A STUDY OF THE PERIPATETIC *MECHANICA*

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This thesis is submitted for the degree of
Doctor of Philosophy
July 2021

Declaration

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Chapter 5 has been reworked and expanded from an essay submitted in November 2018 for the MPhil in History and Philosophy of Science and Medicine at the University of Cambridge.

This thesis does not exceed the word limit of 80,000 words set by the History and Philosophy of Science Degree Committee.

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Abstract

This study aims to understand the aims and methods of a less-studied work from the early Peripatos, the *Mechanica*. I argue that the *Mechanica* (*Mech.*) was an application of natural philosophy to the technical sphere of mechanics. The primary aim is to give causal explanations of various puzzling phenomena in this domain. While the author uses lettered diagrams and specialised, geometrical language to achieve this aim, the arguments should not be described as mathematical or demonstrative. Rather, *Mech.*'s explanations are fundamentally physical, causal and analogical.

In Chapter 1, I describe the structure of *Mech*., underscoring a degree of coherence across its 35 problems. I provide evidence for dating *Mech*. to the early Hellenistic period (late 4th – early 3rd c. BCE) and against the attribution to Aristotle. I also take issue with two standard arguments against that attribution: the claim that *Mech.*'s understanding of natural motion differs from Aristotle's, and G.E.L. Owen's claim that *Mech*. applies the notion of motion and speed at an instant. I then situate *Mech*. in its intellectual context through a survey of earlier Greek mechanics and mathematical investigations of motion. At the end of the chapter, Note A summarises *Mech*.'s structure, while Note B examines passages in Aristotle's certainly authentic works sometimes thought to represent a theory of mechanics.

In Chapter 2, I argue that *Mech*.'s analysis of radial rotation as the combination of two rectilinear motions should be understood as claiming that two motions are present in a rotating radius, rather than as treating the component motions as useful fictions. To show this, I examine Aristotle's approach to composed motions across several works. I argue that Aristotle's accounts of change in *Physics* 3 and 5 imply a distinctive, realist view of component motions, according to which it is a fact that the rotating radius two simultaneous motions rather than a single motion along the same path. I then examine supporting evidence in passages concerning both celestial and sublunary motions.

In Chapter 3, I explore two further considerations that arise from Aristotle's statements on types of locomotion and their compositions. First, I consider how we should understand Aristotle's division of all motion into straight, circular and mixed. Then I explore the limits of the possible presence of distinct motions in a single object, through examining Aristotle's claim that no contrary motions can be simultaneously present in a body.

In Chapter 4, I undertake a close reading of *Mech*. problem 1, showing that problem 1's arguments draw on the resources of geometry to support a basically physical agenda and to deliver a causal explanation. In light of the arguments of Chapters 2-3, I argue that problem 1's analysis targets radial rotation, which is distinguished from celestial circular motion by the simultaneous presence of two rectilinear motions in the rotating radius. I defend the explanatory potential of *Mech*.'s causal notion of constraint (ἔκκρουσις) and I explore an unresolved tension between the characterisation of the motions as radial and tangential (849a6-849a19, 852a8-13) and their different representation in a diagram (849a19-849b19).

Chapter 5 studies the explanatory strategies of the less-studied problems 4-22, with a focus on their use of lettered diagrams and specialised language. I argue that these problems fundamentally rely on analogies, a kind of reasoning distant from formal geometry, but that they use the specialised language and lettered diagrams of geometry to support these analogies. Since the arguments are analogical rather than deductive, *Mech*.'s method should not be identified with the demonstrative ideals of Aristotle's *Posterior Analytics*.

Chapter 6 examines the paradox of *Mech*. problem 24, known as the *Rota Aristotelis*. This paradox challenges problem 1's claims about rotation and thus threatens to overturn *Mech*.'s explanatory project. I show that the author's aim is not to provide a geometrical explanation. Rather, he draws two distinct puzzles from the paradoxical phenomenon and answers each of them with a solution based on physical principles. This further substantiates my argument over the previous chapters that *Mech.* is not so much a mathematical work as an application of natural philosophy to the technical sphere of mechanics.

Chapter 7 argues that *Physics* 7.4's startling claim that circular and rectilinear motions are incomparable may represent an earlier attempt to solve the *Rota Aristotelis* paradox. I criticise three alternative explanations of *Phys*. 7.4's claim and show how *Phys*. 7.4's argument would make sense as a response to the paradox.

In Chapter 8, I summarise the arguments of previous chapters.

Acknowledgements

I wish to express my gratitude to my supervisor, Liba Taub, who has guided and supported my research, commented on countless chapter drafts, and always been exceptionally generous with her time and advice. I am also grateful to my advisors, Dániel Margócsy and Rebecca Flemming, for their many helpful suggestions on earlier drafts of these chapters. My thanks also go to Nick Jardine, who took an enthusiastic interest in my project and whose comments helped bring this study into its final form. I should like to thank my examiners, Professors Serafina Cuomo and István Bodnár, for their detailed and helpful suggestions.

I should like to thank all the staff at the Whipple Library, and particularly Jack Dixon who has always been quick to acquire the books I need. I would also like to thank Monte Johnson and Jean De Groot for their advice and encouragement, Henry Mendell for generously sharing unpublished material, Sonja Brentjes for advice on Arabic sources, and Christian Loeben of the Museum August Kestner, Hannover, for information on an artefact from Friedrich Wilhelm von Bissing's collection.

Audiences at Manchester, Exeter, Athens and Cambridge heard material from this thesis and their comments pushed me to clarify my arguments. I am grateful to Robinson College for generously contributing to conference expenses. My research would not have been possible without financial support from the AHRC through a Doctoral Training Partnership.

Finally, I am particularly grateful to my family and to my friends in Cambridge and elsewhere.

Contents

1 Introduction **1**
1.1 An overview of the *Mechanica* 1
1.2 Authorship and date 5
1.3 Influences 16
1.4 Debts to earlier scholarship 28
1.5 Chapter overview 30
Note A: A key to the problems of the *Mechanica* 33
Note B: Some comments on Aristotle's so-called 'mechanics' 34

2 Aristotle on composed motions **44**
2.1 Introduction 44
2.2 A puzzle about composed motions 47
2.3 Aristotle's account of κίνησις 50
2.4 Identity of κινήσεις 53
2.5 Multiple motions in Aristotle's physical explanations 61
2.6 Conclusion 65

3 Further themes in Aristotle's account of motions **67**
3.1 Introduction 67
3.2 Classifying motions 67
3.3 Simultaneous contrary motions 74
3.4 Conclusion 83

4 The Balance and Lever in the *Mechanica* **84**
4.1 Introduction 84
4.2 The quadrilateral of motions (848b9-848b25) 86
4.3 Inconstant ratios (848b25-35) 92
4.4 A rotating radius has two motions (848b35-849a6) 93
4.5 Radial constraint (849a6-849a19) 95
4.6 The Rotating Radius Principle (849a19-849b19) 102
4.7 Dynamics or kinematics? 105
4.8 Problems 2 and 3 107
4.9 Shortcomings of the *Mechanica*'s analysis 108
4.10 A model for eternal motions?. 113
4.11 Conclusion 116

5 Analogical explanation through language and diagrams **117**
5.1 Introduction 117
5.2 Analogy and the ideal of demonstrative science 119
5.3 Diagrams and technical language 121

5.4 Experience and idealisation 127
5.5 Conclusion 130

6 The *Rota Aristotelis* Paradox **132**
6.1 Introduction 132
6.2 An overview of the paradox 134
6.3 The *Mechanica*'s statement of the paradox 136
6.4 Comparison of sources 144
6.5 The solutions in the *Mechanica* 146
6.6 Conclusion 153

7 A new interpretation of *Physics* 7.4 **155**
7.1 Introduction 155
7.2 The Construction Interpretation 156
7.3 The Measurement Interpretation 159
7.4 The Cosmological Interpretation 162
7.5 The Wheel Interpretation 167
7.6 Conclusion 171

8 Conclusion **173**
8.1 Summary of the argument 173
8.2 Postscript 175

Bibliography **176**

Chapter 1: Introduction

1.1: An overview of the *Mechanica*

The *Mechanica* attributed to Aristotle teems with ideas that challenge modern conceptions of 'Aristotelian' science. It contains the earliest clear instantiation of what modern physicists call the 'parallelogram rule', an analysis of motion on a circular path into radial and tangential components, and the idea that diverse natural and artificial phenomena can be brought together and understood in terms of the lever and the balance. Is this the essentially qualitative natural philosophy of Aristotle with which we are familiar?

Few modern scholars have examined the *Mechanica* in detail.[1] This is surprising not only in view of the inherent interest of the text, but also given the starring role allotted to mechanics in traditional as well as more recent accounts of the Scientific Revolution and of the differences between ancient and modern science.[2] The fact that the *Mechanica* fell in the nineteenth century into the limbo of 'pseudo-Aristotle' may bear some responsibility for its neglect. In the absence of close analysis, the *Mechanica* has elicited wildly differing evaluations from modern scholars. Paul Tannery found it 'a disorderly and unmethodical collection of very diverse questions, in the solutions of which the author has not been able to leave any mark of his own originality.'[3] For G.E.L. Owen, the *Mechanica* far surpasses Aristotle in its 'almost Newtonian insights'.[4] A poorly organised and derivative school text or the research notes of a proto-Newton? The need for closer study is apparent.

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[1] English-language studies have appeared only since the 2000s.

[2] For example, Burtt 1925, Koyré 1957, Dijksterhuis 1961, Dear 1995.

[3] 'il s'agit d'un recueil, sans ordre et sans méthode, de questions très diverses à la solution desquelles l'auteur n'a pas su imprimer le sceau d'une originalité personnelle' (Tannery, 1915). For similarly negative assessments, see Montucla 1797, vol. 1, 125: 'Ils trouveront sans doute que la plûpart des explications qu'il donne sont entièrement fausses, et que la principale et la première est tout-à-fait ridicule.'; Whewell 1837, Vol.1, 51: 'in scarcely any one instance are the answers, which Aristotle gives to his questions, of any value.; Rose 1854, 192: 'quaestionum minutias et confusionem'; Knorr 1982a, 101n.27: 'the clumsy execution of the physical analyses throughout the *Mechanics* would call into question its ascription to a highly competent physical thinker, as Strato surely was… One might rather suppose that the *Mechanics* was produced by a scholar of no remarkable insight under the shadow of a keen scientific intellect.'

[4] Owen 1976, 8-9. For praise of the *Mechanica*, see also Guido Ubaldo 1577, *praef.*: 'Aristotelemque potius philosophorum coryphaeum imitemur, cuius mechanici amoris ardorem acutissimae illae mechanicae quaestiones posteris traditae satis declarant: qua quidem laude Platonem magnifice superauit.'; Cantor 1894, 241: 'Die sogenannte Mechanik des Aristoteles würde, sagen wir, seines Namens nicht unwürdig sein. Ein Schriftsteller des XVIII. S. [= Montucla] hat zwar darüber so ziemlich das entgegengesetzte Urtheil gefällt, dürfte jedoch damit vermuthlich allein stehen... ein solches Werk ist wahrlich keines antiken Schriftstellers unwürdig, mögen auch einige Fragen in demselben nicht richtig beantwortet sein. Zu diesen nicht richtig beantworteten Fragen gehört eine [=problem 24], welche schon überhaupt gestellt zu haben einen feinen mathematischen Geist verräth.'; Farrington 1949, 46: 'a brilliant attempt to bring a great range of human

The text consists of an introduction (847a11-848a38) followed by a sequence of 35 question-and-answer units called ‘problems’ (προβλήματα).[5] These attempt to answer a broad range of questions about the powers and effects of bodies in motion, such as:

> How can a small rudder move a huge ship?
>
> Why do things thrown from a sling travel further than things thrown by hand?
>
> Why are pebbles on the beach round?
>
> How do nutcrackers work?[6]

Many collections of ‘problems’ from Greco-Roman antiquity are rather heterogeneous. The problems may be tied together by a general theme – Plutarch’s *Platonic Questions* answers various questions about the interpretation of Plato – but each of the solutions stands alone, without a unifying explanatory principle, or result building on result.[7] By comparison, the *Mechanica* is remarkably unified. The introduction, itself an unusual feature in a book of problems, lays out a clear vision of what mechanical problems are about and how they are to be solved. First, the author describes what mechanical problems are about:

> People wonder at things that occur according to nature of which the cause is unknown, and at things that occur contrary to nature that happen through skill to the advantage of humans…For example when the lesser overpower the greater, and what has small natural motion moves great weights, and almost all the problems that we call mechanical.[8]

activities within the scope of mathematical explanation’; De Gandt 1982, 123-24: ‘Je ne connais pas d’exemple qui atteigne ou surpasse cette perspicacité avant le *de Vi centrifuga* de Huyghens et les travaux de Newton.’); White 1984, 177: ‘a practical, down-to-earth writer, with a shrewd grasp of the fundamentals of his subject’.

[5] The author uses the term ‘problems’ at 847a24.

[6] Paraphrasing problems 5, 12, 15, and 22.

[7] This characterises the books of the pseudo-Aristotelian *Physical Problems*. On ‘problems’ as a genre in the Peripatos, see Bodnár 2015 and Taub 2015.

[8] 847a11-25: Θαυμάζεται τῶν μὲν κατὰ φύσιν συμβαινόντων, ὅσων ἀγνοεῖται τὸ αἴτιον, τῶν δὲ παρὰ φύσιν, ὅσα γίνεται διὰ τέχνην πρὸς τὸ συμφέρον τοῖς ἀνθρώποις... τοιαῦτα δέ ἐστιν ἐν οἷς τά τε ἐλάττονα κρατεῖ τῶν μειζόνων, καὶ τὰ ῥοπὴν ἔχοντα μικρὰν κινεῖ βάρη μεγάλα, καὶ πάντα σχεδὸν ὅσα τῶν προβλημάτων μηχανικὰ προσαγορεύομεν. Translations are my own unless otherwise noted.

This describes with reasonable accuracy the scope of the following problems which, with only three exceptions, aim to explain the use of human skill and craft to achieve certain ends otherwise more difficult or impossible.[9] Next, the author sets out his method of explanation:

> These are not entirely the same as physical problems nor separated, but common to both the speculations of mathematics and those of natural science. For the 'how' (τὸ ὡς) is clear through mathematics, the 'about what' (τὸ περὶ ὃ) through natural science.
> …
> What happens in the balance is referred back to the circle, what happens in the lever is referred back to the balance, and almost all other things that happen in relation to mechanical motions are referred to the lever.[10]

Again, this corresponds fairly closely to what follows. Problem 1 ('Why are larger balances more accurate than smaller ones?) is answered by explaining the balance in terms of what I, adapting a phrase of Jean De Groot, shall call the 'Rotating Radius Principle': on a rotating radius, a point further from the centre moves faster than a nearer point.[11] Problem 3 (Why can small forces move great weights by means of a lever?) is answered by explaining the lever in terms of the balance.[12] Most subsequent problems (20 out of 32) offer explanations that rely on analogies to the lever, balance, or Rotating Radius Principle.

When the *Mechanica* answers questions like those above (e.g. How can a small rudder move a huge ship?), three levels of explanation are involved:

> LEVEL 1. It is shown that the thing to be explained is analogous to a lever or balance or rotating radius.

[9] The exceptions are problems 15, 30 and 35.
[10] 847a25-847b2: ἔστι δὲ ταῦτα τοῖς φυσικοῖς προβλήμασιν οὔτε ταὐτὰ πάμπαν οὔτε κεχωρισμένα λίαν, ἀλλὰ κοινὰ τῶν τε μαθηματικῶν θεωρημάτων καὶ τῶν φυσικῶν· τὸ μὲν γὰρ ὡς διὰ τῶν μαθηματικῶν δῆλον, τὸ δὲ περὶ ὃ διὰ τῶν φυσικῶν… τὰ μὲν οὖν περὶ τὸν ζυγὸν γινόμενα εἰς τὸν κύκλον ἀνάγεται, τὰ δὲ περὶ τὸν μοχλὸν εἰς τὸν ζυγόν, τὰ δ' ἄλλα πάντα σχεδὸν τὰ περὶ τὰς κινήσεις τὰς μηχανικὰς εἰς τὸν μοχλόν. I discuss the translation of τὸ ὡς and τὸ περὶ ὃ below.
[11] De Groot 2014 calls this the 'Moving Radius Principle'; others (e.g. Krafft 1970, Bodnár 2011b) call it the 'principle of concentric circles'. Problem 1 uses three different formulations of this principle, one comparing the speeds of points on a single radius, one comparing the speeds of radii, one comparing the speeds of the end-points of radii. The first of these is found only in the preface. De Groot 2014, 21-31 comments on several formulations of the principle.
[12] Whether the lever is properly explained by or reduced to the balance in problem 3 is more controversial (see Chapter 4). At a minimum, we can say that *Mech.* characterises the lever as a balance with unequal arms suspended from below.

LEVEL 2. Its properties can therefore be explained by the Rotating Radius Principle.

LEVEL 3. This principle holds because a radius' rotation results from two motions – one radial, one tangential. Provided they have the same radial motion, points nearer the centre undergo more radial motion and hence their tangential motion is proportionally reduced.

Levels 2 and 3 are provided in Problems 1-3. In answering later problems, usually only a level 1 explanation is spelled out. But explanations at this level depend on the further levels of explanation. Thus problems 1-3 in an important way provide the basis of the explanatory programme.[13]

At the end of this chapter, Note A provides an overview of the 35 problems and how they fit into this programme. Problems 1-3 have a special status, setting up the foundations. Of the remaining 32 problems, 20 offer level 1 explanations while 5 raise questions about the underlying explanations of levels 2 and 3, even questioning the project's legitimacy. There are seven outlying problems that do not relate to this programme.[14] These are all found towards the end of the text, from problem 25 onwards; I will not examine them in detail. The primary aim of this thesis is to elucidate the aims and methods of the explanatory programme that runs through our text from the introduction to the end of problem 24. My claim that there is a coherent plan behind *Mech.* is supported by a number cross-references within the problems.[15]

Although I have claimed that the *Mechanica* is fairly coherent down to and including problem 24, I do not mean to imply that this text was composed by only one author. It is

[13] In referring to a 'programme' or 'project' I underscore the relatively high degree of coherence of this problem text, where most problems share the aims and methods outlined in the preface.
[14] They can be loosely related to issues raised by problems within the scope of the programme, e.g. problem 25 partly concerns breaking wood (compare problems 14 and 16); problem 31's thought that it is easier to move something in the direction in which it is moving might recall part of problem 8; problems 32-34 discuss projectiles, also the subject of problem 12.
[15] Problem 3 refers to the introduction (850a3: 'Why do small powers move large weights by means of the lever, just as was said at the beginning, even when the lever has added its weight?') and also to problem 1 (850b3-4: 'The cause is that said before, that the [line] further from the centre describes a larger circle.') Problem 9 refers to problem 1 (852a19-20: 'just as we said larger balances are more accurate than smaller ones.'). Problem 27 refers to problem 1 (857a3: 'this has been shown earlier'). Less directly, problem 3, 850a30-32 echoes the introduction, 847b13-16.

possible that more than one writer was responsible for problems 1-24 but, if so, those writers shared common principles, methods, and research interests. For the sake of simplicity, I will speak of 'the author' and the reader may add 'or authors' where appropriate.

1.2: Authorship and date

The *Mechanica* has been transmitted separately from most of the Aristotelian corpus in 31 manuscripts, none earlier than the late thirteenth or early fourteenth century. It is attributed to Aristotle in most manuscripts. One manuscript, P^t (*Par. Graecus* 2507), expresses some scepticism.[16] In the Renaissance it was generally held to be an authentic work, with only one or two doubters.[17] Valentine Rose (1854) was influential in casting doubt on the text's authenticity.[18] Modern scholars are divided on the question of authorship, with some favouring Aristotle, others a Peripatetic philosopher in the decades following Aristotle's death, and some identify Strato of Lampsacus, head of the Peripatos after Aristotle's immediate successor Theophrastus (from c.287–269 BCE).[19]

Writing on mechanics was already attributed to Aristotle in antiquity.[20] The ancient lists of Aristotle's works in Diogenes Laertius and the *Vita Hesychi*, both largely derived from a third century BCE source, mention a Μηχανικόν. An Arabic list attributed to Ptolemy al-Gharib includes a *Mechanical Problems*. The *Vita Marciana*, which ultimately derives from an epitome of Ptolemy al-Gharib's *Life of Aristotle*, refers to Μηχανικὰ προβλήματα. In his commentary on the *Categories*, Simplicius divides Aristotle's theoretical writings into three groups, the theological, the natural, and 'the mathematical, such as the geometrical and mechanical books he wrote'.[21] Olympiodorus and Elias mention works on optics and mechanics as examples of Aristotle's mathematical output.[22] Athenaeus, a first-century BCE

[16] αἰτιολογία τῆς τῶν μηχανικῶν ἐνεργείας ὥς τινες λέγουσι Ἀριστοτέλους. Van Leeuwen 2010, 196 (cf. 2016, 31) dates this manuscript to 1370-1380.
[17] Cardano *Opus novum de proportionibus numerorum, motuum, ponderum, sonorum, aliarumque rerum mensurandarum* (Basel, 1570) and Patricio *Discussiones Peripateticae,* Tomus I, Liber III (Venice, 1571).
[18] McLaughlin 2013, 2-3 conveniently summarises Rose's arguments.
[19] Gohlke 1957, Nobis 1966, Krafft 1970, Bottecchia Dehò 2000 and Van Leeuwen 2016 suggest Aristotle is the author (Krafft thinks an early work, Golhke a late, advanced work); Forster 1913, Owen 1985, De Gandt 1981, Schiefsky 2009, Bodnár 2011a and 2011b, and De Groot 2014 propose an early Peripatetic. Moody and Clagett 1952, Clagett 1957, Drachmann 1963a, Gottschalk 1965, Fleury 1993, Laird and Roux 2008, and Dosch and Schmidt 2018 favour Strato. Winter 2007 implausibly suggests Archytas.
[20] I am here indebted to McLaughlin 2013 and Van Leeuwen 2016, 19.
[21] *In Cat*. 4.26.
[22] Olympiodorus *Introduction to Aristotle's Logic*, 7 Busse: 'the mechanical and optical problems' (τὰ Μηχανικὰ καὶ Ὀπτικὰ προβλήματα). Elias *Commentary on Porphyry's Isagoge* 116 Busse: 'his mathematics

writer on war machinery, mentions Aristotle as one of several theoretical writers who might be consulted by beginners in machine-construction but whose works have little practical use.[23] Athenaeus does not, however, say that the named authors wrote books explicitly devoted to mechanics.[24]

Aristotle nowhere indicates in his certainly authentic works that he had personally undertaken to investigate mechanics. When in the *Meteorology* he summarises the range of natural investigations to be undertaken, there is no trace of mechanics.[25] Some scholars have argued that Aristotle would not have been interested in mechanics. For example, Rose claimed that Aristotle was essentially uninterested in practical matters.[26] This kind of argument has not found recent supporters. It assumes a questionable assessment of mechanics as practical rather than theoretical.[27] Also, regular illustrations drawn from the manual crafts in Aristotle's works may cast doubt on Rose's generalisation about his interests. Setting this argument aside, let us consider the main arguments against inauthenticity upheld today.

The *Mechanica* has been seen as taking un-Aristotelian positions on three theoretical issues: (1) the meaning of κατὰ φύσιν and παρὰ φύσιν (natural and unnatural); (2) whether a body can truly be said to be changing at an instant; (3) the nature of circular motion. None of these points, however, is decisive. In fact, in each case I find the supposed divergence from Aristotle's own views has been exaggerated.

First, the issue of natural and unnatural motions. *Mech.* problem 1 applies the terms κατὰ φύσιν and παρὰ φύσιν respectively to the tangential and rectilinear components of rotating radial motion six times.[28] Several scholars have claimed that this is inconsistent with

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such as the *Optics* and *Mechanics* ascribed to him' (τὰ δὲ μαθηματικὰ αὐτοῦ ὡς τὰ Ὀπτικὰ καὶ Μηχανικὰ αὐτῷ βιβλία γεγραμμένα).

[23] Someone writing on practical matters should set to work, 'having carefully understood himself on the basis of the famous Delphic precept, rather than the works of Strato and Hestiaeus and Archytas and Aristotle and the others who have written works similar to theirs. For younger devotees of knowledge they would be useful [as a training] in elementary principles; but for those already wanting to do something they would be altogether irrelevant and detached from practical thinking.' Trans. Whitehead and Blyth 2004.

[24] As Whitehead and Blyth 2004, 69 seem to assume.

[25] The same is true of other curriculum surveys, for example at the end of *MA* and *IA*.

[26] Rose 1858, 192. Cf. Forster 1913: 'Whilst the scientific standpoint of the *Mechanica* is certainly Peripatetic, the writer's interest in the practical application of the problems involved is quite un-Aristotelian.'

[27] *Mech.* is interested in theoretical explanation rather than practical applications. The author almost never gives details of objects material or measurements and several phenomena are described misleadingly (see Chapter 5).

[28] 849a15-16, 849a19-21, 849b3-4, 849b5-6, 849b10-12, 849b18-19. The text of the first passage is disputed. Apelt's edition and many recent commentators adopt Van Cappelle's conjecture. Micheli 1995, 63-74 and Van

Aristotle's use of these terms.[29] If the author means that the tangential component is always natural throughout every stage of every rotation, then that is indeed a notion of natural motion very different from Aristotle's. On that view, the tangential component motion of a shot whirled around in a sling is natural, whereas for Aristotle the only natural motion of a heavy shot is towards the centre of the world. None of our later sources indicate that such a radical re-conceptualisation of natural and unnatural change had been proposed in the Peripatos, but then our later sources do not refer to any of *Mech*. problem 1's distinctive ideas.

In any case, there is no compelling reason for seeing this unusual notion of natural and unnatural motion in *Mech*.[30] The use of these terms for tangential and radial motions in problem 1 can be understood with reference to the particular case under consideration, a balance-arm weighed downwards from a horizontal position. In that case, the tangential motion is 'natural' in precisely the Aristotelian sense, being the downwards motion of a heavy object.[31]

So it is arguable that the identification of the component motions as natural and unnatural plays no explanatory role. The terms κατὰ φύσιν and παρὰ φύσιν may be mere labels, with no function in the argument.[32] In all manuscripts, but not in most modern editions, the diagram corresponding to this part of problem 1 takes the orientation of a balance-beam moving down

Leeuwen 2016, 13-18, favour the majority manuscript reading but do not, in my view, sufficiently explain the other five references to κατὰ φύσιν and παρὰ φύσιν motion. I discuss this further in Chapter 4.

[29] E.g. Vilain 2008, 153-54; Berryman 2009, 109; Schiefsky 2009 57n.15**;** Bodnár 2011a, 449.

[30] We should bear in mind the introduction's use of κατὰ φύσιν and παρὰ φύσιν conforms to the standard Aristotelian use of those terms; for further commentary, see Schiefsky 2007.

[31] This interpretation is favoured by Guevara 1627, 51, Krafft 1970, 33, De Gandt 1982, 122, Anders 2013, 125-26. The cost of this interpretation is that problem 1's explanation would need to be retouched when carried over to other cases, such as the lever (problem 3) and the sling (problem 12), where the tangential motion is not natural. I am arguing that such modifications are trivial, since the labels 'natural' and 'unnatural' are inessential to the explanation.

[32] Arguably, it is the radial motion's status as constraint that matters rather than its status as unnatural. Otherwise, what I in Chapter 4 call the 'constraint principle' would also apply to the quadrilateral of motions, since at least one of the two motions to which a body is subject must be unnatural. The difficulty, to which I shall return in Chapter 4, is that the motion will be both natural and tangential only for a moment, at the start of the balance arm's descent. Bodnár 2011a, 449 writes that the use of the labels is 'substantially different from what we find in other Aristotelian contexts. This remains so, even if we were to admit that the use of the expression "according to nature" is conditioned by the particular example the *Mechanics* sets out to explain in this problem.' This is because 'the contrast between the two components, between the natural tangential motion and the radial motion contrary to nature is asserted about every phase of the motion on a circular trajectory.' I have not found that the labels 'natural' and 'unnatural' in the six passages cited in n.28 are directly applied to every phase of the rotation. The πάσῃ of 849a14 more likely means lines of every size than at every phase of rotation.

from a horizontal position.[33] If the ancient diagram took this orientation, the use of κατὰ φύσιν and παρὰ φύσιν with reference to lines in the diagram would more clearly have been anchored to the specific example of the balance.[34] Further, problem 8 offers a similar analysis of rotation into radial and tangential component motions, but here the tangential motion is the result of pushing and so there is no claim that this could be a natural motion.[35] This suggests that one need not see the tangential component motion as natural in such an analysis, which could equally well procede on the assumption that both motions are unnatural. In summary, I find that *Mech*.'s use of the terms κατὰ φύσιν and παρὰ φύσιν is less conclusive as evidence against authenticity than some commentators have claimed.

The second supposed point of tension is whether a thing can truly be said to be changing or resting at an instant. G.E.L. Owen claimed more than once that a major reason for the poverty of Aristotle's 'dynamics' lay in his response to Zeno's Arrow paradox. Aristotle had said that 'nothing is moving in an instant'.[36] Owen took this as a denial that anything can truly said to be changing or resting at an instant:

> 'Unable to talk of speed at an instant, Aristotle has no room in his system for any such concept as that of initial velocity, or what is equally important, of the force required to start a body moving. Since he cannot recognize a moment in which the body first moves, his idea of force is restricted to the causing of motions that are completed in a given period of time. And, since he cannot consider any motion as caused by an initial application of force, he does not entertain the Newtonian corollary of this, that if some force F is sufficient to start a motion the continued application of F must produce not just the continuance of the motion but a constant change in it, namely acceleration.'[37]

In *Mech*., by contrast, Owen found that 'circular motion is resolved into two components… And the remarkable suggestion is made that the proportion between these components need

[33] Van Leeuwen 2016, 115, 122, 154-57, 206-9.
[34] We cannot know the orientation of the diagram in antiquity, but the manuscript orientation has the virtue of matching the subject-matter of the problem; the orientation chosen by modern editors is arbitrary. Diagrams in Greek mathematical papyri sometimes give visual representations of the objects discussed (e.g. PSI III, 186), so a diagram *could* have refocussed attention on the balance by depicting e.g. its stand and pans, but we cannot tell.
[35] 852a11-12: τὴν μὲν γὰρ εἰς τὸ πλάγιον αὐτοῦ κίνησιν ὠθεῖ τὸ κινοῦν.
[36] *Phys*. 6.3, 234a24: οὐθὲν ἐν τῷ νῦν κινεῖται.
[37] Owen, 1958, 161-62. This thesis was repeated without significant modification in Owen 1965, 148 (the denial of motion at an instant 'bedevilled the course of dynamics'); Owen 1970; Owen 1976; Owen 1985.

not be maintained for any time at all, since otherwise the motion would be in a straight line.' What is remarkable here is the idea 'of a point having a given motion or complex of motions at an instant and not for any period, however small.'[38]

This powerful interpretation of Aristotelian mechanics can be challenged on a number of issues.[39] First, it is questionable whether rejection of change at an instant by Aristotle could have the importance for the history of dynamics that Owen implies. Ancient mathematicians sometimes did consider speeds at instants. Ptolemy has a procedure for calculating the speed of the Moon at an instant.[40] This did not lead the ancients or their medieval successors closer to a Newtonian conception of force. In fact, it is arguable that Newtonian mechanics is neutral on the issue of motion at an instant.[41]

Secondly, the text of *Mech.* does not *explicitly* say that anything has a motion at an instant. In problem 1, a rotating radius is said to move with two motions that are ἐν μηθενὶ λόγῳ μηθένα χρόνον ('in no ratio for any time'). The motions do not maintain a fixed ratio to each other for any time interval. If they did, it is argued, the radius would trace a rectilinear path during that interval. The Aristotelian term for 'instant' (νῦν) is not applied. All that is needed is that, whichever periods are specified, the ratio of motions will be different for any two of them. Although the author does not put things in these terms, one might suppose that at least one of the motions must be varying in speed during any time interval, provided their directions remain the same. Is that any more in tension with the thesis that nothing is changing at an instant than is the Aristotelian belief that some moving things accelerate and decelerate? I shall return in Chapter 4 to the author's reticence to elaborate on the implications of his claim that the two motions are ἐν μηθενὶ λόγῳ μηθένα χρόνον.

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[38] Owen 1970, 256.

[39] Penner 1970, Appendix II and Sorabji 1976, 87 challenge parts of Owen's assessment. Sherry 1986 criticises Owen's claim that Aristotle should have accounted for ordinary talk of instantaneous speeds as homonymous uses of 'motion' (cf. Owen 1958, 161: 'an unjustified departure from usage: it deprives us of a… common idiom for us and for the Greeks.') Sherry suggests that talk of motion or speed at an instant would not have been 'common idiom' for Greeks of Aristotle's day.

[40] *Almagest* 6.4. See Pedersen 2010, 90-91 for a clear exposition in modern algebraic notation.

[41] White (1991, 177-79) makes this point by defining instantaneous velocity in terms of the Cauchy-Weierstrass ε-δ definition of limit. The claim is then about what modern physicists recognise as 'Newtonian mechanics' rather than Newton's ideas. There is a danger to explaining the history of Greek mechanics in terms of failure to grasp a single idea. Another common assessment (e.g. Barbour 2001, 34-41) holds that the chief obstacle to progress was a failure to consider limiting cases such as vacuums and frictionless planes. While this may be true for Aristotle, Hero was aware of the effects of friction and what would happen in its absence (Hero, *Mechanica* 1.20) and of course plenty of Greek thinkers were more willing than Aristotle to entertain the possibility of a vacuum, the Epicureans for example. Single factor explanations seem unlikely to take us far.

Finally, one can question whether Aristotle unequivocally denied that it makes sense to speak of change or rest at an instant. As Benjamin Morison has pointed out, the key phrase, 'nothing is moving *in* an instant', is ambiguous. It could mean either 'nothing is moving *at* an instant' or 'nothing gets any motion done *during* an instant', and Aristotle himself sometimes relies on the intelligibility of talk of change and rest at an instant.[42] Recent studies have shown how Aristotle's replies to Zeno's paradoxes can be understood without ascribing to him the denial of change at an instant.[43]

The third issue on which *Mech*. has been thought to differ from Aristotle is the nature of circular motion. In *Phys.* 8 and *DC* 1, Aristotle says that there are two types of simple motion, circular and straight, and there is also 'mixed' motion. *Mech.* seems to take a different view, that circular motion is a mixture of two straight motions. This is entirely at odds with Aristotle's conception of circular motion, so Aristotle cannot be the author.[44]

The key objection to this argument has been aptly articulated by István Bodnár: 'Even though Aristotle argues in *De Caelo* 1 that circular motion is simple, this does not necessarily apply to all the constrained circular motions there are. The simplicity of the natural celestial revolutions… may very well allow for the presence of composite forced revolutions.'[45] My arguments in Chapters 2-4 reinforce this point: there is a distinction to be made between true circular motion as found in the heavens and radial rotation. To the best of my knowledge, it has not previously been noted that this distinction is marked on the terminological level. Aristotle calls simple circular motion either ἡ κύκλῳ κίνησις, ἡ κύκλῳ φορά, or κυκλοφορία. *Mech*.'s author carefully avoids these terms. His object of analysis is not circular motion, but ἡ γράφουσα τὸν κύκλον, 'the [line] describing the circle' or ἡ ἐκ τοῦ κέντρου γράφουσα τὸν κύκλον 'the line from the centre describing the circle'. For convenience, I refer to this as 'the rotating radius'.[46] I take this as an indication that *Mech*.'s account of motion on a circular path did not pose a direct challenge to received views on the simplicity of circular motion. It is, incidentally, unlikely that *Mech*.'s account was intended to apply to celestial rotations. Not

[42] Morison 2013, 179-80.
[43] See Magidor 2008 on the Arrow paradox and Cohoe 2018 on the Dichotomy paradox.
[44] Owen 1970, 256; Berryman 2009, 109.
[45] Bodnár 2011a, 449.
[46] There is no single term in Greek equivalent to 'radius'. Sidoli 2003 discusses Greek expressions for 'radius' in plane and solid geometry.

only is there no solid 'radius' connecting the heavens to their centre of rotation but, in a finite cosmos, there is literally no place towards which a tangential component in the sphere of fixed stars could be directed.

A terminological difference from Aristotle's certainly authentic works has, to the best of my knowledge, not previously been mentioned in relation to the question of authorship. *Mech.*'s introduction says 'the 'that' (τὸ ὣς) is clear through mathematics, the 'about what' (τὸ περὶ ὃ) through natural science.' The crucial expressions, τὸ ὣς and τὸ περὶ ὃ, are not applied in Aristotle's certainly authentic works in a similar context.[47] There are also two more minor terminological differences from Aristotle's works.[48]

On a number of more minor issues, *Mech.* could be seen as departing from Aristotle's main line, though none of these cases is particularly persuasive. *Mech.*'s author thinks that there are imperceptible motions;[49] his explication of 'faster' involves an un-Aristotelian sense of τόπος;[50] he thinks that when one pushes something already moving in the opposite direction, 'some of the mover's power is subtracted, even if it is much faster';[51] he is aporetic about the causes of projectile motion.[52]

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[47] Aristotle's terms for 'explanation' include τὸ διότι, ὁ διὰ τί, ἡ αἰτία, τὸ αἰτίον, even ἡ ἀρχὴ. The author of *Mech.* knew these phrases and used them freely through the text (e.g. 848b2, 855b32). Ferrini 2010, 243 comments that 'l'uso sostantivato di *hos* e del nesso *peri ho* è raro in Aristotele', citing *EN* 7.4, 1146b14-17 ('Our investigation starts with the question whether what distinguishes the self-controlled person and the un-self-controlled is type of object or manner (πότερον ὁ ἐγκρατὴς καὶ ὁ ἀκρατὴς εἰσι τῷ περὶ ἃ ἢ τῷ ὣς ἔχοντες τὴν διαφοράν), I mean whether lack of self-control is marked off just by having to do with a particular type of things, or rather just by manner, or whether it is a combination of both (πότερον τῷ περὶ ταδὶ εἶναι μόνον ἀκρατὴς ὁ ἀκρατής, ἢ οὒ ἀλλὰ τῷ ὥς, ἢ οὒ ἀλλ' ἐξ ἀμφοῖν)', trans. Rowe); *GC* I.5, 320a26 ('So it is obvious that there are differences in the ways in which change occurs in things which come to be, alter, and grow, and not only in the respects in which <such changes occur> (διαφέρει οὐ μόνον περὶ ὃ ἀλλὰ καὶ ὣς)', trans. Williams; cf. 320a11-16 where ὁ τρόπος is substituted for τὸ ὣς: πότερον μόνως ἐν τῷ περὶ ὅ ἐστιν αὐτῶν ἡ πρὸς ἄλληλα διαφορά… ἢ καὶ ὁ τρόπος διαφέρει τῆς μεταβολῆς). See also *APo*. 1.33, 89a36: There can be correct and incorrect beliefs about the same object, τὸ αὐτὸ γὰρ ὅτι ἄνθρωπος, τὸ δ' ὡς οὐ τὸ αὐτό ('For it is the same because man <is the same>, but the manner is not the same', trans. Barnes); *Phys*. 5.4, 228b25-27 where τὸ ὣς of a change is 'the manner' (Ross) and Aristotle's example is speed.
[48] *Mech.* uses κυλίνδω, Aristotle uses κυλινδέω for 'to roll'. In Aristotle πέφυκεν κινεῖσθαι describes a tendency for natural motion but in *Mech*. 24 it seems to describe something moving naturally (see §6.5).
[49] Wardy 1990, 318n.26, commenting on *Phys*. 7.5's threshold proviso, suggests that Aristotle does not countenance imperceptible motions ('If nothing *perceptible* happens, nothing happens'), but elsewhere Aristotle seems to allow for imperceptible changes (*De Sensu* 6, 445b30-446a15; cf. Sorabji 2004).
[50] See Note B at the end of this chapter.
[51] Contrast Aristotle's views on 'swamping' (see references in Hussey 1991, 221n.24); on the other hand, *De Sensu* 7, 447a21-24 is close to *Mech*. here.
[52] Problems 32-34, especially 32 858a16-17: ἢ ἄτοπον τὸ ταῦτ' ἀπορεῖν, ἀφέντα τὴν ἀρχήν (cf. Lloyd 1987a, 155-58). Aristotle discusses projectile motion at *Phys*. 4.8, 215a14-19, 7.2 243a20-243b2 and 8.10, 266b27-267a12.

The principal reason why I think *Mech.*'s author is more likely an early Peripatetic than Aristotle himself is that *Mech.*'s methodological terminology has no direct parallel in Aristotle's certainly authentic works. The appearance of various concepts in problem 1 that are unparalleled in Aristotle's works (e.g. motions in no ratio for any time, radial constraint, both to be discussed extensively in Ch.4) may also suggest a different author. These considerations are evidently not conclusive. If *Mech.*'s reference to temple wheels (848a23-25) could be tied to early Hellenistic Egypt (see below), that would clearly speak in favour of post-Aristotelian authorship.

The author probably wrote in the late fourth or early third century BCE. This early date is suggested, though not conclusively shown, first, by *Mech.*'s geometrical terminology. Some terms in *Mech.* are vaguer or less standardised than is typical in later writers. For example, problem 1 refers to what is likely a rectangle using the term τετράπλευρον, which for Euclid and later geometers is the most general term for a quadrilateral. [53] The author, in common with pre-Euclidean writers such as Hippocrates of Chios (fl. late fifth century BCE) and Aristotle, often uses complex designations in referring to lettered diagrams. For example he will designate a point by the formula τὸ ἐφ' οὗ A instead of the shorter τὸ A found in Euclid and many later authors.[54] However, complex designations are also found in some post-Euclidean writers and so cannot be used as unproblematic evidence of a date before Euclid became influential.[55]

Further, *Mech.* does not use the Archimedean notion of centres of weight even where it might simplify or strengthen an argument.[56] Although Archimedes' dense and highly technical works probably had a small readership in antiquity, later commentators on Aristotle were

[53] Some further examples drawn from Heiberg 1904 and Heath 1949: problem 1 uses περιφερές for either a general curve or for part of a circle; problem 3's statement of inverse proportionality would be expressed differently by Euclid; problem 5, 851a13 says an angle subtended by a certain base 'sits' on that base (καθῆσθαι) while Euclid would have said it 'stands' (βεβηκέναι); problem 8, 851b21-24 refers to the 'angle of contact' between a wheel and the surface it rolls on as an 'angle', but Euclid 3.6 proved that such an "angle" is not really an angle at all; problem 8, 851b27-28 uses the expression πρός ὄρθιον (perpendicular) rather than πρός ὀρθὰς or ὀρθή πρός.

[54] *Mech.*, like Aristotle, uses both forms.

[55] Vitrac 2002, 252-4, Acerbi 2020, 51-60.

[56] See Drachmann 1963b. Although it is often assumed that Archimedes developed the notion of centres of weight, Hero *Mech.* 1.24 says that Archimedes refined a pre-existing theory, which, according to the Teubner edition, Hero ascribes to the Stoic philosopher Posidonius (Nix and Schmidt, 1900, pp. 62-65). Chronologically this is nonsense: Posidonius lived long after Archimedes. Clagett 1959, 50 rejects the Teubner emendation but does not suggest an alternative reading. I think it is more likely that a correct name has been garbled in transmission and translation than that Hero was seriously mistaken on the origins of the theory. *Mech* problem 29 might be seen as relying on an implicit, intuititive notion of centres of weight.

familiar with Archimedes' technical inquiries, results, and concepts, including centres of weight.[57] Yet there are no centres of weight in *Mech*. Other Archimedean ideas are missing too: there are no spirals in problem 35's discussion of whirling water, and the screw is not among the simple machines investigated.

The late third-century BCE mechanical author Philo of Byzantium, the first-century BCE Roman architect Vitruvius, and the (probably) first-century CE mechanical author Hero of Alexandria all seem to draw on sources similar to *Mech*.[58] For example, Philo in his book on war machines recaps some ideas from his lost book on levers. In doing so, he deploys a pattern of argument found in *Mech.* (*Belopoeica* 59):

> ἐπεὶ γὰρ οἱ μείζονες κύκλοι κρατοῦσιν τῶν ἐλασσόνων τῶν περὶ <τὸ> αὐτὸ κέντρον κειμένων, καθάπερ ἐν τοῖς Μοχλικοῖς ἀπεδείξαμεν, διὰ δὲ τὸ ὅμοιον καὶ τοῖς μοχλοῖς ῥᾷον κινοῦσι τὰ βάρη, ὅταν ὡς ἐγγύτατα τοῦ βάρους τὸ ὑπομόχλιον θῶσιν (ἔχει γὰρ τὴν τοῦ κέντρου τάξιν· προσαγόμενον οὖν πρὸς τὸ βάρος [δὲ] ἐλασσοῖ κύκλον, δι' οὗ τὴν εὐκινησίαν συμβαίνει γίνεσθαι)· τὸ αὐτὸ δὴ νοητέον ἐστὶ καὶ περὶ τὸ ὄργανον. ὁ γὰρ ἀγκών ἐστι μοχλὸς ἀντεστραμμένος· ὑπομόχλιον μὲν γὰρ γίνεται τὸ ἓν <μέσῳ τοῦ τόνου> μέρος αὐτοῦ, ἡ δὲ τοξῖτις νευρὰ τὸ βάρος, ἥτις ἐξ ἄκρου τοῦ ἀγκῶνος ἐχομένη τὸ βάρος ἐξαποστέλλει.
>
> 'Larger circles overpower smaller ones fixed about the same centre, as we demonstrated in our *Principles of Leverage*; similarly, men move loads with levers more easily when they place the fulcrum as close as possible to the load (the fulcrum, of course, performs the function of the centre; when brought close to the load it decreases the circle and easy movement is the result). Therefore, the same principle must be applied to the piece of artillery. The arm [in a catapult] is an inverted lever; the fulcrum is the part of the arm in the middle of the string; the (apparent) load is the bow-string, which dispatches the (actual) load and is attached to the end of the arm.' (trans. Marsden with modifications)

[57] See, for example, Simplicius *On Aristotle's De Caelo 2* 543, 549-550; *On Aristotle's Physics 1,* 59-60, *On Physics 7*, 1110; Olympiodorus *On Aristotle's Meteorology* 119, 211.
[58] See especially Philo, *Belopoeica* 59; Vitruvius *De Architectura* 10, *praef.*-3; Hero *Mech*. 2.34.

There are three points of similarity to *Mech.*: (i) the central role given to unequal concentric circles in explaining the lever; (ii) the term μοχλὸς ἀντεστραμμένος;[59] (iii) the practice of supporting an argument by correlating parts of the device to be explained to parts of the lever. This passage applies to a part of a complex machine a style of explanation that *Mech.* used to understand simple machines, though that does not necessarily imply that *Mech.* was written earlier. Vitruvius and Hero both include lists of question-and-answer units. Some of these overlap in content with *Mech.* but dependence on *Mech.* itself has not been proven.[60] Philo, Hero and Pappus share that view that there are five simple machines: the wheel and axle, the lever, the compound pulley, the wedge, and the infinite screw.[61] This is a more rigid and schematic classification of simple machines than is found in *Mech.*, where the lever and balance hold a privileged if rather loosely defined position. This may again suggest a date for *Mech.* some time before Philo's *floruit*.

The artefacts studied by *Mech.* are mostly very ancient, for example the oar, the sail and the wheel.[62] Only a few post-date the earliest extant Greek literature. An artefact described in *Mech.*'s introduction may potentially be helpful for dating, though it is difficult to draw firm conclusions on the current evidence. *Mech.*'s author describes a system of small metal wheels in contact such that if one wheel is turned clockwise, the adjacent wheel turns anticlockwise, the wheel adjacent to that clockwise again, and so on.[63] It is unclear if the wheels have interlocking teeth or if they work by friction. The author says such arrangements of wheels are found in temples. In 1901, Paul Tannery suggested that the *Mechanica* might have been written in Egypt, since other ancient writers refer to purifying wheels found in Egyptian

[59] *Mech.* uses the similar term μοχλὸς ἀνεστραμμένος to describe the steelyard (854a10-11). As in the steelyard, the effort in the catapult is input nearer to the fulcrum than the weight which is moved, which is the inverse of what happens in a regular lever.

[60] Micheli's (1995, 115-119) scepticism is appropriate. I am also cautious not to compare too closely *Mech.* problem 1 with the 'principle of concentric circles' attributed to Archimedes, Philo and Hero (Pappus *Collectio* 8, 1068.19-23: ἀπεδείχθη γὰρ ἐν τῷ περὶ ζυγῶν Ἀρχιμήδους καὶ τοῖς Φίλωνος καὶ Ἥρωνος μηχανικοῖς, ὅτι οἱ μείζονες κύκλοι κατακρατοῦσιν τῶν ἐλασσόνων κύκλων, ὅταν περὶ τὸ αὐτὸ κέντρον ἡ κύλισις αὐτῶν γίνηται.) That principle is not in fact equivalent to any explicit statement in *Mech.* (*Mech.* problem 1 is about speeds of radii, not powers of circles). To judge from Hero *Mech.* 2.7, Hero's approach at least was very different from *Mech.*'s (for a start, he does not analyse rotation into two motions).

[61] See especially Pappus *Collectio*, 8.31, 1115 Hultsch: Πέντε τοίνυν οὐσῶν δυνάμεων δι' ὧν τὸ δοθὲν βάρος τῇ δοθείσῃ βίᾳ κινεῖται… ἀποδέδοται δὲ ὑπὸ τοῦ Ἥρωνος καὶ Φίλωνος καὶ διότι αἱ προειρημέναι δυνάμεις εἰς μίαν ἄγονται φύσιν, καίτοι παρὰ πολὺ διαλλάσσουσαι τοῖς σχήμασιν. See also Hero *Mech.* 2.1-20.

[62] Oars were found with the Cheops ship (c.2650 BCE) and sails are depicted in Egyptian art from c.3100 BCE (McGrail 2009). Some Greco-Roman writers claimed that ancient tools were relatively recent inventions. For example, Pliny *HN* 7.198 makes implausible suggestion that the architect Theodorus of Samos (c.550-520 BCE) invented the lever.

[63] 848a23-25: κατασκευάζουσί τινες ὥστ' ἀπὸ μιᾶς κινήσεως πολλοὺς ὑπεναντίους ἅμα κινεῖσθαι κύκλους, ὥσπερ οὓς ἀνατιθέασιν ἐν τοῖς ἱεροῖς ποιήσαντες τροχίσκους χαλκοῦς τε καὶ σιδηροῦς.

temples.[64] In the same year, the German Egyptologist Friedrich Wilhelm von Bissing published details of a copper wheel he had acquired 'im Kunsthandel zu Theben'.[65] The wheel (7 cm diameter, 2.5 cm thick) is mounted on an axle in a copper box (9 cm long, 4 cm high, 6 cm deep). A hieroglyphic inscription on the box is now all but illegible. After writing to several museums which acquired portions of von Bissing's collection, I found that the wheel is now in the Museum August Kestner in Hannover. A recent exhibition catalogue dates it to the twenty-fourth or twenty-fifth Dynasty.[66] Lacking any information on its provenance or archaeological context, it is difficult to reach firm conclusions about the original function of this artefact.

Andrew Wilson has recently revived Tannery's suggestion that *Mech.* was written in Egypt, arguing that it was 'probably written at Alexandria between about 280 and 260 B.C'. Wilson adds to Tannery's point about Egyptian temple wheels that early investigations into the principles of gearing may have been taking place in early third-century Alexandria.[67] The caveat here is that our evidence for the early history of gearing is meagre.

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[64] Reprinted in Tannery 1915, citing Hero *Pneumatics* 1.32 and also Clement of Alexandria *Stromata* 5.672 (=5.8.45). Hero describes a type of wheel found 'in Egyptian temples' (ἐν τοῖς Αἰγυπτίων ἱεροῖς) called a 'purifier' (ἁγνιστήριον, 2.32); when rotated it dispensed purifying water. Clement quotes the grammarian Dionysius Thrax (fl. c. 170 BCE) 'concerning the symbol of little wheels' (περὶ τοῦ τῶν τροχίσκων συμβόλου). Dionysius claimed that 'the revoling wheel in the temples of the gods, drawn from the Egyptians' (ὅ τε τροχὸς ὁ στρεφόμενος ἐν τοῖς τῶν θεῶν τεμένεσιν εἱλκυσμένος παρὰ Αἰγυπτίων) is a symbol. Clement then quotes some Orphic verses which suggest an interpretation of the wheels as representing the turns of fate. It is not clear if the verses were also quoted by Dionysius. The word τροχίσκων is the same diminutive of τροχός used by *Mech.*. Outside mechanics, τρόχισκος often means a pill or pastille. Apollodorus, *Poliorcetica* 155.9 uses the term for the wheels of a ram; Blyth 2005, 135 finds this usage 'odd', citing *Mech.* as a rare parallel. The Arabic version of Philo's *Pneumatica* provides further evidence: 'A water wheel for ablution and purification, near a *mosque* or temple… [T]he wheel is of copper. The ancients used many constructions of this type. When they were about to enter a temple, they sprinkled their robes with water from this wheel. Then they put their hands to the wheel; they believed that touching the copper had a purifying effect. The wheel turned with regular, continuous motion, and whistled. The motion and the whistling drew attention to it, when people entered the temple. It stopped when one touched it; when one released it, it started again and turned as before.' (trans. Prager 1974, 226-8). A square copper container with a wheel in it is placed in a temple doorpost. It is powered by falling water concealed in the doorpost: 'People believe the movement comes from anything but the water.' (*ibid.*) The authenticity of this portion of the Arabic Philo is contested, though recent studies have favoured a third-century BCE origin (Lewis 1997, Schomberg 2008). Note that both *Mech.* and Philo emphasise ignorance of the cause of motion. On the other hand, it should be noted that Hero and Philo describe single wheels, whereas *Mech.* describes a system of wheels and does not say that worshipers touch them or that they are believed to purify. Plutarch, attempting to explain Numa's injunction to turn around while worshiping, suggests a comparison to 'Egyptian wheels' (*Numa* 14: εἰ μὴ νὴ Δία τοῖς Αἰγυπτίοις τροχοῖς αἰνίττεταί τι καὶ διδάσκει παραπλήσιον ἡ μεταβολὴ τοῦ σχήματος, ὡς οὐδενὸς ἑστῶτος τῶν ἀνθρωπίνων, ἀλλ᾽ ὅπως ἂν στρέφῃ καὶ ἀνελίττῃ τὸν βίον ἡμῶν ὁ θεός).

[65] Capart and Bissing 1901.

[66] Fitzenreiter et al. 2014, Kat. II.40, 316.

[67] Wilson 2008, 338. Peripatetics had a strong presence in Alexandria in the early third century BCE. See Von Staden 1989, 39, 97.

These considerations do not conclusively demonstrate a *terminus ante quem*, but they collectively point to a date before the mid-third century. Could the author be identified as Strato of Lampsacus, as some have suggested? One of the few things we know about Strato's life is that, before leading the Lyceum from 287/86, he served as tutor to the young Ptolemy II Philadelphus in Alexandria. If *Mech.*'s description of a system of wheels found in temples can be tied to Alexandria, this might also suggest Strato or someone close to him. However, it is difficult to say much for or against this ascription to Strato since the only independent evidence we have of Strato's work consists of second-hand reports and fragments. None of the distinctive ideas known to us through these have been securely identified in the *Mechanica*. The main positive consideration is that the list of Strato's works in Diogenes Laertius includes a Μηχανικόν.[68] Strato is not a bad guess, but it is little more than a guess.

1.3: Influences

Among the ancient Greek texts on mechanics still extant, the *Mechanica* is one of the earliest.[69] Yet it was written – if our dating is correct – several decades after the Pythagorean philosopher and politician Archytas of Tarentum (fl. c. 400-350 BCE) is said to have founded the mathematical discipline of mechanics. What did it owe to earlier investigations? I will approach this question by distinguishing four possible influences: (i) mechanical inquiry; (ii) technological advances; (iii) mathematical astronomy; and (iv) the inquiry into nature.

We are told by Diogenes Laertius that Archytas 'was the first to bring mechanics to a system by applying mathematical principles; and he first employed mechanical motion in a geometrical construction, namely, when he tried, by means of a section of a half-cylinder, to find two mean proportionals in order to duplicate the cube.'[70] None of Archytas' writings

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[68] This appears in the same line as the title Περὶ τῶν μεταλλικῶν in manuscripts B and P. Some editors have emended to Περὶ τῶν μεταλλικῶν μηχανημάτων. The more elaborate arguments of Gottschalk 1965 for Stratonian authorship have been effectively dismantled by Bodnár 2011a.

[69] De Groot 2014 (49) suspects that *Problems* 16 is earlier since *Mech.*'s arguments are more detailed and involve innovative notions such as 'constraint'. These observations are pertinent but hardly conclusive; a relative dating of *Mech.* and *Problems* 16 may not be possible. *Mech.* is the earliest to describe itself as concerned with 'mechanics', a term not used in *Problems* 16, which is entitled 'ΟΣΑ ΠΕΡΙ ΤΑ ΑΨΥΧΑ' ('Concerning inanimate things')

[70] Οὗτος πρῶτος τὰ μηχανικὰ ταῖς μαθηματικαῖς προσχρησάμενος ἀρχαῖς μεθώδευσε καὶ πρῶτος κίνησιν ὀργανικὴν διαγράμματι γεωμετρικῷ προσήγαγε, διὰ τῆς τομῆς τοῦ ἡμικυλίνδρου δύο μέσας ἀνὰ λόγον λαβεῖν ζητῶν εἰς τὸν τοῦ κύβου διπλασιασμόν. *Lives of Eminent Philosophers* 8.4.83, with Kühn's emendation μαθηματικαῖς for μηχανικαῖς (trans. Hicks with modifications). Vitruvius *De Arch* 7, praef.14 and Athenaeus Mechanicus (cited above) both mention Archytas as having contributed to mechanics but do not identify specific achievements.

survive intact. Our knowledge of mechanics in the fourth century BCE is dependent on later reports which require careful handling. In addition to textual problems (did Diogenes write 'mathematical principles' or 'mechanical principles'?), and the question of our sources' reliability, there is the issue of what 'mechanics' meant. Derived from the noun μηχανή ('trick', 'contrivance', 'plot', 'machine'), the terms μηχανική (τέχνη) and τὰ μηχανικὰ could cover a diverse set of activities and were understood differently by different authors.

