) = 93.42$, $p = .000$ ($p < .05$), partial $\eta^2 = .46$ (very large effect size[11]).

[11] Effect sizes are standardised measures that demonstrate the magnitude or power of a test statistic. Eta-squared, including partial eta-squared, indicates the proportion of variance in the dependent variable that is related to a factor. That is, a main effect or interaction, excluding other sources. Guideline for two-way ANOVA (partial η^2): .02 = small; .13 = medium; .26 = large (Cohen & Manion, 1994).

The *main effect between groups* shows that there is a significant effect in Music Aptitude scores *between* intervention groups yet with a small effect size: $F(5, 108) = 2.63$, $p = .033$ ($p < .05$), partial $\eta^2 = .11$ (small effect size). Indeed, due to randomisation, it was expected that the gain scores *between* groups might be small. In other words, differences in children's gains between groups such as Melody and Rhythm were similar, that is, children in different groups improved to a similar degree.

To clarify, the result in this test that is most relevant to the primary research question – mean improvement in children's Music Aptitude scores over time, that is, the main effect of time – is statistically significant, with a very large effect size. Referring to Figure 20 above, one can see that the average starting and ending scores for the six groups are somewhat different from each other, yet as noted, the main phenomenon examined for this study regards whether the scores have increased for all groups from pretest to posttest.

Music is in the National Curriculum for England and Wales (English National Curriculum, 2011), although the suggested methods within the framework are not mandatory and it is often treated as a low priority subject (for example, Russell-Bowie, 2009). Therefore, it could be expected that variation in musical skills may exist at pretest with greater uniformity being achieved after nine months of weekly music lessons, particularly when the same instructor is teaching, as in this study.

The scores of the Form Implicit group, due to a higher starting level, demonstrated a phenomenon similar to a ceiling effect in improving to a lesser degree in comparison to the other four groups that improved to a large degree though there was still room to grow, as was demonstrated by the rhythm groups. All groups nonetheless displayed relatively similar levels of improvement.

6.3.3 Summary of Results in the Music Aptitude Tests

This portion has reported the results of Music Aptitude tests taken at the beginning and end of the school year.

Overall, students in all groups performed significantly higher on age-standardised Music Aptitude posttests (M = 115.33, SD = 7.79) than on age-standardised pretests (M = 105.53, SD = 10.51). Age-standardised mean scores went from 105.53 to 115.33, going from slightly more than 5 points (5.53) above the population mean at an "average/age-appropriate" level to level to 15.33 points above, which is well above the mean normative value for this age group. In other words, all groups' margins of improvement are significantly higher on posttests than students of the same age in the UK are, as shown by the standardised scores.

The *main effect of time* in ANOVA shows a statistically significant difference in Music Aptitude test scores from pretest to posttest, $F(1, 108) = 93.42$, $p = .000$ ($p < .05$), with a very large effect size ($p = .000$, partial $\eta^2 = .46$).

The *main effect of group* shows that there is a slight statistically significant effect in Music Aptitude scores *between* intervention groups yet with a small effect size: $F(5, 108) = 2.63$, $p = .033$ ($p < .05$), partial $\eta^2 = .11$ (small effect size).

Tukey post hoc tests, showing differences between groups in estimated marginal means, were not significant, ($p > .05$), because groups improved by relatively similar degrees due to a similar intervention, as was expected.

To conclude this section, these statistically significant improvements in Music Aptitude scores are key to the foundation of this study, since the main hypothesis states that mathematical scores will improve following nine months of musical training. It is essential therefore to have shown the effectiveness of the music lessons through demonstrating substantial advances beyond the age norms in musical cognition.

6.4. Analysis of the Spatial Reasoning Tests

This section shows whether children improved in Spatial Reasoning scores beyond norms. As with the Musical Aptitude tests, these scores have also been age-standardised, yet in this case the test was standardised on a UK population rather than a US population (also with 100 being the norm average). To review, standardisation is done in order to determine improvement beyond that which may occur due to normal maturity. Therefore, any improvement in standardised test scores indicates that students improved to a higher degree than other children did of the same age in the UK on average.

Most students' mean Spatial Reasoning scores have improved over time beyond norms or the population mean. This evidence supports the argument that spatial reasoning ability may improve with musical experience as discussed in the literature review.

6.5. Two-way Mixed ANOVA for Spatial Reasoning Test

6.5.1 Pre-testing for Parametric Suitability

Outliers and homogeneity of variance

There are no outliers in the data, as illustrated by the boxplot in Appendix 1. Spatial Reasoning standardised scores are normally distributed for all intervention groups at both time points, as assessed by the Shapiro-Wilk's test ($p > .05$). (See Appendix 1 for corresponding table.)

There is not homogeneity of variances for the pretest scores, as assessed by Levene's Test of Homogeneity of Variance, ($p < .05$). Yet Welch's more robust test ($p < .001$) is not significant at this level, $p = .014$. Therefore, similar to the Music Aptitude test, the ANOVA is run nonetheless, as both are strict (Welch as well as the ANOVA itself, particularly with the Bonferroni correction), while specific significance interpretations are made with caution (Field, 2009, p. 155).

Homogeneity of Co-variance

There is not homogeneity of co-variances, as shown by Box's test of equality of covariance matrices, $p = .000$ ($p < .001$). The primary interest, however, is in improvement levels from pretest to posttest.

Once again, it is notable that neither the Music Aptitude tests nor the Spatial Reasoning tests have homogeneity of variances or co-variances at pretest, yet they do at posttest. As mentioned before, these cognitive domains are not formally regulated in schools and therefore variation at the beginning of the intervention is perhaps to be expected. Therefore, the shift from non-homogeneity at pretest to homogeneity at posttest may be due to the musical intervention.

As before, even without homogeneity of co-variances, it can be acceptable to run the ANOVA (Field, 2009, p. 604).

Reliability Analysis

The Spatial Reasoning pretest and posttest subscales have acceptable internal consistency, $\alpha = .62$ and $\alpha = .63$ respectively.

Key: Acceptable = $.60 \leq \alpha < .70$ (Kline, 2000; George & Mallery, 2003). See Appendix 1 for full reliability table (Kline, 2000; George & Mallery, 2003).

6.5.2 Results of the Two-way Mixed ANOVA for Spatial Reasoning Test

Results of the two-way mixed ANOVA performed on the Spatial Reasoning test are given in this section and are accompanied by visual depictions of the findings.

Profile Plots Indicating Gains

The graph below indicates that student age-standardised scores of all groups in the sample increased above the population scores. The graph also illustrates a strong *main effect* for *time and group*, as is shown in the statistics below. This suggests that consistent music lessons over time (nine months) contributed to improvements in Spatial Reasoning scores for all groups, as hypothesised.

Melody Explicit and Rhythm Explicit groups started out at a higher level than the other groups within the study and at a markedly higher level than the average child of the same age within the UK population as well (6.29 and 9.44 points above, respectively). Therefore, both improved to a lesser degree than other groups perhaps due to a ceiling effect phenomenon.

Melody Implicit results are distorted due to the mysterious occurrence at posttest that was described in Chapter 5.

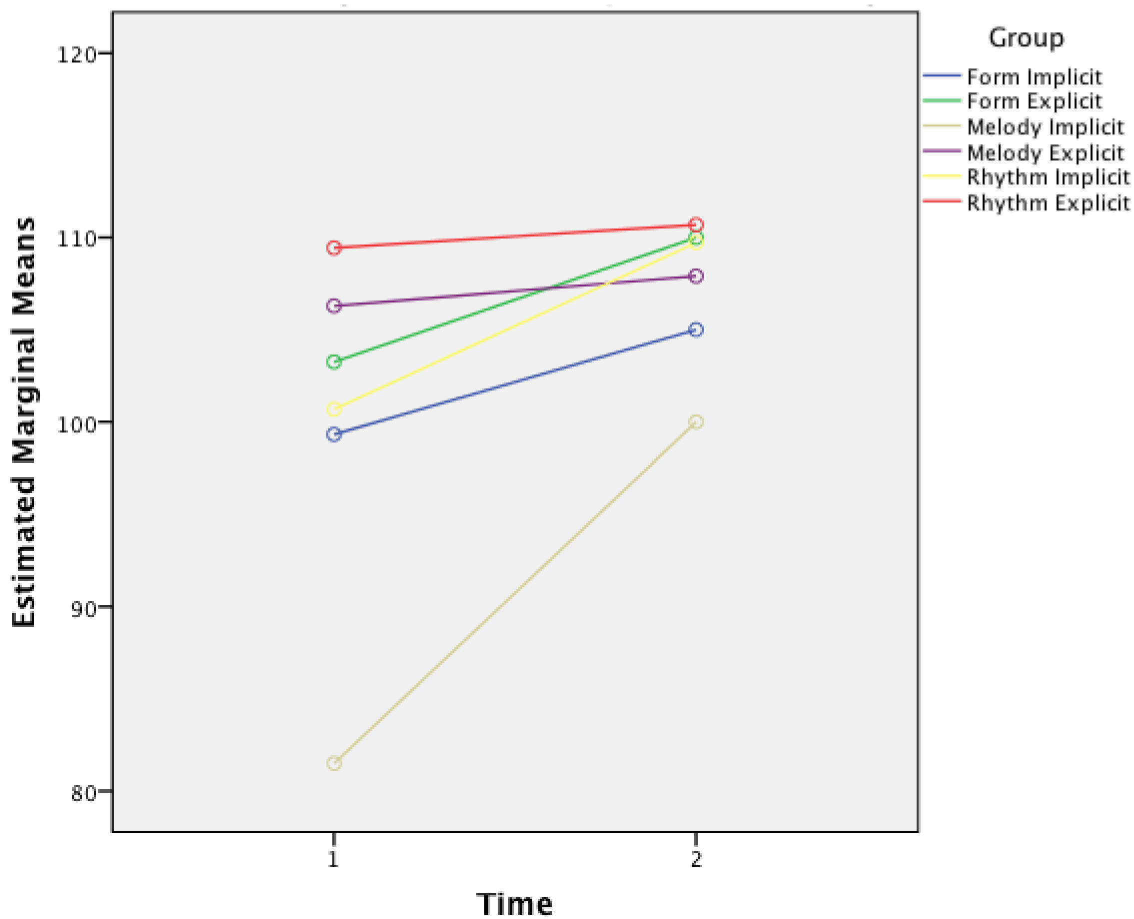


Figure 18 This shows change in the treatment groups' Spatial Reasoning scores over time.

Figure 18 above shows that children's Spatial Reasoning scores increased considerably. These scores reveal a statistically significant estimated marginal mean gain from pretest to posttest, (-7.00), p = .000, (p < .001). This supports the hypothesis that spatial reasoning is a particularly relevant cognitive construct, sharing aspects of both musical and mathematical cognition, therefore often serving as a cognitive bridge.

The main effect of *time* shows a statistically significant difference in Spatial Reasoning test scores from pretest to posttest, $F(1, 104) = 24.32$, $p = .000$ ($p < .05$), partial $\eta^2 = .19$ (medium effect size).

As illustrated in figure 21 above, there is a strong *main effect of time*. This shows a statistically significant difference in Spatial Reasoning test scores from pretest to posttest, increasing from an estimated marginal means of 99.76 at pretest to 106.76 at posttest; $F(1, 104) = 24.32$, p =

.000 (p <.05), partial $\eta^2 = .19$ (medium effect size). Therefore, there is a statistically significant estimated marginal mean gain in Spatial Reasoning scores from pretest to posttest (7.00), $p = .000$, ($p < .05$). This suggests that consistent music lessons over time (nine months) may have contributed significantly to improvements in Spatial Temporal Reasoning scores for all groups, as hypothesised.

The *main effect of group* shows that there is not a statistically significant difference in Spatial Reasoning scores *between* intervention groups $F(5, 104) = 2.05$, p>.05 (.079), partial $\eta^2 = .09$ (small effect size).

This non-significant effect regards mean differences at pre- and post- time points, not gain scores. On the contrary, as seen above, Spatial Reasoning mean gain scores show relatively large increases for all groups from pretest to posttest, which are statistically significant.

Similar to the Music Aptitude scores, the Spatial Reasoning scores were less balanced at pretest, with greater uniformity being achieved by posttest. (In contrast, it will be shown that Mathematics scores were more balanced across groups at both points.)

To review, the two groups with higher starting levels (Melody Explicit & Rhythm Explicit) improved to a small degree, and the Melody Implicit group (with only two taking the posttest, hence making the group itself somewhat of an outlier) improved to a very large degree. The other three groups improved to a large degree.

Additionally, as with the Music Aptitude test, this increase in uniformity in two of the groups could also be due at least in part to the consistent music lessons across the student population. This may have helped increase the balance of spatial awareness scores at posttest, therefore explaining the shift from non-homogeneity at pretest to homogeneity in Spatial Reasoning tests at posttest as well.

6.5.3 Summary of Results in the Spatial Reasoning Tests

On average, students in all groups performed significantly higher on age-standardised Spatial Reasoning posttests (M = 106.76, SD = 11.29) than on age-standardised pretests (M = 99.76, SD = 12.92). The age-standardised mean scores went from 99.76 to 106.76, with a mean gain of 7.0 points, going from slightly below the national average to 6.76 points higher than the mean normative value for this age group.

The *main effect of time* in ANOVA, that is, the difference in Spatial Reasoning test scores from pretest to posttest, is statistically significant with a medium effect size which is summarised here: $F(1, 104) = 24.32$, $p = .000$, (p <.05), partial $\eta^2 = .19$ (medium effect size).

As noted above, the statistically significant estimated marginal mean gain in Spatial Reasoning scores from pretest to posttest (7.00, $p = .000$), supports the hypothesis that spatial reasoning is a particularly relevant cognitive construct, sharing aspects of both musical and mathematical cognition, therefore often potentially serving as a cognitive bridge.

The *main effect of group* shows that there is a statistically significant difference in spatial reasoning scores between intervention groups $F(5, 104) = 2.05$, $p = .033$ ($p < .05$) (.079), partial $\eta^2 = .09$ (small effect size). This small effect regards mean differences at pre- and post- time points, as was expected, not mean gain scores, which did increase significantly.

Tukey post hoc tests, showing differences between groups in estimated marginal means, were not significant, ($p > .05$), because overall, groups improved by relatively similar degrees as each other.

In review, the two groups with the ceiling effect phenomenon (Melody Explicit & Rhythm Explicit) improved to a small degree, and the Melody Implicit group (with only two taking the posttest) improved to a very large degree. The other three groups improved to a large degree. Again, this increase in uniformity in two of the groups could also be due at least in part to the consistent music lessons across the student population.

As a whole, student scores in Spatial Reasoning tests in this sample increased well above the population mean scores. In other words, on average, all groups improved beyond other UK children of the same age, who function as controls in this study.

6.6 Analysis of the Mathematics Tests

This section is dedicated to showing whether children improved in Mathematics scores beyond norms. As noted with the Spatial Reasoning tests, the scores have been age-standardised to the UK population (with 100 being the norm average) in order to determine improvement beyond that which may occur due to normal maturity.

Students' mean Mathematics scores have indeed improved over time well beyond norms, or the population mean, therefore based upon the statistically significant results, the main hypothesis is shown to be true.

6.7. Two-way Mixed ANOVA for the Mathematics Test

6.7.1 Pre-testing for Parametric Suitability

Outliers and Homogeneity of Variance

There are no outliers in the data, as illustrated by the boxplot in Appendix 1. Mathematics standardised scores are normally distributed for all interventions at both time points, as assessed by the Shapiro-Wilk's test ($p > .05$) (See Appendix 1 for corresponding table.).

There is homogeneity of variances, as assessed by Levene's Test of Homogeneity of Variance, pretest, $p = .95$; posttest, $p = .44$ ($p > .05$) also shown in Appendix 1.

Homogeneity of Co-variance

There is homogeneity of co-variance, as assessed by Box's test of equality of covariance matrices, $p = 1.00$ ($p > .001$).

(See Appendix 1).

Reliability Analysis

The Mathematics pretest items have excellent internal consistency, $\alpha = .92$. Likewise, the Mathematics posttest items also have excellent internal consistency, $\alpha = .90$.

Excellent = $\alpha \geq .90$ (Kline, 2000; George & Mallery, 2003). See Appendix 1 for full reliability table (Kline, 2000; George & Mallery, 2003).

6.7.2 Results of the Two-way Mixed ANOVA for the Mathematics Test

Results of the two-way mixed ANOVA performed on the Mathematics test are given in this section and are accompanied by visual depictions of the findings.

Profile Plots Indicating Gains

Figure 22 below illustrates that all groups improved over time on standardised Mathematics scores and mean scores of all children in the sample increased above the norms or population scores. This graph indicates a strong main effect for Time and Group, with Form Implicit being the least dramatic.

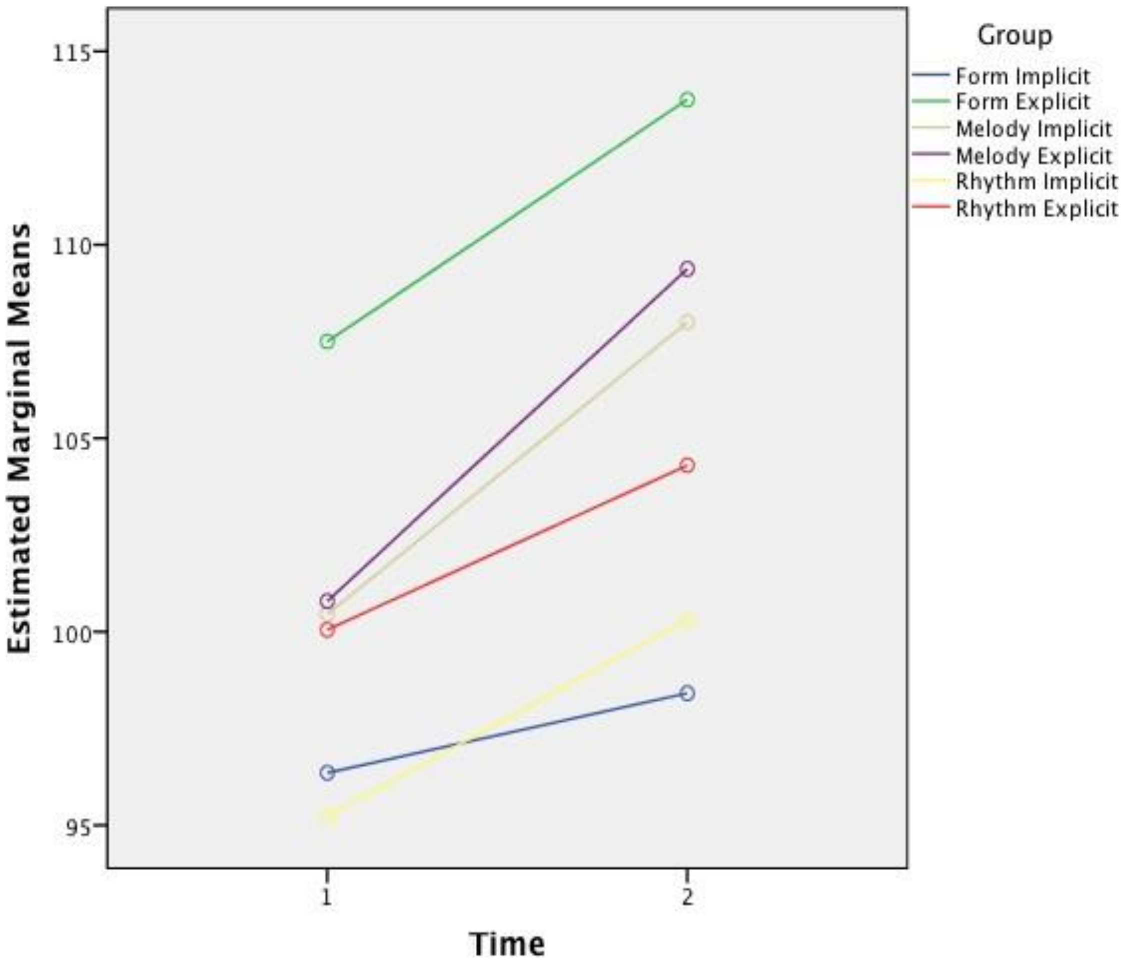


Figure 19 This shows change over time in the treatment groups' Mathematics scores.

Figure 19 above illustrates that all groups significantly improved over time on students' standardised Mathematics scores. Therefore, any improvement in these standardised test scores indicates that students improved to a higher degree than other children did of the same age in the UK on average. To review, the "control group" has a population score of 100 for the age-standardised pretest and posttest scores, which is the mean standardised score in the UK. One can see here that scores of all groups in the sample increased well above the population mean scores.

As illustrated, there is a strong *main effect of time*. This *main effect of time* shows a statistically significant difference in Mathematics test scores from pretest to posttest, increasing from an estimated marginal means of 100.06, SD = 8.73 at pretest to 105.69, SD = 9.16 at posttest; $F(1, 125) = 53.29$, $p = .000$ ($p < .05$), partial $\eta^2 = .30$ (large effect size). Therefore, there is a statistically significant estimated marginal mean gain in Mathematics scores from pretest to posttest (5.63), p =

.000, (p < .05). This suggests that consistent music lessons over time (nine months) contributed significantly to improvements in Mathematics scores for all groups, as hypothesised.

The *main effect of group* shows that there is a statistically significant difference in Mathematics scores between intervention groups $F(5, 125) = 3.04$, $p = .001$ ($p < .05$) partial $\eta^2 = .11$ (small effect size). Yet since all groups had a similar intervention (music lessons using the same songs though with different arrangements and instruments) albeit with different emphases (form, melody, rhythm) and slightly different levels of mathematical explicitness (implicit, explicit), it was not expected that there would be large differences in change over time between groups.

When looking at individual pairs, one can see that the Explicit groups improved to a greater degree than the Implicit groups on average. Yet only Form groups showed statistically significant Mathematics gain score differences between Implicit and Explicit groups. Nonetheless, those effects were not strong enough to change the overall gain score significance levels. Referring to Figure 22 above, one can see that the average starting and ending scores for the six groups are different, yet scores have increased similarly for all groups from pretest to posttest. This increase in gains is statistically significant in terms of time but not as large in comparison to each other.

Since these scores are UK population age-standardised, one can therefore see that, on average, all groups improved well beyond other UK children of the same age.

6.7.3 Summary of Results in Mathematics Tests

Overall, students in all groups performed significantly higher on age-standardised Mathematics posttests (M = 105.69, SD = 9.16) than on age-standardised pretests (M = 100.06, SD = 8.73). Age-standardised mean scores went from 100.06 to 105.69, going from virtually the population mean for the participants' age level to 5.69 points above the mean normative value for this age group. As noted before, this indicates that all groups' margins of improvement are significantly higher on posttests than students of the same age in the UK, as shown by the standardised scores.

Again, the increase from an estimated marginal means of 100.06 at pretest to 105.69 at posttest; $F(1, 125) = 53.29$, p =.000 (p <.05), partial $\eta^2 = .30$ (large effect size) is statistically significant with a large effect size, therefore strongly supporting the main hypothesis. The estimated marginal mean gain in Mathematics scores from pretest to posttest was (5.63), p = .000, (p < .05).

As noted before, this statistically significant *main effect of time* suggests that consistent music lessons may have strongly contributed to improvements in Mathematics scores for all students on average. Additionally, mean gain scores in each of the six groups improved from pretest to posttest and all gains in both raw and standardised Mathematics scores were statistically significant except

Form Implicit.

The *main effect of group* shows that there is a statistically significant difference in Mathematics scores *between* intervention groups, though a small effect size $F(5, 125) = 3.04$, $p = .001$ ($p < .05$) partial $\eta^2 = .11$ (small effect size).

Yet again, referring to Figure 22, one can see that the average starting and ending scores for the six groups are different, yet scores have increased for all groups from pretest to posttest. *This increase is statistically significant.* Additionally, all groups increased their scores by approximately the same amount (within error) during the study.

In the Tukey post hoc tests for Mathematics, Form Explicit showed statistically significant higher levels of differences compared with Form Implicit (.038) and Rhythm Implicit (.015) at the .05 significance level.

As noted, when looking at individual pairs, the Explicit groups do improve to a slightly greater degree than the Implicit groups on average. This supports the hypothesis that even briefly yet consistently noting the connection between music and mathematics to students can yield slightly stronger results. As noted, only one pair (Form groups) showed a significant difference in Mathematics gain scores between explicit and implicit groups, with the explicit group improving to a much higher degree. Form Explicit also displays a significantly larger improvement gain over Rhythm Implicit as well as Form Implicit.

To conclude this section, these strong changes in age-standardised Mathematics scores are key to this study, as the main hypothesis states that mathematical scores will improve following nine months of musical training. It was essential to have shown the effectiveness of the music lessons through demonstrating substantial advances beyond the age norms in musical cognition as a foundation, yet the ultimate dependent variable is the Mathematics scores. Therefore, by demonstrating statistically significant effects with a large effect size (partial $\eta^2 = .30$) in mathematical achievement, the null hypothesis is clearly rejected and the primary hypothesis is strongly supported.

6.8. Analysis of the Proportional Mathematics Tests

This section focuses on a subcategory within mathematics, proportional mathematics. This subcategory was chosen for a full analysis for two reasons: 1) It is hypothesised in this thesis that music training – particularly that which features rhythm – can enhance proportional mathematics achievement, and 2) There are enough items in the Mathematics test that fit into this category to run an ANOVA.

Questions selected, such as those involving fractions and division, require some understanding of proportion. Proportional questions comprise 24% of all questions (11 out of 45 questions). A new variable was created comprising these, which was used in these statistical analyses. Raw scores were used because separate portions of the Mathematics test were not age-standardised. On the next two pages note the MaLT mathematical item category table as well as the proportional category table, both created in order to answer the research questions, primarily question 2, which focuses on specific aspects of musical and of mathematical thinking. Also see Appendix 1 for the full test.

Table of Categories in the MaLT 8 Test

Mathematics Test: 8 Categories

PR/CN /NF= Pattern Recognition/ Counting/ Number Facts	**NF/AS/MD = Number Facts/ Addition-Subtraction/ Multiplication**	**FR/PS = Fractions/ Proportional Problem Solving (incl. division)**	**DZ/PS = Decimals-Zeros (as in place values)/ Problem Solving**
1a, 1b (* see note above)	3a, 3b	1,2) 4a, 4b	6
		3) 10	
2	7	4) 12	8
		5, 6) 13a, 13b	
		7) 19	
5		8) 27	14
18		9) 29	28
		10) 31	
33a, 33b	31	11) 34	

CA/AS/MD/PS= Calculation/ Addition-Subtraction/ Multiplication/Problem Solving	**ME/TI/ CN = Measurement/ Time/ Counting**	**SH/GE = Shape/ Geometry**	**SH/LM = Shape/ Language-Meaning (Not to be analysed)**
17a, 17b, 17c	9a, 9b	25	15 ('half turn')
20	11	32a, 32b	24 ('litres,' etc.)
21a, 21b	16		
22	23		
26	30		

Table 13 Above are the different categories within the Mathematics test, which was used in this study. There are 11 questions that relate to proportion (that are featured below). A new variable was created comprising these for use in the statistical analyses.

Proportional Questions

Questions that Involve Proportional Thinking: Fractions & Proportional Problem Solving (Including Division)
1,2) 4a, 4b
3) 10
4) 12
5, 6) 13a, 13b
7) 19
8) 27
9) 29
10) 31
11) 34

Table 14 These 11 questions require some understanding of proportional concepts.

Below it will be shown that students' mean Proportional Mathematics scores overall improved at a statistically significant level with a large effect size.

6.9. Two-way Mixed ANOVA for the Proportional Mathematics Test

6.9.1 Pre-testing for Parametric Suitability

Outliers, Homogeneity of Variance and Co-variance

There are outliers in the data, as illustrated in the boxplot in Appendix 1. Nevertheless, this particular data set is used in this Proportional Mathematics ANOVA so that it can be appropriately and accurately compared to the overall Mathematics ANOVA, of which this is a subset. Proportional Mathematics scores are not normally distributed for any of the interventions at neither time point, with slight significance levels ($p < .05$) for two groups at pretest (Form Implicit and Melody Implicit) and one group at posttest (Rhythm Implicit) as assessed by the Shapiro-Wilk's test. The corresponding table is shown in Appendix 1. Note discussion above regarding the choice to do the ANOVA with these very small violations of normality.

There is homogeneity of variances at both time points, as assessed by Levene's Test of Homogeneity of Variance, ($p > .05$). Additionally, there is homogeneity of co-variances, as assessed by Box's test of equality of co-variance matrices, $p = .91$ ($p > .001$).

Reliability Analysis

The Proportional Mathematics pretest items have good internal consistency, $\alpha = .76$. Likewise, the Proportional Mathematics posttest items also have good internal consistency, α

= .79.

Good = .70 ≤ α ≤ .90. (Kline, 2000; George & Mallery, 2003). See Appendix 1 for full reliability table (Kline, 2000; George & Mallery, 2003**)**.

