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Abstract

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1 Introduction

Someone in 1980s America might have interacted with two dozen different companies in the course of a typical day. In the near future, however, it is not unthinkable for a young person to wake up in an Amazon-sourced apartment[1], check the news on her Facebook Feed and then hail a Google-operated self-driving car on her Apple iPhone to pick up fresh groceries at an Amazon Supermarket. She might then meet some friends for lunch – paid for with Apple Pay – before working from home on her Apple Macbook, collaborating with her co-workers via Google Sheets or on a server hosted on Amazon Web Services. In the evening she might order-in dinner via Deliveroo[2], chat with her parents over Microsoft's Skype and then unwind over in-house content on Amazon Prime, or with a good book on her Amazon Kindle.

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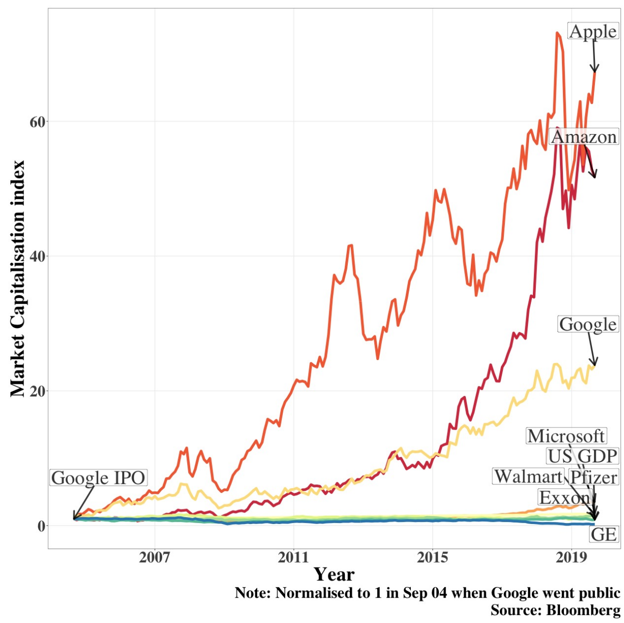
[1] See e.g. https://www.forbes.com/sites/alyyale/2019/07/23/amazon-enters-the-real-estate-game-launches-smart-tech-heavy-homebuying-program/

[2] Deliveroo is a popular food delivery company for which Amazon is the lead investor.

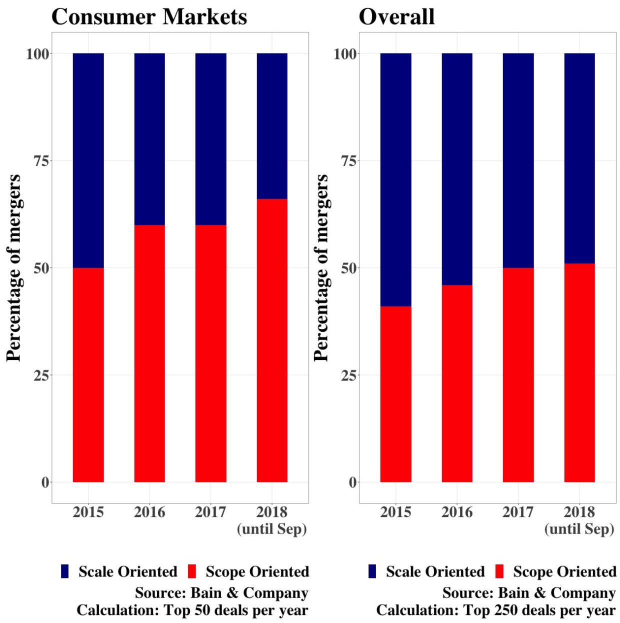
The expansion of internet firms (Amazon, Apple, Facebook, Google and Microsoft) into new markets[3] has been accompanied by their exponential growth far outstripping GDP (see Figure 1 (a)).[4] Apple is now valued at nearly 5% of the US's $20.5 trillion GDP, a far larger proportion than AT&T and Standard Oil were at their peeks (Temin and Galambos, 1987).[5] Acquisitions underlie all this. The activity of internet firms in this regard is so pronounced that it has changed the overall trends of M&A activity and the exit model for startups. In the US, more than 80% of all successful venture capital liquidation events are now through the M&A route compared to about 40% in the 1990s (Thomson Financial/National Venture Capital Association), while Bain & Company, a management consultancy, note that 2018 was the first year global M&A activity was dominated by 'scope' deals (taking firms into new lines of businesses) rather than 'scale' deals (allowing firms to build market power in markets they already operate in) as shown in Figure 1 (b).[6]

Figure 1: Growth of market capitalization and Scope oriented M&As

Notes: Panel (a) plots the quarterly market capitalization of Amazon, Apple, Google, Microsoft, Exxon Mobil, Walmart, Pfizer, General Electric, as well as US GDP. The numbers are normalized to 1 in September 2004 when Google went public. Panel (b) shows a trend of scope-oriented M&As (the lower bar in red) over 2015-2018.



(a) Market capitalization since September 2004



(b) Scope oriented M&As on the rise

These observations naturally motivate a series of salient questions: Will internet firms continue to grow and develop a presence in ever more markets? Are such conglomerates an inevitable

[3]See, for example, https://www.wsj.com/articles/amazon-is-leading-techs-takeover-of-america-1497653164

[4]Within a larger category, the Fortune 100 (the 100 companies with the highest revenue) have also seen their proportion of nominal GDP rise from about 33 percent in 1994 to 46 percent in 2013. See e.g. https://fivethirtyeight.com/features/big-business-is-getting-bigger/

[5]Also see e.g. https://www.businessinsider.com/americas-most-valuable-companies-over-the-last-100-years-2017-11

[6]Used with permission from Bain & Company. See e.g. https://www.bain.com/insights/M-and-A-in-disruption-2018-in-review/

feature of modern industrial organization? In this paper, we provide a framework for thinking systematically about modern industrial structure, and show how it can be applied to uncover some of the key underlying forces shaping modern industry. Our results suggest that as technological advances precipitate fundamental changes in markets, these can serve as an impetus for large conglomerates to suddenly emerge. Under stronger, but still natural assumptions, this emergence is not merely a possibility, it is inevitable—suggesting such conglomerates are here to stay.

The foundations for our work come from the resource-based view of competitive advantage in the management literature, pioneered by Wernerfelt (1984).[7] The fundamental idea is that different firms have different immutable and scarce resources or core competencies, and it is these competencies that deliver competitive advantage and profits. Such competencies might include, for example, unique forms of human capital, production know-how, patents, a strong brand value, and so on. In such models, the immutability of such competencies is crucial. Otherwise, competitors would develop those they are missing and then any competitive advantage would be short-lived. We endow firms with competencies and let these be reorganized through *mergers*, *demergers*, *disposals*, *procurements*, and *entries*.[8] Our interest are the *stable industry structures*, in which there are no profitable demergers, disposals, procurements or mergers, and how they depend on the environment.

While a given competency will deliver competitive advantage for a firm in some markets, it will not be valued by all markets. For example, a team of molecular biologists might provide a firm with a competitive advantage in biotech markets, but they are unlikely to be a source of competitive advantage for the firm in markets for financial services. The competitiveness of a firm in a given market depends on its *relevant competencies*—the set of competencies which it both possesses, and are valued by the market.

Viewed through the lens of a resource-based view of competitive advantage, the recent acquisition of Whole Foods by Amazon for \$13.7B would have made little sense twenty years ago—Amazon and Whole Foods would have had little opportunity to gain competitive advantage by combining their competencies. This is no longer the case. The merger has allowed Amazon to combine its digital and e-commerce competencies with the Whole Foods brand and its network of stores. Amazon's proprietary data on the shopping habits and interests of Whole Foods customers can help adverts and offers be better targeted (as is now standard for grocery retailers), while its vast logistical and distribution network are valuable for offering online grocery shopping. More broadly, it seems that a myriad of markets are beginning to value some of the same competencies, ones often held by internet firms. For instance, the automobile industry, in its race towards driverless cars, has begun to prize computer science

[7] This is one of the two preeminent theories. The other is due to Porter (2008). The resource-based view of competitive advantage has been deployed very widely in the management literature. For example, it has been used to explain partnerships in the oil and gas industry (Garcia et al., 2014), acquisitions in the nanotechnology industry (Youtie and Kay, 2014), and so on.

[8] We will use the term 'procure' to refer to cases where unassigned competencies are taken on by a firm. This is as opposed to the term 'acquire', which we use interchangeably with 'merger' – this preserves the conventional usage of these words e.g. 'mergers and acquisitions' typically refers to a process between two separate firms.

talent to complement its engineering mainstays. Such cars pose an immense technical hurdle: they require both accurate and near-instantaneous image recognition, as well as sophisticated decision making systems as they interact and adapt to human drivers and pedestrians. Unsurprisingly, these systems are trained on enormous amounts of data with any array of artificial intelligence techniques which, in turn, has given internet companies a valuable competitive advantage. Google[9], for instance, has leveraged its machine learning competencies from Google Brain (the company's AI research team), and DeepMind (which it acquired in 2014) to emerge as a leader in the industry—it is the first company with driverless cars operating on public roads. Other internet conglomerates are not far behind: Apple[10], Amazon[11], and Baidu[12] have all emerged as competitors in the industry. Although the case of driverless cars is particularly acute, other such examples abound: wearable technology and textiles; household appliances connected to the internet; smartphones and payments; offices and cloud computing; the list goes on.

Can changes in technology that lead markets to value more of the same competencies account for the emergence of internet conglomerates? To make progress with the problem we represent the competencies that firms have and the competencies markets value as a pair of hypergraphs[13]. The question of what the set of stable industry structures looks like then becomes a question of which firm hypergraphs are stable for a given market hypergraph. Representing the problem as one of hypergraph formation allows us to draw on an extensive body of work in graph theory, from which we use both concepts and results. We then provide a rudimentary theory of how industry structures change in response to such changes in the connectedness of markets.

We first find an upper bound on the size of the largest firm in all stable industry structures. This bound holds very generally and captures the received wisdom from the finance and management literature that businesses should focus on their core-competencies and not enter unrelated markets. As market connectivity increases, this bound tends to change abruptly and even small changes in connectivity can drive severe increases in the upper bound. We demonstrate this by drawing on results from the random hypergraph literature.

We then show that when the cost of maintaining competencies is not too convex, the upper bound is not just hypothetical, but *tight*—we show by construction there always exists a stable industry structure in which the size of the largest firm is equal to the upper bound.

A fundamental force driving the sudden transition in the size of the largest firm is the phe-

[9]Waymo, a leader in the self-driving car market, originally started within Google and remains under the umbrella of Google's parent company, Alphabet.

[10]Apple recently acquired Drive.ai, a self-driving car startup, which seems to lend some credence to the much-circulated rumour that Apple has been working on developing its own self-driving cars for some time.

[11]Amazon has made numerous forays into the field of driverless vehicles. It is running public trials with autonomous delivery robots and is collaborating with Toyota working on developing multi-functional vehicles which might move goods and people.

[12]Baidu, a Chinese internet conglomerate, recently revealed its fleet of driverless vehicles has collectively clocked 1.2 million miles autonomously in urban environments across 13 cities in China.

[13]Hypergraphs are generalisations of graphs in which, instead of just being a pair of nodes, an edge can be formed from any subset of the nodes.

nomenon that mergers can beget further mergers. For example, many of Google's more recent acquisitions might not have been profitable if not for its preceding ones: if, for instance, it was still narrowly focused on providing a search engine, then the billion dollar acquisition of Waze in 2013, a GPS navigation software app, might not have been a shrewd business decision. However, Google had by then already acquired Zip Dash, Where2 and Keyhole Inc (all in 2004) to develop Google Maps. The acquisition of Waze thus allowed Google to further augment the existing competencies it had already developed to strengthen its foothold on the mapping business. This example underscores the intuition for the phase transition result: there is a key connectivity threshold for markets (based on the competencies they jointly value) which, once passed, allows merger opportunities to cascade leading to "giant" firms.[14]

Finally, under a stronger complementarity assumption on holding more relevant competencies for a given market, specifically supermodularity in the profit functions, we find a lower bound on the size of the largest firm in all stable industry structures. This lower bound becomes tighter as more capabilities (or capabilities valued by more markets) become scarce. Even with just a small number of scarce capabilities, our upper and lower bounds can coincide.

While it seems intuitive that as markets become more connected, new synergies can emerge which in turn drive growth in firm size, this conclusion is by no means forgone. In particular, we let firms have a convex cost of maintaining competencies, so that the marginal cost of maintaining an additional competency increases as a firm gets larger and maintains more competencies. Further, new synergies can affect the relative competitiveness of firms in affected markets. Indeed, we show by example, that it is possible for all firms to get smaller (hold fewer competencies) as markets become more connected due to these competitive effects. At the same time, antitrust authorities constrain merger activity.[15] The fact that large firms can and must emerge in spite of these countervailing forces is somewhat surprising, and suggests that the forces we have identified are fundamental.

1.1 Related Literature

We now place our work in the broad context of several related literatures, but reserve discussions and specific comparisons to relevant portions of our paper.

Hypergraphs are a generalization of the networks used in the networked-markets literature (e.g., Kranton and Minehart, 2001; Elliott, 2015; Nava, 2015; Condorelli et al., 2016; Bimpikis et al., 2019).[16] In this literature, the network encodes a constraint set on who can transact with whom. Our hypergraph approach generalizes this by capturing how competitive different firms are in different markets. In terms of competition across markets, the closest papers are Nava (2015) and Bimpikis et al. (2019) which both model Cournot competition. In terms

[14]We use the term 'giant' here because of the link with the technical term "giant component" from the random hypergraph and random graph literatures.

[15]We also include the possibility that a merger which reduces competition in a market can reduce the profits of the merging firms. This is a feature of Cournot competition known as the Cournot paradox. We make assumptions directly on profit functions that nest Cournot competition as a special case.

[16]See Goyal (2017) for a survey.

of network/hypergraph formation, the most closely related papers are Kranton and Minehart (2001) and Elliott (2015). In these papers, however, access is provided by specific investments rather than through mergers to acquire competencies as in our paper.[17] Despite these analogies, the competency-based approach we take in this paper allows us to employ new techniques to ask different questions—we consider industry structure across the economy in a way that the networked-markets literature has not previously done.