Some examples will illustrate the point. Philo of Byzantium's *Mechanical Collection* included books on levers, port construction, catapults, pneumatics, theatrical automata, fortification, siege warfare, and military stratagems.[71] The fifth-century CE Platonist Proclus says that mechanics includes the making of war machines; the creation of marvels through air, weights or ropes; the study of equilibria and centres of weight; sphere construction (i.e. making models of the heavens); and 'in general every art concerned with the moving of material things'.[72] Hero of Alexandria took a radical view on the constituent 'parts' of mechanics: 'The mechanicians around Hero say that there are a discursive and a manual part of mechanics; the theoretical part is composed of geometry, arithmetic, astronomy and discourses about nature, the manual part of work in metals, architecture, carpentry and painting and of manual practice in these.'[73] 'Mechanics' in antiquity had no unifying essence or definition and authors took widely differing views on what the various parts of the science were.

What did Diogenes mean when he said that Archytas had introduced mathematical principles to mechanics? The slipperiness of the term 'mechanics' makes it impossible to answer with certainty, but we can still consider the possibilities. He may have had in mind Archytas' solution of the problem of doubling the cube.[74] This achievement was doubly 'mechanical'. In the first place, the use of motions in geometrical constructions (e.g. rotating a plane figure to construct a solid) could be seen as 'mechanical'.[75] Secondly, methods of cube duplication

[71] Following the reconstruction of Drachmann 1963a.
[72] Proclus *Comm. in Eucl.* 41, trans. Morrow.
[73] Pappus *Collectio* 8, 1023.13-1024.2 Hultsch, trans. Cuomo 2000, 93
[74] Archytas' solution is preserved in Eutocius' *Commentary on Archimedes' On the Sphere and Cylinder 2*, 3.84.12–88.2 Heiberg. See Huffman 2005, 342–401 and Menn 2015 for commentary. In Diogenes, a καὶ separates the report on the problem from the claim that Archytas systematised mechanics; this may or may not be epexegetic.
[75] Diogenes Laertius says Archytas used 'mechanical motions' to construct solid shapes.

were of interest to ancient writers on the construction of war machines since one practical context in which the problem arose was the scaling-up of model machines.[76]

An alternative possibility is that Diogenes or his source meant to refer to Archytas' investigations of motion, but here we have only a few brief and ambiguous references to go on. Archytas reportedly claimed that motion is caused by inequality,[77] and that there is a 'proportion of equality' present in natural motion that produces circles and curves.[78] We also know that Archytas compared the effect on acoustic pitch of striking or blowing an instrument with varying intensity to the effect of varying strength on projectiles' motion. The stronger the throw, the further the distance travelled; similarly, the stronger the breath, the higher the pitch.[79] Whether these oddments are samples of a unified theory of motion is unclear and there are various possibilities for filling in the details.[80]

On any interpretation, Archytas' attempts to explain motions in terms of quantitative relations such as inequality and proportion may have been seen as putting mechanics on mathematical principles. We should not assume, however, that Archytas would have recognised a distinct inquiry by the name of 'mechanics' (μηχανική). When Archytas surveys the mathematical disciplines, mechanics and optics are not mentioned: 'Indeed concerning the speed of the stars and their risings and settings as well as concerning geometry and numbers and not least concerning music, they handed down to us a clear set of distinctions. For these sciences seem to be akin.'[81] It is possible that Archytas considered mechanics and optics as parts of geometry but there is no reason to think he distinguished them as disciplines in any way, even as sub-disciplines.[82]

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[76] Philo (*Belopoeica* 51-52) and Hero (*Belopoeica* 114-119) present methods of cube-duplication in the context of catapult construction; cf. Pappus *Collectio* 8 1028. 18–21. Eratosthenes stressed the value of practical utility in cube duplication methods (see below).

[77] Eudemus fr. 60 Wehrli = Simplicius *In Phys*. 3.2, 431.4-431.16. We do not know if Archytas connected this to the balance.

[78] ps-Aristotle *Physical Problems* 16.9 asks why trunks and stems of plants and limbs of animals are round in shape. The answer (915a25-32): 'Is it, just as Archytas said, because the proportion of equality (τὴν τοῦ ἴσου ἀναλογίαν) is present in natural movement (for he said that all things are moved in proportion), but that this proportion alone bends back on itself (ταύτην δὲ μόνην εἰς αὑτὴν ἀνακάμπτειν), so as to make circles and curves (κύκλους… καὶ στρογγύλα), whenever it comes to be.' (trans. Mayhew)

[79] fr. B1.

[80] See Krafft 1970 144-146; Huffman 2005, 78n.13, 516-40; and De Groot 2014, 195-213.

[81] Porphyry, *On Ptolemy's Harmonics* 1.3 (trans. Huffman) περί τε δὴ τᾶς τῶν ἄστρων ταχυτᾶτος καὶ ἐπιτολᾶν καὶ δυσίων παρέδωκαν ἁμῖν διάγνωσιν καὶ περὶ γαμετρίας καὶ ἀριθμῶν καὶ οὐχ ἥκιστα περὶ μουσικᾶς. ταῦτα γὰρ τὰ μαθήματα δοκοῦντι ἦμεν ἀδελφεά.

[82] Apuleius (*Apologia* 11-15) attributes to Archytas the optical theorem that the angle of incidence and angle of reflection are equal. Huffman 2005 and Schofield 2014, 85-87 doubt the reliability of such late reports that

Our next piece of evidence for the development of mechanical theory in the fourth century comes from the first-century BCE Epicurean Philodemus. According to Philodemus, Plato oversaw advances in mechanics as well as other mathematical sciences in his Academy:

> 'At this time the mathematical sciences (τὰ μαθήματα) were also greatly advanced, with Plato being the architect of this development; he set problems for the mathematicians, who in turn eagerly studied them. In this way, *metrologia* (the theory of proportions?) and research on definitions reached their peak, as Eudoxus of Cnidus and his students completely revised the old theory of Hippocrates of Chios. Especially great progress was made in geometry, as the methods of analysis and of diorismos were discovered. Optics and mechanics also were not (left in contempt)...'[83]

For all his interest in mathematics, Plato never refers to a science of mechanics – or optics, for that matter.[84] To what activities, then, might Philodemus be referring? Methods of cube duplication are again a possibility. In fact, the problem of cube duplication was closely associated with the Academy by some writers. Eratosthenes and Plutarch tell a story in which the people of Delos were instructed by an oracle to double the size of one of their altars and turned to the Academy for help (this is why the problem is sometimes called 'the Delian problem').[85] Although this story is probably a later fabrication, there were three solutions associated with the early Academy: a solution by means of a specially designed instrument falsely ascribed to Plato, a solution by Eudoxus, and Menaechmus' (c.380 – 320 BCE) solution by means of conic sections.[86]

As with Archytas, the study of motion is a second field of inquiry that may have been perceived as contributing to mechanics. In the Academy, this took the form of the general

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Archytas made significant contributions to the nascent sciences of optics and mechanics, whereas Burnyeat 2005 is willing to believe them.

[83] Herc. Pap. 1021, column Y. trans**.** Zhmud, 1998**;** cf. Gaiser 1988, 152.

[84] Burnyeat 2005 suggests the omission is deliberate and polemical.

[85] Eratosthenes' 'Letter to Ptolemy' in Eutocius' commentary on Archimedes' *On the Sphere and the Cylinder* (Heiberg 1881, vol.3, 102-114, with the Delian story at 104.17-106.1); Plutarch *On the E at Delphi* 386. For broader discussion of Eratosthenes' letter, see Taub 2008 and Leventhal 2017.

[86] On these three solution methods, and against the Platonic provenance of the method that bears his name, see Knorr 1986, 57-66. Netz 2003, 500-509 suggests that the attribution of a duplication method to Plato was a response to a deliberate mistake in the Divided Line analogy of *Rep*. 6, 509d–511e. Broadie 2020, 17n.6 claims there is no error in the Divided Line.

analysis of change, the classification of various kinds of spatial motion, some interest in their properties, and the investigation of the results of composing these motions.[87] The last of these is most pertinent to the *Mechanica* but our most detailed evidence concerning the composition of motions in the fourth century relates to models of planetary motion. It seems unlikely that these models as such would have been regarded as part of mechanics rather than astronomy and I postpone discussion for the moment.

The simplest Platonic classification of motion, common to the *Theaetetus* (181b8-d7) and *Parmenides* (138b-c), distinguishes motion from one place to another (χώραν ἐκ χώρας) from movement in the same place by rotating about a fixed centre or central axis (ἐν τῷ αὐτῷ στρέφηται).[88] In Book 10 of the *Laws*, Plato makes a further distinction between two kinds of motion from one place to another (ὅσα φορᾷ κινεῖται μεταβαίνοντα εἰς ἕτερον ἀεὶ τόπον): translation (καὶ τοτὲ μὲν ἔστιν ὅτε βάσιν ἑνὸς κεκτημένα τινὸς κέντρου, 'having a base of one point'), and rolling (τοτὲ δὲ πλείονα τῷ περικυλινδεῖσθαι).[89] The *Laws* again distinguishes from these rotation (ἡ τῶν ἑστάναι λεγομένων κύκλων στρέφεται περιφορά) and, importantly, observes a characteristic property of rotations:

> Those which have the quality of being at rest at the centre move in one location, as when the circumference of circles that are said to stand still revolves… And we perceive that motion of this kind, which simultaneously turns in this revolution both the largest circle and the smallest, distributes itself to small and great proportionally, altering in proportion its own quantity; whereby it functions as the source of all such marvels as result from its supplying great and small circles simultaneously with harmonizing rates of slow and fast speeds—a condition of things that one might suppose to be impossible.[90]

[87] On these topics, see Skemp 1942 and 1967, Post 1943, Mourelatos 1981.
Post's claim that 'The motion of a moving wheel was no doubt studied by Heracleides Ponticus' (1943, 301) relies on the assumption that Heraclides thought Venus encircled the Sun. That interpretation was first proposed by Martin 1849, 120-121, 426-428 and developed by Schiaparelli. It is based on a misreading of Calcidius' commentary on the *Timaeus* (Neugebauer 1972: Calcidius' text is obscure but more likely means that Venus is sometimes ahead of and sometimes behind the Sun). On Heraclides, see also Heath 1913, 249-83 and the contributions by Bowen, Todd and Keyser in Fortenbaugh and Pender 2009.
[88] Cf. *Rep*. 4, 436d4-e6 on the spinning top.
[89] The distinction between rotating and rolling was adopted by Aristotle *DC* 2.8. *Mech*. problem 24 is a detailed study of rolling; see Chapter 6. Whether this puzzle owes anything specifically to the Academic study of motion is currently impossible to say.
[90] *Laws* 10, 893c-d, trans. Bury.

This is the earliest known articulation of the Rotating Radius Principle that is so crucial to *Mech.*'s explanatory programme. The claim that this property is responsible for 'all such marvels' resembles the emphasis on θαῦμα in *Mech.*'s introduction. The *Laws* does not say much about the other crucial idea in *Mech.* problem 1, the composition of motions. There is a brief comment on what happens when bodies moving in opposite directions collide:

> And whenever one such object [= something moving from place to place] meets another, if the other is at rest, the moving object is split up; but if they collide with others moving to meet them from an opposite direction, they form a combination which is midway between the two (τοῖς δ' ἄλλοις ἐξ ἐναντίας ἀπαντῶσι καὶ φερομένοις εἰς ἓν γιγνόμενα μέσα τε καὶ μεταξὺ τῶν τοιούτων συγκρίνεται.)[91]

It is not difficult to see how such investigations of motion could have been construed as contributions to 'mechanics', and they may form part of the conceptual background to *Mech.* problem 1. In the *Timaeus,* Plato discusses other topics that could be associated with mechanics such as weight and projectile motion.[92] Even so, I suspect the solutions to the problem of cube duplication are more likely what lie behind Philodemus' reference to 'mechanics'.[93]

In Aristotle's works we encounter, at last, references to a field of inquiry called 'mechanics'.[94] In the *Posterior Analytics*, mechanics and optics are named as sciences subordinate to geometry, just as harmonics is subordinate to arithmetic.[95] Working this out in more detail, Aristotle says that optics is subordinate to (plane) geometry and mechanics to solid geometry (*stereometria*).[96] The reference to solid geometry may suggest that Aristotle had taken cube duplication as a paradigm of mathematical mechanics.[97]

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91 *Laws* 10, 893e, trans. Bury.
92 *Timaeus* 63b-c, 80a-c.
93 Cube duplication is a recurring concern in Hellenistic writers on mechanics, whereas the emphasis on motions, their path shapes, and their compositions is strong in *Mech.* and weaker in other authors. Plato's theories of weighing and projectile motion are qualitative.
94 Besides these one should note *Pol.* 7.11 1331a1-2, 14 mentioning recent advances in missiles and siege engines (τὰ βέλη καὶ τὰς μηχανὰς εἰς ἀκρίβειαν πρὸς τὰς πολιορκίας) and *Pol.* 1336a10-12 describing the use in some societies of *mekhanika organa* for straightening childrens' limbs.
95 *APo*. 1.9, 76a16-25. Demonstrations in subordinate sciences make use of principles from the sciences to which they are subordinated.
96 *APo*. 1.13, 78b32-39.
97 For cube-duplication in mechanical works, see Hero *Bel.* 114-9, *Mech.* 1.11, Philo *Bel.* 51-2 with 56 on transferring from scale models to full-size machines, Pappus *Coll.* 8, 1070.1ff. (also 3, 30.3-68.16).

Although it is unknown when the term 'mechanics' was first used to denote a mathematical science, we have seen that there were two main strands of mathematical mechanics in the fourth century BCE: efforts to duplicate the cube and studies of spatial motion.[98] It is the latter, including earlier formulations of the Rotating Radius Principle, that is pertinent to *Mech*. Non-mathematical ideas about the operation of machines are another important part of the background, though our information is limited. The Hippocratic *On Fractures* 31 (late fifth / early fourth century BCE), describing the use of levers for reducing fractures, groups levers together with pulleys and wedges on the basis that they allow people to accomplish tasks otherwise difficult or impossible.[99] This is similar to the account of μηχανή we have seen in *Mech*.'s introduction.[100]

Technological change may have been one source of stimulus for the development of mathematical and non-mathematical thought about machines in classical Greece.[101] Architectural projects, particularly the construction of large temples, may have involved the organised use of weight-lifting devices as early as the sixth century BCE.[102] In discussing the pulley, *Mech*.'s author refers to its use 'in construction work'.[103] Later, the invention of the catapult, traditionally dated to 399 BCE in Syracuse, may have encouraged reflection on how machines work. Catapults of various designs quickly spread across the Greek world and there were deliberate efforts to improve on older models and develop new ones.[104] By the mid-fourth century, the new war machines were familiar enough that references to them and to

[98] To some extent the two may have influenced each other (cf. the motions in Archytas' cube duplication).
[99] 'Of all the apparatus contrived (ἅρμενα μεμηχάνηται) by men these three are the most powerful (ἰσχυρότατα) — the turning of the pulley, use of a lever, and use of a wedge (ὄνου τε περιαγωγὴ καὶ μόχλευσις καὶ σφήνωσις). Without some one, indeed, or all of these, men accomplish no work requiring great force. This lever method (ἡ μόχλευσις), then, is not to be despised, for the bones will be reduced thus or not at all.' Trans. Withington with modifications. The singular ὄνου suggests a simple pulley (from our perspective, this does not strictly offer mechanical advantage as such, though it makes the task of lifting easier by changing and steadying the direction of pull). Compare *On Joints* 72-74: 'It seems to me that no joint is incapable of reduction with these mechanical forces.' (trans. Withington). Bliquez 2015, 40, 202-205 discusses ancient textual sources on bone levers. Jackson 2005, 110-111 summarises the Roman archaeological evidence.
[100] On the other hand, the Hippocratic author does not arrange the simple machines in a hierarchy as in *Mech*.
[101] Cuomo 2007, ch.3 offers several important suggestions regarding the catapult's impact on knowledge. See also Schiefsky 2009, 49-50.
[102] Coulton 1974 and 1977.
[103] Problem 18, 853b10-11: καὶ ἐν τοῖς οἰκοδομικοῖς ἔργοις ῥᾳδίως κινοῦσι μεγάλα βάρη.
[104] For various reconstructions of the early history of the catapult, see Marsden 1969; Garlan 1974; Cuomo 2007, ch.3; Rihll 2007; on later catapults, Baatz 1994.

specialists in their manufacture and operation could be found in a variety of sources, including philosophical texts.[105]

We have noted the connection between catapult-construction and the problem of cube duplication, but the invention of the catapult and the *formulation* of the geometric problem are not to be related as cause and effect. The problem predates the invention of the catapult, and Hippocrates of Chios had by the late fifth century already made some headway by reducing the problem of cube duplication to the problem of finding two mean proportionals. Still, invention of the catapult may have encouraged the ongoing search for duplication methods. Practical utility was claimed to be a desideratum for a duplication method by Eratosthenes (c.285 – 194 BCE), who criticises the solutions of Archytas, Eudoxus and Menaechmus as less practical than his own method.[106] How these earlier geometers saw their solutions in relation to war machine construction is unknown, but there is no reason to think they would have shared Eratosthenes' negative judgement of their own achievements. Aside from pressing in new directions the old problem of cube duplication, the catapult may, as a particularly striking example of the increased power available through machinery, have prompted reflection on the causes of amplified power through machines.

Our earliest texts on mechanics, *Mech*. and *Problemata* 16, do not mention catapults. It is clear from the chronology and wide diffusion of catapults, as well as from references to catapults elsewhere in the Aristotelian corpus, that the lack of references to advanced technology in *Mech.* is an authorial choice, not the result of ignorance.[107] Perhaps it was thought appropriate to work out the principles of the simplest machines before attempting to understand the complex. At the same time, the author's theoretical orientation may have meant his interest was in answering foundational questions only. [108] Whether Hellenistic

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[105] Plato *Gorgias* 512b refers to the μηχανοποιός who sometimes saves cities (πόλεις γὰρ ἔστιν ὅτε ὅλας σῴζει); Aristotle *EN* 1111a10-11 uses the example of someone accidentally firing a catapult (δεῖξαι βουλόμενος ἀφεῖναι, ὡς ὁ τὸν καταπέλτην); ps-Ar *Constitution of the Athenians* 42.3 refers to teachers (διδασκάλους) of catapult-firing (Rhodes 1992 *ad loc*. cites later epigraphic evidence for specialist teachers); there is a reference to catapults at ps-Aristotle *On Things Heard* 880b13-14.

[106] Eratosthenes (= Eutocius, 106.1-8, 112.19-114.2 Heiberg) says his method will be useful for those wanting to construct 'catapults and stone-throwing machines' (καταπαλτικὰ καὶ λιθοβόλα ὄργανα).

[107] Problems 12, 32 and 34 concern projectiles but do not mention the catapult. Later Hellenistic writers on mechanics, such as Hero and Biton, did not always feel the need to describe the most recent technology (see Cuomo 2002 and Rihll 2007, 142, 164-65 for different explanations).

[108] *Mech.*'s introduction celebrates the practical advantages of μηχανή, but at the same time the emphasis on θαῦμα (wonder) and the search for causes may signal that the investigation is theoretical rather than practical (cf. *Met.* A.2, 982b11-21, 983a11-23).

mechanical theory had any appreciable impact on the technology of the Hellenistic and Roman periods is an interesting question that I will not attempt to address.[109]

A second invention may have shaped the development of mechanical theory: the steelyard, or balance with unequal arms.[110] There are two main varieties. The so-called 'Roman steelyard' has a scale-pan on one arm, usually very short, and a sliding weight on the other, usually much longer; the arms are separated by a fixed pivot. The so-called 'Danish steelyard' or 'bismar' has a scale-pan on one arm, a fixed weight on the other, and a sliding pivot. In both types the balance-beam is gradated so the user can calculate the difference made by the adjustment of the sliding weight or sliding pivot. The production and use of these gradations – in either type of steelyard – requires some understanding of how equilibrium of unequal weights results from their being positioned at precise distances from a pivot. In other words, the steelyard is closely related to the mechanical principle now known as 'the law of the lever', a precise statement of which is given in *Mech*. 3.[111] The invention of the steelyard in Greece may predate the *Mechanica* by more than a century.[112] It has been suggested that this technological development may have played a role in the discovery of the 'law of the lever'.[113] Given the lack of direct evidence for the discovery of the 'law of the lever', this is an interesting and entirely plausible but ultimately speculative hypothesis.[114]

I have already mentioned that the composition of motions was studied in fourth-century astronomy and now it is time for a more detailed discussion. The significance for *Mech*. is

[109] Schürmann 1991 argues that the impact of mechanical theory has been traditionally underestimated.
[110] The English term 'steelyard' is thought to derive from the Steelyard (*stalhof*) in London where Hanseatic merchants traded (*OED*, 'steelyard, n.1', 'steelyard, n.2').
[111] The term 'law of the lever' is potentially misleading. Greeks generally did not use our legal metaphor in describing natural regularities or mathematical generalities. And if the original context for the principle was the unequal-armed balance, the association with the lever is itself an intellectual achievement (cf. Berryman 2009, 64). The principle did not have had a standard short-hand name in antiquity. Knorr 1982a uses the more suitable term 'general principle of equilibrium'. Micheli 1995, 83 dissents from the consensus that *Mech*. problem 3, 850b1-2 states the law of the lever.
[112] Aristophanes seems to refer to a steelyard of the 'Danish' type in a play first performed 421 BCE (*Peace* 1240-49); the device was presumably well-known to the Athenian public that made up the audience (cf. Schiefsky 2009, 46).
[113] See in particular Heidel 1933, 62; Renn and Schemmel 2000; Büttner and Renn 2016, Renn and McLaughlin 2018. *Mech*. problem 20 aims to explain a kind of steelyard referred to as a φάλαγξ, but the description does not precisely match the Danish/bismar model. Instead of a sliding pivot, the device described seems to consist of a bar with fixed strings attached at certain intervals; suspended by any of these and so there is a fixed number of possible pivots (cf. Knorr 1982a, Appendix A).
[114] The very fact that the 'law of the lever' is not attributed by any Greek or Roman source to a specific person could be taken to suggest that its origins lay among practitioners (see Damerow et al. 2002). Some clarification might be gained – though definitive confirmation or disconfirmation of the hypothesis seems unlikely – through further research on the history of weighing devices. The most comprehensive history is still Ibel 1908.

obvious: *Mech*.'s explanations are based on an account of what happens when a single thing undergoes two motions simultaneously.[115] Our evidence does not allow us to determine when and for what purpose Greek thinkers first explicitly recognised and studied the composition of motions, but this notion clearly played a major role in mid-fourth century models of planetary motion.[116]

Plato's *Timaeus* contains a simple example of this programme: the uniform rotation of the sphere of fixed stars from East to West (the Circle of the Same) is composed with a rotation along the zodiac from West to East (the Circle of the Different) to yield the planets' lagging behind against the background of fixed stars.[117] A more elaborate model is attributed to Eudoxus of Cnidus (c.390–c.340). Our most detailed report comes from Simplicius, writing about nine centuries later. Although the aims and technical details of Eudoxus' model are disputed, it must have involved some understanding of what results when multiple uniform circular motions are combined.[118] In particular, Eudoxus showed that a figure-of-eight shape called the *hippopede* ('horse-fetter') could be produced by composing uniform rotations of three spheres. If the investigation of composed motions had reached that stage by the mid-fourth century, it seems likely that the simpler 'parallelogram rule' had in some form been stated or recognised as an assumption.[119]

Finally, we come to *Mech.*'s relation to earlier investigations of nature. I will argue in this thesis that *Mech*. is more closely engaged with natural inquiry than has previously been recognised. This engagement is apparent in the form that *Mech*.'s questions take, laying stress on the challenge posed by the phenomena to a basic principle of causation that was a

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[115] Particularly in problem 1, but also in problems 8, 23 and 24.
[116] In geometry, special curves introduced for solving problems were sometimes defined by composing two or more motions. This practice is certainly attested from the later third century BCE, but cannot be reliably traced back further. It was traditionally maintained that the quadratrix, a curve that can be used to trisect the angle and square the circle, was introduced by the sophist Hippias of Elis in the fifth century BCE, since Proclus calls it 'the quadratrix of Hippias' (*Commentary on Book 1 of Euclid's Elements*, 272 Friedlein), but Knorr (1986, 80-86) has argued that the quadratrix was likely not introduced until the Hellenistic period.
[117] Among the many studies of Plato's accounts of the structure of the heavens, see Dicks 1970, ch.5, Mourelatos 1980, Knorr 1990.
[118] Aristotle *Met*. Λ.8 reports that he assigned a specific number of homocentric spheres to each heavenly body. Our fullest report is due to Simplicius' commentary on *DC* 2.13 (*In De Caelo 492.31 ff.*) Eudoxus' fragments are collected by Lasserre 1966. Among the various reconstructions of Eudoxus' model, see Mendell 1998 and 2000, Yavetz 1999.
[119] Plutarch *Marcellus* 14.5-6, an unreliable source that must be handled with care, claims that the art of mechanics was founded by Archytas and Eudoxus, but when it comes to examples, Plutarch discusses cube duplication rather than the study of motions. Some authors saw a special connection between mechanics and astronomy, notably Vitruvius *De Architectura* 10.4, and a certain Anatolius as reported in ps-Hero *Definitiones*, chapter 38.7 (164.21 - 166.3 Heiberg).

rare point of agreement among Greek investigators of nature, namely that an effect cannot be greater than its cause.[120] If my muscles are too feeble to lift a large rock, how am I able to achieve this by using a lever or a pulley? The input strength is still the same, but the effect is greater. And the longer the lever, the greater the potential effect. A remark on the lever's amplification of causal power is attributed to Archimedes: 'Give me a place to stand and I will move the Earth.'[121] But it did not take Archimedes to ask how such amplification should be possible.

The principle that a cause cannot exceed its effect may date back as far as Xenophanes.[122] It features in Plato's introduction of Forms as causes in the *Phaedo* (99d-102a). Aristotle's two most explicit statements of the principle are not easily applicable to changes of place,[123] but he can elsewhere be seen to assume that extension.[124] Mechanical phenomena form a class of apparent counterexamples to the principle.[125] *Mech.*'s introduction proposes to explain cases 'in which the lesser overpower the greater, and things with small preponderance move great weights'.[126] The problems repeatedly emphasise the smallness of the efficient cause compared to the magnitude of the effect. For example, Problem 5 asks 'Why does the rudder, which is small and at the end of the ship, have so great a power that the great magnitudes of ships are moved by a small tiller and the capacity – and even this is slight – of a single person?' For all its technical and mathematical details, *Mech.*'s inquiry is worth the journey

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[120] On this principle see in particular Lloyd 1976.
[121] Cf. Dijksterhuis 1958, 14-17.
[122] It is attributed to Xenophanes (late sixth – early fifth century BCE) by the pseudo-Aristotelian *De Xenophane*. The argument attributed to him, that whatever exists cannot have come to be, assumes as a premiss that the cause of something's coming to be must be at least as great as the thing generated; cf. Barnes 1979, 88-89.
[123] *A.Po*. 72a29-30 and *Met*. α, 993b24-26.
[124] It is assumed in the argument of *Phys*. 7.1 that if one movement results from another, then the first must be at least as great as the second (242a47-50). It is arguably implicit in *Phys*. 7.5's assumption that a given δύναμις has a proportionate effect. See also *DC* 1.11, 281a6ff. on the finitude of a given power's maximum effect.
[125] This may be a reason why some have doubted *Mech.*'s authenticity (cf. Knorr 1982, 101n.27: 'Aristotle views a force as somehow inherent in the weight of a body and so naturally limited… a view clearly devoid of the mechanician's insight that force may be multiplied indefinitely via mechanisms.'; Berryman 2009, 187-191). I think Knorr goes too far in suggesting Aristotle was unaware of the effects of mechanisms, even if he did not account for them in the *Physics.* Aristotle mentions the lever at *Physics* 8.4, 255a21-22 and 8.6, 259b18-20 (cf. *MA* 7, 701b24-28) and we have seen the basic idea of mechanical advantage as early as the Hippocratic *On Fractures*. Berryman, following Fleury 1993, 138, sees the legend of Archimedes' ship-hauling as a polemical allusion to Aristotle's alleged ignorance, citing *Phys*. 7.5, 250a16-19, but the story may rather recall Odysseus' use of μοχλοί to bring his raft to the water (*Odyssey* 5.261).
[126] ἐν οἷς τά τε ἐλάττονα κρατεῖ τῶν μειζόνων, καὶ τὰ ῥοπὴν ἔχοντα μικρὰν κινεῖ βάρη μεγάλα. As noted by Vernant 1957, this is comparable to Protagoras' characterisation of rhetoric as the art of making the weaker argument the stronger (Aristotle *Rhet.* 2.24, 1402a24 = DK80 B6b: τὸ τὸν ἥττω δὲ λόγον κρείττω ποιεῖν τοῦτ᾽ ἔστιν) and Aristophanes *Clouds* 112-115.

for any inquirer into nature, since defusing these problems will fortify the basic account of causation.

A further way in which the inquiry into machines complements the inquiry into nature is by accounting for some important models of motion.[127] Aristotle frequently refers to ships and their parts for the sake of illustration or analogy. Birds' tails 'are for directing the flight, like rudders in ships';[128] a bird's breast is 'sharp so it is strong (εὔτονος) like the prow of a light ship… so it can drive apart the air it encounters';[129] insects do not have tails and so each 'moves like a rudderless ship';[130] they use their small and weak wings 'like a cargo ship trying to make its journey by means of oars'.[131] Tiny motions within an animal are what produce the perceptible movements of its parts: 'a small change occurring in the origin [of movement] produces many great changes further away; just as, when the tiller is moved only momentarily, a great motion of the prow results.'[132] So long as there remains any mystery about how ships are able to achieve these motions, the explanatory power of these analogies will be obviously limited. Conversely, by explaining the operation of ships, *Mech*. (especially problems 4-7 which focus on ships and their parts) may strengthen and enrich attempts to understand animal motion.[133]

To summarise, several references to mechanical investigations in the fourth century BCE are likely about solutions to the geometrical problem of doubling the cube, which was important for the construction of war machines. Military engineering was an important part of mechanics throughout antiquity.[134] Cube duplication is not a concern in the *Mechanica*, which focusses on the explanation of simple machines, not war machines. Arguably, complex military machinery should still be seen as part of the background for any writer on 'mechanics' in the early Hellenistic period. Further, as we have seen in Philo, accounts of simple machines such as the lever could be applied to individual parts of complex machines,

[127] See especially Bodnár 2004 on the relation of *Mech*. to *MA* and *IA*.
[128] *IA* 10, 710a1ff.
[129] *IA* 10, 710a30ff.
[130] *IA* 10, 710a8ff.
[131] *IA* 10, 710a18-20. The comparison of wings and oars was traditional (*Od*. 11.125 = 23.272; Hesiod *Op.* 628).
[132] *MA* 7, 701b25-28. There are many more instances. For example, *Phys* 2.2 distinguishes knowledge of matter from knowledge of form by contrasting the craftsman's knowledge of the rudder, which is limited to its matter, to the helmsman's knowledge, which is of the appropriate form for a rudder.
[133] Note that *Mech*. problem 30 does tackle a problem about animal motion ('Why do people stand up by making an acute angle between the lower leg and the thigh and between the thigh and the trunk?').
[134] The list of seven μηχανικοί in column 8 of the *Laterculi Alexandrini* names military engineers only (Diels 1904).

such as catapults. But simple machines were also of interest in themselves, and it is likely that the *Mechanica* built on earlier discussions that do not survive. The Hippocratic *On Fractures* classified the turn of the pulley (ὄνου περιαγωγὴ), 'levering' (μόχλευσις) and 'wedging' (σφήνωσις) as three powerful tools that allow people to do what was otherwise impossible, a conception echoed by *Mech.*'s preface. *Mech.*'s author takes a wider though less clearly defined view of the tools that provide mechanical advantage, and he goes further than the Hippocratic writer in his desire to explain and unify the power of these devices. Both his questions and answers are clearly influenced by Aristotle's investigations of nature. The style of explanation pursued in problem 1 probably has significant debts to fourth-century investigations of composed motion, about which we know relatively little.

1.4: Debts to earlier scholarship

The modern interpretation of *Mech.* can be said to begin with Pierre Duhem who presented the arguments of *Mech.* problems 1-3 as deriving the 'law of the lever' from proportionalities found in Aristotle's *Physics*, especially 7.5.[135] This reading held sway over many twentieth-century historians of science, despite Carteron's early objections.[136] The leading study in the Duhemian tradition, Krafft 1970, interprets ἰσχύς as a notion of "effective weight"[137] and further argues that *Mech.* presents mechanics as a 'tricking of nature' ('Überlistung der Natur').[138] François De Gandt (1982) decisively criticised this approach by showing that the language and ideas of *Phys.* 7.5 cannot in fact be found in *Mech.* problem 1.[139] Duhem's reconstruction is a valid deduction, but it goes far beyond anything actually asserted in the text.

If Duhem's approach was misguided, what should replace it? The most detailed alternative so far is due to Jean De Groot.[140] De Groot draws attention to the role of the Rotating Radius

[135] Duhem 1906, vol.1, 5, 109, 357; vol.2 291-3. On Aristotle's proportionalities, see Note B at the end of this chapter.
[136] For example, Clagett 1959, Krafft 1970, Marsden 1971, 175, and Knorr 1982a accept the main elements of Duhem's interpretation. Carteron 1923, cf. Carteron 1975. Drabkin 1938, n.30 expressed reservations: 'Duhem has shown that the law of the lever as stated… is deducible from the dynamical formulations discussed above… It may be doubted whether Aristotle himself made this precise deduction.'
[137] Thus, on Krafft's interpretation, *Mech.*'s ἰσχύς becomes an anticipation of Jordanus Nemorarius' *gravitas secundum situm*.
[138] See also Krafft 1973. Vernant 1957 saw a 'combat de la *technê* contre le *phusis'* in *Mech.*'s preface. Wardy 2005, 83 finds in *Mech.* 'a refreshingly *anti*-natural, as it were "heroic" conception of τέχνη'. Hadot 2004, ch.10 sees mechanics as a kind of trickery. This line has been effectively criticised by Schiefsky 2007.
[139] De Gandt 1982.
[140] See De Groot 2009 and 2014. Schiefsky 2009 reaches similar conclusions on some points.

Principle in *Mech.*'s explanations of the balance and lever. In her view, this principle is explained through an analysis of circular motion that is kinematic, i.e. it emphasises the geometrical properties of motion to the exclusion of causal factors.[141] De Groot also undertakes a sustained comparison of *Mech*. problem 1 and *Problems* 16.[142]

The studies mentioned so far have primarily been concerned with the explanations of the balance and lever (problems 1-3). The preface and the remaining thirty-two problems have attracted less attention. Mark Schiefsky (2009) has studied aspects of their argumentative strategies. Michael Coxhead (2012) has argued that the quotation from Antiphon in *Mech.*'s preface brings out similiarities between poetry and mechanics as *technai*. John Anders (2013) has suggested connections with Aristotle's comments on 'problems' in the *Posterior Analytics* and noted a possible pun on ἀφίημι in problem 32. Monte Johnson (2017) has raised important questions about the method of problems 4-22 and argued that they follow the ideal of demonstration laid out in Aristotle's *Posterior Analytics*. He has also convincingly shown that *Mech.*'s explanations are consistent with teleology. Christopher Frey has noted shared methods and assumptions between *Mech*. problem 30 and the authentically Aristotelian *IA* 9, 708b26–709a7.[143]

There has been remarkably little analysis of *Mech.* problem 24, the so-called 'Wheel of Aristotle'. Drabkin 1939 remains the leading study; a recent paper by Dosch and Schmidt (2018) defends the solution offered in *Mech*.

Van Leeuwen (2016) has recently produced a new *stemma codicum* and a critical edition of all diagrams, based on a new examination of the manuscripts. At the same time, she has shown that the archetype of the diagrams is Byzantine, so the edition cannot be used as evidence for what the diagrams of the original text of the late fourth or early third century BCE, although it has value both as a corrective to modern assumptions about diagrams and a guide to the Byzantine reading of *Mech*. Diagrams in modern editions such as Van Capelle 1812 and Hett 1936 were based on the editors' ideas of what the diagrams *should* look like

141 De Groot makes several further arguments which I cannot address here. For example, she suggests that Aristotle's concept of *dunamis* arose from studying the *dunamis* of mechanical devices such as the lever, and she argues against G.E.L. Owen's 'endoxic' interpretation of Aristotle's *phainomena*.
142 Schiefsky 2009 reaches a similar general conclusion to De Groot on the essentially kinematic character of the *Mechanica*.
143 Frey 2021, 198-200.

rather than on the manuscript diagrams. The present thesis does not answer Van Leeuwen's call for a new edition, but I have borne her cautions in mind while relying primarily on Apelt's text.[144] Hett's Loeb translation is unreliable.[145] I have found Forster's 1913 translation more accurate and the accompanying notes are useful. Translations of *Mech*. in this thesis are my own, unless otherwise noted.

There is no full commentary in English. I have made use of two Italian commentaries.[146] Van Capelle's 1812 notes are still fundamental and I have found earlier commentators often insightful, in particular Monantheuil (1599), Guevara (1627), and Zucchi (1649).[147]

1.5: Chapter overview

At the start of this introduction, I distinguished three explanatory levels that operate in *Mech*. At the most superficial level ('Level 1'), in problems 4-22, explananda are shown to be analogous to a simple model (lever, balance, or moving radius). These simple models are in turn explained in problems 1-3 by the Rotating Radius Principle ('Level 2'). And at the foundation, the Rotating Radius Principle is explained through an analysis of what results from combining two motions in one subject ('Level 3'). The chapters of this thesis address these levels of explanation from the bottom up, starting with the composition of motions.

Chapter 2 argues that the two motions in the Level 3 explanation of the Rotating Radius Principle are not merely useful fictions but are rather distinct changes present in a rotating radius. That may sound improbable to modern ears. On several modern views of what motion is, a single body cannot literally have several simultaneous motions. By contrast, I argue, that notion makes sense for Aristotle. I show that a distinctive approach to composed locomotions arises from Aristotle's account of change in the *Physics*: component motions are real and

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[144] Van Leeuwen's chief concern is contamination by George Pachymeres' 13th century paraphrase of *Mech*. Guided by Van Leeuwen's analysis of the tradition, I decided that Apelt's text is sounder than the more recent Bottecchia 1982. An advantage of the latter, however, is its critical edition of the scholia on *Mech*.
[145] A few examples: on p.345 a gloss ('*i.e.*, that in which AX moves to AΘ') is not clearly separated from the translation; Hett's translation of problem 2, 850a12-14 does not match his punctuation of the Greek; p.353 has 'equal parts' for ἄνισα (850a34).
[146] Bottecchia Dehò 2000 and Ferrini 2010.
[147] Rose and Drake 1971 survey the Renaissance editions, translations and commentaries. On *Mech.*'s influence in the fifteenth and sixteenth centuries, see also De Gandt 1986, Laird 1986, 2001, Helbing 2001, Hattab 2005, Vilain 2008.

distinct. I then examine supporting evidence from across the *Physics*, *De Caelo*, *Meteorology* and *Metaphysics* when discussing the composition of motions.

Chapter 3 explores two further considerations that arise from Aristotle's statements on types of locomotion and their compositions. First, I consider how we should understand Aristotle's division of all motion into straight, circular and mixed. Then I explore the limits of the possible presence of distinct motions in a single object, through examining Aristotle's cliam that no contrary motions can be simultaneously present in a body.

Chapter 4 argues that problem 1's arguments draw on the resources of geometry to support a basically physical agenda and to deliver a causal explanation. The explanations here, as elsewhere in *Mech*., may be reasonably described as an application of the principles of natural inquiry to the technical sphere of mechanics. Drawing on my conclusions about composed motion in Chapters 2-3, I argue that problem 1's analysis targets radial rotation, which is distinguished from celestial circular motion by the simultaneous presence of two rectilinear motions in the rotating radius.

Chapter 5 studies the explanatory strategies of the less-studied problems 4-22, with a focus on their use of lettered diagrams and specialised language. I argue that these problems fundamentally rely on analogies. Thus, the method of *Mech.* cannot be straightforwardly identified with the method of demonstration outlined in Aristotle's *Posterior Analytics*. This chapter calls for an expanded view of the uses of diagrams in Greek science, beyond the deductive geometry of Euclid, Archimedes and Apollonius.

Chapter 6 examines the paradox of *Mech*. problem 24, which since the Renaissance has been known as the *Rota Aristotelis* ('Wheel of Aristotle'). This paradox challenges the claims made about rotations in problem 1 and thus threatens to overturn *Mech*.'s explanatory programme. The paradox is framed in geometrical terms, but I show that the author's aim is not to provide a mathematical explanation, as a comparison with Hero of Alexandria's discussion of the same paradox confirms. Rather, *Mech*.'s author draws two distinct puzzles from the paradoxical phenomenon and answers each of them with a solution based on physical principles.

Chapter 7 returns to Aristotle's works, arguing that *Physics* 7.4's startling claim that circular and rectilinear motions are incomparable may represent an earlier attempt to solve the *Rota Aristotelis* paradox. I criticise three alternative explanations of *Phys*. 7.4's claim and show how *Phys*. 7.4's argument would make sense as a response to the paradox.

I resist applying to ancient texts the modern division of mechanics into kinematics, statics and dynamics. The Greek appearance of these terms is misleading; they are modern coinages.[148] The Greek word δυναμικός means 'powerful', but its use in the name of a science – the *Scientia Dynamica* – is due to Leibniz.[149] There is no such Greek word as *κινηματικός, but there are several terms deriving from the verb κινεῖν ('to move'), such as κίνησις ('motion') and κινητικός ('movable'). It was from this stem that André-Marie Ampère's introduced the term *la cinématique* in his 1834 essay on the classification of the sciences.[150] 'Statics' has a stronger claim to antiquity since Plato refers to a στατική τέχνη (*Charmides* 166b, *Philebus* 55e), but by this he probably meant the art of determining objects' weights rather than a theoretical investigation of equilibrium conditions.[151] Several historians have nevertheless taken these distinctions as a starting-point for interpreting pre-modern mechanics. Clagett's remark on this approach is telling: 'In treating the content of medieval mechanics I have adopted the convenient but somewhat anachronistic division of mechanics into static, kinematics, and dynamics. Concepts and proofs important for all three of these divisions often appear during the Middle Ages in the same work and are intertwined one with another.'[152] Something similar could be said for antiquity.

[148] Meli, 2006, 8 has argued that these terms should not be used for pre-nineteenth century mechanics. Schiefsky 2009, 44-45 expresses a similar view.
[149] See especially his *Specimen Dynamicum* (1695).
[150] *Essai sur la philosophie des sciences, or Exposition analytique d'une classification naturelle de toutes les connaissances humaines*, vol.1, 50-53. The term *Phoronomie*, coined by Hermann in his *Phoronomia, sive De viribus et motibus corporum solidorum et fluidorum libri duo* (1716), was used by Kant in a roughly equivalent sense in his *Metaphysische Anfangsgründe der Naturwissenschaft* (1786) – though Hermann's original usage is closer to our dynamics. In a letter to *Nature* in 1892, the Cambridge mathematician W.H. Besant argued, against then-prevailing usage, that 'phoronomy' should be preferred to 'kinematics'; approving replies were printed in the next issue (*Nature* XLV, 462-63, 486). Incidentally, Besant's main reason for preferring 'phoronomy' was that over thirty years earlier 'the late Dr. Donaldson, a well-known Greek scholar of the time' (probably John William Donaldson, 1811-1861) had suggested to him that 'the word κινέω involved the idea of the cause of motion, and therefore that it ought not to be used when the idea of causation is to be completely set aside.'
[151] Elsewhere he uses other terms e.g. *Rep*. 10, 602d (ἱστάναι).
[152] Clagett 1959, xxiii.

Note A: A key to the problems of the *Mechanica*

1. Why are larger balances more accurate than smaller ones?	10. Why is the balance easier to move without a weight than with one?	19. Why is it that an axe does not cut wood when a weight is placed on top, but does when swung?	28. Why are swing-beams on wells made the way they are?
2. Why does a balance suspended from above return to its original position, but not one suspended from below?	11. Why are loads easier to carry by rollers than by carts?	20. How does a steelyard weigh heavy meat?	29. Why do two men carrying wood not feel the weight evenly?
3. Why do small powers move great weights through a lever?	12. Why do projectiles move further from a sling than from a hand?	21. Why do doctors remove teeth more easily with a tooth-puller than by hand?	30. Why must people form acute angle between lower leg and thigh and between trunk and thigh to stand up?
4. Why do rowers in the middle of the ship contribute most to its movement?	13. Why are larger spindles and windlasses easier to move?	22. Why do nutcrackers crack nuts more easily without striking them?	31. Why is it easier to move what's moving than what is at rest?
5. Why does the small rudder move the huge ship?	14. Why is it easier to break wood on one's knee when holding its ends than holding it near the knee?	23. The Rhombus Puzzle	32. Why do projectiles stop?
6. Why does a ship with a higher mast travel faster?	15. Why are pebbles round?	24. The 'Wheel of Aristotle'	33. Why does a body travel when the motive force doesn't follow it?
7. Why do sailors slacken the part of the sail near the bow in unfavourable wind?	16. Why are longer pieces of wood weaker?	25. Why are beds built six feet by three feet and why are they not corded diagonally?	34. Why do projectiles not travel far?
8. Why are round bodies easiest to move?	17. Why are big and heavy bodies able to be split by a small wedge?	26. Why is it more difficult to carry wood on the shoulders by the end than by the middle?	35. Why do things carried in eddying water finish at the centre?
9. Why is moving things easier and faster with larger circles?	18. Why can one lift great weights with a double-pulley?	27. Why are longer weights harder to carry on the shoulder?	

Key: yellow = simple models; blue = explanation in terms of simple models; green = further study of issues from problems 1-3; white = unrelated to main programme.

Note B: Some comments on Aristotle's so-called 'mechanics'

Interpreters of Aristotle have debated whether his claims about motion that take an apparently mathematical form constitute a theory of mechanics. It is hardly possible to write on Aristotelian or pseudo-Aristotelian mechanics without touching on these issues, though this is well-worn ground and I shall aim to be brief rather than comprehensive.[153]

We must be doubly cautious. We must be on guard against anachronism in translating the terms of a distant theory; this is widely appreciated for concepts such as force and mass, but it is less often acknowledged that our notions of speed and velocity are complex historical products.[154] We also should not assume that Aristotle shared the aims, questions or methods of modern scientists.

First, I shall address Aristotle's views on speed and acceleration; next, I shall consider his statements about 'power' (δύναμις) and heaviness (βάρος) in relation to speed.

Speed and acceleration

Aristotle does not explicitly work out a concept of speed, let alone a vector quantity of velocity, in terms of a relationship between time and distance (or displacement). The Greek nouns often translated as 'speed' or 'quickness', τάχος and ταχυτής, are not common in Aristotle.[155] Rather, Aristotle offers criteria for when one change is faster than, slower than, or equally quick as another.

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[153] The bibliography on these topics is vast. Among the most important studies are Duhem 1906, Hardcastle 1914, Carteron 1923, Cornford 1931, Ross 1936, Drabkin 1938, Sambursky 1956, Hahm 1976, De Gandt 1982, Knorr 1982a and 1982b, Hussey 1983, Owen 1986, Lloyd 1987a, Wardy 1990, De Gandt 1991, Hussey 1991, White 1991, Gregory 2001, Ugaglia 2015, Rovelli 2015, Yavetz 2015. I am grateful to Henry Mendell for sharing unpublished material on *Phys*. 7.5. I do not even touch here on such important topics as reversals of direction and projectile motion. On the former, see Sorabji 1976, White 1991, 54-62, and Cohoe 2018. On projectiles, see Wolff 1978, Manuwald 1985, Wolff 1987.

[154] On force and mass, in addition to the contributions mentioned in the previous note, see Jammer 1957 and 1961, and Hesse 1961. Carteron's incisive discussion of speed in Aristotle and *Mech.* (1923, 1-10) was regrettably omitted in the 1975 English translation of its chapter. Clagett 1956, though primarily concerned with a medieval author, makes several perceptive observations on Greek treatments of speed and motion (see especially p.77: 'Autolycus and most Greek mathematicians give comparative rather than metric definitions... It is not surprising, then, that scarcely any of the Greek authors arrived at the idea of velocity itself as a number or a magnitude'). Kuhn 1964/1977 studies changing notions of speed over several centuries. More recently, Mendell 2007 and Sattler 2017 and 2020 have brought fresh insight to some of these topics. Among commentators on the *Mechanica*, Krafft 1970, 71 and Micheli 1995, 52 explicitly recognise that the author does not share our conception of speed.

[155] Waschkies 1991, 171. Socrates offers a definition of ταχυτής at *Laches* 192b: 'I call the ability to accomplish many things in a short time 'quickness', whether in speech or running or all the other cases.' (τὴν ἐν ὀλίγῳ

Phys. 6.2 contains the main discussion of these criteria, where they are introduced primarily to serve in arguments for the continuity and finitude of change. At the start of the chapter, we read that 'necessarily, the faster moves in an equal time a greater [distance] and in less [time] equal [distance] and in less [time] more'.[156] Here are three conditions which hold when one moving thing is faster than another. Since if A is faster than B, B must be slower than A, the criteria also serve to determine when one thing is slower than another. These conditions for 'faster' and 'slower' over the duration of a motion (we might say they amount to a criterion for 'overall' speed) are in tension with any criterion of instantaneous speed. Contradictions are liable to emerge in situations where something which is slower by one criterion overtakes something that is faster by another criterion .[157] Aristotle also says that one thing is equally quick (ἰσοταχές) as another when it moves an equal distance in an equal time.[158] In another context, Aristotle first applies this criterion for being equally quick, then replaces it with a stricter requirement: 'but let the equally quick be undergoing *the same* change in an equal time'.[159]

In either form, this sense of ἰσοταχές compares two motions, and Aristotle usually applies his criteria to the comparison of whole changes and not subdivisions within a given change. In another sense, ἰσοταχές applied to one motion means motion at a constant speed. Aristotle does not explicate that sense of the term here, although he does state a condition which could be taken to characterise uniform motion: 'If [the moving thing] always traverses a magnitude equal to BE in an equal time, and this [BE] measures out the whole [i.e. the whole magnitude traversed is an exact multiple of BE], then the whole time in which it traverses [the magnitude] will be finite.'[160] In *Phys*. 6.7, ἰσοταχές and especially the adverb ἰσοταχῶς are

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χρόνῳ πολλὰ διαπραττομένην δύναμιν ταχυτῆτα ἔγωγε καλῶ καὶ περὶ φωνὴν καὶ περὶ δρόμον καὶ περὶ τἆλλα πάντα.)

[156] 232a25-27. Aristotle adds 'some people define 'faster' in this way' (καθάπερ ὁρίζονταί τινες τὸ θᾶττον). Knorr 1982b, 120 suggests that these τινες include Eudoxus. For parallels to *Phys*. 6.2's criteria, cf. *Phys*. 4.10, 218b14-18; 4.14, 222b33-223a4.

[157] Kuhn 1977, 254 identified this condition and called motions which satisfy it 'quasi-uniform'. Galileo clearly exposed the contradictions that arise when the criteria are applied to motions that do not satisfy this condition (cf. Drake 1967, 22-27).

[158] 232b14-20: ἔτι δ' εἰ πᾶν ἀνάγκη ἢ ἐν ἴσῳ ἢ ἐν ἐλάττονι ἢ ἐν πλείονι κινεῖσθαι, καὶ τὸ μὲν ἐν πλείονι βραδύτερον, τὸ δ' ἐν ἴσῳ ἰσοταχές, τὸ δὲ θᾶττον οὔτε ἰσοταχὲς οὔτε βραδύτερον, οὔτ' ἂν ἐν ἴσῳ οὔτ' ἐν πλείονι κινοῖτο τὸ θᾶττον. λείπεται οὖν ἐν ἐλάττονι, ὥστ' ἀνάγκη καὶ τὸ ἴσον μέγεθος ἐν ἐλάττονι χρόνῳ διιέναι τὸ θᾶττον.

[159] *Phys*. 7.4, 249b4-5: ἀλλ' ἔστω ἰσοταχὲς τὸ ἐν ἴσῳ χρόνῳ τὸ αὐτὸ μεταβάλλον. Here ἔστω signals that a revisionary stipulation is being introduced.

[160] 233b4-6: εἰ γὰρ ἀεὶ τὸ ἴσον τῷ BE μέγεθος ἐν ἴσῳ χρόνῳ δίεισιν, τοῦτο δὲ καταμετρεῖ τὸ ὅλον, πεπερασμένος ἔσται ὁ πᾶς χρόνος ἐν ᾧ διῆλθεν.

used to indicate that a change has constant speed.[161] Other passages in the Aristotelian corpus use a variety of terms: ὁμοταχες and ὁμοταχῶς, ὁμοτόνως, and ἰσοδρομεῖν.[162]

Wilbur Knorr plausibly suggested that in such passages Aristotle is borrowing from mathematicians and astronomers rather than developing his own ideas. In particular, *Phys.* 6.2's expression of the condition for uniform motion is similar to Autolycus of Pitane's (fl. late 4th c. BCE) postulate on moving ὁμαλῶς in his astronomical work *On the Moving Sphere*,[163] and one can also compare Archimedes on moving ἰσοταχέως in his *On Spirals*. [164]

In *Physics* 4.12, 220b32-221a1, Aristotle claims that time, rather than speed, is the measure of motion.[165] However, Aristotle sometimes implicitly treats speed as the measure of motion.[166]

In at least two places, Aristotle seems to treat speed or quickness (τάχος) as a quantity. *Phys.* 4.8, 215b7-8 describes a ratio (λόγον) of one speed to another (τὸ τάχος πρὸς τὸ τάχος).[167]

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[161] 237b26-28: ὅτι μὲν οὖν εἴ τι ἰσοταχῶς κινοῖτο, ἀνάγκη τὸ πεπερασμένον ἐν πεπερασμένῳ κινεῖσθαι, δῆλον. 237b4-6: ἐάν τε ἰσοταχῶς ἐάν τε μὴ ἰσοταχῶς μεταβάλλῃ, καὶ ἐάν τε ἐπιτείνῃ ἡ κίνησις ἐάν τε ἀνιῇ ἐάν τε μένῃ, οὐθὲν ἧττον. In the same chapter, Aristotle also uses the terms 'uniformly' and 'non-uniformly' (ὁμαλῶς and ἀνωμάλως, 238a21-22). Uniformity is distinguished from constant speed: a motion can be of constant speed and yet non-uniform due to its path shape. Aristotle explains what is meant by uniform and non-uniform change at *Phys.* 5.5, 228b19-30. Uniform motion is important in Aristotle's account of the heavens (cf. *Phys*. 8.9, 265b11-16). Cf. De Gandt 1991, 100, 103.

[162] Some examples. ὁμοταχες: *DC* 2.8, 289b9, *Phys*. 7.4, 249a8; ὁμοταχῶς *Phys*. 6.6, 236b35.; ὁμοτόνως 15.5, 911a14; ἰσοδρομεῖν *Problems* 16.3, 913a38, 16.12, 915b10

[163] Ὁμαλῶς λέγεται φέρεσθαι σημεῖα ὅταν ἐν ἴσῳ χρόνῳ ἴσα τε ἢ καὶ ὅμοια μεγέθη διεξέρχηται· ἐὰν δὲ ἐπί τινος γραμμῆς φερόμενόν τι σημεῖον ὁμαλῶς δύο γραμμὰς διεξέλθῃ, τὸν αὐτὸν ἕξει λόγον ὅ τε χρόνος πρὸς τὸν χρόνον ἐν ᾧ τὸ σημεῖον ἑκατέραν τῶν γραμμῶν διεξῆλθεν καὶ ἡ γραμμὴ πρὸς τὴν γραμμήν. ('A point moves uniformly when it traverses equal/similar magnitudes in equal times. If a point moving uniformly along a line traverses two lines, then the time will have the same ratio to the time, in which the point traverses each line, as the line to the line.')

[164] *On Spirals* 1, trans. Netz 2017, 36: 'If a certain point is carried along a certain line, moved at uniform speed with itself, and two lines are taken in it <=the original line>, the <lines> taken shall have to each other the very same ratio which the times <have to each other, =the times>, in which the point passed through the lines.' (Εἴ κα κατά τινος γραμμᾶς ἐνεχθῇ τι σαμεῖον ἰσοταχέως αὐτὸ ἑαυτῷ φερόμενον, καὶ λαφθέωντι ἐν αὐτᾷ δύο γραμμαί, αἱ ἀπολαφθεῖσαι τὸν αὐτὸν ἑξοῦντι λόγον ποτ' ἀλλάλας ὅνπερ οἱ χρόνοι, ἐν οἷς τὸ σαμεῖον τὰς γραμμὰς ἐπορεύθη.) Archimedes does not explicitly *define* ἰσοταχέως but assumes that what moves ἰσοταχέως covers equal distances in equal times. Dijksterhuis 1958, 140-41 suggests the proposition follows trivially for commensurable distances, so the point of the proof is to show that it also holds for *incommensurable* distances.