6.9.2 Results of the Two-way Mixed ANOVA for Proportional Mathematics Test

Results of the two-way mixed ANOVA performed on the raw Proportional Mathematics test scores are given in this section and are accompanied by visual depictions of the findings.

Profile Plots Indicating Gains

Figure 18 below illustrates that all groups improved over time on raw Proportional Mathematics scores. The graph indicates a strong main effect for *time*, with Form Implicit being only slightly less dramatic than the others, which is interesting given that Form Implicit's overall mathematics gain was the only group without statistically significant results at any level (.001 or .05). This test produced the most consistent and dramatic results of all tests across all groups as shown by the upwardly linear trajectories.

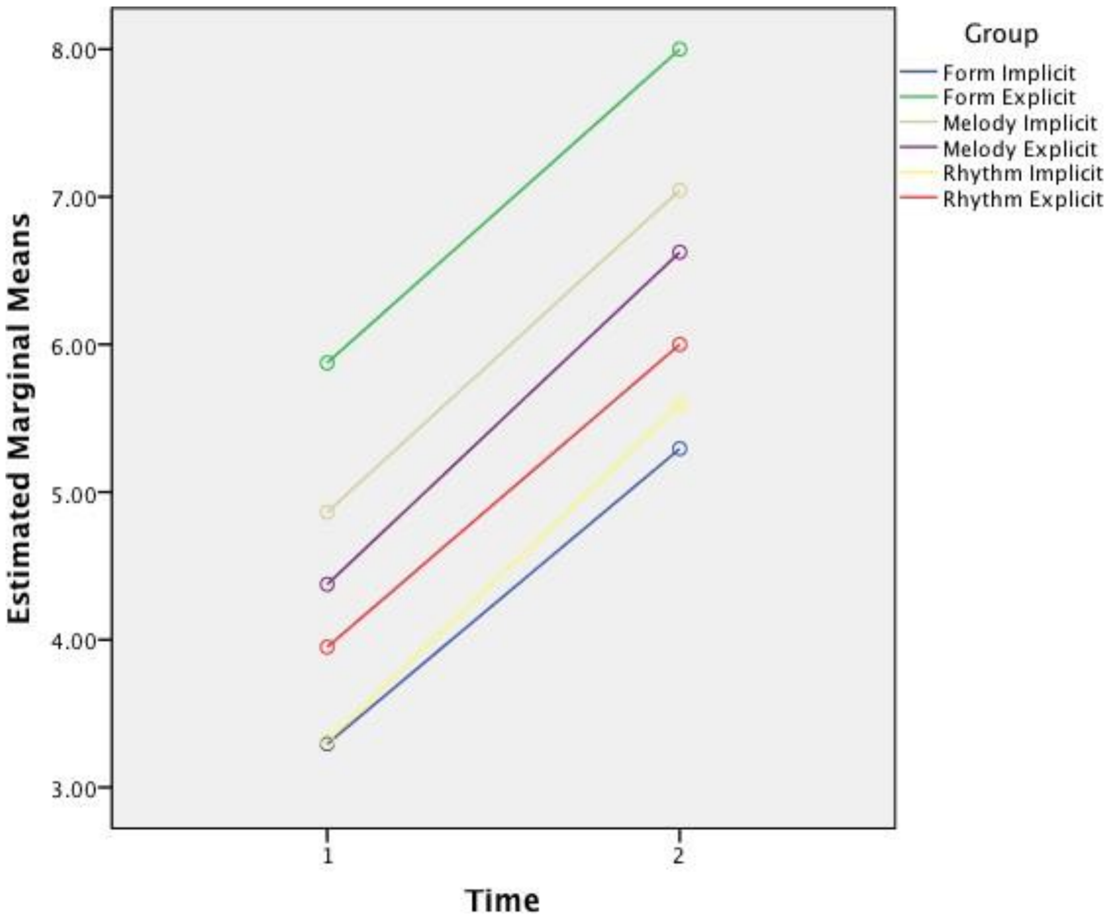


Figure 20. This shows change over time in the treatment groups' Proportional Mathematics test scores.

As noted, Figure 20 above shows that all groups similarly improved greatly over time on Proportional Mathematics scores. This strong *main effect of time* is shown in the statistics below. This powerfully suggests that consistent music lessons over time (nine months) contributed to improvements in Proportional Mathematics scores for all groups, as hypothesised.

The *main effect of time* is statistically significant in Proportional Mathematics test scores, $F(1, 125) = 133.48$, $p = .000$ ($p < .05$), partial $\eta^2 = .52$. This is a very large effect size as .26 is considered large, according to guidelines for partial η^2, as shown on page 154 (Cohen, 1992, p. 157; Cohen, Cohen, West & Aiken, 2005, p. 95). Note that the effect size for the overall standardised mathematics scores is .30. This strongly supports the hypothesis stated in this thesis that musical training may particularly enhance proportional mathematics skills.

The *main effect of group* is statistically significant in Proportional Mathematics scores *between* intervention groups: $F(5, 125) = 3.19$, $p = .001$ ($p < .05$), partial $\eta^2 = .11$ (small effect size).

Referring to the graph above, one can see that the average starting and ending scores for the six groups are different, yet Proportional Mathematics scores for all groups have increased in a similar trajectory and by approximately the same amount (within error) for all groups from pretest to posttest, more closely than any of the other comparisons have changed. Again, note that the increase in scores from pretest to posttest is statistically significant, also strongly supporting the hypothesis.

As pointed out in the overall mathematics ANOVA report, as well as the other ANOVA reports, all groups had the same intervention (music lessons using the same songs) albeit with different emphases (form, melody, rhythm) and different levels of mathematical explicitness (implicit, explicit). Yet similar to the Mathematics results, in the Tukey post hoc tests for Proportional Mathematics, Form Explicit showed statistically significant higher levels of differences compared with Form Implicit (.028) and Rhythm Implicit (.015) at the .05 significance level.

6.9.3 Summary for Results in Proportional Mathematics Tests

The *main effect of time* is statistically significant in Proportional Mathematics test scores, $F(1, 125) = 133.48$, $p = .000$ ($p < .05$), partial $\eta^2 = .52$, which is a very large effect size. This result greatly supports the hypothesis that musical learning particularly enhances proportional mathematics skills.

The *main effect of group* is statistically significant in Proportional Mathematics scores. In the Tukey post hoc tests for Proportional Mathematics, Form Explicit showed statistically significant higher levels of differences compared with Form Implicit (.028) and Rhythm Implicit (.015).

Table 15 below illustrates the percentage of overall mathematics gains that Proportional Mathematics (e.g., fractions, division) questions held for each group as well for all groups. Again, raw scores are used because this portion of the Mathematics test has not been age-standardised.

Proportional Mathematics Gains as a Percentage of Overall Mathematics Gains

	Prop. Gains (Raw)	Overall Gains (Raw)	Subset Percentage
Form Implicit	2.00	5.94	34% (10% above 24%)
Form Explicit	2.13	8.00	27% (3% above 24%)
Form Groups	**2.06**	**6.94**	30% (6% above 24%)
Melody Implicit	2.18	8.18	27% (3% above 24%)
Melody Explicit	2.25	9.17	25% (1% above 24%)
Melody Groups	**2.22**	**8.70**	26% (2% above 24%)
Rhythm Implicit	2.28	7.69	30% (6% above 24%)
Rhythm Explicit	2.05	7.40	28% (4% above 24%)
Rhythm Groups	**2.19**	**7.58**	29% (5% above 24%)
All Groups	**2.17**	**7.81**	28% (4% above 24%)

Table 15. This illustrates the percentage of overall mathematics gains that questions involving Proportional Mathematics (e.g., fractions, division) held for each group as well as for all groups.

As shown in Table 15 above, raw scores were used with Proportional Mathematics because this category of the Mathematics test had not been age-standardised. Note that Proportional Mathematics questions comprise 24% of all questions (11 out of 45 questions) on the Mathematics test. The first percentage in the column on the right compares the percentage of the subset gains (subset = Proportional Mathematics) in relation to the percentage of questions this subset comprises. This shows specific Proportional Mathematics gains in relation to overall gains in Mathematics in light of the weight this subset holds in the test, thereby emphasising the strength of the Proportional gains while further supporting the hypothesis.

6.9.4 Further Discussion of Proportional Mathematics Results

Organisation of the Table

Under "Subset Percentage," the percentage of the overall mathematics gains that Proportional Mathematics gains comprise for each group are noted. In parentheses, the percentage of gains above the maximum that might normally be expected given the percentage (24%) of proportional questions relative to overall mathematics questions is shown.

Purpose

An examination of the degree of proportional mathematics improvement relative to the overall mathematics improvement for each group is made here in order to investigate the strength of the relationship between each musical intervention and understanding of proportional concepts. The findings suggest that the relationship between musical understandings and proportional understandings is strong, which supports the hypothesis that musical training enhances this link, thereby leading to higher scores in proportional mathematics particularly and in mathematics overall. Further discussion will be in the following chapter.

6.10. Closing Notes for Quantitative Analysis

6.10.1 Tukey Post Hoc Tests for All Measurements

The mathematics scores, including proportional mathematics, show the strongest significance levels in differences *between* groups in estimated marginal means in the Tukey post hoc tests. These involved Form Explicit in all cases. Form Explicit showed statistically significant higher levels of differences compared with Form Implicit (.038) and Rhythm Implicit (.015) in Mathematics scores and with Form Implicit (.028) and Rhythm Implicit (.015) in Proportional Mathematics scores at the .05 level. Explicit groups overall improved slightly beyond test achievement levels of Implicit groups, suggesting that explicit reference to music-mathematics connections my help, yet the main enquiry regards whether the experience of learning music itself would implicitly have an impact on children's learning and understanding of mathematical concepts, regardless of whether or not the music-mathematics connection is made explicitly. This appears to be the case given the statistically significant gains in all groups, with small differences between the Explicit and Implicit groups, though differences have been shown.

6.10.2 Overall Results

Overall, student achievement levels in all three tests – Music Aptitude, Spatial Reasoning and Mathematics (including the Proportional Mathematics subset) improved beyond the achievement levels of other children their same age in the UK, as revealed by the statistical analyses results of all standardised tests scores.

Below, Table 16 illustrates this phenomenon via the means gains and significance levels produced by the ANOVA test.

Estimated Marginal Mean Gains by Test

Test	ANOVA Estimated Marginal Mean Gain/Standard Error	Significance
Music Aptitude	105.53 to 115.33 = 9.80 (.94) (.70) (1.01)	*p* = .000 (*p* < .05)
Spatial Reasoning	99.76 to 106.76 = 7.00 (1.75) (1.61) (1.43)	*p* = .000 (*p* < .05)
Mathematics	100.06 to 105.69 = 5.63 (1.13) (1.27) (.77)	*p* = .000 (*p* < .05)
Proportional Mathematics	4.28 to 6.43 = 2.15 (.23) (.25) (.19)	*p* = .000 (*p* < .05)

Table 16

Table 16 above shows that all students on average improved beyond the achievement levels of other children their same age, as revealed by the statistical analyses results of all standardised tests scores as well as the raw scores for the proportional subset of the mathematics test. This table illustrates this phenomenon via the means gains and significance levels produced by the ANOVA test. As shown, all means gains in this study are statistically significant and therefore strongly support the hypothesis put forth in this thesis, which states: *Music education via the voice as the primary instrument can enhance mathematical achievement*.

6.14. Conclusion

This chapter, through detailed reporting of the analyses performed upon the data for this research, reveals numerous details while also giving an extensive overview of the changes in student awareness and learning over time that will help to answer the research questions put forth in this thesis and to support the hypotheses. The following chapter will discuss these results in terms of the questions as well as the hypotheses for this thesis.

Chapter 7 Interpretation & Discussion

7.1. Introduction

The previous chapter focused on the detailed results of the statistical analyses performed. These were conducted in order to answer the research questions for this study.

In this chapter, the theoretical foundation and study will be reviewed and summarised. Then the interpretation of the results and discussion of the findings will follow.

7.2. Summary of the Theoretical Foundation and Study

7.2.1 Theoretical Foundation Revisited

As noted in Chapter 1 and periodically thereafter, this research investigates the potential for music education to enhance children's mathematical understanding and therefore, finally their achievement through repeated activation across the brain for related cognitive challenges shared by these two domains – music and mathematics.

The theoretical foundation is therefore based upon theories of *brain plasticity* (Foscarin, Rossi & Carulli, 2011; Leuner & Gould, 2010; Münte, Altenmüller, & Jäncke, 2002; Pascual- Leone, Amedi, Fregni & Merabet, 2005; Wan & Schlaug, 2010), *functional connectivity* (Fingelkurtsa, Fingelkurtsa & Kähkönenb, 2005; Guye, Bartolomei & Ranjeva, 2008; Jenkins, 2001; Park, Park & Polk, 2013) and *distributive processing* (McIntosh, 2000).

As stated in the first chapter, the theory of *brain plasticity* suggests that the capability and propensity of the brain to adapt to experiences and variations in the environment is an integral result of evolution available throughout the lifespan (Pascual-Leone, et al., 2005) and that neural growth and structural change occur due to learning experience (Leuner & Gould, 2010; Foscarin, et al., 2011). Therefore, it follows and was speculated in this thesis that music learning can result in noteworthy changes in the brain.

Functional connectivity, as stated in Chapter 1, is the neural grouping of activity for the achievement of a complex cognitive task or perceptual process (Fingelkurtsa, et al., 2005).

Extended from the concept of functional connectivity and grounded in the hypothesis that learning and memory are emergent properties of large-scale neural network interactions, the theory of functional distribution, or distributive processing, postulates that brain regions are structurally interconnected, and therefore process information in a distributed way (McIntosh, 2000). As noted before, an essential aspect of this theory is that a region can have different roles across many functions and that each role is influenced by its interactions with anatomically related regions (McIntosh, 2000; Grigg & Grady, 2010).

Due to the interconnected nature of brain activation and functionality, when a brain region is regularly activated to serve a specific cognitive domain, this development can also impact other cognitive domains that are related, such as music and mathematics. Since networks within different regions can operate among varying contexts via recruitment of neural networks throughout the brain, the distributive processing theory supports the idea of cognitive transfer, in which knowledge or ability in one cognitive domain can be used or transferred to another domain, as discussed in more detail in Chapter 2.

To reiterate from before, these related theories of distributive processing and cognitive transfer support the hypothesis that musical learning can enhance mathematical thinking and therefore achievement, due to the shared neural network interactions among similar cognitive skills such as pattern, structural and proportional recognition, as well as creativity and problem solving.

This potential transfer is revealed not only in the tested outcomes of this study, but also in interviews with intervention participants, many who could recognise the link between these two domains even among students in the implicit groups. As pointed out previously, these results further support the possibility that music learning can implicitly raise awareness of number, pattern, structure and proportion without the necessity of explicit reference to these concepts.

Structural proximity could be an important factor underlying the transfer effects shown across different cognitive domains in this research. Indeed, this thesis indicates that cognitive functions particularly occurring in the same brain region further facilitate transfer from one cognitive domain to another. Therefore, as noted in Chapter 1, it is likely that when multi-modal integration areas in the frontal and parietal regions such as those surrounding the intraparietal sulcus (IPS) are repeatedly accessed, cross-modal influence on other behavioural or cognitive processes can utilise neural networks functioning during both music and mathematics processing.

To repeat from the first chapter, numerous studies have indicated that intensive music training can result in intraparietal sulcus modulations (Schlaug, et al., 2001, 2003, 2005, 2009, 2012). As noted, this area also is associated with numerical representation and operations (Cohen et al., 2007; Dehaene et al., 1998; Piazza et al., 2007; Pinel et al., 2004). Therefore, these shared regions use the same neural resources, principally for the understanding of and mental manipulation of symbolic representations, both musical and mathematical. Therefore, to emphasise, certain sensorimotor and cognitive enhancements beyond music that have been associated with music learning may be due in part to functional connectivity, distribution and plasticity of related neural networks (Fingelkurtsa, et al., 2005; McIntosh, 2000; Pascual-Leone, et al., 2005; Wan & Schlaug, 2010, 2012).

Parietal cortex, inter-hemispheric transmission and mathematical performance. As discussed in Chapter 1, the work by Schlaug et al demonstrated a causal link between music training and corpus callosum growth is relevant for this study, since the corpus callosum facilitates inter-hemispheric information transfer. As noted before, recent research revealed a positive correlation between inter-hemispheric functional connectivity and mathematical performance (Park, Park & Polk, 2013). Dehaene, et al have shown that parietal cortex is central to numerical cognition, yet since this region is also engaged in verbal, spatial and attentional functions it also may contribute to calculation. (Dehaene, Piazza, Pinel, & Cohen, 2003). As the right parietal region is primarily involved in basic quantity processing and the left parietal region in precise number processing and numerical operations, Park and colleagues' study established the importance of functional connectivity between the right and left parietal cortex for numerical processing that involves both fundamental number representation and learned numerical operations.

In the study with Park et al, as noted in Chapter 1, mathematical results also correlated with the degree of functional connectivity across participants, while activity within each brain region did not. This study indeed accentuates the importance of parietal functional connectivity in numerical processing and suggests that arithmetic processing relies upon communication between hemispheres of the parietal cortex. Furthermore, it indicates that this communication influences the degree of numerical competence (Park, Park & Polk, 2013).

As pointed out previously, even though the full experience of music is due to activity in widely distributed areas, relevant neural networks are nevertheless dedicated to specific aspects of musical processing. Perception, production and analysis of detailed patterns and structure of rhythm and pitch relationships happen mainly in the left hemisphere, while timbre, melodic contour (Warren, 1999, p. 571) and metrical extraction (Levitin, 2006, p. 169) are processed primarily in the right (Peretz & Zatorre, 2005). Therefore, to repeat from Chapter 1, the understanding of specific musical patterns and structures is analogous to the understanding and processing of precise numbers and numerical operations, all of which are processed mainly in the left hemisphere. The processing of general melodic contour could be analogous to the mathematical processing of basic quantity (Dehaene, et al., 2003; Park, et al., 2013), which are both processed in the right hemisphere.

Finally, Posner and Rothbart's statement regarding the growing recognition among prominent neuroscientists that "classroom interventions can alter neural networks related to cognition" in ways that manifest beyond the specific domain of instruction (Posner & Rothbart, 2005, p. xx) is particularly relevant to this thesis. The results of this study confirm this collective acknowledgement.

7.2.2 The Present Study

This study aimed to contribute to the literature by focusing on specific musical and mathematical elements, activated principally through the medium of singing. Set in five primary schools, this real-world, quasi-experiment took place over nine months, the course of a full school year. Nearly 200 children aged seven-eight years, in seven school classes (forming six groups), experienced structured 40-minute weekly music lessons, congruent with National Curriculum objectives for music but with specific foci. This design employed two independent variable categories: musical focus (form, melody or rhythm) and mathematical emphasis in teaching (implicit or explicit). In all other respects, lesson content was kept as constant as possible by using the same songs in all classes, though using different arrangements. Pretests and posttests in standardised behavioural measures of musical, spatial and mathematical thinking were administered to all children, and comparisons between student scores of each test were analysed via two-way mixed ANOVAs. These revealed positive significant gains in the majority of comparisons over normative progress for groups with all musical emphases and both teaching conditions, with a slightly greater effect in the mathematically explicit lessons as hypothesised.

To monitor, validate and contextualise this study, several procedures were undertaken: 1) all music lessons were video recorded, 2) classroom mathematics lessons were observed, 3) interviews were conducted with a subsample of children in each group and 4) questionnaires were completed by classroom teachers.

7.2.3 Discussion and Interpretation of the Findings

It was hypothesised that regular structured music lessons over a substantial period, given by a specialist in music education for children would result in improvements in both musical and mathematical cognitive processing. The three research questions explore this hypothesis from several perspectives in order to examine the potential effect of the specific teaching foci employed. These are musical modes (form, rhythm, melody) and mathematical explicitness (implicit, explicit). The findings will be discussed below in relation to each research question. Here, as a reminder, are the research questions, which will be discussed individually below.

Research Question 1

How far, if at all, can school-based musical learning primarily via the voice improve mathematical attainment?

Research Question 2

Does a focus on specific musical elements while teaching enhance understandings of possible corresponding mathematical concepts?

Research Question 3

Does teaching music with brief yet explicit references to hypothesised mathematical correlates enhance children's corresponding mathematical skills, and if so, does it do so more than teaching music without these references?

7.3. Discussion of Primary Research Question

How far, if at all, can school-based musical learning primarily via the voice improve mathematical attainment?

7.3.1 Main Findings

The results of the present study show that music lessons featuring singing and taught by a specialist indeed can promote a markedly higher degree of improvement in mathematical achievement than expected based on test norms related to and following improvement in music aptitude and spatial reasoning abilities as hypothesised. Measuring these results appropriately is dependent upon standardised tests. These tests, scrutinised in Chapter 4, are again, Music Aptitude: *Primary Measures of Music Audiation* (Gordon, 1979, 1986, 2002), Spatial Reasoning: *Spatial Reasoning Year 3* (Smith & Lord, 2002) and Mathematics: *MaLT 8* – (Mathematics Assessment for Learning and Teaching for age 8 (Murray, Williams, Wo & Lewis, 2005, 2010).

As shown in the previous chapter, the statistical analyses of all tests conducted for this study reveal positive statistically significant gains in most comparisons over normative progress, particularly in music and mathematics, and completely in proportional mathematics for all six groups, which include all combinations of three musical mode emphases (Form, Melody and Rhythm) and two teaching conditions (Implicit and Explicit).

In assessing *four* cognitive measures (music aptitude, spatial reasoning, overall mathematics performance and proportional mathematics performance) for all *six* subgroups, *twenty-four* mean gains outcomes were measured and analysed. As detailed in Chapter 6 and noted above, only five out of the 24 subgroup outcomes were not statistically significant (*Music Aptitude*: Form Implicit and Melody Explicit; *Spatial Reasoning*: Rhythm Explicit and Melody Explicit; *Mathematics*: Form Implicit). All Proportional Mathematics outcomes were statistically significant. In other words, 19 out of 24 mean gains from pretest to posttest showed statistical significance.

All musical mode groups showed statistically significant mean gains on all tests in at least one subgroup with one exception: Melody groups did not show statistically significant mean gains on Spatial Reasoning tests. The sub-hypothesis that Melody groups would show statistically significant gains in Spatial Reasoning is the singular one which was not supported by behavioural evidence in this study. Yet, as noted in Chapter 6, circumstances did not allow for valid measurement of this musical mode group on this test, as only two students completed the test in one of the two subgroups (Melody Implicit).

Due to holding initial scores that were towards the top end of the standardised distribution for their age group, students in three of the five groups that showed “nonsignificant” effects (i.e., three of the 24 potential test-group configurations) had less tendency for improvement on certain tests. The tests-group arrangements with relatively minimal mean gains perhaps due to higher initial scores were Music Aptitude-Form Implicit, as well as Spatial Reasoning-Rhythm Explicit and Spatial Reasoning-Melody Explicit. This phenomenon could have been blamed on the tests themselves, yet the tests allowed for these groups to score higher, therefore potential maturity-level cognitive constraints could explain the smaller gains for students already near the top range in their age group. Therefore, within their age-range, students in these three cases may have had limited room to grow.

7.3.2 Further Discussion

As noted, Spatial Reasoning is theorised as having the potential to be a cognitive bridge between Musical and Mathematical cognitions. Justifications for these links were discussed and evidence from prior research was presented in the literature review (Freudenthal, 1984; Graziano, Peterson, & Shaw, 1999; Rauscher, Shaw, Levine, Ky & Wright, 1994; van Nes & de Lange, 2007).

The intervention included 40-minute lessons only once per week, yet there were nonetheless statistically significant results across all measures of cognitive attainment, revealing a strong relationship between the intervention and the outcome.

Within the mathematics education literature, the importance of counting comprehension has been noted as a prerequisite ability for understanding fundamental mathematical concepts including and beyond counting. When children first appear to count, they may simply be repeating a series of words, as one might do when learning a song. This is sometimes called pseudo-counting. A clear understanding that each successive number refers to the total amount of something, whether intangible or tangible, is often not understood until later development (Gelman & Gallistel, 1978).

In the seminal paper by Gray and Tall (1994), the authors differentiate between process and concept in mathematical thinking and introduce the term "procept" to describe the merging of the two which occurs with more advanced development in mathematics. Relevant to this discussion, they note that "the process of counting is encapsulated in the concept of numbers" (p. 3). In identifying the calculation processes of children who are more advanced in mathematics, they noted that these children group together separate pieces of information such as "3+4=7" in flexible ways in order to most efficiently solve a problem. Therefore, they no longer rely on counting as they instead chunk number relationships into known and therefore usable parts.

Since counting is fundamental to generating most music, at least on an intuitive level, it is reasonable to suggest that when this ability is learned within a structured musical realm, it could transfer to and reinforce learning to count within other situations in life, supporting foundational steps needed to solve mathematical challenges in school. Music is also "rich in relationships" (Gray & Tall, 1994, p.2). This phrase had been used by these authors with regard to mathematics. Therefore, more advanced musicians often group individual beats into larger units in the same way that high achievers in mathematics intuitively chunk pieces of information together into meaningful structures.

Therefore, it is perhaps equally appropriate to note while discussing higher levels of both mathematical and musical cognitions and manifest entities Piaget's idea that:

> …the whole of mathematics may therefore be thought of in terms of the construction of structures,...mathematical entities move from one level to another; an operation on such 'entities' becomes in its turn an object of the theory, and this process is repeated until w

reach structures that are alternately structuring or being structured by 'stronger' structures. (Piaget 1972, p. 70 as cited in Gray & Tall, 1994, p. 2)

Most relevant to children's lives is the goal, as stated in England's national curriculum guide (The English National Curriculum, 2010), for learned understandings in mathematics education to transfer to usage in other subjects as well as in daily life. One must take into account, however, that feeling the beat does not necessarily mean that a full understanding of counting exists. Yet when properly performing a piece that has been arranged with more than one instrument (including the voice as an instrument, hence all songs in this study) and more than one structural part – as with most songs in this study – some knowledge of the function of counting in relation to overall form must come into play, even if only at an unconscious level.

As was demonstrated during fieldwork activities in all classes and noted in Chapter 5, it was essential for students to feel the rhythmic pulse yet also to keep an accurate count of the beats throughout each musical piece because most of the pieces were arranged with requirements for entering at certain sections on particular counts. Other cues existed, however, such as a change in the melodic pattern or accompaniment. Yet sometimes similar sections would be followed in different ways, therefore making an awareness of the counts even more important for accurate assessment of one's standing within the piece. Furthermore, with practice, feeling the general pulse of a musical piece usually becomes automatic. Yet especially for entrances in more complex works, a heightened awareness of pulse via counting is often essential in order to recognise cues for entering each section whether one is singing, playing an instrument or dancing.

Every piece the students learned had a specific arrangement beyond the simple repetition of verses. Most of the songs were from folk literature, and yet I created a more complex arrangement if one was not already inherent in the piece, using patterns expressed via the voice, an instrument or dance. If children did not precisely follow the musical beats, though they were provided with other cues at times, they risked coming in at the wrong time and therefore standing out in their class or "messing up" the music. This was a challenge that gave them incentive to be on alert during what nevertheless appeared to be an exciting and fun process, as seen by their faces and confirmed by the interviews as well as the teacher questionnaires. The children's newly discovered capabilities, in most cases, were fully demonstrated at the final concerts given at each school. This not only delighted the students, but also impressed parents, teachers and head teachers, as expressed by some students in interviews, as well as many observers who commented favourably afterwards as well as two of the head teachers, who expressed surprise at the volume of memorisation and learning that took place.

By activating cognitive awareness related to numeracy and organisation beyond what simpler arrangements and requirements may have done, heightened arrangements and performance requirements encouraged results that further support the hypothesis that school-based musical learning leads to enhanced mathematical processing as measured by standardised tests. Additionally, when an instructor focuses on specific musical elements while teaching, whether via the voice as in this study, or via other instruments, specific mathematical categories as well as overall mathematical attainment can improve as shown by statistically significant mean gains in test scores.

7.4. Research Question 2

Does focusing on specific musical elements while teaching enhance understandings of possible corresponding mathematical concepts?

The three sub-questions listed below were also proposed, though the last two were simply reasoned conjecture.

- Does emphasis of music structure (form) comprehension particularly enhance pattern recognition, structural recognition and creative problem-solving abilities?
- Does emphasis of pitch relationships (melody) while teaching music particularly enhance pattern recognition as well as geometrical and spatial-temporal understandings?
- Does rhythmic emphasis while teaching music particularly enhance counting ability and pattern recognition as well as proportional and spatial-temporal understandings?

In order to properly answer these questions, appropriate statistics needed to be performed. Many of the mathematical subcategories could not be analysed because the number of test items was too small. Therefore, only two mathematical concept categories were analysed: Spatial-temporal Reasoning, using a separate test (Smith & Lord, 2002) and Proportional Mathematics, a subcategory within the MaLT test (Murray, Williams, Wo & Lewis, 2005, 2010).