To the best of our knowledge, we are the first to utilise hypergraphs to model the competencies firms have and the competencies markets value. There are only a few papers in the economics literature which use hypergraphs.[18] The closest to our paper is Malamud and Rostek (2017) who also use hypergraphs to represent markets. They model financial exchanges, and competition across them, as a hypergraph. However, both their approach to modelling markets, and the questions they address are markedly different. Apart from the hypergraph approach, there is also relatively little work in economics (in contrast to the management literature) that thinks about firms or markets as being endowed with differing sets of competencies. Four exceptions are Nocke and Yeaple (2007), Nocke and Yeaple (2014), Sutton (2012) and Goyal et al. (2008). Goyal et al. (2008) study R&D collaborations while Sutton (2012) maps firms' competencies to countries' wealth, and uses this as the basis to study the economics of globalization. Also in an international trade setting, Nocke and Yeaple (2007, 2014) analyze the role of firm heterogeneity in issues such as cross-border mergers and acquisitions, and the international organization of production. In their setting, firms can have two types of competencies, and their endowments of these affect the trade-offs they face when confronted with various decisions. They find that this heterogeneity can explain several correlations in the international trade data. There is also a theory of the firm literature that studies the investment incentives created by different distributions of asset ownership, where assets might be interpreted broadly in a way analogous to our competencies (e.g., Grossman and Hart, 1986; Hart and Moore, 1990). While the focus of these papers is on investment incentives and the provision of costly effort, there is no effort choice in our model.[19] Finally, an empirical literature on hedonic utilities uses data to identify the characteristics of firms that matter for a range of outcomes (e.g., Bartik, 1987).

The central questions we pose on the emergence of internet conglomerates are contemporary, though the analysis of industry structure goes back to at least Chandler (1962). Chandler was motivated by the emergence of large corporations and argued that strategic growth arose from an awareness of opportunities—created by changing population, income and technology—and the need to employ existing or expanding resources more profitably. We build on his work, incorporating key forces he identified such as economies of scale. The proliferation of conglomeratization in the late 1960s and early 1970s also inspired substantial work. Explanations for

[17]Akerlof and Holden (2016) can also be viewed in this context. They study a networked market for financing investment projects and find that some managers, whom they term as "movers and shakers" endogenously become central.

[18]Hypergraphs have mainly been used to represent communication structures among agents and to study the coalitions that form (see e.g., Myerson, 1980; van den Nouweland et al., 1992; Slikker et al., 2000). Dziubiński and Goyal (2017) use hypergraphs to study attack and defense networks.

[19]Our theory is not, nor is it intended to be, a theory of the firm. We do not explain what is made within a firm and what is bought in through the market from suppliers.

conglomeratization identified in this literature include the overconfidence of managers (Roll, 1986), managers empire-building and pursuing other personal objectives (Jensen, 1986; Shleifer and Vishny, 1989; Morck et al., 1990), growth maximisation (Mueller, 1969), risk reduction for managers (Amihud and Lev, 1981), synergies in management and production (Matsusaka, 1993), transaction costs minimization (Teece, 1982), and the financial benefits of "winner-picking" across industries (Stein, 1997). Our approach emphasizes the value of conglomerate mergers as a means of acquiring complementary competencies to augment competitiveness in different markets. We do not claim that this was a major factor in earlier periods of conglomeratization, but do think it is applicable to the emergence of internet conglomerates. This is consistent with more recent work on conglomerates. Using text-based analysis of product descriptions, Hoberg and Phillips (2017) find that conglomerates operate in industries with related products.[20]

Our work also relates to a rich literature in industrial organization on mergers and their regulation. Merger reviews typically focus on the trade-off between the emergence of market power, and potential efficiency gains (Williamson, 1968; Chatterjee, 1986; Larsson and Finkelstein, 1999). Several sources for possible efficiency gains have been identified: mergers can reallocate capital from unprofitable projects, replace unproductive managers and correct mis-valuations (Jovanovic and Rousseau, 2002; Shleifer and Vishny, 2003; Jovanovic and Rousseau, 2008); market power acquired through mergers might improve innovation incentives (Spulber, 2013); or firms might merge to realize financial or production synergies (Lewellen, 1971; Fluck and Lynch, 1999; Segal and Whinston, 2007). Our focus is on production synergies. Previous work considering production synergies have tended to concentrate on integrating the efficiency gains from these synergies into merger evaluations without directly evaluating the size of the efficiency gains (see, for example, Farrell and Shapiro, 1990; Whinston, 2007). We undertake the complementary exercise of providing a theory of such synergies capable of identifying which mergers will generate large production synergies, and which will not. Empirical work suggests that synergies are an important factor in merger decisions and their success (Larsson and Finkelstein, 1999; Devos et al., 2008; Hoberg and Phillips, 2010; Makri et al., 2010; Bena and Li, 2014).

Finally, our work relates to a new literature on the rise of large firms with market power. Crouzet and Eberly (2019) attribute rising industry concentration to intangible capital such as intellectual property, branding, and software, while Bessen (2017) and Lashkari et al. (2018) both find that proprietary information technology can explain much of the observed rise in market concentration in US and French firms respectively. These findings are consistent with our theoretical results. In particular, Section 6 demonstrates how scarce capabilities – which proprietary information technology can be – valued by many intersecting markets can generate merger synergies prompting a giant firm to emerge. Theoretical explanations for these phenomena (see, e.g. Autor et al. (2020), Aghion et al. (2019), and Luttmer (2011)) have primarily

[20]Our paper also connects to a literature studying the firm size distribution in industry structures. For example, Angelini and Generale (2008) study the impact of financial constraints on firm size distribution and conclude that financial constraints cannot be considered the main determinant of the firm size distribution evolution in developed economies.

focused on aggregate-level approaches. By contrast, we explicitly model the capabilities held and valued by each firm and market respectively.

The rest of this paper is organised as follows: in Section 2, we formally introduce our model. In Section 3, we use a motivating example to illustrate our main results. In Section 4, we prove an upper bound and also sharp transition of the upper bound on the size of the largest firms in stable industry structures. In Section 5 we show that the upper bound is also tight. In Section 6, we complement the upper bound result with a lower bound on the size of the largest firm in stable industry structures – if the largest firm is below this bound, the firm hypergraph is necessarily unstable. Section 7 demonstrate robustness by way of simulations. Section 8 outlines how our framework might be deployed for future work.

2 Model

2.1 Firm Competencies

There is a finite set of firms $\{1, \dots, n\}$ and a finite set of markets $\{1, \dots, m\}$. We start with endowing each firm i with a set of *competencies*. We consider competencies as something specific such as a top biotechnological research team, a piece of proprietary information technology[21] or a patent. These competencies are the hard-to-imitate potential sources of competitive advantage possessed by a firm. The finite set of competencies in the economy is denoted by $\mathcal{B}$. Each competency $b \in \mathcal{B}$ is assigned to a unique firm or remains unassigned. We let S denote the set of unassigned competencies, and $F_i \subseteq \mathcal{B}$ be the subset of competencies held by firm i.

We associate each competency with a unique *capability*. We consider capabilities as something more abstract than competencies but which can be derived from some corresponding competencies—for instance, expertise at biotechnological research instead of a specific biotechnological research team. We denote the finite set of capabilities in the economy by $\mathcal{A}$. While each competency is associated with a unique capability, each capability can be associated with multiple competencies. We move between the capability and competency spaces with a many-to-one matching correspondence $\mu : \mathcal{A} \cup \mathcal{B} \rightrightarrows \mathcal{A} \cup \mathcal{B}$, which satisfies the standard conditions: (i) $\mu(a) \subseteq \mathcal{B}$ for all $a \in \mathcal{A}$; (ii) $\mu(b) \subseteq \mathcal{A}$ with $|\mu(b)| = 1$ for all $b \in \mathcal{B}$; and (iii) $b \in \mu(a)$ if and only if $\mu(b) = \{a\}$ for all $b \in \mathcal{B}$ and all $a \in \mathcal{A}$. We also use $\mu(B) := \{a \in \mathcal{A} : \mu(a) \cap B \neq \emptyset\}$ to identify the set of capabilities associated with the set of competencies $B \subseteq \mathcal{B}$ and $\mu(A) := \{b \in \mathcal{B} : \mu(b) \in A\}$ to identify the set of competencies associated with the set of capabilities $A \subseteq \mathcal{A}$. We assume throughout and without loss that $\mu(\mathcal{B}) = \mathcal{A}$, i.e., there is at least one competency associated with each capability. Because of the many-to-one matching correspondence μ between the capability and competency spaces, any action undertaken by one or more firms in the competency space $\mathcal{B}$, such as mergers and demergers (defined as below), can be always mapped into to a corresponding action in the

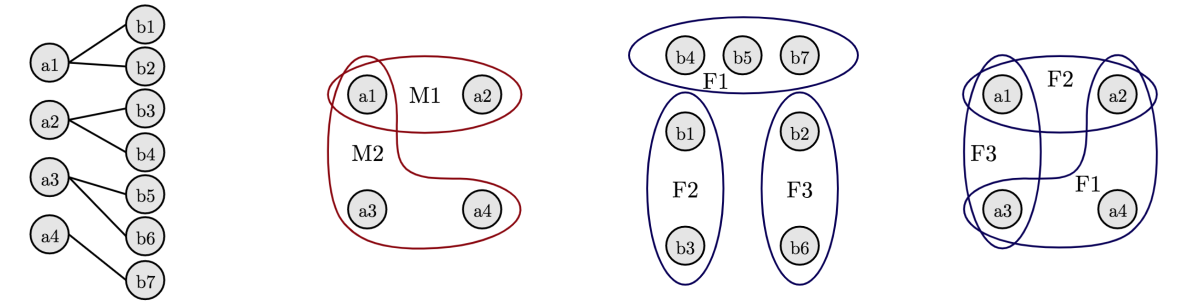
[21] Bessen (2017) finds that proprietary information technology, by increasing the productivity of top firms, can help explain recent increases in industry concentration.

capability space $\mathcal{A}$. Therefore, with some abuse of terminologies, we often speak of the actions taken by firms directly in the capability space and refer to a firm i's *capabilities*, by which we mean $\mu(F_i)$.

Each capability is a potential source of competitive advantage in some, but typically not all, markets. We let the capabilities a market j values be denoted by $M_j \subseteq \mathcal{A}$. The competitiveness of a firm in a market is then determined by the relevant capabilities they, and other firms are able to deploy in that market. To capture the relevant capabilities firm i is able to deploy in market j we define $\theta_{ij} := \mu(F_i) \cap M_j$.

We allow each competency to be deployed across multiple markets. This is realistic because many competencies – information technology know-how, brands, business processes, databases, patents, supplier networks, customer relationships, etc. can all typically be deployed in one market without substantially diminishing their possible value in others.[22]

Figure 2: Representation of a simple industry structure



(a) Matching μ. (b) Market hypergraph. (c) Firm hypergraph. (d) $\mathcal{A}$ representation.

It is convenient to represent the competencies firms have and the capabilities markets value with hypergraphs[23]. The market hypergraph is given by $H_M := (\mathcal{A}, \{M_1, M_2, ...M_m\})$ while the firm hypergraph is $H_F = (\mathcal{B}, \{F_1, F_2, ...F_n\})$. Figure 2 (a) illustrates the mapping $\mu(a_1) = \{b_1, b_2\}, \mu(a_2) = \{b_3, b_4\}, \mu(a_3) = \{b_5, b_6\}, \mu(a_4) = \{b_7\}$. The markets $M_1 = \{a_1, a_2\}$ and $M_2 = \{a_1, a_3, a_4\}$ are represented in Figure 2 (b). The firms $F_1 = \{b_4, b_5, b_7\}, F_2 = \{b_1, b_3\}, F_3 = \{b_2, b_6\}$ are represented in Figure 2 (c). Unlike the market hypergraph, the edges of the firm hypergraph must be disjoint—each competency can be held by at most one firm although multiple firms will hold competencies that are viewed in the same way by markets (i.e., are associated with the same capability). It will sometimes be convenient to represent the firm hypergraph over the capability space rather than competency space to make direct comparisons with markets possible. In such cases, each edge is defined over the capabilities corresponding to the competencies each firm possesses. For instance, in Figure 2 (d), firm 1 is represented by the edge $\{a_2, a_3, a_4\} = \mu(\{b_4, b_5, b_7\})$. Notice some information is lost in this representation since a firm might hold multiple competencies corresponding to a single capability. Nevertheless,

[22] These competencies can be viewed as forms of intangible capital. The increasing importance of intangible capital has been documented and studied by, for example, Brynjolfsson et al. (2008) and Crouzet and Eberly (2019).

[23] A hypergraph is a generalization of a graph in which the edges can be any non-empty subset of the vertices, as opposed to being constrained to a pair of vertices.

this representation will be helpful for thinking about profitability and competitiveness across markets.

Properties of hypergraphs. Consider a hypergraph $H(A, \{E_i\}_{i=1}^n)$ defined over the nodes A with n non-empty edges. Two nodes $a_1, a_2 \in A$ are *adjacent* if $a_1, a_2 \in E_i$ for some i. A *path* between two nodes $a_1 \in A$ and $a_k \in A$ is a tuple $(a_1, E_1, a_2, \ldots, E_t, a_{t+1} = a_k)$, such that for $1 \leq i \leq t$ and $E_i \in H$, $\{a_i, a_{i+1}\} \in E_i$. A subset A_i of A is *path connected* in H if there is a path between any two nodes of A_i. Two subsets of A, A_i, A_j, are *disconnected* if there is no path between any of the nodes in A_i and any of the nodes in A_j. All pairs of hyperedges in the firm hypergraph H_F are disconnected. A subset A_i of A in H is *isolated* if A_i and $A \setminus A_i$ are disconnected. All edges in the firm hypergraph H_F are isolated. A subset A_i of A in H is *self-connected* if A_i is path connected and all the connecting edges are subsets of A_i. A subset A_i of A in H is a *component* if A_i is self-connected and isolated. A hypergraph $\hat{H} = \{\hat{A}, \{\hat{E}_i\}_i\}$ is a *subhypergraph* of $H = \{A, \{E_i\}_i\}$ if and only if each edge of $\hat{H}$ is non-empty and can be extended to an edge in H by adding nodes of $A \setminus \hat{A}$, i.e., $\{\hat{E}_i\} \subseteq \{E_i \cap \hat{A}\}$. We say the subhypergraph $\hat{H}(\hat{A}) = \{\hat{A}, \{\hat{E}_i\}\}$ of $H = \{A, \{E_i\}\}$ is *induced by* the nodes $\hat{A} \subseteq A$ if $\hat{E}_i = E_i \cap \hat{A}$ for all i. $\mathcal{H}(A)$ is the *set of all possible hypergraphs* fixing the set of nodes A.

We will use the notation $|F_{max}|$ to refer to the cardinality of the number of competencies held by the largest firm.

Components. It will be helpful to explicitly define components for the market hypergraph. Given a market hypergraph $H_M \in \mathcal{H}(\mathcal{A})$, let $\{C_1 \ldots C_p\}$ be the set of all components and $\mathcal{P} = \{1 \ldots p\}$. As there is a finite set of capabilities, there exists a component with weakly more capabilities than any other. We denote the number of capabilities in a largest component of a hypergraph by $|C_{max}|$.