[165] Sattler 2017 and 2020, ch.8-9 has suggested that this may in part be due to Aristotle's account of measurement in *Met*. I.1 which can account for simple measures such as length and duration, but not complex measures such as speed.

[166] For example, in *Phys.* 7.4, Aristotle assumes that motions are compared in terms of their speed rather than their times (cf. De Gandt 1991, 99-100). Sattler 2020, ch.8 argues that Aristotle's reply to Zeno at *Phys*. 6.2, 232b20–233a31 'provides the basis for a complex measure of speed' (384).

[167] 'Let one speed have to the other the same ratio which the density of air has to the density of water.' (ἐχέτω δὴ τὸν αὐτὸν λόγον ὅνπερ διέστηκεν ἀὴρ πρὸς ὕδωρ, τὸ τάχος πρὸς τὸ τάχος.)

Phys. 6.2, 233b21-22 invokes a ratio of speed (ὁ λόγος τοῦ τάχους). It would therefore be misleading to say that Aristotle had *no* notion of speed.[168]

Aristotle was aware of acceleration and deceleration but did not attempt a mathematical description.[169] In *Physics* 6.7, he argues that a motion along a finite line cannot continue for an infinite amount of time. He first shows that this is impossible if the motion is uniform (i.e. of constant speed), but then incorrectly claims that it is impossible even if the speed is non-uniform.[170] Was the failure to describe acceleration mathematically the cause of this blunder? A similar confusion in *DC* 1.6, where Aristotle argues that an infinite body would necessarily have infinite weight even if non-uniformly distributed, suggests that the source of the error is not *per se* a failure to analyse acceleration. More likely, Aristotle reached his conclusion in both cases by assuming that an infinite whole cannot have an infinite part.[171]

The author of the *Mechanica* shares Aristotle's criteria for comparing speeds. In problem 1, after stating that larger balances are more accurate because larger radii move faster, the author comments: ''Faster' is said in two ways: for we call [something] faster both if [it] traverses an equal place in less time, and if [it traverses] more in an equal [time].'[172] The latter sense is pertinent to problem 1. A phrase echoing Aristotle's formula for sameness of speed features in *Mech.* 24, though whether it is an expression of the same thought or of another idea will be discussed in Chapter 6.

To summarise briefly, *Mech.* has a similar conception of speed to Aristotle. Aristotle's speed is not the same as ours. One important difference is that Aristotle does not have theoretically articulated notion of instantaneous speed. Yet there is no need to deny that Aristotle has *any*

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[168] Lang 1998, 141-42 goes too far in arguing that 'Aristotle's definition [of motion] is incompatible with the requirements of speed as a concept.' Carteron 1923, 3 more carefully puts it that speed was not an autonomous concept for Aristotle.

[169] Among the passages in which Aristotle refers to acceleration and deceleration, see *Phys*. 5.6, 230b21-6; 8.9, 265b12–14; *DC* 1.8, 277a27-b8; 2.6, 288a19–22; 3.2, 301b16–30.

[170] This is mistaken because some decelerating motions fail to traverse a particular finite distance in any finite amount of time. For example, a motion decelerating so that in the n-th unit of time it traverses 2^{-n} units of distance will never traverse a distance of 1 unit.

[171] Knorr 1982b argued for this interpretation. White 1991, 64-69 is in basic agreement, though he also argues that Aristotle is correct to maintain that no finite line is traversed in an infinite time. One should compare the argument of *Phys*. 3.5, 204b19-22.

[172] 848b5-8. This use of 'place' (τόπον) is surprising in light of Aristotle's account of place (*Phys*. 4.1-5). Strato apparently understood place as three-dimensional extension (Sharples 26b, 27a). *Mech.* applies this understanding of 'faster' to the motion of a point at the tip of a rotating radius.

notion of speed. Although he often talks in terms of 'faster' and 'slower', he sometimes speaks of τάχος which may be treated quantitatively.

Power, weight and speed

Let us turn now to Aristotle's statements about the effects of power and weight on motion, which have been taken by some readers form the core of a theory of 'dynamics'. The relevant passages are scattered across the *Physics* and *De Caelo*. Some concern natural and others unnatural motion, some are more precise (e.g. that double the weight takes half the time) while others are less so (e.g. that a heavier weight takes less time).[173]

It must be conceded by all parties that Aristotle gives no explicit indication that he viewed these statements as contributions to mechanics.[174] In my view, though I shall not argue the point here, Aristotle's statements about power, weight and speed are best understood in terms of their immediate contexts. They are introduced to support dialectical arguments about the principles of natural philosophy, for example to deny the possibility of a void (*Phys*. 4.8), or of an infinite body (*DC* 1.6-7).[175]

Aristotle's precise claims about weight and speed are, of course, false. Some commentators point out that heavier objects of the same size and shape do in fact fall faster in viscous media, though not in a vacuum.[176] However, this line cannot make sense of Aristotle's more precise claim that what is twice as heavy will cover the same distance in half the time.[177] Hussey has suggested that Aristotle may have had in mind the starting speeds of weights on

[173] Imprecise statements on natural motion: *DC* 1.8, 277b3-5; 2.13, 294b4-6; 3.5, 304b13-19; 4.1, 308b18-19, 27-28; 4.2, 309b12-15. Precise statements on natural motion: *Phys.* 4.8, 216a13-16; *DC* 1.6, 273b30-274a2; 2.8, 290a1-2, 3.2, 301a28-32. Imprecise statements on unnatural motion: *DC* 1.7, 274b33-275a14, 275a20; 3.2, 301b1-16. Precise statements on unnatural motion: *Phys*. 7.5, 249b27-250a9; 8.10, 266a13-b24. It is important to note that some of these passages are concerned with change in general and not only change of place (Owen 1986, 321).

[174] They do fall within the exceptionally broad definition of mechanics which Pappus attributes to followers of Hero (*Collectio* 8.1-2).

[175] In this I agree with Owen 1986 and Lloyd 1987a, 217-26. Owen's (1986, 327-28) acute observation that the proportionalities concerning forced motion are essential while those concerning natural motion are inessential (that is, Aristotle has other equally potent arguments and so could dispense with the proportionalities), should be borne in mind. A difficulty for this line is that *Phys*. 7.5's aims are obscure. Owen's (1986, 327) observation that the chapter supplies a premise assumed by *Phys*. 8.10 does not take us far in accounting for *Phys*. 7.5's length or its details. As Owen himself notes (1986, 330), *Phys.* 8.10 seems to ignore the threshold proviso of *Phys.* 7.5. I am not persuaded by Wardy's (1990, ch.8) admittedly speculative (335) suggestion that the chapter is a defence of 7.1's argument against an infinite chain of changes.

[176] E.g. Toulmin and Goodfield 1965, 99-102. This defence has recently been revived and clarified by Ugaglia 2015 and Rovelli 2015.

[177] O'Brien 1995, 48.

an equal-armed balance. On the balance, the stronger weight prevails and 'in Aristotle's terms, this means that the rotation induced by the stronger power is *faster* than the other one.' This hypothesis can account for the more precise claim, since unit weights balance one double unit weight, so the double weight is twice as fast.[178] The suggestion is plausible, if somewhat speculative. The behaviour of weights on the balance would have been familiar to anyone in Aristotle's audience and would be a good source of dialectical premises.

I shall not dwell on the various challenges that face those who would reconstruct an Aristotelian 'dynamics', for example the lack of a clear explanation for acceleration in Aristotle.[179] Those problems and a range of possible if speculative suggestions have been discussed at length in the literature cited. Instead, I would like to address a less-discussed issue. The precise proportions found in Aristotle played a central role in the medieval science of weights, in the works of Jordanus de Nemore and others. Were they also applied to the study of levers, balances and machines – in a word, *mechanics* – in Greek antiquity?

For the case of Aristotle, the answer seems to be 'no'. The statements also find no application in the *Mechanica*, except perhaps in an imprecise way in problem 24.[180] *Pace* Duhem, they play no role in the explanations of problems 1-3. Interestingly, it is in other mechanical authors, more distant from the Peripatos, that statements similar to Aristotle's are applied. Philo of Byzantium used the supposed fact that heavier weights fall more quickly as an illustration in a dispute about the number of springs that should be used in a catapult.[181] Hero's *Mechanica* attempts to explain why greater weights fall in a shorter time.[182] However, neither Philo nor Hero commits himself to Aristotle's more precise claim that double the weight falls twice as fast.

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[178] This is perhaps an advantage over Carteron's hypothesis that Aristotle was generalising from observations such as that heavier objects have greater impacts after a fall.
[179] Hussey's (1991, 238) suggestions are perhaps the best that can be done for acceleration.
[180] See Chapter 6.
[181] *Belopoeica* 69 claims that a two-mina weight falls much faster than a one-mina weight, but also faster than two, three, or even more weights of one mina fastened together, since the combined speed of a collection of one mina weights will not be more than the speed of a single weight. Compare Galileo's famous thought-experiment about falling bodies in *Two New Sciences* (Crew and Salvio 1954, 63ff.).
[182] *Mechanica* 2.34, question d (Nix and Schmidt, 176). Hero also asks (question e) why a broad object falls slower than a round object of equal weight. On the other hand, Hero argues against an Aristotelian view in *Mechanica* 1.20.

Some Latin and Arabic texts apply more precise Aristotelian ideas in deductions of mechanical principles, and particularly of the so-called 'law of the lever'. These texts have been suspected to derive from Greek originals, but this remains controversial, and further investigation here by Arabists is needed. Briefly, our texts are:

a) *De Ponderoso et Levi*, attributed to Euclid.[183]
b) *Kitab al-Qarstun*, attributed to Thabit ibn Qura (836-901).[184]
c) *Liber Karastonis*, attributed to Thabit ibn Qurra and translated by Gerard of Cremona in the twelfth century.[185]
d) *Fragment on the Roman Balance*, also known as *Excerptum de Libro Thebit.*[186]

Text (a) is a short and likely fragmentary work consisting of nine postulates and five or six theorems.[187] The nine postulates belong in three groups of three. Each group follows the same pattern: one postulate defines equality or sameness of a property, the next defines inequality or difference, and the last explains what possessing a greater amount of the property means. The first group concerns the size (*magnitudo*) of bodies, the second their power (*uirtus, fortitudo*), and the third their kind (*genus*). *Uirtus* is defined as a property inherent in bodies, rather than one externally applied, and may correspond to the Greek term *rhope*.[188]

The first theorem is similar to the statements found in Aristotle: 'Of bodies which traverse unequal places in equal times, that which traverses the greater place is of greater power.'[189] The subsequent theorems develop further relations between the properties defined at the outset. While the systematisation of such ideas about motion in postulates and theorems may recall axiomatic mathematical treatises, formal proofs are not supplied for all theorems, and

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[183] See the edited text in Moody and Clagett, 1952, 21-31. The *Liber de Ponderoso et Levi* was printed in several early modern editions of Euclid's works, such as David Gregory's 1747 *Euclidis quae supersunt omnia*. Sarton, 1927, 156 thought the work's use of a notion of specific weight meant that it must postdate Archimedes. I do not think there is anything particularly Archimedean about the work's definition of *genera corporum*. Hahm 1976 treats it as authentically Euclidean. There is an Arabic version in al-Khazini's *Book of the Balance* 1.3.
[184] Jaouiche 1976.
[185] Moody and Clagett 1952, 77-117.
[186] Moody and Clagett 1952, Appendix 1; Knorr 1982a, Appendix C.
[187] The sixth theorem was edited by Buchner from a manuscript known only to him.
[188] This would fit the work's title, which means 'Book on the Heavy and the Light'. However it is not clear from the extant text if the author thought, like Aristotle, that there was a distinct natural motion upwards, or whether, like Strato, he thought the upwards motion of light bodies was not due to a distinct property 'lightness', but rather to displacement by heavier bodies.
[189] *corporum que temporibus equalibus loca pertranseunt inequalia, quod maiorem pertransit locum, maioris esse virtutis*.

the text does not apply these ideas to the explanation of the balance, lever, or any other machines.[190] The diagram-letters follow the Greek alphabet, with one possible exception.[191]

Texts (b) and (c) are clearly closely related and are often described as two versions of the same work. [192] Both aim to solve a problem about the equilibrium conditions for what is nowadays termed a 'Roman' steelyard: 'given a beam of uniform weight, suspended from a point other than its midpoint, to determine that weight which when suspended from the end of the shorter arm brings the beam into equilibrium.'[193] The two versions, (b) and (c), differ in a number of respects. For example, two propositions of (c) are missing in (b). The two works sometimes give different proofs for the same proposition, where (b) invariably has the more rigorous proof. For example, in the case of the crucial proposition 6, (b)'s proof is quite precise and valid while (c)'s proof is circular.[194] Text (c) begins with a brief prefatory letter from Thabit to an unnamed addressee (*o frater*). Thabit and his addressee had apparently struggled to understand a work entitled *Cause karastonis.* Thabit judged that the obscurities were due to translation and poor copyists (*permutationem linguarum interpretum et vicissitudiness* [*sic* Moody and Clagett 1952, 88] *manum scriptorium*) and explains that he has attempted to clarify the *Cause karastonis* in the text following his letter. There is no prefatory letter in (b).

Despite these differences, the two versions address the same problem through roughly the same sequence of propositions and, importantly, begin with the same theorem: 'The ratio of two distances traversed by two moving things in two [equal] times is equal to the ratio of the force of one moving thing [which traverses] the uniform [= *mustawiya*] distance to the force of the other moving thing.'[195] This is the same idea we saw in (a) and in Aristotle. Thabit

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[190] It may be that an earlier version of the *Liber de Ponderoso* contained proofs that were left out at some stage in transmission. In any case they are not difficult to supply.

[191] *A, B, G, D, E, Z, H, T.* The exception is the letter *V* in theorem 1. Could this be a transliteration of Arabic *wau*?

[192] The nature of their relation is controversial. Buchner, 1920 thought (c). was adapted by its translator but Gerard of Cremona was typically a faithful translator. Knorr, 1982a hypothesised that (b) was an Arabic translation of a Greek work while (c) represents Thabit's editorial efforts on it. Brentjes 2020 suggests that (b) and (c) are two versions of Thabit's own work, the latter being adapted to a didactic context.

[193] Knorr 1982a, 7. Knorr calls this the 'problem of the weighted beam'.

[194] See Knorr 1982a, 49-56.

[195] I translate from the French translation of Jaouiche, 1976, 147 (see also the critical notes on p.170). Compare the corresponding postulate in *L.Kar.*: *dico ergo quod omnium duorum spaciorum que duo mota secant in tempore uno, proportio unius ad alterum est sicut proportio virtutis motus eius quod secat spacium unum ad virtutem motus illius secantis spacium alterum.*

uses it to prove the so-called ‘law of the lever’ before tackling the more complex problem of equilibrium conditions for a weighted beam of unequal arms.[196]

Moody and Clagett suspect that text (d) is either an excerpt from Thabit or else the original *Cause karastonis* which Thabit revised. It contains all but one of (c)’s propositions, but in reverse order. Paragraphs 7-9 prove the ‘law of lever’ in the same way as (b) and (c), starting from the proportionality of powers and distances, applying it to arcal displacements at the ends of a balance beam, and then showing that arcs are proportional to arm lengths.

Could (b) and (c) derive from a Greek original, perhaps the *Cause karastonis* mentioned by Thabit? Text (b)’s proof of theorem 6 (that equilibrium is preserved if the weight uniformly distributed along one arm of an immaterial unequal-armed balance is replaced by an equal weight suspended from the original weight’s midpoint) uses a sophisticated version of the so-called method of exhaustion that has been dubbed the ‘method of compression’.[197] Wilbur Knorr took the application of this technique to suggest that (b) represents the work of a Greek mathematician, translated and edited by Thabit.[198]

Sonja Brentjes has criticised Knorr’s argument, observing that there are four mathematical works by Thabit which demonstrate his competence in applying the so-called ‘method of exhaustion’ and ‘axiom of Archimedes’.[199] In these cases Thabit uses the exhaustion technique as the ‘method of approximation’ rather than the ‘method of compression’, yet it is prejudiced, Brentjes suggests, to argue that Thabit could not also have applied the latter.[200] Brentjes nevertheless agrees that theorem 6 of (b) likely derives from a Greek source, but for

[196] Duhem effectively read this method of proof of the ‘law of the lever’ back into *Mech*.
[197] Jaouiche 1976 numbers this theorem ‘proposition IV’. The term ‘method of compression’ comes from Dijksterhuis, 1987, 130-133 who classified the convergence methods used by Archimedes. The ‘method of approximation’ is that found in Euclid’s *Elements* Book 12.2, 5, 10-12, 18 and Archimedes’ *Quadrature of the Parabola* 18-24 and *Measurement of the Circle* 1. The ‘method of compression’ is not found earlier than Archimedes. It involve inscribing and circumscribing sequences of rectilinear figures in and around a curvilinear figure. Dijksterhuis distinguishes two subvarieties. In ‘compresion by difference’, it is proven that the difference between the bounding figures can be made less than any finite magnitude (e.g. *Quadrature of the Parabola* 16, *Spiral Lines* 21-23, *Conoids and Spheroids* 19-20, and *Method* 15). In ‘compression by ratio’, it is proven that the ratio between the bounding figures is closer to unity than any ratio (e.g. *Sphere and Cylinder* 1.2-6).
[198] More specifically, Knorr argued that Thabit’s *Kitab al-Qarastun* may derive from part of Archimedes’ lost *On Balances*. For an earlier reconstruction of *On Balances*, see Drachmann 1963b.
[199] Brentjes 2020. Brentjes refers to Thabit’s works on (i) the quadrature of the parabola; (ii) parabolic bodies of revolution; (iii) two lines that meet when they include a non-right angle; (iv) the trisection of the angle. All are found in MS Paris Bibliothèque nationale de France, Arabe 2457.
[200] Brentjes 2020, 46.

the different reason that she has identified a number of Graecisms in the Arabic text.[201] Since there are no Graecisms in the early propositions of the work, her preferred hypothesis is that Thabit edited an earlier Greek work on the unequal-armed balance which contained theorem 6, and that Thabit gave this work a new foundation by deriving the general principle of equilibrium from the proportionality of powers and times.

Not being an Arabist, I am unable to delve deeper. It is time to reflect on this material. We cannot rule out that some of Aristotle's proportions were applied to the understanding of the balance and lever in antiquity.[202] Supposing they were, what could we conclude from that? Were writers on mechanics after Aristotle under the philosopher's spell, or was Aristotle himself drawing on ideas already applied by mathematicians, as he seemed to be doing in *Phys*. 6.2? What it would mean for our understanding of *Mech*., or of Aristotle for that matter, if we were to see the *Kitab al-Qarstun* as its near contemporary? So little of early Greek mechanics has survived that is easy for us to assume that Archimedes was the first to put mechanics on axiomatic-deductive foundations, and to see the pseudo-Aristotelian *Mechanica* as a stumbling first step towards a demonstration of the conditions of equilibrium for a balance. What if Archimedes' proofs concerning the balance were not the first but rather the culmination of an earlier tradition of proofs of equilibrium conditions, perhaps like the *Kitab al-Qarstun*, or like another Arabic text likely derived from a Greek source, the *Book on the Balance* attributed to Euclid?[203]

Posing these questions forces us to recognise the differences between the projects of *Mech*. and of Archimedes and other writers interested in deducing equilibrium conditions. Archimedes in *Plane Equilibria* assumes that the balance beam rotates when unbalanced, but he does not attempt to describe how or explain why this happens. He deduces from this assumption, together with several others, the necessary and sufficient conditions for equilibrium. *Mech*.'s author states the conditions of equilibrium in passing but (*pace* Duhem) does not attempt a precise proof or explanation. The primary interest is finding the causes of the motion of the balance and lever.

[201] Brentjes 2020, 158 and personal communication.
[202] In his commentary on *Phys*. 7.5, Simplicius mentions Archimedes and an instrument, τὸν καλούμενον χαριστίωνα, perhaps the steelyard known to Arabic authors as the *qarastun* (*Commentary on Aristotle's Physics*, 1110, 2-5).
[203] I do not have space here to discuss this text, which offers a deduction of equilibrium conditions for a balance that differs both from Archimedes' proof in *Plane Equilibria* and from the proof in the *Kitab al-Qarstun*. See the translation and commentary in Clagett 1959, 24-30 and analysis in Knorr 1981, ch.7 and Appendix F.

Chapter 2: Aristotle on composed motions

2.1: Introduction

Mech. 1 asks, 'Why are larger balances more accurate than smaller ones?'[1] The answer offered suggests that a tiny angular motion is more easily noticed in a larger balance-arm since its tip moves faster. The tip of a larger-balance arm moves faster because points further from the centre on a rotating radius move faster. Most of *Mech.* 1 is devoted to explaining this principle. Strikingly, we are told that 'the cause of these things is that the [line] describing a circle moves with two motions'.[2] The full explanation that follows is complex and our interpretation must wait until Chapter 4. At this early stage, however, questions already arise. Should we understand the author's claim literally? Can a single thing *really* move with two motions at the same time? Or are these two motions mere theoretical fictions?

Our answers to the above questions will inform whether we see *Mech.* problem 1's analysis as causal or purely kinematic, and so will affect our understanding of its aims, and whether we see it as potentially applying to the circular motions of the heavens. Since problem 1 provides the foundations for subsequent explanations, these issues will also fundamentally shape our understanding of *Mech.*'s overall project.

It is the task of this chapter to offer some answers. I will argue that the component motions in *Mech.*'s analysis of the balance are not theoretical fictions and hence the explanation is not purely kinematic.[3] I see four possible reasons for thinking the component motions are theoretical fictions: (i) an assumption that *Mech.* contains the modern 'parallelogram of velocities'; (ii) a reading of 847a27-847b1 as claiming that mathematics rather than physics provides the explanations; (iii) the thought that, since a body has only one place at a time, it can have at most one motion at a time; and (iv) concerns about the causal terminology in *Mech.* 1, especially in 849a6-849a19. It should be clear that (i) is hopelessly anachronistic.[4] I

[1] The assumption that larger balances are more accurate is itself questionable (see Chapter 5).
[2] 848b10-11: αἴτιον δὲ τούτων ὅτι φέρεται δύο φορὰς ἡ γράφουσα τὸν κύκλον.
[3] The latter conclusion follows from the Aristotelian thought that any change is driven by an efficient cause which acts throughout the duration of the change. So if a body has a motion, there is an efficient cause of that motion. And if a body has two distinct motions, there is an efficient cause of each motion. The doctrine is not only Aristotle's: it found unusually wide acceptance among Greek thinkers, the Epicureans being a notable exception (cf. Sorabji 1988, 219).
[4] See Note B to Chapter 1 and §2.2 below.

have challenged (ii) in Chapter 1 and will continue to do so in Chapters 4-6. Part of my aim in this chapter is to show that Aristotle and his followers would not have been persuaded by (iii).[5] I will address (iv) in Chapter 4, in the course of a close reading of *Mech*. problem 1.

The idea that a single body can undergo multiple motions at once may sound strange, even nonsensical, to modern ears. Yet the assumption that a body can undergo at most one motion at a time is a historical product, the result of developments in science and philosophy since the seventeenth century. In ordinary speech we often attribute distinct motions to a single body at the same time, for example in describing a diver who performs a somersault while falling towards the water. So the claim that a body can undergo at most one motion at a time is revisionary, forcing us to abandon aspects of our everyday, pre-reflective conceptions of the world. In section 2.2 of this chapter, I will develop this contrast further in the context of a puzzle about composed motions. In sections 2.3-4, I will argue that Aristotle's account of motion in the *Physics* is best understood as implying that bodies can undergo several motions simultaneously. In section 2.5, I will show that this interpretation is supported by passages from Aristotle's scientific works which offer explanations that rely on claims about the number of motions that a body has. At the same time, we will see that these passages (in contrast to *Mech*.) do not apply geometrically precise rules for the composition of motions.

Although there is a considerable scholarly literature examining Aristotle's views on motion, there have been relatively few attempts to understand his views on the composition of motions.[6] A note of caution is appropriate, for two main reasons.

First, Aristotle does not address the issue of composed motions head-on and the composition of motions plays a minor role in the *Physics*. This can be explained in part by the particular aims and interests of that work. In many passages, Aristotle ignores not only the possibility of composed motion, but also more generally the possibility for motions to have paths of different shapes. On such occasions, he considers only the simplest case of motion along a single linear path since this is suited to the clarification of specific concepts: time is assumed to be one-dimensional; all examples of alteration are understood on a one-dimensional, linear

[5] In §2.2-4 below I discuss various arguments for (iii).
[6] Hussey 1991, 220-222 is a notable exception; see also Carteron 1923, 5-10; Miller 2017, 161-3. Among the numerous studies of Aristotle's general account of change, I have particularly benefitted from Kosman 1969, Penner 1970, Waterlow 1982, Coope 2009, Rosen 2012 and Cohoe 2018; on locomotion in particular, I am endebted to Berti 1985, Morison 2002, Bowin 2009, and Odzuck 2013.

model, e.g. from cold to hot, from low to high musical pitch.[7] There are occasional allusions to the complexities introduced by the fact that motion takes place in three-dimensional space, to use a modern phrase. For example, Aristotle states that motion is non-uniform if it takes a non-uniform path, one for which it is not the case that any two equal parts are congruent, and gives as examples ‘an angled line or a spiral’.[8] He considers whether sameness of path shape is a necessary condition for the sameness in species of motions.[9] And he refers to circular and ‘mixed’ motion in *Phys*. 8.8-9, though of these only the former receives extended attention.[10]

Secondly, I shall approach the topic of composed motions via Aristotle’s general account of change outlined in the *Physics*. The difficulty is that there is still disagreement on the interpretation of this account. It would be virtually impossible to remain neutral on these issues while discussing the upshots of this account for cases of composed motions and I will not attempt to do so. I will argue in the terms of the interpretation of the *Physics* account that I favour.[11]

We should not expect to recover well-worked out theory of composed motion from Aristotle’s works. Some of the tensions and difficulties I will discuss in this chapter and Chapter 3 may suggest otherwise. I do not pretend to have reconstructed such a theory. My aim is to show that questions about the number of simultaneous motions occurring in a single body are intelligible within the framework of Aristotle’s natural philosophy. Thus *Mech*.’s author can be said to address an Aristotelian question in problem 1, even if his answer is not one endorsed by Aristotle.

[7] Aristotle’s theory that colours result from certain ratios of black and white is today known to be false. But it is no problem for Aristotle’s one-dimensional treatment of alterations that he does not distinguish multiple dimensions of colour (e.g. hue, saturation, brightness) since each dimension could be treated as a quality in its own right.
[8] *Phys.* 5.4, 228b15-25.
[9] See §2.4 below.
[10] I discuss Aristotle’s ‘mixed’ motion in Chapter 3.
[11] I suspect that similar arguments could be made on the basis of other interpretations of *Physics* 3’s account.

2.2: A puzzle about composed motions

A sailor walks across the deck of a cruising ship. How many motions does the sailor undergo? We may be tempted to say two motions are involved, the ship's and the sailor's. We have just referred to two ways of moving ('a sailor *walks* across the deck of a *moving* ship') and it seems clear that the sailor is affected both by walking and by the ship's motion. So our ordinary ways of describing such situations could be taken to assume that a body can undergo more than one motion at the same time.

Yet if a body is in only one place at a time, how can a body undergo two motions, that is two changes of place, simultaneously? Would that not imply being in two places at a time, which is impossible? Or, if not, what difference could there be between the two motions? Another point of view, then, maintains that the counterpart to the truism that a body is only in one place at a time is that a moving body passes through only one ordered succession of places, one path, over a time interval. This is the position taken by Descartes.[12] On this view, the sailor has one motion which is not identical with either his walking or his being moved by the ship; it is some third thing, a resultant motion, defined by the continuous succession of places occupied by his body over time. A body can be *thought of* as having two simultaneous motions, but these motions are no more than convenient fictions.

Consider again the view that the sailor truly has two motions. Does the direction in which he walks matter? If he walks towards the stern at precisely the same uniform speed as that with which the ship advances, he will remain in (roughly) the same place.[13] Both the walking and the ship's motion still seem to be there, but the body which is supposed to be undergoing both motions now remains in the same place. Can such a body be said to be moving at all? I set this problem aside for now, and will return to it in Chapter 3.

[12] Descartes *Principia* II.25-32: 'when we understand by motion that translation which takes place from the vicinity of contiguous bodies, since only one [group of] bodies can be contiguous to the same mobile at the same moment of time, we cannot attribute to this mobile many motions at the same time, but only one… although it is often useful to separate in this way one motion into many parts, to perceive it more easily, nevertheless, absolutely speaking, one should number only one motion in any body' (trans. Mahoney). For context, see Garber 1992, ch.6. This is not to deny that a body could be said to undergo two motions at the same time in the sense that its *parts* move differently. I am focussing on the motion of the body as a whole.

[13] 'Roughly' only because through walking his limbs will still swing, his head bob, and so on. One could instead imagine the sailor carries an object perfectly level while walking as described; such an object will remain in exactly the same place (cf. Sextus Empiricus *Adv.Math*. 10.55-57).

A third possibility is that the sailor undergoes three motions: walking, being moved by the ship, *and* the resultant motion of combining these two. Yet in that case, the rule for combining motions to produce a resultant should presumably apply again to all three motions, and so the sailor would traverse *twice* the distance actually moved. Thus this third view seems initially less promising.[14] In this chapter I will focus on the first two possibilities.

Whether one should view only the component motions or only the resultant motion as real will depend in part on one's account of what motion is. To see how one's account of motion may influence one's view of component motions, consider a common modern account, the 'at-at' theory of change. According to the 'at-at' theory, 'Motion consists merely in the fact that bodies are sometimes in one place and sometimes in another, and that they are at intermediate places at intermediate times.'[15] On this view, the sailor's motion consists merely in the succession of places he occupies over time. Since the sailor is in only one place at any time, there will be only one succession of places that he occupies over any time-interval making up one path of motion. Thus only the resultant motion has a claim to reality.

I suspect there is another reason why many people today would tend on reflection to resist treating component motions as real, even if they do not endorse the 'at-at' theory.[16] We tend to think of motion as a *state* of having non-zero velocity.[17] We say that a body is 'in motion' if it has non-zero velocity and that it is 'at rest' only if its velocity is zero.[18] On such

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[14] It is in principle possible to distinguish motions which compose according to the parallelogram rule from motions which do not. Alternatively if, as in modern physics, the parallelogram of *motions* is abandoned for the parallelogram of velocities and the parallelogram of forces, then questions about the number of motions will have some independence from issues of composition.

[15] Russell 1918, 83-84.

[16] Dissenters from the 'at-at' theory may hold a range of views. Some might take change as primitive (e.g. Bostock 1996, xxxi: 'change is one of the basic and fundamental concepts in natural science, and cannot be defined in terms of anything more fundamental'). Tooley 1988 argues that instantaneous velocity is an irreducible intrinsic property and gives it a functional definition via the Ramsey-Lewis method (Bigelow and Pargetter 1989 and Lowe 2002 take similar views). The main objection to such an account is that treating velocity as intrinsic conflicts with modern physical theories in which velocity is frame-relative – not only special relativity, but also formulations of classical mechanics in neo-Newtonian, or 'Galilean', spacetime; see the detailed criticisms of Butterfield 2006. Lange has proposed treating instantaneous velocity as a dispositional property, noting that this has the surprising consequence that 'if a body is moving now at 5 cm/s not in virtue of its current categorical properties, but in virtue of where it would be were it to continue to exist, then two bodies could be thoroughly alike now, as far as their categorical properties are concerned, and yet possess different velocities now' (2005, 460).

[17] Penner 1970, 411-424 gives a clear explanation of why Aristotle's account of change excludes the treatment of motion as a state.

[18] This is true even if we cannot measure absolute velocities. *Mutatis mutandis*, it holds true for relative motion. In speaking of the velocity of a spatially extended body, we typically focus on the velocity of its centre of mass; there may be some vagueness in cases where the body's centre of mass is at rest but other parts of the body are moving.

occasions, 'motion' is not used as a count noun, and asking how many motions a body has seems to make little sense. Its velocity at any time can be decomposed into arbitrarily many component vectors but we have no reason to privilege any of these equivalent sets of vectors, except perhaps the single resultant vector. Our answer to the question 'how many motions does this moving body have?' is therefore likely to be one, or we will deny that there is any fact of the matter: a body is 'in motion' or 'at rest' but there is no fact about the number of motions it has.

This does not exhaust the range of accounts of change and motion currently alive, but I think the above are reasonably typical responses of reflective, scientifically informed people today upon being asked how many motions a body can undergo simultaneously.[19] What I wish to emphasise is that some of the reasons for these responses are informed by modern scientific developments.[20] The one argument we have seen that does not depend on modern scientific theory is Descartes': a body is in only one place at a time, so it can have at most one motion at a time. I have not found an ancient version of this argument, yet since Aristotle accepted the premise but (so I will argue) denied the conclusion, we should ask how he would respond to Descartes' argument. The key to understanding Aristotle's view and how he might have responded to the Cartesian argument lies in his account of change in the *Physics*.[21]

[19] Recent writers on the metaphysics of events offer alternatives. Bennett 1988, ch.10 insists that, at least in some cases, two motions can truly be said to occur in the same spatiotemporal zone, e.g. the spin of a top and its simultaneous movement across the table; there is, Bennett suggests, also a third event, the 'fusion' of these two, the top's whole movement. Bennett is sceptical about the possibility of a systematic theory of events (cf. the earlier attempts of Kim 1976, Lombard 1986). By contrast, Quine's austere account of events follows the spirit of Russell's account of change: 'Physical objects...are not to be distinguished from events... Each comprises simply the content, however heterogeneous, of some portion of space-time, however disconnected and gerrymandered.' (Quine 1960, 70).
[20] The at-at theory draws on the calculus to construct instantaneous velocity. The identification of motion with velocity and the conception of velocity as a state are inspired by modern physical theories.
[21] Although Descartes is remote in time from Aristotle, the comparison of the two is productive. Partly this is because Descartes' definition of motion in the *Principia* involves a surprisingly Aristotelian notion of place: 'motion… is the translation of one part of matter, or of one body, from the vicinity of those bodies that are in direct contact with it and are viewed as at rest to the vicinity of others.' (*Principia* §5, trans. Mahoney).

2.3: Aristotle's account of κίνησις

Aristotle's views on motion are contained in his general discussion of change (κίνησις) in the *Physics*.[22] In contrast to the Eleatics and perhaps also to Plato, Aristotle believes that change is a feature of reality. And in contrast to the atomists, who reduce all change to motion, Aristotle thinks that there are many irreducibly different kinds of change. He recognises change with respect to four categories: quality (alteration, ἀλλοίωσις), quantity (increase αὔξησις and decrease φθίσις), substance (coming-to-be γένεσις and ceasing-to-be φθορά), and place (motion, φορά).[23]

Nevertheless, motion has a special status. Aristotle says that his general term for change, κίνησις, is properly used only of motion.[24] He argues for general conclusions about change by taking locomotion as the paradigm case.[25] And he argues that motion is prior, in several senses of priority, to other kinds of change.[26] Understanding motion is crucial for Aristotle's inquiry into nature. The cosmological investigations of the *De Caelo* focus on the natural motions of the simple bodies – earth, water, air, fire and especially the 'first body' which makes up the heavens. The *Meteorology* investigates the motions of winds and meteors. The investigation of animals includes the study of their motions (*De Motu Animalium, De Incessu Animalium*). Even screening off mechanics, Aristotle's investigations of the natural world cover a wide range of complex motions, varying not only in the range of possible destinations but also in the multitude of ways of reaching places and passing through them.[27]

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[22] κίνησις is traditionally translated as 'motion' which is misleading since in common English usage 'motion' exclusively means change of place. 'Change' is less misleading but hardly unproblematic, since there are occurrences we might call changes that are not strictly κινήσεις in Aristotle's sense. An important example is substantial change (coming-to-be or ceasing-to-be) which does not count as κίνησις (*Phys*. 5.1 224b35ff., 5.2, 226a23). Another example is discontinuous or 'jerky' change (e.g. from being-in-contact to not-being-in-contact, between which there is no continuum of intermediate states). Aristotle's more general term for change, covering both κίνησις and not-κίνησις, is μεταβολή. 'Process' avoids some of these problems but can sound rather stilted and there is no corresponding verb to match κινεῖν ('procede' means something different and in any case cannot be transitive). I translate κινεῖν*,* κίνησις etc. as 'change' except where it is used exclusively for change of place, for which I use the terms 'locomotion' and 'motion' interchangeably.

[23] These are the kinds of change Aristotle recognises in *Phys*. 5.2. At *Phys.* 5.2, 226a32-226b1, Aristotle says that φορά strictly applies only to 'things that change their place only when they have not the power to come to a stand, and to things that do not move themselves locally.' (trans. Hardie and Gaye). Simplicius glosses this as 'things which change like inanimates' (921.18-19, trans. Urmson).

[24] *Phys.* 8.9, 266a1-2.

[25] See especially *Phys.* 6.4-5.

[26] *Phys.* 8.7; see Odzuck, 2013.

[27] Especially diversity of path-shape and means of transport. Aristotle draws attention to this diversity in passages I consider below.

One way to understand complex movements is through analysis into component motions.[28] It is important to distinguish the idea that a body may undergo two or more motions at once from the precise geometrical account of how motions compose (the 'parallelogram rule'). I am arguing in this chapter that Aristotle applies the former; the latter is found in *Mech*. but not explicitly in Aristotle's certainly authentic works.

According to Aristotle, change is 'the actuality of what is potentially, as such'.[29] There is no room here for full treatment of the interpretative challenges on this obscure expression.[30] I will only sketch my favoured interpretation. Aristotle's idea is that things in the world have potentials to be in different states: a lump of bronze has the potential to be a statue, my hot cup of tea has the potential to be cold, and I have the potential to be in the University Library. A thing is changing for Aristotle when its potential to be another state is in some sense actual. The 'as such' phrase in Aristotle's formula identifies the appropriate sense of actuality: change is the actuality of a potential *as a potential*. A thing's potential to be in a state is actual in a different sense when the thing has reached that state. The statue is not the actuality of the bronze's potential to be a statue, as a potential, since when the bronze is actually a statue it is no longer potentially a statue. When the bronze is becoming a statue, it is still potentially a statue and not an actual statue, and its potential is actual in the sense that it is being manifested rather than being merely dormant. So a thing is changing when its potential for being in a state, to borrow Ursula Coope's phrase, is making a difference in a way directed at being in that state.[31] It is for this reason that Aristotle says that change is incomplete: it is directed at a state that lies beyond it. When that state has been obtained, the change is no more.

Change is the incomplete fulfilment of a thing's potential to be in a different state. So motion, which is change of place, is the incomplete fulfilment of a thing's potential to be in a different place. A motion is therefore defined in part by its the place towards which it is directed, its *terminus ad quem*.

[28] We saw in Chapter 1 that by the mid-fourth century Greek astronomers had begun investigating the composition of circular motions. In particular, Eudoxus showed that complex planetary motions could result from the composition of two circular motions.
[29] *Phys.* 3.1, 201a10-11: ἡ τοῦ δυνάμει ὄντος ἐντελεχεία, ᾗ τοιοῦτον.
[30] In my understanding of *Physics* 3's account, I follow Kosman 1969, Waterlow 1982, and Coope 2009.
[31] Coope 2009, 282-83.

To return to our earlier example, the sailor is moving to the other side of the deck when his potential to be on the other side of the deck is making a difference in a way that is directed at being on the other side of the deck. And he is moving in the direction of the ship's travel when his potential to be at some place further along in that direction is making a difference in a way directed at his being in that place. What is important for our immediate purposes is that motion is *not* defined by the succession of places occupied by the sailor's body over time.[32] This leaves room for questions about the number of simultaneous motions a body is undergoing, since a body's potential to be in one place and the same body's potential to be in another place might both be making a difference directed at those places at the same time.

One group of remarks in Aristotle's general discussions of change that do not apply to change of place are those in which Aristotle speaks of change in terms of the acquisition of form.[33] Motion does not involve acquisition of form because place is not a form. Place is not a form because it is separable (*Phys*. 4.2, 209b30-31). This separability is clear in the phenomenon of replacement. I fill the jug with water, the water replaces the air: Aristotle thinks any satisfactory account of place must make sense of the idea that the water now occupies the same place the air previously occupied.[34] This is one reason why change of place is prior to other kinds of change: it least affects the form of the thing changed.[35] This is furthermore the reason why the heavens' motion does not entail their destructibility: 'for change does not imply for them, as it does for perishable things, the potentiality for the opposite, which makes the continuity of the motion distressing'.[36]

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[32] Time is absent from *Physics* 3's account of change.

[33] The thesis that change involves acquisition of form should be distinguished from the thesis that it involves *transmission* of form, i.e. that if *a* changes *b* then *a* must already possess the form which *b* acquires as a result of that change. This thesis about transmission of form, sometimes called the 'principle of synonymy', may have an even more restricted scope than the thesis about acquisition of form. *Met.* Z.9, 1034b16-19 seems to restrict it to substantial change only. *Met*. Λ.3, 1070a4-5 states it only for substances. See Bodnár and Pellegrin 2006; Bowin 2009, 50-55; Coope 2015, 251-53. That *a*'s moving *b* does not involve the transmission of a particular form from *a* to *b* fits observed facts about motion. To push a cork into a bottle I do not need to have squeezed myself in the bottle's neck first. It is significant that motion is absent from lists of the types of change when Aristotle's discussion focusses on transmission of form. For example, *Phys*. 3.2, 202a3-11 mentions change in substance, quality and quantity, but not in place: 'That which produces change will always carry some form, either 'this' or 'of such a kind' or 'so much', which will be the principle of, and responsible for, the change, when it produces change' (trans. Hussey).

[34] Morison 2002, 20-25.

[35] *Phys*. 8.7, 261a20-23: ἥκιστα τῆς οὐσίας ἐξίσταται τὸ κινούμενον τῶν κινήσεων ἐν τῷ φέρεσθαι· κατὰ μόνην γὰρ οὐδὲν μεταβάλλει τοῦ εἶναι, ὥσπερ ἀλλοιουμένου μὲν τὸ ποιόν, αὐξανομένου δὲ καὶ φθίνοντος τὸ ποσόν.

[36] *Met.* 9.8, 1050b24-27 (trans. Tredennick): οὐ γὰρ περὶ τὴν δύναμιν τῆς ἀντιφάσεως αὐτοῖς, οἷον τοῖς φθαρτοῖς, ἡ κίνησις, ὥστε ἐπίπονον εἶναι τὴν συνέχειαν τῆς κινήσεως. Cf. Bodnár 1997, 111.

2.4: Identity of κινήσεις

We have now seen that Aristotle's account of motion in terms of actuality and potentiality allows for us to truly describe the sailor as simultaneously walking and being conveyed by his vessel. But *Physics* 3's account does not give us criteria for identifying and distinguishing changes. Undoubtedly, the sailor is simultaneously walking and being conveyed, but it might be thought that 'is walking' and 'is being conveyed' are two descriptions of the same motion. In this section, I will examine what Aristotle has to say about the conditions under which two changes are the same or different. Here are the conditions as presented in *Physics* 5.4:

> 'There are three classes of things in connexion with which we speak of motion, the 'that which', the 'that in which', and the 'that during which'… Of these three it is the thing in which the motion takes place that makes it one generically or specifically, it is the thing moved that makes the motion one in subject, and it is the time that makes it consecutive: but it is the three together that make it one without qualification: to effect this, that in which the motion takes place (the species) must be one and incapable of subdivision, that during which it takes place (the time) must be one and unintermittent, and that which is in motion must be one – not in an accidental sense.'[37]

We may summarise this by saying that two changes, x and y, are one without qualification if and only if:

(i) x and y are in the same subject non-accidentally
(ii) x and y have the same start time and the same end time
(iii) x and y are the same in indivisible species

What does it mean for two changes to be the same in indivisible species? In *Phys*. 5.4, Aristotle mentions different levels at which changes can be classified. At the highest level are 'differences in genus'. Aristotle's examples are of changes in different categories: spatial motion, qualitative alteration, quantitative growing and shrinking. Within each category there are differences in species. *Phys*. 5.4 is clear that x and y are the same in indivisible species only if x and y have the same terminus *a quo* and the same terminus *ad quem*. Hence white-

[37] *Phys*. 5.4, 227b23-32 (trans. Hardie and Gaye)

to-black alteration and black-to-white alteration are different in species.[38] This is a necessary condition for sameness in indivisible species, and in the categories of quality and quantity it may also be sufficient. But in the special case of motion, Aristotle suggests that this is insufficient for sameness in indivisible species:

> One might ask whether change is one in *eidos* when the same changes from the same to the same, for example one point from this place to this place again and again. But if that were so, circular motion and straight motion would be the same [in *eidos*] and so would rolling and walking.[39]

This passage introduces two extra considerations: the shape of the path taken between the *termini*, and what we might call the ‘manner’ of travel. It is not entirely certain from the context of *Phys.* 5.4 whether Aristotle takes the identity of straight and circular motion and of rolling and walking to be a *reductio ad absurdum* or simply the working out of the commitments of one possible view, though the former may seem more likely. The main argument of *Phys*. 7.4 claims that rectilinear and circular motion are different in species because straight lines and arcs are different in species (249a13-21), but that chapter is more aporetic in tone when discussing the difference made by the manner of motion:

> Spatial motion has species according to the species of the lines on which it moves (and sometimes if the manner is different, for example if feet, walking, if wings, flying - or is this wrong but the motion is different by its shapes?)[40]

However, in two other passages Aristotle straightforwardly states his allegiance to the view that differences in manner of motion constitute differences in species of motion:

> For every movement takes time and is for the sake of an end and is complete when it has made what it aims at… In their parts and during the time they occupy, all movements are incomplete, and are different in kind from the whole movement and

[38] *Phys*. 5.4, 227b7-9: οἷον χρώματος μὲν εἰσὶ διαφοραί—τοιγαροῦν ἄλλη τῷ εἴδει μέλανσις καὶ λεύκανσις.
[39] *Phys*. 5.4, 277b14-18 (trans. mine): ἀπορήσειε δ' ἄν τις εἰ εἴδει μία ⟨ἡ⟩ κίνησις, ὅταν ἐκ τοῦ αὐτοῦ τὸ αὐτὸ εἰς τὸ αὐτὸ μεταβάλλῃ, οἷον ἡ μία στιγμὴ ἐκ τοῦδε τοῦ τόπου εἰς τόνδε τὸν τόπον πάλιν καὶ πάλιν. εἰ δὲ τοῦτ', ἔσται ἡ κυκλοφορία τῇ εὐθυφορίᾳ ἡ αὐτὴ καὶ ἡ κύλισις τῇ βαδίσει.
[40] *Phys*. 7.4, 249a16-19 (trans. mine): καὶ γὰρ ἡ φορὰ εἴδη ἔχει, ἂν ἐκεῖνο ἔχῃ εἴδη ἐφ' οὗ κινεῖται (ὁτὲ δὲ ἐὰν ᾧ, οἷον εἰ πόδες, βάδισις, εἰ δὲ πτέρυγες, πτῆσις. ἢ οὔ, ἀλλὰ τοῖς σχήμασιν ἡ φορὰ ἄλλη;).

from each other. For the fitting together of the stones is different from the fluting of the column, and these are both different from the making of the temple… So, too in the case of walking and all other movements. For if locomotion is a movement from here to there, it, too, has differences in kind – flying, walking, leaping, and so on. And not only so, but in walking itself there are such differences; for the whence and whither are not the same in the whole racecourse and in a part of it, nor in one part and in another, nor is it the same thing to traverse this line and that; for one traverses not only a line but one which is in a place, and this one is in a different place from that. [41]

Very likely, too, there are other attributes, which, though they come under the same general head, exhibit specific differences;—for example, the locomotion of animals: of which there are plainly more species than one—e.g. flight, swimming, walking, creeping.[42]

The indivisible species of motion is defined not only by the termini of motion, but also by the shape of the path between them and by the particular manner of motion. We are nonetheless left wondering how some of these finer distinctions are to be drawn. Sameness and difference of path is reasonably clear, but what constitutes sameness or difference of manner of motion? Is jogging down the street the same or different in manner, and so in species, from skipping down the street? What about differences in gait? Aristotle’s answers are not clear cut.[43]

[41] *EN* 10.4, 1174a19-31 (trans. Ross): ἐν χρόνῳ γὰρ πᾶσα κίνησις καὶ τέλους τινός, οἷον ἡ οἰκοδομική, καὶ τελεία ὅταν ποιήσῃ οὗ ἐφίεται… ἐν δὲ τοῖς μέρεσι καὶ τῷ χρόνῳ πᾶσαι ἀτελεῖς, καὶ ἕτεραι τῷ εἴδει τῆς ὅλης καὶ ἀλλήλων. ἡ γὰρ τῶν λίθων σύνθεσις ἑτέρα τῆς τοῦ κίονος ῥαβδώσεως, καὶ αὗται τῆς τοῦ ναοῦ ποιήσεως… ὁμοίως δὲ καὶ ἐπὶ βαδίσεως καὶ τῶν λοιπῶν. εἰ γάρ ἐστιν ἡ φορὰ κίνησις πόθεν ποῖ, καὶ ταύτης διαφοραὶ κατ' εἴδη, πτῆσις βάδισις ἅλσις καὶ τὰ τοιαῦτα. οὐ μόνον δ’ οὕτως, ἀλλὰ καὶ ἐν αὐτῇ τῇ βαδίσει· τὸ γὰρ πόθεν ποῖ οὐ τὸ αὐτὸ ἐν τῷ σταδίῳ καὶ ἐν τῷ μέρει, καὶ ἐν ἑτέρῳ μέρει καὶ ἐν ἑτέρῳ, οὐδὲ τὸ διεξιέναι τὴν γραμμὴν τήνδε κἀκείνην· οὐ μόνον γὰρ γραμμὴν διαπορεύεται, ἀλλὰ καὶ ἐν τόπῳ οὖσαν, ἐν ἑτέρῳ δ’ αὕτη ἐκείνης. δι’ ἀκριβείας μὲν οὖν περὶ κινήσεως ἐν ἄλλοις εἴρηται, ἔοικε δ’ οὐκ ἐν ἅπαντι χρόνῳ τελεία εἶναι, ἀλλ’ αἱ πολλαὶ ἀτελεῖς καὶ διαφέρουσαι τῷ εἴδει, εἴπερ τὸ πόθεν ποῖ εἰδοποιόν.
[42] *PA* 1.1, 639a30-b3 (trans. Peck): ἕτερα δ’ ἴσως ἐστὶν οἷς συμβαίνει τὴν μὲν κατηγορίαν ἔχειν τὴν αὐτὴν διαφέρειν δὲ τῇ κατ’ εἶδος διαφορᾷ, οἷον ἡ τῶν ζῴων πορεία· οὐ γὰρ φαίνεται μία τῷ εἴδει· διαφέρει γὰρ πτῆσις καὶ νεῦσις καὶ βάδισις καὶ ἕρψις.
[43] One suggestion, due to Charles 1984, ch. 1, is that x and y are same in indivisible species if and only if x and y are realisations of the same type of capacity, where sameness and difference of capacities is to be established by scientific investigation, not *a priori*. This is an interesting suggestion, but it lacks clear textual support. Charles cites *Phys*. 5.4, 228a13-14 for support, but this is inconclusive (see Heinaman 1987, 311). Aristotle’s *De Incessu Animalium* could be seen as delineating different ‘manners’ of motion, but it is unclear whether the distinctions drawn there are final.

Phys. 5.4 also tells us that, because the same change can occur quickly or slowly, quickness and slowness are not species or differentiae of change, and hence differences of weight or lightness are not species or differentiae of change either.[44]

Having established these criteria, let us return to our sailor. His walking and his being carried are located together in the same thing, his body. By the criteria just examined, are they the same change or not? Arguably, the motions in our example will struggle to meet any of the three criteria for identity.

A boat journey is of course likely to be longer than a sailor's walk and in that case condition (ii) will not be met. But clearly this will not hold in every case of interest, and in the case of the sailor we can imagine a short boat journey or a slow walk.

More importantly, the motions fail to meet (i) because they are in the same subject only accidentally. The sailor's being carried is accidental to him. The person on a ship is one of Aristotle's stock examples of accidental change. Saying a sailor moves or a nail moves because the ship moves is like saying that the pale moves because Coriscus moves. Simplicius' comments on this issue are helpful:

> 'For when Coriscus turns black and walks, being himself numerically one, he seems to change in two ways at once. But being one is incidental to the white Coriscus, since Coriscus is incidentally white. That is why the changes are two, even though they occur in a continuous time, for the thing changing also is not one as such but was taken incidentally. For Coriscus does not change in both ways in the same respect, but each of them in different respects, as if two things were changing. For Coriscus grows black in respect of being white, but walks in respect of being a pedestrian, being this as such, but white incidentally.'[45]

Finally, the motions are not the same in species. There are two points of difference. In the first place, walking and being carried by the ship are surely different manners of moving, though, as we have seen, Aristotle' confidence in this requirement wavers. Secondly, these

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[44] *Phys*. 5.4, 228b25-30: οὐκ εἴδη κινήσεως οὐδὲ διαφοραὶ τάχος καὶ βραδυτής ('quickness and slowness are neither species nor differentiae of change').

[45] *Commentary on Physics* 5, 855.11-19, trans. Urmson in Lautner 2014, 85.

changes do not share the same termini. The terminus *ad quem* of the sailor's walking is the other side of the deck. The terminus of his being carried is some place in the direction of the ship's travel. Although the sailor may come to occupy both places at the same time, they are conceptually different.

The alternative would be to claim that the final location of the sailor, defined in relation to the world as a whole, is the terminus of the single resultant motion, but this cannot be Aristotle's view. What one might call the 'GPS location' actually reached by the sailor can play no role in explaining the process or processes which led him to it.[46] If that GPS location were the true terminus of his motion, that would imply that his motion would necessarily cease at that proper place when reached, since motion is an incomplete actuality. This is not the case. The sailor could have walked slower or faster to the other side of the deck. And in those cases, his eventual proper place would be different since the boat would have carried him a greater or lesser distance in its direction of travel. And yet the motion must be the same when the sailor walks faster or slower, since we have seen that for Aristotle speed is not a differentia of change.[47] The sailor's occupying that particular GPS location is an accidental outcome, to be explained in terms of the composition or interference of two goal-directed processes.[48]

The sailor is only one case. How far can we generalise? In particular, do motions which are not intentional or natural retain their identity when composed? A leaf floats downstream and is at the same time blown by the breeze towards the riverbank – one motion or two? In such cases, my approach to these topics through Aristotle's account of change runs into trouble. Accidental change is an area where Aristotle's account of change is weak.[49] However, other aspects of Aristotle's thought may suggest a similar conclusion applies. The resultant-only

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[46] See *PA* 1.1, 641b23-25 on explanatory function of the end (*telos*) towards which a change proceeds 'so long as nothing hinders it'.

[47] Morison 2002, 55-66 distinguishes a 'circumscriptive' notion of containment (the container surrounds the contained on all sides) from a 'receptive' notion (the container is a spatial interval and the contained occupies parts of it) and argues that Aristotle is not interested in the latter, citing Aristotle's arguments against the *diastema* theory of place in *Phys.* 4.4. However, see Rosen 2012, 76-77: '[I]t is implausible that we always aim to arrive at a definite person-sized place. Someone might walk with the aim of arriving at the Acropolis, while being indifferent to where exactly on the Acropolis her walk will take her. If people sometimes move with a proximate end no more specific than that of arriving in the Acropolis, then, according to the second approach, people sometimes go to the Acropolis *per se*.'

[48] Arguably this is an accidental outcome of the kind described in *Phys*. 2.5, where Aristotle's example is one person encountering upon another in the market and recovering his debt, though both parties went there for shopping, not to meet each other or to settle the debt.

[49] Waterlow 1982, 127-29; Coope 2009, 289.

approach would upset the simplicity of Aristotle's understanding of causal powers. Aristotle thinks that an irrational causal power can bring about only one effect (*Met*. Θ.2, 1046b4-7). So it would be strange to say that two causal powers acting on a body from different angles jointly realise a single change along the diagonal, because neither is a power for putting something in that place.

To recap, motions are the same if they: (i) are in the same subject non-accidentally, (ii) have the same start time and the same end time, (iii) are the same in indivisible species. Identity in indivisible species requires the sameness of termini, sameness of the path between these termini, and further the sameness of manner (which is left somewhat vague). I have argued that, by these criteria, component motions retain their distinct identities. In particular, component motions are typically different in species and in the same subject only accidentally, and thus they fail to meet criteria (i) and (iii).