Of the hypothesised musical mode-mathematical concept cognitive correlates that could be measured, several yielded statistically significant outcomes:

Form: *Structural awareness* and *pattern recognition* were both hypothesised to improve for this group, though these were not analysed specifically due to limitations in the test materials.

Conceptual overlaps exist between meanings of both terms as well as with elements of proportional awareness and spatial reasoning, however, and these outcomes were both statistically significant.

Melody: As noted before, *spatial reasoning* was hypothesised to improve, yet there were not statistically significant results on this test for this group. Two possible reasons are that the results were skewed for one of the subgroups (Melody Implicit) due to the lack of participants (only two) and the other subgroup (Melody Explicit) started out at a high level for their age group, leaving little room to grow given their maturity level.

Rhythm: *Proportional understanding* was hypothesised to improve and indeed this was statistically significant. *Spatial-temporal reasoning* was also hypothesised to improve and this was statistically significant for both Rhythm groups together, though Rhythm Explicit was not statistically significant perhaps due to higher starting levels as discussed before.

Table 2 on page 95 of Chapter 4 demonstrates the hypothesised relationships between variables. On the following page, a third column revealing the outcomes has been added to the initial table. The initially surprising finding here was that all six subgroups showed statistically significant gains in *proportional understanding* even though the rhythm groups were the only ones hypothesised to show such gains. (Also see Chapter 6, Table 15, p. 172 and Table 16, p. 174.)

Hypothesised Relationships between Variables with Outcomes

Independent Variable (Music Teaching Focus & Music Learning)	Primary Dependent Variable (Mathematics Learning)	Outcome as Shown by Significance Levels of Mean Score Gains
Form/Structural Focus, Awareness & Music Learning	1. Pattern recognition 2. Structural awareness and categorisation ability 3. Creative and analytical problem-solving skills	1. Not measurable in this study (Yet elements of *proportional* understanding and *spatial reasoning* overlap with pattern recognition in some aspects. These were both statistically significant.) 2. Not measurable in this study (Yet elements of *proportional* understanding and *spatial reasoning* overlap with structural awareness. These were both statistically significant.) 3. Not measurable in this study
Melodic/Harmonic Focus, Awareness & Music Learning	1. Pattern recognition 2. Geometrical understanding 3. Spatial & spatial-temporal reasoning	1. Not measurable in this study (Yet proportion, an overlapping construct, was statistically significant) 2. Not measurable in this study 3. Implicit – statistically significant (yet very small sample therefore not usable) Explicit – spatial reasoning not statistically significant possibly due to ceiling effect yet proportional was. Mean of both groups – not statistically significant.

Rhythmic Focus, Awareness & Music Learning	1. Pattern recognition 2. Counting 3. Spatial-temporal reasoning 4. Proportional understanding: fractions, division	1. Not measurable in this study (yet see below regarding proportion) 2. Not measurable in this study (yet improvement was apparent behaviourally) 3. Implicit – statistically significant. Explicit – not statistically significant possibly due to ceiling effect. Both – statistically significant 4. Statistically significant (separate groups as well as both together)

Table 17

The music teaching focus, awareness and learning independent variables and hypothesised mathematical learning dependent variable cognitive correlates are shown in Table 17 above. "Not measurable" is indicated for the categories that could not be analysed statistically in this study due to the small numbers of questions within that category on the MaLT test.

Reflecting upon the prevalence of proportions within music, it now seems that proportion is indeed ubiquitous within all operations, whether *rhythmic*, *melodic* or *structural*. Initially, I had only considered the obvious proportions within the realm of *rhythm*, such as fractions, which are inherent within the rhythmic divisions in each bar (or measure). The strength of the results in proportion is therefore an unanticipated finding, and worth some discussion in order to speculate upon possible reasons why the results in proportional understanding were strong for the other two groups (Melody and Form).as well as Rhythm

Two musical aspects of the Melody group are: melody and *harmony*. These also contain specific proportions in every relationship, whether horizontally in melody or vertically in harmony, including within one note via the overtone series. To note just one instance of many, the proportion of a major third interval (4 semitones apart) appears in most western (hemispheric) pieces, as this interval comprises the first two notes from the root upward of a major chord, the basis of western *harmony*. One could take this as a reference proportion and all other juxtapositions of proportions in relation to this can tell the story of each musical piece.

From one perspective, *form* addresses proportion in a broader sense, with ideas such as the size relationship between the main parts of a song (such as parts A and B) and a less integral part of a song, such as a bridge (part C) or of the harmonic or textural differences and comparative relationships between parts. For example, the question of *form* may address the general idea that part C might be shorter in duration or thinner in harmonic richness or instrumental production than part A but the opposite in comparison to part B. Though the proportions can be measured exactly, such as part C hypothetically being half the duration of part A, the relatively lengthy timeframe between parts may not induce one cognitively to process these proportions *in situ*.

Yet the relationships between different patterns that comprise the structure of a piece can be defined in specific terms and occur within a timeframe conducive to the cognitive processing of existing proportions. For example, a motif containing a four-note pattern may be introduced in a piece, and it may repeat with varying extensions or productions (such as instrumental productions, i.e., first time a flute, next time the voice). Beethoven's famous 5th Symphony took the motif to new heights, and is displayed throughout the symphony. There are distinctive rhythmic and melodic features of this motif, which contains eight notes in two phrases of four notes each. This motif begins with dah dah dah daaaah: rhythmically – *short short short long* and melodically – *same same same down*. Then it is repeated, starting a major 2nd (2 semitones) below the first starting point with the rhythmic pattern staying exactly the same (dah dah dah daaaah) and the melodic pattern staying the same in general shape (same same same down) but changing in the specific intervallic relationship between the third and fourth notes of each 4-note phrase within this motif. This relationship goes from a descent of a major third (4 semitones) in the first phrase to a descent of a minor third (3 semitones) in the second phrase. See Figure 22 below, which shows the standard music notation for the main motif.

Figure 22 Main motif of Beethoven's Symphony no. 5 in standard music notation.

In this piece, Beethoven keeps one proportional element constant within the primary motif: proportion of time, i.e., the rhythmic pattern. Yet only one aspect of melodic proportion stays constant – the direction of the melodic contour – while the exact intervallic distance varies slightly, as in between the first two bars of each phrase: the melody first descends the proportion of a major 3rd, and then in phrase 2, the melody descends the proportion of a minor 3rd. Furthermore, this motif is musically sequential (as in repeating a pattern yet starting each phrase either higher or lower than the previous one), with phrase 2 starting a major 2nd below phrase 1.

In the extensions built upon the main motif, Beethoven varies specific rhythmic and melodic proportions yet maintains the general shape of the phrases. For example, the timing of the phrases speeds up in the extended motif in that the fourth note is not held in the first two sequences within the motif but instead is the same rhythm as the first three notes and then a 3rd sequential phrase is added: this example is a related sequential pattern in which each 3-bar phrase starts on a different interval from the previous one and continues to evolve using mathematically related patterns.

Beethoven's symphony could be expressed numerically with x being a rest, 0 being the starting point pitch of the motif (in terms of the structural layout of the proportions), and the subsequent numbers representing the number of semitones from the base note, or starting pitch of the motif, with a minus designating any note lower than the starting note 0. Slashes are inserted in between each short phrase and the superscript numbers denote duration of hold (in relation to the duration of the initial note, which is given the duration value of 1. See below:
x 0 0 0 -4^4 /x -2 -2 -2 -5^8 / x 0 0 0 -4/ 2 2 2 1 /8 8 8 6^5 / 0 0 0 -5/ 1 1 1 0/ 10 10 10 7^5 etc.

Above, the music is shown purely as numbers, from which one can derive patterns. Yet there is an added proportional dimension due to the circular nature of music that is based upon the natural overtone series, with octaves being at frequency ratios of 2:1 and expressed in Western Hemispheric music as being 12 semitones apart. Therefore, an even more intricate and layered definition of form emerges to include all the numerous patterns and levels of proportions of a piece. Beethoven's work is based upon specific proportional changes between rhythmic and melodic elements; therefore, examining his piece in more numerical detail illustrates how form is directly related to specific proportional relationships containing rhythmic, melodic and harmonic elements, the latter of which I did not even touch upon in this demonstration.

Additionally, the mini-form within the larger form of the overall piece is one more dimension that is possible to analyse. Beethoven expertly constructed his work in a multi- dimensional and logical, yet surprising fashion at times. Perhaps that is why the piece is so appealing to many. Finally, one could look at the interplay of structural, melodic, harmonic and rhythmic perceptions and conceptualisations among different brain regions to further highlight the extensive neural activation invoked by music. For example, melodic contour is normally perceived in the right hemisphere and structural analysis in the left hemisphere (Warren, 1999, p. 571; Peretz & Zatorre, 2005) though often many regions across the brain are involved in processing this information (Levitin, 2006; Warren, 1999).

Returning to the children's musical experiences in my research, as previously noted, the simpler, broader concept of the overall form of a piece, such as ABA, includes proportion. Also, awareness of the different sections requires an awareness of patterns within each. For example, the African children's song Gbodi O that was used in the fieldwork is in ABA form. The A portion is four times as long as the B portion, therefore showing that even in the broader global sense of form, proportion is still an ingredient in labelling the form of a work.

It should be noted that students in the Form groups also participated in a small amount of rhythm work even though the emphasis in discussions and arrangements of the pieces was on Form. Instruments were added for this group in order to balance out the groups since rhythm and melody groups used instruments (melody – glockenspiels and rhythm – percussion) The Form groups also used percussion instruments, though the primary instrument – the voice – was melodic and there was more discussion regarding form concepts than the other groups had regarding their foci since they were playing instruments related to these foci more. Therefore, the awareness of proportional specificity for the Form groups in this study could have been influenced not only by form work but also by melodic work (as all groups were due to the singing) and the supplementary rhythm work, including some rhythm chart work, in which they also participated.

Additionally, it has been shown that even though only the Rhythm Explicit groups were encouraged to notice the association between their rhythm charts and fractions, they clearly were not the only groups who exhibited strong improvement in proportional understanding. All groups did, as discussed before. Therefore, the hypothesis that music learning implicitly encourages certain types of mathematical awareness is supported in the results. *Pattern recognition* improvement was hypothesised to correlate highly with all groups, as music, by its very nature and definition is normally composed of patterns throughout, whether the larger patterns of a structure or the intricate patterns within a melody for example, therefore informing the analysis of a work's form and relating to proportion in ways just discussed. Exceptions to this phenomenon exist, however, such as in contemporary music literature. Unfortunately, there were not enough test items in the pattern recognition category of the MALT tests to allow for a proper statistical analysis.

As specified before, *Spatial-temporal reasoning* improvement was hypothesised to correlate highly with the Melody and Rhythm groups, yet the Melody Explicit group had a high initial mean score, leaving little room to grow when considering maturity level and the Melody Implicit group, though showing a statistically significant gain, was not large enough for an accurate assessment. Form groups, though not hypothesised to do so, and nonetheless showed statistically significant gains in spatial-temporal reasoning. In considering this again, Form was hypothesised to show strong improvement in structural awareness and categorisation ability as well, which are cognitive skills related to spatial-temporal understanding. As noted – though Form groups neither experienced rhythm work to the same degree as Rhythm groups, nor were taught the same explicit mathematical relationships as the Rhythm Explicit group – they did have some exposure through working with charts and playing percussion instruments.

It has been maintained that all groups learned the same songs and all songs contained the three elements of music discussed throughout. Yet importantly, each musical mode group focused upon a specific musical mode more than the others, whether it was form, melody (specific pitch relationship and contour) or rhythm. Additionally, it is interesting that there were some definite differences, supporting the concept that learning took place in specific cognitive domains as conjectured in the sub-hypotheses within Research Question 2.

To wrap up the discussion regarding Question 2, two particularly interesting findings are: Form groups improved in the subcategory of Proportional Mathematics slightly more than Melody or Rhythm groups relative to each group's overall improvement. This was initially surprising in that Form groups seemed to be a group for which proportional awareness would not necessarily be strong. Part A in many pieces may differ from Part B to differing degrees, but not to specific degrees, while both melodic and rhythmic relationships are inherently specific, whether in terms of a harmonic chord, a melody or in the rhythmic patterns throughout a musical work. Yet as discussed, the form of a musical piece can contain many layers of proportional relationships within it – from melodic to harmonic to rhythmic.

To reiterate, a second reason Form groups may have improved a bit more dramatically in Proportional mathematics *relative to their overall gains* than other musical mode categories is that, as all students needed to also play an additional instrument to differentiate musical mode further, which also served to help accompany the songs, percussion instruments were chosen at the request of one of the class teachers. Therefore, Form group students had more rhythm exposure than the melody groups. This additional exposure may have aided their understanding of proportion and therefore may have functioned as a confound in this study, as the musical modes were by necessity not kept distinctly separate in a purist experimental sense due to the nature of musical conventions and of a real-world study within a school.

Third, this group is the only one that composed. Since the process of composing itself prompts a level of analytical and structural thinking that performing itself may not activate as strongly, this could be another reason why the Form groups gained highly in proportional and spatial temporal understandings.

1) As hypothesised, Rhythm groups did achieve the largest gains in the Proportion subcategory scores of the Mathematics tests. I emphasised earlier that Form groups gained the most in this subcategory *relative to their overall gains* but nonetheless, the Rhythm groups outscored all groups in the Proportion category.

Therefore, the results support the hypothesis that focusing on specific musical elements while teaching will enhance understandings of corresponding mathematical concepts.

Research Question 3

Does teaching music with brief yet explicit references to hypothesised mathematical correlates enhance children's corresponding mathematical skills, and if so, does it do so more than teaching music without these references?

As hypothesised, both parts of Question 3 were only mildly supported by the test results. The mild expectation of a difference in learning was based upon the assumption that learning music itself would implicitly enhance numerical awareness. This was also supported by the interview results discussed later in this chapter. Explicit lesson groups in two of the three musical mode groups achieved greater mean score gains in Mathematics tests than Implicit lesson groups. The musical mode group in which the Explicit group did not achieve higher mean score gains was the Rhythm group. This could be because the Explicit group started out with a higher mean score (standardised score: 100.05) than the Implicit group (95.22). The Explicit group still achieved a higher posttest score (104.30) than the Implicit group's posttest score (100.31) but the mean standardised score gain for the Explicit group was slightly smaller (4.25) than the gain for the Implicit group (5.09), though both gains were statistically significant. This could have reflected limitations due to cognitive maturity and experience levels at that age.

As was noted in Chapter 6, Explicit groups overall did improve slightly beyond test achievement levels of Implicit groups, indeed suggesting that explicit reference to music-mathematics connections may help. Yet as noted above, the main enquiry regards whether the experience of learning music itself would implicitly have an impact on children's learning and understanding of mathematical concepts, regardless of whether the music-mathematics connection is made explicitly.

Notably, Form Explicit group's mean score gains were higher than Form Implicit group in all tests – Musical, Spatial and Mathematical, with statistical significance in both Music and Mathematics scores. Also, considering the Proportional Mathematics mean score gains noted when discussing Question 2, perhaps it can be seen that these relatively large improvements between Form subgroups (explicit versus implicit) illustrate the potential power of explicitly learning broader concepts of form in combination with more specific concepts within musical-numerical relationships. Being aware of the multiple layers of proportion that exist within musical structures may indeed increase both awareness of the music-to-mathematics connection as well as achievement in numerical tasks.

As seen in Chapter 6, mean score gains for Explicit groups were lower, however, for some of the Music Aptitude and Spatial Reasoning tests. It was not surprising that the Explicit groups sometimes did not gain more than the Implicit groups in Music Aptitude since explicitness in noting mathematical correlates while teaching fundamentally had no bearing on the music teaching. Additionally, since spatial-temporal reasoning is hypothesised as a bridge between musical aptitude and mathematical aptitude and explicit references to mathematical concepts may not directly affect overall levels of spatial-temporal reasoning, it follows that improvement in spatial reasoning may hold a similar pattern to music aptitude.

Additionally, as pointed out in the Chapter 6 ANOVA report, since all groups had the same intervention (music lessons using the same songs) albeit with different emphases (form, melody, rhythm) and different levels of mathematical explicitness (implicit, explicit), it was not expected that the interaction term (*group*time*) would be significant, though it was considered to be worth testing.

Yet again, similar to the Mathematics results, in the Tukey post hoc tests for Proportional Mathematics, Form Explicit showed statistically significant higher levels of differences compared with Form Implicit and Rhythm Implicit at the .05 significance level.

Nonetheless, the results above help to answer Research Question 3 in the affirmative. Teaching music with brief yet explicit references to hypothesised mathematical correlates does slightly enhance children's corresponding mathematical skills, and it has been shown to do so more than teaching music without these references to varying degrees among the specific groups studied.

The quantitative data were the core data for this research in an attempt to specify objectively a potentially causal relationship between music learning and mathematics achievement. Ideally, a future study would use a mathematics test that more closely matches the relevant concepts and has enough questions so that the hypothesised relationships can be measured more reliably. This would further counter the tendency towards confirmatory bias.

As noted in Chapter 4, this study was further monitored and contextualised through video recording the lessons, interviewing students and administering a questionnaire to teachers. Furthermore, these monitors supported the validity of the study by monitoring it from another perspective and also gave voice to all participants by including student and teacher perspectives. Since this research is quantitative, these supplementary data were not designed to be analysed. Nonetheless, they will be discussed briefly here.

7.5. Contextual Voices

7.5.1 Interviews with Students and Questionnaires for Teachers

As noted in Chapter 4, a representative sample of students (approximately 30%, therefore over 50 in total) from each experimental group (Form Implicit, Form Explicit, Melody Implicit, etc.) were interviewed about their musical and mathematical experiences as well as their thoughts regarding the connection between the two. This was done in order to contextualise the study and gain insight into the children's experiences and thoughts from their own perspectives and in their own voices. Short interviews were conducted with students individually from students of mixed achievement levels and both sexes. The interviewees were chosen in consultation with the teacher, and lasted about ten minutes each, as noted in the design and assessment tools chapter (Chapter 4). Yet due to the size of the intervention and data, this study focused on and analysed only quantitative data. Therefore, the interview questions with further discussion are not discussed here, but can be found in Appendix 3.

As noted, all teachers participating in the study completed a questionnaire to further contextualise and monitor the study. These responses helped me gain more information about their overall teaching, students' musical or mathematical experiences outside of school, as well as additional school activities in and beyond the classroom. Furthermore, it was an opportunity to acquire the perspective of each teacher regarding their own teaching of music and mathematics in addition to their experience and assessment of the intervention.

The teachers' responses in the questionnaires showed that during the fieldwork timeframe, the amount of music in the schools and the amount of music in after-school programmes were similar across all schools. Teachers' musical experiences were similar as well. St. Lucy's (as noted before, the name was changed in order to protect the privacy of the schools) had the only teacher who played an instrument (guitar).

In terms of mathematical school experience and teacher expertise, it appeared that all teachers spent virtually the same amount of time on mathematics – one hour per day with homework assignments every one-two weeks.

Further discussion as well as the questionnaire are in Appendix 3. One note, however, will be made here due to its relevance to proportional understanding. In the questionnaire given to the teachers at the end of the intervention (therefore the academic year), four out of seven teachers said proportional mathematics (fractions or division) was their least favourite aspect of mathematics to teach because of the difficulty for students. Only one other teacher chose another concept – time – as her least favourite for the same reason (perceived difficulty). Interestingly, this teacher's class comprised the Melody Explicit group, which scored slightly lower than the other five groups in Music Aptitude, yet this group scored well in Mathematics. Perhaps this teacher's slightly higher enthusiasm for teaching challenging topics in mathematics made up for a slight disadvantage they may have had, according to this thesis, due to her class' slightly lower score in music.

Research on the comprehension of fractions has shown that there is a need for teacher awareness of effective student-centred ways to teach fractions. As many students lack the understanding that natural numbers and rational numbers (fractions) are fundamentally different concepts, it is important for educators to begin teaching fractions from a place that young learners understand, particularly as clear understanding of proportional mathematics is a crucial foundation for higher forms of mathematics (Nunes et al., 2006). An example cited by Nunes et al. notes that children may think that "1/3 of a cake is smaller than 1/5 because 3 is less than 5" (p. 2) yet they readily realise that the same cake shared among three people versus five will result in more cake for the former group. Therefore, situational teaching of fractions can be most effective.

Music education as embodied, experiential learning is therefore a helpful, often powerful way to teach mathematical concepts such as fractions. For example, teaching the concepts of metre and rhythmic patterns is an effective way to teach fractions, as I found in my fieldwork with the Rhythm Explicit group. One example in Chapter 5 described my use of rhythm boxes showing the number of beats per bar (or measure). Therefore, for a chart with six beats, the symbol "x" was inserted for beats that should be drummed. When 3 out of 6 boxes were filled with "x's" I would write 3/6 on the board after students answered the question "How many beats should be played out of six?" Then I would ask "what is another way to name this" or "what is the equivalent fraction" once they had started to learn this term in school. The answer, 1/2, was not just a number game; they could see that half of the boxes had "x's" in them and half were empty, therefore grasping the concept more clearly. Music education can move knowledge from mere theory to practical application and therefore solidify and deepen understanding.

An obstacle to learning fractions relates to an existing belief that natural number knowing can interfere with fractional knowing, yet recent research (Steffe & Olive, 2010) suggests that "children's fractional knowing can emerge as accommodations in their natural number knowing" (p. 5). The authors hypothesise that if a "new way of knowing is constructed using a previous way of knowing in a novel way, the new way of knowing can be regarded as a reorganization of the previous way of knowing" (p. 5). It is suggested in this thesis that music can be used as a novel tool to reorganise old and introduce new ways of knowing.

7.6. Conclusion

One of the biggest findings in this study is that statistically significant mean score improvement was shown by all six subgroups in proportional mathematics and by five out of six subgroups (all but Form Implicit) in overall mathematics. All of the three main musical mode groups as a whole (Form, Melody and Rhythm) achieved statistically significant score gains in both proportional mathematics and overall mathematics.

At the end of Chapter 2, the idea that spatial sense may be a potential cognitive bridge between musical and mathematical domains was featured. Yet the results of this study suggest that spatial-temporal reasoning may not be the only cognitive bridge between these two domains. Perhaps proportional understanding could also be considered a cognitive bridge as well, particularly enhanced after engaged musical experiences over time.

When shifting from the researcher's attentional tendency of seeking end gains to looking at process, it was satisfying to see the children enjoying the musical learning process and to realise that perhaps this experience and others like it may enhance their mathematical enjoyment and understandings as well.

Indeed, the possibility that musical learning might help mathematical learning appears to answer a need for children at this age who are beginning to be taught a higher level of mathematics in school, which includes division and fractions. This is also a favourable time for students to learn and absorb more advanced musical concepts, such as metre, rhythm patterns and harmonies, which are also proportional. Therefore, the two subjects are perhaps a particularly serendipitous match during Year 3.

The next and final chapter will include the critical reflection and implications of this study as well as the conclusion.

Chapter 8
Critical Reflection:
Limitations, Implications, Contributions & Conclusion

8.1 Introduction

The previous chapter discussed the findings of this study in depth, mainly through critically revisiting the research questions. This chapter further reflects upon the results of the overall study, while noting ethical considerations, limitations and challenges of this doctoral journey in addition to future research ideas. Implications for children and schools as well as contributions to knowledge conclude this chapter and therefore the thesis.

8.2 Ethical Considerations

As noted in Chapter 4, all research was conducted according to the ethical guidelines of the British Educational Research Association (BERA, 2011). Details are outlined regarding voluntary informed consent, participant knowledge of right to withdraw at any time, confidentiality and that no harm, physically or psychologically, was caused (BERA, 2011, pp. 1-13; BPS, 2009, pp. 1-6).

Two ethical issues arose with this research, which will be noted briefly, as details can be found in Chapter 4. First, though the research question was not withheld, the group identifying information (Form Implicit, and so forth) was not mentioned or promoted. All information, however, was available for the school, teacher, as well as any parent or child at the end of the intervention

Second, the schools, teachers, children and most parents, gave consent for video recordings with varying degrees of use being requested via a letter and permission form for the parents or carers. Participating schools and the teachers gave consent for audio recordings since identifying the students would be difficult with this alone. This letter with accompanying permission form is shown in Chapter 4.

As noted in Chapter 4, classroom teachers were informed of the benefits of video recordings, which were named as such: Recording the lessons keeps the work open for judgment by the teachers, school and parents, while these lesson recordings can also aid in the research. The intent of the research is to help the children as well as the community-at-large.

Only two parents did not grant one or more of the requests. The child whose parents did not grant video recording was never included in any of the videos and the child whose parents did not allow participation in the recital (on religious grounds) participated in another activity at the school during the rehearsal in the auditorium and during the recital as well.

I had been concerned that, due to vulnerable and variable school and classroom schedules, all groups may not receive the same number and duration of lessons, though that was the goal and was important for the research. Over the course of the intervention, however, that concern was quieted as I was able to work with the teachers to ensure the appropriate amount of lessons, which was confirmed by my records at the end of the intervention. Indeed, every child did receive lessons of the same duration and number as all students in the study.

Therefore, as mentioned in Chapter 4, there was equal distribution of teaching and learning time. The only varied element, as required by the research design, was that of specific content allocation in terms of song arrangements, instrumentation and performance styles, though all songs were the same other than the 11th song that was unique to each group and in line with the musical mode emphasis. Additionally, for the explicit groups, there were brief references to the musical mode of focus and its relationship to corresponding mathematical concepts. This seemed to be of relatively little difference for them in terms of educational and musical value.

8.3 Limitations & Challenges

It has been demonstrated that potential *direct* effects of music lessons on mathematical achievements are likely due to similarities in both disciplines such as the existence of patterns and proportions, and subsequently the stimulation of corresponding cognitive activity in both domains. Nonetheless, it indeed must be acknowledged that *indirect* effects of music education may also contribute to academic achievement, including mathematics achievement. As noted before, indirect effects due to musical experiences and learning can include increased levels of focus or of attention span (Posner, Rothbart, Sheese & Kieras, 2008) in addition to self-esteem due to accomplishment and recognition (Burton, Horowitz & Abeles, 2000; Abeles & Sanders, 2007), confidence in presenting in front of others and cooperativity (Abeles & Sanders, 2007).

Often students in the New Jersey study would talk about their experiences performing in concerts. These were a culmination of their music learning in the program, and the effect on their self-esteem, confidence and excitement levels was evident. Had I been able to interview the students in my study at Cambridge after their concert, perhaps their responses might have been more powerful, even though they were already positive. Therefore, if I were to do this study again, I might interview the children three times: 1) at the beginning of the intervention, 2) at the end of lessons yet before the concert (as in this work) and 3) after their concert performances in order to gauge the totality of their experiences, which progressed from learning the first note to performing in the concert at the end of the school year. Yet it was also informative to isolate the responses of the students in this doctoral study to regard solely the lessons and their learning before the intense experience of a concert, which may have overshadowed their responses to the weekly lessons, as the concert indeed appeared to have augmented their overall experience. Nonetheless, interviewing after the concert for this field work was not feasible due to the school schedules.

Additionally, in a future study I would like to ask more directly in what ways music lessons may have made a difference in their lives in general, rather than focusing primarily on how it may have influenced their "maths" learning and their understanding of the music-mathematics link, although this line of questioning was entirely appropriate for this research. It was pleasant, however, that many students whom I interviewed naturally talked about how the intervention affected their lives in addition to answering my specific queries. The responses were not as dramatic as some of the responses in Newark, but this again may be due in part to the fact that they had not had a concert yet. Additionally, many of the students in the New Jersey study came from poor and socially deprived backgrounds, hence the total experience of properly learning an instrument and participating in concerts with a full orchestra was life-changing for many of them.

Though other arts activities also can encourage many of the same indirect effects, such as increased self-esteem or enjoyment in school, they have not been shown to impact directly on mathematical cognition. Each artistic realm has unique qualities that appear to have corresponding influences due to regular practice in that field. Supporting this concept, large across-school studies as well as meta-analyses of arts partnerships in schools have shown that visual art is correlated with thinking outside of the box, dramatic art is associated with compassion as well as communication and music correlates with mathematical ability (Burton, Horowitz & Abeles, 2000).

As noted in the literature review, the potentially far-reaching effects of regular participation in music lessons on student achievement have been shown in other correlational data as well. It would be interesting to track the correlations between the number of quality music lessons per week and mathematics scores in international rankings such as PISA (Programme for International Student Assessment). For example, Japan's primary school curriculum includes two music lessons per week and students on average consistently show high levels of achievement in science and mathematics by early secondary school age, as seen in rankings that are among the top in the world (Ministry of Education, Culture, Sports, Science and Technology, 2016; OECD, 2016). In addition to theoretical justifications and experimental evidence, both given in this thesis, correlational studies could support the hypothesis that a causal relationship between music learning and mathematical achievement might exist.

8.3.1 Design Limitations

This project was carefully designed, implemented and analysed, yet in real-world research such as this it was challenging to create optimal and balanced conditions. A key feature of my project is that it is based a) in real schools and classrooms with all their idiosyncrasies, but b) the use of five different but matched schools should help to reduce the effects of such idiosyncrasies on the overall results. Listed below are several additional areas that were inevitable aspects of real-world research.

