Critical link analysis of a national internet backbone via dynamic perturbation ⋆

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Abstract: Long-haul backbone communication networks provide internet services across a region or a country. The access to internet at smaller areas and the functioning of other critical infrastructures rely on the long-haul backbone high speed services and resilience. Hence, such networks are key for the decision-making of internet service managers and providers, as well as for the management and control of other critical infrastructures. This paper proposes a critical link analysis of the physical infrastructure of the UK internet backbone network from a dynamic, complex network approach. To this end, perturbation network analyses provide a natural framework to measure the network tolerance facing structural or topological modifications. Furthermore, there have been taken into account variations on data-traffic for the internet backbone that usually happen in a typical day. The novelty of the proposal is, then, twofold: proposing a weighted (traffic informed) Laplacian matrix to compute a perturbation centrality measure, and enhancing it by a time-dependent perturbation analysis to detect changes in link criticality within the network, coming from data traffic variation in a day. The results show which are the most critical links at every time of the day, being of main importance for protection, maintenance and mitigation plans for the UK internet backbone.

Keywords: Communication network, perturbation analysis, graph theory, complex networks, dynamic systems.

1. INTRODUCTION

Communication systems are a collection of machines and linking mechanisms that facilitate the transfer of information between many actors. More abstractly, they can be described as a collection of nodes engaging in many-to-many communication (Ding, 2016). Such nodes can be, for instance, router stations, core routers, switches, servers, and computers. They are connected to each other to transmit and share data and information. Communication channels such as cables (twisted wire, coaxial, fibre-optics, optical) and wireless radio-frequencies, are the network links. A communication network has a heterogeneous topology. This comprises a central mesh structure - the core or backbone network - that is densely connected, while reaching the end-user through a branched network (last-mile connection). Backbone networks provide the main paths for the information exchange between nodes, to reach peripheral network areas and end-users.

The physical backbone of a communication network is a fibre-optic, meshed, trunk network providing internet service to a region or a country (Durairajan et al., 2015). As a consequence, it is of crucial importance to maximise its resilience to targeted attacks or unintended disruptions (originated by random or natural causes) through the protection of the most critical nodes such as router stations and exchange servers. Ranking the network nodes by their criticality is of high interest for scheduling further operation and maintenance plans as well as optimising the network performance. In the literature, this task has been widely studied from a complex networks perspective, which means we use a graph with consistent mathematical properties, representing some physical, real-world attributes (Van Mieghem, 2014).

Complex networks have shown to be key for analysing communication networks performance and resilience. In this regard, we highlight the work of Kuipers (2012), which gives an overview of algorithms that impart the network the ability to maintain operations under failure of one or more network assets. Related to this paper is the work of Shatto and Cetinkaya (2017). Within it, the authors use the Laplacian spectra, a network scientific concept, to analyse the strength against targeted attacks. Another example is the work of Jiao et al. (2019), where the authors use features extracted from the normalised Laplacian spectra to decompose internet graph networks.

This paper proposes a novel approach for the link criticality and resilience assessment of a backbone communication network. This is based on complex network analysis. Critical link analysis has been investigated for the resilience assessment of several critical infrastructures such as water (Herrera et al., 2015; Ayala-Cabrera et al., 2019), transport (Wang et al., 2016; He et al., 2018), and electric power

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networks (Fang et al., 2016; Moussa et al., 2017), among others. Most of these studies use percolation theory (Stauffer and Aharony, 2018) to define and detect criticality through an iterative link removal process. We propose a critical link analysis, using the weighted Laplacian matrix of an internet backbone network to create a variant of the perturbation centrality index developed by Ceci and Barbarossa (2018). This measure is of special relevance for the analysis herein, as it naturally follows to compute how resilient the structure of any graph or network is, when facing some modification (perturbation) to its edges and vertices. The weighted version informs the index on the intensity of the network flow. As such traffic flow varies along time, the proposal adds a second novelty introducing a temporal analysis (Huang et al., 2017) of the weighted perturbation centrality. The UK backbone communication network is the case-study selected to check the suitability of the proposed framework.