The topic of composed motions in Aristotle has not been extensively discussed in earlier scholarship. The most important contributions are due to Edward Hussey, who addressed the topic of composed motion twice. In his 1983 commentary, he referred to component motions as 'virtual', but affirmed that Aristotle allows for their existence, and that 'where there are 'powers' there are (virtual) changes, and where there are motions there are 'powers' being applied.'[50] This is close to the position for which I have been arguing. However, in his 1991 essay, Hussey took a revised position:

> 'In the simple case of a man in a boat, or a sphere mounted on another sphere, there are two things in motion, and two different component motions; here it is the compounded motion which appears to be not real, a sort of useful fiction. But if there is only one object, being moved in two different ways at once, the compounded motion seems to be the only actual one.'[51]

Hussey still recognises that a single body can have many motions at once and that, at least in some cases, component motions are objective features of the physical world and not the convenient fictions of theorists. So far we are in agreement. However, Hussey's revised

[50] Hussey 1983, xviii, 197-98.
[51] Hussey 1991, 221-222.

account is more restrictive than the position I am arguing for. He requires 'two things in motion' for component motions to be real and in his two examples one of these things is *in* or *on* the other. For cases where this is not the case ('if there is only one object'), there is at most one motion. Hussey does not spell out in detail his reasons for this restriction. I expect that he would justify it in terms of Aristotle's account of place in *Physics* 4.1-4.[52]

The relevant idea here may be that a body has many *per aliud* places but only one *per se* place; so it can have many *per aliud* motions but only one *per se* motion at a time. Aristotle recognises that, in a sense, an object can be in many places at once. I am in the Whipple Library, the Department of History and Philosophy of Science, the New Museums Site, Cambridge, the United Kingdom, and so on. Only one of my many places, the smallest, is my *proper* place, that which does the most to locate me and answer the question 'Where am I?' Aristotle identifies a thing's proper place with the inner limit of its containing body.[53] My proper, primary, or *per se* place is, for example, the inner limit of the air around me in the library. My other places are *per aliud*; I occupy them because I am in some intermediate thing which is in them.[54]

Thus, the sailor's proper place (his position in relation to the ship) is different at different times. Meanwhile his place *per aliud* is different at different times because the ship's proper place (its position in relation to the sea) is different at different times. Something moves *per se* if it changes position relative to its immediate, smallest container, and moves *per aliud* if one of its containers changes position relative to a further container. Schematically, a body x has two continuous motions over a time-stretch T just in case (1) x is in y; (2) y is in z; (3) x's position in relation to y is different at every instant of T; (4) x's position in relation to z is different at every instant of T. But if x is not in a moving container, it has no *per aliud* motion, hence Hussey's restriction for the case where there 'is only one object'. Our ultimate interest is in such a case, the balance-beam of *Mech*. problem 1.[55] On Hussey's interpretation, we would be required to treat the beam's two motions as fictional.

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[52] Hussey 1991, 220n.21: 'Aristotle grounds the distinction between absolute and relative motion, needed here, on the theory of place (*Phys*. IV. 1-4).' What follows is my own reconstruction of Hussey's line of thought. based on this suggestion and Hussey's (1983) comments on *Phys.* 4.1-4.

[53] *Phys*. 4.4, 210b34-211a1: ἀξιοῦμεν δὴ τὸν τόπον εἶναι πρῶτον μὲν περιέχον ἐκεῖνο οὗ τόπος ἐστί; cf. 212a5: τὸ πέρας τοῦ περιέχοντος σώματος.

[54] Cf. Hussey 1983's ad *Phys.* 4.2, 209a31; Morison 2002, 59-61.

[55] Later problems of *Mech.* are typically also concerned with 'only one object'.

First, a comment on this rather than an objection. This interpretation relies heavily on the notion of containing. This notion is vague, and this fact entails a particular kind of vagueness about whether certain motions are occurring or not. When a man is inside a boat he is surrounded by it. When he stands on the deck he is not surrounded, but presumably still counts as in some sense 'in' the ship. Aristotle says that we are on the earth and at another time that we are in the air, but we are not completely surrounded by either.[56] Objects are not always strictly encircled on all sides by a single body and Aristotle's discussion of place seems ill-equipped to handle this fact.

Now to the objection. The argument I have offered above for Hussey's later interpretation relies on essentially the same argumentative move as Descartes, to assume that the facts about a body's motion (or motions) are determined by the places it occupies over time. This assumption is alien to Aristotle's account of change. Further, the Cartesian argument seems to assume that a body moving along a path towards its terminus is located at each intermediate place on its path for at least an instant, though it is debatable that a moving thing is ever actually located at the intermediate places on its path. In Aristotle's second reply to Zeno we read that intermediate locations are potentially but the start and end are actually.[57] As Sarah Waterlow has written:

> 'The rolling object is not nowhere, but nor is it at any moment somewhere either, in the full-blooded sense in which it was and will be somewhere before and after the passage… Rolling is an actuality whereby the subject is not actually anywhere, nor yet nowhere, but (surely) potentially somewhere.'[58]

For independent reasons, Aristotle's account of place seems unlikely to shed light on the trajectories of moving objects.[59] Notoriously, that account struggles to make sense of motion.[60] Aristotle applies it only once in his scientific works (*DC* 4.3). Some commentators have suggested that the account of place may have had a more limited aim, to define only the

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[56] *Phys.* 4.2, 209a33-b1; 4.4, 211a23-29; cf. Sedley 2012, 189-190.
[57] *Phys*. 8.8, 262b8-264a6, especially 262b31-263a1.
[58] Waterlow 1982, 130.
[59] Morison 2002 develops an ingenious interpretation according to which the *unmoved* surrounding body of *Phys*. 4.4 is the universe as a whole. While this eliminates some of traditional difficulties for Aristotle's account of place, I doubt that it is what Aristotle had in mind. For various critical comments along these lines, see Bostock 2006, Sedley 2012, 184-86 Algra 2014, 20-21.
[60] Hussey's own statement of the difficulties (1983, xxx, 117-118) is instructive.

termini of motions rather than their trajectories, or perhaps only to make sense of natural place.[61]

In summary, I have argued that Aristotle's general discussions of change in *Physics* 3.1-2 and 5.4 suggest that component motions should be understood as distinct processes, not as merely useful fictions. There is nothing to prevent a single body from having several motions at the same time. I considered and rejected Hussey's 1991 suggestion that component motions are actual 'if there are two things in motion' but fictitious 'if there is only one object' for reasons connected with Aristotle's theory of place in *Physics* 4.1-5. I objected that this would only follow on the assumption that a body's motion is determined by the places successively occupied by body along its path, and that this assumption should not be attributed to Aristotle. Let us now look beyond the *Physics*, to see what evidence can be found for or against these suggestions in Aristotle's other writings.

2.5: Multiple motions in Aristotle's physical explanations

In this section I review several passages in Aristotle's scientific works that speak of bodies as undergoing many motions simultaneously. In these passages, the number of motions a body undergoes is expressed by saying that it moves '*n* κινήσεις' or '*n* φοράς' where *n* is the specific number of motions it undergoes, or an indeterminate quantifier (e.g. 'many', 'few').[62] *Mech.* generally follows this practice, but in one case the standard text offers the alternative form 'ἐν *n* φοραῖς' for which I have found no parallel.[63] There is no indication in the texts that these claims are not to be taken literally – no statement that component motions are useful but that in reality there is only one motion at a time. I suggest that they can and should be taken literally. The first group of passages I shall consider concern the numbers of motions possessed by various heavenly bodies.[64]

The first section of *DC* 2.12 asks why different heavenly bodies move with different numbers of motions, noting that there is no correspondence between a heavenly body's place in the heavens' concentric ordering and its number of motions. The fixed stars have only one

[61] Sedley 2012, Algra 2014.
[62] E.g. *DC* 2.12, 291b29-292a1, *GC* 2.10, 336a33.
[63] *Mech.* Problem 1 848b24 φερόμενον ἐν δύο φοραῖς.
[64] A complication is that the rotations of heavenly bodies are not directed towards end-states.

motion, but the innermost heavenly bodies, the Sun and Moon, have fewer motions (ἐλάττους… κινοῦνται κινήσεις) than some of the planets that have an intermediate position. Aristotle's tentative suggestion involves an analogy between kinds of sublunary organisms and kinds of heavenly bodies.[65] We tend, he says, to think of the heavenly bodies as 'mere bodies or units, occurring in a certain order but completely lifeless', but we should think of them 'as partaking of life and initiative'.[66] In the extended analogy, the Earth corresponds to plants, which have no (or very few) motions; the Sun and Moon correspond to people who have a relatively limited range of activities; the other planets correspond to people who engage in a wide range of actions in pursuit of various goals; the fixed stars correspond to the person who has just one activity, and the Unmoved Mover corresponds to a person in the best state, who has no need to act.

This passage supports my argument in two ways. First, in this passage Aristotle takes the fact that the heavenly bodies have different numbers of motions as a given fact in need of explanation. Secondly, the heavenly bodies' motions are compared to animal motions which are independent from one another. This strongly suggests that in saying that, for example, the Moon has fewer motions than a more distant planet, Aristotle is not merely claiming that the distant planet has a more complex (single) motion but rather that it has a larger number of independent motions. Aristotle emphasises the variety of human actions (πράξεις) compared to animal actions, not the complexity of individual actions; the analogy could hold even if animal actions were typically more complex in some sense than human actions, so long as they did not aim at a more diverse set of ends. A planet's several motions are as genuinely distinct as the motions a human can undertake in pursuing various ends.

The latter part of *DC* 2.12 addresses a different question. Why is it that the outermost sphere contains many fixed stars, but each of the planets moves with several motions of its own?[67] Aristotle's answer draws on his ideas about the number of motions belonging to each body:

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[65] At *DC* 2.12, 292a15ff. Aristotle stresses the difficulty of investigating objects as distant as the heavenly bodies. See Leunissen, 2010, pp. 165-68.

[66] *DC* 2.12, 292a20-21 (trans. Guthrie). It is debatable whether Aristotle means that the spheres are literally alive or merely that we should think of them as if they were alive.

[67] It is possible to read this in terms of homocentric celestial spheres, although they are not explicitly mentioned in this context. From that point of view, the question is why the outermost sphere contains many fixed stars but each system of lower spheres contains only one star, attached to the innermost sphere of the system, and moving with many motions corresponding to the spheres that make up that system.

‘This then is Nature’s way of equalizing things and introducing order, by assigning many bodies to one motion, and to the one body many motions.’[68]

In *Met.* Λ.8 Aristotle argues that the number of unmoved movers is equal to the number of motions in the heavenly bodies, and that in turn is equal to the number of spheres, each performing one component motion.[69] If the number were simply equal to the number of heavenly bodies, each heavenly body having only one motion, its determination would be trivial. Yet, as Aristotle says, ‘That the motions are more in number than the things which move, is clear even to those who have engaged in the subject to a moderate extent; for each of the wandering stars is moved in respect of more than one motion.’[70] By ‘the things which move’, Aristotle here refers not to the spheres, which are in fact equal in number with the movers (see 1074a14-a31), but rather to the planets and also to the fixed stars taken as a single collective.

These passages concerning the heavenly bodies support my argument only up to a point. Multiple motions are clearly attributed, but these are all cases where, as Hussey put it, ‘there are two things in motion, and two different component motions’.[71] These passages do not tell us anything about cases of a single object in motion, where Hussey sees the component motions as imaginary and I see them as real. For cases of a single object, we must turn to the sublunary domain.

The second group of passages I will now examine concern sublunary phenomena. The component analysis of motions is applied in this region too, and is apparently applied to cases where only a single object is considered. In the *Meteorology*, Aristotle explains the path commonly taken by shooting stars in terms of their moving with two motions:

> διὰ δὲ τὴν θέσιν τῆς ἀναθυμιάσεως, ὅπως ἂν τύχῃ κειμένη τοῦ πλάτους καὶ τοῦ βάθους, οὕτω φέρεται ἢ ἄνω ἢ κάτω ἢ εἰς τὸ πλάγιον. τὰ πλεῖστα δ᾽ εἰς τὸ πλάγιον διὰ

[68] *DC* 2.12, 293a2-4 (trans. Guthrie): Ταύτῃ τε οὖν ἀνισάζει ἡ φύσις καὶ ποιεῖ τινὰ τάξιν, τῇ μὲν μιᾷ φορᾷ πολλὰ ἀποδοῦσα σώματα, τῷ δ' ἑνὶ σώματι πολλὰς φοράς. There are further references to the numbers of motions possessed by heavenly bodies in 2.14, 296a34-296b6, where Aristotle argues against the hypothesis that the earth moves.
[69] *Met.* Λ.8 1074a14-22.
[70] *Met.* Λ.8, 1073b8-10, trans. Judson: ὅτι μὲν οὖν πλείους τῶν φερομένων αἱ φοραί, φανερὸν τοῖς καὶ μετρίως ἡμμένοις (πλείους γὰρ ἕκαστον φέρεται μιᾶς τῶν πλανωμένων ἄστρων). I cannot here do justice to the difficulties of this passage, on which see Lloyd 2000, Bodnár 2005 and Judson 2015.
[71] Hussey 1991, 221-222, discussed above in §2.4.

τὸ δύο φέρεσθαι φοράς, βίᾳ μὲν κάτω, φύσει δ᾽ ἄνω· πάντα γὰρ κατὰ τὴν διάμετρον φέρεται τὰ τοιαῦτα. διὸ καὶ τῶν διαθεόντων ἀστέρων ἡ πλείστη λοξὴ γίγνεται φορά.

'The motion is upwards, downwards or sideways according to the position of the exhalation and whether it happens to lie vertically or horizontally. The motion is most often sideways because it is a combination of two motions, an impressed motion downwards and a natural motion upwards, and bodies under these conditions move obliquely. Therefore the movement of shooting stars is commonly transverse.'[72]

Clearly, Aristotle uses the idea that a thing can move with more than one motion (δύο φέρεσθαι φοράς). This passage has sometimes been seen as an application of the geometrically exact 'parallelogram rule', but it is unclear whether κατὰ τὴν διάμετρον should be taken as 'along the diagonal' of a parallelogram.[73] There is no reference to a quadrilateral or a lettered diagram and the component motions seem to be contrary (κάτω… ἄνω) rather than at angles.[74] It is possible that the passage was originally accompanied by a diagram; that would be one way to make sense of the reference to a διάμετρος.[75] But this is highly speculative, and in any case a diagram depicting a διάμετρος may have been no less qualitative than the text itself.

GA 5.3, 782b18-23 exemplifies how qualitative Aristotle's explanations by composed motions could be. Aristotle suggests that hair becomes curly by being bent by two motions (κάμπτεται γὰρ διὰ τὸ δύο φέρεσθαι φοράς) of the smoky exhalation within it, one earthy and one hot.[76] This unmistakably qualitative example serves as a reminder that explanations in terms of the composition of motion need not involve geometrical precision.

Also relevant are passages where Aristotle takes care to note that a body moves with one motion only, implying that it would be possible for such a body to move with more than one motion. The main purpose of *Meteorology* 3.1 is to explain two violent wind phenomena, the

[72] *Mete.* 1.4, 342a24-28, trans. Lee. Hussey (1983, 197 and 1991, 220) relates this to the 'parallelogram rule' in *Mech.* problem 1.
[73] Hussey 1991, 220 understands it in this way; Berryman 2009, 99-100 is more cautious.
[74] Hussey notes that the violent downward motion may be only *approximately* downwards
[75] A diagram does not need to be lettered. See Taub 2003, 103-115 and 2017, 100-110 on explanation through diagrams in the *Meteorology*.
[76] This is one of two explanations given for curly hair. The other suggests that hair contracts and curls when it is dried out and loses moisture.

ἐκνεφίας and the τυφῶν.[77] To begin with, however, Aristotle addresses an analogous phenomenon which is smaller and more easily observable: eddies of wind (δῖνοι / δῖναι πνεύματος) which occur 'when the wind is forced from a wide place into a narrow place, in gateways or streets'.[78] He explains that the first part of the stream of wind meets with resistance and so cannot move forward, but it is pushed from behind by the rest of the stream and so it is forced sideways. 'This happens to each succeeding part of the stream, till finally it forms one thing, and this is a circle; for any figure possessing a single motion must itself be single.'[79]

In all these cases, specifying the number and direction of motions involved in producing a phenomenon is part of giving a causal explanation of that phenomenon. In the sublunary examples, several motions are identified in situations where there is only one moving object. In Chapter 4, I will argue that *Mech.*'s claim that a rotating radius undergoes two motions simultaneously is similarly part of a causal explanation.

2.6: Conclusion

In the puzzle about the sailor, there is a tension between our ordinary ways of describing what happens and the thought, tempting for *us*, that just as a body can only be in one place at a time, so it can only undergo one change of place at a time. I suggested that modern science and philosophy are responsible for the latter view's appeal. I then argued that the account of change Aristotle offers in the *Physics* implies a different answer, one on which component motions are real. Next, I argued that Aristotle adopts this approach across his works when discussing the composition of motions (*Physics, De Caelo, Meteorology, Generation of Animals, Metaphysics*). I also noted that, although in the case of astronomy he is influenced by geometrical models, some of Aristotle's component analyses of motion are vague and imprecise and do not apply the 'parallelogram rule'.

Aristotle was committed to a set of facts about motion that we moderns typically do not accept. To illustrate this, each of the two diagrams below represents the uniform motion or

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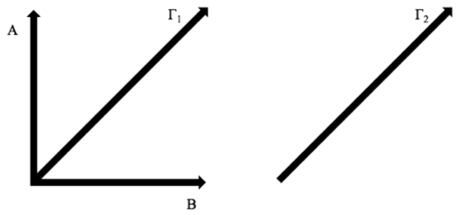
[77] Translated by Lee as 'hurricane' and 'whirlwind' respectively.
[78] *Mete.* 3.1, 370b18-19, trans. mine: ὅταν ἐξ εὐρέος εἰς στενὸν βιάζηται ὁ ἄνεμος ἐν πύλαις ἢ ὁδοῖς
[79] *Mete.* 3.1, 370b25-27, trans. Lee with modifications: καὶ οὕτως ἀεὶ τὸ ἐχόμενον, ἕως ἂν ἓν γένηται, τοῦτο δ' ἐστὶ κύκλος· οὗ γὰρ μία φορὰ σχήματος, τοῦτο καὶ αὐτὸ ἀνάγκη ἓν εἶναι

motions of a single object.[80] In the left-hand diagram, the object moves along path Γ_1 as a result of two component motions, A and B. In the right-hand diagram, the object's motion along path Γ_2 is simple; it cannot be analysed into further components. Aristotle would say that in the case of Γ_1 the object moves with two motions, but in the case of Γ_2 the object moves with only one motion. This because each component motion A, B of Γ_1 is a κίνησις in its own right, since each is the actuality of a potentiality.



Γ_1 is the result of two efficient causes, one acting towards A, the other acting towards B, whereas Γ_2 has only one efficient cause, acting towards the end-point of Γ_2. The peculiar consequence is that a body can be moving with several motions yet be in almost all observable respects identical to a body moving with only one motion. In an important way, the truth about motions in the world is not evident to sensation. On Aristotle's account of change, many facts about motion will not be immediately apparent to us. The truth is accessible, but to determine how many changes are occurring in a given situation, one needs a broader understanding of the patterns of natural causation.

I have not addressed later developments of the topics I have discussed, but there is scope for future work here. How did ancient commentators on Aristotle understand composite motion? We have evidence for the views of at least Xenarchus, Alexander, Simplicius and Philoponus. Studying their disagreements may shed further light on the development of thought about composed motions in Greek science.[81]

[80] I introduce this figure for the sake of exposition. Note that Aristotle's own discussions of composed motions in the sublunar region are not as geometrically precise as the above diagram might suggest.

[81] Xenarchus criticised Aristotle, arguing that if every simple motion belongs to a simple body, then every mixed motion will belong to a mixed body; but there are infinitely many mixed motions and only finitely many bodies (Simplicius, *In De Caelo*, 23.11-15). Simplicius responded that 'the mixed motions are not infinite in their forms either, unless it is because they occur again and again, as bodies do' (ibid. 17-19); Simplicius' point is that Xenarchus' conclusion is not absurd since the infinitely many bodies do not need to exist at a single time. Commenting on *DC* 1.2's reference to the mixed motions of mixed bodies, Alexander wrote that, 'Motions are not mixed in the same way as bodies are. For simple bodies exist together with one another in a mixture, but in the case of motion the prior motion does not survive the second one in such a way that we can say that this has been mixed with this.' (ibid. 17.10-14). According to Alexander, component elements in a mixed body do not retain their individual motions as components of the mixture's motion; one motion dominates and the other is destroyed. Alexander's comments on the ὑπέκκαυμα suggest that he did not oppose the idea that a single body can have two or more motions at a time. Simplicius held that composite motion as in shooting stars is single, but not simple (ibid. 17.5-10).

Chapter 3: Further themes in Aristotle’s account of motions

3.1: Introduction

The previous chapter argued that Aristotle’s account of motion can make literal sense of a body’s undergoing two or more motions simultaneously. This is the main claim that will inform my reading of *Mechanica* problem 1 in Chapter 4. The discussion of Aristotle’s views on less-examined topics such as path shape and composition of motions raise some further considerations which, although not bearing directly on *Mech*. problem 1, are still relevant and worth examining. In section 3.2, I examine Aristotle’s classification of motions as straight, circular, and mixed. In section 3.3, I return to a question I raised in Chapter 2. Does the orientation of one motion with respect to another affect the way in which they compose? In particular, can something simultaneously undergo motions in contrary directions?

3.2: Classifying motions

In the *Physics* (8.8-9) and *De Caelo* (1.2), Aristotle states that every motion is either straight, circular, or mixed.[1] The term here for ‘mixed’, μικτή, should be distinguished from the term συνθέτη, sometimes translated as ‘composite’, which Aristotle applies to *successive* motions that are not really unified, for example when a body moves backwards and forwards on a finite straight path.[2] What is mixed motion? There seem to be two possibilities: either the simultaneous occurrence of two or more motions, or the simultaneous action of a power that is itself ‘mixed’.[3] It is possible that Aristotle uses one term for both.

Although they offer similar classifications of motion, *Phys*. 8.8-9 and *DC* 1.2 differ in aims and approach. *DC* 1.2 aims to show that the heavens are made from a special kind of matter

[1] *Phys*. 8.8 261b28-39: πᾶν μὲν γὰρ κινεῖται τὸ φερόμενον ἢ κύκλῳ ἢ εὐθεῖαν ἢ μικτήν. 8.9 265a13-15: πᾶσα γὰρ φορά, ὥσπερ καὶ πρότερον εἴπομεν, ἢ κύκλῳ ἢ ἐπ’ εὐθείας ἢ μικτή. ταύτης δὲ ἀνάγκη προτέρας εἶναι ἐκείνας. *De Caelo* 1.2 268b17-21: Πᾶσα δὲ κίνησις ὅση κατὰ τόπον, ἣν καλοῦμεν φοράν, ἢ εὐθεῖα ἢ κύκλῳ ἢ ἐκ τούτων μικτή· ἁπλαῖ γὰρ αὗται δύο μόναι. Aristotle speaks of motion (κίνησις/φορά) as ‘straight’ (εὐθεῖα) and ‘mixed’ (μικτή), but ‘in a circle’ (κύκλῳ), but we should not infer that Aristotle means. to define simple circular motion by its path. Hence I prefer to translate ‘circular motion’ rather than ‘motion in a circle’. Cf. *Phys*. 8.8 262a15-16: οὐ γὰρ ταὐτὸν κύκλῳ φέρεσθαι καὶ κύκλον; Plato, *Parmenides* 145b: Καὶ σχήματος δή τινος, ὡς ἔοικε, τοιοῦτον ὂν μετέχοι ἂν τὸ ἕν, ἤτοι εὐθέος ἢ στρογγύλου ἤ τινος μεικτοῦ ἐξ ἀμφοῖν.

[2] In Simplicius the distinction is blurred.

[3] The latter possibility is important for the case of mixed bodies which contain tendencies for both upwards and downwards motion, since Aristotle is thought to prohibit simultaneous contrary motions (see below). The former possibility seems necessary if, as Aristotle’s language suggests, mixed motion is in some sense *mixed from* other motions, and particularly if there are to be mixes of circular and straight motions (see below).

that is not found in the sublunar region and that naturally moves with uniform circular motion about the centre of the universe.[4] In this context, Aristotle restricts his considerations to particular subclasses of possible circular and straight motions: ‘By “circular motion” I mean motion around the centre, by “straight,” motion up and down. “Up means away from the centre, “down” towards the centre.’[5]

The overall argument of *Phys.* 8 aims to show that, since change in the world is eternal and every changed thing is changed by something else, and since there cannot be an infinite regress of movers, there must be at least one eternal unmoved mover. *Phys*. 8.8’s place in this plan is as part of a sub-argument to the effect that if change is eternal there must be at least one eternal motion. Building on 8.7’s conclusion that any eternal change must be a motion, 8.8 argues that among motions only uniform circular motion can be eternal. The argument is more abstract than that in *DC* 1.2, where Aristotle quickly specifies that by ‘straight motion’ he means motion towards or away from the centre of the world while ‘circular motion’ means motion around the centre. *Phys*. 8.8 assumes that the world is finite but does not rely on further assumptions about its geometric structure or about simple bodies’ natural motions.[6]

The main challenge in interpreting these passages is deciding what is meant by ‘mixed’ motion. I shall focus on three possible interpretations. According to what I call the Broad Interpretation, a body moves with mixed motion if and only if it simultaneously undergoes two or more simple motions. These simple components need not be of different kinds. For example, a body undergoing two rectilinear motions would count as moving with mixed motion. This is denied by the two alternative interpretations. The Narrow Interpretation says that a body moves with mixed motion if and only if it simultaneously undergoes two or more motions, of which at least one must be circular and at least one must be straight. Finally, the Line Interpretation claims that a motion is mixed if and only if it follows a mixed line (what this means is explained below).[7] Let us now turn to the texts.

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[4] Aristotle’s preferred term for the stuff of the heavens is ‘the first body’; as *DC* 1.3 explains, ‘aether’ was the traditional name. However, it has long been conventional to refer to Aristotle’s first body as ‘aether’. Aether theory contradicted the standard view (e.g. Plato’s *Timaeus*) that the heavenly bodies are fiery.

[5] 268b20-22, trans. Guthrie: Κύκλῳ μὲν οὖν ἐστιν ἡ περὶ τὸ μέσον, εὐθεῖα δ’ ἡ ἄνω καὶ κάτω. Λέγω δ’ ἄνω μὲν τὴν ἀπὸ τοῦ μέσου, κάτω δὲ τὴν ἐπὶ τὸ μέσον. See Alexander’s comment (Simplicius, *In De Caelo* 14.31-15.1).

[6] There is, for example, no reference to ‘the centre’. *Phys*. 3.5 effectively argued that the world is finite.

[7] Proclus (*In Euc.* 104.1-5) seems to have held the Line Interpretation. Wildberg’s (1988) reading of *DC* 1.2-3 assumes a version of the Narrow Interpretation.

Phys. 8.8 argues that only circular motion can be eternal through a process of elimination. Since the world is finite, any rectilinear motion must have a finite path. An eternal rectilinear motion could therefore only be a case of repeated traversal of a finite path. Most of 8.8 consists of arguments that such a motion could be continuous. Virtually no space is given to mixed motion. Aristotle gives his reason for this imbalance at the outset:

> Ὅτι δ' ἐνδέχεται εἶναί τινα ἄπειρον, μίαν οὖσαν καὶ συνεχῆ, καὶ αὕτη ἐστὶν ἡ κύκλῳ, λέγωμεν νῦν. πᾶν μὲν γὰρ κινεῖται τὸ φερόμενον ἢ κύκλῳ ἢ εὐθεῖαν ἢ μικτήν, ὥστ' εἰ μηδ' ἐκείνων ἡ ἑτέρα συνεχής, οὐδὲ τὴν ἐξ ἀμφοῖν οἷόν τ' εἶναι συγκειμένην.
>
> Let us now say that it is possible for there to be an infinite, one, and continuous motion, and that this is circular motion. For everything moves either in a circle or in straight or mixed. So if one of these is not continuous, neither is that combined from both.[8]

I have translated ἢ κύκλῳ ἢ εὐθεῖαν ἢ μικτήν ambiguously to reflect the Greek. The adjectives 'straight' and 'mixed' qualify a feminine noun, but we could supply either 'line' (γραμμή) or 'motion' (κίνησις). Aristotle argues that if either circular or straight motion is not continuous, the motion composed 'from both' will not be continuous.[9] The sense of the argument and the use of ἐξ ἀμφοῖν (261b30) suggest that the composite motion of the apodosis must be a combination of at least one circular and at least one straight motion.[10] On the other hand, it is not obvious that this is what Aristotle meant by 'mixed' in the tripartition of all motion into straight, circular and mixed. For one thing, he uses different terms (μικτή *versus* συγκειμένη). For another, if this were what he initially meant by 'mixed', his claim that the classification is exhaustive (πᾶν… τὸ φερόμενον) would be false. The Sun does not have circular motion, rectilinear motion, or a combination of these two. As we have seen, Aristotle thinks it has a complex, spiralling motion that results from multiple circular components. It would be strange if Aristotle's apparently exhaustive classification of motions failed to account for something as familiar and important as the Sun's motion. That seems to

[8] 8.8, 261b27-31 (trans. mine).
[9] I assume that ἐκείνων and ἐξ ἀμφοῖν refer to both circular and straight motion.
[10] What in Aristotle's world has such a motion? Only one suggestion comes to mind, but it seems unlikely Aristotle had it in mind in *Phys*. 8 or *DC* 1. In *Mete*. 1.3 Aristotle describes the fiery upper region of the atmosphere immediately below the sphere of the Moon, which he calls ὑπέκκαυμα. He claims that this is carried in a circular motion by the heavenly rotation that touches it (340b32-41a12). Some bits of fire in or near this region may thus experience a mixture of circular and rectilinear motion.

be a consequence of the Narrow Interpretation. Even if the classification is only intended to cover natural motions, the Sun's motion would seem to be a counterexample.[11]

I think it is more likely that the initial mention of 'mixed' motion in the tripartite classification is more general than the mention of composed motion in the apodosis, and that it includes circular-circular, circular-straight and straight-straight mixes. If that is right, Aristotle takes for granted that circular-circular and straight-straight mixes will be continuous or discontinuous according as their components are. I shall discuss these assumptions in Chapter 4 after we have seen a possible objection. My current point is that the Broad Interpretation offers a more charitable reading of *Phys*. 8.8 than the alternatives.

The opening sentences of 8.9, arguing that circular motion is prior to all other motions, are not committed to the idea that mixed motion must involve both circular and straight components:

> Ὅτι δὲ τῶν φορῶν ἡ κυκλοφορία πρώτη, δῆλον. πᾶσα γὰρ φορά, ὥσπερ καὶ πρότερον εἴπομεν, ἢ κύκλῳ ἢ ἐπ' εὐθείας ἢ μικτή. ταύτης δὲ ἀνάγκη προτέρας εἶναι ἐκείνας· ἐξ ἐκείνων γὰρ συνέστηκεν. (8.9, 265a13-16)
>
> 'It is clear that circular motion is primary among locomotions. For all locomotion, as we have said before, is circular or on a straight line or mixed. These must be prior to this, since it is made up from them.' (trans. mine)

The Greek here is less ambiguous than the passage of 8.8: μικτή must qualify φορά. This does not decisively rule out the Line Interpretation, but we would expect μικτῆς if mixed motions were defined in terms of mixed lines.[12] Note also that mixed motion is said to consist ἐξ ἐκείνων, 'of these', rather than ἐξ ἀμφοῖν 'of both' as Aristotle might have said if the Narrow Reading were correct. However, this passage can be understood adequately on any of the candidate interpretations.

[11] One could defend the Narrow Interpretation by suggesting that circular-circular and straight-straight mixes are accounted for in the categories of circular and straight motion respectively. However, Aristotle attributes properties such as uniformity, which some circular-circular mixes do not have, to the class of circular motions without qualification.

[12] E (Par. Gr. 1853) and K (Laur. 87.24) have εὐθεῖα for ἐπ' εὐθείας.

We now turn to *DC* 1.2:

> Πᾶσα δὲ κίνησις ὅση κατὰ τόπον, ἣν καλοῦμεν φοράν, ἢ εὐθεῖα ἢ κύκλῳ ἢ ἐκ τούτων μικτή.
>
> All change with respect to place, which we call locomotion, is either straight or circular or mixed from these. (268b17-18, trans. mine)

We have already seen that in, *DC*, 'circular' means around the centre, and 'straight' means towards or away from the centre. In this passage, εὐθεῖα and μικτή qualify κίνησις and mixed motion is ἐκ τούτων rather than ἐξ ἀμφοῖν as it might have done on the Narrow Reading.[13] Aristotle next makes a brief statement offering partial justification of his classification: 'The reason is that these, the straight and the circumferential, are the only simple magnitudes.'[14]

The connection made between simple motions and simple lines might suggest a similar correspondence between mixed motions and 'mixed lines', although Aristotle does not make the connection. In fact neither Plato, nor Aristotle, nor Euclid explicitly use the category of a mixed line. Proclus presents two classifications of lines due to Geminus,[15] the second of which (111.9-20, 112.16-113.3) distinguishes simple (ἁπλῆ) from mixed (μικτή).[16] Simple lines are divided into 'making a figure' (σχῆμα ποιοῦσα), the circle, and 'indeterminate' (ἀόριστος), the straight line. The classification of mixed lines is much more complex and encompasses virtually all other lines used by Greek geometers. Interestingly, the cylindrical helix is classed as a mixed line despite being homoeomeric, as Apollonius of Perga had proven in the late third century.[17] A more decisive problem for the Line Interpretation is that it becomes difficult to see how *Phys*. 8 could reach conclusions about the discontinuity of some mixed motions from claims about the discontinuity of rectilinear motions.

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[13] *Pace* Guthrie (1939, 11: 'either straight or circular or a compound of the two') and Wildberg (1987, 44: 'either rectilinear or circular or a combination of the two').
[14] Αἴτιον δ' ὅτι καὶ τὰ μεγέθη ταῦτα ἁπλᾶ μόνον, ἥ τ' εὐθεῖα καὶ ἡ περιφερής (268b19-20, trans. my own).
[15] Geminus probably wrote in the early first century BCE (see Jones, 1999).
[16] See the convenient tree diagram in Heath, 1956, p.161.
[17] A line is homoeomeric if and only if any part can be made to coincide with any other. The circle, straight line and cylindrical helix are the only homoeomeric lines in three dimensions. Xenarchus of Seleuceia seems to have taken Aristotle's 'simple' to mean homoeomeric (Simplicius, *In De Caelo* 13.25–26).

Aristotle next draws an analogy between simple and mixed bodies and simple and mixed motions: ‘Since some bodies are simple and some composed of them… it must be that also motions are some of them simple and some mixed in some way, and that the simple [motions] belong to the simple [bodies], and the mixed to the composite, and they move in accordance with the dominant.’[18] The simple bodies indeed have motions on simple lines: aether in a circle, the rest in straight lines up or down. The term ‘composite (σύνθετα) bodies’ is less familiar. Simplicius suggests that Aristotle here refers to the observable bodies we commonly call earth, water, air and fire, in contrast to the pure forms of the elements which we never observe, but it is possible that he means any inanimate body that is a mixture or juxtaposition of elements.[19]

Here again I find the Broad Interpretation (a body moves with mixed motion if and only if it simultaneously undergoes two or more simple motions) offers the most charitable interpretation. Scholars who have pursued the Narrow Interpretation have found Aristotle’s arguments in *DC* 1.2-3 badly mistaken, if not incoherent.[20] In *DC,* Aristotle argues for a sharp separation of the heavens from the sublunar world. His arguments for the aether are the key to this cosmic diptych. But if the separation really is sharp, heavenly circular motion would not mix with sublunary rectilinear motions. So there would be no natural mixed motions, despite Aristotle’s claims in this chapter. A second problem is that the composite bodies Aristotle describes would not have mixed motions since they only have rectilinear components. And again, it is difficult to account for the composite motions of the heavenly bodies on the Narrow Interpretation of the scheme.[21]

[18] Ἐπεὶ δὲ τῶν σωμάτων τὰ μέν ἐστιν ἁπλᾶ τὰ δὲ σύνθετα ἐκ τούτων… ἀνάγκη καὶ τὰς κινήσεις εἶναι τὰς μὲν ἁπλᾶς τὰς δὲ μικτάς πως, καὶ τῶν μὲν ἁπλῶν ἁπλᾶς, μικτὰς δὲ τῶν συνθέτων, κινεῖσθαι δὲ κατὰ τὸ ἐπικρατοῦν. (268b29-269a2, trans. my own).
[19] Simplicius, *In De Caelo*, 16-17. I suspect the reason Aristotle uses σύνθετα rather than μικτά is that he generally reserves μικτ- words for a special kind of homogeneous mixture. Here, however, he means more generally bodies that do not consist of one element only. On the other hand, when he uses μικτά in recalling the point at 269a28. Alexander (as reported by Simplicius, *In De Caelo* 37.13-15) suggests the mixed bodies in question are in the upper atmosphere, assuming the ὑπέκκαυμα theory of *Mete*. 1.3. This is very difficult to integrate with *DC* 1’s arguments.
[20] The Narrow interpretation states that a body moves with mixed motion if and only if it simultaneously undergoes two or more motions, of which at least one must be circular and at least one must be straight.
[21] Wildberg 1988 (47-48, 51) draws these conclusions but views them as defects in Aristotle’s account: ‘Aristotle speaks of composite natural movement in terms of rectilinear and circular movement. The incompatibility is apparent, for if this view is interpreted on the level of body, a composite movement requires an underlying body which is composed of a terrestrial and the celestial element.’

The interpretation I favour claims that mixed motions result from the composition of two or more simple motions, or powers for simple motions, but that these can be all straight or all circular. If this is correct, there can be mixed motions despite the sharp separation of the heavens and sublunar world, since a motion composed of two simple rectilinear motions is a mixed motion.[22] This makes better sense of *Phys*. 8 and *DC* 1.2 than the alternatives.[23]

Aristotle does not attempt to justify the classification of motions as straight, circular and mixed, though I have noted its Platonic provenance.[24] Since it features as a dialectical premise, it should have seemed a plausible assumption. It resonates with the fundamental role of the circle and straight line in fourth-century geometry.[25] Some modern commentators charge that the scheme is seriously mistaken and unable to account for complex curves such as conic sections. These commentators typically assume the Narrow Interpretation.[26] On the Broad Interpretation, the objection loses its strength. Aristotle never mentions conic sections but his scheme, on the Broad Interpretation, could at least in principle accommodate motions along such section.[27] Parabolic motion can result from two rectilinear motions, one uniform and one uniformly accelerating, as was shown by Galileo; and elliptical motion can be shown to result from rectilinear motions. More generally, classical (Newtonian) mechanics accounts for all motions in terms of rectilinear velocities and forces. A physical theory based on straight motions alone can go a long way. The problem is not that Aristotle's scheme cannot accommodate such motions, but that he had no way of showing how it could.

[22] This seems to be what Simplicius had in mind (*In De Caelo*, 16-17).

[23] Philoponus *In Mete*. 65.37 calls the result of two rectilinear motions 'mixed'. See also Galileo *Two World Systems:* 'This eventually forces people to say that even motion made along the same straight line is sometimes simple, and sometimes mixed. Thus the simplicity of the motion no longer corresponds to the simplicity of the line alone.' (trans. Drake 1967, 17).

[24] *Parmenides*, 145b3-5, discussed in Chapter 1.

[25] Mendell 1986, 363: 'The structure of constructions as being from the simple figures and as constructible by means of basic tools is, in part, a pragmatic fact. Until the mid-fourth century, alternatives did not exist. Among the philosophers, this pragmatic fact of the limitations of construction becomes part of ontological fact.'

[26] Graham 1999, 135: 'But to say that all curves can somehow be derived from the straight line and the circle is to make a bold claim which needs argument. There are of course many complex curves which cannot in any rigorous sense be reduced to a combination of the straight line and the circle.'; Wardy 1990, 272: 'Why then does Aristotle introduce the notion of a kinesis which is a hybrid of motion along a line and a curve, a notion which is apparently incoherent by his own lights? Perhaps he has carelessly confused the unnatural movement of a projectile with the innate movement of a compound body… on any story, a ballistic parabola will prove an embarrassment.'

[27] This is an interesting omission, since the conic sections had been discovered (Mendell 1986 and 2004).

3.3: Simultaneous contrary motions

In Chapter 2, section 2, I raised the question of whether a body can undergo motions in opposite directions at the same time. The difficulty I shall address in this section is that although Aristotle seems to deny that this is possible, since he denies generally that anything can undergo opposite changes at the same time, there are nonetheless some cases where it seems he should accept that this is what occurs.

Sextus Empiricus describes a situation in which it seems that simultaneous contrary motions occur.[28] His aim in doing so is to refute a definition of motion as 'transition from place to place, either of the whole body or of parts of the whole', for in such cases the body is not thought to go out from the place it is in.[29] Sextus gives the example of someone walking astern on a ship, carrying an upright rod. 'In the case thus supposed,' he says, 'there will certainly be transitional motion, but the moving object will not go out from the place wherein it is either wholly or in part… It is, then, possible for a thing which does not quit the place wherein it is either wholly or in part to move transitionally.'[30]

Galen discusses another example and offers an argument for what Sextus assumes, that the body in question is in fact moving. In Galen's example, there is only one object: 'Imagine a lofty bird which appears to be staying in the same place. Should one describe it as motionless, as though it happened to be suspended from above, or as moving upwards to the same extent as the weight of its body carries it downwards? I think the latter is more correct. If you killed the bird or destroyed its muscular tension, you would see it fall quickly to the ground. That makes it plain that the bird was evenly counterbalancing its innate downward inclination due to the weight of its body by the upward motion resulting from its soul's tension.'[31]

Paradoxically, the hovering bird must move to stay in the same place.[32] What would Aristotle make of these cases? On the interpretation I have presented, it seems that he should agree

[28] *Adv.Math.* 10.55-57

[29] Trans. Hankinson 2015.

[30] Trans. Bury.

[31] *On Muscular Movement* 4.402.12-403.10K = LS 47K (trans. Long and Sedley). Galen dismisses as irrelevant a question as to whether the bird truly stays in the same place or whether it in fact is constantly vibrating with rapid and minute up and down motions in turn.

[32] 'Well, in *our* country,' said Alice, still panting a little, 'you'd generally get to somewhere else—if you ran very fast for a long time, as we've been doing.' 'A slow sort of country!' said the Queen. 'Now, *here*, you see, it

with Galen’s conclusion and also Galen’s argument for it. That counterfactual argument shows that the bird’s potential to be upwards is incompletely actual: it is being manifested, making a difference, and this is evident when its strength is destroyed. Unfortunately, our answer cannot be so simple. Aristotle several times says that the same thing cannot undergo contrary changes at the same time.[33] To understand what this claim means, and how much of a problem it poses, we should examine its applications by turn.[34] First, I will consider the applications to qualitative change, and afterwards turn to motion.

At the close of the *De Anima*’s account of perception, Aristotle discusses how we discriminate between perceptible objects, both heterogeneous like white and sweet, and homogeneous like white and black. In the first place, he argues that there must be a single thing which distinguishes white and sweet. Next, he argues that this single thing which distinguishes perceptible objects must be inseparable and indivisible, and that it must discriminate perceptible objects at a single time and not at separate times. He then raises an objection to the idea that a single thing could discriminate homogeneous perceptibles like white and black: ‘But it is impossible for the same thing *qua* indivisible to be changed with contrary changes simultaneously, and in an indivisible time.’[35]

This thought is echoed in *De Sensu* 7, 448a1-5, and here we have a brief argument for the principle: ‘Again, if movements of contraries are themselves contrary, and if contraries cannot subsist together in the same indivisible subject, and if contraries, e.g. sweet and bitter, come under one and the same sense-faculty, we must conclude that it is impossible to discern them simultaneously.’[36] If contrary changes were in the same thing at the same time, then contraries would be in the same thing. And, as Aristotle argues in *Met*. Γ.3-8, that is

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takes all the running *you* can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!’ (Carroll, *Through the Looking-Glass*, ch.2)

[33] *Phys.* 8.8, 264a7-22; *DC* 2.13 295b11-16; *DA* 3.2 426b29-31; *De Sensu* 7 448a1-5. Plato used a similar principle to argue for the tripartition of the soul in *Rep*. 4. 436b-437a: ‘the same will not do or suffer contraries, at least with regard to the same and towards the same, at the same time.’ On this passage, see Robinson 1971.

[34] Hussey 1983, xvii-xviii and Coope 2009, 290 see this as a significant counterexample to Aristotle’s account of change: ‘Aristotle takes it for granted that something cannot be moving in two opposite ways at the same time… so he cannot reply that in such a case the stone remains stationary in virtue of undergoing two opposite motions that cancel each other out. To answer this objection Aristotle would have to spell out a sense of “incompletely actual” in which a potential for F was incompletely actual when (and only when) there was a change towards F.’

[35] *DA* 3.2, 426b29-31: ἀλλὰ μὴν ἀδύνατον ἅμα τὰς ἐναντίας κινήσεις κινεῖσθαι τὸ αὐτὸ ᾗ ἀδιαίρετον, καὶ ἐν ἀδιαιρέτῳ χρόνῳ.

[36] Trans. Beare with slight modification. Ἔτι εἰ αἱ τῶν ἐναντίων κινήσεις ἐναντίαι, ἅμα δὲ τὰ ἐναντία ἐν τῷ αὐτῷ καὶ ἀτόμῳ οὐκ ἐνδέχεται ὑπάρχειν, ὑπὸ δὲ τὴν αἴσθησιν τὴν μίαν ἐναντία ἐστίν, οἷον γλυκὺ πικρῷ, οὐκ ἂν ἐνδέχοιτο αἰσθάνεσθαι ἅμα.

something absolutely impossible.[37] The occurrence of contrary changes in the same thing at the same time is ruled out by ‘the securest principle of all’[38] since changes towards contraries are themselves contrary.[39]

Let us now turn to the passages which bring the prohibition against simultaneous contrary change to bear on spatial motion. In *DC* 2.13, 295b11-16, the prohibition features as a premise in an argument that is not Aristotle’s and which he in fact rejects, so it is doubtful that this application of the principle can offer insight into Aristotle’s positive commitments. Anaximander and certain unnamed others claimed that the earth rests at the centre of the world due to uniformity (ὁμοιότης).[40] The argument from uniformity explains the earth’s being at rest from its position at the centre of the world. What is at the centre (a) has no impulse for motion up, down or sideways; (b) is related similarly to the limits.[41] Furthermore, (c) it is impossible to move towards opposites simultaneously.[42] Therefore, the earth, which is at the centre, is necessarily at rest.[43] Aristotle raises two objections. He alleges that (a) is false and complains that the theory cannot explain why earth falls towards the centre when displaced.[44] He does not explicitly reject (c), but the rival cosmologists are represented as applying (c) in a way that Aristotle could fault, since the argument assumes diametrically opposed limits of the world are opposites. Since what is at the centre is equidistant from the world’s limits, it has no sufficient reason to move in one direction rather than another. So if it moved in one direction it would move in all directions; but it is impossible to move towards opposites simultaneously. Therefore it does not move at all. Aristotle would deny that diametrically opposed limits of the world are mutual opposites; they are parts of the periphery which is the opposite of the centre.

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[37] e.g. *Met.* Γ.3, 1005b27-28: μὴ ἐνδέχεται ἅμα ὑπάρχειν τῷ αὐτῷ τἀναντία.

[38] *Met.* Γ.3, 1005b18: πασῶν βεβαιοτάτη ἀρχή.

[39] That thought is expressed not only in *De Sensu* 7, but also in *Categories* 14, 15b1ff. (trans. Ackrill): ‘Change in general is contrary to staying the same. As for the particular kinds, destruction is contrary to generation and diminution to increase, while change of place seems most opposed to staying in the same place—and perhaps to change towards the contrary place (upward change of place, for example, being opposed to downward and downward to upward).’

[40] We can only guess who else Aristotle has in mind: possibly Plato, who deploys arguments from uniformity at *Phaedo* 108e4-109a7 and *Timaeus* 62d12-63a2.

[41] ὁμοίως πρὸς τὰ ἔσχατα ἔχον.

[42] ἅμα δ’ ἀδύνατον εἰς τὸ ἐναντίον ποιεῖσθαι τὴν κίνησιν.

[43] ὥστ’ ἐξ ἀνάγκης μένειν.

[44] Aristotle rejects (a) through a thought-experiment: fire placed at the centre would not stay still but move to the circumference. He does not explain why it would move along one radial path rather than another.

The principle's application in *Physics* 8.8 is more complex. In this chapter, Aristotle presents five arguments to support his claim that there cannot be an eternal, continuous, and single motion on a finite straight line. The third argument (264a7-22) may be summarised as follows:

1) If X is moving continuously and is not deflected from its original path, and during this motion X arrives at B, then X was moving towards B from the start of its motion.
2) If X is moving continuously and eternally back and forth along a finite straight line from A towards Γ, it is also moving towards A from Γ.[45]
3) Therefore X is moving with contrary motions simultaneously, A to Γ and Γ to A.[46]
4) Furthermore, 'it is moving from what it is not in'.[47] This presumably means that X, setting out at A, simultaneously moves from Γ to A despite not yet being at Γ.
5) 'If, then, this is impossible (εἰ οὖν τοῦτ' ἀδύνατον), it must rest at Γ.'[48]

The concluding sentence of this argument is less clear than we might like. What does the 'this' (τοῦτ') pick out as impossible? Some commentators take (3) and (4) as distinct consequences, which are both independently ruled out as impossible.[49]

There are two alternatives. The singular 'this' (τοῦτ') suggests that Aristotle rejects only one conclusion as impossible. This could be either (4) or the conjunction of (3) and (4). In the latter case, his point would be that it is impossible for a body to undergo simultaneous contrary motions in a specific sense: one in which the start-points of the simultaneous contrary motions are distinct. He does not mean to deny that the grey can blacken and whiten simultaneously.[50] And (4) is not a separate conclusion that Aristotle regards as impossible. Its role in the argument is to specify that (3) describes a case of simultaneous contrary motion that is not like the case of something grey simultaneously blackening and whitening. When

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[45] τὸ δὴ ἀπὸ τοῦ Α [ἐπὶ τὸ Γ] φερόμενον, ὅταν ἐπὶ τὸ Γ ἔλθῃ, πάλιν ἥξει ἐπὶ τὸ Α συνεχῶς κινούμενον. ὅτε ἄρα ἀπὸ τοῦ Α φέρεται πρὸς τὸ Γ, τότε καὶ εἰς τὸ Α φέρεται τὴν ἀπὸ τοῦ Γ κίνησιν (264a14-17)

[46] ὥσθ' ἅμα τὰς ἐναντίας· ἐναντίαι γὰρ αἱ κατ' εὐθεῖαν (264a17-18)

[47] ἅμα δὲ καὶ ἐκ τούτου μεταβάλλει ἐν ᾧ οὐκ ἔστιν (264a18-19)

[48] εἰ οὖν τοῦτ' ἀδύνατον, ἀνάγκη ἵστασθαι ἐπὶ τοῦ Γ (264a19-20)

[49] For example, Graham 1999, 150-51 claims that Aristotle draws two distinct conclusions, 'M is moving in contrary directions' and 'M is moving from a point where it has not been', and says both are impossible. Cf. Ross 1936, 450: 'if then rectilinear motions are contrary, and a thing cannot move with contrary motions at once, that which is moving from A to Γ cannot simultaneously be moving from Γ to A'.

[50] On this interpretation, Aristotle is not vulnerable to Bostock's (1996, 296) objection: 'We should get the same, apparently absurd, result by applying Aristotle's reasoning to a movement, in a single direction, from A via C to somewhere else.'

the grey subject simultaneously blackens and whitens, both changes have the state that the subject is actually in, grey, as their start-point.[51] That is not the case for the hypothesised body in 8.8 and Aristotle's reason for stating (4) is to specify carefully his objection. *Phys.* 8.8 does not offer incontrovertible evidence for a blanket objection to simultaneous contrary motion, but only to simultaneous contrary changes that require their common subject to be moving from a state which it is not in. However, even if Aristotle does make the general assumption that the same subject cannot undergo contrary motions simultaneously (cf. *Phys*. 8.7, 261b5–7), that assumption is not essential to his purposes since the argument in which it features is one of five for the same conclusion.

Let us now turn to *Phys*. 5.5, where Aristotle asks what it is for two changes, x and y, to be contrary. His answer is that x and y are contrary only if their start-points are contrary and their ends-points are contrary.[52] A body subject to equal pulls in contrary directions is not like this. It is like a body at the midpoint C of line AB that moves from C to A and moves from C to B. Even if the two motions' ends, A and B, are contrary, their starts certainly are not, for they are the same, C. So far it seems that the body subject to equal pulls in contrary directions experiences motions in contrary directions but not contrary motions.

Unfortunately, there is a complication in *Phys*. 5.5's account which prevents this straightforward solution from working. Aristotle says that changes from opposites to an intermediate (μεταξὺ) should also be counted as changes to opposites because the change 'uses the intermediate as an opposite'.[53] He does not explicitly say that changes from the intermediate to the opposites should be counted as opposite changes, but this further claim is strongly implied by his examples: grey acts as black in grey-to-white and white-to-grey changes, but as white in black-to-grey and grey-to-black changes (229b14-21). It may follow that something grey cannot whiten and blacken at the same time. Although the start-point of these changes is the same shade of grey, it acts as black for the whitening and as white for the blackening so that the changes are opposite. Now, one might suppose that the same account should apply to the case of the body at C undergoing a motion from C to A and an equal

[51] Even if this state 'acts as' something different in each case (*Phys*. 5.5, 229b14-21; see discussion below) or even if one were to say that each change has a different start-point.
[52] κίνησις μὲν δὴ κινήσει ἐναντία οὕτως ἡ ἐξ ἐναντίου εἰς ἐναντίον τῇ ἐξ ἐναντίου εἰς ἐναντίον (229b21-22). *Phys.* 5.5's arguments suggest that this is a necessary condition for contrariety. Whether it is also a sufficient condition is less clear. *DC* 1.4, 271a5-10 (discussed below) suggests that other conditions must be met.
[53] ὡς ἐναντίῳ γὰρ χρῆται τῷ μεταξὺ ἡ κίνησις (229b16).

motion from C to B. Could one not say that the midpoint C acts as B for the C-to-A motion and as A for the C-to-B motion? In that case, we should expect the same conclusion, that the body at C undergoes opposite motions.

One might think that the cases are not analogous and the conclusion does not follow. The opposition of white and black is independent of any particular change between them, but the 'opposites' involved in locomotion are not generally like this. Aristotle thinks the centre and periphery of the world are naturally opposed, and so natural motions up and down are opposite.[54] But this is a special case. When a stone moves horizontally, the termini of its motion are only coincidentally opposites. Their opposition is parasitic on the structure of some particular change between them and it is only relative to that change that anyone has reason to think of them as opposites.[55] This seems to be the case for the termini of all finite locomotions, except the natural motions of sublunary simple bodies.[56] That this is true can be seen from the fact that any start-point A of a motion to B can also serve as the start-point for a motion to a third point C. Indeed anywhere can serve as the start-point for infinitely many changes to infinitely many end-points. So if A and B's being the termini of a change were sufficient to establish that they were objective opposites rather than opposites relative to the structure of the change in question, then A and B and indeed any start-point would have an infinity of opposites. This is absurd, because each thing *per se* has only one opposite or none.[57] Something can only have more than one opposite if, like grey, its role as an opposite is relative to and dependent on particular changes. C is spatially intermediate between A and B in the sense of occupying the middle position of a straight line between them, but there is an infinity of pairs of points to which C bears this relation, and its bearing this relation to a pair of points does not make them contraries, otherwise every spatial point would be the opposite of every other, which is surely false.[58]

This may sound reasonable, but it does not fit Aristotle's earlier definition of 'opposite' in change of place (*Phys*. 5.3, 226b32-34 trans. Waterfield): 'In change of place, 'opposite'

[54] 'Motions on a straight line oppose each other on account of the places: for up and down are a difference and opposition of place.' *DC* 2.4, 271a3-5.
[55] By 'structure' I have in mind two particular features of a change: (i) its having definite termini, (ii) the ordering of before and after.
[56] I exclude these elemental motions from the following argument.
[57] Ἀλλ' ἓν ἑνὶ ἐναντίον (*DC* 1.2, 269a14). "Doesn't the centre of the world have multiple opposites, every place on the periphery of the universe?" The periphery is one in definition as the natural place of fire.
[58] In fact every point will be the opposite of every other point since it is possible to draw a straight line between any two points in a sphere.

refers to that which is furthest away in a straight line, because a straight line is the shortest distance between two points and is therefore limited; so it acts as a measure, just as anything limited does.' In *Phys*. 5.5 229b6-10 we read that 'movement upwards is taken to be the opposite of movement downwards (since the end-points are opposed on the dimension of length), movement to the right is taken to be the opposite of movement to the left (since the end-points are opposed on the dimension of breadth), and movement to the front is taken to be the opposite of movement to the back (since here too there are opposite end-points).' (trans. Waterfield). Here, motions are contrary although their start-points and end-points are not opposites independently of the changes in question.

We should try a different approach. Perhaps, for the principle that nothing can undergo simultaneous contrary motions to apply, the subject of the two motions must be the same *non-coincidentally*. The sailor walking astern has two motions, but is the proper subject of only one of them, his walking. He is only coincidentally being carried in the direction of the ship. The bird is the proper subject of its self-motions but the proper subject of its downwards tendency is its heavy bodily matter. The bird is not unqualifiedly the same as its matter, so when the bird is hovering it will not be true to say without qualification that 'the same thing is moving with opposite motions'.[59]

In line with this suggestion, in *Phys*. 8.4 Aristotle draws a sharp contrast between an animal's movements and its body's movements, noting that an animal's self-motion is always natural for the animal as a whole but may be unnatural for its body:

> What is moved by itself is moved by nature … That is why the animal as a whole moves itself by nature; however, its body may be moved both by nature and contrary to nature. For it makes a difference with what sort of movement it happens to be moved and from what element it is composed.[60]

[59] *DA* 2.1 is the classic statement of Aristotle's hylomorphism. For a similar dissolution of a different puzzle about motion, see my discussion of *Mech.* 24 in Chapter 6.
[60] *Phys*. 8.4, 254b14-20, trans. Graham: τό τε γὰρ αὐτὸ ὑφ' αὑτοῦ κινούμενον φύσει κινεῖται… διὸ τὸ μὲν ζῷον ὅλον φύσει αὐτὸ ἑαυτὸ κινεῖ, τὸ μέντοι σῶμα ἐνδέχεται καὶ φύσει καὶ παρὰ φύσιν κινεῖσθαι· διαφέρει γὰρ ὁποίαν τε ἂν κίνησιν κινούμενον τύχῃ καὶ ἐκ ποίου στοιχείου συνεστηκός.