- In the natural, real-world setting used here, it is not feasible to have completely random assignments. Therefore, this research is more accurately described as a quasi-experimental study (Campbell & Stanley, 1963). The schools, however, were selected to be as similar to each other as possible on key variables. Hence, as noted in more detail in Chapter 4, all groups were selected from mixed-gender state primary schools in Cambridge.
- The arguments for and against including a set of control classes, as opposed to reliance on using the standardised test norms for comparative purposes, were presented in Chapter 4. Although a parallel set of control groups would be a check on the validity of the test norms for this particular sample of children, and their inclusion would be a refinement of the present design, the work force and resources of time were beyond the scale of this study, with a single researcher, and would require several teachers to teach the music lessons. Such a design could be used in a larger scale study.
- Ideally there would have been more time for the teaching, yet even within this limitation (40-minute classes per week for full school year) students still showed significant improvement.
- The potential language benefit of memorising songs, which would reinforce the learning of English, while also promoting other languages and cultures, could also be a possible confound in the research when attempting to pinpoint precisely the source of the mathematical improvements.

8.3.2 Monitoring Limitations

Though this was extensive in some respects such as the videotaping of virtually all lessons and concerts, and recording of all interviews, it was limited in terms of monitoring students' activities and lessons, musical and mathematical, outside of the intervention lessons other than the mathematics lessons I observed (one for each classroom teacher involved in the study).

8.3.3 Measurement Limitations

Chapter 4 included the critiques of the tests, musical aptitude, spatial reasoning and mathematics, in detail. I have already noted that to counter the tendency towards confirmatory bias, future studies should use tests that more closely match the relevant concepts and that have enough questions in certain categories so that the hypothesised relationships can be measured with greater validity.

As noted in Chapter 4, for this study, related control skills such as attention and memory were not tested due to feasibility in terms of time and money. Furthermore, in addition to answering the research questions more directly, it was more straightforward and feasible to test for skills that might improve as a *direct* effect of the musical learning.

8.3.4 Data Management Limitations

The amount of data used for this research was extensive, particularly the quantitative data, with nearly 200 participants who each took three pre-tests and three posttests averaging approximately 40 questions per test. Therefore, there were over 45,000 items to mark and then to input into SPSS for analysis. With double-marking, there were effectively over 90,000 items. Given the large number of items and operations, such as marking, inputting and then analysing, there is the possibility that errors may have occurred, even with the double-marking precaution. *SAT scores.* As outlined and justified in detail in Chapter 4, the most recent Standard Assessment Tasks (SAT) scores in numeracy for the participating children were collected in addition to SAT scores from students who were at the end of Year 2 in the prior year. There were, however, four main problems with this data:

- Since these tests for Key Stage 1 were administered and assessed by the classroom teachers, there was missing data as well as general inconsistencies, such as quite different ways of marking, recording and reporting the results of these tests. Many of the teachers had a column for their own assessment next to the students' initial marks on the test. As noted in Chapter 4, these adjusted assessments sometimes indicated the mark that the teacher felt the student *should* have received, based upon their expectations or opinions.
- One teacher openly expressed contempt for the tests, which may have influenced subsequent care in recordkeeping.
- As highlighted in Chapter 4, and partially repeated here due to the importance of the problem, in one case. I noticed that some "final" scores had been inserted by the teacher, which had been raised from the actual performance scores and yet some "final" scores had been lowered from the actual performance scores. As previously expressed, I became quite concerned when I saw

that the name next to the raised scores of one student was an English name and the name next to the lowered scores of another student was Bangladeshi. Therefore, there was some evidence of potential systematic racist bias in the assessments in one class, which further undermined the validity of the scores. Investigating this concern is beyond the scope and focus of this study, as noted before.

In addition, the reliability and validity of SAT scores in general have been extensively questioned (Harlen, 2004; Baird & Black, 2013).

Therefore, for the above reasons, and because the uniformly administered and marked standardised tests used in this study provided a large amount of reliable and valid data, the SAT scores were not formally analysed.

Monitoring and contextual data, such as recordings, interviews, observations and questionnaires for this study served as monitors for validation as well as supporting documentation to contextualise the study and acknowledge the participants. As this was a large quantitative study, this data was not designed to be analysed due to the lack of time and resources. In the future, however, I would like to conduct a mixed methodology research or purely qualitative research that examines mathematical understandings through observing and working with children on musical and mathematical challenges and analysing their processes in overcoming them.

8.3.5 Analyses Limitations

The analysis could have been more complex and even more specific, yet with a three-way interpretation of such a large intervention, clarity might have suffered. The two-way ANOVA provides an overarching picture that illustrates the story and connections clearly, making interpretation more straightforward.

8.3.6 Interpretation

Differences in the mathematics scores may have been due to differences in the quality of the mathematics teaching, though differences were not observed during the lessons.

8.4. Potential Biases and Influences

I have mentioned the potential bias that exists with this study due to my strong motivation to show the power of music and to affect school curriculum policy. Other potential biases that may exist are noted below.

8.4.1 Researcher-musician

Related to the afore-mentioned bias, as a musician I may have a stronger assumption regarding the power of music as well as a stronger desire to prove it to others. Additionally, as a professional singer as well as an experienced music teacher with a lot of enthusiasm, the influence as a teacher for the field work may have been more robust than might be typical for a primary classroom environment. One way to reduce this threat to validity would have been to have one or more other teachers who were not trained musicians, nor convinced of the power of music, to teach the lessons. This would require resources well beyond the scope of this study and would not be in the best interests of the participating children.

8.4.2 Researcher-Teacher

Another potential source for bias is that I was both the researcher and teacher yet “action research” is a valid form nonetheless, which is more widely practiced than before. While this had advantages, for example having the same teacher across all conditions, therefore contributing to the consistency of the intervention, it was necessary to take precautions in the research to reduce its potential influence. These are noted below in my critique and reflection on the research design.

8.4.3 Confirmatory Bias

As a scientist, I was aware of the potential for biases and sought to reduce their threats to validity with a consistently rigorous design, methodological practice and analysis. In addition to the design, practice and analysis, another strategy to reduce these potential biases was to audio and video record all lessons in order to observe and reflect critically upon my practice as a teacher and researcher.

In future research I would further counter the tendency towards confirmatory bias by using mathematics tests that more closely match the hypothesised cognitive abilities. Given the limitations of the tests themselves, there was a danger of “over-interpreting” results based on only a few items per category, such as pattern recognition or proportional mathematics. This would ensure that the hypothesised relationships would be measured more reliably.

8.5 Interpreting Behavioural Results in Terms of Inner Processes

It is difficult to analyse the nature of mathematical thinking or any other type of thinking, even when there is strong behavioural evidence. One can only track and document the behaviour and explore the possible motivations and thinking behind the acts. Although there may be a probable behavioural outcome, a result may not occur, due to other constraints. Accurately measured, cognitive tests must correspond, at least in part, with a certain amount of understanding even though they do not tell the whole story. Therefore, achievement and thinking might not fully align and tests may likely under-estimate or over-estimate cognition.

It seems reasonable to suggest that the higher the construct validity of the tests, the greater the likelihood of some neural correspondence. As noted in Chapter 4, however, the tests, though carefully selected did have limitations with respect to the specific musical, spatial, or mathematical construct that they purported to measure. As suggested above, it is possible therefore that the tests underestimated or failed to “locate” cognitive developments.

8.6 Implications

8.6.1 Implications for Future Research

In reviewing the intent of this study, to help children develop both their creative and numerical- logical selves through music learning and experience, further research that seeks the most effective educational policies and practice within primary schools is needed. The reflections in this chapter have included some ideas for changes in this research design. Below are listed some additional ideas towards improving the current design, as well as ideas employing a new design:

❖ This study appears worth repeating as the results were strong, but it would be interesting to see whether there would be similar results with another sample and with another qualified teacher. The five schools used in this research were purposely similar to each other, with relatively low socioeconomic status, though not as prevalently underprivileged as with the children in the Newark, New Jersey study. The number of students allowed for some variability in the sample, yet would there be similar results with other similar samples in other cities?

❖ As seen by the results and reflections in the measurements section, it would be beneficial to do this study again, making sure that all groups took the spatial reasoning test, ideally one that has been tested further and therefore appropriately updated. I would administer all tests if allowed. Again, in this study, Spatial Reasoning for the Melody Implicit group was only one of two tests that I did not personally administer. (The other test was the Mathematics test for one-half of the Rhythm Implicit group. Recall that this group joined with another school to create this one subgroup due to small numbers within each class.) Nonetheless, one full Melody group (Explicit) took all of the dependent variable tests and the Implicit Melody group took all, except that there were only two for the Spatial Reasoning test. Analysed together, however, the sample size was in the range of the recommended minimum number of 30. Therefore, the analyses looking at the groups together were valid, albeit with a smaller number than the other groups. The opportunity to compare the Explicit Melody group to the Implicit Melody group on Mathematics attainment was nevertheless equal to the other groups.

❖ Ideally a bespoke standardised and age-appropriate mathematics test would have been created to focus precisely on the specific mathematical concepts considered likely to be influenced by musical experience, therefore optimising construct validity. This test would include enough test items in each relevant mathematical category in order to create an adequate amount of data to answer the concept-specific research questions unequivocally. Therefore, it would include even more proportional questions, plus more problem solving and pattern recognition questions. The tests would not require specific linguistic/cultural knowledge or special terminology such as the names of measurements, like “litre” for example. While this would have been the ideal, test development and standardisation would be a project in itself, not feasible to include in the present time frame.

❖ Since focusing primarily on standardised tests provides a relatively narrow interpretation, a future study could use less quantitative data and more qualitative case studies for longitudinal study of the children’s development related to their musical and mathematical experiences.

❖ Research similar to this study but with expanded qualitative data could be conducted. This could include case studies and more in-depth interviews periodically or at least once before and once after the final concert.

❖ Having demonstrated change behaviourally, it would be fascinating to look directly at brain activity using EEG methodology and changes over time, both functional and structural using child- friendly fMRI. (See Schlaug, et al., 2001; 2004; 2008).

❖ Noting the correlation between high musical and mathematical abilities, one could look at potential long-term effects of individuals’ musical background and chosen specialisation in mathematics by surveying university students for details of their musical background. One could perhaps even look at the differences in brains between students in different areas of study such as history and mathematics using fMRI. They would need to be from the same population as much as possible so that all other variables are the same.

8.6.2 Implications for Children & Schools

During the course of my research, and after the completion of my fieldwork, Hallam published a book entitled *The Power of Music* (2015). This inclusive research synthesis includes a broad range of quantitative and qualitative studies of music experiences and possible transfer. It supports many of the benefits of music in other curricular and psychological areas that have been identified independently in this study. Hallam (2015, p. 29) noted that evidence concerning music and mathematics is mixed and that earlier studies in particular are primarily correlational and even though studies with the potential to show causality are increasing, there is still a need for more experimental studies.

As active participation in music through singing or playing an external instrument requires sustained attention, often taking place over a longer duration than some children may tolerate in a mathematics lesson for example, this could be an effective alternative avenue for providing support for general attentional development, through extended cognitive practice for organising units in an orderly fashion, as musical production requires cognitively and physically managing units of time through following the musical beat structures within a piece.

An additional implication of the results of this study is that if mathematical understandings can be enhanced by including a quality music curriculum, then this inclusion should be considered by policy makers and curriculum designers as a viable means for supporting numeracy achievement. Even if this hypothesis were not true, numerous other benefits of music learning have been documented such as increasing children's attention span and motivation for school in general (Posner, Rothbart, Sheese & Kieras, 2008; Abeles & Sanders, 2007). Additionally, it is often enjoyable and fulfilling, as noted in the introduction and reflection portion of this chapter. Therefore, if learning music is an enjoyable method at least supporting not only musical ability but also the understanding of counting and numerical relationships – as shown by the majority of students in the fieldwork classes throughout the year, and corroborated by teacher questionnaires and student interviews for this study as well – then this alone could be enough reason for schools and teachers to consider regular music inclusion when designing curricula and lesson plans for both music and mathematics.

Therefore, I propose that these four areas be reviewed considering this and other similar research:

- ❖ Policy for music & mathematics in schools
- ❖ Curricula of music & mathematics in schools
- ❖ Music education inside and outside of schools
- ❖ Mathematics education inside and outside of schools

8.7 Contribution to Knowledge

My contribution to knowledge fills in missing gaps in the existing body of research because of its size (medium-scale), duration (longitudinal, over nine months) and setting (real-world in schools) study that may be the only study looking at specific elements within music (form, melody and rhythm) while seeking potential cognitive correlates that could be enhanced due to long-term and engaged experiences in music with a qualified teacher.

New theoretical perspectives may have been added due to speculations put forth in this thesis. For example, linking the work of Schlaug and colleagues (Schlaug, et al., 1994, 1995, 2008) who found that extended training in music for young children resulted in thickening of the corpus callosum, therefore aiding speed of inter-hemispheric information transmission, with work by Park, Park and Polk (2013), who showed a positive correlation between inter-hemispheric functional connectivity and mathematical achievement may add to the body of knowledge through connecting research in different fields together.

Unique teaching approaches were used in the field work for this study in the differentiation between modes of music and the inclusion for half of the participants of discussions on the link between music and mathematics. New pedagogical approaches for teaching mathematics might be inspired by this work so that children who may not have desire or confidence to do mathematics might be interested due to a new-found link between the two domains, one of which, they may already have interest. This may result in new perspectives on learning potential and in unique applications to the school curriculum or individual teachers' classroom lesson plans.

Finally, the statistically significant results of this unique design in themselves can further contribute to knowledge in the field and help answer questions regarding the link between the oft-questioned pair of domains while adding confidence in the potential for a causal relationship. Perhaps the particularly strong results for mean gains in proportional understandings can spark an interest among researchers and teachers to look more specifically at ways to encourage mathematical thinking in children or even in learners across the life span.

8.8 Conclusion

This present long-term, real-world research has addressed current objectives for children's academic success in areas such as mathematics and science, while at the same time providing thorough training and enjoyment through singing to promote their musical development. As noted above, the project has sought to demonstrate through a strong, quasi- experimental design that musical training can enhance mathematical performance. It also has shown that this effect is increased slightly when the music- mathematics link is made, even while briefly and lightly, yet nonetheless explicitly. Therefore, as hypothesised, the musical training, taking place within the everyday curriculum on a regular weekly basis for the duration of a school year, indeed did improve the children's mathematical attainment, while explicit, but brief verbal references to the mathematical correspondences slightly enhanced this improvement.

I was inspired to conduct this research with a sense of urgency because of the many school districts in the United States, the United Kingdom and elsewhere that are narrowing the curriculum and limiting school extracurricular activities, including music to focus on performance in high- stakes core curricular tests (Alexander, 2011; Berliner, 2011; Nicholls & Berliner, 2007; Wyse, McCreery & Torrance, 2010).

If it can be shown that musical training enhances logical thinking, policy-makers may consider retaining or even increasing music education in schools. The current curricula in other countries, however, such as Japan and Hungary were highlighted earlier in this chapter for the challenging nature of their music education systems. Perhaps it is no coincidence that the distinguished music educators and philosophers, Suzuki and Kodaly, whose compelling arguments regarding the importance of music education as part of the regular curriculum in schools, were from these countries.

Today, perhaps it is musicians and media stars, such as Gareth Malone of *The Choir*, or James Rhodes, of *Don't Stop the Music*, whose goal is that every primary school in England will have a specialist music teacher and that every child in England will have the chance to learn an instrument who will have a similar impact in England. The success of the "Sing Up!" movement (singup.org) in English schools, contemporaneous with the research reported here regarding the improvement of learning and socio-emotional development, supports the present thesis.

Concerns about the narrowing of the primary curriculum, and the growing and suffocating ethos of performativity in schools demands prominent voices, such as Kenneth Robinson, Lou Aronica (Robinson & Aronica, 2015) and others, who urgently yet eloquently are calling for more space for creativity in schools. It is hoped that the present study can provide scientifically rigorous support for the emerging awareness of the importance of music and the arts in schools, principally for their own sake, but also because they support children's learning in other disciplines, such as mathematics.

As stated in Chapter 1, my attempted contribution to knowledge in the field lies in highlighting the impact that vocal-based music education in primary schools can have on children's lives and in specifying detailed musical and mathematical relationships that may inspire new thinking about curriculum design, influence effective pedagogical practice, and have a positive and productive impact on the educational process.

The findings of this project raise a number of hypothetical questions. What if one's pattern and proportional recognition as well as problem-solving skills are improved via music lessons, which in turn support mathematical thinking? What if executive functions such as sustained attention, inhibitory control, working memory and cognitive flexibility are also developed via music lessons? By seeking creative ways to help children learn and develop, we are serving their needs responsibly and potentially much more effectively. Ultimately, if the effects of music education can demonstrate that it has the power and potential to enhance children's academic achievement levels, as well as other areas in their lives, policy-makers in England, in the United States and in other countries may be persuaded to give music a bigger space in the curriculum, perhaps seeking not only higher rankings in mathematics achievement, but also greater well-being and richer life experiences for children. The ripple effects of the potential benefits of holistic learning experiences should not be underestimated, as these can extend to their families and communities as well as society at large.

Figure 22 The photos above are from one of the concerts at the end of the fieldwork. As the concert featured songs from around the world, children were encouraged to wear clothing from around the world as well. The teacher followed suit.

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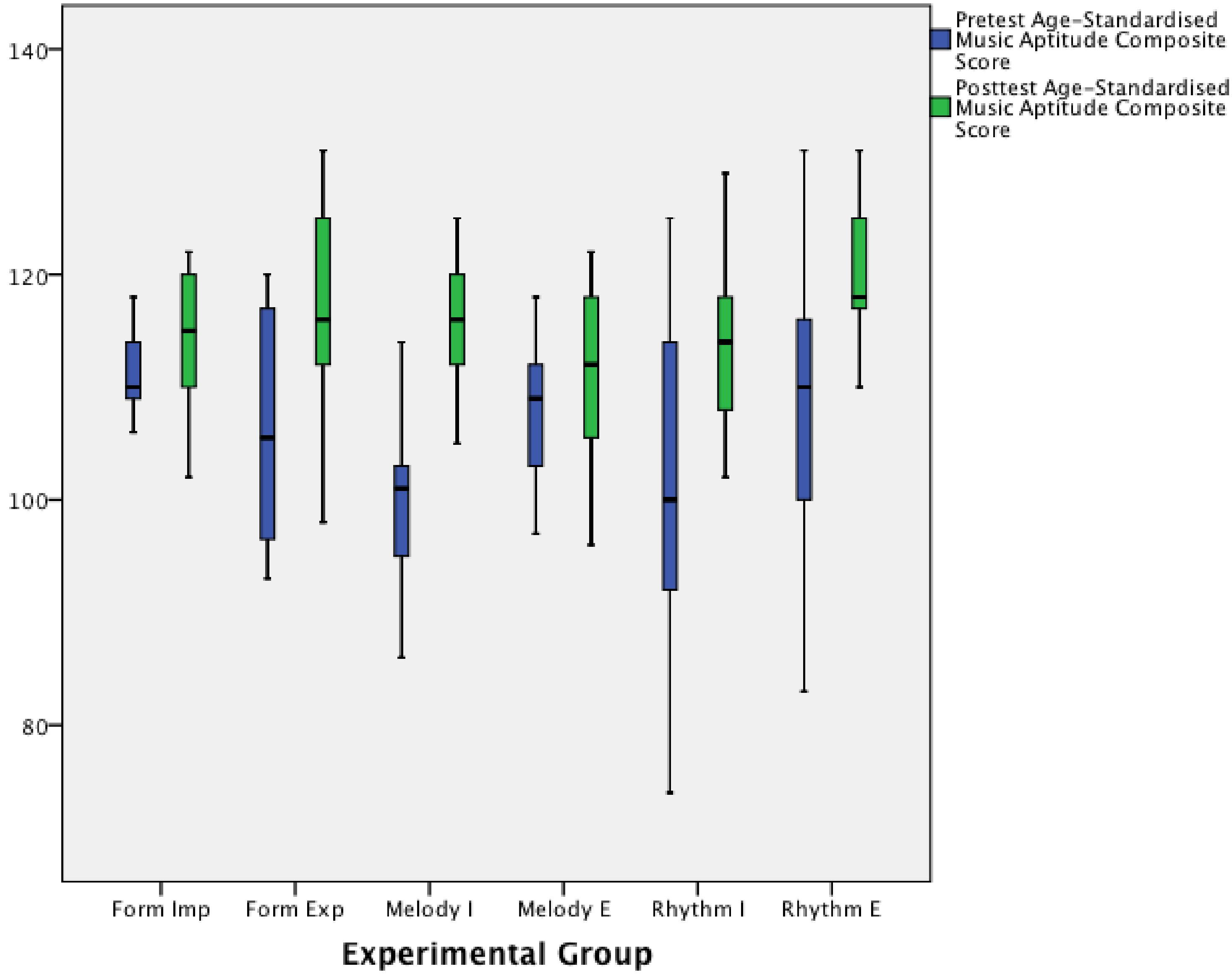
Appendix 1
Statistical Materials for Chapter 6

Pre-ANOVA Assumptions for Music Aptitude Tests & Cronbach's alpha Reliability Analysis

Below is a list of the items that have been checked for each standardised test.

1. Outliers – Boxplot
2. Normality of Distribution – Shapiro-Wilk's Test
3. Homogeneity of Variances – Levene's Test and Welch's Test
4. Homogeneity of Covariance – Box's Test
5. Reliability Analysis – Cronbach's alpha (α).

Checking for Outliers: Boxplot for Music Aptitude Tests



There are no outliers in the Music Aptitude data, as illustrated by the boxplot above.

Checking Music Aptitude Tests for Normality of Distribution:

Shapiro-Wilk's Test

	Experimental Group	Shapiro-Wilk		
		Statistic	df	Sig.
Pretest Age-Standardised Music Aptitude Composite Score	Form Implicit	.95	15	.596
	Form Explicit	.91	20	.055
	Melody Implicit	.97	18	.836
	Melody Explicit	.96	20	.474
	Rhythm Implicit	.97	22	.746
	Rhythm Explicit	.95	19	.396
Posttest Age-Standardised Music Aptitude Composite Score	Form Implicit	.91	15	.143
	Form Explicit	.96	20	.534
	Melody Implicit	.96	18	.558
	Melody Explicit	.94	20	.275
	Rhythm Implicit	.94	22	.154
	Rhythm Explicit	.96	19	.524

Music Aptitude Standardised Scores are normally distributed for all interventions at both time points, as assessed by the Shapiro-Wilk's test ($p > .05$) above.

Homogeneity of Variances for Music Aptitude Tests:

Levene's Test[a] and Welch's Test

Test of Homogeneity of Variances (Levene)

	Levene Statistic	df1	df2	Sig.
Pretest Age-Standardised Music Aptitude Composite Score	4.60	5	121	.000
Posttest Age-Standardised Music Aptitude Composite Score	.89	5	120	.490

Robust Tests of Equality of Means (Welch)

	Statistic[a]	df1	df2	Sig.
Pretest Age-Standardised Music Aptitude Composite Score	6.35	5	55.56	.000
Posttest Age-Standardised Music Aptitude Composite Score	3.93	5	55.25	.000

a. Asymptotically F distributed.

There was not homogeneity of variances for the pretest scores, as assessed by Levene's Test of Homogeneity of Variance, $p < .05$ and Welch's more robust test, $p < .001$. The ANOVA is run nonetheless, as it is fairly robust, while specific significance interpretations are made with caution (Field, 2009, p. 155; Lund, 2013).

Testing for Homogeneity of Covariance for Music Aptitude Tests:

Box's Test

Box's Test of Equality of Covariance Matrices[a]

Box's M	49.05
F	3.11
df1	15
df2	53378.63
Sig.	.00

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.

a. Design: Intercept + Group
Within Subjects Design: Time

As noted in Chapter 6, there was not homogeneity of covariances, as shown by Box's test of equality of covariance matrices, $p < .001$). When musical modes are assessed separately, Form groups are the groups that do not have homogeneity of covariance at pretest and therefore have contributed to the overall lack of homogeneity. The primary interest, however, is in improvement levels from pretest to posttest.

Reliability Analysis for Music Aptitude Tests –

Cronbach's alpha (α).

Music Aptitude Pretest: Cronbach's Alpha Based on Standardised Items	Music Aptitude Posttest: Cronbach's Alpha Based on Standardised Items	Number of Items
.88	.78	3

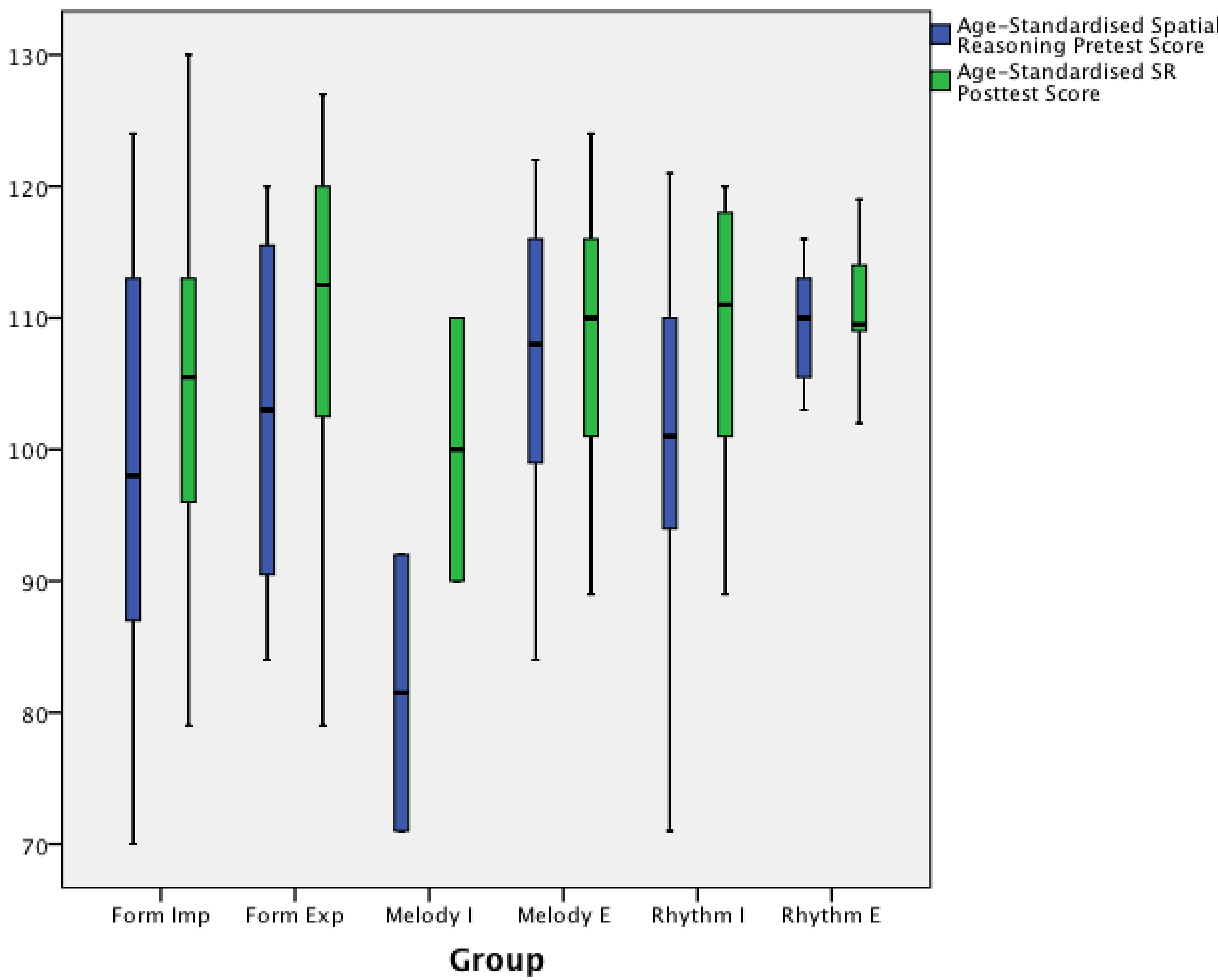
The Music Aptitude pretest subscales have very good internal consistency, α = .88. Likewise, the Music Aptitude posttest subscales also have good internal consistency, α = .78, though not as high as the pretest subscale reliability score for this sample.

The general guideline for illustrating internal consistency using Cronbach's alpha is shown below (Kline, 2000; George & Mallery, 2003).

Cronbach's Alpha	Internal Consistency
$\alpha \geq .90$	Excellent (High-Stakes testing)
$.70 \leq \alpha < .90$	Good (Low-Stakes testing)
$.60 \leq \alpha < .70$	Acceptable
$.50 \leq \alpha < .60$	Poor
$\alpha < .50$	Unacceptable

Pre-ANOVA Assumptions for Spatial Reasoning Tests & Cronbach's alpha Reliability Analysis

Checking for Outliers: Boxplot for Spatial Reasoning Tests



There were no outliers in the data, as illustrated by the boxplot above.