2. NETWORK PERFORMANCE UNDER PERTURBATION

This section provides a theoretical basis within complex networks and perturbation theory to build towards the proposed methods.

2.1 Graph theory framework

Let us denote a node set as $\mathcal{V} = \{v_0, v_1, \ldots, v_{n-1}\}$, and the edges between them as $\mathcal{E} = \{e_0, e_1, \ldots, e_{m-1}\}$, such that we have a graph, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of n nodes and m edges. $\mathcal{G}$ can be directed or undirected, and supposing it is unweighted, capture structure in the adjacency matrix $\boldsymbol{A} = [a_{ij}]$. Here $a_{ij} \in \{0, 1\}$, such that when $(i, j) \in \mathcal{E}$, that is, i and j share an edge, $a_{ij} = 1$, else $a_{ij} = 0$. In directed networks, $a_{ij} \neq a_{ji}$, since $(i, j) \neq (j, i)$, however in undirected networks, $a_{ij} = a_{ji}$, such that $\boldsymbol{A}$ is symmetric.

Supposing the addition of a unique weight to each edge, we add the weight set, $\mathcal{W} = \{\omega_0, \omega_1, \ldots, \omega_{m-1}\}$, such that $\mathcal{G} = (\mathcal{E}, \mathcal{V}, \mathcal{W})$. Given this bijection between $\mathcal{E}$ and $\mathcal{W}$, we may replace $\boldsymbol{A}$ with the weighted adjacency matrix, $\boldsymbol{W} = [w_{ij}]$, where $w_{ij} \geq 0$ and zero when $(i, j) \notin \mathcal{E}$. Directedness or undirectedness manifest on weighted networks in the same manner as they have been previously defined to do. Since we only consider simple networks, there are no self loops, such that $a_{ii} = w_{ii} = 0$ always. Physical and performance characteristics of every link may vary, so we work with weighted graphs.

The Laplacian matrix (Mohar et al., 1991) is another matrix that closely captures network structure. It is used because of the interesting properties arising from its spectrum, which is the set of eigenvalues with their multiplicities (Mohar, 1997). It is defined as $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$, where $\boldsymbol{D} = [d_{ij}]$, and d_{ii} is the degree of node i, and $\boldsymbol{D}$ is 0 everywhere off the leading diagonal. For weighted networks, we define the Laplacian by $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{W}$. The multiplicity of eigenvalues equalling zero for $\boldsymbol{L}$ determines the number of connected regions, known as components, in $\mathcal{G}$. This means that it is connected if the first eigenvalue, $\lambda_1 = 0$ and if the second eigenvalue is $\lambda_2 > 0$. The size of λ_2 determines how strongly connected $\mathcal{G}$ is, and is thus called the algebraic connectivity (Von Luxburg, 2007). To allow for comparison between different network graphs, a normalised version of the Laplacian matrix, $\boldsymbol{L}_n$, is widely used as it is symmetric and positive definite. The normalised Laplacian follows the expression of Equation (1).

$$\boldsymbol{L}_n = \boldsymbol{D}^{-1/2} \boldsymbol{L} \boldsymbol{D}^{-1/2} = \boldsymbol{I} - \boldsymbol{D}^{-1/2} \boldsymbol{A} \boldsymbol{D}^{-1/2}. \qquad (1)$$

2.2 Perturbation and graph spectra analysis

This paper proposes a critical link analysis based on a perturbation network framework. This is a graph operation, shifting the state of $\mathcal{G}$ by the addition of a perturbed graph, $\delta\mathcal{G}$, such that

$$\tilde{\mathcal{G}} = \mathcal{G} + \delta\mathcal{G}, \qquad (2)$$

where $\tilde{\mathcal{G}}$ is the new state after perturbation. We can represent this as a matrix operation upon the eigendecomposition of a Laplacian matrix, $\boldsymbol{L}$, and it is achieved by modifying (perturbing) the properties of a small percentage of $\boldsymbol{L}$, using $\delta\boldsymbol{L}$, as Equation (3) shows.