2.2 Stable industry structures

Firms' profits in a market j are determined by the distribution of capabilities relevant for that market across the firms. Specifically, firm i's profit in market j is described by the function $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) : \Theta_j^n \to \mathbb{R}_{\geq 0}$, where $\Theta = 2^{M_j}$ is the set of possible relevant capabilities a firm can have in market j and $\boldsymbol{\theta}_{-ij} \equiv \{\theta_{kj}\}_{k \neq i}$ is a vector of relevant capabilities firms other than i have for market j. Firm i' *gross profits* are just the sum of their profits across the markets. Firm i' *net profits* are their gross profits less their cost of maintaining their competencies,

$$\pi_i(H_F, H_M) := \left(\sum_{j \in \mathcal{M}} \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) \right) - \kappa(|F_i|).$$

Given any initial pair of non-empty firm and market hypergraphs (H_M, H_F), we are interested in the set of stable industry structures, in which firms are unable to take actions that would increase their net profits. We allow firms to take the following actions to change the competencies they have:

Procurement. A procurement undertaken by a single non-empty firm i lets the firm procure any subset of the unassigned competencies $B \subseteq S$, endowing the resultant firm with competencies $F_i' = F_i \cup B$. The set of procured competencies B now exits the set of unassigned competencies such that $S' = S \setminus B$.

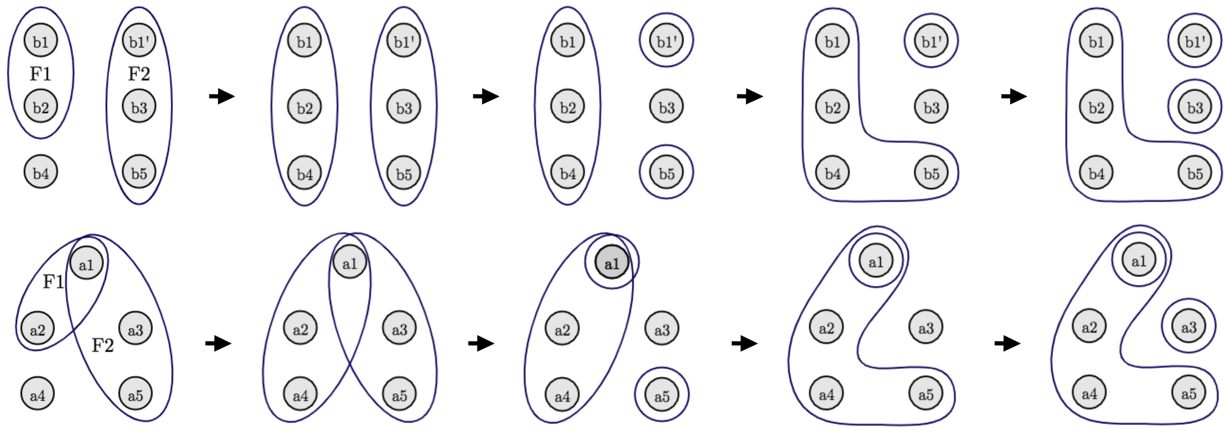
Demerger. A demerger undertaken by a single non-empty firm i lets the firm reorganise into one or more firms $\{F_1, \ldots, F_n\}$ while also disposing of unwanted competencies such that

$$\{\cup_{l\in\{1\ldots n\}} F_l\} \cup \{S'\} = \{F_i \cup S\}$$

where $S' \supseteq S$ is the set of unassigned competencies after the firm i has undertaken its demerger. When a demerger generates only one resultant firm F_i', we must have $F_i' \subset F_i$, and we refer to the action as a *disposal*.

Figure 3: Sequence of procurements, demergers, mergers, and entry

Note: Top row: representation in the competency space ($\mathcal{B}$); bottom row: representation in the capability space ($\mathcal{A}$). We have matching $\mu(a_1) = \{b_1, b_1'\}$ and $\mu(a_i) = b_i$ for $i = \{2, \ldots, 5\}$. Each action is relative to the previous hypergraph.



(a) Initial (b) Procurement (c) Demerger (d) Merger (e) Entry

Merger. A merger between two non-empty firms i and k combines i's and k's capabilities, and also permits i and k to reorganise any duplicate capabilities they have by spinning off firms and/or discarding of unwanted duplicate capabilities (see conditions (i)-(iii) below). Formally, a merger between firms i and k generates a set of firms $\{1, \ldots, n\}$ and set of unassigned competencies S' such that

(i) $S' \supseteq S$;

(ii) there exists a firm $l \in \{1\ldots n\}$ such that $\mu(F_l) = \mu(F_i) \cup \mu(F_k)$;

(iii) $\{\cup_{l\in\{1\ldots n\}} F_l\} \cup \{S'\} = \{F_i \cup F_k \cup S\}$;

(iv) firms $\{F_1, \ldots, F_n\}$ could not have been generated by a (single) demerger.

Any profitable merger that violates condition (iv) is either also a profitable demerger (and so already a profitable permitted deviation) or else one firm is sponsoring the unprofitable demerger of the other. Condition (iv) rules out such deviations, which we view as inconsistent with antitrust principles.[24]

For an industry structure (H_F, H_M), a merger between firms i and k creating firms $1, \ldots, n$ and leading to the industry structure (H'_F, H_M) is strictly net profitable when it strictly increases the joint net profits of firms i and k (i.e., $\sum_{l \in \{1 \ldots n\}} \pi_l(H'_F, H_M) > \pi_i(H_F, H_M) + \pi_k(H_F, H_M)$). Similarly, any other deviation by a firm i that replaces firm i with the (possibly empty) set of firms $\mathcal{F}$ is strictly net profitable if $\sum_{l \in \mathcal{F}} \pi_l(H'_F, H_M) > \pi_i(H_F, H_M)$.

Entry. An entry creates a new firm l endowed with competencies $F_l \subseteq S$.

Definition (**Stability**)**.** *We say an industry structure (H_M, H_F) is stable if and only if there is no strictly net profitable procurement, demerger, merger, or entry. An industry structure is unstable if it is not stable.*

We impose several weak conditions directly on firms' profit functions which we maintain throughout.

Assumption (Primitives on profits)**.** *Let $I(\boldsymbol{\theta}_j) = \{i \in \mathcal{N} : \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) > 0\}$ denote the set of firms operating in market j. Let $\boldsymbol{\theta}_{Ij} := \{\theta_{ij}\}_{i \in I}$ denote the capabilities these firms have that are valued by market j. We assume that the profit functions $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ satisfy the following conditions:*

(i) ***Firms with no capabilities make 0 gross profits.*** *$\pi_{ij}(\emptyset, \boldsymbol{\theta}_{-ij}) = 0$ for all $\boldsymbol{\theta}_{-ij} \in \Theta^{n-1}$.*

(ii) ***Firms which are not operating in market j do not influence profits in market j.*** *For all $\boldsymbol{\theta}_j$ and $\boldsymbol{\theta}'_j$ such that (i) $I(\boldsymbol{\theta}_j) = I(\boldsymbol{\theta}'_j)$ and (ii) $\boldsymbol{\theta}_{Ij} = \boldsymbol{\theta}'_{Ij}$, we have that $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) = \pi_{ij}(\theta'_{ij}, \boldsymbol{\theta}'_{-ij})$ for all i.*

(iii) ***Labels do not matter.*** *For any bijection $b : \{1, \ldots, n\} \to \{1, \ldots, n\}$ (changing the labels of the players while holding their capabilities fixed), $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) = \pi_{b(i)j}(\theta_{b(i)j}, \boldsymbol{\theta}_{-b(i)j})$.*

(iv) ***Weaker competitors increase gross profits.*** *Let $\boldsymbol{\theta}_{-ikj} := \{\theta_{lj}\}_{l \neq i,k}$. For all $k \neq i$, if $\theta'_{kj} \subset \theta_{kj}$ then $\pi_{ij}(\theta_{ij}, \theta'_{kj}, \boldsymbol{\theta}_{-ikj}) \geq \pi_{ij}(\theta_{ij}, \theta_{kj}, \boldsymbol{\theta}_{-ikj})$ with strict inequality if and only if $\pi_{kj}(\theta_{kj}, \boldsymbol{\theta}_{-kj}) > 0$ and $\pi_{ij}(\theta_{ij}, \theta'_{kj}, \boldsymbol{\theta}_{-ikj}) > 0$.*

Assumption (Increasing and convex maintenance costs)**.** *Defined as a scalar function in the competency space $\mathcal{B}$, $\kappa(0) = 0$, κ is strictly increasing and has increasing differences (i.e., $\kappa(x) - \kappa(x-1) > \kappa(x-1) - \kappa(x-2)$ for all $x \geq 2$).*

This captures a conglomeratization cost associated with maintaining many competencies. It may reflect the scarcity of management time or inability of the firm to tailor their corporate culture towards maintaining a broad set of competencies.

[24] In Section 6.2 we show how a stronger, systematic antitrust policy based on the market-by-market requirement that consumer surplus is non-decreasing in any market can be incorporated into our framework without affecting our main results.

There are no fixed costs other than competency maintenance costs. Thus holding the competencies of a firm fixed, a firm will never choose to operate in a market in which it would make a loss, and will always choose to operate in a market in which it would make a strictly positive gross profits. We thus refer to a firm as being active in a market if it is making strictly positive profits in that market (as we discuss in Appendix A, in a nested Cournot model, there will be no firms making exactly zero profits in a market while also making strictly positive sales).

We view our assumptions above regarding firm profit functions and competency maintenance costs as weak regularity conditions and will maintain these throughout.

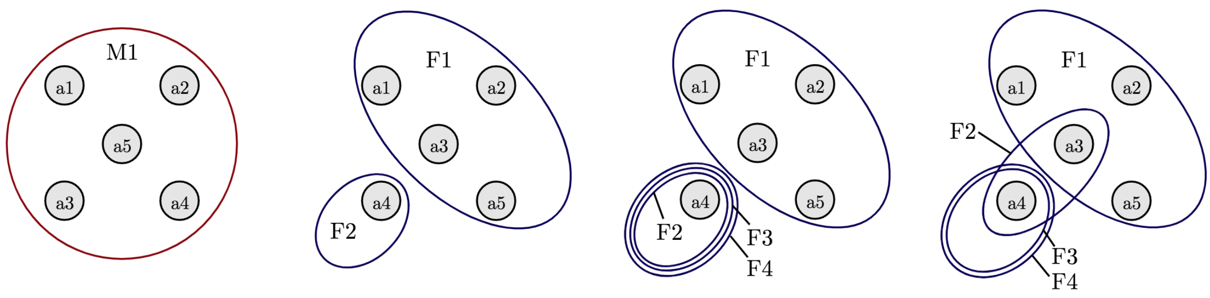
We will occasionally work with $\pi_{ij}(H_F, H_M)$ in lieu of $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$, noting that the hypergraph pair (H_F, H_M) is always sufficient to recover $(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ via the many-to-one matching correspondence μ. When it is unlikely to cause confusion, we will also drop the arguments. If $\mu(F_i) \cap M_j \neq \emptyset$, we will say that i *possesses relevant capabilities* for market j. If $\pi_{ij} > 0$, we will say that i is *operating in* j. We let $J_{i,\pi>0}$ denote the set of markets that firm i operates in.

2.3 Flexibility of the model

The set of competencies is intentionally left abstract, and intended to capture anything that might deliver competitive advantage for a firm. This general interpretation of competencies is in line with Wernerfelt (1984). Some concrete examples of competencies that a firm might hold include a brand or reputation, patents, know-how related to a production process, the corporate culture of a firm, human capital of various forms, relationships with supplier or customers, a distribution network, a customer base, information about customers, and so on. Our approach does not require any fixed interpretation of how competencies deliver competitive advantages (for example, by reducing marginal costs, or increasing consumers' willingness to pay).

Allowing multiple competencies to be associated with a given capability, paired with our assumptions on profits, allows our model to capture heterogeneities across capabilities in the spirit of the management literature which often speaks of whether, and to what extent capabilities are valuable, especially rare, and synergistic for a given market (see, for instance, Barney (1991)).

Figure 4: Heterogeneity across capabilities in value, rarity, and synergies



(a) H_M. (b) H_F. (c) H'_F. (d) H''_F.

Consider Figure 4. Panel (a) shows the market hypergraph, while Panels (b)-(d) show alternative firm hypergraphs. Accommodating the idea that some capabilities might be much more valuable to possess than others, in Panel (b) firm 2 might make higher profits in market 1 than firm 1. At the same time, multiple firms holding this capability can erode its value—in Panel (c) firm 1's profits might instead be higher than firm 2's. Some capabilities can also be highly complementary. For example, in Panel (d), the capabilities a_3 and a_4 might generate sufficient synergies such that firm 2 makes substantial profits while none of the other firms can profitably compete in and so become inactive in market 1.

This flexibility allows our framework to capture substantial heterogeneity across markets. At one end of the spectrum, some markets can be "winner-take-all" with the most competitive firms able to capture the whole market (see, for instance, Autor et al. (2020)). This might be driven by strong complementarities between relevant capabilities for the market, significant network effects or low search frictions which facilitate strong price based competition. At the other end of the spectrum, some markets can accommodate many firms with strong and weak firms coexisting (see, for instance, Shaked and Sutton (1983)). Our formulation nests these two extremes, as well as everything in between. More importantly, it allows markets of different types to coexist within the same framework.

Finally, our assumptions on profits are sufficiently weak to accommodate Cournot competition as a special case (see Appendix A).

3 A Motivating Example

In this section, we provide a simple example to help illustrate our main results. Additional details are in Supplementary Appendix II.