Checking Spatial Reasoning Tests for Normality of Distribution:

Shapiro-Wilk's Test

	Group	Shapiro-Wilk		
		Statistic	df	Sig.
Age-Standardised Spatial Reasoning Pretest Score	Form Implicit	.95	22	.313
	Form Explicit	.92	24	.065
	Melody Implicit			
	Melody Explicit	.94	21	.192
	Rhythm Implicit	.96	25	.499
	Rhythm Explicit	.95	16	.477
Age-Standardised SR Posttest Score	Form Implicit	.97	22	.786
	Form Explicit	.94	24	.124
	Melody Implicit			
	Melody Explicit	.95	21	.302
	Rhythm Implicit	.92	25	.055
	Rhythm Explicit	.94	16	.377

Spatial Reasoning Standardised Scores are normally distributed for all intervention groups at both time points, as assessed by the Shapiro-Wilk's test ($p > .05$) above.

Testing Spatial Reasoning Tests for Homogeneity of Variances: Levene's Test[a] and Welch's Test

	F	df1	df2	Sig.
Age-Standardised Spatial Reasoning Pretest Score	5.01	5	104	.000
Age-Standardised SR Posttest Score	2.70	5	104	.025

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group Within Subjects Design: Time

Welch's Robust Tests of Equality of Means

		Statistic[a]	df1	df2	Sig.
Age-Standardised Spatial Reasoning Pretest Score	Welch	5.42	5	10.31	.014
Age-Standardised SR Posttest Score	Welch	.72	5	10.25	.673

a. Asymptotically F distributed.

There was not homogeneity of variances for the pretest scores, as assessed by Levene's Test of Homogeneity of Variance, $p = .000$ ($p < .001$), yet Welch's more robust test, though at $p = .014$, was significant at the .001 level, but was not significant at the .05 level. The ANOVA is run regardless, as it is fairly robust, while specific significance interpretations are made with caution (Field, 2009, p. 155).

Checking for Homogeneity of Covariance for Spatial Reasoning Tests:

Box's Test of Equality of Covariance Matrices[a]

Box's M	47.52
F	3.79
df1	12
df2	55863.80
Sig.	.000

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.
a. Design: Intercept + Group Within Subjects Design: Time

There is not homogeneity of covariances, as shown by Box's test of equality of covariance matrices, $p = .000$ ($p < .001$). The primary interest, however, is in improvement levels from pretest to posttest.

It is notable that neither Music Aptitude tests nor Spatial Reasoning tests have homogeneity of variances or covariances at pretest, yet they do at posttest. These cognitive domains are not formally regulated in schools. Therefore, it is not surprising that there would be variation at the beginning of the intervention. It is therefore possible that the shift from non-homogeneity at pretest to homogeneity at posttest is related to the musical intervention.

As noted before, even without homogeneity of covariances, ANOVA is often run, as it is fairly robust (Field, 2009, p. 604), though the interaction effect should be interpreted cautiously. Fortunately, interactions between groups are not key to this study and were not expected to be significant. The question regarding the effectiveness of music-mathematics explicitness is addressed by comparing the means of Implicit and Explicit group scores to each other directly, particularly noting the differences within the same musical mode.

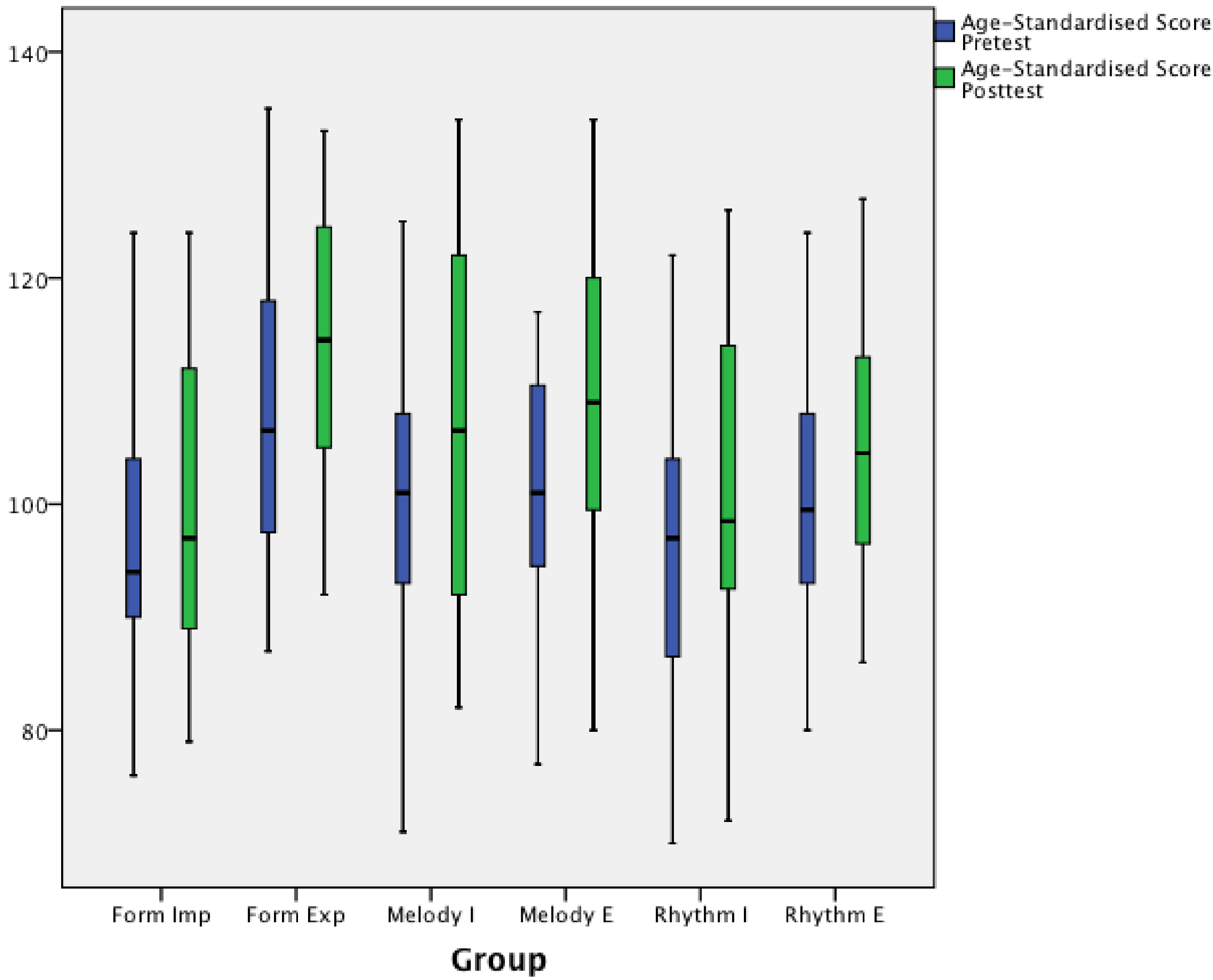
Reliability Analysis for Spatial Reasoning Tests – Cronbach's alpha (α).

Spatial Reasoning Pretest: Cronbach's Alpha Based on Standardised Items	Spatial Reasoning Posttest: Cronbach's Alpha Based on Standardised Items	Number of Items
.62	.63	4

The Spatial Reasoning pretest and posttest subscales have acceptable internal consistency, $\alpha = .62$ and $\alpha = .63$ respectively. Acceptable = $.60 \leq \alpha < .70$ (Kline, 2000; George & Mallery, 2003).

Pre-ANOVA Assumptions for Mathematics Tests & Cronbach's alpha Reliability Analysis

Checking for Outliers: Boxplot for Mathematics Scores



There are no outliers in the mathematics test data, as illustrated by the boxplot above.

Checking Mathematics Scores for Normality of Distribution: Shapiro-Wilk's Test

		Group	Shapiro-Wilk		
			Statistic	df	Sig.
Standardised Score Pretest	Age-	Form Implicit	.97	17	.881
		Form Explicit	.97	16	.874
		Melody Implicit	.97	22	.744
		Melody Explicit	.95	24	.253
		Rhythm Implicit	.96	32	.362
		Rhythm Explicit	.98	20	.864
	Age-	Form Implicit	.94	17	.290
		Form Explicit	.95	16	.530
		Melody Implicit	.96	22	.431

Standardised Score Posttest	Melody Explicit	.97	24	.694
	Rhythm Implicit	.95	32	.163
	Rhythm Explicit	.96	20	.614

Mathematics Standardised Scores are normally distributed for all interventions at both time points, as assessed by the Shapiro-Wilk's test (p > .05) above.

Testing for Homogeneity of Variances of Mathematics Scores:

Levene's Test[a]

	F	df1	df2	Sig.
Age-StandardisedSco	.23	5	125	.950

rePretest	AgeStandardisedScorePosttest	.96	5	125	.441

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

There is homogeneity of variances, as assessed by Levene's Test of Homogeneity of Variance, p = .95/.44 (p > .05).

Testing for Homogeneity of Covariance of Mathematics Scores: Box's Test

Box's M	4.81
F	.31
df1	15
df2	52298.35
Sig.	1.00

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.

There is homogeneity of covariance, as assessed by Box's test of equality of covariance matrices, $p = 1.00$ ($p > .001$).

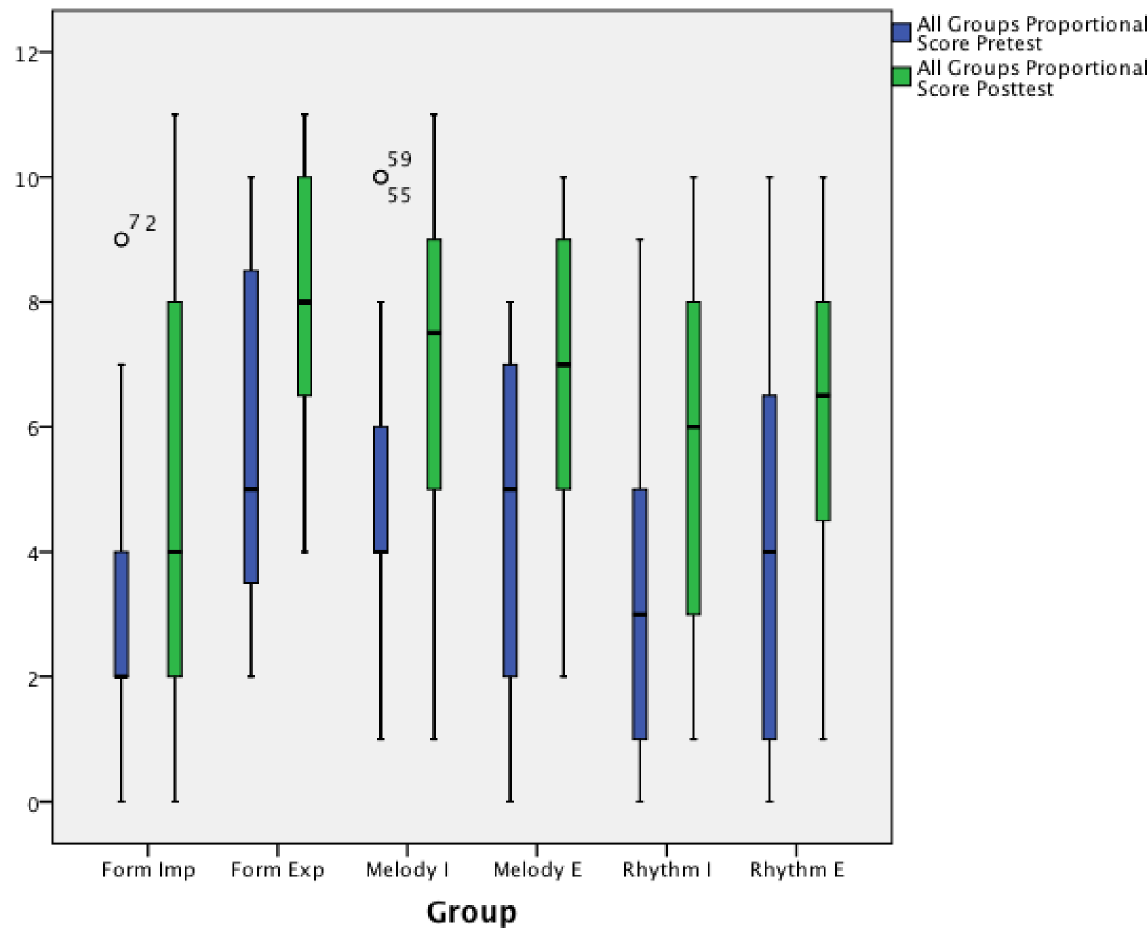
Reliability Analysis for Mathematics Tests – Cronbach's alpha (α).

Mathematics Pretest: Cronbach's Alpha Based on Standardised Items	Mathematics Posttest: Cronbach's Alpha Based on Standardised Items	Number of Items
.92	.90	45

The Mathematics pretest items have excellent internal consistency, α = .92. Likewise, the Mathematics posttest items also have excellent internal consistency, α = .90. Excellent = α ≥ .90 (Kline, 2000; George & Mallery, 2003).

Pre-ANOVA Assumptions for Proportional Mathematics Tests & Cronbach's alpha Reliability Analysis

Checking Proportional Mathematics Tests for Outliers: Boxplot



There are outliers in the Proportional Mathematics data, as illustrated by the boxplot above.

Checking Proportional Mathematics Tests for Normality of Distribution: Shapiro-Wilk's Test

	Group	Shapiro-Wilk		
		Statistic	df	Sig.
	Form Implicit	.87	17	.022
	Form Explicit	.91	16	.102
	Melody Implicit	.90	22	.041
All Groups Proportional Score Pretest	Melod	.92	24	.063

	y Explicit			
	Rhythm Implicit	.94	32	.062
	Rhythm Explicit	.95	20	.311
	Form Implicit	.95	17	.401
	Form Expli	.92	16	.164

	cit			
	MelodyImplicit	.94	22	.223
All Groups Proportional Score Posttest	MelodyExplicit	.93	24	.080
	RhythmImplic	.93	32	.052

	it Rhythm Explicit	.95	20	.321

Proportional Mathematics Scores are not normally distributed for any interventions at neither time point as assessed by the Shapiro-Wilk's test (p > .05) above.

Checking Proportional Mathematics for Homogeneity of Variances: Levene's Test

	F	df1	df2	Sig.
Proportional Score Pretest	.55	5	125	.740
Proportional Score Posttest	1.61	5	125	.163

There is homogeneity of variances at both time points, as assessed by Levene's Test of Homogeneity of Variance.

Testing Proportional Mathematics for Homogeneity of Covariance: Box's Test

Box's Test of Equality of Covariance Matrices[a]

Box's M	8.61
F	.55
df1	15
df2	52298.35
Sig.	.91

Tests the null hypothesis that the observed covariance matrices of the dependent variables are *equal across groups.*

There is homogeneity of covariances, as assessed by Box's test of equality of covariance matrices, p = .91 (p > .001).

Checking Mathematics Tests for Reliability Analysis: Cronbach's alpha (α).

Proportional Mathematics Pretest: Cronbach's Alpha	Proportional Mathematics Posttest: Cronbach's Alpha Based on Standardised Items	N of Items
.76	.79	11

The Proportional Mathematics pretest items have good internal consistency, $\alpha = .76$. Likewise, the Proportional Mathematics posttest items also have good internal consistency, $\alpha = .79$. Good $= .70 \leq \alpha \leq .90$. (Kline, 2000; George & Mallery, 2003).

Appendix 2
Tests Used for Doctoral Study

Gordon Primary Measures of Music Audiation: Tonal and Rhythmic p. 271

Spatial Temporal Reasoning assessment p. 273

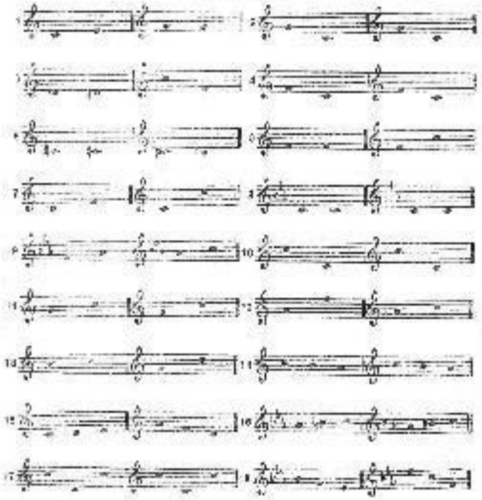
MaLT 8 Mathematics assessment p. 295

PRIMARY MEASURES OF MUSIC AUDIATION

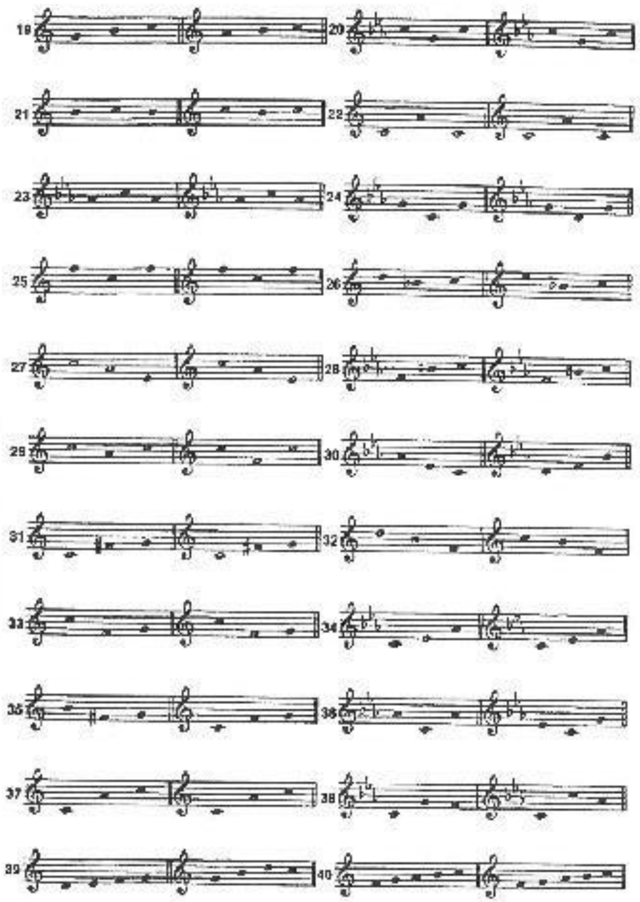
TONAL TEST

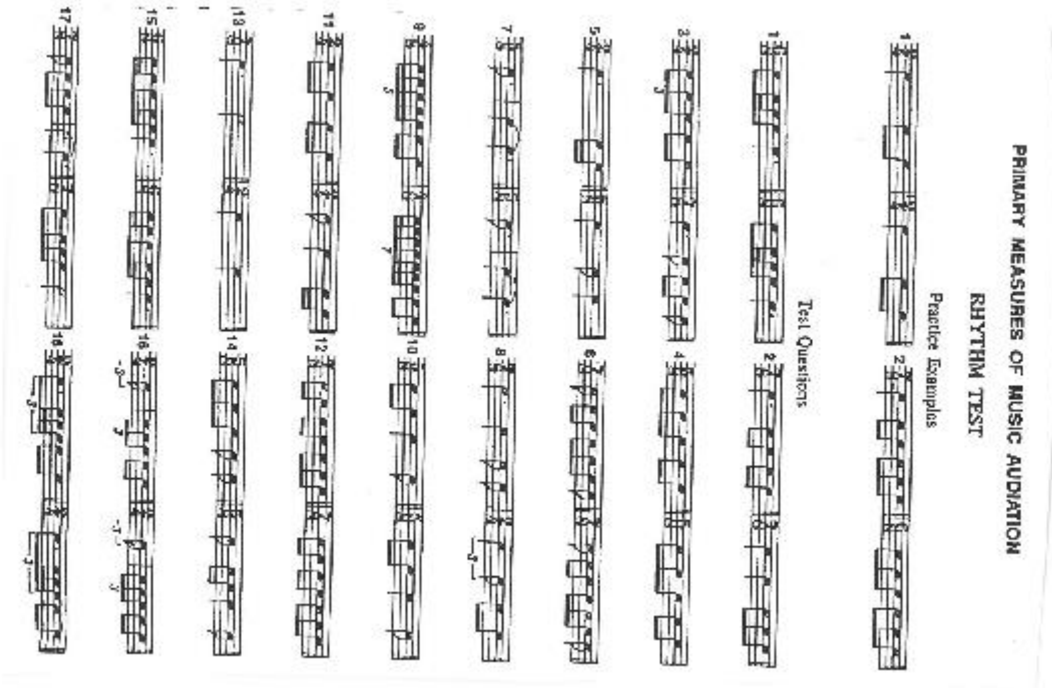
Practice Examples

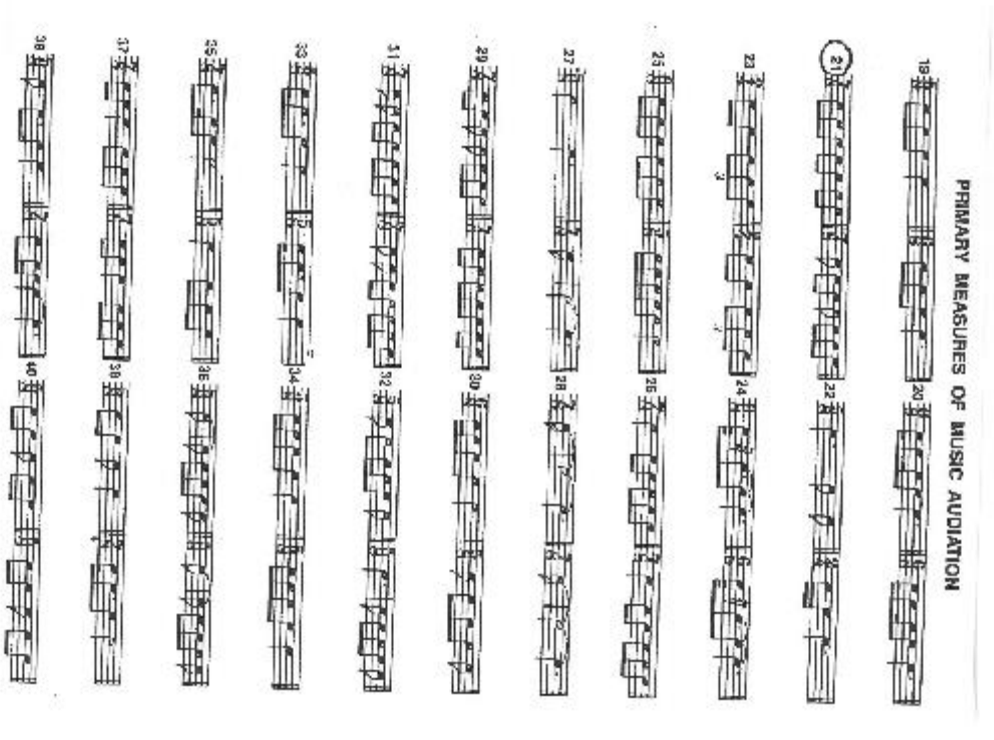
Test Questions



PRIMARY MEASURES OF MUSIC AUDIATION



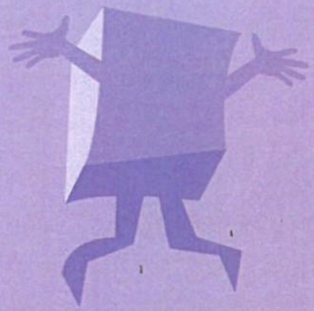




Note – Transferring test documents into word produced inaccurate and missing figures at times. Nonetheless, one can see many examples that are clear and complete.

PUPIL BOOKLET

GL assessment
the measure of potential



Spatial Reasoning

Name

School

AGE 8 and 9

Windows

This test is about imagining what you would see from inside a shop.
You will be shown the outside of a shop window, with a drawing on it.
You must imagine how it would look if you went into the shop and looked at the window from inside. **Here** is an **example.**

Example 1

This is the view from the street.

A

8

These are the four possible views from inside the shop. -+

Which is the correct

one? C is the correct

answer,
so letter C is circled.

D

Example 2

Here is another example.

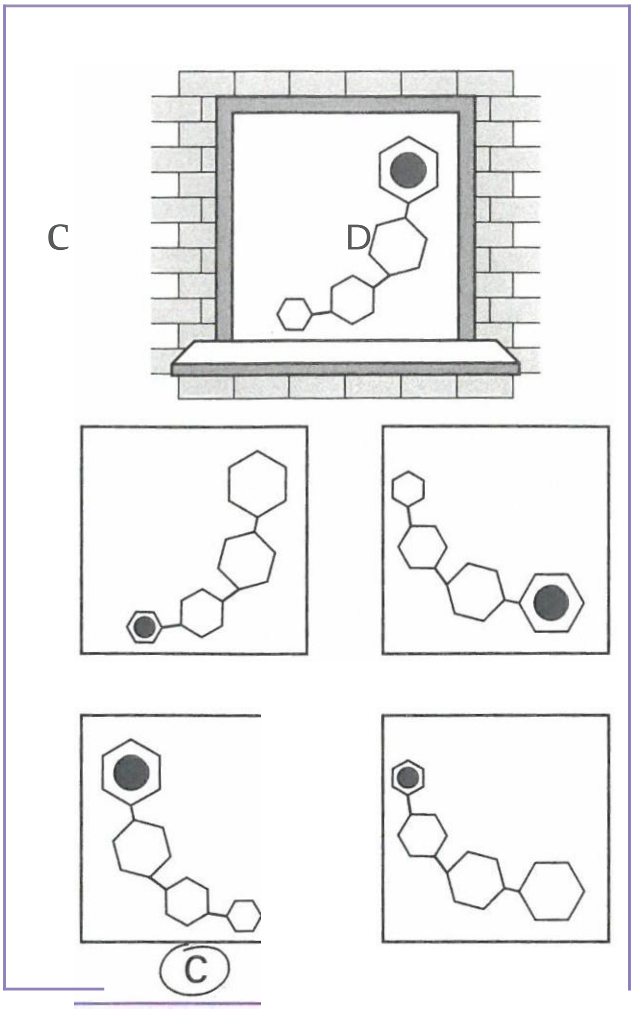
This is the view from the street.

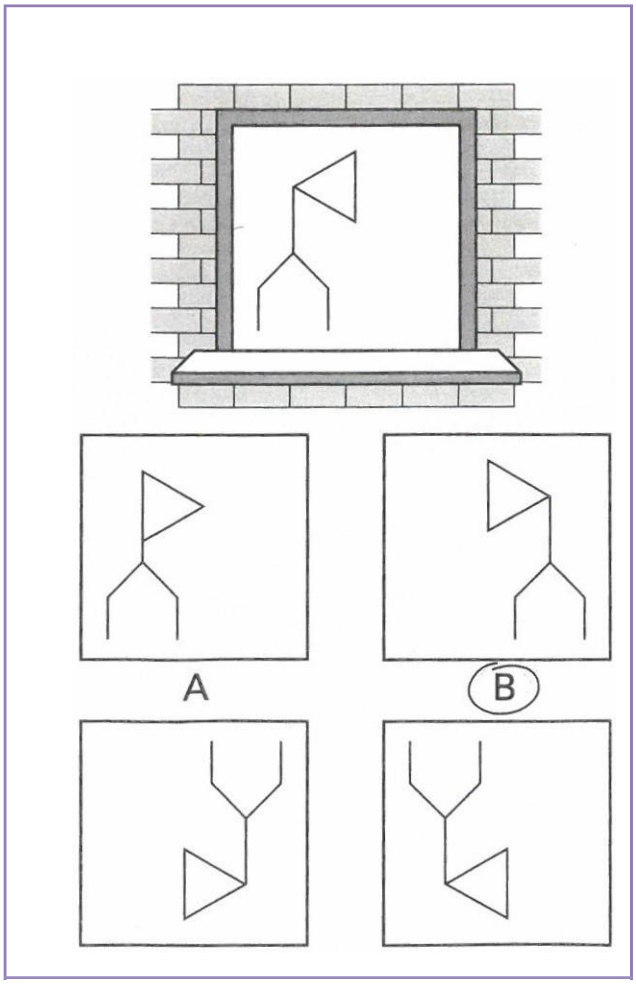
These are the four possible views from inside the shop.-+

Which is the correct one?

8 is the correct answer, so letter 8 is circled.

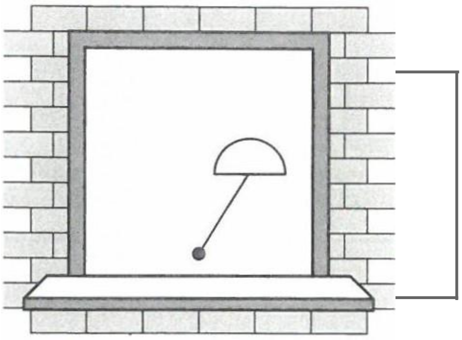
2

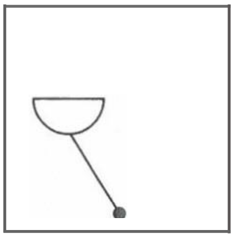




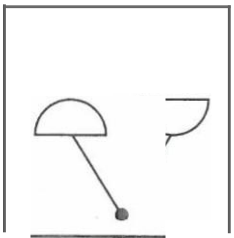
Now try these two practice questions.