$$\tilde{\boldsymbol{L}} = \boldsymbol{L} + \delta\boldsymbol{L}. \qquad (3)$$

The expression of Equation (3) will serve as a basis for network metrics and analysis considering a disruption or failure. One may cause perturbations from a node-centric rather than an edge-centric perspective, which would be achieved by modifying the edge set incident to the perturbed node. The perturbation analysis that we propose is in large part building on the work of Ceci and Barbarossa (2018). It shows that in the case of removing a single edge, denoted r, bounded be vertices u_r and v_r, the perturbation matrix, δL, can be rewritten as in Equation (4).

$$\delta\boldsymbol{L}(r) = -\boldsymbol{a}_r \boldsymbol{a}_r^t, \qquad (4)$$

where $\boldsymbol{a}_r$ is a vector of length n, the number of nodes, and is all zero except for $a_r(u_r) = 1$ and $a_r(v_r) = -1$. Following Ceci and Barbarossa (2018), the perturbed eigenvalues can be computed in relation to their unperturbed counterparts by following Equation (5) for $\tilde{\lambda}_i$. The same can be done for perturbed eigenvectors by following Equation (6) for $\tilde{\boldsymbol{q}}_i$.

$$\tilde{\lambda}_i = \lambda_i + \Delta\lambda_i(r) \approx \lambda_i + \boldsymbol{q}_i^t \delta\boldsymbol{L}(r) \boldsymbol{q}_i; \qquad (5)$$

$$\tilde{\boldsymbol{q}}_i = \boldsymbol{q}_i + \Delta\boldsymbol{q}_i(r) \approx \boldsymbol{q}_i + \sum_{j \neq i} \frac{\boldsymbol{q}_j^t \delta\boldsymbol{L}(r) \boldsymbol{q}_i}{\lambda_i - \lambda_j} \boldsymbol{q}_j, \qquad (6)$$

where $\Delta\lambda_i(r)$ is the perturbation of the i-th eigenvalue and $\Delta\boldsymbol{q}_i(r)$ is the perturbation of the i-th eigenvector, corresponding to the removal of the edge, r.

There exists a centrality measure, developed in Ceci and Barbarossa (2018), called perturbation centrality. The measure is based on the sum of the K smallest eigenvalues of the Laplacian matrix. Equation (7) shows the perturbation centrality expression in the case of removing the edge r.

$$\rho_K(r) = \sum_{i=2}^{K} |\Delta\lambda_i(r)| \tag{7}$$

Since the smallest eigenvalues of the Laplacian inform on graph connectivity, Equation (7) describes the variation in graph connectivity after the removal of an edge, r. This measure is therefore sensitive to edges that bridge neighbourhoods.

3. DYNAMIC PERTURBATION ANALYSIS

Among the primary contributions of this paper is a method to analyse networks perturbations over time, with two main novelties. The work is pioneering on considering a generalisation of the perturbation concept by variations on the weighted Laplacian matrix rather than edge/s removal or addition. The edge removal is, then, a particular case for which the associated weight is equal to zero. The second novelty presented herein is the analysis of such a weighted Laplacian perturbation over time.

3.1 Weighted perturbation analysis

Weighted perturbation analysis does not necessarily remove edges from the graph but rather varies their weights. In the work of Poignard et al. (2018), the authors propose to perturb the weights of the edges of a given graph to obtain a Laplacian matrix with two convenient properties - simpler eigenvalues and an algebraic connectivity, λ_2, with associated eigenvector, $\boldsymbol{q}_2$ with all nonzero entries. We work with weighted Laplacian matrices, where weight perturbation in an edge represents, a partial variation of the importance or performance of a given edge. Note that the space of edge removal or addition operations are a subset of of those that perturb weights. This is because if one supposes binary edge weights, fixing an edge weight to zero, or assigning nonzero weight to a zero edge, are operations that are isomorphic to removal or addition. This suggests that Equation (4) may be extended into the general case, so that it is re-computed using the weighted Laplacian, $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{W}$, consequently perturbed by proportionally adjusting a given edge, r, incident on nodes u_r and v_r, by $\beta(r)$, where $\beta \in [0, 1]$. We therefore define the modification matrix, $\delta\boldsymbol{L}$, through Equation (8), such that