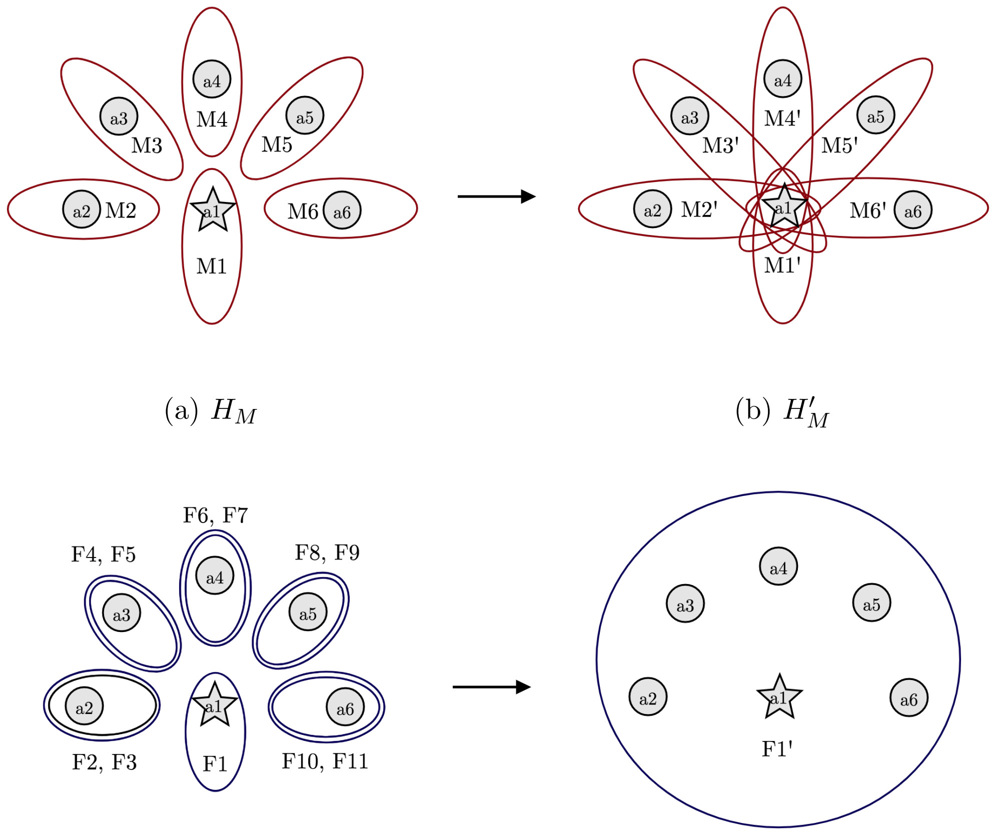
Example 1. *First consider Figure 5 below. Panel (a) illustrates the initial market hypergraph, with market* 1 *valuing capability* a_1*, market* 2 *valuing* a_2*, and so on. We will assume* $S = \emptyset$ *and there are two competencies corresponding to* $a_2, a_3, \ldots a_6$*, and one competency corresponding to* a_1*, which in light of its scarcity, we represent with a star.*

Our first result (Proposition 1) shows that the size of the largest component on the market hypergraph always imposes an upper bound on the size of the largest firm in any stable industry structure. For instance, when this is small as in Figure 5 (a), the size of the largest firm is necessarily small—the industry structure given by (H_M, H_F) *in Figure 5 (a) and (c) is stable. For instance, there are no synergies between markets* 1 *and* 2*, and so a merger between firms* 1 *and* 2 *generating a firm with capabilities* $\{a_1, a_2\}$ *would make identical gross profits as before across markets* 1 *and* 2*, but bear increased capability maintenance costs. As such, this merger is strictly net unprofitable.*

As all markets begin to value more, and more of the same capabilities, the size of the largest component of the market hypergraph—and hence our upper bound—can increase abruptly. Figure 5 (b) illustrates this by considering the case in which all markets begin to value the capability

a_1. *For instance, markets* $2, 3, \ldots 6$ *might be for different consumer products, while market* 1 *is the market for social networking and capability* a_1 *is user data. The expansion of* H_M *to* H'_M *might be facilitated by technological advances permitting better analysis of the data collected, or better opportunities to deploy the data to target advertisements or discounts for consumer products. After this change in the capabilities valued by markets, the largest component of the market hypergraph goes from containing a single capability to containing all capabilities. While this expansion of* H_M *to* H'_M *is rather specific, the abrupt transition is by no means knife-edge—quite generally, there is a phrase transition in the size of the largest component of the market hypergraph even when markets value more capabilities uniformly at random (Proposition 2).*

Figure 5: Large firms emerging in response to markets valuing more capabilities



(a) H_M (b) H'_M

(c) H_F (d) H'_F

Our next result shows that our upper bound on the size of the largest firm is always tight—there must exist some stable industry structure for which the largest firm is the size of the largest component on the market hypergraph (Theorem 1). In this example, in response to the market hypergraph expanding, firm 1 *finds it profitable to sequentially merge with firms* $2, 4, 6\ldots$. *For the parameter values we consider in the Supplementary Appendix II, the remaining firms in markets* $2'$ *to* $6'$ *are unable to keep competing profitably in these markets and so exit by disposing of their sole capability. The resultant stable industry structure is given by* (H'_M, H'_F) *in Figures 5 (b) and (d). While this example is very specific the result is general: we show for any feasible* *mapping between the capability and competency spaces, stable industry structures achieving the bound exist.*

We also turn to the question of necessity. Our final result establishes a lower bound on the size of the largest firm, and shows that the presence of scarce capabilities can necessitate the emergence of giant firms (Proposition 3). In H'_M of this example, a_1 is scarce and is valued by all markets. This serves as a source of synergies linking all markets and makes the emergence of a firm which competes in all markets inevitable—the possible synergies are only fully realized when a single giant firm holds all possible capabilities. For the more general case, consider the market subhypergraph induced by the set of scarce capabilities. Then for any component of this market subhypergraph, consider the markets belonging to this component. We show that all capabilities (including non-scarce ones) valued by these markets must be held by the same firm. This provides our lower bound on the size of the largest firm. Even with only a few scarce capabilities, our lower and upper bounds can coincide, as in this example. Scarce capabilities are essential for establishing a lower bound—Example 2 in Appendix B shows that absent scarce capabilities, it is possible for all firms to become smaller as markets value more capabilities.

4 An upper bound on the size of firms

In the introduction, we briefly discussed the expansion of internet conglomerates into the supermarket and automobile industries. We argued that these forays into new markets could be rationalized by these markets beginning to value the capabilities held by internet firms, generating new synergies which were not present fifteen years ago. This, and a myriad of similar observations motivate the question of whether markets valuing more, and more of the same capabilities, can explain the sudden emergence of large conglomerates.

This is a challenging problem. While our model allows for significant heterogeneities in how capabilities interact to deliver competitive advantages across different markets, it also makes equilibrium adjustments complex. The space of possible firm and market hypergraphs grows rapidly as the cardinality of $\mathcal{A}$ and $\mathcal{B}$ increase,[25] while there will typically be multiple stable industry structures. All this means there is little hope for us providing a complete characterization of the set of stable industry structures, and even if we were able to do this we would need some way of selecting between them. Finally, as we show in Appendix B and discuss above, even in very simple cases firms can respond to markets valuing more capabilities by maintaining fewer competencies. Despite these challenges, we will show that a surprising amount can be said about stable industry structures.

Consider a demerger. Demergers always save on competency maintenance costs. In the case where some competencies are disposed, this reduces the number of competencies which require maintenance. Even if no competencies are disposed, the convexity of the competency maintenance costs guarantees that a demerger will reduce competency maintenance costs. On the

[25] For example, the set of all possible firm hypergraphs is of cardinality $|\mathcal{H}(\mathcal{B})| = B_{|\mathcal{B}|+1}$ where B_n is the Bell number defined recursively such that $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_n$ and $B_0 = B_1 = 1$. So, for $|\mathcal{B}| = 50$, $|\mathcal{H}(\mathcal{B})|$ is of order 10^{48}.

other hand, synergies in markets the initial firm operated in may be lost, and this will reduce the gross profits that are obtained in such markets. Demergers can also lead the newly created firms to compete against each other—which can have ambiguous effects on the joint profitability of the merger.[26] Lemma 1 provides sufficient conditions for a demerger to weakly increase gross profits, and thus strictly increase net profits.

Lemma 1. *Let $\hat{A}_i := \cup_{j \in J_{i,\pi>0}} M_j$ denote the set of capabilities valued by markets firm i operates in. There exists a demerger that firm i can undertake that weakly increases its gross profits (i.e., $\sum_j \pi_{ij}$ weakly increases) if any of the following three conditions hold*

(i) $\mu(F_i) \not\subseteq \hat{A}_i$,

(ii) $|F_i| > |\mu(F_i)|$,

(iii) the market subhypergraph $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$ contains more than one component.

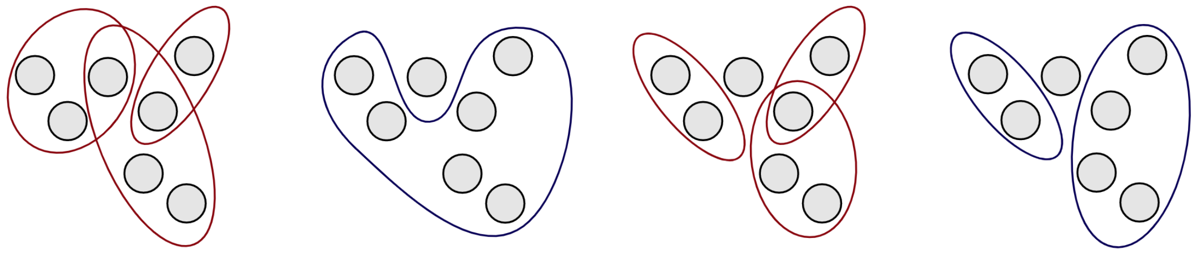
Lemma 1 is simple but powerful. As demergers which leave gross profits unaffected are net profitable (because of the convexity of competency maintenance costs), it allows us to get a first handle on the set of stable industry structures. First, in any stable firm hypergraph, firms cannot hold unused competencies—either by holding competencies that are not valued by any market the firm competes in (condition (i)), or by holding multiple competencies that correspond to the same capability (condition (ii)). Second, when conditions (i) and (ii) are met, firms can never find it optimal to hold a combination of competencies if there is a way to partition them without destroying some synergies (condition (iii)). This is illustrated in Figure 6, where it is net profitable for firm i to demerge into firms k and l. This demerger is along the component boundaries of the induced subhypergraph $\hat{H}_M(\mu(F_i))$ and thus generates a set of firms such that for every market firm i competed in prior to the demerger, there is an equally strong competitor post-demerger. This idea resonates with the received wisdom from the management literature – firms should focus on activities aligned with their core competencies, rather than spread themselves too thin. We now employ Lemma 1 to provide an upper bound on the size of the largest firm in any stable firm hypergraph.

Proposition 1. *In all stable industry structures the number of competencies maintained by the largest firm is weakly less than the number of capabilities in the largest component of the market hypergraph ($|F_{max}| \leq |C_{max}|$).*

Proof. Without loss of generality, suppose that firm i is a largest firm (i.e., $|F_{max}| = F_i$). In a stable industry structure no firm, including i, can have a strictly net profitable demerger. Hence, by Lemma 1(ii), $|F_i| = |\mu(F_i)|$. Next, recall that $\hat{A}_i := \cup_{j \in J_{i,\pi>0}} M_j$ and note that by Lemma 1(i), $\mu(F_i) \subseteq \hat{A}_i$, and so $\mu(F_i) \cap \hat{A}_i = \mu(F_i)$. Finally, by Lemma 1(iii), the market subhypergraph $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$ must contain a unique component, and so $|\mu(F_i) \cap \hat{A}_i| \leq |C_{max}|$. Combining these conditions $|F_{max}| = |F_i| = |\mu(F_i)| = |\mu(F_i) \cap \hat{A}_i| \leq |C_{max}|$. □

[26] For example, in a Cournot setting, such a demerger can be profitable because it, in effect, helps commit the newly created firms to produce more than the firm they were created from and take a greater market share all else equal.

Figure 6: An example of a demerger along components on the subhypergraph induced by $\mu(F_i)$



(a) H_M (b) $\mu(F_i)$ (c) $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$ (d) $\mu(F_k), \mu(F_l)$

4.1 Phase transition of the upper bound

We have found an upper bound on the size of the largest firm in terms of the size of the largest component in the market hypergraph. As markets value more of the same capabilities the size of the largest component will increase and our upper bound on the size of firms will be relaxed. However, if these changes are only gradual this mechanism would have a hard time accounting for the rapid expansion of internet conglomerates. A natural question to then ask, is whether we should expect a sudden change in this upper bound. To explore this question, we need to add some structure to how the market hypergraph evolves. We do this by modeling it as a random hypergraph. This provides a natural benchmark and will illustrate how small changes in connectivity can massively relax our upper bound on firm size.

For simplicity, we consider a standard random hypergraph model which yields a neat closed-form characterization though as it will become clear, these results hold far more broadly. We let each edge (market) of size i in our random market hypergraph occur independently from each other, and independently from edges of other sizes, with probability p_i. We denote the random hypergraph model by $\mathcal{R}(\mathcal{A}, \mathbf{p})$ with $\mathbf{p} = (p_1, p_2, \ldots, p_t)$ where t is the largest edge size permitted. We define the *degree*[27] of a node $a \in \mathcal{A}$ in a hypergraph $H(A, \{E_i\}_{i=1}^n)$ as the number of node-edge pairs (a_i, E_i) such that $\{a, a_i\} \subseteq E_i$. The expected degree of a random hypergraph $H \in \mathcal{R}(\mathcal{A}, \mathbf{p})$ is[28]

$$\mathbb{E}[d(|\mathcal{A}|)] = \sum_{k=2}^{t} (k-1) \binom{|\mathcal{A}|-1}{k-1} p_k.$$

We use the following result from the random hypergraph literature (Schmidt-Pruzan and Shamir, 1985).

Proposition 2 (Schmidt-Pruzan and Shamir, 1985). *The upper bound $|C_{max}|$ has the following*

[27]This is sometimes referred to as 'vertex degree' in the hypergraph literature to distinguish it from alternative generalizations of degree as used in the networks literature.

[28]The equation is intuitive. For a given node, the binomial coefficient ($|\mathcal{A}|-1$ choose $k-1$) gives the number of different possible edges of size k that include the node in question (along with $k-1$ other nodes). Multiplying this by p_k and $k-1$ gives the contribution of edges of size k to the node's expected degree, and summing over k gives the expected degree.

properties: There exists a finite constant $\bar{d}$ such that

(i) [subcritical case] if $\mathbb{E}[d(|A|)] < \bar{d}$, then $|C_{max}| = O(t \log n)$

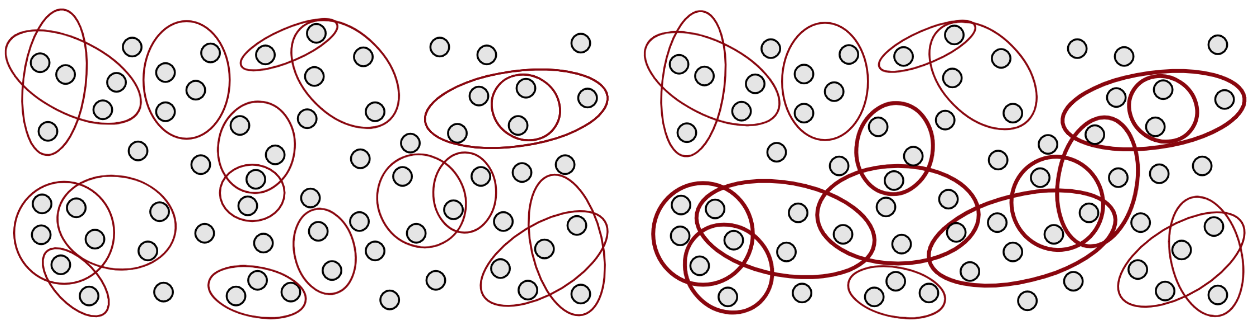
(ii) [critical case] if $\mathbb{E}[d(|A|)] = d$, then $|C_{max}| = O(n^{2/3})$

(iii) [supercitical case] if $\mathbb{E}[d(|A|)] > \bar{d}$, then $|C_{max}| = O(n/t)$

Proposition 2 shows there is a critical threshold for the connectivity of markets. If the expected degree of the random market hypergraph is above this threshold (supercritical case) then the largest firms in stable industry structures can be large and contain a constant fraction of $\mathcal{A}$. If the expected degree of the random market hypergraph is below this threshold (subcritical case) then all firms in stable industry structures are necessarily small and contain a vanishing fraction of $\mathcal{A}$.