Practice 1

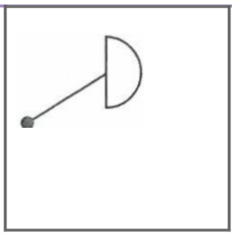




A

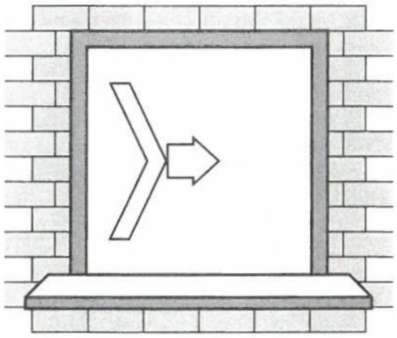


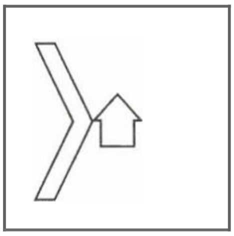
B



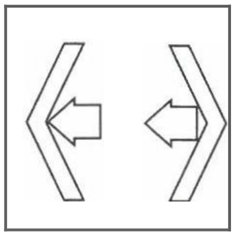
C

Practice 2

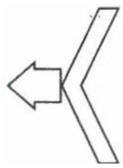




A



8

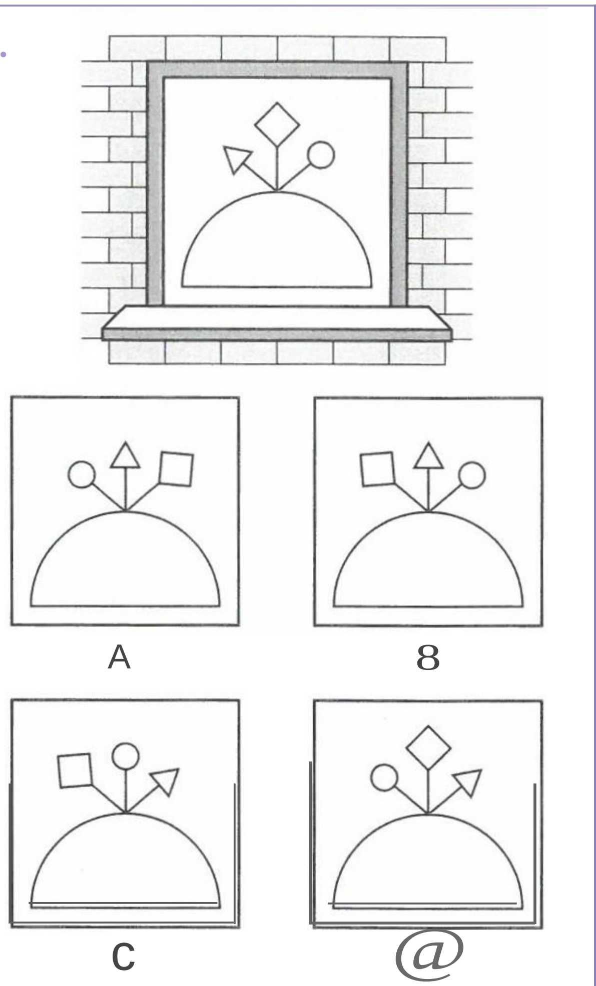


D

Remember:
Choose what you would see from inside the shop.

Do NOT turn over.

1.



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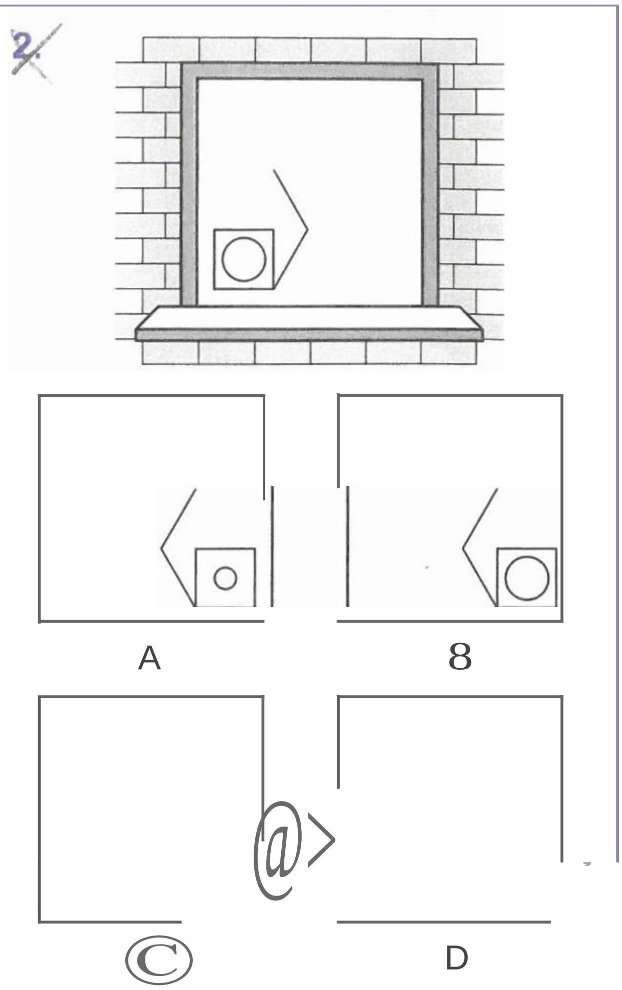

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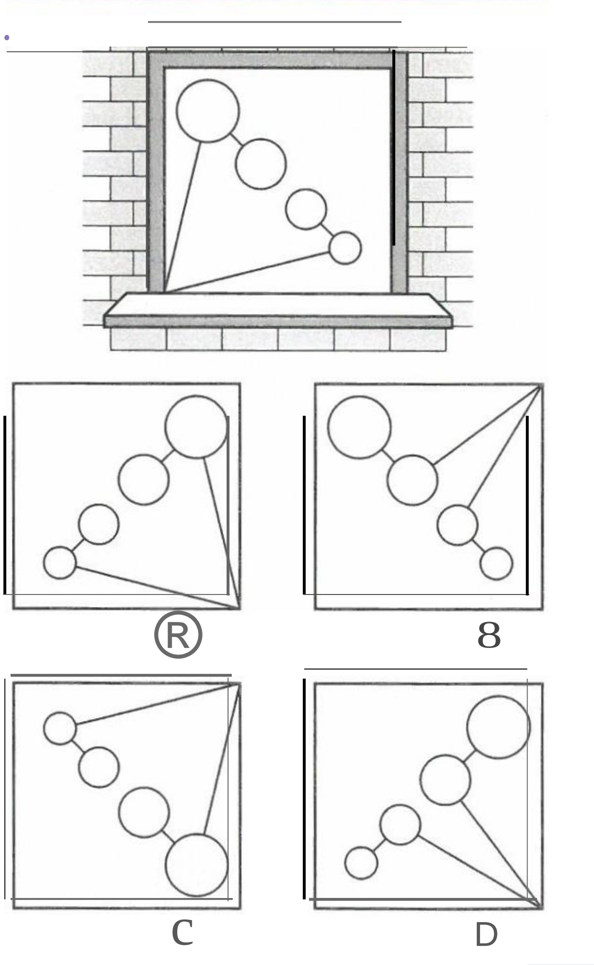

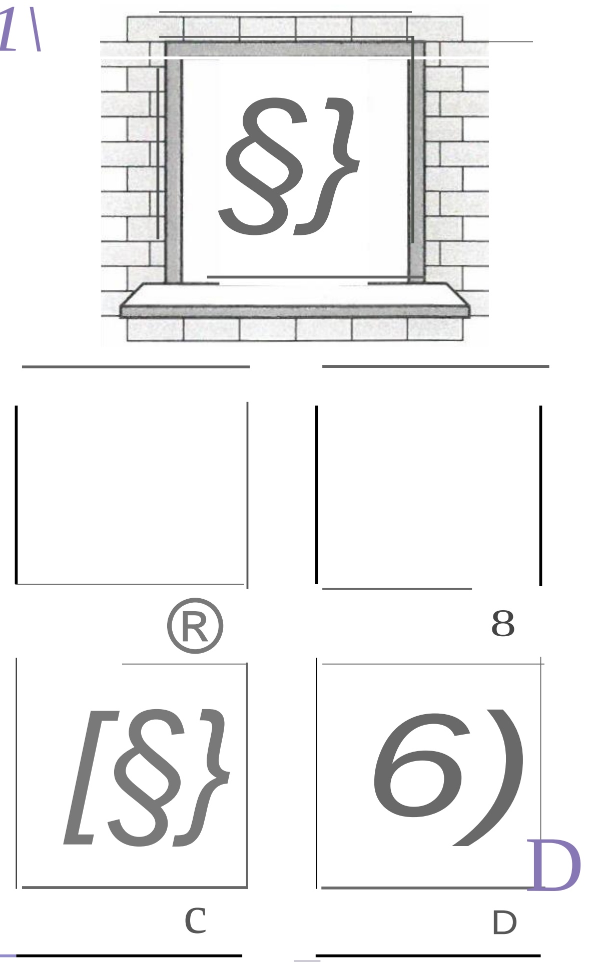

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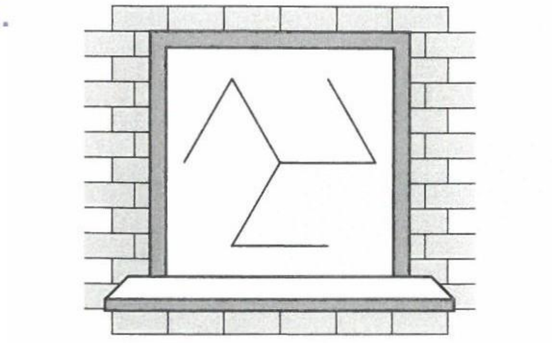
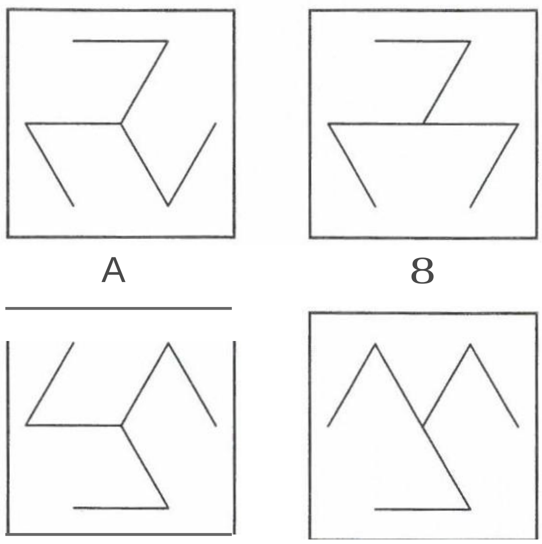


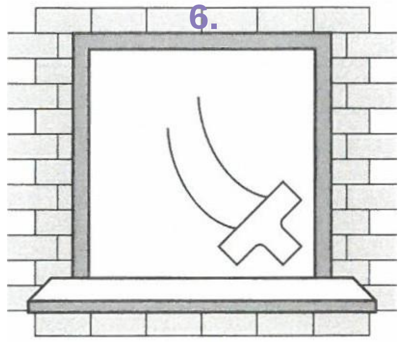


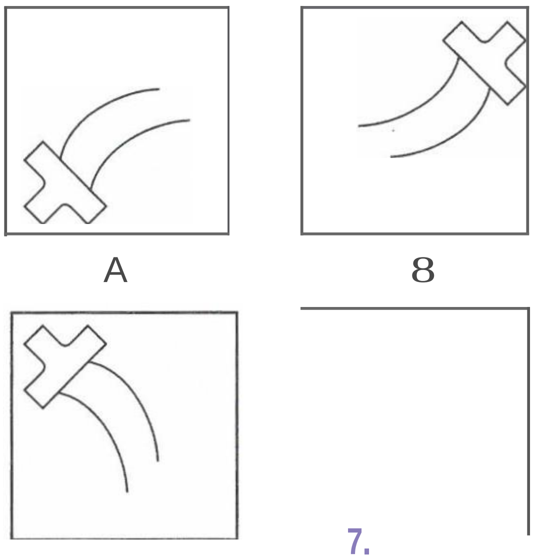
A

8

C

6.

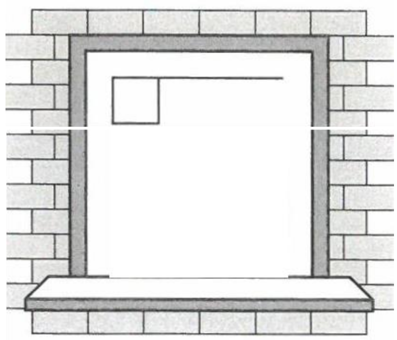




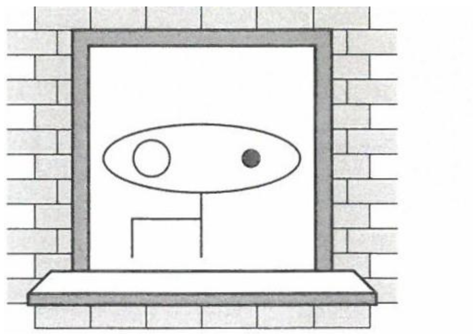
A

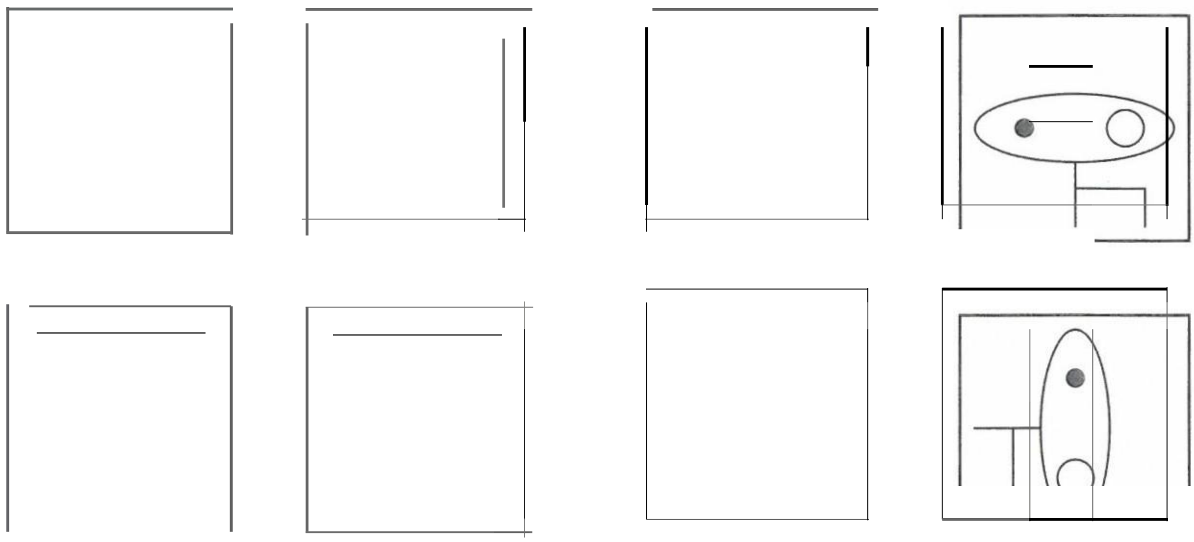
8

7.

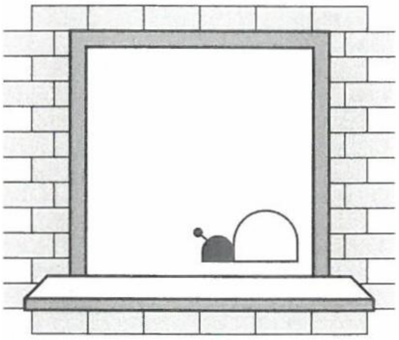


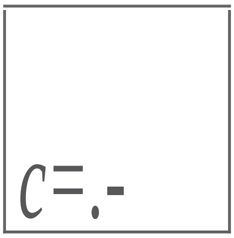
Turn over.

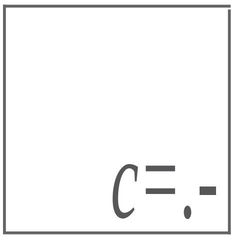




9.

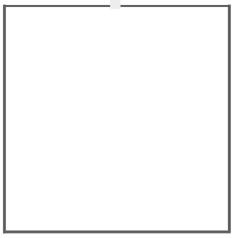






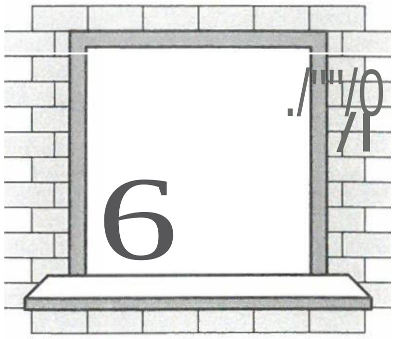
8

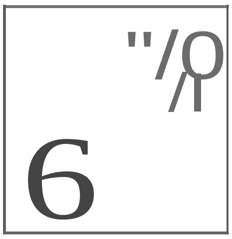
C



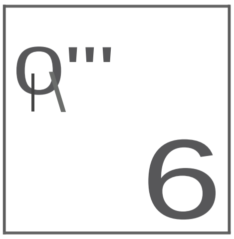
D

10.

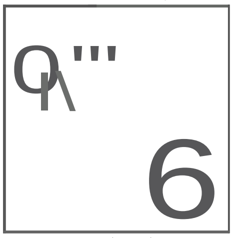




A



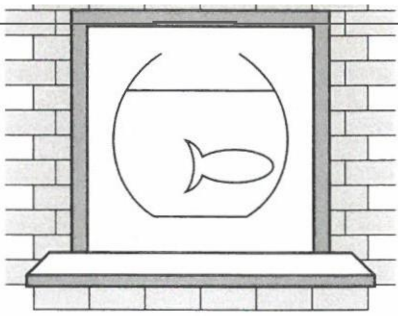
8

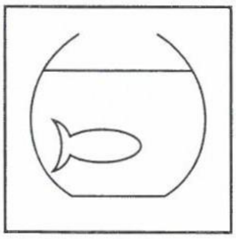


(C)

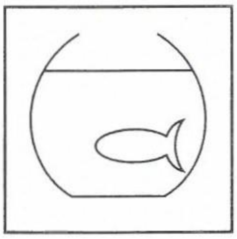
D

11.

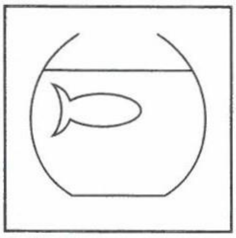




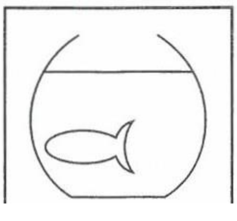
A



8

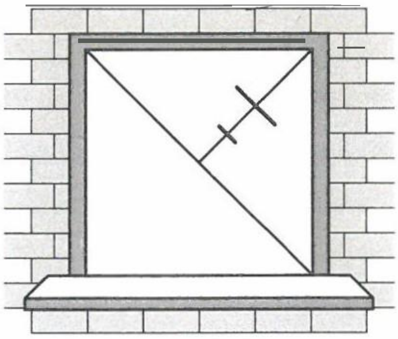


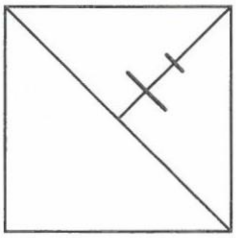
C



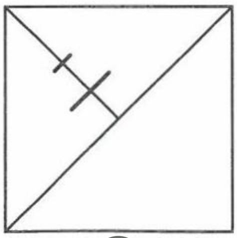
@

1j:

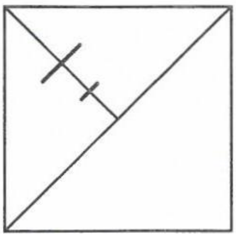




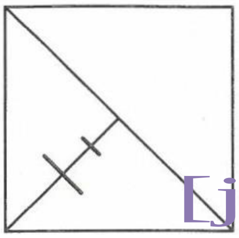
A



®



C



O

Hidden shapes

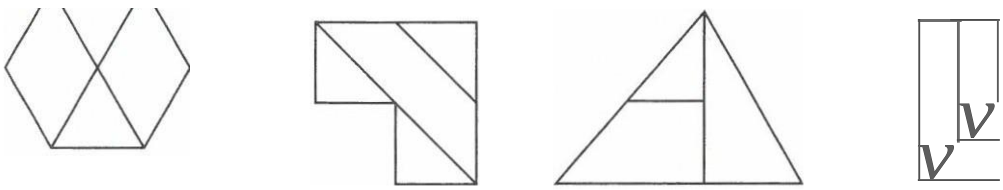
This test is about hidden shapes. Each question is a shape.

It is hidden in one of four patterns. These patterns are called A, B, C and D.

You have to find where a shape is hidden and circle the letter for that pattern. The shape will be exactly the same size and way round. You won't need to imagine it turned around or flipped over.

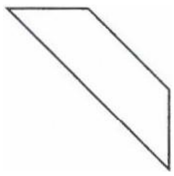
Here is an example.

A 8 C D



Example 1

A O C D



It is hidden here. -+

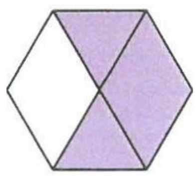
Here is another example.

Example 2

O B C D

D

It is hidden here. -+



Now try these two practice questions.

Practice 1

A B C CO

Practice 2

A 0 C D

6

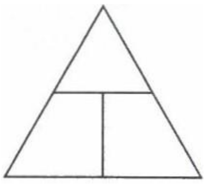
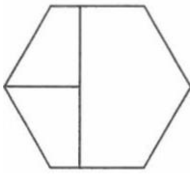
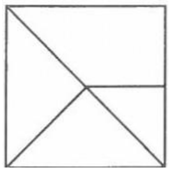
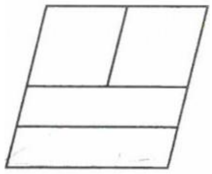
Remember to look for: The same shape

The same size

The same way round

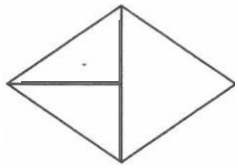
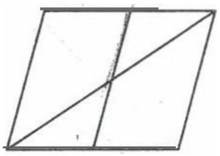
All the edges must be marked on the pattern.

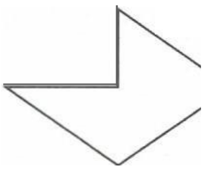
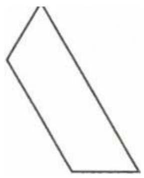
A 8 C D

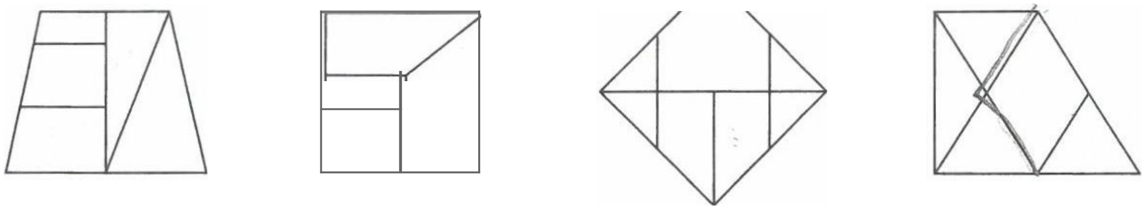
1. A ® c D	2. A 8 @ D L	3. (i) 8 C D [2 C
4. A 8 @ D	A 8 C @ LJ	6. A 8 c @ cfl

A 8 C D

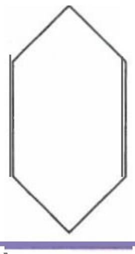
[2J   

l(A 8 @) D 	8. A 8 C @ 	9. 8 C D LJ [[
0. A 8 @ D O	11. A 8 C @	12. A 8 (9 D [[[

A B C D



13. A B {) D



14. A B C @

IT

1X A B @ D

O/

[]

G1

16. {£) B C D

q

17. A B C @

18. A B @ D

G

~~15~~ 13

Do NOT turn over.

Jigsaws

This test is about doing jigsaws in your head. Each question has some jigsaw pieces.

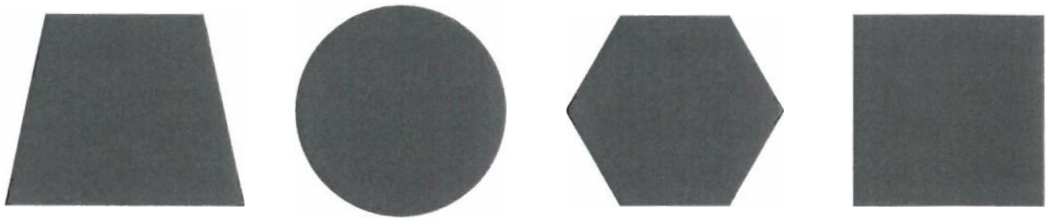
The pieces can be put together to make a finished design.

These designs are called A, 8, C and O and are shown at the top of a question page.

You have to work out which design you could make with the pieces and circle the letter for that design.

Here is an example.

A	8	C	D



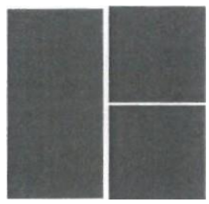
Example 1

A

8

C

It is shown here. -+



O is the only design that you could make with these pieces, so O is circled.

Here is another example.

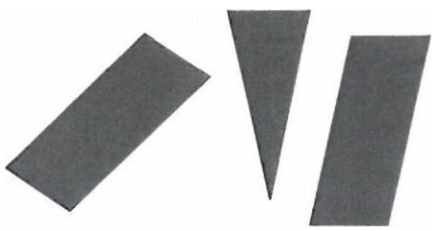
Example 2

O

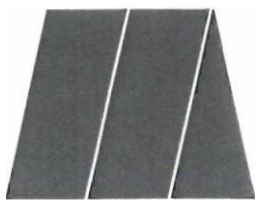
8

C

D



It is **shown here.-+**



A is the only design that you could make with these pieces, so A is circled.

Now try these two practice questions.

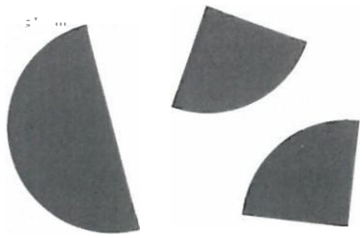
Practice 1

A

8

C

D

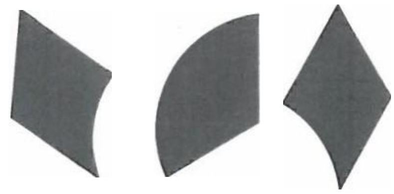


Practice 2

A

B

D



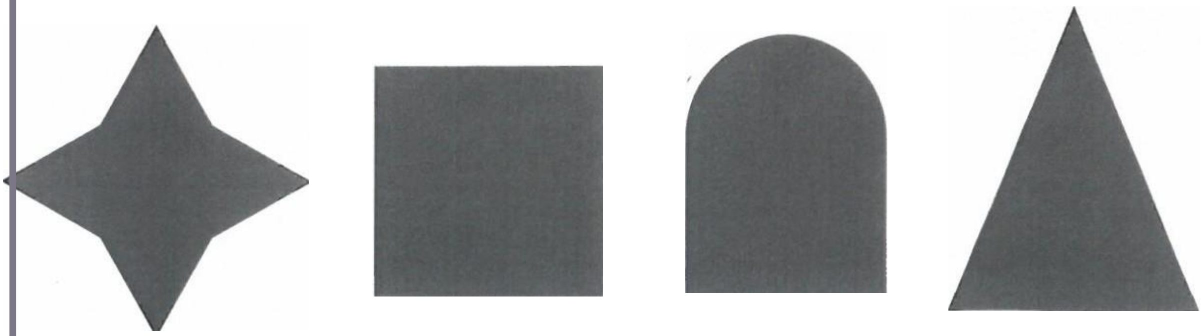
Remember:

Only one of the designs can be made from each set of jigsaw pieces.

You can imagine moving the pieces around on the page, but you cannot flip them over. The jigsaw would be the same size as the pieces put together.

Do NOT turn over.

A B C D



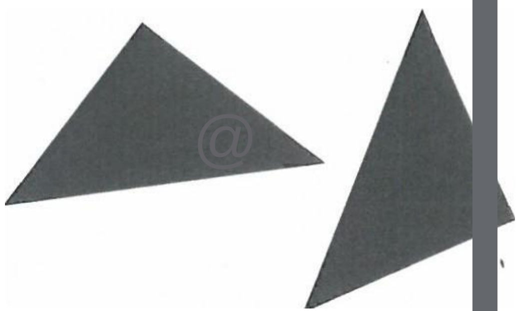
1.