$$\delta\boldsymbol{L}(\beta, r) = -\boldsymbol{w}_r^{\beta}\boldsymbol{w}_r^{t,\beta}, \tag{8}$$

with $\boldsymbol{w}^{\beta}$: $\mathcal{V} \to \mathbb{R}$ as all zero except for $\boldsymbol{w}_r^{\beta}(u_r) = \sqrt{\beta}$ and $\boldsymbol{w}_r^{\beta}(v_r) = -\sqrt{\beta}$.

As Equation (5) and Equation (6) define perturbation in the discrete case, we may also define them in the corresponding weighted context, as in Equation (9) and Equation (10).

$$\tilde{\lambda}_i = \lambda_i + \Delta\lambda_i(\beta, r) \approx \lambda_i + \boldsymbol{q}_i^t\delta\boldsymbol{L}(\beta, r)\boldsymbol{q}_i; \tag{9}$$

$$\tilde{\boldsymbol{q}}_i = \boldsymbol{q}_i + \Delta\boldsymbol{q}_i(\beta, r) \approx \boldsymbol{q}_i + \sum_{j \neq i} \frac{\boldsymbol{q}_j^t\delta\boldsymbol{L}(\beta, r)\boldsymbol{q}_i}{\lambda_i - \lambda_j}\boldsymbol{q}_j, \tag{10}$$

where $\Delta\lambda_i(\beta, r)$ is the perturbation of the i-th the eigenvalue and $\Delta\boldsymbol{q}_i(\beta, r)$ the perturbation of the i-th eigenvector, corresponding to a perturbation of size β upon the edge r. We redefine perturbation centrality in the weighted case in Equation (11).

$$\rho_K(\beta, r) = \sum_{i=2}^{K} \mid \Delta\lambda_i(\beta, r) \mid \tag{11}$$

3.2 Temporal perturbation centrality

Temporal networks can be represented as an ordered sequence of graphs taken at regular time stamps, $t \in \mathcal{T}$ (Holme and Saramäki, 2012). In addition to vertices, edges, and their weights, we now have another parameter that defines graph states, which is time, such that $\mathcal{G} = (\mathcal{E}, \mathcal{V}, \mathcal{W}, \mathcal{T})$, as in our graph theoretical framework. We can claim that each new timestamp involves a perturbation to the original graph - no change simply suggests an all zero perturbation. Extending Equation (2) to involve consecutive perturbation, we define time progression as

$$\mathcal{G}(\mathcal{E}, \mathcal{V}, \mathcal{W}, t+1) = \mathcal{G}(\mathcal{E}, \mathcal{V}, \mathcal{W}, t) + \delta\mathcal{G}(\mathcal{E}, \mathcal{V}, \mathcal{W}, t). \tag{12}$$

Given that Equation (12) is isomorphic to (2), it is clear that shifts from stamp t to $t + 1$ can be defined in terms of Laplacian perturbation, as in Equation (3).

Combined with the weighted extension of perturbation analysis, introduced in Subsection 3.1, we therefore propose a novel dynamically-weighted perturbation centrality metric. The advantage of such a measure is that, since each timestamp is dependent on the last, a complete computation of the Laplacian spectra is only necessary once at $t = 1$, so it is then possible to just update the evolution in time of the eigenvectors and eigenvalues. Figure 1 shows the overall process of decision making support based on temporal perturbation centrality.

After an intialisation step, Figure 1 shows that, for time stamps such that $t > 1$, we need only update the eigenvalues and eigenvectors as dependant on new weights coming from information of the flow passing by the network. From a computational perspective, this is of great value, since the Laplacian spectrum need only be computed once, after which only the Laplacian spectrum of the perturbation matrix needs computation at each timestamp. This is advantage since perturbations are all zero but for an affected link.