Figure 7: Giant component of the market hypergraph

Notes: The figure illustrates two hypergraphs defined over the same space. Only the hypergraph in (b) has a giant component (the constituent edges are bolded). Both hypergraphs have the same number of edges, and each edge is of sizes between 2 and 6. The hypergraph in (b) can be reached by expanding the size of edges in the hypergraph in (a).



(a) Without a giant component

(b) With a giant component

5 Tightness of the upper bound

Proposition 1 established an upper bound on the size of the largest firm. Proposition 2 then showed this upper bound can abruptly transition as markets become increasingly connected. In this section, we show the upper bound is tight.

We have thus far made only very weak assumptions to the profit function. To show the upper bound is tight we need to make some additional assumptions which impose more structure on the value of additional relevant capabilities, and how capabilities might interact with each other to generate competitive advantages.

Assumption 1 (Lower bound on the value of additional capabilities)**.** *If firm i obtains more relevant capabilities for market j ($\theta'_{ij} \supset \theta_{ij}$), then i's profits in j weakly increases:*

$$\pi_{ij}(\theta'_{ij}, \boldsymbol{\theta}_{-ij}) - \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) \geq 0.$$

Further, if firm i initially made positive profits in market j ($\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) > 0$) or no firms compete in market j ($\pi_{kj}(\theta_{kj}, \boldsymbol{\theta}_{-kj}) = 0$ for all $k \in \mathcal{N}$), then the increase in profits from every additional capability is bounded from below by the cost of maintaining a single capability:

$$\pi_{ij}(\theta'_{ij}, \boldsymbol{\theta}_{-ij}) - \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) > \kappa(1),$$

where $\theta'_{ij} \supset \theta_{ij}$ and $|\theta'_{ij}| = |\theta_{ij}| + 1$.

Assumption 2 (Complementary capabilities)**.** *For any $\mathcal{F} \subseteq \mathcal{N}$ such that for any $i, i' \in \mathcal{F}$, $\theta_{ij} \cap \theta_{i'j} = \emptyset$, merging firms $\mathcal{F}$ into a single firm l increases gross profits:*

$$\pi_{lj}(H'_F, H_M) \geq \sum_{i \in \mathcal{F}} \pi_{ij}(H_F, H_M),$$

where $\theta_{lj} = \bigcup_{i \in \mathcal{F}} \theta_{ij}$, the pre-merger industry structure is (H_F, H_M) and the post-merger industry structure is (H'_F, H_M). Further, the inequality is strict if there exists $i, i' \in \mathcal{F}$ such that $\theta_{ij} \neq \emptyset$, $\theta_{i'j} \neq \emptyset$, and $\max\{\pi_{ij}, \pi_{i'j}\} > 0$.

We view Assumption 1 as relatively weak. Although it imposes some structure on the value of additional relevant capabilities, this is certainly in the spirit of capabilities being key sources of competitive advantage. More precisely, it requires that a firm already profitably operating in a market values the ability to deploy an additional relevant capability by at least $\kappa(1)$, the minimum it could ever cost to maintain that capability. It is worth noting that Assumption 1 and the maintained assumption that firms which do not operate in a market do not influence the profitability of those which do, together imply that for any market $j \in \mathcal{M}$, if at least one firm $i \in \mathcal{N}$ holds relevant capabilities for j, then in a stable firm hypergraph there must be some firm that operates in the market. Assumption 2 is stronger, but is also natural in our context. It requires that the competencies associated with different capabilities are complements[29]—they are more valuable held together than separately. Appendix A shows that Assumptions 1 and 2 are consistent with Cournot competition. Moreover, we shown in Appendix A.1 that all our assumptions are satisfied by a very simple linear specification of Cournot competition.

Lemma 1 showed that a firm has a weakly gross profitable demerger if they hold competencies which are not useful in the market they compete in ($\mu(F_i) \supset \hat{A}_i$), hold more than one competency associated with the same capability ($|F_i| > |\mu(F_i)|$) or hold competencies associated with capabilities that are in different components of the market subhypergraph $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$. Under Assumption 2, these conditions are also necessary for there to be a weakly gross profitable demerger.

Lemma 2. *Suppose that Assumption 2 holds. Then there does not exist a demerger that firm i can undertake that weakly increases its gross profits (i.e., $\sum_j \pi_{ij}$ weakly increases) if all of the following three conditions hold*

(i) $\mu(F_i) \subseteq \hat{A}_i$,

[29]In contrast, competencies of the same type (i.e., associated with the same capability) are substitutes.

(ii) $|F_i| = |\mu(F_i)|$, *and*

(iii) the market subhypergraph $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$ *contains a single component.*

For much of our analysis, we will restrict the convexity of κ. There are two principal reasons for this. First, sufficiently convex competency maintenance costs will inevitably prevent the emergence of large firms. This can obfuscate other forces (e.g. synergies, changes to the competitive landscape and, in Section 6.2, antitrust) that are of interest. Second, the very existence of large internet conglomerates each spanning a myriad of markets suggests that κ cannot, in practice, be very convex. We will say that a result holds for all κ of *sufficiently low convexity* if there exists a $\varepsilon > 0$ such that the result holds for all κ satisfying $\kappa(|\mathcal{B}|) - \kappa(|\mathcal{B}| - 1) - \kappa(1) < \varepsilon$.

Finally, we make our notion of tightness of the upper bound precise. We say the upper bound is *tight* if for any $H_M \in \mathcal{H}(\mathcal{A})$ and any set of competencies such that there is at least one competency associated with each capability, there exists a stable industry structure with $|F_{max}| = |C_{max}|$.[30]

Theorem 1. *If Assumptions 1 and 2 hold, then for any non-empty* $\mathcal{A}$, $\mathcal{B}$, *any matching* μ, *and any* $H_M \in \mathcal{H}(\mathcal{A})$, *the upper bound* $|C_{max}|$ *is tight for sufficiently low convexity of* κ.

As well as showing that our upper bound on firm size is tight, Theorem 1 establishes existence of a stable firm hypergraph. Even this is far from trivial. The space of hypergraphs is large and discrete and it is straightforward to show that there are cycles of profitable deviations which makes a proof based on Tarski's fixed point theorem unlikely to work.[31] Further, while Lemmas 1 and 2 reduce the space of possible stable industry structures somewhat, even in simple examples there remain many possibilities, and these possibilities will depend on the details of the situation that we have not specified—i.e., our assumptions are too weak to be able to say for sure whether one such hypergraph is definitively stable or unstable.

Consider the example shown in Figure 8 below. Panel (a) shows the set of capabilities that exist and the market hypergraph. We assume there are exactly two competencies associated with each such capability. Panels (b) - (f) illustrate 5 of the approximately 700,000 possible firm hypergraphs.[32] The firm hypergraphs shown in Panels (b) - (d) are necessarily unstable:

[30] We could have added the condition that it must be possible to feasibly reach the stable firm hypergraph that achieves the bound from any initial firm hypergraph through a sequence of mergers, demergers, procurements, or entry. For any initial firm hypergraph we can let each firm sequentially dispose of their competencies, and then let firms in the stable industry structure enter sequentially.

[31] See Jackson and Watts (2001).

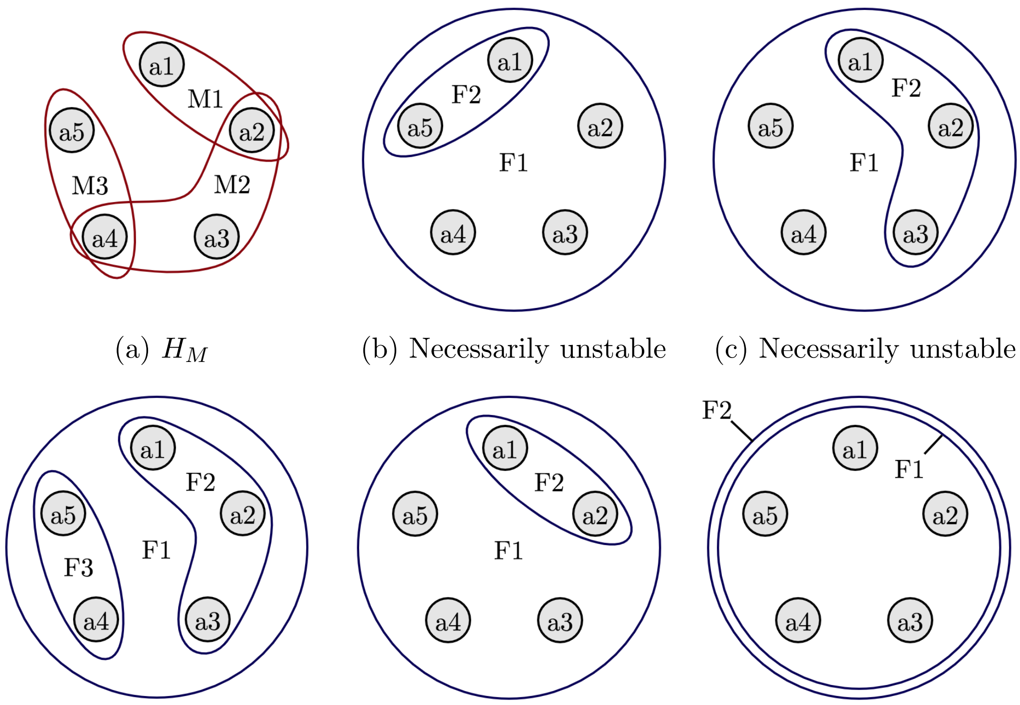
[32] If there are no unassigned competencies, the number of ways to partition the ten competencies among different firms is given by the Bell number $B_{10} = 115,975$. If there are s competencies in the set of unassigned competencies, there are $\binom{n}{s}$ ways of forming this set. For each such set there are then B_{n-s} ways of assigning the remaining competencies to firms. Thus we have

$$\sum_{s=0}^{n} \binom{n}{s} B_{n-s} = \sum_{k=0}^{n} \binom{n}{k} B_k = B_{n+1}.$$

So the number of firm hypergraphs is $B_{11} = 678,570$.

In Panel (b), firm 2 has a profitable demerger by Lemma 1. Panel (c) is slightly more involved. If firm 2 does not compete in market 2, then by Lemma 1 it has a profitable demerger (in this case, a disposal of capability 3). If firm 2 does compete in market 2, then suppose it procured capability 4. Then, by Lemma 2, this newly created firm would not have a profitable demerger, including disposals and in particular the disposal of capability 4. Hence the initial procurement of capability 4 must have been profitable. Now consider Panel (d). Suppose neither firm 2 or 3 has a profitable demerger. This implies that firm 2 competes in market 1 and 2 and makes non-negative net profits, while firm 3 competes in at least market 3 and makes non-negative net profits. But Lemma 2 then implies that firms 2 and 3 have a strictly net profitable merger for all κ not too convex. As such, the merger strictly increases gross profits in market 2 and these gains dominate additional capability maintenance costs. In contrast, Panels (e) and (f) show two of the many firm hypergraphs which *might* be stable—our assumptions are too weak to either rule out or guarantee stability. Nonetheless, even in light of this indeterminacy, Theorem 1 guarantees the existence of at least one stable industry structure for sufficiently low convexity of κ.

Figure 8: A few potential industry structures



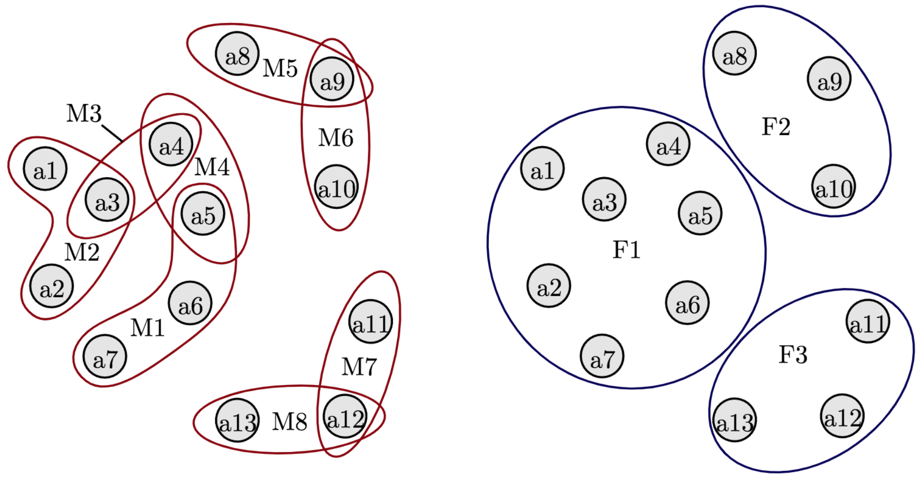
(a) H_M (b) Necessarily unstable (c) Necessarily unstable

(d) Necessarily unstable (e) Possibly stable (f) Possibly stable

The proof of Theorem 1 is involved and deferred to Appendix C. However, we offer some intuition and a rough outline here. Our approach is constructive, and informed by Lemmas 1 and 2.

Starting with the empty firm hypergraph, and thus all competencies assigned to the set S, we create a set of firms with competencies corresponding to the components of the market hypergraph (the first iteration). Note that one of the created firms will have $|C_{max}|$ competencies. If after this exercise the set S is empty (which occurs if and only if there is a unique competency corresponding to each capability i.e. $|\mathcal{B}| = |\mathcal{A}|$), then this firm hypergraph will be stable for sufficiently low convexity of κ. An example is pictured below in Figure 9. By Assumption 1, each firm created will find it profitable to compete in all markets it has relevant capabilities for. Thus, for κ not too convex, there are no profitable demergers by Lemma 2. Further, by Lemma 1 there are no profitable mergers—any merger would generate a firm hypergraph in which there is a strictly net profitable demerger undoing it. Finally, as there are no unassigned competencies, there are no possible procurements or entries.

