Supplementary information for: *f*-electron hybridised Fermi surface in magnetic field-induced metallic YbB_{12}

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November 18, 2021

This supplementary information considers non-Lifshitz-Kosevich (LK) quantum oscillation amplitude temperature-dependence for gapped models in comparison to the LK temperature-dependence for gapless models.

Supplementary note 1: Model simulations

To distinguish between gapless and gapped models of quantum oscillations in the unconventional insulating phase, here we simulate the quantum oscillation amplitude for various gap sizes. We use the formulation of refs. (*1, 2*); the ratio of the first harmonic between the gapped state and the normal state is:

$$\frac{M_{\mathrm{g}}}{M_{\mathrm{n}}} = \frac{\sinh(X)}{X} \int_0^{\infty} \cos\left(\frac{X\mu}{\pi}\right) \partial_{\mu} \left(\frac{\mu}{\sqrt{\mu^2 + (\Delta/T)^2}} \tanh\left(\frac{\sqrt{\mu^2 + (\Delta/T)^2}}{2}\right)\right) \mathrm{d}\mu, \tag{1}$$

where μ is the chemical potential, Δ is the isotropic gap size, and X is the temperature damping coefficient given by $X = 2\pi^2 k_{\mathrm{B}} T m^*/e\hbar B_0$. Here, k_{B} is Boltzmann's constant, T is temperature, m^* is the quasiparticle effective mass, e is the electron charge, $\hbar$ is the reduced Planck constant, and $B_0 = \mu_0 H$ is the applied magnetic field (*3*).

If we set $T = X\omega_c/(2\pi^2)$ and $\Delta/T = 2\pi^2\Delta/\omega_{\mathrm{c}}X = \pi\delta/X$, we find:

$$\delta = \frac{2\pi\Delta}{\hbar\omega_{\mathrm{c}}}, \tag{2}$$

where ω_{c} is the cyclotron frequency. We therefore find the ratio of the first harmonic between the gapped state and the normal state to be:

$$\frac{M_{\mathrm{g}}}{M_{\mathrm{n}}} = \frac{\sinh(X)}{X} \int_0^{\infty} \cos\left(\frac{X\mu}{\pi}\right) \partial_{\mu} \left(\frac{\mu}{\sqrt{\mu^2 + (\pi\delta/X)^2}} \tanh\left(\frac{\sqrt{\mu^2 + (\pi\delta/X)^2}}{2}\right)\right) \mathrm{d}\mu. \tag{3}$$

Gapped model simulations of the non-LK form of quantum oscillation amplitude at low temperatures are shown in the lower inset to Fig. 4a for various gap sizes (i.e. various sizes of δ), compared with the LK growth in quantum oscillation amplitude at low temperatures for gapless models (i.e. $\delta = 0$).

Supplementary note 2: Model comparisons with experimental data

Upper insets to Fig. 4a and Fig. 4b of the main text show the growth in quantum oscillation amplitude of the 700 T frequency in magnetic torque and 800 T frequency in electrical resistivity plotted against X^2, respectively, in the unconventional insulating phase of YbB_{12}. The LK exponential low temperature growth of the measured quantum oscillation amplitude observed for both electrical transport and torque magnetisation is in striking contrast to the non-LK finite temperature activation expected for gapped models of quantum oscillations (lower inset to Fig. 4a).

For the insulating regime of YbB_{12} in which temperature dependent quantum oscillations are measured, the isotropic gap size at 40 T is given by $2\Delta \approx 15$ K (*4*), which yields $\delta \approx 12$ for $m^*/m_e = 7$ for the quantum oscillation frequencies shown in Fig. 4. Simulations with various values of δ are shown in the lower inset of Fig. 4a in the main text (*1, 2, 5*). For the gapless case ($\delta = 0$), quantum oscillation amplitude simulations show an exponential LK growth at low temperature, while for the gapped case (finite δ, shown for values up to $\delta = 10$, similar to YbB_{12}), quantum oscillation amplitude simulations show non-LK finite activation behaviour at low temperature. A comparison of measured quantum oscillation amplitude growth at low temperature with model simulations thus evidences neutral gapless excitations in the unconventional insulating phase of YbB_{12}.

Supplementary note 3: Low temperature model expansion

A further simplification may be yielded at low temperatures by using a low temperature expansion. We perform a series expansion of the term $\sinh(X)/X$ corresponding to the temperature damping term R_T in the Lifshitz-Kosevich (LK) formula that describes the temperature dependence of quantum oscillations for particles obeying the Fermi-Dirac distribution (*3*).

For small T, a series expansion of the temperature dependence term yields:

$$R_{\mathrm{T}} \approx 1 - \frac{X^2}{6} + O\left(X^4\right). \tag{4}$$

The quantum oscillation amplitude therefore linearly increases with decreasing X^2 approaching the zero T limit. The low temperature growth in quantum oscillation amplitude is captured by the relative change of quantum oscillation amplitude at a finite temperature $A(T)$ with respect to the amplitude at the lowest measured temperature A_0, given by:

$$\begin{aligned} 1 - \frac{A(T)}{A_0} &= \frac{A_0 - A(T)}{A_0} \\ &= \frac{X^2}{6}. \end{aligned} \tag{5}$$

A plot of $(A_0 - A(T))/A_0$ against X^2 would therefore yield a straight line with a gradient equal to $1/6$ at low temperatures for low-energy excitations within the gap. In contrast, in the absence of low-energy excitations, gapped quantum oscillation models would yield a much reduced change in amplitude as a function of X^2 at low temperatures well below the gap temperature scale (Lower inset to Fig. 4a) (*1, 2, 5*). A simplified comparison to distinguish between gapless and gapped forms of measured quantum oscillation amplitude is thus provided by this low temperature expansion.

Supplementary References

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