One suitable way to work with temporal networks is to extract a weighted perturbation centrality time series per each network link/node, over a cycle of time (day) of the data traffic. Boccaletti et al. (2014) and Kivelä et al. (2014) previously declare that working with time-based marginals for centrality measures is a particular case of a multilayer network, where new layers are created at each time stamp, and propose computing marginals by layer and node. Building on this, Taylor et al. (2017) presented time-based marginals for eigenvector centrality measures. Lv et al. (2019) go on to present a PageRank centrality for temporal networks. The current proposal takes a different approach, instead relying on the weighted Laplacian via its

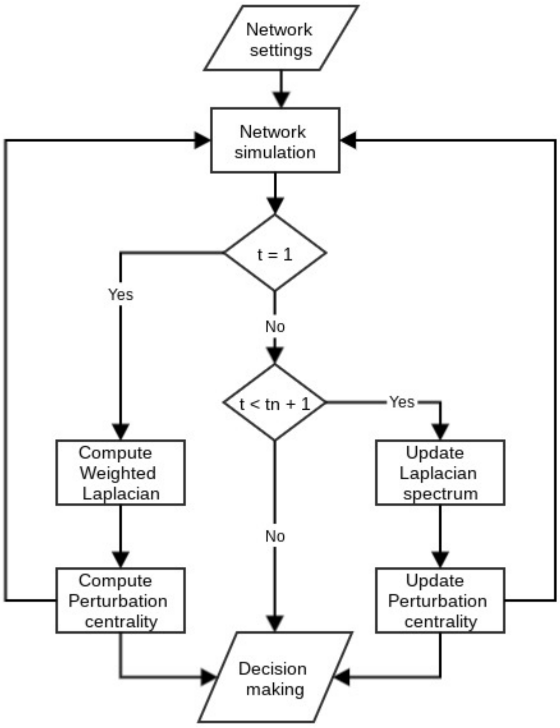

Fig. 1. Flowchart of any decision making using temporal perturbation centrality

perturbation and how this measure varies over time with the intensity of the network flow. Related methods have used Laplacian perturbation to identify node criticality (Liu et al., 2015), however these are not temporal methods.

In the proposed method, Equation (11) is extended to a temporal context, which creates

$$\rho_K(\beta_t, r) = \sum_{i=2}^{K} \mid \Delta\lambda_i(\beta_t, r) \mid, \tag{13}$$

where Equation (13), is used to populate the sequence, $\{\rho_K(\beta_t, r)_{t=1}^{T}\}$. This, instead of a single value assigned to each node, gives n many times series, one for each node, all of equal length and increment.

4. CASE-STUDY OF THE UK BACKBONE NETWORK INFRASTRUCTURE

This work is part of the Engineering and Physical Sciences Research Council (EPSRC) and BT Prosperity Partnership project: Next Generation Converged Digital Infrastructure. Today, the BT long-haul backbone network for the UK comprises 103 nodes; representing super hub, regional hub, and metro router stations. Besides, there is a total of 309 links (fibre-optic cables) connecting those nodes. In addition to the information about the network layout and physical elements, we also have information about the data traffic on a typical weekday. The traffic information on this weekday is taken every 2 minutes at each of the nodes of the network. Figure 2 shows the network layout and the average traffic at each link. This is obtained by network flow simulation considering

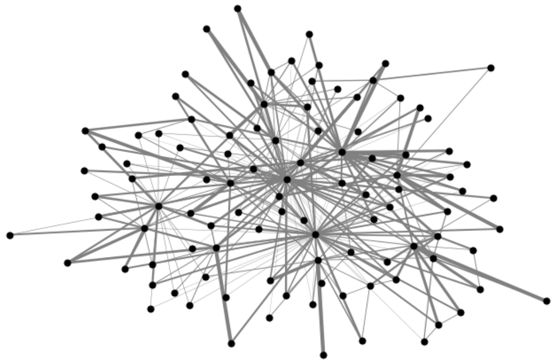

Fig. 2. Backbone for the UK communication network. Links are weighted by the average data traffic measured for 1 typical weekday. Geographical information is withheld to preserve anonymity.