A

B

C

D



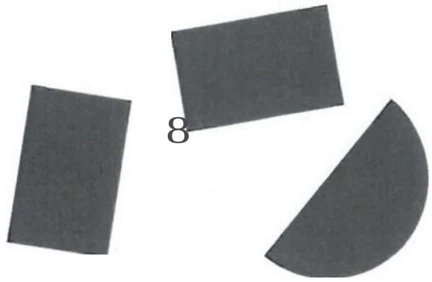
2.

A

8

C

D



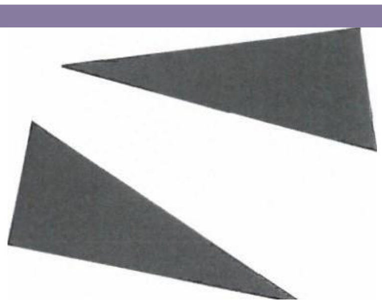
3.

A

8

C

D



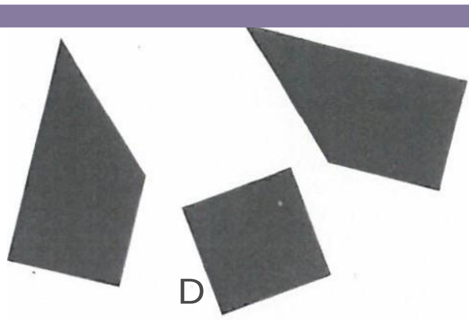
4.

A

B

C

D



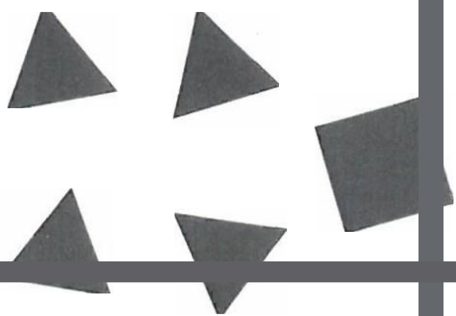
5.

A

8

C

D



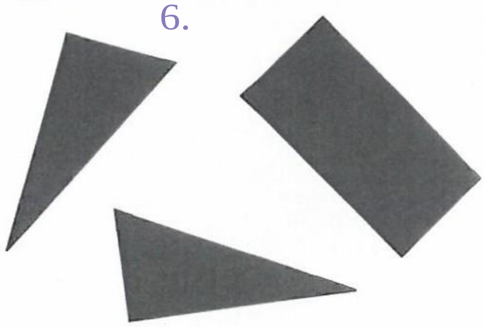
6.

A

B

C

D

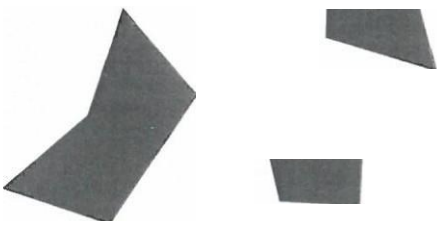


A

B

C

D

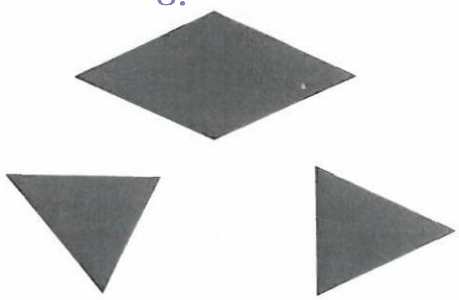


8.

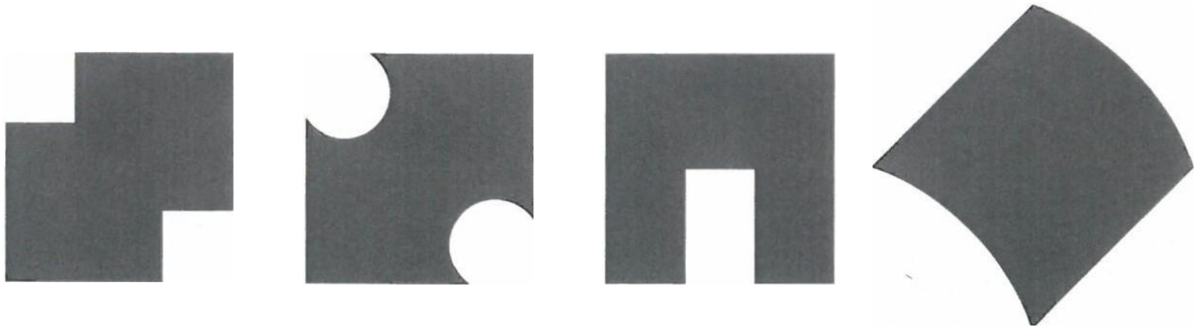
A

C

D



A B C D



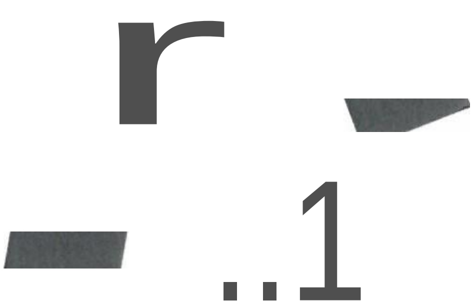
9.

A

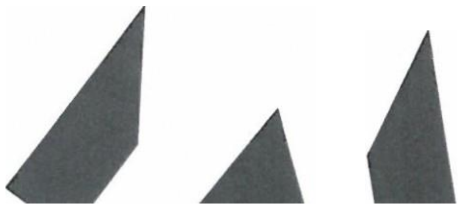
B

C

D



10.



B

C

D

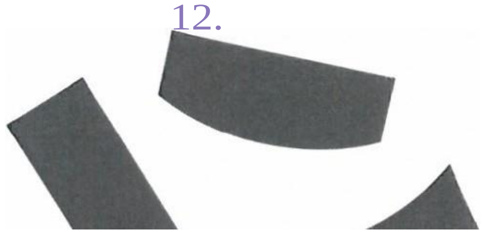
A

B

C

D

12.



A

B

C

D

13.

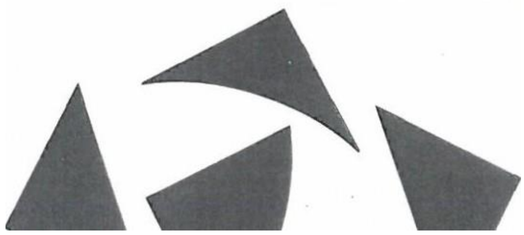
A

B

C

D

14.



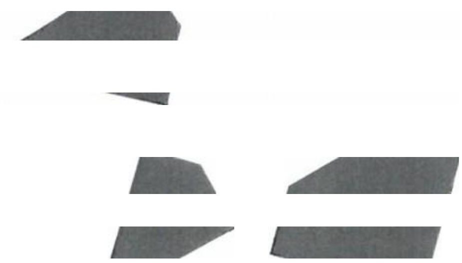
A

B

C

D

15.



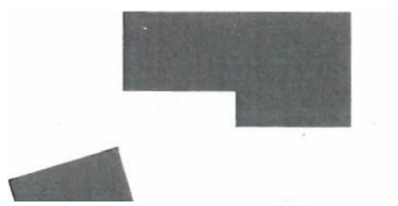
A

B

C

D

16.



A

B

C

D

Do NOT turn over.

Stacks

This test is about stacks of shapes.

Each question is a stack of shapes. The shapes are called A, 8, C, D and sometimes E. They are piled on top of each other in a stack or heap.

You have to find which shape is at the very bottom of the stack.

Then you can circle its letter next to that stack.

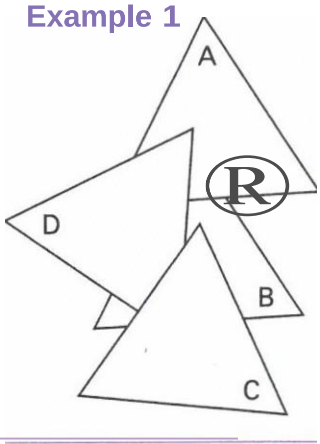
Here is an example.

Example 1

A

C

D

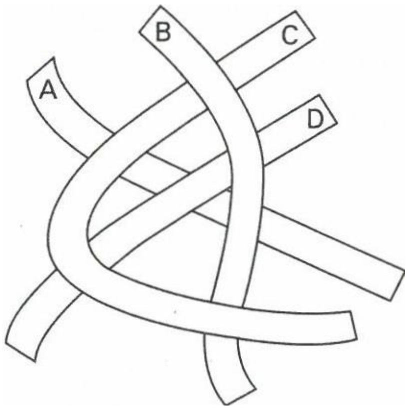


Here is another example.

Example 2

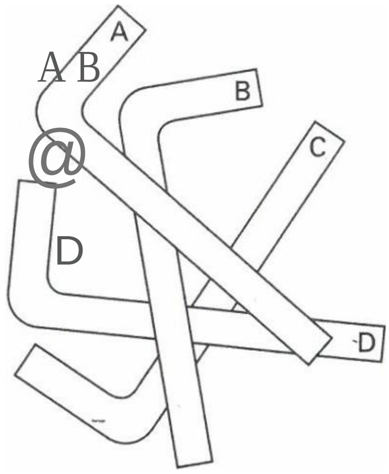
A

B C D

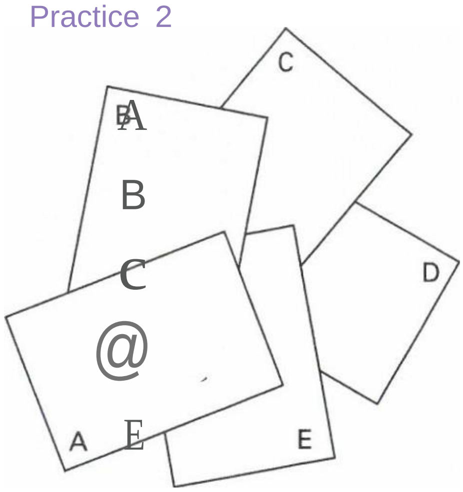


Now try these two practice questions.

Practice 1

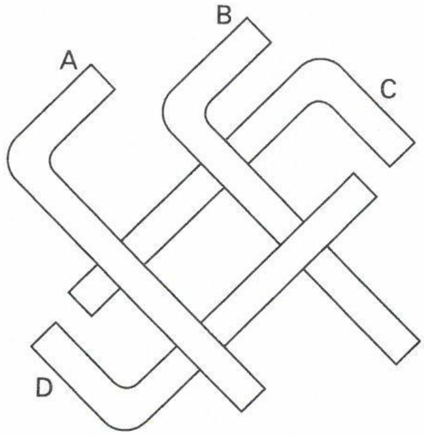


Practice 2



Remember:
Look for the shape at the very bottom of the stack.

1.



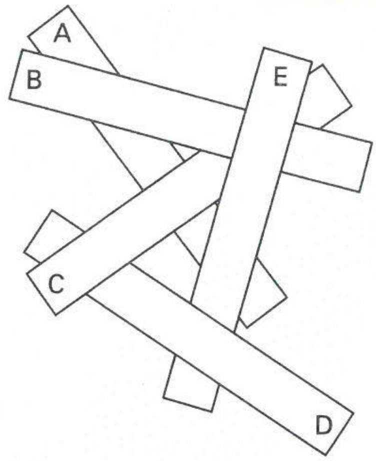
A

B

C

D

2.



A

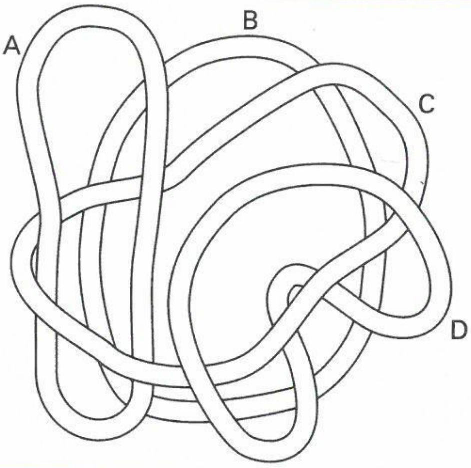
B

C

D

E

3.



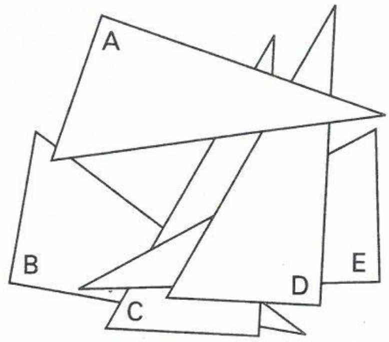
A

B

C

D

4.



A

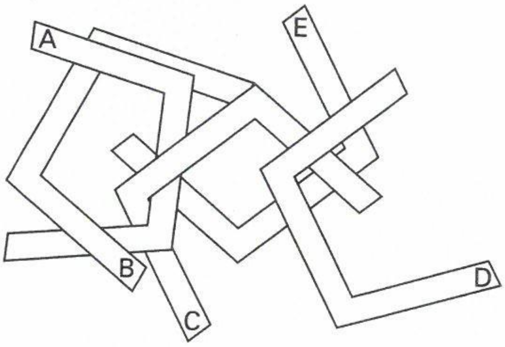
B

C

D

E

5.



A

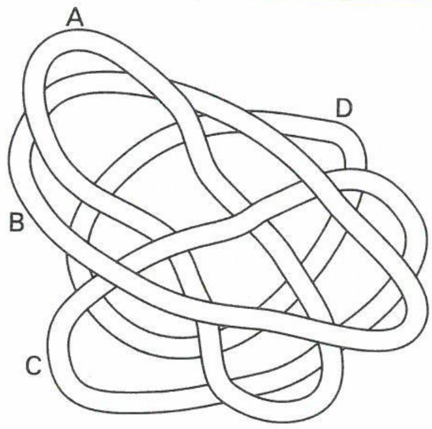
B

C

D

E

6.



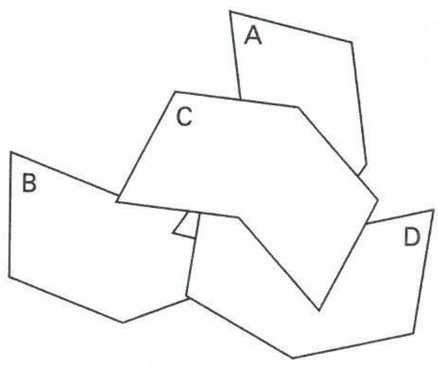
A

B

C

D

7.



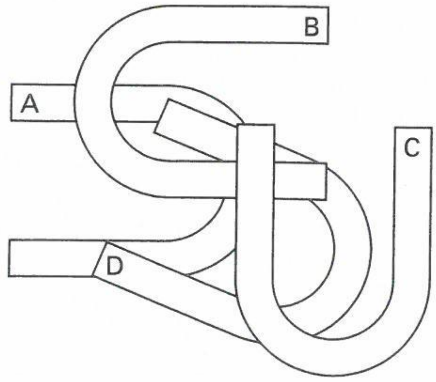
A

B

C

D

8.



A

B

C

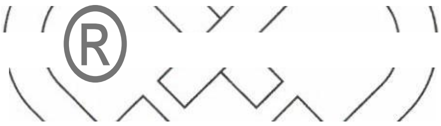
D

9.

A

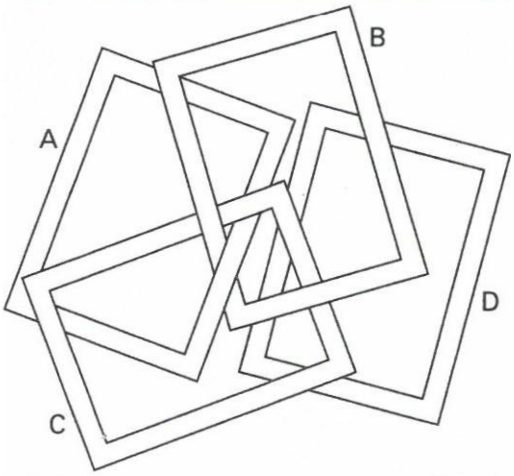
C D E

11.



8 C D

A



C

D

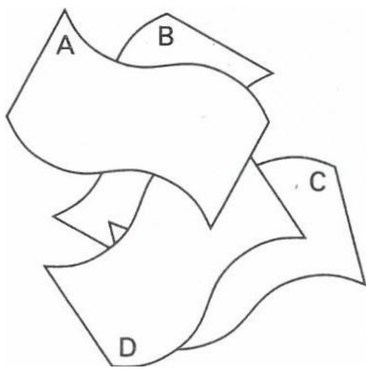
15.

A

@

C

D

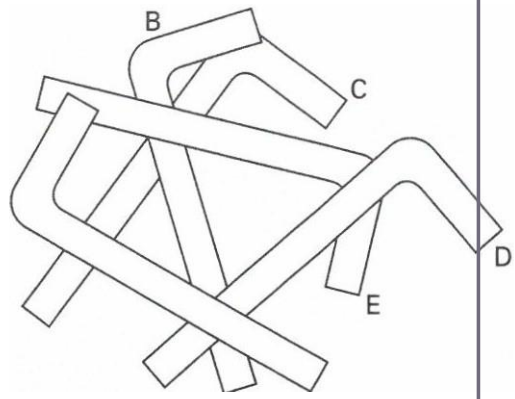


10.

A

8

8



D

E

B

O

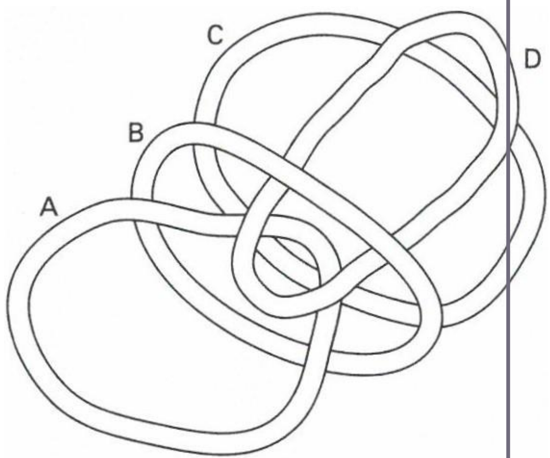
12.

A

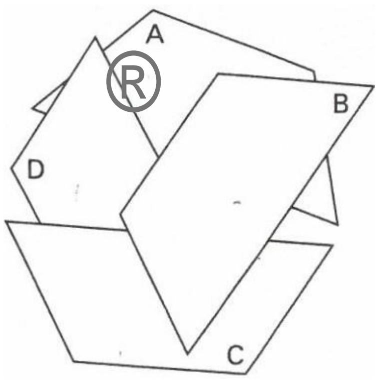
8

©

D



14.



8

C

D

D

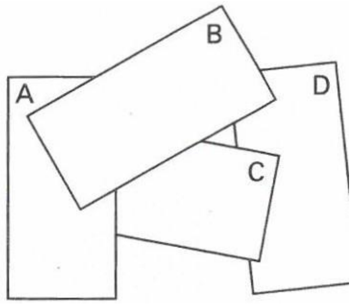
Q

A

8

(£)

D



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Note – Transferring test documents into word produced inaccurate and missing figures at times. Nonetheless, one can see many examples that are clear and complete.

Mathematics Assessment

MaLT 8

for Learning and Teaching

FIRsT NAME t\a.MJes

LAST NAME

SCHOOL Queen Edith primary School

CLASS Miss Alexander DATE OF TEST 13/7/11

GIRL/BOY DATE OF BIRTH November 26th 2002

TOTAL RAW SCORE:...................... AGE: ....YEARS........... MONTHS STANDARDISED SCORE:................. NC LEVEL:.......................................

You will need:

- A pen or pencil
- A rubber

Remember:

- Try each question but if there is any one you cannot do, go on to the next one and come back to it later.
- Do any working on the booklet itself.
- If you make a mistake, cross (or rub) it out and write the correct answer clearly.
- When you have finished, go back and check your answers.
- Ask your teacher if you are not sure what to do.

1

Fill in the missing numbers to complete the counting patterns.

(a) [}3] [E]I sz I I |<=>Z|

(b) j605I

2

Circle the number that is nearest to **92** in value.

100 99 82 90 920

3

Write the numbers in the boxes to make these correct.

(a) **60** + 40 − 100

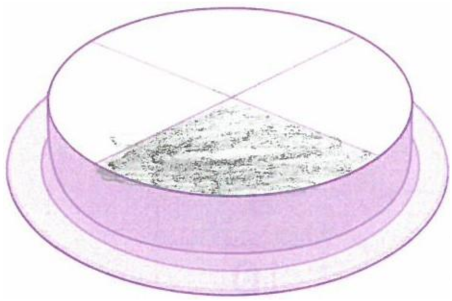
(b) **79** + ___, − 100

1. 21..;

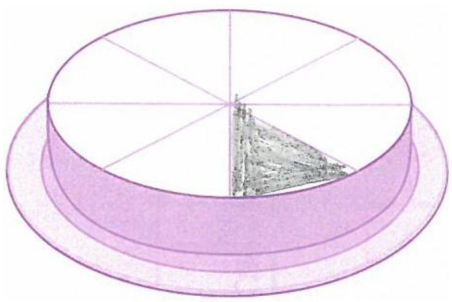
TOTAL

4

Shade **one-quarter** ($\frac{1}{4}$) of each cake. (a)



(b)



5

Fill in the missing numbers to complete the pattern.

140	120	100	80	60	40	20

6

Put circles round **both** the numbers in which the 3 represents 3 'hundreds'.

7301   70310   310   703   306

TOTAL

7

Draw a line for each number fact. The first has been done for you.

3+6=

4 + 7 =

20 − 7 =

7
8
9
10
11
12
13
14
15
16
17
18
19
20

8

A bar of chocolate costs 60p.
Circle how much **three** bars of chocolate cost.

18p

£1.08

£18

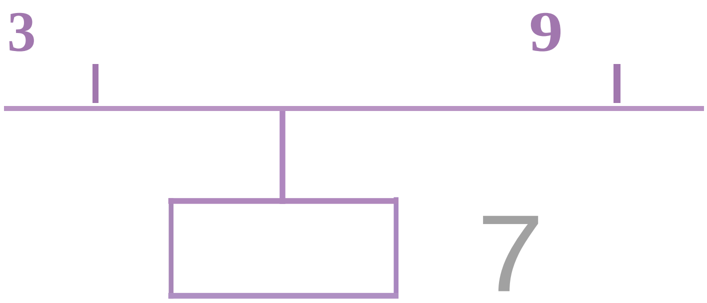
0

£1.80

9

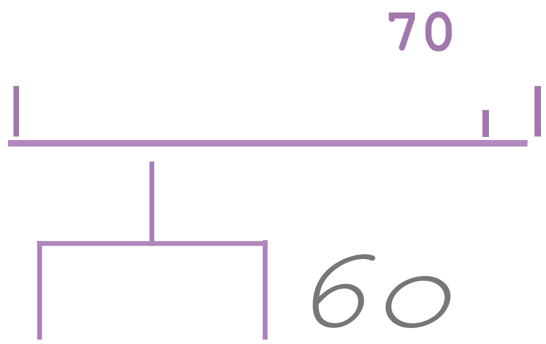
(a) Write in the box the number which is exactly **half-way** between

3 and 9.



7

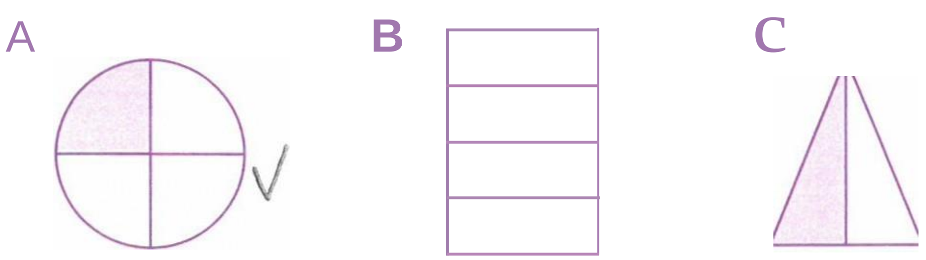
(b) Write in the box the number that is half-way between 50 and 70.



60

10

Tick (./) the shape or shapes that have **one-quarter** ( ) shaded.



D

E

TOTAL

11

A mug weighs 170 grams, rounded to the nearest ten grams. How much could the mug really weigh?

The mug could weigh 180 grams

12

Circle the numbers that are multiples of **5.**

30 41 95 37 1 51 5

13

Write your answers in the boxes: (a) Double 24 48

(b) Double 38 76

14

A pencil costs 35p.
How many pencils can I buy with £1.00? Write it in the box.

I can buy 2 pencils with £1.

TOTAL

15

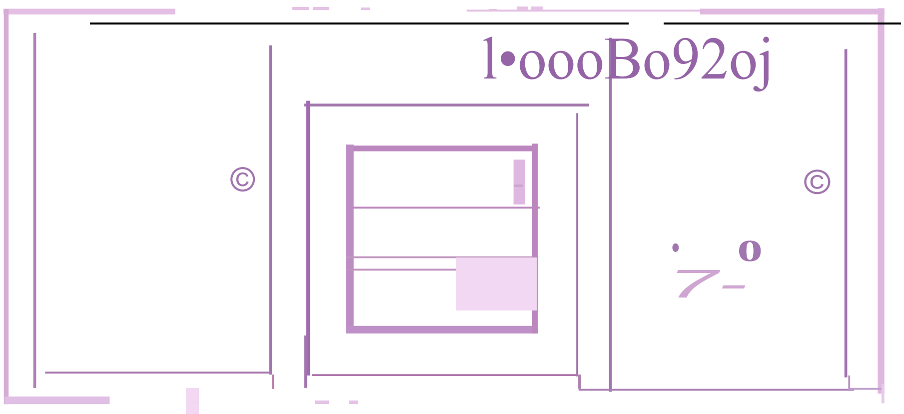
This card is pinned by its corner. Jack turns it half a turn.

Tick (✓) the card's position after the half-turn.

A B C D

16

Samantha puts a cake in the oven at this time: 09:20



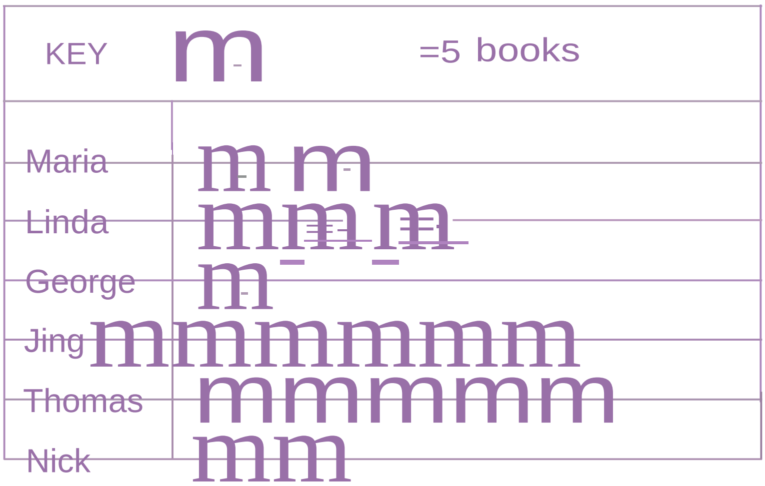
The cake was taken out of the oven 50 minutes later. At what time did the cake come out of the oven?

10:10

TOTAL

17

The pictogram shows the number of books six friends bought this year.



(a) How many books did Linda buy this year?

15 books'

(b) How many **more** books did Thomas buy than George?

20 books'

(c) Two friends bought the **same** number of books.
Circle the names of the two friends.

Maria Linda George Thomas @

18

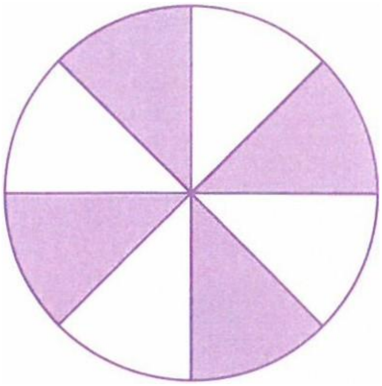
Fill in the missing numbers in this counting pattern.

930 900 870 840 810 780

TOTAL

19

Look at this circle:



Tick **all** of the statements that are true:

A of the circle is shaded *Ill* V

B of the circle is shaded X

C $\frac{2}{4}$ of the circle is shaded V

D $\frac{1}{4}$ of the circle is shaded *X*

E $\frac{4}{8}$ of the circle is shaded ✓

20

Fill in the missing number.

36 + 36 + 36 + 36 + 36 + 36 + 36 + 36 + 36 + 36 = 36 x I

11X6=68

11X30=320

TOTAL

21

For each number sentence, tick (✓) for correct or draw a cross **(x)** for wrong.
The first one has been done for you.