As a solution to the problem of simultaneous motion, this distinction is somewhat speculative, since I have not found it explicitly used for that purpose in Aristotle.[61] It is also only a partial solution. Cases where the contrary changes share a subject that is the same without qualification could still pose a problem. An alternative open to Aristotle would be to abandon the assumption that changes between contraries are contrary. I have argued that this would have only minor consequences for his theoretical investigations.[62] The issue does not directly bear on the interpretation of *Mech*. problem 1, our topic in Chapter 4, since the composed motions considered problem 1 are all orthogonal, and Aristotle is clear that orthogonal motions are not contrary.[63]

Finally, let us turn very briefly to Aristotle's account of celestial motions, since this involves what may appear to be simultaneous contrary motions. According to *Met.* Λ.8, 1073b38-1074a5, the complex motions of each planet are produced by a unique system of nested concentric spheres, with the planet attached to the innermost sphere of its system. To prevent the motions produced by a planet's system of spheres from being transmitted to every lower system, there is a further system of 'rewinding' spheres located beneath each planetary system which neutralise the motions of the higher planetary system.[64] Rewinding spheres achieve this by moving with two apparently opposite revolutions, one transmitted to them by a higher sphere, and one that they perform in order to cancel this out. The motion of a rewinding sphere and the motion inherited from the planetary sphere might seem to be contrary, since they are circular motions of equal speed yet in opposite directions about a common axis. If that were so, they might strengthen my suggestion about cases like the sailor on the ship, as another case of simultaneous contrary motions in a single subject, where one of these motions belongs to that subject properly and one has been transmitted to it.[65] Yet

[61] Further queries can be raised but not answered. In our discussion of species of change in Chapter 2, we saw that sameness of start-points and end-points was sufficient for identity in categories of change other than motion. *Phys*. 5.4 touched on the further conditions for the case of motion only briefly, almost as an afterthought. *Phys*. 5.5's discussion of opposite motions focusses entirely on start-points and end-points. Have further conditions for the case of motion been overlooked in this chapter? Is a straight motion from A to B opposed to a curved motion from B to A or only to a straight motion from B to A? In general, are the path-shapes and manners of motions relevant to their opposition, just as they were to their identity? The distinction drawn in *Phys*. 8.4, 254b14-20 also raises the question of how to understand spontaneous, automatic or reflex motions in living things.

[62] We have seen that his arguments in *DC* 2.13 and *Phys*. 8.8 do not require this principle.

[63] *Phys*. 8.8, 262a12.

[64] See Bodnár 2005 for a discussion of some of the difficulties with this arrangement.

[65] The cases are not exactly analogous since the sailor need not be carried by the ship, but each celestial sphere is necessarily carried by its higher spheres.

Aristotle insists that ceslestial motions have no contraries and hence this case is different from the sublunary cases I have been considering. It is worth briefly examining why.

In *DC* 1.4, Aristotle denies that circular motions have contraries, as part of a broader argument for the eternity and inalterability of the heavens.[66] 'It might be thought,' Aristotle says, 'that the same thing which has been said of rectilinear motion applies to circular, namely that the motion from a point A in the direction of a point B is the contrary of the motion from B to A.'[67] This is not so. For one thing, between two points there is only one straight line but an unlimited number of circular arcs (271a9-10: the operative assumption here is that contraries must come in pairs**).** For another, motion through a full circle involves returning to the start-point, so the end-point and the start-point are the same. In that case, the start-points and end-points cannot be contrary and so the motions cannot be contrary (recall *Phys*. 5.5's claim that two changes are contrary only if their start-points are contrary and their end-points are contrary). Further, if there were contrary motions in the heavens, one of them would be superfluous. But 'God and nature do nothing in vain' (271a33).

A different argument that celestial motions cannot have contraries could be constructed from *Phys*. 8.9. Here it is claimed that celestial motions do not have distinct start-points and end-points:

> [T]he points of a circumference are undefined. For why should any point of the curve be more of a limit than any other? Each point is at once a beginning and a middle and an end, so that the moving body is at the beginning and the end always and never. That is why in a sense a revolving sphere both moves and is at rest; for it occupies the same place. The reason is that all these attributes belong to the centre point: it is the beginning and the middle and the end of the magnitude.[68]

An argument based on this would be similar to the third argument of *DC* 1.4 in rejecting the contrariety of start-points and end-points in circular motion. The key difference is that an argument based on *Phys*. 8.9 would assume that the circular motion is eternal, which is what

[66] See also *DC* 1.3, 270a18-21.
[67] 271a5-8 trans. Guthrie.
[68] 265a32-265b4 trans. Graham.

is at issue in *DC* 1.4. Also, while *Phys*. 8.9 locates the start-point and end-point together at the centre of revolution, *DC* 1.4 locates them on the circular path itself.

Aristotle's rewinding spheres are automatically exempt from his ban on simultaneous contrary motion because their motions are not truly contrary. This celestial exemption relies on special considerations about circular motion, and so it cannot be transferred to the cases of sublunar, rectilinear motions with which we began.

3.4: Conclusion

In this chapter I raised two questions. First, what does Aristotle mean by 'mixed motion' and is his classification of motions defensible? Secondly, do some simple cases such as a man walking towards the stern of a moving ship force Aristotle to violate his prohibition against simultaneous contrary motions in a single subject? Although it must be admitted that definitive answers are not available, I suggested that the category of 'mixed motion' may be wider than some commentators allow, and that Aristotle's prohibition against simultaneous contrary motions may apply only to motions that are non-accidentally in the same subject.

Several later Greek natural philosophers followed Aristotle's classification of motions, even as Hellenistic geometers defined and studied an expanding range of higher-order curves.[69] Questions about simultaneous contrary motions seem to have continued to prompt debate in later natural philosophy, particularly among the Stoics. On the one hand, the 'tensile motion' that gives material bodies their individuality and coherence was said to consist of an outwards motion and an inwards motion.[70] On the other hand, we are informed that some Stoics took a different approach from Plato to incontinence: rather than divide the authoritative part of the soul (τὸ ἡγεμονικόν), Chrysippus and his followers claimed that it is indivisible and oscillates imperceptibly quickly between two alternatives.[71]

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[69] For example, Chrysippus fr.492 = Stob. *Ecl.* 1.165.15; Apollodorus apud Stob. *Ecl*. I p. 166, 24.

[70] There are at least three possible interpretations of tensile motion: (1) simultaneous contrary motion (Sambursky 1959, 21-48); (2) vibration; (3) circulation (Hensley 2020).

[71] Plutarch *On Moral Virtue* 446F–447A, on which see Sorabji 2002. Note that Galen alluded to oscillation as a possible solution to the puzzle of the hovering bird. A version of this rapid oscillation hypothesis relating to the subjectively simultaneous perception of two things is dismissed in *De Sensu* 7 but endorsed in the pseudo-Aristotelian *De Audibilibus*, 803b34-804a8.

Chapter 4: The Balance and Lever in the *Mechanica*

4.1: Introduction

In this chapter I return to *Mech.* Problem 1. Most of Problem 1 consists of an attempt to explain the Rotating Radius Principle through a series of claims about the results of composing two rectilinear motions (848b9-849b19). It is this that makes Problem 1 the longest problem in *Mech*.. This investigation of composed motions culminates in an argument that every radius rotating about one of its endpoints moves with two rectilinear motions, one tangential and one towards the centre, which are in a constantly changing ratio, and that the shorter radius undergoes more motion towards the centre for a given amount of tangential motion. However, the discussion begins with simpler ideas, gradually developing ideas about the composition of two motions. We may distinguish four main claims:

i. The moved thing necessarily moves on a straight line when it moves [with two motions that are] in a certain ratio.[1]
ii. This motion is associated with the diagonal of a corresponding quadrilateral diagram, the sides of which represent the two component motions.[2]
iii. What moves with two motions that are not in any ratio for any time cannot move in a straight line and must take a curved path.[3]
iv. The [line] describing a circle moves with two motions, one radial and one tangential, that are not in a fixed ratio for any time.[4]

Problem 1 is the most detailed study of composed motions in the Aristotelian corpus. It poses a number of interpretative challenges and has been the focus of most scholarship on *Mech*.. The analysis of a rotation into two rectilinear motions goes against our expectations, since Aristotle's *Physics* or *De Caelo* emphasise the simplicity of circular motion. The striking idea that a body could have two rectilinear motions in constantly changing ratio has been taken to violate Aristotle's supposed denial of motion (and so of speed) at an instant or his sharp division between circular and rectilinear motion.[5]

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[1] 848b10-11 ὅταν μὲν οὖν ἐν λόγῳ τινὶ φέρηται, ἐπ' εὐθείας ἀνάγκη φέρεσθαι τὸ φερόμενον.
[2] 848b23-25 φανερὸν οὖν ὅτι τὸ κατὰ τὴν διάμετρον φερόμενον ἐν δύο φοραῖς ἀνάγκη τὸν τῶν πλευρῶν φέρεσθαι λόγον.
[3] 848b34-35 περιφερὲς γίνεται δύο φερόμενον φορὰς ἐν μηθενὶ λόγῳ μηθένα χρόνον.
[4] 848b35-36 ὅτι μὲν τοίνυν ἡ τὸν κύκλον γράφουσα φέρεται δύο φορὰς ἅμα, φανερὸν.
[5] These differences were discussed in Chapter 1. To my knowledge, no other ancient author claims that a body can move along a circular path as a result of some combination of rectilinear motions, or vice versa. In the

One could be forgiven for thinking *Mech.*'s analysis appears over-complicated. Does the Rotating Radius Principle itself not answer the question about balances? And since the truth of this principle was widely accepted, why did it need a long and, in places, obscure explanation? The whole problem might have been dealt with in a couple of lines by a quick statement of the principle. Again, *Physical Problems* 16.3 argues that if one part of a body travels faster than another, it must move in a circle, since this is the only shape in which points that always remain opposite can pass along unequal lines in the same time. So one might expect *Mech.*'s author to infer, from the fact that points within a rigid beam move at different speeds (849a11-19), that they must move in a circle. Instead, we have a difficult passage based on a claim of proportionality which is introduced without justification (849b1-19). Why did *Mech.* take a more difficult path than seems necessary?

This chapter offers a close reading of Problem 1, which I have divided into five passages. I emphasise two ideas throughout. First, one might ask if ἡ γράφουσα τὸν κύκλον in (iv) above means the line (γραμμή) or the motion (κίνησις) that describes a circle. The textual evidence points towards supplying γραμμή, where the line in question is a sweeping radius (rather than the circumference).[6] Secondly, I will carry forward the results of Chapter 2, where I examined evidence for Aristotle's 'component realism'. I will argue that *Mech.* problem 1's claim that a balance-beam undergoes two motions simultaneously is best understood literally. These two points have a significant upshot. They suggest that the domain of Problem 1's analysis may be more restricted than is usually recognised. Commentators often treat Problem 1 as analysing *all* motion that could be described as 'circular'.[7] Rather, I suspect it is primarily a study of bodies rotating around internal points located within their surfaces.[8] The analysis might not, in that case, apply to the heavens.

thirteenth century, al-Tusi showed how a body can take a rectilinear path as a result of two circular motions in a combination that is now known as the 'Tusi couple'.

[6] For example, 848b8-10 ἡ δὲ μείζων ἐν ἴσῳ χρόνῳ γράφει μείζονα κύκλον· ὁ γὰρ ἐκτὸς μείζων τοῦ ἐντός. αἴτιον δὲ τούτων ὅτι φέρεται δύο φορὰς ἡ γράφουσα τὸν κύκλον. See also 849a26, ἡ AB γράφουσα κύκλον, where AB is a radius. Krafft 1970, 81 also favours this reading. Contrast De Groot 2014, 239: 'It seems most likely that the author intends *kinesis*.'

[7] A notable exception is Bodnár 2011a, 449.

[8] The radius is a line within the body connecting a moving part to the relatively fixed internal centre of rotation.

4.2: The quadrilateral of motions (848b9-848b25)

In this passage the author aims to show that something moving in a fixed ratio traverses a rectilinear path. This basic proposition serves a springboard for the more adventurous suggestion in the following section, that something moving with two motions that are not in fixed ratio has a curved path.

> αἴτιον δὲ τούτων ὅτι φέρεται δύο φορὰς ἡ γράφουσα τὸν κύκλον. ὅταν μὲν οὖν ἐν λόγῳ τινὶ φέρηται, ἐπ’ εὐθείας ἀνάγκη φέρεσθαι τὸ φερόμενον, καὶ γίνεται διάμετρος αὐτὴ τοῦ σχήματος ὃ ποιοῦσιν αἱ ἐν τούτῳ τῷ λόγῳ συντεθεῖσαι γραμμαί.
>
> ‘The reason for these things is that the [line] describing a circle moves with two motions. When, then, the moved is carried in a certain *logos*, it necessarily moves on a straight line, and this is the diameter of the figure which the lines constructed in this *logos* make.’[9]

The discussion begins without explaining what it means for something to move ἐν λόγῳ τινὶ. In mathematics, a λόγος is a ratio between two numbers or magnitudes.[10] The author has just mentioned two motions (δύο φορὰς) and these seem to be the relata of the λόγος in question. The lines which are in the same λόγος (ἐν τούτῳ τῷ λόγῳ) somehow represent those motions. Some might interpret this λόγος as a relation between the motions’ speeds,[11] but the author does not say as much.[12] We should also note that the author does not assume that the motions are uniform, but only the weaker condition that they are in some (constant) ratio.

The author refers to the figure as a τετράπλευρον (848b20, literally a ‘four-sider’). This term is not found in Aristotle’s certainly authentic works.[13] For Euclid, it is the most general term for a quadrilateral.[14] Several commentators have read this passage as an instantiation of the

[9] I assume that γράφουσα refers to the line, not the motion (see §4.1 above).
[10] Euclid, *Elements* 5 def. 3 Λόγος ἐστὶ δύο μεγεθῶν ὁμογενῶν ἡ κατὰ πηλικότητά ποια σχέσις (‘A ratio is a sort of relation in respect of size between two magnitudes of the same kind.’, trans. Heath).
[11] Aristotle refers to a λόγος between speeds at *Phys*. 4.8, 216a8-11 and 6.2, 233b22, which I discussed in Note B to Chapter 1.
[12] As noted by Schiefsky 2009, 55-56n.12.
[13] See Heiberg 1904, 15, who conjectures that Euclid had personally coined the term τετράπλευρον. This is speculative, given the loss of all complete texts of pre-Euclidean geometry.
[14] Euclid *Elements* 1, def. 19: Σχήματα εὐθύγραμμά ἐστι τὰ ὑπὸ εὐθειῶν περιεχόμενα, τρίπλευρα μὲν τὰ ὑπὸ τριῶν, τετράπλευρα δὲ τὰ ὑπὸ τεσσάρων, πολύπλευρα δὲ τὰ ὑπὸ πλειόνων ἢ τεσσάρων εὐθειῶν περιεχόμενα (‘rectilineal figures are those that are bounded by straight lines: *tripleura* by three, *tetrapleura* by four, amd *polupleura* by more than four straight lines.’) Euclid’s terms for defined subvarieties of quadrilateral are

‘parallelogram rule’, though two lines in a given ratio do not determine a unique parallelogram since the angle between them may vary. There seem to be two possibilities for understanding the author’s τετράπλευρον. Either the author means a parallelogram, but has omitted to discuss the role of the angle between them in determining the figure and its diameter, or the author specifically has a rectangle in mind, in which case the discussion is restricted to bodies undergoing motions that are perpendicular to each other.[15]

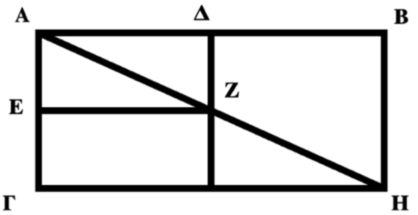

Fig. 1: Hett’s diagram for 848b9-848b25.

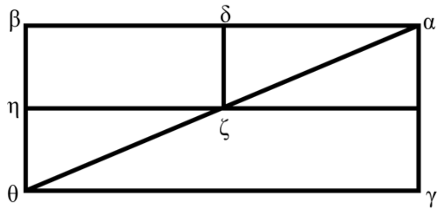

Fig. 2: Van Leeuwen’s (2016, 150) reconstruction of the archetype diagram for 848b9-848b25.

One consideration that suggests the author intended a rectangle is that he claims the converse of the initial thesis also holds: the thing moving along the diagonal with two motions must move with the ratio of the sides.[16] Unless the scope is restricted to rectangles this is false.[17] There are infinitely many parallelograms that share a given diagonal and these differ in the ratio of their sides (see fig. 3).

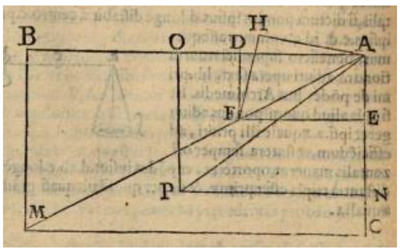

Fig. 3: Benedetti’s (1585, 152) diagram for 848b9-848b25

(*Elements* 1. def. 22): τετράγωνον (square), ἑτερόμηκες (rectangle), ῥόμβος (rhombus), ῥομβοειδὲς (rhomboid) and τραπεζεῖον (trapezium). He does not define the παραλληλόγραμμον (parallelogram) but uses the term anyway (e.g. *Elements* 1.35).

[15] All manuscripts have a rectangle in the diagram.

[16] φανερὸν οὖν ὅτι τὸ κατὰ τὴν διάμετρον φερόμενον ἐν δύο φοραῖς ἀνάγκη τὸν τῶν πλευρῶν φέρεσθαι λόγον. (848b23-25)

[17] As was pointed out by Benedetti 1585, 152.

Further, the author's interest in Problem 1 is in explaining radial rotation and for this an understanding of how perpendicular motions compose is sufficient.[18] Radial rotation is claimed to consist of a tangential motion and a radial one, and these are perpendicular. If problem 1 focusses on the special case of a rectangle rather than a parallelogram, that is a matter of avoiding unnecessary complications for the problem at hand. The parallelogram features in Problem 23, which focusses on comparing the speeds of the component motions to the speed of the resultant motion.[19]

Medieval manuscripts of Greek geometric texts often contain 'overspecified' diagrams. For example, a theorem about triangles in general may be accompanied by a diagram of an equilateral triangle, a theorem about all pentagons by a diagram of a regular pentagon. Whether medieval manuscripts are a sound guide to the diagrammatic practices of the late fourth or third century BCE is doubtful, but if this medieval practice was a continuation of ancient practices, then a diagram depicting a rectangle would have done little to clarify the text's ambiguous τετράπλευρον.[20] As we will see, later passages of problem 1 are terse and their relations to each other unclear. Ambiguity may be the norm rather than the exception.[21]

Three other Greek writers explain the results of combining two motions through a quadrilateral. For two of these writers, the quadrilateral is a rectangle rather than a general parallelogram. Thus Hero of Alexandria describes two motions of uniform speed composing in a rectangle.[22] Philoponus, commenting on the passage of *Mete.* 1.4 discussed in Chapter 2,

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[18] Cf. Mourelatos 1981, 9, n.21: 'The text envisages only the special case of a rectangle of motions, since it uses this construction as a step toward showing that circular motion has a tangential and a centripetal component.'
[19] Problem 23 refers to its quadrilateral as a ῥόμβος.
[20] On overspecified diagrams in medieval manuscripts, see Saito and Sidoli, 2012. The oldest complete manuscript of Euclid's *Elements*, Vaticanus gr. 190, includes a rectangular diagram for Euclid 1.44, a theorem about parallelogram. Yet we should be careful not to assume that the medieval manuscripts are a clear window through which we can observe ancient practices, for which papyri may be our best evidence.
[21] This may suggest that the *Mechanica* was written for a relatively small audience, for whom some of these ambiguities would not have been so troubling, either because they shared certain assumptions with the author or because the author was able to explain his work to them. A 'lecture notes' hypothesis does not seem unreasonable for problem 1.
[22] I rely on Cohen and Drabkin's translation from the Arabic. Hero *Mechanica* 1.8: 'We shall now prove that a point moved by two motions, each of uniform velocity, may traverse unequal distances [in a given time]. Let ABDC represent a rectangle with diagonal AD. Let point A move with constant velocity along line AB, and let line AB [at the same time] move with constant velocity along lines AC and BD. Let the time which point A takes to reach B be equal to the time which line AB takes to reach CD. I say that point A in a given time moves along two unequal lines. Proof: When line AB has moved for a given time, and has reached the position EF, point A, which moves along line AB, will, at the given time, also be on line EF. And there is a constant proportion. The ratio of line AC to line AB (i.e. to line CD) is equal to the ratio of line AE to the line extending from point E to the point moving on it. But AC:CD = AE:EH. Therfore the point moving on line AB will, at H, be on line AD, the diagonal. Similarly it can be shown that the point moving on line AB is always moving along

refers to a rectangle (τετράγωνον).[23] And when Philoponus comments on *Physics* 8, 262a12 ('sideways motion is not the opposite of upwards motion') he illustrates Aristotle's point by imagining motions along the side of a rectangle.[24]

By contrast, Sosigenes, as quoted by Simplicius, refers to a parallelogram. Sosigenes is explaining the composition of motions of homocentric heavenly spheres. He starts with examples of motions about the same pole: if two such motions are in the same direction, the resultant motion will have the sum of their speeds. He then explains what happens if the spheres are about different poles:

> 'For then the speeds will not be compounded in this way, but in the way it is usually proved (δείκνυσθαι εἴωθεν) in the case of a parallelogram (ὡς ἐπὶ τοῦ παραλληλογράμμου) where the motion along the diagonal is produced from two motions, one of a point moving on the length of the parallelogram, and one of this length itself drawn down in the same time through the breadth of the parallelogram. For the point and the side of the length which has been drawn down will be together at the other end of the diagonal; and the diagonal will not be equal to <the sum of> both of the lines which are broken at it, but it will be less, so that also the speed of the compound will be less <than the sum of the two speeds> although it is compounded of the two.'[25]

line AD, and traverses, in a given time, both lines AB and AD. But AD and AB are unequal. Therefore a point moving with a constant velocity will, in a given time, traverse two unequal lines. But the motion of the point on line AB is, as we have pointed out, a simple motion, whereas its motion on diagonal AD is composed of (1) the motion of line AB on the two lines AC and BD and (2) the motion of A along line AB. Therefore point A will in a given time and with constant velocity traverse two unequal lines. Q.E.D.' (trans. Cohen and Drabkin 1948, 223)

[23] 'Suppose you were to imagine two ants [moving] on some solid surface, coming face to face with equal force. As they come together from opposite places, and meet, contact and push each other, they are no longer borne along the same straight line as previously, but will be forced by the collision to move with an oblique and crosswise motion... For the cross-section of rectangles (τῶν τετραγώνων) is crosswise with respect to the sides by which the rectangle (τὸ τετράγωνον) is formed. So, he likens the sideways ejection of the shooting stars to the motion along the cross-section of objects which previously moved along the sides of the rectangle but have been pushed out by each other at the corner by a mutual collision so that they get carried off at a diagonal. Assume a rectangle ABCD, with its cross-section, i.e. diagonal, AD; let two ants of equal strength be moving, one from C to A and another, again, from B to A. When they are at A and neither gets the better of the other as they push each other, they are shoved off the sides of the rectangle, and being deflected, they get carried off along the cross-section AD.' (trans. Kupreeva 2014). Proclus' commentary on Plato's *Republic* also uses the example of an ant, though in a different way, to explain composite motion (2, 234.9).

[24] *in Phys.* 842.18ff.: ὑποκείσθω γὰρ τετράγωνον χωρίον καὶ κινούμενά τινα τὸ μὲν ἐκ τῆς μιᾶς πλευρᾶς ὡς ἐπὶ τὸ κάτω, τὸ δὲ ἐκ τῆς ἑτέρας ἐπὶ τὰ πλάγια. ὅταν συνέλθωσι ταῦτα περὶ τὴν γῆν, οὐ στήσουσιν <ἄλληλα> ἀλλὰ συνωθήσουσι καὶ λοξὴν κινήσουσι κίνησιν κατὰ τὴν διάμετρον τοῦ χωρίου.

[25] *In De Caelo* 200.21ff., trans. Mueller 2005.

Interestingly, Sosigenes speaks plainly of the speeds (τὰ τάχη) where *Mech*. seemed reticent. The heavenly spheres have uniform speeds, so it is clear what is meant by the τάχος of each motion. Further, Sosigenes, like Hero, is interested in comparing the speed of the resultant to the speeds of the components. This is not at issue in *Mech*. Problem 1 where the focus is on the path-shape of the resultant motion, straight or curved, rather than on its magnitude.[26] Sosigenes' expression 'it is usually proved' (δείκνυσθαι εἴωθεν) implies that the ideas he describes are common knowledge. This is to some extent confirmed by the variety of the texts I have cited.

An aside in Plutarch's *On the Correct Manner of Listening* may offer a further indication of the familiarity of the parallelogram. Plutarch criticises young men who show off their learning at lectures by asking hair-splitting technical questions about mathematics such as 'what motion along the side or diagonal is' (43A-B: τίς ἡ κατὰ πλευρὰν ἢ κατὰ διάμετρον κίνησις), perhaps a reference to the ideas we have been examining.[27]

Finally, the technique for describing the composition of motions in this passage deserves comment. Two techniques were deployed by Greek writers when describing composed motions. One could consider either: (1) the intersection of two moving lines; or (2) a point's motion along a moving line. For example, the quadratrix was described by technique (1) as the line described by the intersection of a rotating line and a line with a certain rectilinear motion.[28] Archimedes defined the spiral by technique (2) as the line described by a point moving along a line that is rotating.[29] Different passages of *Mech*. apply different techniques. Problem 1 (see Figs. 1-2 above) adopts technique (1):

> ἔστω γὰρ ὁ λόγος ὃν φέρεται τὸ φερόμενον, ὃν ἔχει ἡ ΑΒ πρὸς τὴν ΑΓ· καὶ τὸ μὲν ΑΓ φερέσθω πρὸς τὸ Β, ἡ δὲ ΑΒ ὑποφερέσθω πρὸς τὴν ΗΓ· ἐνηνέχθω δὲ τὸ μὲν Α πρὸς τὸ Δ, ἡ δὲ ἐφ' ᾗ ΑΒ πρὸς τὸ Ε. εἰ οὖν ἐπὶ τῆς φορᾶς ὁ λόγος ἦν ὃν ἡ ΑΒ ἔχει πρὸς τὴν ΑΓ, ἀνάγκη καὶ τὴν ΑΔ πρὸς τὴν ΑΕ τοῦτον ἔχειν τὸν λόγον.

[26] The magnitude of the resultant is, however, a central issue in *Mech*. problem 23.
[27] Contrast Babbitt's suggestion in the Loeb: 'When a body moves are its various positions determined by the position of its diagonal (i.e. interior lines) or of its exterior lines?'
[28] Pappus *Collectio* 4 prop.30 (250.33-252.25 Hultsch).
[29] Archimedes *Spiral Lines*, preface.

For let the *logos* with which the moved thing is moved be that which the [line] AB has to the [line] AΓ. And let the [line] AΓ move towards the [point] B and the [line] AB move towards the [line] HΓ. And let the [point] A travel towards the [point] Δ and the [line] on which [are] AB towards the [point] E. If, then, the *logos* of the motion is that which the [line] AB has to the [line] AΓ, it is necessary that the [line] AΔ has this *logos* to the [line] AE.

On the other hand, Problem 23 (see fig. 4 below) uses technique (2):

φερέσθω γὰρ ἐπὶ τῆς AB τὸ μὲν A πρὸς τὸ B, τὸ δὲ B πρὸς τὸ A τῷ αὐτῷ τάχει· φερέσθω δὲ καὶ ἡ AB ἐπὶ τῆς AΓ παρὰ τὴν ΓΔ τῷ αὐτῷ τάχει τούτοις. ἀνάγκη δὴ τὸ μὲν A ἐπὶ τῆς AΔ διαμέτρου φέρεσθαι, τὸ δὲ B ἐπὶ τῆς BΓ

For on the [line] AB, let A move towards B and B towards A with the same speed, and let the [line] AB move on AΓ alongside ΓΔ at the same speed as them. Necessarily, A moves on the diagonal AΔ, and B on BΓ.

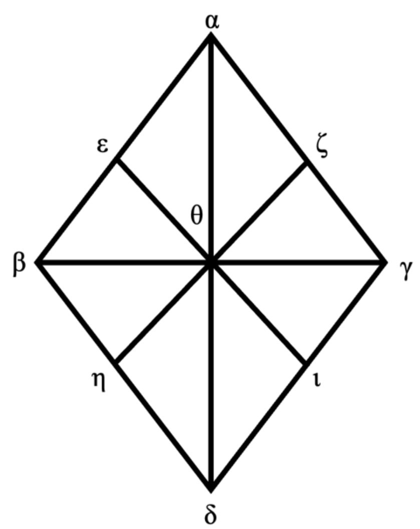


Fig. 4: Van Leeuwen's (2016, 230) reconstruction of the archetype diagram for problem 23.

4.3: Inconstant ratios (848b25-35)

This passage introduces the idea that the component motions could be 'in no ratio for any time', and argues that the combined motion would be curved (849a34: περιφερὲς).[30] This is shown indirectly, by arguing that the combined motion could not possibly be straight (848b27: ἀδύνατον εὐθεῖαν εἶναι τὴν φοράν). This argument assumes what was established in the previous section, that if something moves with two motions along a straight line, its component motions are in a fixed ratio.

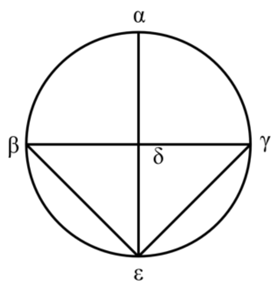
I have presented evidence that the quadrilateral of motions was a fairly widespread idea in Greek technical literature. By contrast I have found no parallels for the idea that a thing could move with two motions that are in no fixed ratio for any period of time (δύο φερόμενον φορὰς ἐν μηθενὶ λόγῳ μηθένα χρόνον). It must be emphasised that there is no attempt in *Mech.* to describe precisely or even in rough quantitative terms how the ratio of component motions varies over time. Since the argument is indirect, no example is worked out.

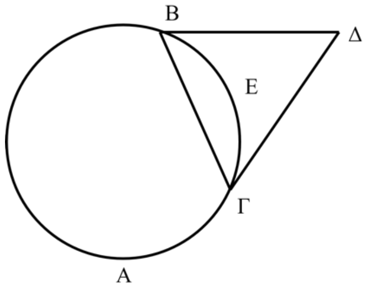
It has sometimes been suggested that this passage contradicts Aristotle's views on motion at an instant. It is assumed that the concept of motions' being in no fixed ratio for any period of time implies that they are in a different ratio at every instant. To make sense of that, one would need to accept the notion of motion or speed at an instant, and yet it has traditionally been thought that Aristotle rejected motion at in instant. Although it is true that there is no trace in Aristotle of *Mech.*'s notion of motions being in no ratio for any period of time, I argued in Chapter 1 that it is not certain that Aristotle denied that anything can truly be said to be changing at an instant. I also argued that the claim of the present passage, that the motions are in no fixed ratio for any period of time, does not require the introduction of a ratio of motions at an instant. All that is needed is that, whichever periods are specified, the ratio of motions will be different for any two of them.

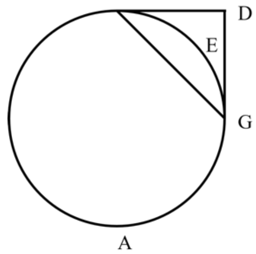
[30] Commentators disagree on whether περιφερὲς here means curves generally or circles and their arcs. The former is of course what one *should* say, but I cannot see anything that settles which of these meanings our author intended.

4.4: A rotating radius has two motions (848b35-849a6)

This passage argues that a rotating radius moves with two motions. There is disagreement over the text and diagram, and therefore over the argument.







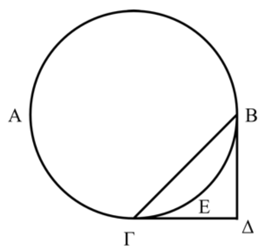


Fig. 5: Van Leeuwen's (2016, 204) reconstruction of the archetype diagram for 848b35-849a6.	*Fig. 6: Hett's diagram for 848b35-849a6.*	*Fig. 7: Mendell's (2002) preferred diagram for 848b35-849a6.*	*Fig. 8: Another possibility, reflecting the orientation as the balance*

> ὅτι μὲν τοίνυν ἡ τὸν κύκλον γράφουσα φέρεται δύο φορὰς ἅμα, φανερὸν ἔκ τε τούτων, καὶ ὅτι τὸ φερόμενον κατ' εὐθεῖαν ἐπὶ τὴν κάθετον ἀφικνεῖται, ὥστε εἶναι πάλιν αὐτὴν ἀπὸ τοῦ κέντρου κάθετον. ἔστω κύκλος ὁ ΑΒΓ, τὸ δ' ἄκρον τὸ ἐφ' οὗ Β φερέσθω ἐπὶ τὸ Δ· ἀφικνεῖται δέ ποτε ἐπὶ τὸ Γ. εἰ μὲν οὖν ἐν τῷ λόγῳ ἐφέρετο ὃν ἔχει ἡ ΒΔ πρὸς τὴν ΔΓ, ἐφέρετο ἂν τὴν διάμετρον τὴν ἐφ' ᾗ ΒΓ. νῦν δέ, ἐπείπερ ἐν οὐδενὶ λόγῳ, ἐπὶ τὴν περιφέρειαν φέρεται τὴν ἐφ' ᾗ ΒΕΓ.
>
> That the [line] describing the circle moves with two movements simultaneously is clear from these things, and that the thing moved in accordance with a straight [line] arrives at the perpendicular [line], so that that [line] from the centre is perpendicular again. Let there be a circle ΑΒΓ, and let the end-point on which [is] B move to the [point] Δ. Then it arrives at some time at Γ. If, then, it had moved in the *logos* that ΒΔ has to ΔΓ, it would have moved over the diagonal on which [are] ΒΓ. Now since [it moves] in no *logos*, it moves on the circumference on which [are] ΒΕΓ.[31]

The difficulties in this passage begin with the text's underdetermination of the diagram. We are not told where point Δ is located. It is also not immediately clear where A, B, E and Γ lie

[31] Translation adapted from Mendell 2002. Modern diagrams for this passage do not usually represent a radius drawn from the centre of the circle to B. This might give the impression that the author is not concerned specifically with a radius in this passage, but the reference to τὸ δ' ἄκρον τὸ ἐφ' οὗ Β indicates that B is implicitly understood as the tip of a radius.

on the circle, though E certainly lies between B and Γ. The archetype diagram is nonsensical.[32] Van Leeuwen observes that several manuscripts read ἡ ΒΔ ΕΓ rather than ἡ ΒΔ πρὸς τὴν ΔΓ.[33] She argues that the latter reading derives from Pachymeres' paraphrase of *Mech*. Van Cappelle suspected κατ' εὐθεῖαν was a corruption but also doubted earlier emendations.[34] Forster and Heath proposed to delete κατ' εὐθεῖαν.[35] That may be an overreaction. The immediately preceding passage of Problem 1 argued that a body undergoing two rectilinear motions can follow a curved path. Referring to what traverses a circular path as τὸ φερόμενον κατ' εὐθεῖαν is an unusual way of putting things, and certainly has potential to be misleading, but it is not inconsistent with the author's line.[36] I shall now explain how I understand the argument of this passage.

We assume the end-point of the rotating radius has a tangential motion. This is represented by the line B to Δ. However, it turns out that in completing this motion the end-point does not reach the point on tangent labelled by Δ, but rather the point on the circumference labelled by Γ. It may seem puzzling that the motion 'to Δ' results in arrival at another point, Γ. This can be explained by assuming it underwent another, simultaneous motion, one towards the centre. The final two sentences (εἰ μὲν οὖν ἐν τῷ λόγῳ… φέρεται τὴν ἐφ' ᾗ ΒΕΓ) argue indirectly that the two motions are not in a fixed ratio. If the motions had a fixed ratio, the end-point would move from B to Γ along a straight line, which is called a diagonal (διάμετρον). Since the motion does not take a straight path, but rather a curved path along the circumference ΒΕΓ, the motions must not be in a fixed ratio.

The text tells against the interpretation of ΒΕΓ as a general arc of any length (as in Hett's diagram, fig. 6 above), rather than a quadrant. ἐπὶ τὴν κάθετον ἀφικνεῖται means 'arrives at the perpendicular [line]'. Perpendicular to what? A plausible construal is perpendicular to the radius' original position. In that case, the radius has traversed a quadrant (or three quadrants,

[32] Of the above figures, figs. 6 and 7 are closest to my understanding of this passage, but this is only a guess.
[33] Van Leeuwen, 2016, 151-54.
[34] Van Cappelle, 1812 158: 'Locus hic valde est obscurus et, ni fallor, corruptus. In variantibus lectionibus ad calcem Editionis Syllburgianae adjectis, duae inveniuntur emendationes; nempe, ut pro τὸ φερόμενον κατ' εὐθεῖαν legatur ἡ φερομένη κατὰ τὴν περιφέρειαν, vel ut μὴ φερομένη. Sed neutra lectio omnem difficultatem tollit, etsi prior vulgatae praeferenda videatur. Hoc enim voluisse videtur Aristoteles, ideo patere radium moveri duobus motibus, secundum et contra naturam, quoniam motu suo per circumferentiam tandem ad talem situm perveniat, ut sit priori situi suo perpendicularis. Quod non fieret, si simplici motu per lineam rectam moveretur.'
[35] Heath, 1949, 231.
[36] Note that the tip of the radius is said to move to Δ (849a3-4: τὸ δ' ἄκρον τὸ ἐφ' οὗ Β φερέσθω ἐπὶ τὸ Δ). Here I agree with De Groot 2009, 32-33 (cf. De Groot 2014, 229-33).

but it is clear from the rest of the passage that the author does not have this in mind).[37] Hett translates ἐπὶ τὴν κάθετον ἀφικνεῖται as 'is along a perpendicular'. This escapes the conclusion that the radius has traversed a quadrant, but does not seem an acceptable rendering of ἀφικνεῖται.

The interpretation I have just offered involves two assumptions: (1) that the end-point of the rotating radius has a tangential motion; (2) that if it does not follow a tangential path, it must have a second motion and this motion is radial. What justifies these assumptions? The assumption that the end-point of the rotating radius has a tangential motion may be given by the specific example considered in Problem 1: the balance. A weight in a scale-pan naturally moves 'downwards', towards the centre of the earth.[38] If the radius in this passage corresponds to one of the balance's arms, then this basic fact about weights in balances implies a tangential motion 'downwards'. The diagrams of problem 1 could be drawn at an orientation to reflect this.[39] In the context of Problem 1, then, this assumption seems justified. The assumption of radial motion towards the centre is less obvious and I take it that part of the task of the following passage (treated below in §4.5) is to substantiate this assumption. Furthermore, a focus on the specific case of the balance could explain why the author discusses rotation through a quadrant: a weighted balance arm could fall through a quadrant and arrive at a position roughly perpendicular to its original, horizontal position, but it could not fall through just *any* arc.

4.5: Radial constraint (849a6-849a19)

In this section, the author suggests that the point rotating on a lesser radius is constrained more by the centre and so moves slower. My discussion will take two stages. First, I shall present Apelt's text and discuss two textual issues. Secondly, I shall address the passage's explanatory value and the status of ἔκκρουσις. Here is Apelt's text, with the controversial passages underlined:

[37] What would the diagonal (849a5: τὴν διάμετρον) be in that case?
[38] The weight's motion will be both natural and tangential only for an instant at the start of the its descent.
[39] As suggested, for example, by Guevara 1627, 51; Krafft 1970, 26; De Gandt 1982, 122; and Van Leeuwen 2016, 153-4. Note Van Leeuwen's caution on her reconstructed archetype diagram for 848b35-849a6 (2016, 154): 'I will by no means argue that this is the figure intended by the author of the text.'

ἐὰν δὲ δυοῖν φερομένοιν ἀπὸ τῆς αὐτῆς ἰσχύος τὸ μὲν ἐκκρούοιτο πλεῖον, τὸ δὲ ἔλαττον, εὔλογον βραδύτερον κινηθῆναι τὸ πλεῖον ἐκκρουόμενον τοῦ ἔλαττον ἐκκρουομένου· ὃ δοκεῖ συμβαίνειν ἐπὶ τῆς μείζονος καὶ ἐλάττονος τῶν ἐκ τοῦ κέντρου γραφουσῶν τοὺς κύκλους. διὰ γὰρ τὸ ἐγγύτερον εἶναι τοῦ μένοντος τῆς ἐλάττονος τὸ ἄκρον ἢ τὸ τῆς μείζονος, ὥσπερ ἀντισπώμενον εἰς τοὐναντίον, ἐπὶ τὸ μέσον βραδύτερον φέρεται τὸ τῆς ἐλάττονος ἄκρον. πάσῃ μὲν οὖν κύκλον γραφούσῃ τοῦτο συμβαίνει, καὶ φέρεται κατὰ τὴν περιφέρειαν, τὴν μὲν κατὰ φύσιν εἰς τὸ πλάγιον, τὴν δὲ παρὰ φύσιν εἰς τὸ κέντρον. μείζω δ' ἀεὶ τὴν παρὰ φύσιν ἡ ἐλάττων φέρεται· διὰ γὰρ τὸ ἐγγύτερον εἶναι τοῦ κέντρου τοῦ ἀντισπῶντος κρατεῖται μᾶλλον.

If of two [things] moved by the same strength one is constrained more and one less, it is reasonable that the more constrained move slower than the less constrained. This seems to happen in the greater and lesser of the [lines] describing circles from the centre. For on account of the end-point of the lesser's being nearer what rests than the [end-point] of the greater, as being[40] restrained in a contrary direction, the end-point of the lesser moves more slowly towards the middle. This happens in each [line] describing a circle: it moves along the circumference, naturally sideways, non-naturally towards the centre. The lesser always moves more non-naturally: for because it is closer to the restraining centre it is more controlled.

The first textual issue concerns 849a13. Forster 1913 re-punctuates: ὥσπερ ἀντισπώμενον εἰς τοὐναντίον ἐπὶ τὸ μέσον, βραδύτερον φέρεται τὸ τῆς ἐλάττονος ἄκρον 'as if restrained in a contrary direction, towards the middle, the end-point of the lesser moves more slowly.' This seems preferable, and both Berryman and De Groot follow Forster too.[41] The point required by the argument is that the lesser radius' rotational motion is slower, not that one of its components is slower.[42]

The second textual issue concerns 849a15-17. Apelt follows a conjecture due to Van Cappelle, but most manuscripts read: φέρεται τὴν μὲν κατὰ φύσιν κατὰ τὴν περιφέρειαν, τὴν δὲ παρὰ φύσιν εἰς τὸ πλάγιον καὶ τὸ κέντρον ('It moves naturally along the circumference,

[40] Alternatively, 'as if'.
[41] Berryman 2009, 110n.24; De Groot 2014, 237.
[42] τοὐναντίον seems to be used loosely. Cf. *Phys.* 8, 262a12 ('sideways motion is not the opposite of upwards motion').

unnaturally sideways and towards the centre') and this is the reading favoured by Bekker. Taken alone, this sentence seems to admit two interpretations depending on whether one takes τὴν δὲ παρὰ φύσιν εἰς τὸ πλάγιον καὶ τὸ κέντρον to identify one or two motions: (A) There are only two motions, a natural motion along (or according to) the circumference and a non-natural motion towards the centre (the centre is 'sideways' for what rotates around it; if I walk clockwise round a tree it is always on my right side). (B) There are two non-natural motions, one sideways (tangential) and one towards the centre, and one natural motion along the circumference. Against (A), problem 1 is clearly concerned with a tangential as well as radial motion and εἰς τὸ πλάγιον apparently indicates tangential motion in problem 8, 852a12-13.[43]

The Aldine and several later Renaissance editions (e.g. Mononatheuil 1599) contain the reading φέρεται τὴν μὲν κατὰ φύσιν τὴν δὲ παρὰ φύσιν κατὰ τὴν περιφέρειαν εἰς τὸ πλάγιον καὶ τὸ κέντρον. This is also found in one manuscript, V^3 (*Barb. Gr.* 22), and was favoured by the most recent editor, Bottecchia Dehò.[44] This reading leaves room for interpreting the tangential motion as natural and the radial motion as unnatural, but does not express that idea as clearly as van Cappelle's conjecture. In light of Sicherl and Van Leeuwen's arguments that V^3 is a copy of the Aldine text, it carries no independent weight.[45]

Micheli and Van Leeuwen have defended the majority manuscript reading which identifies circumferential motion as κατὰ φύσιν.[46] Van Leeuwen writes, 'The author of the *Mechanics* is not concerned, however, with the natural motion of physical objects… he is interested in finding out which motion is natural to the specific properties of a mechanical object. If we look at circular motion in a wheel, we notice that it is natural to the properties of a round object to move along its circumference.'[47] Again, 'In the specific context of mechanics, a natural circular motion can be defined as a motion that is natural to the properties of a round object.'[48] If I understand correctly, Van Leeuwen's suggestion is that, in addition to the sense of natural motion found in Aristotle's natural philosophy, there are further notions of characteristic ("natural") motions specific to certain kinds of objects. Objects of a certain

[43] τὴν μὲν γὰρ εἰς τὸ πλάγιον αὐτοῦ κίνησιν ὠθεῖ τὸ κινοῦν, τὴν δὲ ἐπὶ τῆς διαμέτρου αὐτὸς κινεῖται. Here, motion εἰς τὸ πλάγιον is the result of pushing, and so it is not claimed to be natural.
[44] Bottecchia 1982, Bottecchia Dehò 2000
[45] Sicherl 1997, 95-96, Van Leeuwen 2013. Bottecchia thought V^3 was an exemplar of the Aldine.
[46] Micheli 1995, 64-65; Van Leeuwen 2016, 12-18. Carteron 1923 8n.28 also favoured the ms reading.
[47] Van Leeuwen 2016, 16.
[48] Van Leeuwen 2016, 17.

shape have certain characteristic ways of moving. For example, cylinders characteristically roll (κυλίνδω is the verb 'to roll').[49] One could identify this broader notion of "natural" motion with what Aristotle calls *per se* motion. Artefacts have privileged motions that they are essentially capable of undergoing: a ship is by definition capable of travelling over water; anything incapable of this would not count as a real ship.

The majority manuscript reading is problematic regardless. For one thing, what would the contrasting sense of 'unnatural' motion be? The tangential and radial component motions hardly seem uncharacteristic in the sense just described, since they are required if the rotation is to happen. If the body is essentially capable of rotating, and rotation essentially involves these tangential and radial motions, then is the body not essentially capable of them?

Further, Problem 1 goes on to use the labels 'natural' and 'unnatural' a further five times.[50] Better sense is made of these subsequent passages on the understanding of the labels 'natural' and 'unnatural' common to both those who follow Van Cappelle's conjecture and those who follow V^3. One such passage is 849a38-849b4 (fuller discussion in §4.6 below):

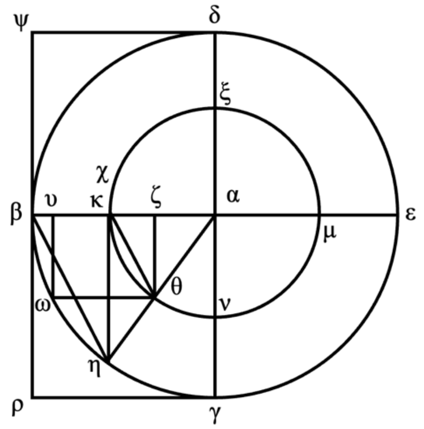


Fig. 9: Van Leeuwen's (2016, 14, 206) reconstruction of the archetype diagram for 849a19-849b19.

ἐν ὅσῳ δὴ χρόνῳ ἡ ΑΘ τὴν ΧΘ ἐνηνέχθη, ἐν τοσούτῳ χρόνῳ ἐν τῷ κύκλῳ τῷ μείζονι μείζονα τῆς ΒΩ ἐνήνεκται τὸ ἄκρον τῆς ΒΑ. ἡ μὲν γὰρ κατὰ φύσιν φορὰ ἴση, ἡ δὲ παρὰ φύσιν ἐλάττων· ἡ δὲ ΒΥ τῆς ΖΧ.

In the time in which ΑΘ is carried through ΧΘ, the end-point of BA has been carried more than BΩ in the greater circle. For the natural motion is equal, but the unnatural is less; and BY is less than ZX.

The point of this passage is to explain why the greater radius BA has traversed a greater arc BH than the arc XΘ which the smaller radius AX has traversed in the same time. It would fail to do that if 'the natural motion' referred to the traversal of the arc BH. For one thing that would be to assume what is to be explained, and

[49] *Mech.* uses κυλίνδω, Aristotle uses κυλινδέω (e.g. *HA* 9 (7), 586b26, 8 (9), 612b24). Neither Micheli nor van Leeuwen give examples, but within *Mech.* one might compare 848a34-35 (ταύτην οὖν λαβόντες ὑπάρχουσαν ἐν τῷ κύκλῳ τὴν φύσιν) and *Mech*. 24's expression πέφυκεν κινεῖσθαι.
[50] 849a19-21, 849b3-4, 849b5-6, 849b10-12, 849b18-19.

for another it would falsely suggest that BH and XΘ are equal. Rather, 'the natural motion' refers to what is represented in the diagram by YΩ and ZΘ, and 'the unnatural motion' refers to what is represented by BY and ZX. In fact, this is how Van Leeuwen understands the passage.[51] The majority manuscript reading for 849a15-17 does not give the sense required by later passages of problem 1.

Returning to the passage describing the effects of radial constraint (849a6-849a19), we may now raise the question of how it relates to the rest of problem 1. De Groot views this passage as 'overlaid' on the main argument and criticises its explanatory value: 'Yet how would a force of constraint located at the center of the circle aid a *physical* explanation of the lever? First, how does a point, the centre of the circle, constrain movement, since points have no power? Surely it is something about the rigid beam of the lever that does the constraining.'[52] I agree that there is a tension between this passage and other passages of problem 1 (see §4.9), but I do not think its explanatory value can be so easily dismissed. Aristotle indeed denies that extensionless points can have powers in the context of arguing against a suggestion that the heavens are moved by the celestial poles.[53] Although the unmoved mover must be extensionless if it is to have infinite power, it cannot be a point. Thus Aristotle must deny that a point could have a power in the way the unmoved mover must: independently and in its own right. This might leave open the possibility that a point can have power in a dependent sense. Imagine a perfectly sharp pin, tapering off to an exact point. The point at the end of the pin has the power to pierce, but only because it is part of an extended material body with certain properties (hardness, rigidity, capable of being picked up and pushed, etc.). To transfer to the case of the balance, one could argue that the centre of the balance beam has its power for constraining because is part of an extended material body with certain properties.

This is not to deny that *Mech.*'s account of ἔκκρουσις is somewhat mysterious.[54] It seems strange to call the radial component of a solid beam's rotation about a pivot a 'motion'. No part of the beam ever comes nearer to the centre of rotation, towards which the motion is supposedly directed. Further, ἔκκρουσις only seems to occur when another motion, the

[51] Van Leeuwen 2016, 155: 'for the same natural motion (ΘZ=ΩY), the unnatural motion is larger in the smaller circle (X>BY)'. Van Cappelle's notes (1812, 159) do not object to the majority manuscript reading but rather take issue with the vulgate of V^3, the Aldine and other early editions.
[52] De Groot 2014, 238.
[53] *MA* 3, 699a20-21. See the recent discussion in Coope 2020, 256-57.
[54] The term is, incidentally, a *hapax* in the Aristotelian corpus.

tangential motion, is occurring in the beam. For these reasons, ἔκκρουσις is an unusual kind of motion, one that is dependent on another motion and which does not bring its subject towards the place at which it is directed.

Why, then, does the author insist that ἔκκρουσις is a motion (φορά)?[55] An alternative would be to characterise constraint in terms of a power for resistance to motion, rather than a power for motion.[56] What is important is the fact that the centre has a firmly fixed position and that it is at rest. Would it not be more plausible to say that the centre's power to resist being moved from its position is responsible for ἔκκρουσις? Powers to resist motion are only manifested when some power for motion is exerted, and this seems to be the case for ἔκκρουσις. The author seems to equate ἔκκρουσις with ἀντισπᾶσθαι, a term that elsewhere in *Mech*. refers to resistance.[57]

The answer may be that ἔκκρουσις differs from powers for resistance to motion in some way that makes it better characterised as a motion. Powers to resist, to judge from Aristotle's examples, either prevent a motion from occuring at all or make it slower than it would otherwise be. By contrast, ἔκκρουσις changes the spatial path taken by the balance beam and, while not preventing the tangential motion altogether, prevents the balance beam from reaching any of the spatial positions it might have attained had it been subject to the tangential motion alone. Although the tip (or any other part) of the beam does not approach the centre, its potential to be at the centre is making a difference to its location throughout the rotation. Thus ἔκκρουσις might seem less like a power to resist and more like the incomplete

[55] Most clearly at 849b4.
[56] On capacities for resistance to motion, see especially *MA* 3, 699a32-699b1: πρὸς δὲ τούτοις δεῖ τὴν ἰσχὺν ἰσάζειν τοῦ κινοῦντος καὶ τὴν τοῦ μένοντος. ἔστιν γάρ τι πλῆθος ἰσχύος καὶ δυνάμεως καθ' ἣν μένει τὸ μένον, ὥσπερ καὶ καθ' ἣν κινεῖ τὸ κινοῦν· καὶ ἔστιν τις ἀναλογία ἐξ ἀνάγκης, ὥσπερ τῶν ἐναντίων κινήσεων, οὕτω καὶ τῶν ἠρεμιῶν. καὶ αἱ μὲν ἴσαι ἀπαθεῖς ὑπ' ἀλλήλων, κρατοῦνται δὲ κατὰ τὴν ὑπεροχήν. (trans. Morison: 'And moreover the force of the thing causing the movement and that of the thing that stays still should be equal. For there is a certain amount of force and power on the basis of which the thing that stays still stays still, just as there is also an amount on the basis of which the thing causing the movement causes the movement. And just as there must be some proportion for motions which are opposed, so too for states of rest. And equal ones are unaffected by each other, but they are overcome in cases of excess.'); cf. the comments of Lefebvre 2004, 131 and Coope 2020. Aristotle mentions powers for resistance to change, and especially to destruction, in *Met*. Δ.12, 1019a26-28 (trans. Kirwan: 'any state in respect of which a thing is wholly unaffectable or unalterable or not easy to change for the worse is called a capacity') and *Met*. Θ.1, 1046a13-15, (trans. Makin: 'the state of not being liable to be acted on for the worse and so as to be destroyed by something else or by itself qua something else—i.e. by an origin of change').
[57] 849a30-31: διὰ τὸ γίνεσθαι μείζονα τὴν ἔκκρουσιν καὶ ἀντισπᾶσθαι. Cf. *Mech*. 31, 858a15.

manifestation of a potential to be in a place.[58] Treating ἔκκρουσις as a motion makes the principle introduced in this passage more familiar. The claim that points nearer the centre experience more constraint is thus comparable to the Aristotelian thought that what is nearest to the mover moves fastest.[59]

Mech.'s remarks about constraint are brief and elliptical. I have argued that they represent an effort to provide a causal explanation. I have indicated how this passage supports the argument of what came before. It substantiates an implicit assumption of the earlier passage 848b35-849a6 (discussed in §4.4 above), that if the weight on a balance does not in fact follow a tangential path, it must have a second motion, which happens to be radial. I also note that 849a30 refers back to the ideas first presented in this passage. In short, this passage is well-integrated in the argument of Problem 1.

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[58] Note also that the radial motion should pass Aristotle's faster/slower test for distinguishing κίνησεις and ἐνέργειαι (*NE* 10.3, 1173a31-b4), since when the radius spins faster, the radial and tangential motions should be proportionally faster.

[59] *Phys*. 8.10, 267b7-8: ἀλλὰ τάχιστα κινεῖται τὰ ἐγγύτατα τοῦ κινοῦντος. Aristotle uses this principle to locate the unmoved mover at the periphery rather than the centre of the world.

4.6: The Rotating Radius Principle (849a19-849b19)

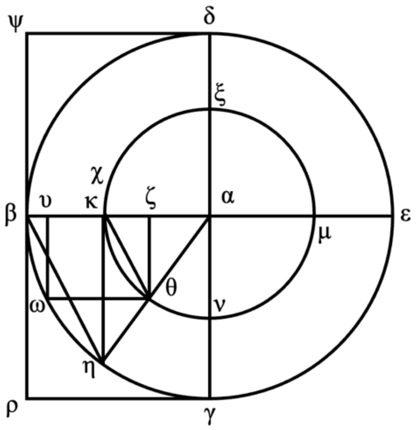

Fig. 9 (as above): Van Leeuwen's (2016, 14, 206) reconstruction of the archetype diagram for 849a19-849b19.

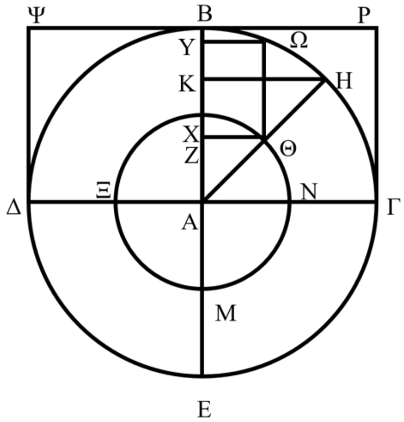

Fig. 10: Hett's diagram for 849a19-849b19.