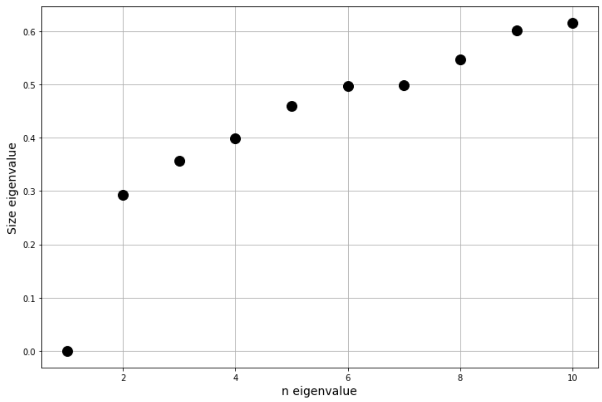

Fig. 3. First 10 eigenvalues of the unweighted, normalised Laplacian matrix of the UK backbone network

the traffic information at each node and the associated network dynamics.

Preliminary analysis of the network show an average node degree equal to 2 (Newman, 2018). This value is computed apart from the node hubs, representing super and regional routers, which have node degree in a range of 15 to 27 (in addition to 2 super hubs of a degree value over 60). Among other possible preliminary analysis, it is specifically relevant to compute the top normalised Laplacian eigenvalues. Figure 3 shows the first 10 eigenvalues, from which it highlights the relatively high value of the second smallest eigenvalue (Fiedler, 1973), $\lambda_2 = 0.29$. This indicates that any given edge, upon removal, has a small chance of splitting the network into clusters. This claim is also reinforced by the value of the spectral gap (Estrada, 2006) for the first 10 eigenvalues. The spectral gap functions as a measure of the surplus of the strength needed to split the network from k to $k + 1$ clusters.

Real traffic data collected from the backbone network is used to run network traffic simulations further. This endows the process with the robustness associated with a re-sampling processes in which traffic is generated by the observed distribution of the 24h demand curve. The network simulation is modified and adapted over the software proposed by Likic and Shafi (2018). Figure 4

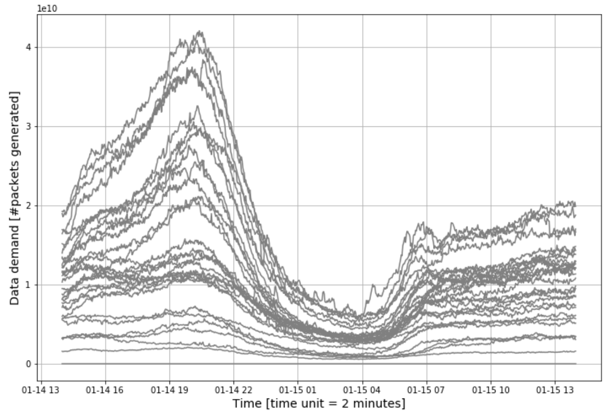

Fig. 4. Time series of 1 day of data traffic over the backbone network nodes

shows 1 day of measures of data traffic over the network nodes. Note that this is a weekday, so the rest of the simulations and inferences made at the present paper are about weekdays (since it is expected that data traffic may have a different profile at weekends and festive days).

Figure 4 shows groups of similar nodes regarding their demand profile. There is also noticeable, for at least 3 of the 4 observed groups, a large difference in data traffic over the day, having a clear peak and valley with a difference of size up to 3.5e10 packets per time unit. Based on the observed data, 20 simulations have been run following the changing parameters of Table 1. The different groups of nodes have also been considered in the data being generated: C1, C2, C3, C4 at Table 1. There have been sent, in average, 1,915,455 out of which, also in average, 833,945 have been received.