Figure 9: Special case with $|\mathcal{B}| = |\mathcal{A}|$

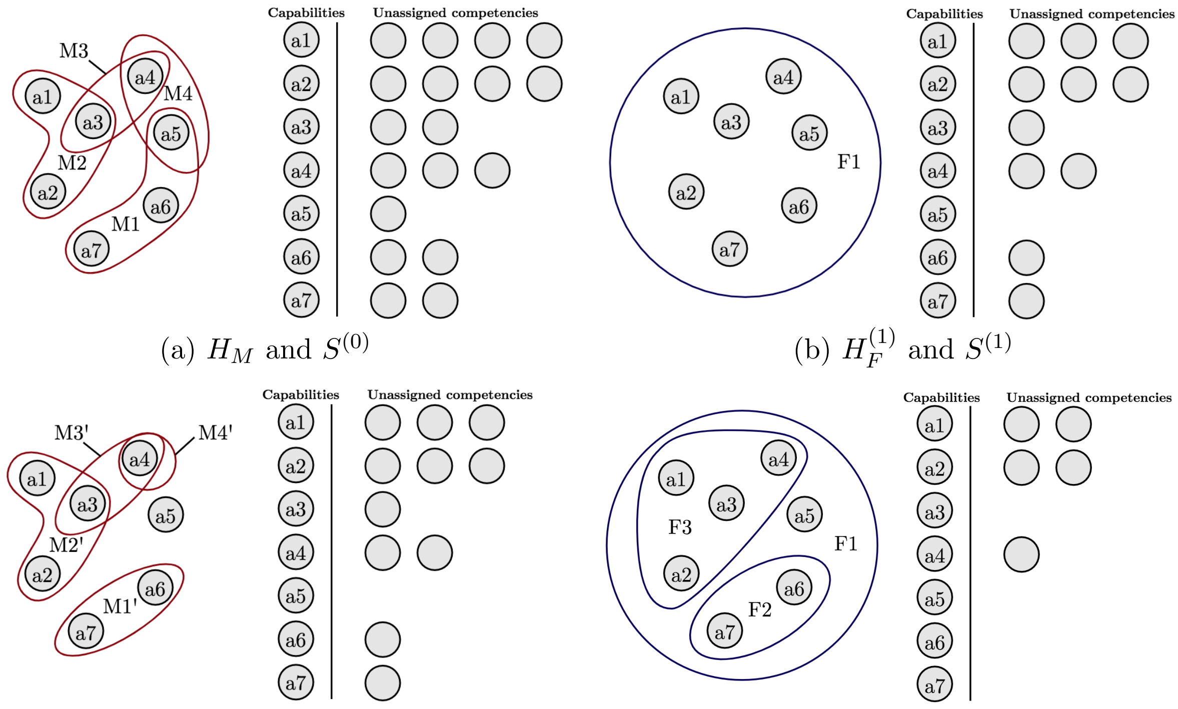


(a) H_M (b) Stable H_F with $|F_{max}| = |C_{max}|$

If instead the set S is non-empty after the first iteration (see Panels (a) and (b) of Figure 10 for an example), we repeat the above exercise to construct a new set of firms. We do this by considering the market subhypergraph restricted to the capabilities $\mu(S)$, and then forming firms with competencies corresponding to each component. Panels (c) and (d) of Figure 10, which we subsequently explain in full, is an example of this process. These newly created firms are not monopolists and so Assumption 1 does not apply and these firms might have a profitable demerger. We let each newly created firm undertake its most profitable demerger (which might just be a disposal). Crucially, it can be shown that the profitability of one firm's choice is independent of other firms' choices because all such firms operate in disjoint sets of markets. Hence this exercise leads to a configuration of firms in which no further demergers are profitable for the firms created in this iteration.

Moreover, there continues to be no profitable demergers for firms created in the first iteration. Although such firms may now face competition and hence receive lower gross profits in some markets, they still find it profitable to compete in the same set of markets as before. This stems from the observation that for a firm i created initially, if it faces a new competitor k in market j, that competitor will have a weak subset of its relevant capabilities for market j. This is because each component of the market subhypergraph on the set of capabilities $\mu(S)$ in the second iteration contains a subset of the capabilities contained in the initial components

Figure 10: An example of hypergraph construction via Algorithm 1 (Part 1 of 2)



(a) H_M and $S^{(0)}$

(b) $H_F^{(1)}$ and $S^{(1)}$

(c) $\hat{H}_M(\mu(S^{(1)}))$

(d) $H_F^{(2)}$ and $S^{(2)}$

of the market hypergraph. Thus, if the new firm k finds it profitable to compete in market j, for κ not too convex, firm i also finds it profitable to continue competing in market j. Hence it can be shown that firm i has no profitable demerger.

To see that there is no profitable merger, we once again draw on the observation that each new firm contains a weak subset of the capabilities held by one of the firms initially created. This means that potential mergers fall into one of two cases: either the firms have disjoint capabilities, or one of the firms has a weak subset of the capabilities held by the other firm. In the latter case any merger will lead to a set of firms that could have been obtained via a demerger—and hence do not need to be considered by condition (iv) for mergers. If instead the firms have disjoint capabilities, because of the way firms were constructed, there are no potential synergies that can be exploited in any market and Lemma 1 implies that any such merger is unprofitable. Proceeding iteratively in the manner described allows us to show a stable industry structure containing a firm with $|C_{max}|$ competencies can always be constructed.

The exposition above offers only the broad strokes of an otherwise involved proof, with the goal of generating some informal intuition. To formalize this approach, we draw upon the following constructive algorithm.

Algorithm 1. *Given $\mathcal{A}$, $\mathcal{B}$, $\mu : \mathcal{A} \cup \mathcal{B} \rightrightarrows \mathcal{A} \cup \mathcal{B}$, and $H_M \in \mathcal{H}(\mathcal{A})$, initialize $H_F^{(0)} = (\mathcal{B}, \emptyset)$, $S^{(1)} = \mathcal{B}$, and $q = 1$. We use notation $X^{(q)}$ to denote the variable X at iteration q of the algorithm.*

(1) Construct the market subhypergraph $\hat{H}_M(\mu(S^{(q)}))$ induced by the capabilities $\mu(S^{(q)})$. Denote its components by $\hat{C}_r^{(q)}$, where $r \in \mathcal{P}^{(q)} = \{1...p\}$ is the index of the components.

(2) Create a set of interim firms F_r from the capabilities $\mu(S^{(q)})$ such that for all $r \in \mathcal{P}^{(q)}$, $\mu(F_r) = \hat{C}_r^{(q)}$ and $|F_r| = |\hat{C}_r^{(q)}|$.[33]

(3) Let all interim firms F_r ($r \in \mathcal{P}^{(q)}$) undertake their respective most profitable demerger.[34] *If the most profitable demerger is not unique, and an interim firm F_r is indifferent between demerging into the firms $\{\mathcal{F}_1, \dots, \mathcal{F}_n\}$, then execute any demerger $\mathcal{F} \in \{\mathcal{F}_1, \dots, \mathcal{F}_n\}$ such that there does not exist $\mathcal{F}' \in \{\mathcal{F}_1, \dots, \mathcal{F}_n\}$ for which (i) for all $i' \in \mathcal{F}'$, there exists a firm $i \in \mathcal{F}$ such that $\mu(F_{i'}) \subseteq \mu(F_i)$; and (ii) $|\cup_{i\in\mathcal{F}} F_i| > |\cup_{i\in\mathcal{F}'} F_i|$.*[35] *Denote the set of firms generated from a demerger of firm F_r by $\mathcal{F}_r^{(q)}$. Denote the set of all firms generated in this step by $\mathcal{F}^{(q)} = \bigcup_{r\in\mathcal{P}^{(q)}} \mathcal{F}_r^{(q)}$.*

(4) Update the firm hypergraph such that $H_F^{(q)} = H_F^{(q)}(\mathcal{B}, \bigcup_{1\le s\le q} \mathcal{F}^{(s)})$.

(5) If $H_F^{(q)} = H_F^{(q-1)}$, return the hypergraph $H_F^{(q)}$ and terminate the algorithm. Otherwise, set $S^{(q+1)} = S^{(q)} \setminus \{\cup_{i\in\mathcal{F}^{(q)}} F_i\}$, add a count to q and move back to (1).

The following lemma is crucial for showing that Algorithm 1 always generates a stable industry structure.

Lemma 3 (Concentric property)**.** *Fix any firm hypergraph $H_F^{(u)}$ created by Algorithm 1. Then for any firm $k \in \mathcal{F}^{(q+1)}$, there must exist $i \in \mathcal{F}^{(q)}$ such that $\mu(F_k) \subseteq \mu(F_i)$.*

Lemma 3 is proven in Appendix C, and is a striking and general feature of the class of hypergraphs generated by Algorithm 1. In particular, it implies

Corollary 1. *For any two firms i, k in any firm hypergraph $H_F^{(u)}$ generated by Algorithm 1, either (i) $\mu(F_i) \cap \mu(F_k) = \emptyset$; or (ii) either $\mu(F_i) \subseteq \mu(F_k)$ or $\mu(F_k) \subseteq \mu(F_i)$.*

Figures 10 and 11 illustrate Algorithm 1 and Lemma 3 at work. Given the market hypergraph H_M shown in Figure 10 (a) we initialize the firm hypergraph to empty and assign all competencies to the set of unassigned competencies (i.e., $S^{(0)} = \mathcal{B}$).

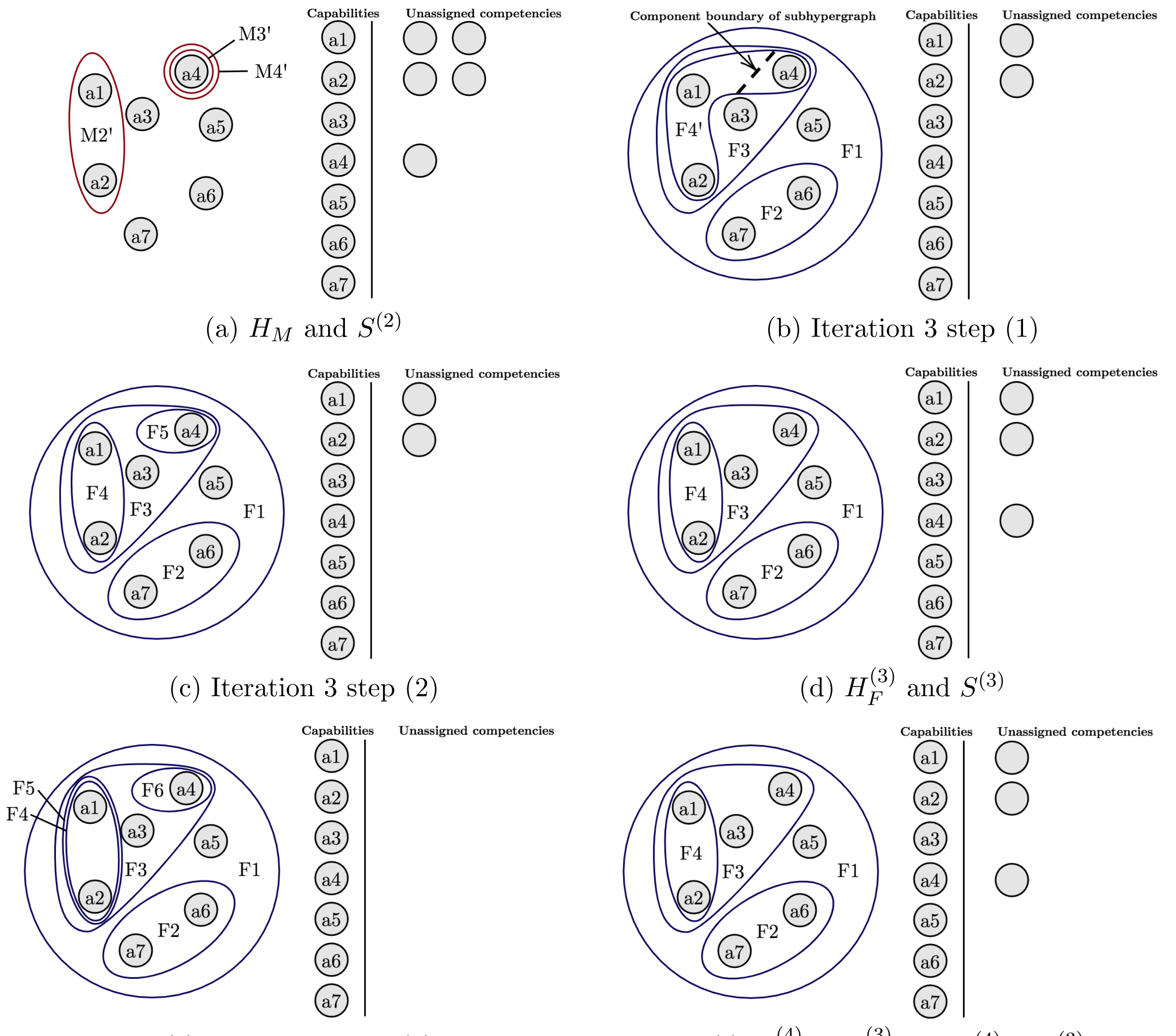
In iteration 1, a giant firm 1 is created with capabilities $\mu(F_1) = \mu(S^{(0)}) = \mathcal{A}$ (Figure 10 (b)). In this case, the market hypergraph has a single component and firm 1 has all capabilities. By Assumption 1, firm 1 has no net profitable demerger and we move to the next iteration.

In iteration 2 step (1), a market subhypergraph is constructed using the capabilities $\mu(S^{(1)}) = \mathcal{A} \setminus \{a_5\}$ (Figure 10 (c)). In step (2), we then partition the capabilities $\mu(S^{(1)})$ along the component boundaries of the market subhypergraph thereby creating firms 2 and 3. Supposing that after this neither firm 2 nor 3 have a profitable demerger, we are ready to move onto the next iteration. The firm hypergraph $H_F^{(2)}$ is reached at the end of iteration 2 and shown in Figure 10 (d).

[33]This implies: (i) each interim firm is matched to a corresponding component; (ii) each interim firm holds the same capabilities as the component it is matched to; and (iii) all interim firms never hold more than a single competency corresponding to the same capability.

[34]The order in which this is done does not matter. The profitability of each demerger is independent of which

Figure 11: Example of hypergraph construction via Algorithm 1 (Part 2 of 2)



(a) H_M and $S^{(2)}$

(b) Iteration 3 step (1)

(c) Iteration 3 step (2)

(d) $H_F^{(3)}$ and $S^{(3)}$

(e) Iteration 4 step (2)

(f) $H_F^{(4)} = H_F^{(3)}$ and $S^{(4)} = S^{(3)}$

In iteration 3 step (1), we first construct the market subhypergraph with the unassigned capabilities $\mu(S^{(2)}) = \{a_1, a_2, a_4\}$ (see Figure 11 (a)), and the component boundary of the market subhypergraph is shown in Figure 11 (b). In step (2), we create two interim firms 4 and 5 along the component boundary of the market subhypergraph (see Figure 11 (c)). In step (3), we let firms 4 and 5 each undertake their most profitable demerger. Suppose firm 4 does not find it net profitable to undertake any demerger, but firm 5's most profitable demerger is to dispose of its sole competency corresponding to capability a_4. The resultant firm hypergraph $H_F^{(3)}$ is shown in Figure 11 (d), and we move to the next iteration.

In iteration 4 step (1), we again construct the market subhypergraph with the unassigned capabilities $\mu(S^{(3)}) = \{a_1, a_2, a_4\}$; in step (2), we create two interim firms 5 and 6 as shown

other demergers are undertaken. Take two firms i and k that are considering demerging. By construction, for every market j, if $\mu(F_i) \cap M_j \neq \emptyset$ then $\mu(F_k) \cap M_j = \emptyset$ and so firm i would choose to operate in a disjoint sets of markets from firm k.