This complex passage consists of two parts. The first part (849a19-b4) argues that, for an equal tangential motion, there is greater radial motion/constraint in the smaller radius. The second part (849b4-19) first takes this preliminary result in conjunction with the constraint principle examined in §4.5. Together, these imply that, given the same tangential input, the smaller radius moves more slowly. Thus the Rotating Radius Principle has been explained. The author then explains why, in an equal time and under the influence of the same tangential input, the larger radius AB moves not only further than the smaller radius AX but to H in particular.

The procedure of the first part has a roughly similar structure to that of geometrical propositions in Euclid.[60] The author begins with the following statement: 'That, of lines from the centre [i.e. radii] describing circles, the lesser [line] moves more contrary to nature (τὸ παρὰ φύσιν) than the greater [line] is clear from the following.' (849a19-21). This statement functions in the argument like a Euclidean enunciation (*protasis*), declaring what is to be proved in general terms. Next, parallel to a Euclidean setting out (*ekthesis*), the author constructs a diagram (849a21-27) using typical third-person imperatives (Ἔστω, ἐκβεβλήσθωσαν, παραπεπληρώσθω). The author then introduces the idea that AB and AX

[60] On the parts of a proposition, see Proclus, *Commentary on the First Book of Euclid's Elements*, 203-210, and Netz 1999b. Compare Wilson 2013, 252-53 on the organisation of Aristotle's explanation of the rainbow in *Mete*. 3.5.

will each complete a full rotation, describing the whole circle (849a27-29), before restating what is to be shown with reference to the diagram (849a30-32), as in a Euclidean specification (*diorismos*) though lacking its first person λέγω: 'AX moves slower than AB, as has been said, because a greater constraint occurs (γίνεσθαι μείζονα τὴν ἔκκρουσιν) and AX is pulled back more (ἀντισπᾶσθαι μᾶλλον).'[61] This is not precisely the same statement as the 'enunciation' which lacked the claim (though it had already suggested) that greater radial motion entails slower rotation.

Now, the author argues that when AB and AX are subject to an equal tangential motion (ΩY = ΘZ), the lesser radius experiences a greater motion towards the centre (BY < XZ). At 849a32-35, the author adds four further lines required for the argument (AΘH, ΘZ, ΘΩ, ΩY, HK), as in a construction (*kataskeue*). In place of a formal proof (*apodeixis*) that BY < XZ, the inequality is stated (849a35-36) and briefly justified by appeal to the general proposition that 'in unequal circles equal straight [lines] drawn perpendicular to the diameter cut off smaller sections of the diameter in greater circles, and ΩY is equal to ΘZ' (849a35-849b1). This proposition is not found in Euclid, though a proof can be supplied making use of *Elements* 3.31 and 6.8.[62] It is unclear whether *Mech.*'s author himself was familiar with the details of such a proof or left the inequality as evident from inspection of the diagram. There is nothing corresponding to a Euclidean conclusion (*sumperasma*). We are carried on to the second part of the passage without pause or comment. This is surprising, since the author has now offered his justification for what is *the* crucial claim in *Mech.*, the Rotating Radius Principle.

The second part has a new aim. Given that the AB radius rotates faster when subject to the same tangential motion, why, in the time taken for AX's tip to move to Θ, does AB's tip end up precisely at H? Or, to put the question in our terms, why is the angular speed the same?

The author assumes that there must be a proportionality between the natural and unnatural motions. He argues that, if this proportionality is to be preserved, then AB must move to H. In the first place, AB must move further than Ω, since if it only reached Ω, the proportion wouldn't hold (849b1-6). Here, the proportionality is applied negatively, to rule out one

[61] Or perhaps, with epexegetic καί, '…, which is to say AX is pulled back more.'
[62] See Heath, 1949, 233.

possibility.[63] Next, the proportionality is applied positively. AB arrives at H because that is its position when the proportionality holds (849b6-10). The justification is extremely compressed: 'ΘZ is to ZX as HK is to KB. And [this is] evident if [lines] are connected from B, X to H, Θ.'[64] In this passage, we have a terse reference to the required construction after a kind of specification, but there is hardly a formal proof. It is left to the reader to realise that the proportionality must hold since triangles BHK and KΘZ are similar.

The proportionality is the key to these arguments but it is introduced without justification.[65] We are left wondering where it has come from. Schiefsky has offered one suggestion: 'Here the assumption that the two points lie along a single radius is crucial. Because we have two points (X and B) that are fixed on the same radius, at the end of the motion, the ratio between the "unnatural" and "natural" components must be the same for both points.'[66] If this were the author's line of thought, we should expect a simpler argument. To be sure, the assumption that X and B are fixed on the same radius will yield the conclusion that AB moves to H, but this could be achieved without introducing any proportionality. The author might have said that the relative positions of points fixed on the radius must be preserved, so if B lay on a straight line with AX it will still do so when X has moved to Θ, and therefore it must be at H.[67] This interpretation cannot account for the line of argument in fact taken. It is also too restrictive. The aim of problem 1 is to compare the behaviour of balances of different sizes

[63] Only one possibility is explicitly discarded this way. The author does not show why AB could not move to a position before Ω or beyond H or to some position between Ω and H. The pattern of negative argument, once illustrated, could be applied to any other case in which the proportionality is not satisfied, though the author does not attempt to prove this or even point it out. At 849b17-19, the author notes that 'if the [line] B traverses is less or greater than HB, the result will not be the same and the natural will not be proportional to the unnatural in both.' The emphasis on travel to Ω is convenient since the point has already been constructed on the diagram and the non-proportionality of the components indicated.
[64] 849b14-16: ἔστι δὲ ὡς τὸ HK πρὸς τὸ KB, τὸ ΘZ πρὸς τὸ ZX. φανερὸν δὲ ἐὰν ἐπιζευχθῶσιν ἀπὸ τῶν BX ἐπὶ τὰ HΘ. Diagrams in modern editions often omit the lines HB, ΘX, though manuscripts display them.
[65] Van Cappelle 1812, 164: 'Haec verba continent totius sequentis ratiocinii fundamentum, quod tamen hic sine ulla demonstratione ab Aristotele ponitur.'
[66] Schiefsky 2009, 58.
[67] Generally, if three points are fixed on a line, the positions of two will determine the position of the third. This simple form of reasoning has parallels in the Aristotelian corpus. *Problems* 16.3 and 12 both ask why bodies of inhomogeneous weight such as loaded dice revolve when thrown. Both assume that if parts of a body move at different speeds, it must be carried in a circle, and 16.12 also assumes that if all parts of a body move at the same speed, it must move in a straight line. The underlying assumption seems to be points retain their relative positions (see especially 16.3: ἐπειδὴ ἐν τούτῳ μόνῳ τῷ σχήματι ταῦτα ἀεὶ κατάλληλα ὄντα σημεῖα ἐν ταὐτῷ χρόνῳ ἀνίσους διέρχεται γραμμάς). Though different in emphasis, *MA* 3, 699a18-20 is another instance of reasoning about what we would call 'rigid' bodies: 'And those who say that no part whatsoever of the sphere which moves in a circle stays still, are correct in this proposal, at any rate: for then it would be necessary either for the whole thing to stay still, or for its continuity to be ripped apart [ἢ διασπᾶσθαι τὸ συνεχὲς αὐτῆς]' (trans. Morison).

under the influence of the same motive power. It would be improper, then, to assume that the points are fixed on the same radius.

The thought behind the proportionality may rather be that ἔκκρουσις is reactive in the sense I suggested above: the capacity for ἔκκρουσις is activated only when another motion occurs in the beam. In *MA* 3, 699a32-699b1 (quoted in n.56), Aristotle asserts that, when a body remains at rest despite a mover's pushing it, there must be a proportionality (τις ἀναλογία ἐξ ἀνάγκης) between powers for motion and powers for rest. Up to a limit, these powers balance out (ἰσάζειν), but 'they are overcome in states of excess' (κρατοῦνται δὲ κατὰ τὴν ὑπεροχήν). The idea seems to be that the mover, provided it is not overwhelmingly powerful, will bring about the activation or exertion of a certain amount of the object's power for rest, in proportion to or equal to the mover's strength.[68] *Mech.*'s author may have followed a similar line of thought: the mover acting tangentially to the beam will bring about the activation or exertion of a certain amount of the beam's power for ἔκκρουσις, in proportion to the tangential motion.[69] In that case, it would be the dependent nature of the exertion of ἔκκρουσις that accounts for the proportionality: ἔκκρουσις is the kind of effect that is activated only in response to, and in proportion to, a motion.[70]

4.7: Dynamics or kinematics?

As I noted at the start of Chapter 2, several interpreters have taken what may be called an instrumentalist view of problem 1's component motions. According to this view, the component analysis of motions is governed by geometric rules but is in a certain sense arbitrary. A body moving along a path can be treated as having one motion along the path, but it can also be seen as having two (or more) motions at angles to the path that compose to produce the appropriate result. All descriptions that yield the appropriate path are equivalent

[68] If the mover is overwhelmingly powerful, then the power for rest is unable to maintain the proportion and the other object will be moved. See Coope 2020, 262. The reference to ἀναλογία might suggest that Aristotle wishes to avoid directly comparing powers of rest with powers of motion. This could be achieved by a proportion such as mover $_1$: mover $_2$ = power of rest $_1$: power of rest $_2$. However, in 699a37 Aristotle refers to the powers as 'equal' (αἱ μὲν ἴσαι ἀπαθεῖς ὑπ' ἀλλήλων).
[69] If, alternatively, the component motions retain their respective directions throughout (an option to be considered in §4.9) and are only radial and tangential, another account for the proportionality will be needed. See De Groot 2014, 195-213 for a thorough discussion of a possible connection to Archytas' 'proportion of equality' reported at ps-Aristotle *Physical Problems* 16.9, 915a25-32.
[70] In unpublished material, Henry Mendell asks whether the proportion is nat_1:$unnat_1$=nat_2:$unnat_2$, or nat_1:nat_2=$unnat_1$:$unnat_2$. He observes that the latter is preferable if one assumes (a) that the majority manuscript reading of 849a15-17 is correct; (b) that *Mech.*'s author required ratios to be homogeneous.

and equally correct. To quote one recent advocate, 'Briefly put, the argument of *Mechanics* 1 is this. Both rectilinear movement and movement in a circle, i.e., the distances covered in each case, *can be* characterized as a ratio of two other movements that are identified by straight lines.'[71] Again, 'By making these points, the author shows that it is possible to contextualize *any* rectilinear motion as the diagonal of a parallelogram. The parallelogram of movements is a kinematic generalization.'[72]

If this were right, *Mech*.'s strategy could not be to identify the motions and causes which underlie the phenomena, since there would no privileged set of motions. The aim would rather be to illuminate relations between kinematic facts, truths about motion that do not take its causes into account. A surprising fact such as the Rotating Radius Principle is rendered intelligible by showing its place in a network of facts about motion, starting from the more readily intelligible 'parallelogram rule'.

I have been arguing that the evidence does not support such a view. From the start of the problem, the author emphasises that there are two motions in the rotating radius. If these were theoretical fictions, mere instruments of kinematic analysis, we should not find efficient causes. Yet our text does refer, however vaguely, to the causes of tangential and radial motion, (respectively, ἰσχύς/βάρος and τοῦ κέντρου τοῦ ἀντισπῶντος). It is notable that *Mech*. never claims to analyse circular motion (ἡ κύκλῳ κίνησις, ἡ κύκλῳ φορά, or κυκλοφορία) but rather what I have called radial rotation (ἡ κύκλον γράφουσα, literally 'the [line] describing a circle'). The choice of terminology seems deliberate. Recent interpreters such as De Groot and Schiefsky have rightly stressed that the notions of 'power' or 'force' involved are not highly theoretical, and this represents a real advance on Duhem, Krafft, and other twentieth-century readers. On the other hand, I have identified two crucial roles for radial constraint: it causes the smaller radius to move more slowly, and it is the nature of constraint as a reactive power that is responsible for the proportionality of natural and unnatural motions. The kinematics does not explain the dynamics, but, to repeat a point from Chapters 1-2, the distinction between dynamics and kinematics is alien to our author.[73]

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[71] De Groot 2014, 225, emphasis mine.
[72] De Groot 2014, 227, emphasis the author's. Compare Carteron 1923, 8n.28: 'Tout mouvement rectiligne se décompose en deux mouvements rectilignes proportionnels (848b1-26), le mouvement circulaire se décompose en deux mouvements rectilignes qui n'ont entre eux aucune proportion (b26-35).'
[73] The ambiguous status of ἔκκρουσις as both a motion and not a fully typical motion is another sign of this.

4.8: Problems 2 and 3

I will now briefly examine problems 2-3, which take us from the balance to the lever. How does the analysis of why larger radii move faster when exposed to the same tangential influence explain how small powers can lift large weights via a lever?

Problem 2 asks why the balance suspended from below does not return to the horizontal after being displaced, while the balance suspended from above does. In part, this problem prepares the way for problem 3, where the lever is characterised as a balance suspended from below. At the same time, problem 2 may show the author's awareness of the gap between his abstractions and physical reality.[74] In problem 1, the author represented the balance by a breadthless line. Now he explains the difference in the balance-beam's behaviour with reference to its breadth.[75]

Problem 3 turns to the lever, setting out to explain how small powers can move great weights. In passing, the author offers what appears to be a precise statement of the 'law of the lever'.[76] This is presented without explicit justification, although the particle οὖν may suggest an inference.[77] It is not unreasonable to suggest that the author may have presented it as a truth derived from observation, perhaps through experience with steelyards, without the backing of a mathematical derivation.[78]

How successful is the explanation of the lever? I agree that the precise 'law of the lever' is not explained, but what about the weaker, more qualitative principle that a power applied further from the centre or fulcrum has a greater effect? Carteron found the explanation 'very weak': 'What ought to have been shown is that the mover can lift ever heavier weights, the further it is from the centre: that is, beyond the geometrical and kinematical necessities, a

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[74] Cf. Aristotle *MA* 1's clear comments on the differences between animal parts and their geometric representations.
[75] Homogeneous weight appears to be assumed but not stated. Similarly, problem 10, by introducing weight, poses a challenge to problem 9's generalisation that bigger wheels are easier to move.
[76] 850b1-2: ὃ οὖν τὸ κινούμενον βάρος πρὸς τὸ κινοῦν, τὸ μῆκος πρὸς τὸ μῆκος ἀντιπέπονθεν. This is not the typical, Euclidean form of expression for inverse proportionality (Heath 1949, 235). Schiefsky 2009, 64 suggests that this passage might express only the weaker idea that any change in the relationship of the weights is compensated by the inverse change in that of the distances. Micheli 1995, 83 is even more sceptical.
[77] Noted by Schiefsky 2009, 65; Renn and McLaughlin 2018, 122.
[78] Micheli 1995, 83 doubts that the author could introduce the 'law of the lever' as an empirical assumption, but *Mech.* problem 24, 855a36-37 introduces a crucial proportionality with an appeal to observation (see Chapter 6).

principle of dynamics should have been distinguished.'[79] Since the same weight moves a longer balance arm moves faster, the same weight also moves a longer balance arm more easily. So extending the length of the lever-arm at the end at which force is applied makes the task of raising a load easier. Carteron saw this as an application of 'faster implies easier', a confusion of what we should call work and speed.[80]

Perhaps the thought is rather the following. A weight applied nearer the centre is constrained more by the centre (it is subject to more ἔκκρουσις) and so exerts a weaker, and hence slower, downward push. A mover's strength applied further from the centre is less constrained and so exerts a stronger downward push. There is no expansion of the mover's power *ad infinitum*. The relevant factor is the relation of the two downward pushes, and this is determined, for a given weight and a given moving power, by the constraint to which each is subject. The apparatus of the lever is only able to *reduce* effects via constraint and the reduction of the load's effect will be relatively greater if the load is closer to the centre than the moving power. Still, it must be stressed that none of this is explicit in the text.

4.9: Shortcomings of the *Mechanica*'s analysis

In interpreting *Mech*. problems 1-3, I have aimed to take due account of the author's conceptual background and assumptions, indicating how they differ from our own. Nevertheless, this account of the balance appears to suffer from several shortcomings, even on its own terms.

The two motions in a rotating radius are said to be (a) tangential and radial, (b) natural and unnatural, (c) represented by lines in the diagram. It is difficult to see how they can be all these things. Assuming that the motion of a heavy thing is downwards, towards the centre of the earth, there is a tension between (a) and (b): the downwards motion of the load on the balance is only both radial and tangential at the very start of its motion, and even then only for an instant. I have suggested that the labels 'natural' and 'unnatural' are inessential to the

[79] Carteron 1923, 14. Another complaint is that the author fails to keep his promise to explain the lever in terms of the balance (848ab12-15). Thus Schiefsky 2009, 64: 'Despite the author's claims... the references to the "center"... signal the circle model, and it is in fact the circular motion principle that supplies the explanation.' Berryman (2009 64-5) claims that the author claims the lever works on the same principle as the balance, but 'does not say how the two are connected.'
[80] Carteron 1923, 15.

explanation, that they are applied only because the author is describing a horizontal balance about to begin its downward descent. That reading dissolves the tension between (a) and (b). The very slight cost is that the labels 'natural' and 'unnatural' would have to be replaced by other terms if the same explanation were to be given for a case where the tangential motion is not natural at the initial instant.[81] Deeper difficulties are posed by the tension between (a) and (c). Let us now turn to that.

Problem 1 began by considering the quadrilateral of motions. In this case, each component motion remains parallel to itself over time. Presumably the component motions could remain parallel to themselves over time even if the motions did not maintain a constant ratio for any time. We are not given a reason to doubt that they could. Yet this property is not shared by the radial and tangential component motions of a rotating radius. Radial and tangential motions are clearly at issue in the passage on radial constraint (849a6-849a19, examined in §4.5). They are emphasised again in problem 8 (852a8-13):

> ἐκ δύο φορῶν γεγένηται ὁ κύκλος… τὴν μὲν γὰρ εἰς τὸ πλάγιον αὐτοῦ κίνησιν ὠθεῖ τὸ κινοῦν, τὴν δὲ ἐπὶ τῆς διαμέτρου αὐτὸς κινεῖται.
>
> The circle is made from two motions… for the mover drives the sideways motion, and [the circle] itself moves with the [motion] along the diameter.

However, in both phases of the argument of 849a19-849b19 (discussed in §4.6), the component motions of the rotating radius are treated as remaining parallel to themselves over time. The diagram (fig. 9) still operates with the rectangular composition of two motions.

There is an apparent rift between two approaches, one which analyses rotation into tangential and radial components, and one which analyses it in terms of rectangular composition, where the components are only radial and tangential at the very start of the motion. Which approach is correct?

[81] Another problem could be pressed. I have written as if there is a first instant of motion at which the tangential motion is natural, but Aristotle denies that there is a first instant of motion. To this it could be replied that *Mech.* is using the labels loosely.

Option (A): Each component motion remains parallel to itself over time and they only align with the circle's radius and tangent at the start. Within the quadrant, the downward thrust remains the same, whereas the deflecting component is greater and greater as the balance arm descends. The advantage of this option is that it gives lines YΩ, ΘZ, BY, XZ in fig.9 a relatively straightforward interpretation. It is of course not the case that the radius moves to Ω via two motions in the ratio of BY to ΩY, for otherwise it would move on a straight line. Rather, each of the lines YΩ, ΘZ, BY, XZ represents the total quantity of motion during the respective rotation. The interpretation of these lines is more problematic on Option (B), as we shall see below. Option (A) faces three main difficulties.

First, it limits the analysis to motion within the quadrant. What happens when the radius continues its rotation past the quadrant is unclear. Perhaps one motion ends and a new one is acquired. In any case, the division of continuous, symmetric radial rotation into quadrants seems to be an artificial imposition, especially for cases of horizontal rotation such as the potter's wheel or the sling.[82]

Second, on this option, the component motions of a rotating radius are not tangential and radial, except at the very start of the motion. This is difficult to reconcile with the emphasis on radial and tangential motions in both problem 1, 849a6-849a19 and problem 8, 852a8-13.

Third, this makes it harder to understand why the proportionality is introduced at 849b4-6. As I suggested above, the proportionality could be accounted for in terms of radial constraint, but that is only possible on Option (B). An alternative suggestion, that the proportionality is justified because the points under discussion are fixed on a single radius, would in principle still be open. However, as I argued above, the assumption that there is a single radius under discussion is not supported by the context.

Option (B): One of the rotating radius' two motions is radial, the other is tangential.
The main advantage is that this fully accommodates problem 1, 849a6-849a19 and problem 8, 852a8-13. Option (B) faces two problems.

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[82] At 849a27-28 the author is clear that the radius 'returns to itself', i.e. it completes a full revolution, not just a quarter revolution.

First, it is unclear on this approach how the lines ΥΩ, ΘΖ, ΒΥ, ΧΖ represent the component motions. ΥΩ and ΘΖ supposedly represent the tangential motions of points B and X respectively. Although they do not make contact with those points, they are at least parallel to the correct direction when the radius is at B. Yet when the radius has reached Ω, ΥΩ is not tangential.[83] One suggestion would be that the lines ΥΩ, ΘΖ, ΒΥ, ΧΖ offer an erroneous but workable representation of the radial and tangential motions. They have the status of an approximation or idealisation. It may be argued that, since Ω, H, and Θ are arbitrary within the quadrant, any distortions introduced by treating each component as if it remained parallel to itself over time can be minimised by making ΥΩ, ΘΖ, ΒΥ, ΧΖ arbitrarily small. Even if that is so, it must be admitted the author has not made any attempt to explain how a conclusion about the lines of the diagram could be transferred to radial and tangential motions.

Second, an earlier section of the problem (848b25-35; see §4.3 above) introduced the idea of motions in no fixed ratio for any time. How can the ratio between the radial and tangential motions be constantly varying, given the symmetry of the circle? I see two ways in which this question can be answered. First, one could argue that there is a break in problem 1's reasoning by the time we reach 849a19-849b19 and that the 'no ratio for any time' thesis is quietly ignored thereafter. Perhaps that idea was mobilised in 848b26-849a6 only to argue for the suggestion that two orthogonal rectilinear motions can compel something to move along a circular path, while a different approach is needed once ἔκκρουσις is introduced (849a6), because ἔκκρουσις is no ordinary motion. When ἔκκρουσις acts orthogonally to another motion, it causes the moving thing to slow down rather than speed up (849a6-849a19), whereas in the rectangle of motions, the diagonal is longer than either side and so the addition of a motion at right angles makes the object move faster.[84]

Alternatively, one could argue that the 'no ratio for any time' claim need not imply that at least one of the motions involved has varying speed. It might also apply to cases where the

[83] As noted by Brown, 1978, 182.

[84] Hine 1984 claims that whether deflection decreases speed or not should depend on the angle between the two motions: '*Mech*. 849a6-9… is wrong when it says that a greater deflecting force acting on a moving body results in slower motion than a lesser deflecting force acting in the same body – for this ignored the difference made by the angle between the two forces.' But all motions in problem 1 are orthogonal; the angle between motions is not considered until problem 23.

relation between the two motions cannot be characterised in terms of a ratio at all.[85] What about the proportionality between natural and unnatural motions introduced at 849b4-6? Does that not assume that two component motions of each radius are in some ratio, albeit one that is constantly varying? Not necessarily. The proportionality can be taken in two ways: $nat_1{:}unnat_1{=}nat_2{:}unnat_2$, or $nat_1{:}nat_2{=}unnat_1{:}unnat_2$.[86] On the latter reading, no radial motion is claimed to be in any ratio with a tangential motion. Even if both ratios, $nat_1{:}nat_2$ and $unnat_1{:}unnat_2$, are constant (and *not* constantly varying), as presumably they would be if they compare, respectively, radial motions and tangential motions, the proportionality would still serve two functions. It would identify a significant fact about how ἔκκρουσις responds to the motion that brings it about, and it would show, through the false but workable assumption that the motions remain parallel to themselves, that, however far Θ rotates due to a given tangential input, H, when subject to the same input, will travel through an equal angular distance.

Each of the above options faces its difficulties. The author gives us no guidance as to how the two approaches should be reconciled or which should be given primacy. That being said, we may not need to choose between them. The author did not resolve the tension himself. It may be worth recalling that the *Mechanica* is a problem text. Like the *Physical Problems*, is often aporetic in tone. Its solutions, which are typically posed as questions ('Is it because…?'), are not presented as definitive.[87] Sometimes more than one answer is offered in response to a single question.[88] The two approaches we have identified in problem 1 may be another case of attacking a problem from more than one angle. On the other hand, it must be admitted that the two approaches are less clearly marked and separated.

There is a final shortcoming, not directly related to the foregoing. It is not clear how problem 1's analysis can be extended to cases where the centre of rotation lies outside the rotating body. Aristotle described in *MA* 7 a child's toy carriage with unequal wheels which moves around in a circle. The centre of rotation will lie somewhere beyond the smaller wheel on a

[85] Monantheuil 1599, 31-33 understood the claim in this way, though for the different reason that he (like Micheli 1995 and Van Leeuwen 2016) took the natural motion as along the circumference.
[86] I owe this observation to Henry Mendell. The statement in Greek is (849b4-6): δεῖ δὲ ἀνάλογον εἶναι, ὡς τὸ κατὰ φύσιν πρὸς τὸ κατὰ φύσιν, τὸ παρὰ φύσιν πρὸς τὸ παρὰ φύσιν.
[87] Admittedly problem 1's solution is an exception: it is *not* posed as a question. Keyser 2020 88-89 suggests that the *Problems* attempts not to provide definite explanations but to show that apparent anomalies can be handled within an accepted theoretical framework.
[88] This occurs in problems 12, 19, 30, 32, 34, 35

straight line connecting the two centres of the two wheels, although Aristotle somewhat surprisingly says that the smaller wheel itself acts like the centre.[89] The author of the *Mechanica* does not explain how his theory should be adapted to cases like this. Simply asserting, like Aristotle, that the part of the body nearest to the centre of rotation *is* the centre would do nothing to clarify matters.

4.10: A model for eternal motions?

Scholars have identified three main points of tension between Problem 1 and the certainly authentic writings of Aristotle. In Chapter 1, I argued that the tension in two of these cases may be merely apparent.[90] The third point of tension is that the treatment of motion on a circular path as a mixture of two rectilinear motions seems to go against the grain of Aristotle's certainly authentic works, particularly the *Physics* and *De Caelo*, where circular motion is treated as simple and uniquely capable of continuing eternally.[91]

It is important to understand the nature of the tension. We have seen that *Mech.*'s claim that some unnatural motions on circular paths are mixtures of rectilinear motions could co-exist with Aristotle's view that the heavens' natural rotation is simple.[92] A more interesting point is that *Phys*. 8.8's arguments that no rectilinear motion can be eternal, and that hence only (simple) circular motion can be eternal, do not clearly rule out the possibility of eternal rectilinear motions that are mixed in the way described by *Mech*. problem 1. *Phys*. 8.8 presents several arguments that there cannot be an eternal motion back and forth along a finite rectilinear distance, but the radial and tangential rectilinear motions that make up *Mech.*'s radial rotation are not motions back and forth along radial and tangential lines.[93]

Could there be a pair of eternal motions composed in the manner described in problem 1?[94] Two issues are involved here. The first is that such an arrangement would require at least one

[89] *MA* 7, 701b4-7: καὶ τὸ ἁμάξιον, ὅπερ <ὁ> ὀχούμενος αὐτὸς κινεῖ εἰς εὐθὺ <πάλιν> καὶ πάλιν, κύκλωι δὲ κινεῖται τῶι ἀνίσους ἔχειν τοὺς τροχούς – ὁ γὰρ ἐλάττων ὥσπερ κέντρον γίγνεται, καθάπερ ἐν τοῖς κυλίνδροις –, οὕτω καὶ τὰ ζῶια κινεῖται.
[90] These two cases were *Mech.*'s use of the terms 'natural' and 'unnatural' and *Mech.*'s supposed assumption that there is motion at an instant.
[91] Relevant passages discussed in §2.4 above.
[92] This has been noted by Bodnár, 2011a, 448-449.
[93] There cannot be rectilinear motion along an infinite distance since, according to Aristotle, the world is finite.
[94] I am grateful to István Bodnár for his suggestions on this section.

eternal rectilinear motion to drive it, filling the role played by the weight acting tangentially in the case of the balance. Something like that might seem possible. Think of a water-wheel being turned by the rectilinear motion of a river. Provided that river doesn't cease to exist, its rectilinear will continue eternally. The river continues to flow because it is part of a larger system of motions in the sublunary sphere, the cycle of elements that imitates the celestial motions.[95] An eternal rectilinear motion might be possible within such a larger system, one in which there are already eternal motions, simple, circular ones, in the celestial realm.

On the other hand, the identity of the river's motion can be questioned by reference to the criteria we examined in §2.4. Recall that for two motions to be identical, they must be in an identical subject. The river with its ever-changing waters is hardly a paradigm of identity over time. In the *Topics* 1.7, 103a14-23: water from the same spring (τὸ ἀπὸ τῆς αὐτῆς κρήνης ὕδωρ) is the same in species, because it is is very similar (τῷ σφοδροτέραν εἶναι τὴν ὁμοιότητα), but it is nevertheless not numerically the same. If the river does not have numerical identity over time, its motion will not either, and so the water-wheel's rotation may not be a single eternal motion but rather a series of overlapping or interlocking finite motions.[96]

Also relevant in this connection are two passages of the *De Caelo* in which Aristotle rejects the idea of an eternal unnatural motion. In the first, his qualm is with the notion that the only continuous and eternal motion might be unnatural: 'it is strange, in fact quite absurd, that being unnatural it should yet be the only continuous and eternal motion, seeing that in the rest of nature what is unnatural is the quickest to fall into decay'.[97] The second passage, in the course of an argument that the existence of the aether necessitates the existence of the earth, appears to state more clearly that 'nothing unnatural is eternal'.[98]

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[95] See *GC* 2.10-11on the cycle of sublunary elements and *Mete*. 1.13 on rivers in particular.
[96] Cf. Heraclitus DK22 B12.
[97] *DC* 1.2, 269b7-10: θαυμαστὸν καὶ παντελῶς ἄλογον τὸ μόνην εἶναι συνεχῆ ταύτην τὴν κίνησιν καὶ ἀΐδιον, οὖσαν παρὰ φύσιν· φαίνεται γὰρ ἔν γε τοῖς ἄλλοις τάχιστα φθειρόμενα τὰ παρὰ φύσιν.
[98] *DC* 2.3, 286a17-28: οὐθὲν γὰρ παρὰ φύσιν ἀΐδιον. *Mete*. 1.3 seems to attribute an eternal circular motion to the highest, fiery part of the sublunary world, τὸ ὑπέκκαυμα ('fuel', 'the inflammable'), which modern scholars often refer to it as the 'firesphere'. The rotating firesphere plays a significant role in Aristotle's explanations of meteorological phenomena (see especially *Mete*. 1.4 and 1.7). Some ancient commentators, for example Xenarchus and Philoponus, took the firesphere to vitiate *DC*'s arguments that the heavens are made of aether rather than fire (see Wildberg 1988, 125-134). Others defended Aristotle. For example, Simplicius distinguished unnatural motion that is contrary to nature (e.g. fire moving downwards) from unnatural motion that is not (e.g. fire rotating around the centre). Simplicius calls the latter kind of non-natural motion ὑπερ φύσιν. Arguably, the firesphere, like the river, is not an eternal thing, since portions of matter constantly join and leave it. In that case, it would have a series of finite motions, not an eternal motion.

No justification is given but one might conjecture that one could be constructed through the 'principle of plenitude' and its denial that there are any eternally unrealised possibilities. Since it is possible for a παρὰ φύσιν to stop, it must at some time stop.

The second issue is that the material object in which the motions are instantiated, for example the water-wheel, will, like all things in the sublunary domain, be subject to wear and tear. If the water-wheel's parts are gradually replaced over time, will it be the same wheel? Arguably the refurbished wheel would be a different thing of the same form or kind, and in that case the motions of the refurbished wheel would be different motions. However, some Aristotelian scholars argue that form is the criterion of identity over time.[99] For Frede, this is related to the fact that Aristotle speaks of form as substance in *Met*. Z.[100] On that view, so long as the water-wheel did not at any stage lose its organisation and function, it would retain its identity. This turns on deep problems in Aristotle's metaphysics, and for this reason, I leave as moot the issue of whether artefacts can be eternal by retaining their identity through repair and renovation, or whether the continuously refurbished artefact is in fact would be a series of things of the same form or kind.

Even as we leave the second issue unsettled, our recognition of the first issue, the fact that problem 1's component motions depend on a rectilinear input motion, brings us some way to seeing how Aristotle could defend his procedure in *Phys*. 8.8. Such an arrangement of composite motions could only be eternal if there were an eternal rectilinear motion, which *Phys*. 8.8 shows to be impossible. Aristotle does not argue against *Mech*.'s arrangement of component motions in *Phys*. 8, but since he had the resources to show that motions in such an arrangement could not be eternal, one cannot draw conclusions from this omission about his awareness or ignorance of the theorising in *Mech.* problem 1.

[99] Frede 1987, 66: 'What makes for the identity of the repaired ship with the original ship is obviously a certain continuity. This is not the continuity of matter, or of properties, but the continuity of the organization of changing matter, an organization which enables the object to function as a ship, to exhibit the behavior of a ship.'
[100] Frede 1987, 64-65.

4.11: Conclusion

I have argued that problem 1 goes beyond what we find in Aristotle's certainly authentic works, but also that its account of composed motion can be understood in terms of Aristotle's account of change. One might say that Aristotle provides the conceptual starting-points and explanatory standards, and the author of *Mech*. develops an innovative account of composed motions against this backdrop.

I have argued that the author assumes what we may term *realism* about component motions and aims to give a causal account of sublunary motion on a circular path, to be distinguished from the simple circular motion of the heavens.

I have suggested that several features of Problem 1's presentation are conditioned by the author's focus on the specific case of the rotating balance. The quadrilateral of motions is likely a rectangle rather than a general parallelogram, since the problem's explanation involves only perpendicular component motions. The label κατὰ φύσιν corresponds to the downwards motion of a weighted balance-arm.

The use of geometrical language and diagrams in problem 1 is relatively informal. One might question whether this should be counted as a contribution to 'mathematics'. It may be more apt to say that a Peripatetic philosopher is drawing on the resources of geometry to support a basically physical agenda and a causal explanation. A detailed comparison of *Mech*. problem 1 and Aristotle's treatments of the halo and rainbow in *Meteorology* 3 would be enlightening.[101] The first two diagrams of problem 1 are underdetermined by the text and consequently the diagrams themselves must carry a substantial part of the argument's weight. In Chapter 5 I shall argue that the explanations of problems 4-22 also rely on visual as well as verbal forms of argument.

[101] I intend to pursue this in future work.

Chapter 5: Analogical explanation through language and diagrams

5.1: Introduction

The primary task of this chapter is to analyse the methods of explanation in *Mech.* problems 4-22.[1] These problems ask questions about the movements and effects of artefacts that are answered with reference to the subjects of problems 1-3: the rotating radius, the balance, and the lever. My first claim is that these explanations are fundamentally analogical. My second claim is that the use of specialised geometrical language and diagrams is essential to establishing the analogies.

My emphasis on language and diagrams recalls the work of Netz, who has argued for their central importance in geometrical arguments.[2] This link to the tools of mathematics is especially interesting in light of the methodological passage of *Mech.*'s introduction, discussed in Chapter 1, which claims that 'the 'how' (τὸ ὣς) is clear through mathematics, the 'about what' (τὸ περὶ ὃ) through physics'.[3] However the function of diagrams in *Mech.* is considerably different from what Netz found in more purely geometrical texts.

Diagrams throughout *Mech.* represent a mix of both abstract (e.g. 855b5: 'ἔστω γὰρ κύκλος...', 'for let there be a circle…') and physical (e.g. 851a17-18: 'ἔστω γὰρ ἡ AB κώπη…', 'for let the [line] AB be an oar…') objects. Accordingly diagrams can function as intermediaries between the physical explananda and the models of the lever and balance with which they are compared. I shall suggest that visual analogy between the two classes of diagram, physical and abstract, is a tool for securing analogies, complementary to the text's verbal content.

By contrast, arguments in the deductive works of Greek geometry are not based on analogy, and no argumentative force is gained by drawing visual analogies between the diagrams

[1] This portion of the text has received less attention in recent scholarship (e.g. De Gandt 1982, De Groot 2009, Schiefsky 2009) which has tended to focus on Problem 1. Van Leeuwen's subsection on 'Cognition of Diagrams' (Van Leeuwen, 2016, pp. 148-57) considers diagrams from problem 1 only. Sections 5.1, 5.3 and parts of 5.2 and 5.5 had developed from an essay submitted in November 2018 for the MPhil in History and Philosophy of Science and Medicine at the University of Cambridge.
[2] Netz 1999a, which focuses on Euclid, Archimedes and Apollonius.
[3] 847a27-847b1.

constructed in mathematical texts. The geometers do make use of analogy in a different way, insofar as they generalise from arguments about particular diagrams to theorems and problems about all cases of the relevant kind, for example, from a proof concerning some triangle ΑΒΓ to a conclusion about all triangles. Mueller raised the problem of the legitimacy of such universal generalisation, which geometers do not attempt to justify.[4] Netz agreed that Greek mathematicians and philosophers had offered no justification of generalisation, but provided an explanation of why their generalisations nevertheless appeared convincing, in an attempt to 'unearth their unsatisfactory theory – even supplying them with the articulation they might have lacked.'[5] Similarly, this chapter will explore why 'unsatisfactory' analogical arguments in *Mech.* seemed reasonable.

In the canonical works of Greek geometry, there is a one diagram for every proposition, and vice versa. Netz's survey of Apollonius' *Conics* III shows that despite much continuity of subject matter, a diagram is constructed afresh for each proposition, including two identical diagrams in propositions 45-46.[6] In *Mech* we shall find precisely the opposite: an economical sharing of diagrams across problems and explananda. I suggest that this may reinforce the text's aim of unification.

The argument of this chapter also contributes to a discussion in the study of Peripatetic science: to what extent did Aristotle's accounts of syllogistic in the *Prior Analytics* and of demonstration in the *Posterior Analytics* set the aims and methods of scientific inquiry undertaken by Aristotle and his followers? Recent debate has largely focussed on the zoological works.[7] Mechanics promises to be an especially interesting case, given its close relation to geometry, the paradigmatic demonstrative science in antiquity.[8]

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[4] Mueller 1981, 11-14.
[5] Netz 1999a, 241.
[6] Netz 1999a, 38-43.
[7] Among some of the most important contributions arguing for reconciliation of the natural investigations and the *Analytics* are Bolton 1987, Gotthelf 1987, Lennox 1987, Lennox 2001 ch.1-4, Leunissen 2010. For the alternative view, see Barnes, 1969, Barnes 1981, Lloyd 1991, and Lloyd 1996.
[8] Cf. Archimedes' *Plane Equilibria*, an example of axiomatic-deductive mechanics.

5.2: Analogy and the ideal of demonstrative science

It has recently been suggested that *Mech.* closely follows *APo.*'s methodological schemata.[9] In its strongest form, that suggestion might seem to sit at odds with my thesis that *Mech.*'s explanations rely on analogy. Before analysing the specific techniques of analogical reasoning deployed in *Mech.*, it is the aim of this section to argue that *Mech.*, for all its use of geometrical ideas, does not apply a method of demonstration.

In purely formal terms, *Mech.* does not have the appearance of a demonstrative text. There is no list of first principles at the outset. Problems 4-24 do not build on one another's conclusions.[10] Lettered diagrams do not feature as objects of continuous deductive reasoning – a point I will elaborate on below. There is no mention of demonstration (ἀπόδειξις), definitions or syllogism.[11] That there is not a single explicit syllogism in *Mech.* is, of course, inconclusive, since it can be argued that *APo.*'s strictures require only that scientific explanations be syllogistically *formalisable* and that whether the formalisation has actually been carried through or not is of little consequence.[12] Nevertheless, the assumption that *Mech.*'s author was interested in formalisability is open to question. Any single valid deductive inference can be presented in syllogistic form, when taken in isolation.[13] So the bare fact that some individual inferences in *Mech.* can be reformulated as syllogisms tells us little about the author's aims and methods.

In fact, attempting a syllogistic reformulation speaks against the suggestion that *Mech.*'s explanations are demonstrative. One could represent Problem 5 of *Mech.* syllogistically as follows:

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[9] Johnson 2017 argues that 'the methodology of *Mech.* is self-consciously modelled on Aristotle's method in the *Analytics*' (134) and that 'Aristotle or some other philosopher self-consciously following the methodology of *APo*. (and following it closer than Aristotle often seems to in other surviving scientific works) authored *Mech*., the first systematic text in the science of mechanics.' (150). See also Anders 2013. Asper 2017, 45 describes *Mech*. as 'a rigorous deductive piece that attempts to derive all solutions of physical problems from one complicated mathematical-physical demonstration of how circles and their radiuses behave. ' Further, 'The author's ambition primarily focuses on subjecting phenomena to rigorous explanation. The rigour is deductive in the sense that Euclidean exposition is.' (47).

[10] Contrast *Elements* Book 1 where twenty propositions form a deductive chain of dependence. Longer deductive chains are an indication that the author has attempted to economise on assumptions or first principles. Schiefsky 2009, 52 provides a tree-diagram which shows the limited dependence relations between *Mech.*'s problems.

[11] The only two uses of a δείκ- stem are inflections of δείκνυμι (problem 25, 856b30; problem 27, 557a33), meaning nothing more theoretical than 'show'.

[12] For arguments of this kind in relation to Aristotle's zoological works, see Gotthelf 1987; Lennox 1987.

[13] C.S. Peirce 1992, 131f. showed that any deductive inference can be cast as a syllogism in *Barbara*.

(1) All levers produce greater movement.

(2) All rudders are levers.

(3) All rudders produce greater movement.[14]

We have seen in the course of Chapter 4 that premise (1) is established in problems 1-3 through indirect and heuristic arguments, rather than demonstrations from first principles.[15] However, my concern in this chapter is with premise (2). Premise (2) is not a first principle and is not demonstrated from first principles. It is secured in the text of Problem 5 through analogical reasoning. If the premises of a syllogism are neither first principles nor conclusions of deductions from first principles, then the syllogism is not a demonstration. Aristotle himself classified analogical arguments as rhetorical and persuasive, sharply distinguishing them from deductively valid syllogisms.[16]

To be sure, no explicit terms of comparison are involved in establishing (2). Artefacts compared to levers are not said to be or behave 'like' levers; rather, the author says that each *is* (ἐστι, γινεται) a lever.[17] The crucial point is that *Mech*. never even attempts to define the lever, so (2) cannot be inferred deductively. In the paradigm case of problem 3, the lever is a tool for lifting heavy objects, where the strength applied is on the opposite side of the fulcrum from the load. From problem 4 onwards, the sense of 'lever' is stretched from this paradigm case to devices that do not share all these properties. Consider the rudder of problem 5. The rudder does not lift anything upwards; the helmsman and the ship he moves are on the same side of the rudder, not on opposite sides.[18] The conditions under which a device may truly be called a lever are never specified.[19] The author justifies a claim that a device is a lever only partially, through the identification of some points of similarity.

[14] Compare Johnson 2017, 136 and *passim*.

[15] For Aristotle, indirect arguments fall short of the ideal status of ἀπόδειξις (cf. *APo.* 1. 26). On the other hand, Euclid and other geometers use indirect arguments freely.

[16] See *Rhetoric* 2.20, 24 on argument from example, and *Topics* 1.13, 17-18, 8.1 on argument from likeness.

[17] Hence 'all rudders *are* levers' in my example.

[18] *Mech*., in common with later Greek writers on mechanics, does not distinguish the three classes of lever taught to modern students of mechanics.

[19] The Hippocratic *On Fractures* 31 gives a more detailed account of the necessary conditions for successful application of a lever: 'One must have iron rods (σιδήρια) made in fashion like the levers (μοχλοὶ) used by stone masons, broader at one end and narrower at the other… Then one should use these, while extension is going on, to make leverage (μοχλεύειν), pressing the under side of the iron on the lower bone, and the upper side against the upper bone, in a word just as if one would lever up violently a stone or log. The irons should be as strong as possible so as not to bend… If, perchance, the upper bone over-riding the other affords no suitable hold for the lever, but being pointed, slips past, one should cut a notch in the bone to form a secure lodgment for the lever.' (trans. Withington with modifications). For the Hippocratic author, too, the paradigm case is lifting heavy objects, such as stones and logs; the medical tools are 'like' these levers. The two necessary conditions noted are

In each analogical explanation, parts of the phenomenon to be explained are assimilated with parts of the relatively abstract systems of problems 1-3. As an example, here is the first analogy to the lever, with constituent parts of the lever underlined (problem 4, ‘Why do rowers amidships move the boat the most?’, 850b11-13):

> ἢ διότι ἡ κώπη μοχλός ἐστιν; ὑπομόχλιον μὲν γὰρ ὁ σκαλμὸς γίνεται (μένει γὰρ δὴ τοῦτο), τὸ δὲ βάρος ἡ θάλαττα... ὁ δὲ κινῶν τὸν μοχλὸν ὁ ναύτης ἐστίν.
>
> Is it because the oar is a lever? For the thole-pin becomes a fulcrum (since it stays in place), the sea [becomes] the weight... and the sailor is the mover of the lever.

In the following section, I shall examine the verbal and visual tools implemented to support these non-deductive inferences.

5.3: Diagrams and technical language

In a Euclidean proof, the text always takes the reader through the construction of the diagram using a standard set of imperatives and subjunctives. *Mech.* adopts a mixed practice. In some cases the diagram is explicitly constructed using these standard construction formulae (e.g. 849a21-25):

> Let there be a circle on which are ΒΓΔΕ (ἔστω κύκλος ἐφ’ οὗ ΒΓΔΕ), and in this another smaller circle on which are ΧΝΜΞ, [both] about the same centre Α. And let the diameters be cast out (ἐκβεβλήσθωσαν αἱ διάμετροι): [the diameters] in the big circle on which are ΓΔ and ΒΕ, and the [diameters] ΜΧ, ΝΞ in the smaller circle.

In other cases there is no explicit construction. The diagram is, as it were, taken for granted. This is the case in two passages we examined in Chapter 4. In setting out the quadrilateral of motions (848b13-23) the author initially only discusses lines’ relations to one another and doesn’t confirm the overall shape until later:

(i) that the lever is rigid and does not bend; (ii) that the lever does not slip relative to the fulcrum. Condition (i) is never implied in *Mech*. A gesture to something like (ii) is made at 850b12 where the analogy between the thole-pin and the fulcrum is justified ‘μένει γὰρ δὴ τοῦτο’ (‘since it stays in place’), repeating almost verbatim the phrase of 849b23. But does the thole-pin of a moving ship remain stationary? Condition (ii) is perhaps more clearly violated in problem 17’s assertion that the wedge is a lever (see further discussion below).

> Let the ratio by which the moved-thing moves be that which the [line] AB has to the [line] AΓ.

The author presupposes a diagram is already available to his readers, and interprets it as a representation of component motions. For readers without a reliable diagram to hand this is initially confusing. The ambiguity is eventually settled at 848b20, which reveals in passing that the shape is a quadrilateral.[20] But in other cases the ambiguity is more serious, and the correct diagram-construction is never resolved by the text.

Van Leeuwen gives the example of problem 1's second diagram construction, which aims to show that the ratio of the two rectilinear motions of a rotating object must continually vary (849a2-3):

> Let there be a circle ABΓ, and let the end-point on which B is be carried to the point Δ.

We are never told whether the point Δ is inside, outside, or on the circumference of the circle, and editors of the text have produced a range of interpretations.[21]

The manuscripts offer no solution. The archetype of their diagrams is probably Byzantine, and hence even van Leeuwen's recent critical edition cannot tell us what the diagrams looked like in the original text of the late fourth or third century BCE.[22] Since we are concerned with mechanical theory in its formative centuries, and not its Byzantine reception, we should consider primarily what can be cautiously gleaned from the text itself.

One consequence of widespread under-specification is the impossibility of reconstructing diagrams with certainty from the text alone. This is acutely problematic for our understanding of *Mech.*'s diagrams since no ancient diagrams survive.[23]

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[20] Whether this quadrilateral is a rectangle or a parallelogram is controversial; see my discussion in Chapter 4.
[21] van Leeuwen 2016, 91.
[22] van Leeuwen 2016, 74, 99.
[23] Netz 1999, 19-26 showed that this is not unusual for diagrams in Euclid and Apollonius.

There are references to lettered diagrams in the preface and in thirteen problems.[24] Strikingly, there are no references to diagrams throughout problems 6-16. Still, as a preliminary assessment we may say the number of diagrams is very high for a text on natural inquiry.[25] The uneven distribution of references to lettered diagrams in *Mech.* raises the question of why they are used in particular problems but not in others. Bearing this question in mind I turn now to close readings of passages that refer to diagrams in Problems 3-22.

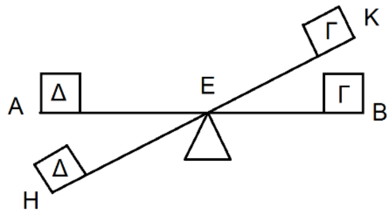

Fig. 1. Hett's diagram for problem 3.

In Chapter 4, I showed that the diagrams of Problem 1 are an essential part of that problem's arguments. The diagram of problem 3 ('Why can small forces move great weights by means of a lever?') does not fit this mould. It appears at first superfluous, or at least merely illustrative and not explanatory. The "Law of the Lever" is stated at 850b1-3, together with the qualitative statement that 'the greater the distance from the fulcrum, the more easily it will move'. Only at the end of the problem, with the explanation already given, does the text describe the construction of a diagram.

In working our way towards understanding *Mech.*'s practice here, let us consider a parallel case. Problem 26 (Why is it harder to carry beams on the shoulders by the end than by the middle?) ends with the construction of a simple diagram lettered A-Γ, which at first seems merely illustrative in the same way as that of problem 3. However this diagram seems to be shared with problem 27, which refers to letters A-Γ of a diagram without constructing it (857a30-33).[26] This sharing of a diagram across discrete textual units contrasts with the one-to-one correspondence of propositions and diagrams in later geometry. Problems 26-27 demonstrate that the author of *Mech.* did not abide by those constraints. Bearing this in mind, let us return to the diagram of problem 3.

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[24] Problems 1-3, 5, 17, 21-27, 30.

[25] There are few or no diagrams in most of Aristotle's authentic writings on natural science – though for notable exceptions see Taub 2017, 100-103 – and likewise in the pseudo-Aristotelian *Physical Problems*. Even in the 'mathematical' Book 15 of the *Physical Problems*, the density of diagrams is significantly lower than in *Mech.* (3 lettered diagrams over 3 Bekker pages, against *Mech.*'s 17 lettered diagrams over 11 Bekker pages).

[26] Problems 26-27 share a diagram in all extant manuscripts (van Leeuwen, 2016, pp. 248-250).

I propose that problem 3's diagram (compare Figs. 1 and 2) may be similarly put to work in subsequent problems, and thus provide the foundation for lever-based explanations. When the author offers an explanation by analogy to the lever, the diagram of problem 3 could serve as a visual aid to the reader trying to make the analogical connections demanded, such as in the ship's oar of problem 4 or the beam of problem 16. The act of turning back from explanations of sundry phenomena to the same single diagram would strongly reinforce the unifying pattern of explanation that is characteristic of *Mech*. Far from being explanatorily useless, this diagram may be at the heart of subsequent explanations by analogy to the lever.

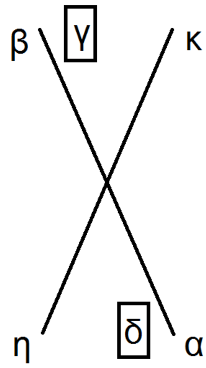

Fig. 2. Van Leeuwen's (2016, 215) reconstruction of the archetype diagram for problem 3

Let us see how problem 3's diagram might support a more complex analogical argument. Problem 17 asks why small wedges split large bodies and answers by explaining that wedges are composed of two levers. This is one of *Mech.*'s less felicitous explanations. A wedge for splitting or cutting, such as that of an axe, does not behave like a lever or pair of levers.[27]

Here, the diagram may have added persuasive force to a weak argument (compare Figs. 3 and 4). The construction begins, 'ἔστω σφὴν ἐφ' ᾧ ΑΒΓ, τὸ δὲ σφηνούμενον ΔΕΗΖ' ('let there be a wedge on which are ΑΒΓ, and the thing-being-split ΔΕΗΖ' 853a27). We have, presumably, a triangle ΑΒΓ representing the wedge, and a quadrilateral ΔΕΗΖ representing the candidate for splitting (see Figures 3-4).[28] The text then uses this lettered diagram in its analogy to the lever, beginning 'μοχλὸς δὴ γίνεται ἡ ΑΒ, βάρος δὲ τὸ τοῦ Β κάτωθεν, ὑπομόχλιον δὲ τὸ ΖΔ' ('the [line] AB becomes a lever, what is below B a weight, and the [area] ΖΔ a fulcrum' 853a28). But the wedge's edges, AB and ΒΓ, are not obviously akin to levers. They do not pivot about the identified fulcrum. In fact, they do not seem to rotate at all. This is especially troubling since *Mech.*'s account of levers is founded on the

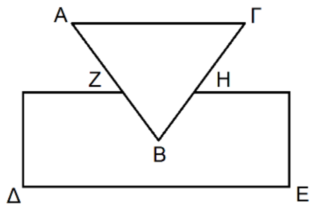

Fig. 3. Hett's diagram for problem 17.

[27] Rather, it can be thought of as two inclined planes.
[28] Hett curiously does not place Z and H at the corners of the quadrilateral in his diagram. This runs contrary to our expectation that letters specify corners or ends of line-segments, and to all manuscript diagrams examined by van Leeuwen 2016, 221-23.

discussion of rotation in problem 1. There is no further justification in the text for this analogy, so why should readers accept it?

The diagram is the key. If in the original diagram the triangle ΑΒΓ intersected with the quadrilateral ΔΕΗΖ, as it does in the diagrams of all manuscripts and editions, visual analogy through the lever diagram could do the explanatory work. The lines ΑΒ and ΒΓ are each met somewhere along their length by a side of the quadrilateral. However the diagram was drawn, at least one of the intersections of ΑΒ and ΒΓ must have been at an oblique angle. It is probable that problem 3's diagram also prominently featured intersecting lines at oblique angles. Such is the case in all manuscript diagrams (see Figure 2) which show two crossed lines AB and HK, representing the lever-bar 'before' and 'after' its action.[29] In that case the visual resemblance of problem 17's diagram to problem 3's may have persuaded the reader, or the writer for that matter, to accept the analogy. We should also note that in both problems the chosen order of lettering labels the designated lever-bar AB, bringing about a further similarity.

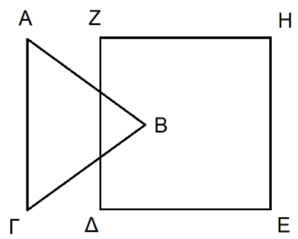

Fig. 4. Van Leeuwen's (2016, 221) reconstruction of the archetype diagram for problem 17

I suggested above that *Mech.*'s author allows sharing of diagrams across explananda. With this in mind, we might find it puzzling that problems 21 and 22, on tooth-extractors and nut-crackers respectively, do not share a diagram, since they are both analysed as double-levers. The diagrams (Fig. 5) differ at the level of what we can make out from the text. While the 'pin' in problem 22's diagram is simply lettered A, the corresponding part of problem 21's diagram seems overburdened with letters: Δ, Θ and Γ. Here van Leeuwen's study of manuscript diagrams is suggestive. All manuscripts give the tooth-extractor a distinctively curved form, while the nutcracker is represented by two intersecting straight lines forming an X-shape.[30] We cannot know the state of the diagrams in antiquity, but the extra letters of problem 21 may have made more sense in a more elaborate diagram. While our other case-studies have focused on the role of

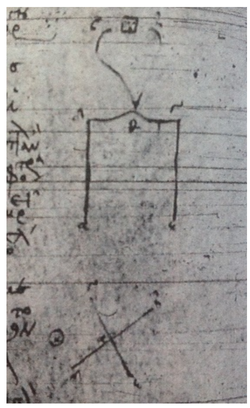

Fig. 5. Diagrams for problems 21-22 in Codex Vat.gr. 1339 f.289r. Image from van Leeuwen 2016, 44.

[29] van Leeuwen 2016, 215-18. This cannot be reconstructed from the text alone which designates AB as a line but the points H and K only individually.
[30] van Leeuwen 2016, 117, 127, a distinction erased in Hett 1936, 379.

diagrams in establishing analogies, problems 21-22 seem to emphasise difference. This may be due to the significant functional difference between the tools under consideration: one is for pulling teeth, the other for cracking nuts.

Problem 5 (Why is the small rudder able to move the huge ship?) contains a series of subsections answering different but related questions. The diagram is constructed towards the end in answer to the puzzle 'why the boat advances further in one direction than the oar's blade travels in the opposite direction' (851a14-28). This is a surprising return to the oar of problem 4 from the rudder with which problem 5 began. After giving a diagrammatic explanation of the oar's power (Fig. 6), the author declares 'and the rudder does the same thing too' (851a28-29: τὸ δ' αὐτὸ καὶ τὸ πηδάλιον ποιεῖ). In the final lines of the problem, the rudder is explained not by analogy to the lever (that came earlier at 850b31-34), but analogy to the oar (851a32-34): 'one must think of where the rudder is attached as the middle of a moved object (τι τοῦ κινουμένου μέσον), and just like the thole-pin is to the oar.'