	23	+	1	3	+	12
(a)	125	+	285	285	+	125
	546		124	124		546
(b)	25	x	12	12	x	25
	400	÷	80	80	÷	400

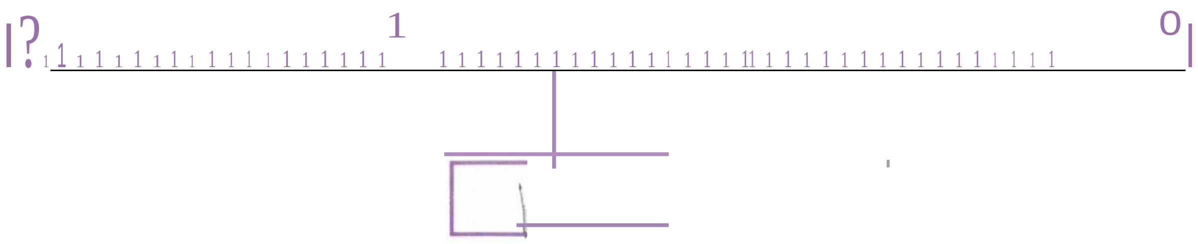
22

Look at the number sentence.
Fill in the box.

$37 - 18 = $ 19

23

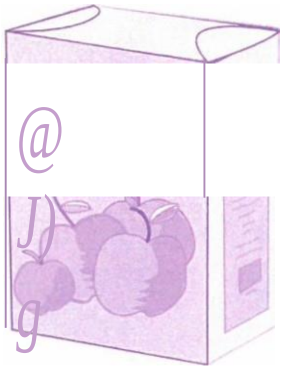
The picture shows a thirty-centimetre ruler.
Write down the measurement shown on this ruler, in centimetres.



TOTAL

24

How much juice does this carton contain? Circle the correct choice.



2 kilometres

2 centimetres

2 apples

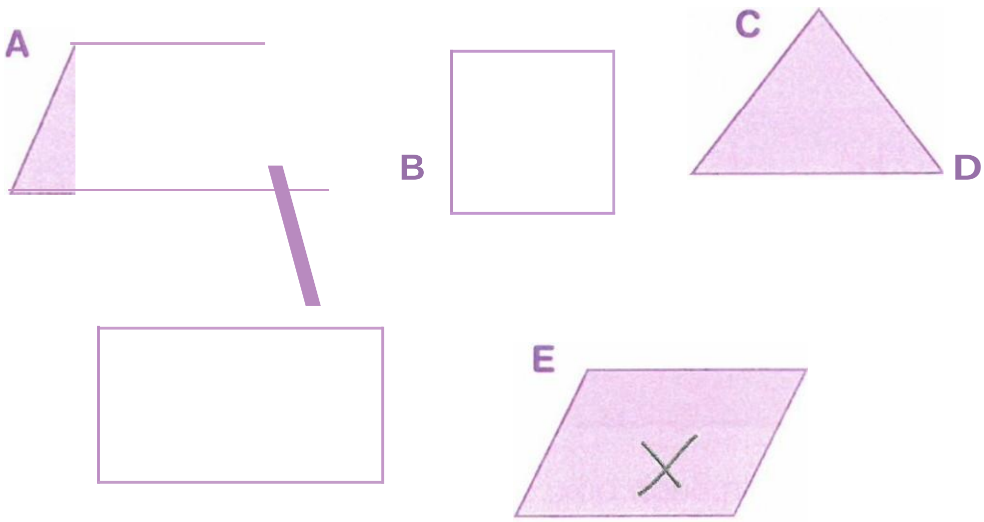
2 litres

2 metres

27

25

One of these shapes has **no** lines of symmetry.
Draw a cross **(x)** on the shape with no lines of symmetry.



26

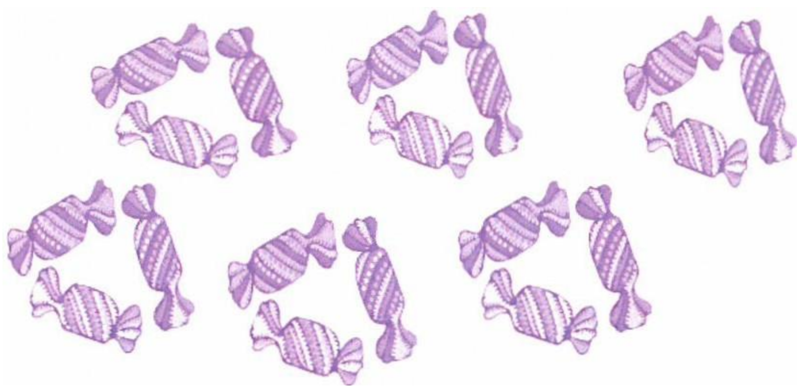
Put a number in the box to make this correct.

36 + 1 1/67 1 = 2o3

36+64=100

TOTAL +k5o -- OC
ZC,<

Kathy ate $\frac{1}{3}$ of these sweets.



27

How many did she eat? ____ sweets

28

Circle the number that equals 8 hundreds, 0 tens and 12 units.

8012 812 **8120** **802** **80012**

29

There are 12 eggs in each box. Jack has 60 eggs.

Which calculation would you use to work out how many boxes Jack has? Circle it

12×60 **12760** **60-12** **60712**

6

0+12

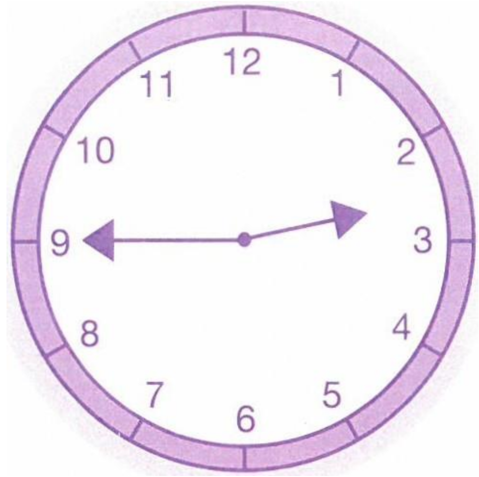
TOTAL

30

What time does this clock show? Circle your answer in the list.

9	2
2	4
3	3
9	1
2 :	1 5

3: 45



3 4 5

31

Circle the **two** multiplication sums that give the **same** answer.

10 x 7

8x5

3x5

G ·

C?:X 16;2)

3

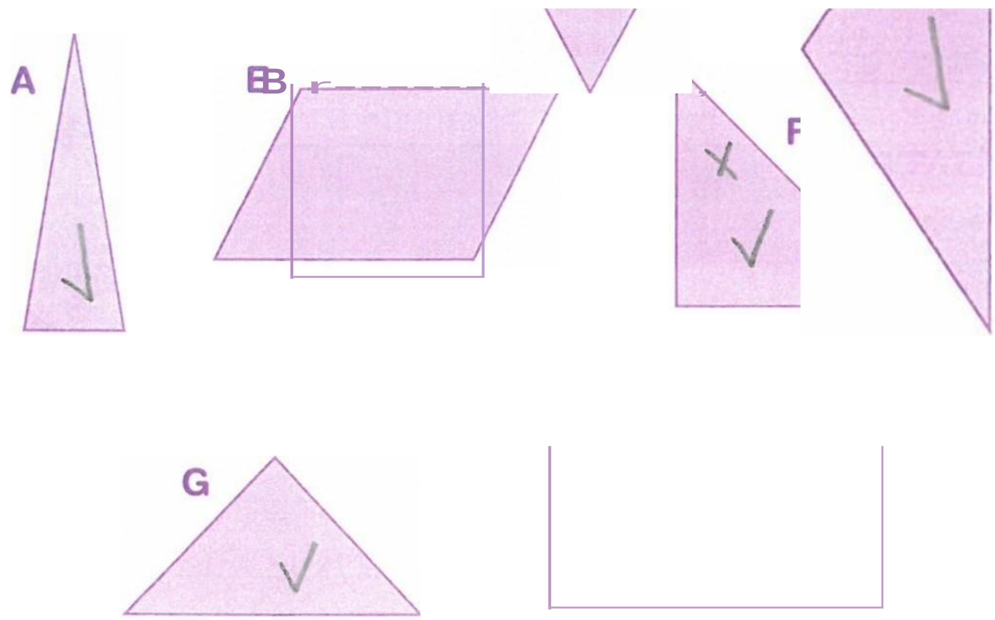
10 x 7 = 70
2 x 10 = 20
4 x 5 = 20

TOTAL

LLook at the shapes:

(a) Tick (v') all the **triangles**.

(b) Draw a cross **(x)** in every shape that has a **right-angle**.



H

Fill in the boxes to make this number sentence correct.

25 $\boxed{5}$ =5

One hundred and fifteen

(a) One hundred and three

(b) Two thousand and fourteen

Appendix 3
Monitoring and Giving Voice

Student Interviews and Teacher Questionnaires

As noted before, the qualitative data for this study provide contextual detail and function as monitors to ensure that the research was consistently in line with the goals set forth as well as ethical guidelines. A fuller description of the procedures and contents of the interviews and questionnaires, including the specific questions, was seen in Chapter 4, while excerpts of both accompanied by a discussion will be in Chapter 7.

Other monitoring tools utilised for this study were teacher discussions and mathematics lesson observations, as well as music lesson audio and video recordings. As documented in Chapter 4, discussion with classroom teachers and observation of their mathematics lessons confirmed that all teachers were following the English National Curriculum and doing their best to follow the numeracy framework though the latter was no longer required at the time. Therefore, mathematical content was similar across all classes. Furthermore, general style of teaching was similar across all classes as well.

Additionally, the recordings of the lessons validated 1) the professional and consistent music teaching style across all classes while also showing 2) the ways in which the different musical modes were emphasised, 3) the manner in which brief musical-mathematical links in the lessons were given to the Explicit groups, as well as 4) student responses to the teaching and music.

Student Interviews

Approximately 50 interviews were conducted with students. Excerpts will follow in Chapter 7. Overall, student interviews were positive. Aligning with observations and teacher questionnaires, the interviews confirmed they enjoyed the lessons. Interviews also revealed that the brief mention of the link between mathematics and music during lessons for the Explicit groups helped children make the links more fully, especially if prompted.

As noted in Chapter 4 and repeated here from Chapter 7, a representative sample of students (approximately 30%, therefore over 50 in total) from each experimental group (Form Implicit, Form Explicit, Melody Implicit, etc.) were interviewed about their musical and mathematical experiences as well as their thoughts regarding the connection between the two. This was done in order to contextualise the study and gain insight into the children's experiences and thoughts from their own perspectives and in their own voices. Short interviews were conducted with students individually from students of mixed achievement levels and both sexes. The interviewees were chosen in consultation with the teacher, and lasted about ten minutes each, as noted in the design and assessment tools chapter (Chapter 4).

Interview Excerpt with Student from Form Implicit Group

Edel: How do you feel about making music?

Pupil (Girl): I like it quite a lot because it's like art; it's all about communicating, really. Edel: In what ways do you like to use music in order to communicate?

Pupil: I prefer short music things.

Edel: What makes a long song not boring?

Pupil: Well, when there's a sudden change, like in "Woodland Wild." Edel: Yes, that's the one we all composed together.

Edel: Can you think of any ways that music and maths are alike?

Pupil: Singing and science… math leads to biology and vocal cords are biological. Music can be very useful.

Edel: Yes. In what ways?

Pupil: Communication and sometimes like when you create a song to describe how you feel. Also, if you get sciencey with it, it sometimes turns into maths.

In the interview above, the student reveals in a sophisticated and insightful way, particularly in light of her age, the primary reason why she likes music. She also admits her preference for "short" music. When expressing what would be necessary for long music to be interesting for her, she drew upon work that we had done as part of the intervention programme. Specifically, the student revealed dimensions of the impact that the group composing experience had upon her. She shared her knowledge of the purpose for form in musical composition when she noted that creating a sudden change prevents a long song from being boring. She had recalled adding a third part (Part C) to the song, "Woodland Wild," which we composed together as a class. We added the third part to make it more interesting and to add a fresh point to the storyline. As noted above, the process of composing itself usually requires analytical and structural thinking that performing music itself may not bring forth as markedly, therefore this experience potentially added to the strength of the mathematical improvement.

When answering my question “In what ways?” in response to the question about the link between music and mathematics, her response was very creative. Since she was in an implicit group, she had not been influenced by any discussions regarding the link, therefore her comments were particularly original. This student’s last sentence about getting “sciencey” and the idea that with this perspective, “it sometimes turns into maths” was insightful. An interview excerpt with a student from the Form Explicit Group follows.

Interview Excerpt with Student from Form Explicit Group

Pupil (girl): [When asked whether she likes maths] I used to be quite bad at maths but now it is getting easier....

Edel: Do you see any connection between music and maths? Pupil: Yeah.

Edel: Why do you think there is [one]?

Pupil: I don't know, but since I've been doing [music] it's been getting better.

Edel: That's what makes me curious about it. What are some ways you think music and maths are alike? …. Anything….

Pupil: I keep forgetting, but...what do we call a song that has three parts, how would we label it?

Edel: ABC. So, if the first part is called A, and then we come back to it later, what do we call that?

Pupil: I [forgot]. Edel: A prime [A’].

Edel: What are some things you don’t like about music class? Pupil: I don’t know.

Edel: What are some things you like about music class?

Pupil: I like Scarborough Fair and I also like Jamaica Farewell. Edel: Aah, and they are very different from each other.

Pupil: Yes, Scarborough Fair is very sad and Jamaica Farewell I keep thinking as a merry song.

Edel: It is interesting that the words are sad but it is an upbeat [sounding] chorus…. What are some of the things you do in school in maths?

Pupil: We do division.... it's fun. Edel: What might make it fun?

Pupil: Like today we did puzzles.... I love puzzles.

Edel: Yes, it's like solving problems. It's [indeed] fun to do that. Pupil: It is kind of like a bit of

mystery.

Edel: Ooh, yes, like you're a detective – like Sherlock Holmes. Pupil: (smiles) Yes.

Edel: Do you like the process? Or the feeling [you have] after you've found the answer? Pupil: Both.

Edel: We can solve problems when we make music too.

In this interview above, the pupil reveals that mathematics has become easier for her after she started learning music, to which she attributes this improvement. The interview also shows that this child had learned a key idea about form when she brought up a main concept regarding structure, though she needed to be reminded of details. Her newfound enjoyment of mathematical processes came up again towards the end of the interview, at which point she said she enjoyed doing division and that puzzles made it fun. We ended by noting problem-solving aspects of mathematics and I pointed out that we can also solve problems with music, thereby reiterating the connection between the two domains in my final remark.

When reflecting upon this interview, particularly during the discussion about parts of a song, I am reminded that perhaps I answered too quickly and should have waited longer for the student to find the answer. When teaching a class, I tend to give pupils more time to find the answer, while giving helpful clues along the way if the pause starts to get lengthy. I felt a bit rushed with the interviews due the short amount of time with each student. Though in many of the interviews I allowed more space for their answers and provided small clues along the way, in the future, I would monitor myself even more. Nonetheless, students still came up with their own ideas quite often, and this student as mentioned showed she had a clear understanding of the idea of form in music, even though she had forgotten one of the labels. At the end of each interview I often parted with a comment to help pupils make the link between music and mathematics further or more clearly, hoping to leave them with some food for thought in this area of enquiry. The last interview excerpt follows, and is with a student from the Rhythm Implicit Group.

Interview Excerpt with Student from Rhythm Implicit Group

Edel: Thank you for helping me with my book. Congratulations on how well you did in the play. How do you feel about making music? Do you like it a lot, medium or not much?

Pupil (boy): Not really sure… medium [I think] Last part is inaudible.

Edel: How does it make you feel? I saw you smiling while you were singing Coulter's Candy, when we were singing and clapping.

Pupil: I would like to go to the dance club after school and I've been taking guitar lessons for about 3 months. I liked singing before too.

Edel: What are some things that you don't like about it? Pupil: I don't know….

Edel: How do you feel about doing maths? Pupil: They're usually complicated.

Some are hard and some are easy. I like the hard ones more; they are more fun to do. The easy ones are short.

Edel: Aah, you like the challenge. How do you feel when you do a difficult one? Like aaah, I just solved a mystery?

Pupil: Yeah.

Edel: Do you think there might be a connection between music and maths? Pupil: There can be.

Edel: In what ways?

Pupil: You can do maths problems that have also got to do with music. You can look at how many syllables are in the song.

Edel: Good idea. Any other ways?

Pupil: How many beats in a song… like 4.

Edel: Yes, many songs have 4 beats per measure or bar for example. Do you remember the African and Japanese songs? How many counts per bar (or rhythm box) are in the introduction of the Japanese song for example? Or in the main part of the African song?

Pupil: I don't remember.

Edel: Let's do some of the Japanese song together. [After completing the first phrase of the song, he jumped in and triumphantly answered.]

Pupil: 6

Edel: That's right. You mentioned that you like solving problems when doing maths; there are patterns and problemsolving opportunities in music too.

The boy in the interview above seemed to have no plans to impress me with how much he liked music at first and was expressionless, even shrugging his shoulders at one point, until I brought up a song he liked in our class. Then his face changed and not holding back his excitement anymore, he shared some of his musical experiences.

Even though this student was in one of the Implicit groups, after being asked about it he responded with good ideas regarding ways mathematics can relate to music (or how numbers can be in music, such as his suggestions that one can count syllables and beats in a song). Additionally, it did not take much prompting for him to understand and extend upon the connection between mathematics and music even further even though this link had not explicitly been made in the lessons.

As noted previously, most children in the interviews were positive about music, including the intervention. This was often not expressed until reminders of a specific song came up, but every student shared something they liked and their sincerity was evident in their faces, though I am aware that some positive reactions may have been influenced by my presence. Not all of the students were positive about mathematics, although most found something they liked about it when it was presented as potentially being fun or connected to music. When asked, all children were able to think of some relationship between the two, whether they had been in one of the Explicit groups, though the majority needed a bit of prompting at first. This consistent response suggests the existence of an inherent connection between the two domains of music and mathematics.

Interview Questions for Year 3 Pupils

Note that the order of asking about math or music first should alternate.

"Thank you so much for talking with me today. I'm writing a book about what children think about certain things they are doing and learning. I'm especially interested in music and maths, so I'd like to know what you think about these things. Would you help me by answering a few questions please? ... Do you mind if I record what you say?

1) How do you feel about **making music**/doing maths ... do you think it's OK, do you like it a lot, or not much?

2a) *(according to the answer)* What do you like about doing **music**/maths? ... and why is that? 2b) What don't you like about it ... and why is that?

2c) *(if answer is 'OK', or 'I don't like it')* What would help you to like it more? ... and why is that?

3a) Have you always liked/disliked **music/**maths for as long as you can remember or has this changed?

3b) *(according to the answer)* In what way has it changed?

Repeat questions 1 and 2 for maths. Alternate the order.
Therefore, if doing six interviews for one class, then do three with music first and three with maths first.

4a) Can you think of any ways that **music and maths** (maths and music) are alike?

4b) Interesting. * Why do you think that?

5a) Do you take lessons in singing, dancing, maths or a musical instrument on your own (without your classmates from school)?

5b) (*if so)* Which do you have lessons in?

5c) (*if not specific*) Are they private or in a group? 6a) Do you like doing **music**/maths in school?

6b) What are some things you like about it?

6c) What are some things you don't like about it?

6d) Did you enjoy the music that we did together this year? 6e) What did you like about it? ... and why is that?

6f) What didn't you like about it? ... and why is that?"

* I said "Interesting." in Q3 so that when I would ask "why," the children wouldn't think that they had given a wrong answer. (Though there are no right or wrong answers, nonetheless I wanted to make sure students would not misinterpret any of the follow-up answers.)

Interview questions and guidelines for PhD fieldwork July 2011.

Commentary on Teacher Questionnaires

Overall, the responses were positive regarding the music programme that was provided for this study. Teachers reported that the students enjoyed the lessons. The questionnaires also revealed other creative activities that were provided by parents, teachers or the school and that all classes had similar durations, frequencies and intensity levels of these activities.

Below is the questionnaire followed by further discussion.

Questionnaire

Regarding Music for the Academic Year 2010-2011

Thank you so much for your support! I have enjoyed working with your lovely class and have learned very much this year. I admire all that you do for the children!

Name* ____

School____

Date____

***** Names of schools, teachers and children will be anonymous.

1) Other than the work I did with the children, what other musical activities did they participate in this year? For example:

a) Did the school put on a musical play or play with music?

i. If so, please describe it briefly. (e.g., Popular style, some dancing, mostly singing, mostly talking with some singing, equal parts acting, singing, dancing, theme/topic of play, duration of performance, preparation time in terms of weeks)

ii. Title/Composer

b) Did your school put on a musical concert of any sort? (If so, please describe.)

c) Did your school have a music teacher (or teachers) come in to do group work such as singing or dancing at any point? If so, for how long and how large was the group? (For example, did the teacher work with your class only or with another year at the same time?)

d) From your knowledge of the children's lives in and out of school, could you say which children in your class have instrumental lessons, in or out of school, and what instruments they play?

e) Do you know which pupils take singing or dancing lessons (either privately, at a studio, or in after-school musical clubs, such as a dance club, singing club, band or choir)?

2) Regarding maths,

a) In general, how often do your students have maths lessons per week and how long is a typical lesson?

b) Do they have maths homework, and if so, about how often and how much?

c) Does your school or community have any special programmes for mathematics training available for the children? If so, approximately how many participate?

d) To your knowledge, do any of your pupils receive private or small group tutoring in maths?

e) Which aspect(s) of maths do you like teaching the most (and why)?

f) Which aspect(s) of maths do you like teaching the least (and why)?

g) Which subject(s) do you like teaching the most (and why)?

h) Which subject(s) do you like teaching the least (and why)?

Would you like to comment on the music teaching this year?

Did you enjoy it or learn anything from it?
If so, is there any idea or activity that you might use in the future or pass on to a colleague, friend, trainee teacher or pupil?
(Perhaps you could share an example or two?)
How much would you say that the class overall enjoyed the music lessons?
Did the students ever refer to the music sessions in other lessons or at other times?
If so, please give an example or two.
Do you have any additional comments or suggestions?

Again, thank you so much for your time and support! Best wishes, Edel

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Teacher Questionnaire for PhD fieldwork July 2011.

Responses to Questions on Teacher Questionnaire:

"Which aspect(s) of maths do you like teaching the least/most (and why)?"

School and Group	**Answers Regarding Proportional Mathematics**
Least Favourite to Teach:	
1. St Peter's (Rhythm Explicit)	"Fractions of numbers – the expectations of the National Strategy are too demanding for this age group."
2. Fremont (Rhythm Implicit 1 of 2)	"Fractions -- children at this age find it a difficult concept."
3. St. Lucy's (Rhythm Implicit 2 of 2)	"Division…so hard to understand for the children."
4. Lady Gytha's (Melody Implicit)	"Fractions"
5. Lady Gytha's (Melody Explicit)	"None"
6. St. Michael's (Form Implicit)	No comment
7. St. Michael's (Form Explicit)	The only other response to this question that was neither "fractions/division" nor "none" was "Time, as find it hard myself."
Most Favourite to Teach:	
Lady Gytha's (Melody Explicit)	"Fractions because it is difficult, but a challenge that can be overcome."
Lady Gytha's Melody Explicit teacher also cited two more mathematical topics she enjoyed teaching, also for the challenge	"Calculation methods because the children learn how to reach answers in maths and they find it satisfying." "Time, because it is very hard for children so to finally crack it and be able to tell the time is great for children."

The teachers' responses in the questionnaires showed that during the fieldwork timeframe, the amount of music in the schools and the amount of music in after-school programmes were similar across all schools. Teachers' musical experiences were similar as well. St. Lucy's (as noted before, the name was changed in order to protect the privacy of the schools) had the only teacher who played an instrument (guitar).

In terms of mathematical school experience and teacher expertise, it appeared that all teachers spent virtually the same amount of time on mathematics – one hour per day with homework assignments every one-two weeks.

The most compelling revelation in the questionnaires regards proportional mathematics as this is the area that over half of the seven teachers in the study said they disliked teaching the most. The teachers who said that proportional mathematics, such as fractions or division, was the least favourite area of mathematics to teach cited this because they said it is difficult for children this age (7-8 years) to understand. The one teacher who said it was her favourite aspect of mathematics to teach cited so for the same reason (difficult for students to understand) yet with a different perspective – that the category of fractions was the most difficult one for students to understand and therefore she liked the challenge.

Therefore, since proportional mathematics showed the most consistently strong gains, this research appears to be especially relevant considering the perceived need for support in this area as revealed by the questionnaires. Furthermore, had teaching fractions been the favourite area for teachers, this may have been a confounding factor. Yet, on the contrary, it was the least favourite for more than half of the teachers.

The primary research question asks whether school-based musical learning can improve mathematical attainment; this study shows that it can and particularly so in the area that may be the most difficult for students. Therefore, the potential for a well-designed music programme to help students understand challenging mathematical concepts such as fractions is promising.

Appendix 4
Letters to Schools and Parents

UNIVERSITY OF CAMBRIDGE
Faculty of Education

08 July 2010

Dear Mr. Hastings,

I am a PhD student at the University of Cambridge in the Faculty of Education and this letter is to seek your permission to carry out my doctoral research with children in your school. The main research question I am asking is whether or not music learning can enhance children's understanding of mathematics.

My interest in this topic stems from my own personal experience as well as research I co-conducted in the USA with the New Jersey Symphony Orchestra in which an extended period of violin training appeared to have unexpectedly positive outcomes on children's scores in literacy and mathematics. For my doctorate I would like to pursue this link between music and mathematics by teaching singing and other musical activities.

I am a qualified, successful and experienced primary and private music teacher, specialising in vocal production and musical expression, and I hold an enhanced disclosure CRB certificate for work with children in the UK as well as equivalent certification for work with children in the US. (See attached resume.) My supervisor at the University of Cambridge, Dr. Linda Hargreaves, will be overseeing this project. She is a Reader in classroom learning and pedagogy and a specialist in primary education and psychology.

I am volunteering to teach music to Year 3 children throughout the 2010/2011 school year. All material and activities will be compatible with the English National Curriculum for music in the schools. In order to measure the effects of the proposed musical activities I would need to test the children's basic skills and musical ability at the beginning and end of the year. All assessments will be appropriate for the age group and consist of widely used standardised tests.

Ideally, I would like to video-record my teaching in a small number of the sessions to be able to monitor and ensure the quality of the research. Permission from the parents/carers and children will of course be requested and children's anonymity and confidentiality will be assured in any publication or report arising from the research. In accordance with the ethical guidelines of the Faculty of Education and British Educational Research Association, children have the right to withdraw from the research at any time.

I know that my request and my offer to teach music, free, in your school for as much of the school year as possible, at least six months, are substantial. However, my questions about potential links between music and mathematics cannot be answered without this extended music experience for the children. I would be very happy to come and explain more about my research to you and your staff or to speak on the phone. I shall telephone in a few days for an initial reaction.

Thank you very much for your time. Feel free to contact me if you have any questions. Yours sincerely,

Edel Marie Sanders
Phone: 01223.528883

Supervisor: Dr. Linda Hargreaves
Email: lh258@cam.ac.uk Email: ems62@cam.ac.uk

UNIVERSITY OF CAMBRIDGE
Faculty of Education

08 December 2010 Dear Parent or Carer,

After more than 10 years as a certified music teacher, I am so delighted to be teaching your child this year! We are learning many songs and discovering exciting musical ideas. We will be presenting a concert of our work in July, which I hope you will be able to attend.

Over the years, I have been continually encouraged by the positive effects that musical training has on multiple aspects of a learner's life. As a research associate at Columbia University, I co-conducted a study of over 500 children (http://www.europeansuzuki.org/web_journal/articles/NewarkEarlyStrings_year5.pdf).
This research revealed a consistent correlation between musical training and logical thinking skills, among other enhanced abilities. In returning to academia to pursue my doctorate at the University of Cambridge, I am exploring that link further.

Therefore, I would like to ask your permission to occasionally video-record your child's musical lessons in order to monitor my teaching. The camera would be focused on me, but may include some of the children at times. However, we can make sure your child is not included in any shots if you would prefer.

In accordance with the British Educational Research Association guidelines, anonymity would be ensured and viewing restricted to teachers and academics solely for educational purposes. It would also be lovely and quite helpful to have a video of their final performance as well. I am quite happy to meet with you at the school to discuss any concerns you may have.

Yours sincerely,

Edel Sanders Ems62@cam.ac.uk

Parental Permission Slip

Please tick within the bracket(s) below to indicate your permission for the following:

I am willing for my child (add name) …………………………………………… to be videotaped for the purposes listed below:

1) Viewing only between my supervisor, Dr Linda Hargreaves and myself. { }
2) Short excerpts of lessons to be used as examples for other educators and researchers. { }
3) Short excerpts of the final concert to be used for educational objectives. { }

I understand that my child's name will not be used in any publication or presentation about this research.

Signed ……………………………………………………….

Please return this slip to the class teacher by 6 January, 2011. Thank you so much!

Holding a “thank you” card from one of the groups of children in my fieldwork.