[35]Since condition (ii) is strictly monotonic, there always exists at least one such demerger.

in Figure 11 (e); in step (3), we let firms 5 and 6 undertake their most profitable demerger. Observe that in iteration 3, a firm identical to firm 6 found it net profitable to dispose of its only competency. Since firm 6 faces identical competition in markets 3 and 4, it must also find it net profitable to dispose of its only competency. Supposing firm 5 cannot generate enough profits in market 2 to sustain the cost of maintaining its two competencies, its most profitable demerger will also be to dispose of all its competencies. The resultant firm hypergraph $H_F^{(4)}$ is shown in Figure 11 (f). Since this is identical to the firm hypergraph at the end of iteration 3 (i.e., $H_F^{(4)} = H_F^{(3)}$), we terminate the algorithm and return this firm hypergraph $H_F^* = H_F^{(4)}$.

The stable industry structures generated by Algorithm 1 capture several realistic features. For instance, multiple large conglomerates can co-exist, competing against each other across many markets. As we observed in the introduction, this is characteristic of contemporary industrial organisation. Second, giant conglomerates can co-exist with smaller and more specialized firms.

6 Necessary existence of a large firm

We have shown the upper bound on the size of firms in stable industry structures is equal to $|C_{max}|$, the size of the largest component of the market hypergraph, and this upper bound can increase abruptly as markets become increasingly connected. We have also found relatively weak conditions under which this upper bound is always tight. In this section, we will establish a *lower bound* on the number of competencies maintained by the largest firm in all stable industry structures. We continue to maintain Assumptions 1 and 2 as well as introduce a final additional assumption. This assumption imposes more structure on the relationship between synergies by assuming that profits are supermodular in deployed capabilities.

Assumption 3. *Let H_F denote the firm hypergraph prior to a merger between firms i and k and H'_F denote the firm hypergraph after the merger between firms i and k in which firms l and h are created with capabilities $\theta_{lj} = \theta_{ij} \cup \theta_{kj}$ and $\theta_{hj} = \theta_{ij} \cap \theta_{kj}$. Then, if $|\mu(F_i)| = |F_i|$ and $|\mu(F_k)| = |F_k|$, $\pi_{lj}(H_M, H'_F) + \pi_{hj}(H_M, H'_F) \geq \pi_{ij}(H_M, H_F) + \pi_{kj}(H_M, H_F)$ with strict inequality if both (i) $\max\{\pi_{ij}, \pi_{kj}\} > 0$; and (ii) $\theta_{ij} \not\subseteq \theta_{kj}$, $\theta_{kj} \not\subseteq \theta_{ij}$.*

Supermodularity is a standard way of incorporating complementarities. In terms of the profit function it requires that it is profitable to create a very strong firm through a merger and spin-off a second weaker firm from the duplicate capabilities.

6.1 Scarce capabilities

We now introduce the concept of scarce capabilities and explore the implications of such capabilities for the necessary presence of a large firm in stable industry structures.

Definition (Scarce capabilities). *A set of capabilities $A \subseteq \mathcal{A}$ is scarce if and only if for all $a \in A$, $|\mu(a)| = 1$.*[36]

[36] We saw in Figure 4 (b) and (c) that the 'rarity' of a given capability can influence the competitive dynamics in associated markets. We use the terminology 'scarce' to refer to the extreme case when $|\mu(a)| = 1$.

There are many examples of scarce capabilities: pharmaceutical patents, proprietary algorithms, certain data, certain human capital, etc. These capabilities in turn offer firms competitive advantages in markets which value them, not simply because they might be valuable, but also because other firms are unable to leverage the same capability. Indeed, firms often go great lengths to ensure such capabilities remain out of reach for their competitors—for example, through protracted and costly patent lawsuits, non-compete clauses for key workers, or resources spent to protect themselves against corporate espionage.

The presence of scarce capabilities has interesting implications for the size of firms.

Proposition 3. *Suppose Assumptions 1 - 3 hold, and some set of capabilities $A \subseteq \mathcal{A}$ are scarce and path connected on the subhypergraph $\hat{H}_M(A)$. Then for sufficiently low convexity of κ, in all stable industry structures, the largest firm must maintain at least $|\{\bigcup_{j\in\mathcal{M}} M_j \,:\, A \cap M_j \neq \emptyset\}|$ capabilities.*

An immediately corollary of Proposition 3 is that in the limit, as all capabilities become scarce, our lower bound and upper bound coincide.

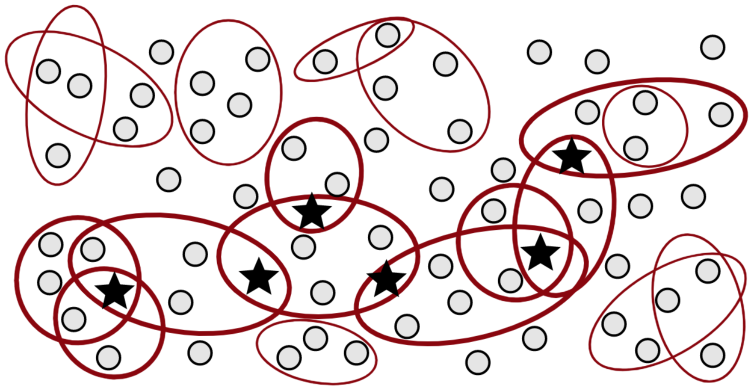
Corollary 2. *Denote the proportion of scarce capabilities with $s := \max_{A\subseteq\mathcal{A}} |A|/|\mathcal{A}|$. Then if $s = 1$, $|\{\bigcup_{j\in\mathcal{M}} M_j \,:\, A \cap M_j \neq \emptyset\}| = |C_{max}|$.*

While the upper and lower bound always coincide as all capabilities become scarce, far fewer capabilities need to be scarce for this outcome to obtain. Figure 12 provides an illustration. This figure takes the example of a market hypergraph with a giant component we saw in Figure 7 (b) and identifies those capabilities that are scarce by using stars to represent them. The markets that value at least one of these scarce capabilities are shown in bold. In this case all the scarce capabilities are path connected on H_M. Proposition 3 then shows that, when capability maintenance costs are not too convex and Assumptions 1 - 3 hold, in all stable industry structures there must exist a firm that holds all these capabilities. Indeed, in this case, these capabilities correspond to those that are in the largest component of the market hypergraph and so the bound we found in Proposition 1 is satisfied in all stable industry structures (and such a stable industry structure exists by Theorem 1).

While Figure 12 just provides an example in which the upper and lower bound on firm size coincide, we study random hypergraphs via simulations in Section 7 below and find that the upper and lower bounds are close to each other when only one in five capabilities are scarce (Figure 14 (a)).

The proof of Proposition 3 is deferred to Appendix C, though the key intuition is as follows. Suppose the set $A \subseteq \mathcal{A}$ is both scarce and connected on the subhypergraph $\hat{H}_M(A)$. Now consider some scarce capability $a_1 \in A$ held by firm 1. This capability not only serves as a source of competitive advantage across markets which value a_1, but also as a source of synergies which firm 1 might draw upon to merge with its competitors. It can be shown this implies for any market which values a_1, firm i must hold all capabilities valued by that market, otherwise it must have some profitable procurement or merger. Now consider another scarce capability

Figure 12: An example of how chains of scarce capabilities induces a lower bound



(a) H_M

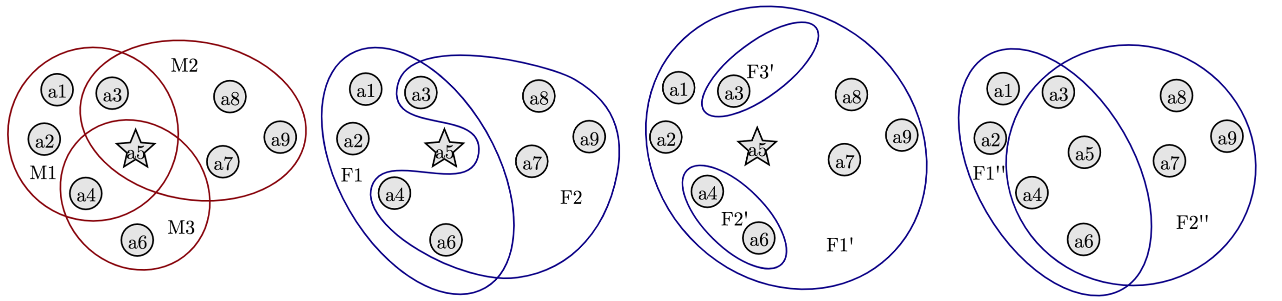
$a_n \in A$ held by firm n. The same argument must also apply, and firm n must hold all capabilities valued by all markets which value a_n. But since $\hat{H}_M(A)$ is connected and $a_1, a_n \in A$, then a_1 and a_n are path connected such that there exists some path $\{a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n\}$ where for $1 \leq i \leq n-1$, $a_i, a_{i+1} \in M_i$. This then implies there must be a cascade of mergers or procurements along this path such that firm 1 and firm n end up merging.

For concreteness, consider the three markets on the bottom left of Figure 12 which has been reproduced below in Panel (a) of Figure 13. Panel (b) shows a corresponding firm hypergraph where firm 1 holds the scarce capability a_5. Now observe either (i) firm 1 or firm 2 operates in market 2, or (ii) both firms 1 and 2 operate in market 2. In both cases, since neither firm has their capabilities relevant to market 2 nested within the other, this merger generates synergies for market 2. In particular, $\theta_{22} = \{a_3, a_7, a_8, a_9\}$ and $\theta_{12} = \{a_3, a_5\}$ and so the only capability firm 2 is missing for market 2 is the scarce capability a_5. Now consider a merger which generates the union of both firms' capabilities while spinning off the intersection as subsidiary firms as shown in Panel (c). This merger must, by the supermodularity of profits, be at least weakly profitable in markets 1 and 3, and strictly profitable in market 2. Hence for sufficiently low convexity of κ, this merger is strictly net profitable.

In contrast, consider an alternate firm hypergraph H_F'' shown in Panel (d) where a_5 is now held by both firms $1''$ and $2''$. Observe that firm $1''$ holds all the capabilities valued by markets 1 and 3, while firm $2''$ holds all the capabilities valued by markets 2 and 3. This implies all mergers between $1''$ and $2''$ create no synergies, and so the emergence of a giant firm in stable industry structures is no longer guaranteed. Nonetheless, we might, in principle, still see an emergence of a giant firm—as before, our assumptions are simply too weak to guarantee this. For instance, we might have multiple copies of $\{a_7, a_8, a_9\}$ which firm $1''$ profitably procures to strengthen its competitiveness in market 2.

Proposition 3 carries two key implications. First, scarce capabilities at the intersection of many markets can function as a crucial source of synergies for conglomeration. Second, although node-adjacency is not a transitive property, chains of adjacent and scarce capabilities can magnify this force. In the limit when all capabilities are scarce ($|\mathcal{A}| = |\mathcal{B}|$), then for sufficiently

Figure 13: An example of how scarce capabilities generate synergies



(a) H_M (b) H_F with scarcity (c) H_F' with scarcity (d) H_F'' without scarcity

low convexity of κ, the number of competencies maintained by the largest firm must be equal to the size of the largest component of the market hypergraph ($|F_{max}| = |C_{max}|$) in all stable industry structures. However, this condition is generally not necessary—Proposition 3 also shows this outcome will still obtain under much weaker conditions. A small set of scarce but widely valued capabilities can suffice. Indeed, instead of going through the somewhat painful calculations in Supplementary Appendix II to support Example 1 from earlier, we could instead of assumed that κ is not too convex and Assumptions 1 - 3 hold and applied Proposition 3. This allows us to immediately conclude that the formation of a giant conglomerate holding all capabilities is inevitable after the scarce capability a_1 becomes valued by all the markets.

6.2 Antitrust

So far we have largely ignored the role antitrust authorities might play in preventing the emergence of large conglomerates. We show now that our conclusions continue to hold when antitrust authorities block mergers that lessen competition. To this end we define a consumer surplus function for each market j, $CS_j(\boldsymbol{\theta}_j) : \Theta^n \to \mathbb{R}_{\geq 0}$, which maps firms' relevant capabilities in market j into consumer surplus. We then assume that the antitrust authorities block any merger which reduces consumer surplus in any market. A merger between two non-empty firms i and k is *permitted* if and only if as a result of the merger, consumer surplus weakly increases across all markets.

Definition (Antitrust Stable). *We say an industry structure (H_M, H_F) is antitrust stable if and only if there is no strictly net profitable procurement, demerger, permitted merger, or entry. An industry structure is antitrust unstable if it is not antitrust stable.*

Note that antitrust stable industry structures are a superset of stable industry structures. Hence Theorem 1 continues to hold when antitrust stable industry structures are considered in place of stable industry structures (and so the hypergraphs which were constructed to prove Theorem 1 are also antitrust stable).

On the other hand, establishing an analog of Proposition 3 which accomodates antitrust stability is more difficult. In principle, antitrust authorities might prohibit mergers and prevent the otherwise inevitable formation of large conglomerates. To make progress with this problem we need to add some structure to the consumer surplus function.

Assumption (Consumer surplus)**.** *We assume that the consumer surplus functions $CS_j(\boldsymbol{\theta}_j)$ satisfy the following (regularity) conditions:*

(i) ***Firms which are not operating in market j do not influence consumer surplus in j.*** *Recall the definition of $I(\boldsymbol{\theta}_j)$ and $\boldsymbol{\theta}_{Ij}$. For all $\boldsymbol{\theta}_j$ and $\boldsymbol{\theta}'_j$ such that (i) $I(\boldsymbol{\theta}_j) = I(\boldsymbol{\theta}'_j)$ and (ii) $\boldsymbol{\theta}_{Ij} = \boldsymbol{\theta}'_{Ij}$, it holds that $CS_j(\boldsymbol{\theta}_j) = CS_j(\boldsymbol{\theta}'_j)$.*

(ii) ***Stronger firms deliver more consumer surplus.*** *If $\theta_{ij} \supseteq \theta'_{ij}$ for all firms i, then $CS_j(\boldsymbol{\theta}_j) \geq CS_j(\boldsymbol{\theta}'_j)$.*

We view these assumptions on consumer surplus function as minimal and maintain them. However, we also make a stronger assumption—that the consumer surplus function is supermodular.

Assumption 4. *Let H_F denote the firm hypergraph prior to a merger between firms i and k and H'_F denote the firm hypergraph after the merger between firms i and k in which firms l and h are created with capabilities $\theta_{lj} = \theta_{ij} \cup \theta_{kj}$ and $\theta_{hj} = \theta_{ij} \cap \theta_{kj}$. Then, if $|\mu(F_i)| = |F_i|$ and $|\mu(F_k)| = |F_k|$, $CS_j(H_M, H'_F) \geq CS_j(H_M, H_F)$.*

This assumption requires the complementarities generated by a merger spill over into higher consumer surplus. Creating two less evenly matched competitors in place of two more evenly matched competitors can reduce competition in the market, so the assumption that consumer surplus is supermodular requires this to be offset by net efficiency gains generated by the merger. Another way of viewing the assumption is that for any merger generating synergies by combining complementary capabilities, there exist remedies that can be offered—and these remedies require the creation of a competitor firm that prevents previously deployed competencies from being hoarded. Appendix A shows that Assumption 4 is consistent with Cournot competition.