There is only one other case of this kind of analogy in *Mech.* explaining one artefact not by the lever or balance but by another artefact.[31] It is likely that the reader is directed to return to the diagram above (Fig. 6), which represents an oar,[32] and reconsider it as representing a rudder. Once again, a single diagram supports more than one explanation.

The phrase 'τὸ δ' αὐτὸ καὶ τὸ πηδάλιον ποιεῖ' ('and the rudder does the same thing too', 851a28-29) is a strategy comparable to Euclid's turn of phrase in *Elements* 1.15, 'ὁμοίως δὴ δειχθήσεται, ὅτι καὶ αἱ ὑπὸ ΓΕΒ, ΔΕΑ ἴσαι εἰσίν' ('it will be shown in a similar way, that also the [angles contained] by ΓΕΒ [and by] ΔΕΑ are equal'). In each case the same argument is asserted to hold in a similar case *mutatis mutandis*,[33] and the move is effective because the required modifications are straightforward and most of the explanation or proof's wording is left untouched, since it uses regimented technical language. To translate problem 5's diagrammatic passage (851a14-28) to apply to the rudder, only three terms need be changed at the very start, where parts of

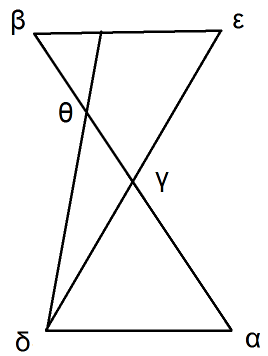

Fig. 6. Van Leeuwen's (2016, 219) reconstruction of the archetype diagram for problem 5

31 Problem 19 explains the axe in terms of the wedge: γίνεται σφὴν ὁ πέλεκυς (853b22).
32 851a16-17 ἔστω γὰρ ἡ ΑΒ κώπη…
33 For discussion of the Euclidean passage see Netz 1999a, 242-43.

the diagram are identified with parts of the oar (851a18-19). The rest of the explanation is couched in technical mathematical language. Indeed special care may have been taken to avoid using oar-specific vocabulary. The author speaks of τὸ ἐν τῷ πλοίῳ ἄκρον ('the end-point in the boat') at 851a26-27 when referring to the diagram, while elsewhere he says ἡ ἀρχὴ τῆς κώπης ('the beginning of the oar', 851a28).

It is surprising that in a problem principally concerned with the rudder the author should privilege the oar in the diagram and leave the rudder's explanation implicit. By doing so the author directs our attention to the easier case. Rudders and steering-oars of the period came in a variety of forms. Often in Greek art we see ships steered by not one but two πηδάλια. Sometimes a tiller (οἴαξ) was used; in double-πηδάλια arrangements this might yoke the two rudders together.[34] These complex arrangements bear less obvious resemblance to the diagram's straight lines than the simple straight rod of an oar. By considering the oar when constructing the diagram, the author eases the process of visual analogy. Meanwhile by simplifying the range of possible rudder-arrangements to the case of a single πηδάλιον and suppressing the tiller,[35] the author allows easy intersubstitution of rudder-terms and oar-terms.

5.4: Experience and idealisation

I have touched on some ways in which *Mech*. simplifies its objects of study. Let us now consider this theme more generally. *Mech.*'s descriptions of phenomena to be explained do not suggest that the author has carried out careful or deliberate observations, or that he is working from reports of observations carried out for a special purpose. On the contrary, the explananda are described in everyday, unspecialised terms, and some of these descriptions seem inaccurate.

Problem 1 asks why larger balances are 'more accurate' (ἀκριβέστερα). Are they? Tradesmen such as jewellers and apothecaries who rely on precise measurement have in recent centuries used relatively small balances.[36] The author's reason for thinking larger balances are more

[34] Irby 2016.
[35] The tiller is mentioned only once, in the initial posing of the question at 850b29.
[36] Baldi 1621, 15. Tartaglia in Drake and Drabkin 1959, 79 concludes that the author is describing a 'mathematical balance'. See Van Cappelle 1812, 169-74 for a clear and precise discussion the effect of weight on the angle formed by a balance.

Fig. 7: The Taleides amphora c. 540-530 BCE, Metropolitan Museum of Art, New York (https://www.metmuseum.org/art/collection/search/254578)

accurate turns out to be that the end of a longer balance-arm undergoes a greater absolute displacement when tipped downwards by a small weight. This is only true, however, if we ignore the weight of the balance-beam itself, but in practice larger balances are typically heavier and thus undergo a smaller angular displacement when moved by the same weight. Perhaps we could supply an appropriate condition: 'Why are larger balances more accurate than smaller ones, *when both balances are of equal weight*?' or 'Whenever larger balances are more accurate than smaller ones, why is this so?' But *Mech.*'s author never alludes to any such specifications and the limits on his statement's generality. Ancient balances were sometimes very large, judging by their representations in art, e.g. the Taleides Amphora (Fig. 7) and the Arcesilaus Cylix (mid-sixth century BCE), but this does not solve the problem.

There is a similar difficulty in Problem 9, which asks why we move things more easily and quickly by means of larger circles, e.g. by larger pulleys. This ignores the fact that larger pulleys are typically heavier. *Mech.*'s author presents his data in a simplified manner, abstracted from the material properties of the devices studied. We do not hear what balances, levers, pulleys, or rudders should be made from. Yet the material composition would be relevant not only for recognising the effect of a device's weight on its utility, but also for appreciating the role of rigidity in the operation of a lever.

Mech. simplifies in other ways too, besides ignoring weight or extension. Problem 6 asks why higher sails move the ship faster. Of course, there is no use in sails lying on the deck. They must be hoisted high to catch the wind. But raising the sails beyond a certain height will slow the ship by causing the prow to dip into the water. The author fails to point out that what he attempts to explain is not universally true.[37] He seems more interested in exploring the reach of the explanatory programme than in the details of his examples.

To be clear, the difficulty here is not abstraction *per se*, but rather that, by ignoring certain factors, the author makes general claims that are false (and false in rather obvious ways) if understood as fully universal.[38] These generalisations can stand only subject to certain qualifications and, from our perspective, the omission of such qualifications seems to pose a problem. Does this amount to a fault on the author's part? That is partly a question of the pragmatics of communication. If I tell you, 'Eating fish is healthy', you will immediately understand what I mean. I do not mean that eating fish is healthy for everyone in every situation. For instance, some people are allergic to fish. Such obvious qualifications do not, under normal circumstances, need to be stated.[39] To expect every qualification and assumption to be made explicit is too demanding. How far such an excuse might apply to *Mech*. is uncertain, as we cannot determine the extent to which our author (and his ancient readers) would have been able, upon questioning, to acknowledge and specify the limitations on *Mech*.'s claims. Thus in problems 1 and 9, we have three options: (i) the author makes an erroneous generalisation, (ii) the author describes a weightless, 'mathematical' balance/pulley, (iii) the author assumes that any qualifications on his generalisations are obvious or irrelevant. In problem 6, only (i) and (iii) are available. In light of my arguments in Chapters 2-4, I find (ii) unlikely. It is difficult to decide between (i) and (iii) and there may be some truth in both.

It would be too hasty to conclude from the lack of recorded observations that *Mech*.'s investigations were uninformed by observation. De Groot has argued that many human practices rely on a tacit 'kinaesthetic awareness' of the Rotating Radius Principle. The angler sweeping his catch out of the water knows how a small motion in his hands will cause the rod and line to swing through the air.[40] One could add that the claim that a rotating radius undergoes a motion towards the centre of rotation (a striking claim since nothing in a rotation

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[37] Forster 1913 *ad loc*. suggests that 'The most probable explanation is that the Greek sailor, being essentially a coaster, preferred a high sail in order to catch the wind which might be cut off by hills and cliffs.'
[38] It is not necessary, however, to interpret 'larger balances are more accurate' etc. as universally quantified.
[39] In Grice's terms, this exemplifies the maxim of quantity, to give as much information as is required *and no more*, and perhaps also the maxim of relation, 'be relevant' (Grice 1989, 26-27). The standards of *scientific* communication today differ from those of ordinary discourse, but it would be a mistake to expect an ancient writer to follow our norms.
[40] De Groot 2014, ch.3.

in fact approaches the centre) may have been informed in some way by experience of physical pulls directed towards fixed points.[41] Yet there are no examples or specific observations in problem 1. The contrast to Aristotle's zoological investigations or to a later mechanical author like Philo is stark. One can also contrast *Physical Problems* 16, which records deliberate, precise observations of a kind not found in *Mech*.[42]

5.5: Conclusion

The success of the explanations in problems 4-22 depends on securing analogies to the lever, balance and rotating radius. I have argued that *Mech*.'s careful use of language and diagrams provides much-needed support for these analogies. *Mech.* thus combines recognised styles of reasoning in Greek science in unexpected ways. This is only to emphasise, in contrast to the arguments of some recent scholarship, that *Mech.* is not a deductive text. Its explanations are not and do not aspire to be either syllogisms nor geometric demonstrations. I do not thereby suggest that *Mech*. rebelled against prescriptive rules set out in Euclid or the *Posterior Analytics*. Rather, *Mech*. shows that the norms of demonstration were less important and influential around 300 BCE than has sometimes been assumed, even in a field as open to influence from mathematics as mechanics.

On the familiar account of diagrams in Greek mathematics, many diagrams in *Mech.* appear merely illustrative, offering no information beyond what is contained in the text. I have argued the diagrams in *Mech.* mutually interact in a form of non-verbal argumentation I have called visual analogy. This account suggests an explanation of the diagrams' uneven distribution across the text: diagrams are constructed either in geometrical arguments,[43] or in explanations that are particularly challenging where their construction allows visual analogies to be drawn.

In considering problem 5, I showed how the formalised language adopted in passages of diagrammatic reasoning allows the author simply to assert that the same explanation would hold for another case, in a manoeuvre with parallels in Euclid.

[41] Think, for example, of a dog straining against the pull of its leash.
[42] See especially *Problems* 16.4, 6, 8, 13.
[43] Problem 1.

Although heuristically valuable, analogical reasoning is not watertight. The analogy involved in Problem 17 is tenuous and Hero of Alexandria later provided a different account of the wedge's power.[44] The positive role of analogy in science is not to confirm theories, although analogical arguments were undoubtedly presented by many Greek thinkers to justify their claims, but rather to suggest new hypotheses and new problems.[45] That heuristic function of analogy, in giving a physical theory an 'open' and progressive character, is exemplified by the *Mechanica*'s exploration of how far three simple models – the rotating radius, lever, and balance – can be extended. It could be said that this makes intelligible the formal presentation of *Mech*. as a 'problems' text. The 'problems' genre is inherently tentative and undogmatic; answers begin 'Is it because…?' and qualifications or even multiple answers are not uncommon. Further, the openness of a problems text, its essential lack of closure or a well-defined theoretical end-point, conveys a sense of ongoing inquiry and almost invites extension and elaboration by later hands.[46] The literary form of *Mech*. complements the explanatory method I have described.

In addition to shedding light on this text's method, I hope to have shown that there may be more to the function of diagrams in Greek scientific texts than has previously been recognised: diagrams can work together with language to support analogical reasoning.

[44] Hero *Mech.* 2.4; cf. Schiefsky 2007, 28-30.
[45] Hesse 1963 is a classic study of the positive function of analogy in science. See also Lloyd 1966 on the uses of analogy in Greek thought to Aristotle.
[46] As certainly happened in the pseudo-Aristotelian *Physical Problems.* As I mentioned in Chapter 1, it is an unconfirmed possibility that the transmitted text of *Mech.* is the work of more than one writer.

Chapter 6: The *Rota Aristotelis* Paradox

6.1: Introduction

Mech.'s preface declares that mechanical 'problems' partake of both mathematical and physical speculations; τὸ ὥς ('the how') is clear through mathematics, τὸ δὲ περὶ ὅ ('the about what') through physics. We have seen in Chapter 4 how in problems 1-3 the author develops models of the balance and the lever, through a study of the composition of locomotions. In Chapter 5, I explained how in problems 4-22 the author applies these models in an attempt to explain a variety of specific phenomena, predominantly the behaviour of artefacts.[1] Problems 23-24 mark a break in this programme. These problems do not concern the effects of particular machines but raise abstract puzzles about the geometrical treatment of locomotions presented in problem 1, on which subsequent explanations have ultimately depended.

I construe problem 24 as not so much demanding explanations as presenting a paradox that threatens the explanatory programme of the *Mechanica*.[2] This paradox is traditionally known as the *Rota Aristotelis* ('Wheel of Aristotle').[3] This concerns the motion of what may seem to be the simplest machine of all, the wheel. A wheel moves by *rolling*.[4] Rather than simply rotating on the spot, like a balance or a lever, rolling also involves linear translation; in Peripatetic terms, it could be said to involve both circular (or rotational) and rectilinear motion.[5]

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[1] The exception is Problem 15 on pebbles; see Note A to Chapter 1.
[2] Bodnár 2011b comments that problem 24 seems 'to run counter to the Thesis of Unequal Circles itself.' Paradoxes played an important role in Greek philosophy, from at least the time of Zeno through to the Hellenistic schools. Some writers seem to have gathered paradoxes in collections. Zeno wrote a book (Plato, *Parmenides* 127 C); Proclus mentions a collection of mathematical paradoxes by a certain Erycinus (68.6-20 Friedlein); Proclus refers to similar collections, including some by Stoic authors (*Comm. in Eucl.* 396-97). The Rota paradox is less general than, Zeno's paradoxes. It concerns a specific kind of locomotion, rolling, rather than change, or at least motion, in general. Further, there is no philosophical doctrine associated with the Rota's paradoxical conclusion as Eleatic monism and the denial of change are associated with Zeno's conclusions.
[3] However, *Mech.* consistently refers to the rolled objects as circles (κύκλοι) rather than wheels (τρόχοι).
[4] Rolling is an exemplary kind of motion in *Phys.* 3.1, 201a18-19 ('housebuilding… learning, healing, rolling, jumping, maturing, and ageing'). In discussing the motions of the heavenly bodies, Aristotle writes that 'there are two *per se* motions of something spherical: rolling and rotating' (*DC* 2.8, 290a10) but then denies that the heavenly bodies roll because the Moon's face is always visible (290a25ff.). It is left unclear whether there are any natural rolling motions.
[5] Rolling and rotating are distinguished in Problem 8, 851b16-22: 'It is possible for a circle to turn in three ways: on a felloe with the centre moving too, like the wheel of a cart turns; or about the centre only, like pulleys, with the centre at rest; or parallel to the plane with the centre staying still, like a potter's wheel turns.' (τριχῶς δὲ ἐνδέχεται τὸν κύκλον κυλισθῆναι· ἢ γὰρ κατὰ τὴν ἁψῖδα, συμμεταβάλλοντος τοῦ κέντρου, ὥσπερ ὁ

Rolling features in several problems of *Physical Problems*, Book 16.[6] Rolling is usually not a direct concern in *Mechanica* problems 1-22, although the wheel (τρόχος) and roller (σκυτάλη) are mentioned in Problems 8-11. These are distinguished in Problem 11 by the fact that the roller has no axle. Pulleys (τροχαλίαι) feature as examples in problems 8-9, and problem 18 is devoted to explaining a system of two pulleys. Although the pulley-wheel is not itself translated when in operation, an account of the pulley's characteristic action must consider the vertical translation of the rope and the load. With a few modifications, problem 24's paradox could be presented for pulleys, but such a variant is not considered in *Mech*.

Nevertheless, problem 24's paradox bears on problem 1's analysis of rotation, since it threatens to make nonsense of the Rotating Radius Principle. The paradox also poses a more general methodological challenge to the project of explaining mechanical phenomena through mathematical analyses of locomotion. The wheel is a very basic machine. If mathematical models of locomotion yield contradictions when applied to the wheel's rolling, then the prospects look dim for *Mech.*'s programme of mechanics as partaking of both mathematical and physical speculations.[7] This paradox's location towards the end of *Mech*. gives it a dramatic sense of urgency. The return to foundational and comparatively abstract issues may be seen as a ring-composition technique for closure.[8] The seven problems of the collection that are unrelated to the programme based on motion on a circular path announced in the introduction appear only after problem 24.[9]

This lack of grand metaphysical conclusions may in part account for the *Rota* paradox's recent neglect. It was not always so. The *Rota* paradox stimulated lively discussion in the sixteenth and early seventeenth centuries, more so than Zeno's collection of arguments. In a pathfinding study written in 1939 and published in 1950, Israel Drabkin surveyed a number of these Renaissance responses to the paradox but gave comparatively little consideration to the significance of the paradox in its ancient context or to the details of *Mech.*'s solution.

τροχὸς ὁ τῆς ἁμάξης κυλίεται· ἢ περὶ τὸ κέντρον μόνον, ὥσπερ αἱ τροχιλέαι, τοῦ κέντρου μένοντος· ἢ παρὰ τὸ ἐπίπεδον, τοῦ κέντρου μένοντος, ὥσπερ ὁ κεραμεικὸς τροχὸς κυλίνδεται.)

[6] *Problems* 16.4-6.

[7] *Mech.* 847a25-27.

[8] Problem 24 is by some way the longest problem in the collection after problem 1.

[9] See Note A to Chapter 1.

Almost all later studies have focussed on early modern treatments of the paradox.[10] A recent study by Dosch and Schmidt (2018) examines *Mech*. 24 in closer detail than previous treatments, but several aspects of the text remain to be explored.

My primary task in this chapter is to present a historical appreciation of this neglected paradox in the context of Peripatetic philosophy. We shall see that *Mech*.'s solution draws on Aristotelian natural philosophy rather than mathematics. Besides *Mech.* only one other ancient discussion of the paradox survives, in Qusta Ibn Luqa's ninth-century Arabic translation of Hero of Alexandria's *Mechanics* (probably first century CE).[11] It is natural to ask what the relationship between these two discussions is. Did Hero derive the puzzle from *Mech*. or from another source, perhaps an earlier one? The fact that we have Hero only in Arabic translation is one major difficulty for discovering the relation between the two. Although I have not been able to find an answer, a comparison of the two discussions is nonetheless enlightening.

6.2: An overview of the paradox

For the sake of clarity and simplicity, let us first consider the paradox unhampered by the complications of our historical sources. Imagine two wheels of different sizes rigidly fixed around a common centre, with the base of the larger wheel resting on a horizontal surface. Because they are fixed together, when the larger wheel rolls along the surface, the smaller wheel moves with it like the hub on a car tyre. We let the smaller wheel rest on the horizontal surface and roll it for one revolution and we find that it travels a distance equal in length to its smaller circumference (call this Case I).[12] We could check this by measuring both with a piece of string. Next, we let the larger wheel roll across the surface until it completes one revolution. The distance it travels across the surface in this time is equal in length to its circumference (call this Case II).

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[10] See, for example, Michel 1964, De Waard 1963, 31, Le Grand 1978, Palmerino 2001, Ferraro, 2009, Arthur 2012, Levey 2020. The Renaissance commentaries themselves are still useful. The paradox features in several publications on recreational mathematics and mathematics education, among them Menninger 1954, 218; Costabel 1968; Ballew 1972; Bunch 1982, 3-12; Gardner 1983, 2-4; Pappas 1989, 202.

[11] The Greek original is lost. Some Greek fragments of Hero's *Mechanica* are preserved in Pappus *Collectio* Book 8.

[12] Part of the larger wheel must hang below the surface; it may help to imagine the wheels as connected by an axis.

In Case II, where the larger wheel rolls out a length equal to its circumference, the smaller circle also rolls out a path of equal length. But in Case I, the smaller circle traces out a smaller path in one revolution, one equal to its own circumference; meanwhile the larger circle traces out a path equal to the smaller circumference. So sometimes a wheel rolls out a path equal to its circumference, but it can also be made to roll out a greater or lesser length. Furthermore all this happens without any sliding or slipping. The smaller wheel in Case II does not slide over any part of the line to keep pace with its larger companion, and the larger wheel in Case I never slips on the spot. All wheels move perfectly smoothly and uniformly.

There are several ways to draw out puzzles. One way is as follows. If any wheel rolling smoothly always traces out a path equal to its circumference, then in both cases the smaller wheel traces out a path equal to its circumference. And in both cases the larger wheel traces out a path equal to its circumference. So in Case II, the larger wheel traces out a path equal to itself and the smaller wheel traces out a path equal to itself. But in either case, the lines traced out by both wheels are equal. So in Case II both paths traced out are equal. Several contradictory conclusions follow: that the larger circle is equal to the smaller circle, that the larger circle is equal to the smaller path, that the larger path is equal to the smaller circle, and that the larger path is equal to the smaller path.

Alternatively, without assuming that a smoothly-rolling always traces out a path equal to its circumference, one can simply ask for an account of why smoothly-rolling wheels traverse different distances in different cases. How can a tiny wheel trace out a great length in one revolution without sliding over stretches of it? And how can a large wheel trace out a short path in one revolution without occasionally slipping on the spot? And, if these things are possible, why do wheels rolled independently tend to trace out paths equal to their circumferences?

A third puzzle concerns the nature of continuous magnitudes. Rolling ‘smoothly’ means that each point on the wheel makes contact with exactly one point on the surface. In other words, there is a one-to-one correspondence between points on the wheel and points on its path across the surface. Therefore, if a wheel can roll smoothly along lines of different lengths, there is a one-to-one correspondence between the points of unequal magnitudes. But how can this be? If I am sharing out pieces of fruit among a group of people and find there is a one-to-one correspondence between people and pieces of fruit, it follows that were there fewer people there would be fruit left over, and that were there more people there would not be enough to go around. Analogously, we might expect that if there is a one-to-one correspondence between points on the wheel and on its path, then were the path shorter or longer, the wheel would necessarily slip or slide.

A version of the first of these puzzles is addressed by Hero. Modern writers have tended to focus on the second and third puzzles. *Mech.*'s author only addresses the second puzzle, and even then only part of it as presented above.[13] Hence Drabkin's complaint that in *Mech.* 'there is no coming to grips with all the difficulties involved… [*Mech.*'s treatment] does not analyse the problem raised by the point-to-point correspondence of the two circumferences of unequal length.'[14]

6.3: The *Mechanica*'s statement of the paradox

Mech. problem 24 is not easy to follow. The passage of thought is seemingly repetitive and beset by sudden shifts of focus. I offer the following analysis of the problem's structure:

1. Introduction (855a28-855b4)
2. Cases I and II (855b5-23)
3. First puzzle: no slipping or sliding (855b23-28)
4. Second puzzle: same speed, different effect (855b28-32)
5. Answering puzzle 1: the same power causes different effects (855b32-856a1)
6. Answering puzzle 1: moved and mover (856a1-32)
7. Answering puzzle 2: the centre is not unqualifiedly the same (856a32-39)

In this section, I will interpret the statement of the paradox (1-4). In the next section, I will explore *Mech.*'s solution (5-7).

6.3.1: Introduction (855a28-855b4)

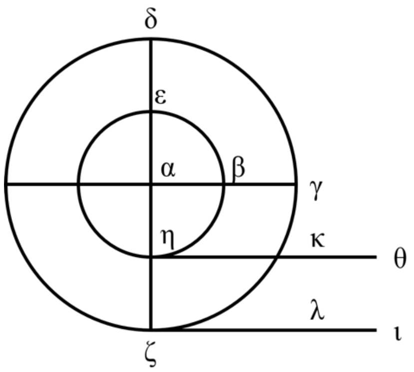
The author opens with a question relating to the paradox: 'There is a puzzle why the larger

[13] The author does not try to explain why a circle rolled independently traces out a distance proportional to its size.
[14] Drabkin 1950, 168-69.

circle rolls out a line equal to the smaller circle, when they are set about the same centre.’[15] The conventional διὰ τί (‘Why?’) of Peripatetic problems writing is prefixed by ἀπορεῖται ‘it is puzzled’. The author first develops this question by showing why the fact described is puzzling. Unequal circles rolled separately traverse paths proportional to their sizes. In other words, a proportionality holds between paths (D1, D2) and circle sizes (μεγέθη C1, C2), viz. D1:D2=C1:C2. The passage justifying this assumption is as follows:

> It is clear that the larger [circle] rolls out more. For the circumference seems to perception to be the angle of its own diameter, the larger [circle’s being] greater, the smaller [circle’s] smaller, so that they will have the very same ratio as the lines that they roll out have to each other, by perception.[16]

Each ratio, that of the circles and that of their paths, is manifest to perception (κατὰ τὴν αἴσθησιν).[17] Whose perception of what? Possibly the reader’s perception of a diagram, though this seems unlikely. There is no textual evidence in the ensuing diagram construction that a diagram presenting the relevant information would have been present.[18] The phrase κατὰ τὴν αἴσθησιν more probably refers to the roots of the proportionality in observations of physical objects like wheels. If so, it is interesting that an abstract, mathematical fact about motion is justified by reference to perception.[19]



[15] 855a28-30: Ἀπορεῖται διὰ τί ποτε ὁ μείζων κύκλος τῷ ἐλάττονι κύκλῳ ἴσην ἐξελίττεται γραμμήν, ὅταν περὶ τὸ αὐτὸ κέντρον τεθῶσι;.

[16] 855a35-855b1: ὅτι μὲν οὖν μείζω ἐκκυλίεται ὁ μείζων, φανερόν. γωνία μὲν γὰρ δοκεῖ κατὰ τὴν αἴσθησιν εἶναι ἡ περιφέρεια ἑκάστου τῆς οἰκείας διαμέτρου, ἡ τοῦ μείζονος κύκλου μείζων, ἡ δὲ τοῦ ἐλάττονος ἐλάττων, ὥστε τὸν αὐτὸν τοῦτον ἕξουσι λόγον, καθ’ ἃς ἐξεκυλίσθησαν αἱ γραμμαὶ πρὸς ἀλλήλας κατὰ τὴν αἴσθησιν.

[17] The author here means unequal circles rolled independently (rather than concentrically), as is clear from the later comment that unequal circles roll out equal lines when they are concentric.

[18] For what they are worth, the Byzantine manuscript diagrams do not attempt to display ratios between paths.

[19] Note that there was no such explicit reference in problem 1’s introduction of the quadrilateral of motions or in problem 3’s introduction of the ‘law of the lever’.

Fig. 1: Van Leeuwen's (2016, 235, 240) reconstructions of the Byzantine archetypal diagrams for problem 24. The left-hand diagram does not significantly differ from diagrams in modern editions. The right-hand diagram may relate to 856a1ff., although the text concerns two unequal circles.

The sense of the key phrase (γωνία …ἡ περιφέρεια ἑκάστου τῆς οἰκείας διαμέτρου) is obscure and has divided commentators. Van Cappelle, Forster and Hett take this expression to mean the angle between the circumference and the diameter, the complement to the so-called 'horn angle'.[20] The difficulty is that the proposition that horn angles are larger in larger circles is strictly unsupportable. One would have to suppose that the author had instead reached this claim informally, perhaps by observing that the two arcs diverge, which is the case for intersecting rectilinear lines just in case they differ by a certain angle.

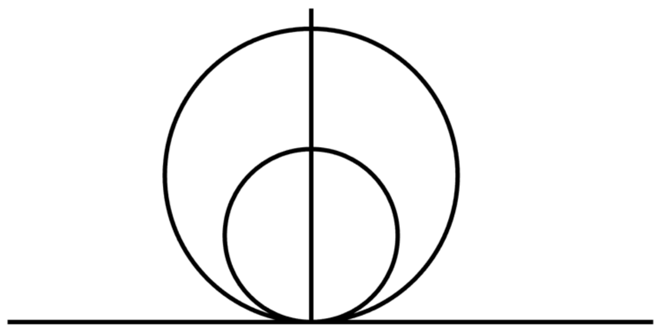


Fig. 2: 'Horn' angles between the diameters and circumferences of differently sized circles

On the other hand, Heath suggested the difficult expression means 'angles subtended at the centre in similar sectors or by similar arcs'.[21] This reading is preferable. It avoids ascribing to the author misconceptions about horn angles and it better matches the theoretical framework of *Mech.*, which is based on the principle that arcs of unequal circles in the same angle are proportional to the radii of those circles.[22] Here, the idea is that there is a proportionality between arcs and paths traced out. The focus on the ratio of arcs rather than simply the ratio of radii or circumferences is the source of the passage's obscurity.

[20] The horn angle is the angle contained by a circumference and its tangent, which is smaller than any acute rectilinear angle. Its complement is the angle contained by a circumference and its diameter, which is greater than any acute rectilinear angle but less than a right angle. *Elements* 3 prop. 16 is the only appearance of the horn angle in Euclid. See also Proclus' comments on *Elements* 1.def.8 (121-128 Friedlein).
[21] (Heath, 1949, p. 249). A similar issue arises in the interpretation of problem 8, 851b38.
[22] The Rotating Radius Principle of Problem 1

It is interesting that the proportionality is given at all. Many modern presentations of the paradox (including Drabkin's and that of §5.2 above) involve the claim that a smoothly rolling circle's path in one revolution is equal to its circumference. This can be checked empirically (κατὰ τὴν αἴσθησιν): it is easy to compare a wheel's circumference to its path by using a piece of string. *Mech.*'s method of comparison, to take separately the ratio of circle sizes and the ratio of path lengths, and to recognise their proportionality, adds a layer of complexity. The reason may be that Aristotle had in *Phys.* 7.4 asserted that straight lines cannot be equal to arcs or circumferences. Our author may have wanted to show that the paradox still arises even if the lengths of straight and circular lines are not directly compared.[23] But the careful talk of proportionality sits alongside phrases that are naturally read as direct comparisons of a straight path to a circle.[24] Our other ancient source for the paradox, Hero, compares the straight and circular lines directly. It is possible that *Mech.*'s proportionality has been imposed on a problem previously formulated in such terms.

6.3.2: Cases I and II (855b5-23)

Having given an initial sketch of the paradox, the author in this section constructs a diagram which clarifies the issue but is never referred to in the solution.[25] The section is introduced with the formulaic phrase 'ἔστω γὰρ κύκλος ὁ μείζων μὲν ἐφ' οὗ τὰ ΔΖΓ…', where γὰρ signals that the diagrams will elucidate the immediately previous statement: that when the circles are concentric they sometimes trace out a path equal to that which the larger traces out, but sometimes one equal to that which the smaller traces out.[26] For all its lettered diagrams, this passage does not come close to offering a proof. Its aim may be simply to clarify, rather than justify, the statement about concentric circles. But this is not mere repetition of the same thought in different terms; new ideas are quietly introduced, for example the contrast between a partial and a full revolution.

[23] This could have motivated the author's choice regardless of whether he thought Aristotle was right: the best paradoxes start from premisses no one would immediately think to dispute. Another possible motive is a desire to account for full revolutions of the rolling circle and also partial revolutions.
[24] In the opening question we have ὁ μείζων κύκλος τῷ ἐλάττονι κύκλῳ ἴσην ἐξελίττεται γραμμήν (866a29-30); see later 855b15-16: ὁ ὅλος κύκλος τῷ ὅλῳ κύκλῳ ἴσην ἐξελιγθήσεται, and 855b27-28: τὸν μὲν μείζω τῷ ἐλάττονι ἴσην διεξιέναι, τὸν δὲ τῷ μείζονι. In each case, Forster translates 'a path equal to that of a smaller circle', etc., but in the last case I find this doubtful.
[25] References to the diagram are confined to 855b5-23.
[26] The author presumably means equal to the paths the circles trace out when rolled independently.

The diagram construction comes first (855b5-8, see Fig. 1 above) and is followed by consideration of Case I, where the smaller circle drives the motion (parallel to Case I of §5.2). Strikingly, the author adopts the first person, ('If I move the smaller circle') rather than an impersonal expression ('If the smaller circle is moved').[27] First he considers a partial revolution, until the radius AB of the smaller circle is parallel to the line HK along which it rolls. The construction passage did not specify the relative positions of points on each circle, so the angle between AH and AB, the angle of this partial revolution, may seem indeterminate.

Modern editions and Van Leeuwen's reconstruction of the Byzantine archetype depict a right angle. Nothing in the passage that constructs the diagram forces this interpretation but there is a later reference at 855b14 to a quarter revolution (τέταρτον).[28] The author notes that after rolling through one quadrant, both circumferences have traversed the same distance, HK and ZΛ being equal. Since a whole revolution is simply four quarter-revolutions, the author draws the conclusion, 'If the quarter part rolls out an equal [line], it is clear that also the whole circle will roll out a line equal to [that rolled out by] the [other] whole circle.')[29]

Next, the author considers Case II, where the larger circle drives the motion (855b17-23). The beginning of this section is marked by another first person singular, 'Similarly, if I move the large circle'. The author points out that, also when the larger circle rolls, the concentric circles revolve together and complete their revolution at the same time. No partial revolution is mentioned.

6.3.5: Puzzle 1: no slipping or sliding (855b23-28)

Discussion of the diagram ends here as the author returns to a general discussion of the paradox. He sharpens the original formulation by identifying two distinct puzzles. The first puzzle is close to the initial formulation: it is puzzling (ἄτοπον) that the larger circle traverses

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[27] Excluding standard formulae (e.g. 'I say…'), the first-person is unusual but hardly unknown in Greek mathematical writing. Eratosthenes uses first-person verbs in finding two mean proportionals using an instrument, the mesolabe (Eutocius, *Commentary on Archimedes' On the Sphere and the Cylinder* 94.15-96.8, with Roby 2016, 55). Cf. Cuomo 2000, 146n.53 on first-person pronouns in Pappus.

[28] This is comparable to the retrospective labelling of a figure as a quadrilateral in problem 1, 848b20 (see Chapters 4-5).

[29] εἰ δὲ τὸ τέταρτον μέρος ἴσην ἐξελίττεται, δῆλον ὅτι καὶ ὁ ὅλος κύκλος τῷ ὅλῳ κύκλῳ ἴσην ἐξελιχθήσεται. Why the author begins with the case of a quarter-revolution is unclear. It may be relevant that problem 1, 849a19-849b19 focussed on rotations within a quadrant. Alternatively, the quadrant may be an illustration of what holds for any angle less than a full revolution.

a path equal to the smaller circle and that the smaller circle traverses a path equal to the larger circle.[30] But these facts are now presented as particularly puzzling given two specific conditions for the rolling motions described: (i) the larger circle does not stand still for any time at the same point for the smaller circle to catch up (it does not slip); (ii) the smaller circle does not skip over any point (it does not slide).[31] A parenthetic remark explains why these conditions hold: 'for both [circles] move continuously in both cases'.[32] The author's understanding of continuous change agrees with Aristotle's in the *Physics*.[33]

These two conditions are the reason why the facts about rolling are so puzzling. The thought may be this: if we roll two wheels separately, we will find it necessary either to have the smaller wheel slide ahead or to have the larger wheel slip on the spot in order to make them traverse equal distances in one rotation.

6.3.6: Puzzle 2: same speed, different effect (855b28-32)

The second puzzle, introduced by ἔτι δὲ and following a similar phrase structure, introduces a new focus, the common centre of the two circles.[34] The author has already four times mentioned the oneness and sameness of the circles' centre.[35] Partly, this was a way to express the circles' concentricity, but it is now given a new significance:

> ἔτι δὲ μιᾶς κινήσεως οὔσης ἀεὶ τὸ κέντρον τὸ κινούμενον ὁτὲ μὲν τὴν μεγάλην ὁτὲ δὲ τὴν ἐλάττονα ἐκκυλίεσθαι θαυμαστόν. τὸ γὰρ αὐτὸ τῷ αὐτῷ τάχει φερόμενον ἴσην πέφυκε διεξιέναι· τῷ αὐτῷ δὲ τάχει ἴσην ἐστὶ κινεῖν ἀμφοτεράκις
>
> It is amazing that, since the motion is always one, the moved centre rolls out in one case a large [distance], in the other case a smaller [distance]. For the same thing moving with the same speed ought to traverse an equal [distance], and to move [it] with the same speed is to move it an equal distance in both cases.[36]

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[30] 856a27-28: τὸν μὲν μείζω τῷ ἐλάττονι ἴσην διεξιέναι, τὸν δὲ τῷ μείζονι, ἄτοπον.
[31] 856a23-26: τὸ δὲ μήτε στάσεως γινομένης τὸ μεῖζον τῷ ἐλάττονι, ὥστε μένειν τινὰ χρόνον ἐπὶ τοῦ αὐτοῦ σημείου… μὴ ὑπερπηδῶντος τοῦ ἐλάττονος μηθὲν σημεῖον
[32] 856a25-26: κινοῦνται γὰρ συνεχῶς ἄμφω ἀμφοτεράκις
[33] Cf. *Phys*. 5.3, 226b27-28: συνεχῶς δὲ κινεῖται τὸ μηθὲν ἢ ὅτι ὀλίγιστον διαλεῖπον τοῦ πράγματος.
[34] Both puzzles begin with a genitive absolute and close with a neuter adjective expressing puzzlement (ἄτοπον / θαυμαστόν).
[35] 855a30-31: ὅταν περὶ τὸ αὐτὸ κέντρον τεθῶσι; 855b3: ὅταν περὶ τὸ αὐτὸ κέντρον κείμενοι ὦσι ; 855b7: κέντρον δὲ ἀμφοῖν τὸ A; 855b10: τὸ αὐτὸ κέντρον κινῶ.
[36] 855b28-30.

Apparently, the centre should travel the same distance in both cases and so the fact it does not presents a puzzle. But it is obscure why we should expect the centre to travel the same distance and the ambiguous wording of this passage poses challenges to interpretation. First, what is meant by 'the motion is always one'? There are at least four options: (a) the circles are always concentric and thus share a single motion; (b) the motion is continuous; (c) the motion is complete; (d) the [angular] speed is the same in both cases. Of these, (a) and (c) seem less likely since (a) is trivial (concentricity was assumed but unstated in puzzle 1) while it is doubtful whether (c) is true or relevant.[37] The syntactic parallel to the genitive absolute conditions in puzzle 1 might suggest (b), with 'one motion' functioning as a convenient shorthand for what had been carefully spelt out above.[38] While (a) and (b) repeat earlier ideas, (d) would introduce a new assumption. Speed and time have not so far featured in the presentation of the paradox; what has mattered in each case is the horizontal distance travelled for a given angle of rotation, not the time taken. Against (d), it would be odd for the new assumption not to be expressed more clearly in terms of sameness of speed.[39]

Sameness of speed is clearly at issue in the second sentence, which attempts to explain why we should expect the centre to traverse the same distance in each case. The reason given is that the same thing moving with the same speed ought to traverse an equal [distance]; '…in an equal time' may be intended, but its omission is unusual and worth noting.[40] However, we are not explicitly given a reason for assuming that the centre moves with the same speed. What is needed for the puzzle to make sense is a way of prompting the expectation that the centre should traverse the same distance in each case.

The author might mean that the centre moves with the same *angular* speed. The point would be that when the centre is rotated through the same angle (e.g. a quarter revolution or a full revolution) in the same time, it travels a different distance in each case. But this would be a

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[37] For the sense in which a change is 'one' if it is complete, see *Phys*. 5.4, 228b11-15.
[38] For Aristotle, every unqualifiedly one change is continuous and every continuous change is one (*Phys*. 5.4, 228a20ff.)
[39] Sameness of speed is not one of the senses in which changes are said to be one in *Phys*. 5.4. Aristotle considers the difference made by uniformity and non-uniformity of speed in a single change (228b15ff.), concluding that non-uniform change is single (because it is continuous) but 'less single' than uniform change.
[40] Cf. *Phys*. 6.2, 232b15-18: ἀνάγκη ἢ ἐν ἴσῳ ἢ ἐν ἐλάττονι ἢ ἐν πλείονι κινεῖσθαι, καὶ τὸ μὲν ἐν πλείονι βραδύτερον, τὸ δ' ἐν ἴσῳ ἰσοταχές, τὸ δὲ θᾶττον οὔτε ἰσοταχὲς οὔτε βραδύτερον, οὔτ' ἂν ἐν ἴσῳ οὔτ' ἐν πλείονι κινοῖτο τὸ θᾶττον; *Phys*. 7.4, 248a16: ὅταν ἐν ἴσῳ ἴσον κινηθῇ, τότε ἰσοταχές; 249b4: ἀλλ' ἔστω ἰσοταχὲς τὸ ἐν ἴσῳ χρόνῳ τὸ αὐτὸ μεταβάλλον.

strange issue to raise. Again, speed seems irrelevant to the comparison of the cases; the distance travelled matters but the time taken does not. Secondly, it seems unreasonable to assume that the same thing will travel the same distance in the same time when moved by the same angular speed. A rolling circle's centre will travel a certain distance, but when the same circle rotates on the spot with the same angular speed, its centre will not travel any distance.

Although Greek writers of the fourth and third centuries BCE (including Aristotle) implicitly use the notion of angular speed, particularly in discussing the heavens, angular speed is usually not explicitly distinguished from linear speed. It is possible that the puzzle relies on a use of τάχος that is ambiguous.[41]

Another possibility is that by τῷ αὐτῷ τάχει φερόμενον the author might mean 'what is moved *by* [a mover with] the same speed'.[42] On this reading, what we have is the idea that if A, moving of itself with a fixed speed, at one time pushes B and at another time pushes C, then, if B and C are in fact the same thing, they should be moved by A an equal distance in an equal time.[43] In the context of the *Rota* paradox, we must imagine a mover bringing about motion at the centre of the concentric circles in Cases I and II.[44] The assumptions would be (i) that the centre of the concentric circles constitutes a single object in the required sense; (ii) that it takes a mover of given speed to roll the centre through a given angle in a given time in each case. What favours this reading is the condition that the φερόμενον be the same; this would be unnecessary if the sentence were simply a statement of what it is for two things to move at the same speed. Further, the first point the author makes in his solution (855b32-856a1, primarily addressing puzzle 1; see 6.5.1 below) is that a mover with a fixed speed moves different objects at different speeds. The effects of movers with fixed speeds are clearly on the agenda.

[41] In that case 'quickness' might be a better translation.
[42] Taking τῷ αὐτῷ τάχει as a dative of agent rather than of manner
[43] This may be the case even if A continues moves at a slower speed when it is pushing another object. In fact, that seems to be the implicature of the condition that the object be the same.
[44] Recall that in Cases I and II the author used the first person in supposing he moved each circle himself. This could be taken to underline the sameness of the mover's speed.

6.4: Comparison of sources

At this point it is worth comparing Hero's statement of the paradox. Hero's *Mechanica* survives only in Arabic translation. I base the following on the available French, German and partial English translations:[45]

> 'And it could also be that the movements of the large circle and the small circle are equally swift even when both are fixed in their motions around the same centre. Let us think of two circles fixed on the same centre, the centre *α*. And let there be a tangent to the larger circle, line *ββ'*. And let us join the points *α* and *β*. Thus line *αβ* will be perpendicular to the line *ββ'*. And the line *ββ'* is parallel to the line *γγ'* and the line *γγ'* is tangent to the smaller circle. Also let us draw from point *α* a line parallel to the other two lines, and this is the line *αα'*. And so if we imagine that the large circle rolls along the line *ββ'*, the small circle passes along line *γγ'*. And when the large circle has already made one revolution, it will appear to us that the small circle has made one revolution also. Thus the position of the circles will be the position of the circles whose centre is at *α'*, and line *αβ* will be in the position of the line *α'β'*. For this reason line *ββ'* will be equal to line *γγ'*. And line *ββ'* is the line which the larger circle traverses when it makes a single revolution. And line *γγ'* is the line along which the smaller circle is rolled in a single revolution. And so the movement of the smaller circle is equally as fast as the movement of the larger circle, because the line *ββ'* is equal to the line *γγ'*, and things which traverse equal distances in equal times have movements of equal swiftness.
>
> In thinking about it, one might consider the conclusion absurd because it is not possible for the circumference of the larger circle to be equal to the circumference of the smaller circle. And so we say that the circumference of the smaller circle has not only rolled on the line *γγ'*, but it has traversed the path of the larger circle along with that circle. And so it happens that the movement of the smaller circle is equal in swiftness to the movement of the larger circle as a result of the two movements. For if we consider the larger circle rolling and the smaller circle not rolling but rather remaining still on point *γ*, then it will lay out the line *γγ'* in equal time; and the midpoint *α* lays out the line *αα'* in this time. But this is equal to both lines *ββ'* and *γγ'*.

[45] Carra de Vaux 1894, Clagett 1959, Nix and Schmidt 1976, 18-23.

Then the proceeding of the smaller circle's rolling makes no difference to the motion and consequently the length of the larger circle's path is the same as that along which the smaller circle moves. So we see that the midpoint traverses this distance without rolling, on account of the motion in which the larger circle moves.'[46]

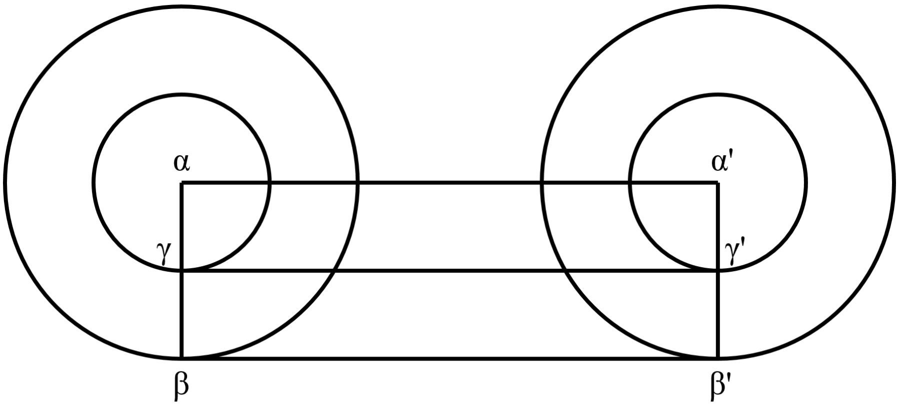

Fig. 3: Diagram accompanying Hero's text in Nix and Schmidt 1976, 16.

Hero opens with a statement of the absurd conclusion he derives from the paradox: that unequal concentric circles move equally quickly. This is a more explicit contradiction of the Rotating Radius Principle than any sentence in *Mech*. 24's treatment of the paradox. Next, he constructs a diagram and explains what each line represents. *BB'* represents the path traced by the larger circle in one revolution and *CC'* represents the path traced by the smaller circle in one revolution. It is clear from the construction that *BB'* and *CC'* are equal. Now things which traverse equal distances in equal times move equally quickly. The circles traverse equal distances in an equal time. Therefore they move equally quickly.

Hero's version of the paradox, though less detailed than *Mech.*'s, is more tightly composed. Its elements are few but they work together to deliver a clearly paradoxical conclusion. *Mech.*'s version lacks the same degree of clarity and coherence. This applies not only to the sequence of discussion, which is knottier in *Mech.*, but also to the conclusions reached. Hero's puzzle clearly relates the issues of the *Rota* to foundational principles in mechanics such as the Rotating Radius Principle. *Mech.*'s discussion is less explicit about the speeds of concentric circles and places the emphasis of its two puzzles elsewhere. The equal-speeds principle appears in the same position in both versions, at the end of the paradox's set-up and

[46] Hero, *Mechanica* 1.7.

immediately before the suggested solution. However, the principle plays a comparatively weak role in *Mech*.

It is particularly interesting that Hero explicitly draws a contradiction from the phenomena of rolling, similar to a puzzle I described in 6.2, whereas *Mech*.'s author notes certain features that are puzzling without drawing a clear contradiction. It is Hero's statement of the contradiction that makes the connection to the Rotating Radius Principle clear. Yet whether Hero's version is discussion is a streamlined and clarified version of *Mech*.'s, or whether *Mech*.'s is an elaboration on a presentation closer to Hero's, remains uncertain.[47]

6.5: The solutions in the *Mechanica*

Mech.'s author addresses the puzzles in the order he presented them. His answer to puzzle 1 is much longer than his answer to puzzle 2.

6.5.1: Answering puzzle 1: the same power causes different effects (855b32-856a1)

The solution begins with a statement that the principle (ἀρχὴ) 'of these things' is the fact that the same and equal δύναμις moves some magnitudes (μέγεθος) faster and some slower.[48] This seems to be borne out by certain ordinary experiences. A horse can pull an empty cart faster than it can pull its full load. It is assumed without question in *Phys*. 7.5's specific assertions that an equal δύναμις moves half the weight double the distance in an equal time, or an equal distance in half the time. *Mech.*'s author illustrates his point as follows:

> εἰ δή τι εἴη ὃ μὴ πέφυκεν ὑφ' ἑαυτοῦ κινεῖσθαι, ἐὰν τοῦτο ἅμα καὶ αὐτὸ κινῇ τὸ πεφυκὸς κινεῖσθαι, βραδύτερον κινηθήσεται ἢ εἰ αὐτὸ καθ' αὑτό[49] ἐκινεῖτο. καὶ ἐὰν μὲν πεφυκὸς ᾖ κινεῖσθαι, μὴ συγκινῆται δὲ μηθέν, ὡσαύτως ἕξει. καὶ ἀδύνατον δὴ

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[47] There are also questions to be asked about our text of Hero. The paradox appears near the start of Book 1, but it is doubtful whether the beginning of this book as transmitted, on the *barulkos*, was the original opening of the work. A version of the *Mechanica* was known in the Arabic world; al-Khazini includes a partial translation in his *Book of the Balance of Wisdom* (see Abattouy 2001). One clue that *Mech*.'s discussion may be a revision of an earlier presentation has been mentioned in 6.3.1 above: the author both seems to offer two different descriptions of the phenomena, claiming both that there is a proportionality between circles and their paths when rolled independently, and that a circle rolled independently traces out a path equal to its circumference.

[48] 856a32-334: ἀρχὴ δὲ ληπτέα ἥδε περὶ τῆς αἰτίας αὐτῶν, ὅτι ἡ αὐτὴ δύναμις καὶ ἴση τὸ μὲν βραδύτερον κινεῖ μέγεθος, τὸ δὲ ταχύτερον.

[49] The manuscripts have αὑτὴ καθ' αὑτὴν.

> κινεῖσθαι πλέον ἢ τὸ κινοῦν· οὐ γὰρ τὴν αὑτοῦ κινεῖται κίνησιν, ἀλλὰ τὴν τοῦ κινοῦντος.
>
> 'If there is something which does not naturally move because of itself, and if something that naturally moves because of itself moves this along with itself, it will move more slowly than if it moved alone by itself. And if it moves naturally but nothing is moved with it, the same will happen. So it is impossible for it to be moved more than the mover, since it is not moved by its own motion but by that of the mover.'

One object moving with its natural motion forces another object to move. Two cases are considered. In the first, the object moved by force does not have a natural motion of its own. In the second, it has a natural motion of its own. In both cases, the object moved cannot move further than the object forcing it to move.

In the first case, does the author mean (a) something artificial and hence without a nature in itself but tending to move in accordance with the natural motions of its material constituents; (b) something with no natural motion at all, even due to its matter; or (c) something with no capacity for self-motion (reading ὑφ' ἑαυτοῦ κινεῖσθαι as 'to be moved by itself'); or (d) something which has a natural tendency to move but is not moving naturally at a particular time?

It is difficult to see how (a) could offer an interesting contrast with the second case. And although ὑφ' ἑαυτοῦ κινεῖσθαι is Aristotle's standard expression for self-motion, there is nothing in this context to suggest that that subclass of animal motions is at issue, so (c) also looks unlikely.

According to (b), the object hypothesised in the passage is neither heavy nor light, nor moves naturally in a circle. [50] Such a body simply rests unless it is moved by something.[51] The

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[50] For Aristotle, heaviness and lightness are tendencies to move downwards and upwards (*DC* 4.3).

[51] It is not entirely clear that a body with no natural motion would lack a nature. Nature is an inner principle of change *and rest* (*Phys*. 2.1). The imagined object might have an inner principle of rest and could be said to have a nature in an extended sense. However, Bodnár 1997 has convincingly argued against 'one-sided' natures that would be only inner principles of motion. I suspect his considerations may be transferrable to the case of natures that would be only inner principles of rest.

suggestion is outlandish. In *De Caelo* 3.2, Aristotle claimed that there could not be a (sublunary) body without heaviness or lightness, arguing that such a hypothesis leads to two absurdities. The author of *Mech.* does not comment on its falsity or absurdity. I have found no independent evidence that any of Aristotle's successors in the Peripatos embraced the notion of an independently existing body without heaviness or lightness. Even the renegade Strato held that *all* bodies are heavy, leaving no room for the kind of body imagined by *Mech*. 24.[52] We may of course be missing part of the story, but it is difficult to see why anyone should have been tempted to posit such objects. If (b) were the correct reading, the author of *Mech*. 24 would be engaged in what we would call a 'thought experiment'.[53] What would be the point? Aristotle says that a body without any ῥοπή is unmovable and mathematical (ἀκίνητον καί μαθηματικόν, *DC* 3.6, 305a24-26), so perhaps the point would be to convince us that the principle can be extended to mathematical entities like circles. Alternatively, the point might be to illustrate that the weight of each wheel is irrelevant to the problem.

All the same, (d) is a more likely interpretation. It is true that Aristotle uses the formula πέφυκεν κινεῖσθαι to describe capacities for natural motion, rather than things moving naturally.[54] That is the main consideration favouring (b). On the other hand, the conditionals of our passage are less vivid (εἰ δή τι εἴη) and vivid (ἐὰν… κινῇ) future, rather than unreal (counterfactual). More importantly, the exotic hypothesis of an object with no natural motion is not necessary for the argument. The author wishes to show that, if A moves B, then B cannot move more than A, whether B is moving naturally or not.[55] Considering a case where B is not moving naturally is sufficient. There is no need to consider cases where B has no natural tendency to move.

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[52] Strato frr. 49-50D Sharples.

[53] Corcilius 2018 studies some thought experiments in Aristotle's authentic works, noting that the Greeks had no category corresponding to our 'thought experiment'. On ancient thought experiments, see also Ierodiakonou 2005.

[54] *Phys*. 8.3, 253a26-28: if it is necessary that there are always some things changing and some things at rest, then ἤτοι τὰ μὲν κινούμενα κινεῖσθαι ἀεὶ τὰ δ' ἠρεμοῦντα ἠρεμεῖν, ἢ πάντα πεφυκέναι ὁμοίως κινεῖσθαι καὶ ἠρεμεῖν. *DC* 1.8, 276b27-29: Πότερον οὖν βίᾳ πάσας ἐροῦμεν κινεῖσθαι καὶ τὰς ἐναντίας; ἀλλ' ὃ μὴ πέφυκεν ὅλως κινεῖσθαι, ἀδύνατον τοῦτο κινεῖσθαι βίᾳ.

[55] The claim that A will move slower if it moves B than if it moves by itself suggests that bodies have some resistence to motion, or at least are capable of slowing motions, independently of their heaviness. See Note B to Chapter 1.

6.5.2: Answering puzzle 1: moved and mover (856a1-32)

In this long passage, the author considers a number of ways in which one circle could move another. The aim, I think, is ultimately to show that Case I and Case II should be understood in terms of one circle playing an active role and forcefully moving the other. The author will tell us that when the smaller circle sits on the surface, the smaller circle causes the larger circle to move, and the motion the larger circle undergoes belongs to and is determined by the smaller circle. When the larger circle sits on the surface, *it* determines the motion.

What is the reason for working through several examples? Perhaps it is to show how one circle can determine the other's motion in cases where they are not concentric. The analysis is less clear when they are concentric, since the two circles are harder to distinguish in comparison to when they are sitting next to each other on the surface. The specific examples considered are: (1) if the smaller circle pushes the larger without rolling, the larger will move as far as the smaller; (2) if the smaller circle pushes the larger while rolling, the larger will move as far as the smaller; (3) if the larger moves the smaller, the smaller will move as far as the greater. The author then moves to generalising: it makes no difference whether the motion is fast or slow; it makes no difference whether one circle surrounds the other, or is fitted inside the other, or is in contact with its circumference. In all cases, one circle is active and one is passive, and the passive moves just as far as the active drives it. It is ultimately this distinction, which explains the difference between Case I and Case II.

6.5.3: Answering puzzle 2: the centre is not unqualifiedly the same (856a32-39)

In returning to puzzle 2, the author now puts it in the mouth of a 'sophistic puzzler' who 'reasons fallaciously'. These terms of criticisms are suggestive.[56] They may call to mind the language of the *Sophistical Refutations*, a work that categorises and examines fallacious arguments that appear to be refutations (ἔλεγχοι).[57] But there is no more specific allusion than this verbal echo, and the fallacy identified is one which Aristotle also discusses in several other texts.[58]

[56] For other uses of παραλογίζεται, see *Phys.* 186a11, 239b5 (against Melissus and Zeno, respectively); *Poetics* 1460a25; *Politics* 1307b35; *Problems* 5.25, 883b8 and almost verbatim at 30.4, 955b16; *Rhet.* 1401b8; 1408a20; *Soph. Ref.*. 171b37. For uses of σοφιστικῶς, compare *Topics* 5.4, 133b16; *Rhet*. 1419a14.

[57] An ἔλεγχος is 'a deduction to the contradictory of a given conclusion' (*Soph. Ref.* 1, 165a2-3, trans. Pickard-Cambridge). Puzzle 2 appears – and merely appears – to deduce a contradiction of the conclusion that the centre traverses paths of different lengths.

[58] E.g. *Met.* E.2, 1026b15ff. In the terms of the *Sophistical Refutations*, it is difficult to decide whether the fallacy identified in puzzle 2 should fall under the fallacy of accident or the fallacy of *secundum quid*.

We are told that the error in puzzle 2 was the tacit assumption that the circles' centre is one and the same without qualification (ἅπλῶς). On the contrary, the centre is the same for both circles accidentally (κατὰ συμβεβηκός), 'like musical and white' (ὡς μουσικὸν καὶ λευκόν). This elliptical comparison to the musical and white assumes familiarity on the part of the reader with the stock examples of Aristotelian literature.[59] The point of the comparison is as follows. A person, say Socrates, who is both musical and white can be described as 'musical' and 'white'. Each of those two terms can then be used to refer to one and the same thing, Socrates. In a sense, then, the musical is white (and vice versa), but this is only true for the sense in which 'the musical' refers to what musical is an accident of, namely Socrates. In the sense in which 'the musical' refers to the accident itself, musicality, the musical is not white.[60] Treating the white and the musical as the same without qualification can lead to all kinds of absurdities (e.g. that musical is a colour).