Table 1. Summary of data packet generation at different time intervals

Time interval	Internal time units	C1	C2	C3	C4
14h-18h	[0,120)	20	15	10	5
18h-22h	[120,300)	40	25	12	5
22h-03h	[300,450)	15	15	5	5
03h-08h	[450,600)	5	3	3	1
08h-14h	[600,720)	15	15	10	5

In addition to tailored dynamics depending on the observed data-traffic, the simulation herein propose a number of advantages such as the possibility to run the period under study many times to get an output statistically robust. In addition, simulation allows to blend telecommunication indices and metrics with complex network analysis. This is actually the case presented in this paper, where the data-traffic, its intensity and variation over time, informs the generation of the current perturbation analysis.

The perturbation over the Laplacian matrix is dynamically informed of the traffic flow at different times of a typical day. As a result we have computed the information about the criticality of the network links under perturbation over such day. Figure 5 shows the UK internet backbone network with its links width depending on how the average perturbation centrality vary over time. This is an expected but important result, given that the data-traffic profile also presents significant variations as it is discussed above on Figure 4. This analysis provides much more information

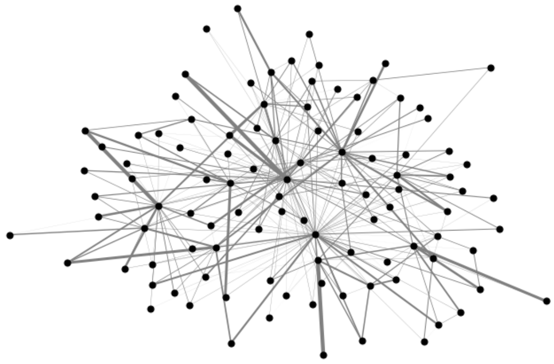

Fig. 5. Backbone for the UK communication network. Links width is proportional to the average of their perturbation for 1 typical weekday. Geographical information is withheld to preserve anonymity.

than the generally calculated static approach, for which the criticality of a link is a constant with respect to the time of day.

Table 2 shows, in average, the top 5 links prone to have a higher average value of the perturbation centrality over the day. The table also shows which are their maximum values of such perturbation and when them happen.

Table 2. Top 5 links in average perturbation centrality and peak critical hours

Link ID	Avg. pertub.	Critical time	Max. perturb.
7	24.2702	22:20	34.9967
39	24.1511	07:28	35.3164
80	24.0111	04:24	38.2808
136	23.0902	11:00	34.1277
62	23.0566	21:38	37.8204

5. CONCLUSIONS

This paper proposes a perturbation analysis of the UK backbone network infrastructure over one normal-day data traffic. The aim is to approach a critical link analysis for advising practitioners and decision makers on optimal asset management and maintenance plans. The results show which links are more critical and at which time of the day. This is highly relevant for assessing network vulnerability and resilience, and so launching protection plans adaptive in time, prioritising network physical assets laying or rehabilitation schemes, optimising capital budgets and the return of investment, and minimising how any failure may have an impact on the customer. These plans and the managerial operations enable awareness to internet providers and companies at higher level, given the also high topological and temporal resolution of the analysis.

The analysis presents two major novelties. First, the use of perturbation analysis as a natural indicator of link criticality and network resilience. To better inform the complex network on telecommunication characteristics, it has been considered for analysis the graph Laplacian matrix weighted by the intensity of data traffic. The second novelty relies on the use of a temporal evolution of such weights to propose different (dynamic) perturbation centrality index depending on the time of the day (and the corresponding variation on data demand). The proposed dynamic perturbation centrality is approached by iterative updates of a first complete computation of the Laplacian spectra. This means that the computational burden is significantly reduced and the application can be straightforwardly adapted to a near real-time approach.

This paper also comes with a future research avenue in which more research should be done. For instance, formalising the graph-theoretic framework via multilayer networks and the use of the supra-Laplacian matrix. Another research direction should focus in the time-series mining of the centrality indices evolution. In this regard, using data coming from week-days and weekends will be of high importance. The reason is that these two periods of the week may behave differently on their data demand.

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