In the case of puzzle 2, there is a single point which is the subject, analogous to Socrates. This point happens to be the centre of the smaller circle and the centre of the larger circle, so it can be referred to as 'the centre of the smaller circle' and 'the centre of the larger circle'. But just as the musical and the white are not the same without qualification, so here the centre of the smaller circle and the centre of the larger circle are not the same without qualification. So from the premise that 'the same thing moving with the same speed ought to traverse an equal [distance]' one cannot deduce that the centre of the larger circle and the centre of the smaller circle ought to traverse an equal distance.[61] In Case I, when the smaller

[59] While neuter, μουσικὸν and λευκόν do not agree with κέντρον. The κέντρον is analogous to the unmentioned subject which possesses these attributes. Cf. Forster's expanded rendering (my emphasis): 'just as *the same thing* may chance to be 'musical' and 'white'.'

[60] Aristotle frequently uses 'musical' and 'white' as examples of the accidental, in all its senses. See *De Int.* 11 21a7-11: 'Of things predicated, and things they get predicated of, those which are said accidentally, either of the same thing or of one another, will not be one. For example, a man is white and musical, but 'white' and 'musical' are not one, because they are both accidental to the same thing.' (trans. Ackrill); *Met.* Z.6, 1031b19ff.; *APo.* 1.4, 73b5 on another sense of accidental: 'what belongs in neither way [sc. of belonging to a subject essentially] I call accidental e.g. musical or white to animal' (trans. Barnes).

[61] Van Cappelle 1812, 262 gives a different interpretation, understanding κατὰ συμβεβηκός in terms of accidental predication: just as humans are not necessarily or for the most part musical or white, the point is not necessarily or for the most part the centre of the larger circle or the centre of the smaller circle; those attributes can be called 'accidental' since they are not essential. While this interpretation is consistent with mine, it does not seem to be what the author intended by κατὰ συμβεβηκός.

circle causes the motion, the centre of the smaller circle governs the motion.[62] In Case II when the larger circle causes the motion, the centre of the larger circle governs the motion.[63]

This solution seems problematic. The author is quick to point out that the circles' centres are not absolutely the same, but does not take care to show that they are different in a way that matters. It is not obvious that the same speeds principle requires that the subject of motion be the same without qualification. It might seem that extensional equivalence is sufficient. After all, items that are spatially coextensive (or co-located or co-positioned, as perhaps we should say for extensionless points) will necessarily traverse the same distance, or remain in the same place, in a given time.

A difficult sentence immediately following the comparison to musical and white attempts either to justify or to clarify the point just made: τῷ γὰρ εἶναι ἑκατέρου κέντρου τῶν κύκλων οὐ τῷ αὐτῷ χρῆται (856a36-37).[64] Taking τὸ κέντρον as the implied subject (cf. 856a34-35), I understand this as saying that the centre 'uses its being the centre of each circle differently'.[65] If this is meant to show that the circles' centres are different in a way that should make a difference to the distance they traverse (note the use of γὰρ), the reasoning seems open to a charge of circularity. Why does the centre traverse different distances? Because it has two attributes. Why do those attributes make a difference to the distance traversed? Because the centre in fact traverses different distances.

The author does not mention in his discussion of the centre's motion the Aristotelian doctrine that what has no parts cannot move except *per accidens*.[66] Perhaps the reason for this omission is that, although problem 24's paradox is couched in abstract, geometrical terms (circles and centres), the author still tacitly has in view the corresponding physical objects (wheels and axles), where the axle corresponding to the centre *does* have parts. Attention to

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[62] It is the ἀρχή (856a38).
[63] 856a38-39. For the notion that an indivisible point can be in a sense two or have two different functions, compare *Phys*. 4.11, 220a10-13; 8.8, 262b24-25; *DA* 3.2 427a9-14.
[64] I read τῷ following Apelt. This appears only in Par. A where other manuscripts have τὸ, which is favoured by Hett and Bottecchia Deho.
[65] Compare Hett, 'For the fact of each circle having the same centre does not affect it in the same way in the two cases.' and Bottecchia Deho, 'perché l'uno e l'altro cerchio hanno si il medesimo centro, ma non usano il medesimo.' *DA* 3.2 427a12-13 uses the same form of the verb (χρῆται) twice, but with the faculty of discrimination as the subject and successively a point (σημείῳ) and a limit (πέρατι), probably the same item, as the objects; *Phys.* 8.8, 262b24-25 uses the perfect tense of the same verb.
[66] *Phys*. 6.10.

the partlessness of the centre would place inappropriate emphasis on a feature of the idealisation.

A pair of recent commentators, Günter Dosch and Ernst Schmidt, have argued that *Mech.*'s solution has been traditionally undervalued and is in fact satisfactory (*befriedigend*).[67] As we have seen, the author distinguishes the way a circle rolls independently from the way it rolls when carried by another circle (856a1-32); he sometimes refers to the way a circle rolls independently as what it does 'naturally' (e.g. 856a19). Dosch and Schmidt argue that calling a circle's motion 'natural' is the obvious thing to do (*naheliegend*) when it rolls independently; likewise we could call a circle's motion 'unnatural' when it is carried by another circle.[68] This is partly explained in terms of modern classical (= Newtonian) mechanics. A circle rolls 'purely', tracing out in one revolution a path on the surface equal to its circumference, when the instantaneous linear velocity of its point of contact with the surface is always zero, and this typically occurs because a linear velocity of zero at the point of contact minimises static friction. If the linear velocity at the point of contact were not zero, constant application of force would be needed to overcome static friction, and in that case the circle would trace out in one revolution either a greater or lesser distance than its circumference.[69] In the context of this modern theory, the authors argue, the labels 'natural' and 'unnatural' also seem obvious (*naheliegend*).[70]

It is undoubtedly important to keep our grip on the modern analysis of rolling, and Dosch and Schmidt's exposition is admirably clear. Nonetheless, I cannot take comfort in the parallels they intimate between this analysis and *Mech.*'s treatment of the paradox. Friction is nowhere mentioned in *Mech.*'s discussion. And in the absence of a worked-out or even roughly sketched account of the effects of static friction, how satisfactory is the part of *Mech.*'s solution which is framed in terms of the way a circle moves naturally? One traditional complaint has been that *Mech.*'s solutions to the paradox are empty.[71] I share the view that the author, by saying that one circle's motion is 'natural' and 'of itself' while the other is moved by it, has essentially re-described the phenomena without accounting for them.

[67] Dosch and Schmidt 2018, 228.
[68] Dosch and Schmidt 2018, 222; cf. 227: 'Die Bezeichnung… ist durchaus angemessen.'
[69] The constant application of force would of course not be necessary in the absence of friction; on an ideal, frictionless ice rink, wheels could easily trace out in one revolution any distance we please.
[70] Dosch and Schmidt 2018, 225.
[71] Klügel 1803, 173: 'Was Aristoteles selbst zur Erklärung der Schwierigkeit beibringt, läuft auf leere Spitzfindigkeiten hinaus und ist keine wahre Auflösung derselben.'

6.6: Conclusion

This chapter has offered a close reading of the text of problem 24. I have argued that this problem opens by launching a subtle challenge to the Rotating Radius Principle, the keystone of *Mech.*'s explanatory programme. The rest of the problem attempts to restore order. The attempt to grapple with the problem involves several striking moves. In the response to puzzle 1, we are invited to consider that, in general, no object that is moved by another object can move faster or further than what is moving it. The response to puzzle 2 involves a division, at least conceptually, of the indivisible point at the centre of the two circles. In both cases, the paradox is resolved by appealing to ideas that belong to natural philosophy, rather than mathematics.

This may be taken as further confirmation of a conclusion reached in the previous chapter, that *Mech*.'s author did not aim to follow the methodological strictures of Aristotle's *Posterior Analytics*. In Chapter 5 I argued that *Mech*.'s explanations are analogical rather than demonstrative. The present chapter suggests another element of Aristotle's scheme has been brushed aside. In the *Posterior Analytics* 1.7, Aristotle prohibits explaining the subject-matter of one science through the principles of another. He allows for a special exception in the cases of mechanics, optics, harmonics and astronomy, which he characterises as subordinate, mixed sciences (1.7, 75b14–17; 1.13, 78b32–79a16). Each is subordinated to a higher, mathematical science (e.g. geometry) one which it depends for its proofs. The lower, physical science states the ὅτι, the facts or phenomena to be explained, but the higher, mathematical science supplies the διότι, the explanation.[72]

Although it does not use those terms, *Mech*.'s preface has often, I would argue mistakenly, been read as aligning the text with this conception of mechanics.[73] In *Mech*. 24 the hierarchical ordering of sciences suggested in the *Posterior Analytics* seems to be inverted. An ostensibly mathematical problem, at any rate a problem about circles that is stated without reference to natures, powers, or other properties characteristic of natural inquiry, is explained through appeal to principles that unmistakably belong to investigations of nature rather than to geometry. The ὅτι is stated in abstract, mathematical terms and yet *Mech*.'s author did not feel the need to have the higher, mathematical science supply the διότι. This should not be

[72] For a fuller analysis, see McKirahan 1978.
[73] τὸ μὲν γὰρ ὡς διὰ τῶν μαθηματικῶν δῆλον, τὸ δὲ περὶ ὃ διὰ τῶν φυσικῶν. I cast doubt on this reading of the preface in §1.2.

very surprising. I argued in Chapter 4 that problem 1's explanation is physical and causal rather than mathematical. The case of problem 24 is only more striking given the higher level of abstraction at which the explanandum is described.

Chapter 7: A new interpretation of *Physics* 7.4

7.1: Introduction

In the previous chapter I compared our two ancient sources for the paradox. In the present chapter I shall show how this appreciation of the paradox may shed light on a disputed chapter of Aristotle's *Physics*.

In *Physics* 7.4 Aristotle asks whether all changes are comparable (συμβληταί) and answers 'no'. Not only are changes in different categories incomparable, so that, for example, an alteration cannot be compared with a generation, but there are also incomparable changes within categories. Surprisingly, Aristotle says that, among changes in the category of place, circular and rectilinear motions are incomparable. His reason for this claim has something to do with the fact that although terms like 'equally quick', 'faster' and 'slower' always mean the same thing, they are different when applied to changes in different species, and circles and straight lines are different species of line.[1]

Scholars agree that Aristotle says this much, but debate his reasons for doing so. Aristotle's belief that there are different species of line is shared with several ancient geometers and philosophers, but the claim that objects belonging to different species are incomparable is not.[2] *Phys*. 7.4 does not mention 'mixed' lines or motions and leaves us unsure to what extent they might be compared.[3]

I shall criticise three past interpretations of *Phys*. 7.4 (I call these the construction, measure and cosmological interpretations) and propose a new one. My aim is not to defend the chapter, but to indicate some serious difficulties in influential readings and to offer an alternative that is more charitable. I do not expect that any interpretation of this difficult text will be free from problems, but I approach the task with three desiderata. First, we should like our interpretation to be charitable: we should assume that Aristotle was not at odds with the

[1] *De Caelo* 1.2 makes a similar distinction between circular and straight lines, adding the further category of 'mixed'.
[2] See Heath 1956, 159-65 on ancient classifications of lines. Euclid (*Elements* 5.def. 3) specifies that ratio is a relation in respect of size between two magnitudes of the same species. There are ten species: curved and straight lines, regions, surfaces, solids and angles. By his lights, curved and straight geometric objects always belong to different kinds so there should be no ratios between them. However the relevance of this is questionable since συμβλητός surely cannot mean 'commensurable'.
[3] See Chapter 3 for my discussion of mixed motions in *DC* 1.2 and *Phys*. 8.8

geometry of his day and that he did not engage in blatant self-contradiction. Secondly, we should like *Physics* 7.4 to have a point. It is possible that *Physics* 7 is something of a scrapbook and I do not commit myself either way on the vexed matter of its overall coherence and integrity. However it is clearly desirable that an interpretation of any of its component blocks should identify a purpose in the context of Aristotle's philosophical agenda. Thirdly, any interpretation should naturally be historically plausible.

I shall suggest that Aristotle's rejection of the comparability of rectilinear and circular motions is motivated by a paradox of motion known as the 'Wheel of Aristotle'. Interpretative disagreement has partly concerned the correct interpretation of Aristotle's συμβλητός, a word not used by Greek mathematicians.[4] I shall not consider in any detail the interpretation according to which συμβλητός means 'commensurable' (σύμμετρος) in the mathematical sense of expressibility as a ratio of two positive integers. It has been satisfactorily shown that this interpretation must attribute to Aristotle a number of erroneous inferences and, in any case, had Aristotle meant to speak of commensurability he could have used σύμμετρος, as he does elsewhere.[5] A similar line of criticism applies to the measure interpretation (see below) according to which Aristotle's use of συμβλητός means or implies μετρητός. On the one hand, this reading would present technical problems; on the other, *Phys*. 7.4 nowhere uses the terminology of measurement which we would expect, were measurement important to his argument.

I think Aristotle's συμβλητός means comparable in the following straightforward sense: *X* and *Y* are comparable if it can be truly said that *X* is equal to, greater than or less than *Y*. Aristotle denies that the speeds of circular and rectilinear motions are comparable in this sense.

7.2: The Construction Interpretation

Some commentators believe that Aristotle makes the mathematical blunder of claiming that a straight line cannot equal a circle in magnitude. 'Of course the circumference *is* equal to a straight line,' writes Bostock, 'and it is astonishing that Aristotle should have thought

[4] There is no entry for συμβλητός in Mugler 1958.
[5] Wardy 1990, 267.

otherwise.'[6] It is indeed hard to know what could motivate this belief; the construction interpretation suggests that Aristotle thought no such line existed either because (i) none had been constructed, or (ii) he believed none was constructible.

The construction interpretation has ancient pedigree. Alexander of Aphrodisias (as reported by Simplicius) apologetically explained that in Aristotle's time the geometrical problem of squaring the circle was still under investigation.[7] Simplicius himself thought that the problem had been abandoned.[8] It is well known that Greek geometers after Aristotle squared the circle by means of a curve called the quadratrix (τετραγωνίζουσα). Unfortunately it is unclear when this method of quadrature was first developed. Formerly, historians ascribed the discovery of this curve, if not necessarily its application to the quadrature of the circle, to Hippias of Elis. The current consensus is that the quadratrix cannot antedate the 3rd c. BCE.[9] So Alexander may well be right that the circle had not been squared in Aristotle's day, although there is little ground for Simplicius' claim that geometers had given up hope. In the *Categories* Aristotle seems open-minded about the possibility of squaring the circle.[10] In the *Eudemian Ethics*, he seems more pessimistic – but was this because the geometers had given up hope, or because of his own conclusions in *Phys*. 7.4?[11] A third possibility is that Aristotle was merely using a stock example of a pointless endeavour.[12]

In any case, the relevance of circle squaring is questionable. Strictly speaking, Aristotle on the present interpretation bans the rectification of the circle, not its quadrature. The two problems are indeed connected. Archimedes' *Dimension of the Circle* proposition 1 reduced the problem of quadrature to the problem of rectification, demonstrating that a circle is equal to the right-angled triangle whose perpendicular sides equal its circumference and radius. So

[6] Bostock 1996, 285.
[7] 'Alexander asserts that he has stated several things about noncomparable motion because it has not yet been proved that a straight line is not equal to a curve but has remained being investigated.' (trans. Hagen 1989, 76).
[8] '[In Aristotle's day,] it was still being investigated whether it is possible for a straight line to be equal to a curve, or rather it had been given up on. And hence the squaring of the circle had not yet been discovered either,' (trans. Konstan 1989, 60-61).
[9] Knorr 1986, 80-86 argued that the quadratrix's application to the problem of squaring the circle should be dated after Aristotle.
[10] *Cat.* 7b29ff.: ἐπιστήμης δὲ μὴ οὔσης οὐδὲν κωλύει ἐπιστητὸν εἶναι· οἷον καὶ ὁ τοῦ κύκλου τετραγωνισμὸς εἴγε ἔστιν ἐπιστητόν, ἐπιστήμη μὲν αὐτοῦ οὐκ ἔστιν οὐδέπω, αὐτὸ δὲ τὸ ἐπιστητὸν ἔστιν. (trans. Ackrill: '[I]f there is not knowledge there is nothing to prevent there being a knowable. Take, for example, the squaring of the circle, supposing it to be knowable; knowledge of it does not yet exist but the knowable itself exists.')
[11] *EE* 2.10, 1226a29-30: διὸ οὐ βουλευόμεθα περὶ τῶν ἐν Ἰνδοῖς, οὐδὲ πῶς ἂν ὁ κύκλος τετραγωνισθείη. τὰ μὲν γὰρ οὐκ ἐφ' ἡμῖν: τὸ δ' ὅλως οὐ πρακτόν (trans. Kenny: 'Hence we do not deliberate about the affairs of India, nor about how to square the circle—the former are not in our power and the latter just cannot be done.')
[12] See Aristophanes *Birds* 1005 (first performed in 414 BCE).

the possibility of one construction entails the possibility of the other. Was Aristotle aware that one problem reduced to the other? Or did Alexander retroject what had become common knowledge by the late 2nd c. CE? For present purposes it does not much matter, since the advocate of the construction interpretation might equally maintain that Aristotle denied the possibility of rectification.[13]

There are however two problems with the construction interpretation. First, the fact that a construction had not yet been produced would not have warranted the conclusion that no such object exists rather than that its existence had not yet been proven or disproven. Secondly, even the impossibility of a construction would not have warranted an existential conclusion. We have no reason to think Aristotle would have been inclined to associate constructability and existence. It is unlikely that such a view could have been borrowed from mathematical practice. Zeuthen's thesis that Greek geometers viewed constructions as existence proofs, although once dominant, is no longer widely accepted.[14] Partly this is because constructions are often better viewed as attempted for their intrinsic interest, and partly because mathematicians sometimes assumed the existence of objects non-constructively, for example in the Eudoxan assumption of the fourth proportional to three given magnitudes in theorems of *Elements* 12, based on tacit intuitions of continuity.[15]

The construction-as-existence reading, or more precisely the straightedge-and-compass-construction-as-existence reading, provides a straightforward explanation of Aristotle's emphasis on the status of circular and straight lines as "simple" (and hence the only possible paths of natural motions, cf. *De Caelo* 1.2, *Phys.* 8.8). An alternative explanation is available: Aristotle called these simple because he believed they were the only homeomeric lines.[16]

[13] Mendell 2004 suspects the equivalence of rectification and quadrature was still unknown in the 4th c. BCE. Simplicius later spoke of Archimedes' rectification in *SL* 18 as a quadrature (*Comm. in Cat.* 7, 192.15-25 Kalbfleisch).
[14] See Zeuthen 1896 with the criticisms of Steele 1936, Niebel 1959, Knorr 1983, and Lachterman 1989. The debate is now conveniently summarised by Thiel 2005 who distinguishes from Zeuthen's thesis the stronger, intuitionistic claim of Becker that Greek geometers identified existence with constructability. From a different angle, Harari 2003 argues that Aristotle's ontology offers no reason to think he saw constructions as existence proofs
[15] Mueller 1981, 230-32.
[16] A homeomeric line is one such that any two equal parts can be made to coincide by superposition. While Aristotle's claim that these are the only homeomeric lines is true for plane geometry, it is false in three dimensions since the cylindrical helix is also homeomeric. However this shape was not studied until Apollonius. See Acerbi 2010.

7.3: The Measurement Interpretation

The measurement interpretation may be concisely sloganised as 'συμβλητός implies μετρητός', comparability implies measurability.[17] It explains Aristotle's conclusion that a straight line cannot be compared to a circle in terms of the account of measurement developed in *Met.* 10.1. According to this interpretation, two magnitudes are comparable only if they are both measurable in terms of a common unit measure. Therefore any two straight lines can be compared since we can use one to measure the other.[18] However we cannot perform the required measurements to compare a curved and a straight line, since there is no unit that fits both. It follows that Aristotle must deny the comparability of any rectilinear geometric object with any curvilinear one.

Unfortunately this interpretation commits Aristotle to mathematical revisionism which he would rather avoid. Aristotle does not formulate his aversion to philosophical legislation over mathematics in the manner of a modern mathematical naturalist such as Penelope Maddy, but a similar tendency is in evidence.[19] He takes care to explain that his conclusions do not challenge the truth of mathematical propositions or the validity of their proofs. In *Phys*. 2.2 we are told that 'nothing false' results from his account of abstraction.[20] In *Phys.* 3.8, Aristotle claims that his rejection of actual infinity 'does not deprive mathematicians of their proofs' since mathematicians only need finite lines of any desired length for their proofs, not infinite lines.[21] However if Aristotle believed that comparability implies measurability in

[17] This approach has been suggested by Ross 1936, 677-78 and Lloyd (as reported by Wardy 1990, 269-70).

[18] By 'can be compared' I do not mean commensurable in the technical sense. I mean that we can say one is larger than, smaller than or equal to the other. Even incommensurable lines may be comparable in this sense (e.g. the diagonal and side of a square).

[19] Maddy 1997.

[20] *Phys*. 2.2, 193b34-35.

[21] *Phys.* 3.7 207b27-34. Whether this is correct for the mathematics of his time remains controversial. The traditional view is that Aristotle's combination of a strictly finite cosmos and an abstractionist philosophy of geometry produces a tension with geometers' reliance on infinitely extendible lines (Cherniss 1935, 34; Solmsen 1960, 173**;** Hintikka 1973, 117-9). Knorr 1982b, 122 concludes, 'Aristotle's theory of the infinite shows remarkable insensitivity to the issues which must have occupied the geometers of his generation.' Hussey 1983, 93-96, 178-79 suggests that Aristotle could accommodate this cosmic upper bound on magnitudes by revising Euclidean geometry so as to avoid references to infinitely extendible lines, by substituting *Elements* 1.32 (that the internal angles of a triangle are equal to two right angles) for the fifth postulate and substituting a localised definition of 'parallel' (e.g. the equal angles property) for def. 23. There is no independent evidence that Aristotle endorsed such a revision, and the proposition of *Elements* 1.32 is treated as a theorem to be proven rather than a first principle at *APr.* 1.35, 48a29-39; *APo.* 1.5, 74a16; *Phys*. 2.9, 200a15ff. and *Met.* Θ.9, 1051a21-31 (the wording of the last passage suggests Aristotle had in mind a proof close to Euclid's; see Heath 1949, 40). Partly for this reason, and partly since Aristotle more than once explicitly states what is now called the 'Archimedean' axiom (*Phys.* 8.10, 277b2-4; *DC* 1.5, 272a1), Hussey's ingenious suggestion seems unlikely. A more promising approach takes issue with the traditional interpretation's assumption that geometrical objects should have physical instantiations from which they are abstracted: 'the real problem here is that some of the lines the geometer needs do not seem to be forthcoming at all… If the requisite lines do not exist, there is

terms of a common unit, he would unambiguously be forced to reject several of the most important mathematical principles, proofs and problems of his day with no prospect of their reinterpretation on philosophically acceptable terms.

In the first place, the measurement interpretation commits Aristotle to the rejection of all rectifications and quadratures of curvilinear figures. Although Aristotle may be comfortable with dismissing the rectification and quadrature of the circle, he should have felt uneasy about this wholesale ban in the century following Hippocrates of Chios' seminal quadrature of lunes. Admittedly this objection is a little difficult since some readers of Aristotle *Phys*. 185a14ff. and *Soph. El.* 171b13ff. have interpreted him as inappropriately rejecting Hippocrates' quadrature. However it is possible to read these passages differently: *Phys*. 185a14ff. rejecting another unknown mathematician's quadrature and *Soph. El.* 171b13ff. criticising as fallacious some other work of Hippocrates.[22]

Secondly, Aristotle would have to reject several geometric proofs which utilise the so-called 'method of exhaustion' associated with Eudoxus.[23] For example, the proof of Euclid *Elements* 12 prop.2 (that circles are to one another as the squares on their diameters) relies on the claim that a circle is larger than its inscribed polygon, but that by doubling the polygon's number of sides one may bring it arbitrarily close to the area of the circle (the 'bisection principle'). On the measurement interpretation Aristotle should object before Euclid even begins bisecting, when he argues that the square inscribed in a circle is greater than half the circle. Similar inequalities between rectilinear and curvilinear figures are crucial to the proofs of propositions 11, 12 and 18 of *Elements* 12.

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nothing to abstract from.' (Hintikka 1973, 22). That is a very strict abstractionist philosophy of geometry and Aristotle's own view may have been more relaxed. According to *Met.* Θ.9, 1051a21-31, geometrical propositions can be proven by constructions carried out *in thought* (cf. Lear 1982, 180; White 1991, 160-61). Concepts of some basic shapes should be acquired by abstraction from their physical instantiations, but once, for example, the straight line and circle have been thus acquired, constructions can procede in thought irrespective of whether there are corresponding physical instantiations. For an alternative approach, compare Kouremenos 1995, 35, 50-53 *et passim*. Another indication of anti-revisionism may be found in the *Metaphysics* M.3 1077b31-33

[22] Hippocrates' quadrature was preserved by Eudemus' *History of Geometry* which was in turn quoted by Simplicius' commentary on *Physics* 1.2. For text and translation see Thomas 1939, 235-53. Mueller 1982 argues for the interpretation given above which is also favoured by Lloyd 1987b.

[23] The ascription to Eudoxus largely rests on Archimedes' report of relevant theorems in the prefatory epistle to *Sphere and Cylinder* 1.

Thirdly, Aristotle appears familiar with the isoperimetric result that the circle bounds the greatest area within a given perimeter length.[24] However on the measurement interpretation this proposition would be unintelligible since the area and circumference of a circle could only be compared with those of other circles.

Fourthly, in *De Caelo* 2.14 Aristotle reports with implicit approval some unknown mathematicians as estimating the earth's circumference at 400,000 stades, i.e. as measuring something circular in terms of a rectilinear unit of length. Indeed, if it were not possible to measure what is curved by what is straight, it seems it would not be possible to measure any distance over land or sea.

Finally, this interpretation would compel Aristotle to deny the comparability of unequal circles, since the arc of a smaller circle cannot measure a larger circle. Arcs of unequal circles are at best similar but never congruent, so they cannot measure each other. On the present interpretation it would follow that they are incomparable. Accordingly we should expect Aristotle to hold that a circular motion cannot be compared to a motion along the path of any circle that is not exactly the same size. This would entail a rejection of the Rotating Radius Principle. Yet, as we have seen, Aristotle himself endorsed the principle, and it played a crucial role in *Mech.*'s explanatory programme.[25]

There is no reason to think Aristotle would reject all of this: Hippocrates' quadrature, Eudoxan 'methods of exhaustion', the Rotating Radius Principle. The assumption 'συμβλητός implies μετρητός' would provide Aristotle with grounds for denying the comparability of circular and rectilinear motions, but at too great a cost. It is also worth noting that to compare sizes and to measure are not the same thing. I do not need to measure the universe to know that it is bigger than the earth and bigger than me, nor need there be any possible measurement procedure. Similarly, I know that the inscribed square is smaller than the circle without measuring either.

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[24] *An.Post.* 2.13 79a15-16, *DC* 2.4 287a23-30. No demonstration of the result appears to have been supplied before Zenodorus in the 2nd c. BCE; see Knorr 1983, 133-34; Knorr 1986, 198.

[25] Plato twice (*Gorgias* 451c5-9, *Phaedo* 98a3-5) has Socrates say that among astronomy's primary tasks was the determination of the relative speeds of the heavenly bodies. Whether the measurement interpretation would also force Aristotle to give up portions of astronomy depends on whether such references are understood in terms of angular or linear speed.

7.4: The Cosmological Interpretation

The cosmological interpretation proposed by Wardy sees the argument of *Physics* 7.4 as motivated by a desire to buttress Aristotle's sharp separation between the celestial and sublunary regions of the cosmos. The former is characterised by ether's eternal circular motion, the latter by the four traditional elements moving on rectilinear paths to their natural places. There is no explicit indication that the circular motion in question in *Phys*. 7.4 is that of the heavens. In fact there is no clear reference to the heavens in *Physics* 7, and mention of the doctrine of the fifth element in any book of the *Physics*.[26] This interpretation requires a distinctive reading of the chapter's opening:

> εἰ δή ἐστιν πᾶσα συμβλητή, καὶ ὁμοταχὲς τὸ ἐν ἴσῳ χρόνῳ ἴσον κινούμενον, ἔσται περιφερής τις ἴση εὐθείᾳ, καὶ μείζων δὴ καὶ ἐλάττων.

Wardy translates as follows:

> 'For if indeed they are all comparable, and things changed an equal amount in equal time are changed at the same rate, there will be a circular motion equal to a rectilinear one, and again circular motions greater and lesser than rectilinear ones.'

The standard reading, 'there will be a circumference equal to a straight line', must be rejected, since there is no straight line in Aristotle's cosmos equal to the circumference of a great circle of the heavenly sphere. Instead Wardy construes περιφερής and εὐθεία as describing types of motion (κίνησις). This may be seen as a virtue, since it avoids ascribing to Aristotle the apparently inept claim that no straight line equals a circumference. On the other hand, Wardy translates the same terms περιφερής and εὐθεία in 248a24-25 as 'circular line' and 'straight one'. This is particularly problematic since in that passage Aristotle claims that a circular line can be smaller than a straight one, which puts strain on the interpretation of περιφερής as designating either the motion or path of the heavens.

[26] Furley 1989, 194 notes the fifth element's absence in the *Physics* and suggests this is a sign of early composition; Falcon 2014, 321 suggests deliberate avoidance.

It is a merit of the cosmological interpretation that it locates *Phys*. 7.4 within Aristotle's broader cosmological project, to the satisfaction of my second interpretative desideratum. However, it faces three difficulties.

First, Aristotle argues for the celestial-sublunar distinction at length in *De Caelo* 1.2-4 and it is doubtful that this doctrine would need further support in the *Physics* and in this way. Wardy's suggestion is that 'since directions of movement are central defining characteristics of Aristotelian simple bodies, he may have forbidden the comparison in question in the belief that tolerating it would obliterate the *essential* difference between 'being the stuff of the stars' and 'being the stuff of the mundane elements'.'[27]

However when Aristotle speaks of the comparability of changes in *Phys*. 7.4 he always means the comparability of their speeds, not directions. And speed is not part of the essence of the simple bodies which are rather defined by their natural places and directions of motion to those places. Nothing in the essences of Aristotle's simple bodies specifies their speed because he does not believe that a simple body's natural motion always has the same speed. The speed of a naturally falling or rising body depends on at least two things: (1) the medium through which the body travels (*Phys.* 4.8 215a24ff.); (2) the heaviness or lightness of the body. Aristotle notoriously claimed that there is a simple proportionality between a body's heaviness or lightness and the time required for it to traverse a given distance: if you double the weight, you halve the time. Moreover, Aristotle notes that the simple bodies' natural motions accelerate, so there is no fixed speed even for the same quantity of the same material in the same medium.[28]

Secondly, the cosmological interpretation does not seem to make adequate sense of Aristotle's analogy to uphill and downhill motions. After denying that circular and rectilinear motions can be of equal speed, Aristotle immediately rejects the suggestion that this is because one is necessarily faster or slower than the other (248a21-23):

> ἄτοπόν τε γὰρ εἰ μὴ ἔστι κύκλῳ ὁμοίως τουτὶ κινεῖσθαι καὶ τοῦτο ἐπὶ τῆς εὐθείας ἀλλ' εὐθὺς ἀνάγκη ἢ θᾶττον ἢ βραδύτερον, ὥσπερ εἰ κάταντες, τὸ δ' ἄναντες.

[27] Wardy 1990, 270-71.
[28] On this proportionality and on acceleration, see Note B to Chapter 1.

'It would be absurd to suppose that the motion of one in a circle and of another in a straight line cannot be similar, but that the one must inevitably move more quickly or more slowly than the other, just as if the course of one were downhill and of the other uphill.' (trans. Hardie and Gaye)

Wardy thinks this analogy draws the following contrast: the same kind of thing (a terrestrial object) can move both downhill and uphill, but it is not the case that the same kind of thing can move in celestial circles and rectilinear paths.[29] Clearly Aristotle would agree with this latter claim, but I cannot see what this has to do with the present issue of inequalities of speed. Why should anyone think that the fact that the heavens are necessarily faster than sublunary bodies implies that they could exchange places? Might not Aristotle's mention of necessity (ἀνάγκη) speak against the possibility of such an interchange?

Thirdly**,** it is unlikely that Aristotle would have thought it ἄτοπον to say that the celestial motions are necessarily faster than any sublunary motion. Not only had earlier investigators claimed as much, such as Anaxagoras who said that the cosmic περιχωρήσις was faster than anything among men (DK59 B9), but Aristotle himself had argued for a similar proposition. Even if he changed his mind, he should not label the old belief absurd without good reason. Finally, the empirical astronomical data available in the 4th c. BCE suggested that the heavenly bodies were the fastest observable things.

Aristotle explicitly argues in *DC* 2.4 287a23-30 that the heavens' motion is the fastest of all changes and uses this thesis in one of a series of arguments for the sphericity of the cosmos.[30] He argues that (1) the heavens are the measure of all changes since their motion is uniquely continuous, unvarying and eternal, (2) in each class of things the measure is its least member, (3) the quickest change is the least [of all changes], therefore (4) the heavens are the quickest of all changes.[31] The difficulties in this argument betray deeper problems in Aristotle's theory

[29] 'The comparison wrongly implies that the same *sort* of thing could engage in both types of locomotion' (Wardy 1990, 271).
[30] The thesis is also mentioned at *Met*. 10.1 1053a8-12.
[31] δῆλον ὅτι ταχίστη ἂν εἴη πασῶν τῶν κινήσεων ἡ τοῦ οὐρανοῦ κίνησις (287a23-30). The argument continues: the condition that the heavens are the fastest is best fulfilled if they enclose the cosmos with the shortest possible path. This seems to be a form of the isoperimetric problem, later formally solved by Zenodorus (Heath 1949, 171). For premise (1) of the argument cf. *Phys*. 4.14 223b12-21 on the heavens as the measure of all change.

of measurement as applied to motion.[32] Yet whatever force the argument has clearly draws on the plausibility of (4) to Aristotle and his audience. It is supported by his observation elsewhere that recent astronomical research has shown the earth's size to be vanishingly small in the cosmos as a whole.[33]

It appears that in *Phys.* 7.4 Aristotle ultimately must deny that the heavens' motion is the fastest of all, since he denies that circular and rectilinear motions are comparable without naming any exceptions. But the uphill-downhill analogy appears at the point in the chapter where that wholesale denial is the thesis to be established and hence cannot possibly serve as his reason for dismissing the hypothesis of necessary inequality as ἄτοπον. *DC* 2.4 is hard to reconcile with any interpretation of *Phys.* 7.4, but especially one that claims Aristotle is particularly concerned with the heavens.[34] If Aristotle were directly to contradict *DC* 2.4's claim, he should offer reasons against it, not dismiss it out of hand as ἄτοπον.

Although in *DC* 2.4 Aristotle here speaks loosely of ἡ τοῦ οὐρανοῦ φορὰ (287a23), he presumably means the motion of the sphere of fixed stars. However it is clear that the lower spheres of the heavens also move faster than anything on earth. For Aristotle the Moon is the lowest and slowest part of the heavens. A quick calculation of even a lower estimate of its speed from early Greek data shows that it moves very fast indeed. The following rough calculation could easily have been performed by Aristotle, but my intention is not to reconstruct an Aristotelian calculation for the Moon's speed. We have no indication that he ever personally studied it. I only wish to illustrate that anyone who wanted could have shown that the Moon moves much faster than anything in our immediate surroundings.

Aristarchus of Samos (fl. c. 280 BCE) offered the surprising underestimate of the Moon's distance at around twenty earth radii.[35] It follows that the Moon's circular orbit is twenty

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[32] See Sattler 2017.

[33] *Mete.* 1.3 339b6-9: ὁ μὲν γὰρ δὴ τῆς γῆς ὄγκος πηλίκος ἄν τις εἴη πρὸς τὰ περιέχοντα μεγέθη, οὐκ ἄδηλον· ἤδη γὰρ ὦπται διὰ τῶν ἀστρολογικῶν θεωρημάτων ἡμῖν ὅτι πολὺ καὶ τῶν ἄστρων ἐνίων ἐλάττων ἐστίν. ('For there is no doubt about the relative size of the earth and of the masses which surround it, as astronomical researches have now made it clear that the earth is far smaller even than some of the stars'), *De Caelo* 2.14 297b23-30.

[34] My suggestion – that the circular motion of interest in *Phys*. 7.4 is the rolling motion exhibited by wheels – may dissolve this apparent contradiction, since there are no rolling motions in the heavens.

[35] Admittedly, earlier thinkers of the sixth and fifth centuries may have located the Moon closer to earth. Among the most important testimonies to early Greek estimates of the Moon's distance, see Plutarch *De Facie* 925A-D, Aetius 2.31 (a chapter titled Περὶ τῶν ἀποστημάτων), ps.-Plutarch *Placita* 2.20-25, with Mansfeld 2000. Anaximander apparently held that the Moon's wheel is 19 times greater than the earth (DK12 A22). It is unclear

times larger than the earth's circumference.[36] Since the Moon completes this orbit approximately once a day, it follows that in one day the Moon travels a distance roughly equivalent to circumnavigating the earth twenty times over.[37] And this is a lower bound. Aristarchus' figures in *On the Sizes and Distances* seem deliberately inaccurate. He assumes that the Moon's apparent diameter is 2°, much larger than the actual value of about ½°. A smaller and more accurate value of the Moon's apparent diameter would increase the value of its distance and hence any estimate of its speed. Aristarchus probably did not truly believe the Moon's apparent diameter was 2° and Archimedes reports that he used the value ½°. The incorrect value in *On the Sizes and Distances* may have been adopted either to emphasise the power of deductive geometrical reasoning over physical facts or to argue that the Sun's illumination in excess of a lunar hemisphere is imperceptible.[38]

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whether this is an estimate of the wheel's radius, diameter, circumference, or some other property (Thibodeau 2017, 97-101 surveys the options, arguing that the estimates 'are simply circular measures of the hoops given as multiples of the earth-sized discs which move along them, measuring as they go', noting that this would set the wheel's distance at roughly 5 earth radii). Anaxagoras estimated that the Moon was the size of the Peloponnese (see Graham 2013 for a recent reconstruction of Anaxagoras' method; assuming an apparent diameter of ½° and a true diameter of 100 km, this would give a lunar distance of over 10,000 km, an orbit of over 70,000 km, and hence a speed of just under 3,000 km per hour). Empedocles apparently held that the Moon's distance from the Sun was twice its distance from the earth. There is no space here to engage in the complex issues that arise from reports of Presocratic views on the sizes and distances of heavenly bodies.

[36] This value is close to one interpretation of the testimony for Anaximander discussed in the previous note, but this may be coincidental.

[37] Aristotle reports some unnamed mathematicians' estimate of the earth's circumference as 400,000 stades (*DC* 2.14 298a15-17). Together with Aristarchus' estimate, this would put the Moon's orbit at around 8,000,000 stades and hence its speed 300,000 stades per hour and over 90 stades per second. Assuming that Aristotle used the Attic foot and stade, this is equivalent to over 55,000 km per hour and over 16 km per second.

[38] Lloyd 1973, 56-57, Van Helden 1985, 8. Some of my objections to Wardy's version of the cosmological interpretation have been anticipated by Benedetti's discussion in *Diversarum speculationum mathematicarum et physicarum liber* (Turin, 1585), capitulum xxxv; translation from Drake and Drabkin 1969, 220-221.: 'But if Aristotle had said that the circular motion of the heavenly bodies was not comparable to the rectilinear motion of the [four] elementary bodies, he would have been right – not because one of these motions is circular and the other rectilinear, but because the celestial motion is regular, not sometimes slow and sometimes fast, but always maintaining one and the same speed, whereas the contrary is true of the motion of the [four] elementary bodies. And a further reason [he might have given] is that there never has been nor will there ever be any of these natural rectilinear motions, as they are called, as swift as the motion of the heaven. For if we wish to consider the diurnal motion of 24 hours, according to the general view, we shall find by calculation that the Moon in quadratures with the Sun, when it is at the equator, moves 500 Italian miles or thereabouts per minute…' Benedetti's counterfactual (*si… dixisset*) indicates his suspicion that the cosmological interpretation does not represent Aristotle's intentions. His arguments are similar to those I have just made, although he interestingly takes them to be *in favour* of the cosmological interpretation since he considers a variant which asserts that heavenly and sublunary motions cannot be equal in speed, or cannot be equal in speed for any extended duration.

7.5: The Wheel Interpretation

I now propose a new interpretation of *Phys*. 7.4 based on the *Rota* paradox studied in Chapter 6. I suggest that Aristotle may have approached this puzzle in *Phys.* 7.4 by distinguishing the circular and rectilinear components of the wheel's rolling motion and arguing that an intolerable paradox follows if we admit the comparison of the component motions' speeds. In contrast to the cosmological interpretation, I suggest that Aristotle is primarily concerned with terrestrial phenomena; he rejects the comparability of circular and rectilinear motions on pain of paradox. It is historically plausible that Aristotle was aware of the Wheel since it is discussed at length in *Mech.* and I have already given reasons for thinking that *Mech.*'s discussion was not the first. Furthermore, this paradox is precisely the kind of topic we should expect to find here, between the Zenonian paradoxes in *Physics* 6.9 and the anonymous further puzzles of motion in 8.8. One may well find the following analysis in terms of speeds an unsatisfactory resolution of the paradox, but this does not invalidate the interpretation since brilliant minds have presented dozens of unsatisfactory analyses of this challenging puzzle.[39]

The opening argument of *Phys*. 7.4 begins with two premises:

I. Two things move with the same speed if and only if they move an equal distance in an equal time (248a11-12: ὁμοταχὲς τὸ ἐν ἴσῳ χρόνῳ ἴσον κινούμενον)
II. Every change is comparable with every other (248a11: ἐστιν πᾶσα συμβλητή)

Aristotle accepts (I) as true, perhaps self-evidently so.[40] Aristotle says that, from these premises, there follows the conclusion, which I leave untranslated for now:

I. ἔσται περιφερής τις ἴση εὐθείᾳ καὶ μείζων δὴ καὶ ἐλάττων (248a13-14)

This is false, so Aristotle concludes that premise (II) must be false. From the present point of view, the περιφερής in question is the wheel's circumference and the εὐθεῖα is its path. So (III) would follow from the premises if the speeds of the component rotation and translation of the wheel's rolling are taken to be comparable. But how should we understand the Greek of the false conclusion (III)? There are two possible readings:

[39] See Drabkin 1950.
[40] Premise (1) is stated in *Physics* 6.2 without argument. We have already seen that this premise plays a vital role in Hero's presentation of the paradox and is also present in *Mech.*'s.

IIIa. There will be a circumference equal to or longer than or shorter than a straight line.

IIIb. There will be a circumference equal to and longer than and shorter than a straight line.

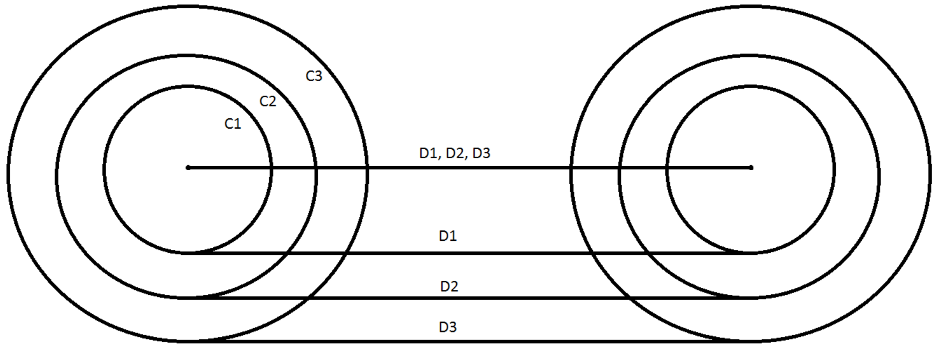
Most translators have taken the καὶ… καὶ construction distributively, along the lines of (IIIa).[41] On this reading Aristotle objects to the comparability of circular and rectilinear motions and lines because of the puzzling phenomena of the *Rota* paradox. Although the (IIIa) reading does little to mitigate the mathematical oddity of the chapter, and is consistent with the construction and measurement interpretations, it does provide at least motivation for the startling claim that different species of line cannot be compared in length.

However, (III)'s καὶ… καὶ construction can also be taken aggregately, as in (IIIb). On the face of it, (IIIb) seems to offer a more compelling reason for Aristotle's rejection of premise (II), since it involves a clear contradiction and is uncontroversially false. By contrast it strikes many readers mathematically incompetent to judge (IIIa) false. We have seen above the difficulties faced by interpretations which would ascribe this view to Aristotle. Presumably the reason for not adopting the conjunctive reading has been a failure to see how it could follow from premises (I) and (II). Such an argument is possible on the wheel interpretation.

Let there be three unequal concentric circles C1, C2, C3 such that C1 < C2 < C3, rigidly fixed about their common centre. Whenever a circle rolls smoothly for one revolution, its centre traces out a path equal to its circumference because the rotation and translation components of motion are equal in speed; these are instances of premises (I)-(II). So when C1 'drives' the motion, its centre traces out path D1 in one revolution and, since the rolling movement is smooth and doesn't involve jumps or stops, D1 is equal to C1's circumference. Likewise for C2, C3 and paths D2, D3. Therefore D1 < D2 < D3. But by (I), the same thing travelling with the same speed must cover the same distance. So all paths traced by the centre in one revolution of the system of concentric circles are equal. Therefore D1 = D2 = D3. Therefore C1 = C2 = C3. So C2's circumference equals D1, D2 and D3. So a circumference

[41] 'we may have a circumference equal to a straight line, or, of course, the one may be greater or less than the other'(Hardie and Gaye), 'a circular distance may be equal to a rectilinear distance, or greater or smaller' (Wicksteed and Cornford), 'the circumference of a circle will be equal to, or longer or shorter than, a straight line' (Waterfield).

(C2) is equal to, greater than and less that a straight line (D2), which is the manifestly absurd conclusion (IIIb). I have introduced a third circle for purposes of exposition, but it is strictly superfluous.



Here is the argument with two circles only:

(1) C1 < C2.
(2) D1 = C1.
(3) D2 = C2.
(4) Therefore D1 < D2 (by (1)-(3)).
(5) But D1 = D2 (by I).
(6) So C1 = C2 (by (2), (3), (5)).
(7) Then C2 < D2 (by (2), (4), (6)).
(8) And C2 > D2 (by (5), (4), (6)).
(9) It follows that C2 is equal to, greater than, and less than D2 (by (4), (7), (8)).

This interpretation allows us to make better sense of Aristotle's uphill-downhill analogy. Let us first emphasise that the words κάταντες and ἄναντες mean specifically 'downhill' and 'uphill' respectively, not 'up' and 'down'. Aristotle consistently uses κάτω and ἄνω to describe the vertical movements of the sublunary simple bodies across his works. His choice of words suggests that he wishes to direct our attention to something else: bodies travelling up and down inclined surfaces.[42] I assume that Aristotle confines his attention to heavy bodies, since 'uphill' and 'downhill' are barely intelligible for the motions of light bodies such as fire and air.[43]

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[42] Compare *Physics* 3.3 and *Met.* 11.9 for the use of κάταντες and ἄναντες, also *Problems* 26.36 which observes that water flows more quickly downhill than over planes.
[43] One could channel smoke through an inclined pipe, but it would be strange to call this uphill motion.

It is false that *anything* moving downhill must be faster than any other thing moving uphill; the tortoise crawling down the hill is slower than the hare bounding up the other side. The uphill-downhill analogy makes sense only if we read in certain assumptions. Moving a heavy body uphill requires more effort than sending it downhill. One might therefore suggest that Aristotle is thinking of a single agent moving the same heavy object uphill and downhill (or at least objects moving of the same or similar weight).

From this point of view, the analogy implies that the circular and rectilinear motions in question are likewise moved by the same power. That is false on the cosmological interpretation, since Aristotle believe that infinite power is required to move the heavens and infinite powers are not found in the sublunary realm.[44] On the other hand, Aristotle might have thought that the rotational and rectilinear components of rolling motion are moved by the same power. As I argued in Chapter 6, *Mech.*'s author makes a similar assumption.[45] Thus it would be ἄτοπον for one of the component motions to be necessarily faster or slower. It would indeed seem strange if a cart-wheel spun round much faster than it proceeded horizontally when both component motions are caused by the same straining horse. Nothing we observe suggests this is the case. If we took the trouble to measure a wheel's circumference and the path traversed in one rotation we would find them roughly equal. Even if there were some deviations from equality in our rough-and-ready measurements we should not conclude that one is necessarily faster than the other.[46]

This interpretation has a further appealing feature. Since, in the scenario of rolling concentric circles, different circular motions correspond to the same rectilinear motion, it turns out that the different circular motions themselves cannot be coherently compared. We begin assuming that they are unequal, but then deduce that they must also be equal. On the interpretation I am suggesting, Aristotle is only able to uphold the difference between unequal circular motions by severing this connection to rectilinear motion.

[44] *De Motu Animalium* 3-4 argues at length that the world cannot contain a *dunamis* as great as that which moves the heavens; therefore the mover of the heavens must be outside the world. Cf. *Phys*. 266a24-b6's argument that there cannot be an infinite power in a finite magnitude.

[45] *Mech.* 24, 855b28-856a32.

[46] Again, contrast the case of the cosmological interpretation where, as we have seen, basic data for estimating the Moon's distance force us to admit that the Moon must move faster than sublunary bodies rise or fall.

7.6: Conclusion

The study of relatively peripheral works in the Aristotelian corpus may shed light on more central works such as the *Physics*. In this chapter, I have suggested that *Physics* 7.4's surprising denial of the comparability of straight and circular motions may have been a response to an earlier statement of the paradox discussed by *Mech*. 24. This suggestion is somewhat speculative, since there is no direct reference to rolling in *Phys*. 7.4, but in this regard it is not much worse off than the alternative interpretations which go beyond the text to explain what drove Aristotle's denial of comparability. I have also identified some problems for three leading alternative interpretations.

It may be objected that (a) *Mech*. 24 does not draw the paradoxical consequence that unequal circles are equal, or that unequal circles move equally quickly, and (b) *Mech*. 24 does not analyse rolling as a complex of straight and circular motions. However, both the paradoxical consequence and the analysis of rolling into two component motions are explicit in Hero's discussion of the paradox. Hero often drew on older sources in writing his texts. It is possible that Hero's discussion of the *Rota* paradox derives from an earlier presentation than that of *Mech*. I noted in §5.5 that Hero's statement of the paradox is more clearly written, and its relevance to the Rotating Radius Principle, which may be the reason why the puzzle features in *Mech.*, is more obvious. Drabkin (1950) suggested that *Mech*. 24 restated an older puzzle. If an earlier version resembled Hero's, then Aristotle may have had a text in front of him that analysed rolling into two components and drew the paradoxical consequence that unequal circles are equal, or that unequal circles move equally quickly. Obviously such speculations go far beyond the evidence.

A final question must be raised: If Aristotle had treated the paradox, why did *Mech*. return to it? Some commentators have suggested that *Mech*. problem 1 implicitly rejects *Phys*. 7.4's doctrine in its analysis of motion on a circular path into rectilinear components.[47] However, I do not see *Mech*. problem 1 comparing a straight motion to a circular motion in terms of faster, slower and equally quick, so I do not think *Mech.*'s author would necessarily reject *Phys*. 7.4's arguments on that score. What we should note is that the two puzzles *Mech*. 24 addresses are not the same as the puzzle addressed by Aristotle (according to my suggestion) and Hero. The author of *Mech*. can thus be seen as addressing different aspects of the

[47] Owen 1970, 256; Knorr 1982a, 101n.27.

paradoxical phenomenon, rather than revising what may have been Aristotle's solution. As is clear from Drabkin's 1950 article, and as I indicated in Chapter 6, the phenomenon described by *Mech*. 24 does not of itself determine a single puzzle or a single question. There are various potential puzzles and paradoxes lurking in it. It may be that once Aristotle had addressed one paradox, *Mech*.'s author addressed two further puzzles.

Chapter 8: Conclusion

8.1: Summary of the argument

I have argued that the *Mechanica* was an application of natural philosophy to the technical sphere of mechanics. The author offers causal explanations of surprising phenomena presented by machines. These explanations ultimately depend on problem 1's analysis of the Rotating Radius Principle. This principle arises because of the manner in which two motions occur simultaneously in one thing. That is a claim that may strike us as strange. On several modern views of what motion is, a body cannot literally have several simultaneous motions. I have shown that, by contrast, this idea makes sense for Aristotle, who understood change as the actuality of a potentiality, *qua* such. The two motions in the rotating radius are not mathematical fictions, but real processes, κινήσεις. Thus I distinguished radial rotation from circular motion both theoretically as well as terminologically and suggested that *Mech.*'s account of radial rotation should not be applied to heavenly circular motions. I also argued that the notion of constraint has a more central role in problem 1's argument than previously recognised, since it is likely constraint that accounts for the proportionality between tangential and radial motion.

In accounting for a broad variety of devices beyond the paradigms of the balance and lever, *Mech.*'s explanations rely on the identification of functionally similar parts of seemingly dissimilar devices. In this reliance on analogical arguments, *Mech.* is closer to Aristotle's zoological inquiries than to deductive geometry. The use of diagrams to support these analogies is one example of the non-deductive use of lettered diagrams in ancient Greek science.

My reading of *Mech.* 24 showed another way in which *Mech.*'s explanatory project is physical rather than purely mathematical. In *Mech.* 24, a paradox challenges the Rotating Radius Principle. I showed that the author answers the paradox by appealing to physical principles. This further substantiates my broader argument that *Mech.* is not so much a mathematical work as an application of natural philosophy to the technical sphere of mechanics.

Finally, I suggested that Aristotle's claim that straight and circular motions are incomparable (*Phys*. 7.4) may have been an attempt to escape the contradictions of an earlier statement of *Mech*. 24's paradox, perhaps a version of the paradox closer to that in Hero's *Mechanica*. This suggestion is tentative. We do not know what motivated Aristotle in *Phys*. 7.4. I criticised three previous interpretations.

To its modern admirers, *Mech.*'s analysis of what I have termed rotating radial motion is a triumph of ancient science, 'almost Newtonian', as G.E.L. Owen put it. Strangely, no later ancient writers whose works have survived used or acknowledged this analysis, even those who echo other aspects of *Mech*.'s approach. For example, Vitruvius (*De Architectura* 10) offers similar analogical explanations and claims that 'circular motion' is the basis of mechanical phenomena. Yet he does not analyse circular motion or suggest that it is anything other than simple, and instead compares rotations in machines to the heavenly rotations. Similarly, Hero of Alexandria deploys analogical explanations and bases his account of the simple machines on circular motion. Although he presents some basic ideas about the composition of motions, circular motion is not analysed into two components. A ninth-century Arabic paraphrase of the first part of *Mech*. left out the analysis of rotating radial motion. What moderns find most praiseworthy, the ancients found least worth preserving. This calls for explanation.

Mech.'s analysis of radial rotation was vitiated by a lack of clarity on some fundamental issues, which I outlined in Chapter 4. It was difficult to see how the ever-changing relation between the tangential and radial motions should be precisely characterised. How can they be constantly changing? The circle is symmetrical and from that point of view it seems that the same characterisation of the radius' components should hold at every instant. Moreover, it was unclear how to reconcile the tension between the motions' characterisation as radial and tangential and their representations in problem 1's diagram. It is partly the *flexibility* of the Aristotelian notion of motion (*kinesis*) that allowed the component analysis to be developed. What we call velocities and what we call accelerations could sometimes fall under what Aristotle called 'motions'. But the analysis leads to a dead end without, at the very least, a well worked-out mathematical theory of acceleration.[1]

[1] When, with the benefit of such a theory, circular motion was re-analysed in the seventeenth century (yielding our familiar formula $a = v^2/R$) none of the parties involved mentioned Aristotle. On circular motion in this later period, see Westfall 1972; Meli 2006, ch.7.

For all its shortcomings, *Mech.*'s analysis may have been valuable in part because it showed how apparently anomalous phenomena could be incorporated within a broader cosmology. Circular motion belongs to the heavens. Below the Moon, inanimate matter naturally rises and falls, and animals use their muscles to push and pull. Within such a framework, explaining how a heavy, solid rod could produce motion on a circular path was a worthwhile task, especially since radial rotation was responsible for many striking effects. Vitruvius, Hero, and the Arabic translator did not share the project of integrating mechanics within a presupposed Aristotelian physical and cosmological framework.

8.2: Postscript

Tartaglia's *Quesiti*, Book 7 opens with the following exchange with Don Diego Hurtado de Mendoza, Charles V's ambassador at Venice:

> *Mendoza:* Tartaglia, since we took a vacation from the reading of Euclid, I have found some new things relating to mathematics.
> *Tartaglia:* And what has your Excellency found?
> *Mendoza:* Aristotle's *Questions of Mechanics* in Greek and in Latin.
> *Tartaglia:* It is quite a while since I saw these, particularly the Latin.
> *Mendoza:* What did you think of them?
> *Tartaglia:* They are very good, and certainly most subtle and profound in learning.
> *Mendoza:* I, too, have run through them and I understood most of them; yet many questions remained with me, which I should like to have more fully explained.[2]

There are still many unanswered questions about the *Mechanica*. This thesis has focussed on some central questions about the text's theory and method. In this conclusion I have indicated some possible implications for research in history and philosophy of science more generally.

[2] trans. Drake and Drabkin 1963, 104

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