Assumption 4 leaves antitrust authorities with a significant role to play. For instance, consider a merger between firms i and k generating a single firm l. As long as there exists some market $j \in \mathcal{M}$ such that (i) $\pi_{ij} > 0, \pi_{kj} > 0$; and (ii) $\theta_{ij} \cap \theta_{kj} \neq \emptyset$ (i.e. the merger is not perfectly synergistic), the merger might be prohibited.

As before, this additional assumption implies scarce capabilities inevitably lead to large conglomerates.

Proposition 4. *Suppose Assumptions 1 - 4 hold, and some set of capabilities $A \subseteq \mathcal{A}$ are scarce and connected on the subhypergraph $\hat{H}_M(A)$. Then for sufficiently low convexity of κ, in all antitrust stable industry structures, the largest firm must maintain at least $|\{\bigcup_{j \in \mathcal{M}} M_j : A \cap M_j \neq \emptyset\}|$ capabilities.*

7 Simulations

There is a large literature on merger simulations (see, for example, Werden and Froeb (2008) and Mermelstein et al. (2020)). The focus of this literature is typically centred on evaluating the precise impact of specific mergers or studying optimal antitrust policy in dynamic settings. By contrast, our aim in this section is primarily to verify that the forces we have identified in the preceding sections continue to operate in a wider class of environments.

We first show sharp transitions in the upper and lower bounds on the market hypergraph continue to obtain without there being a large number of capabilities. We then introduce the firm hypergraph and show large firms do endogenously emerge through simple best response dynamics without imposing specific requirements on the convexity of competency maintenance costs or scarcity of capabilities. In particular, they emerge via a sequence of firm hypergraphs $(H_F^{(1)}, H_F^{(2)}, ..., H_F^{(u)})$ where for all $1 \leq q \leq u$, $H_F^{(q)}$ is reached from $H_F^{(q-1)}$ via a strictly net profitable deviations. In section 5, we noted cycles of profitable deviations might in principle exist. These simulations offer tentative evidence that such cycles are atypical, and that simple sequential best responses tend to converge to stable hypergraphs.[37] Finally, we show via a simple rule-of-thumb that hoarding multiple competencies corresponding to the same capability might be dynamically profitable by suppressing future competition.

7.1 Transition of Upper and Lower Bounds

We defer the details of the simulations to Supplementary Appendix III and instead provide only a coarse overview here. Our broad approach is to draw market hypergraphs probabilistically where all edges of size k realise with uniform probability p_k. This is a generalization of Erdős-Rényi random graph models[38] and we control the expected degree of the random hypergraph. As a first exercise, we simply consider how the size of the largest component varies with the expected degree. Specifically we look for a sharp increase in the size of the largest component with a realistic number of capabilities. Figure 14 (a) shows that sharp transitions do in fact occur and are not just limit phenomena.[39] Further, we continue to let s denote the proportion of scarce capabilities and draw $s|\mathcal{A}|$ scarce capabilities uniformly at random to compute the corresponding lower bound. Figure 14 (a) shows that the lower bound undergoes a similar sharp transition, and that 20% of capabilities being scarce is sufficient for the lower bound to be fairly close to the upper bound.[40]

[37] Over the course of the simulations, we did not find a single instance of such a cycle. The simulation algorithm detailed in Supplementary Appendix III.3 always converged to a stable industry structure.

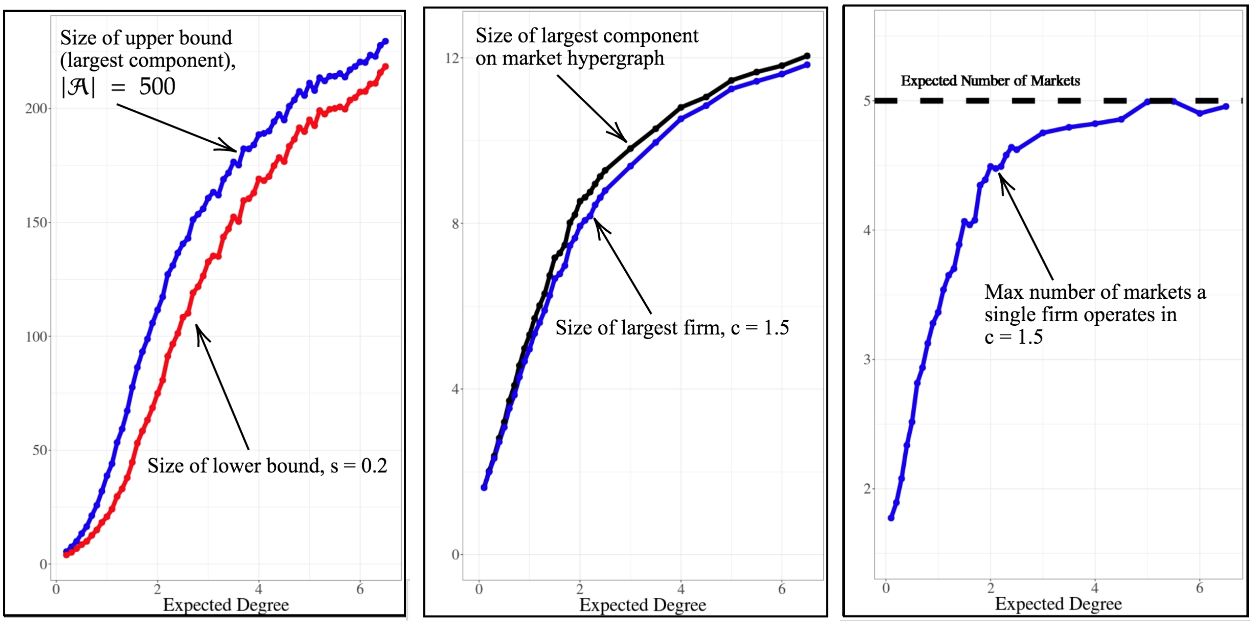
[38] They are a special case where $p_2 > 0, p_k = 0$ for all $k \neq 2$.

[39] If we let certain capabilities be valued by markets with disproportionate probability, this sharpens the transition further (see Supplementary Appendix III).

[40] See Supplementary Appendix III for results on a wider range of $|\mathcal{A}|$ and s.

Figure 14: The emergence of large firms as the expected degree of H_M increases

Note: Averaged over 2500 iterations.



(a) Upper & Lower Bound (b) $\max_{i\in\mathcal{N}} |F_i|$ (c) $\max_{i\in\mathcal{N}} |J_{i,\pi>0}|$

7.2 Giant firms

Next, we introduce firms to the simulations. We also draw initial firm hypergraphs at random and use a linear Cournot model to model competition in markets, setting the maintenance costs of a firm i proportional to $|F_i|^c$ where $c > 1$ is a convexity parameter (the details are again deferred to Supplementary Appendix III). We then sequentially implement the most net profitable deviations available across all firms until we reach a stable industry structure. This is computationally challenging and in practice, it is only feasible to run such simulations with a small number of capabilities (fifteen). Even then, we constrain the space of possible demergers such that (i) a firm can demerge into only two firms rather than any number of firms; and (ii) two firms can only merge by taking the union of their competencies.[41] Figure 14 (b) shows that relatively large firms emerge endogenously even for moderate convexity ($c = 1.5$) of competency maintenance costs,[42] and tracks the upper bound closely. Resultantly, the growth in the size of the largest firm is especially steep in the region with low expected degrees. While the number of competencies held by the largest firm is a natural measure of conglomeration, it is also worth noting that in our simulations, firms grow not simply by procuring capabilities for markets they already operate in, but also find it profitable to undertake 'scope mergers'—those which take it into new markets—as discussed in the introduction. This is reflected in Figure

[41] A firm holding 15 competencies would have up to approximately 1.3×10^9 unique demergers if all demergers were permitted. Further, we do not constrain the capability-competency mapping to be one-to-one, and so there could be many more competencies than there are capabilities, which further further enlarges the space of deviations.

[42] See Supplementary Appendix III for results on a wider range of convexities.

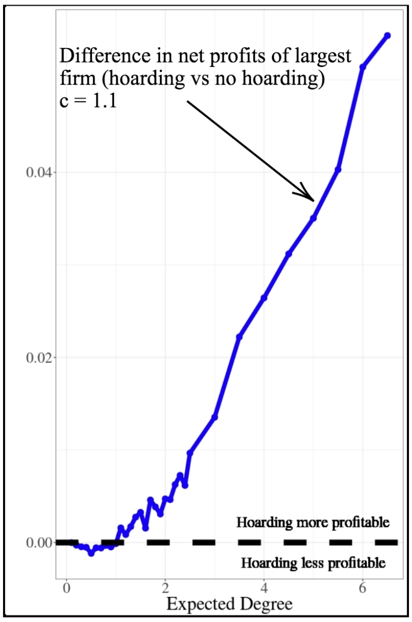
14 (c) where the maximum number of markets a single firm enters also increases sharply with the expected degree of the market hypergraph.

7.3 Hoarding

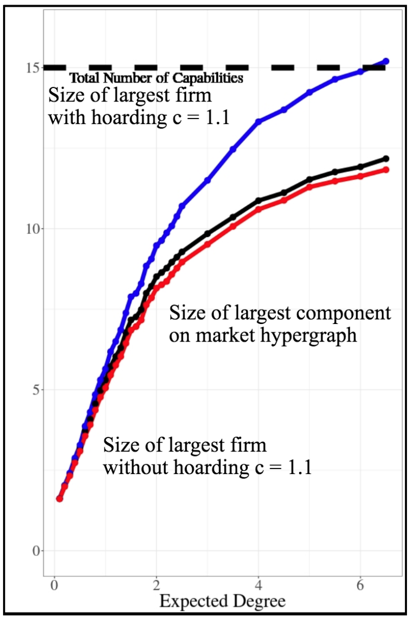
Finally, we use simulations to consider the question of hoarding. This is motivated by possible nascent trends of tech conglomerates to hoard capabilities.[43]

Figure 15: Sharp transitions as expected degree of H_M

Note: Averaged over 2500 iterations.



(a) Diff. in net profits



(b) $\max_{i \in \mathcal{N}} |F_i|$

In our model, duplicate capabilities—multiple competencies corresponding to the same capability—have no myopic value. They are costly to maintain, but do not increase a firm's competitiveness in any market. As such, firms always have a profitable deviation to dispose of them. However, hoarding could, in principle, be dynamically profitable. By hoarding duplicate capabilities, a firm can undermine the strength of competitors in the future by preventing them from procuring the necessary competencies to compete effectively, as well as by precluding potential entries. We model hoarding with a simple and intuitive heuristic: firms search for profitable demergers and disposals over the capability rather than competency space (e.g. for disposals, it chooses to either dispose or retain all copies of the same capability; the details are again deferred to Supplementary Appendix III). Under this rule-of-thumb, hoarding is indeed dynamically profitable, as shown in Figure 15 (a). Moreover, the practice of hoarding effectively makes more

[43]See, for instance, `https://www.economist.com/business/2016/11/05/tech-firms-shell-out-to-hire-and-hoard-talent` which describes how startups struggle to compete with tech conglomerates for scarce AI talent.

capabilities scarce and so by Propositions 3 and 4 contributes to the emergence of giant firms, as shown in Figure 15 (b).[44]

8 Conclusions

We have proposed a capability-based explanation for the sudden and unprecedented emergence of what we have termed internet conglomerates. This helps to close a gap between the economics and management literatures. Our framework is relatively simple, versatile and amenable. While here we use it to study the forces underlying recent trends toward conglomeration, it might also be used to address a range of other theoretical and empirical questions. For example, it could underpin a dynamic analysis of the evolution of industry structure. Such an approach would be able to systematically address issues such as the hoarding of capabilities, which we briefly considered in the context of simulations. This, in turn, has important implications for antitrust policy, particularly regarding conglomerate mergers and regulation of anticompetitive behaviour. Such questions about hoarding inevitably turn on scarce capabilities—our framework offers a tractable means of putting them at the heart of the analysis. More generally, there are interesting questions on joint ventures, the formation of syndicates to bid on large scale projects, and optimal conglomerate partitioning that we believe our framework is well suited to analyse.

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[44]See Supplementary Appendix III for results on a wider range of convexities.

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A Cournot Competition

In this appendix we nest the classical Cournot competition in our model. In particular, we let firms simultaneously choose how much to produce in each market and take a zero output decision of firm i in market j to mean that the firm i does not enter market j. The output choice of firm i in market j is given by $q_{ij} \geq 0$ and the vector $\boldsymbol{q}_i \in \mathbb{R}^m_+$ represents i's entry and output choices in all markets.

We let $Q_j = \sum_{i=1}^n q_{ij}$ be the total output of all firms in market j, and let $P_j(Q_j)$ be the inverse demand function for market j. We make the following assumptions on the inverse demand function.

Assumption A.1 (Inverse Demand). *For all markets j:*

(i) there exists a $\zeta \in (0, \infty)$ such that $P_j(Q_j) > 0$ for $Q_j < \zeta$ and $P_j(Q_j) = 0$ for $Q_j \geq \zeta$; and

(ii) there exists a constant $\alpha < 0$ such that $P_j'(Q_j) < \alpha$ for all $Q_j < \zeta$.

Firm i's profits in market j are:

$$\pi_{ij}(q_{ij}, Q_{-ij}) = (P_j(Q_j) - c_j(\theta_{ij}))\, q_{ij},$$

where $Q_{-ij} = Q_j - q_{ij}$ is the output of firms other than i in market j and $c_j(\theta_{ij})$ is the marginal cost of firm i in market j.

Assumption A.2 (Marginal costs). *$c_j \in \mathcal{C}_j$, where $\mathcal{C}_j$ is the set of cost functions $c_j : 2^{M_j} \to \mathbb{R}_+$ satisfying the following two conditions:*

(i) $c_j(\emptyset) \geq P_j(0)$ for all $c_j \in \mathcal{C}_j$; and

(ii) if $\theta'_{ij} \succeq \theta_{ij}$, then $c_j(\theta'_{ij}) \leq c_j(\theta_{ij})$.

The first condition implies that all firms will only ever be able to profitably enter market j if they have at least one relevant capability for market j and bounds the inverse demand curve from above. The second condition requires i's marginal cost in market j to be weakly lower when i has more relevant capabilities (in set inclusion) for market j. Collectively these conditions imply that $c_j(\theta) \in [c_j(M_j), c_j(\emptyset)]$ for all $\theta \subseteq M_j$.

Remark A.1. *We could instead assume that relevant capabilities increase profits by increasing consumers' willingness to pay, rather than affecting marginal costs, and our results would go through unchanged. Specifically, we could let all firms have identical marginal costs $\bar{c}_j$, but generalize the price firm i receives in market j to $P_j(Q_j) + f_j(\theta_{ij})$, where $f_j : 2^{M_j} \to \mathbb{R}_+$ is a function satisfying the following conditions: (i) $\bar{c}_j \geq P_j(0) + f_j(\emptyset)$; (ii) $\bar{c}_j \geq f_j(M_j)$; and (iii) if $\theta'_{ij} \succeq \theta_{ij}$, then $f_j(\theta'_{ij}) \geq f_j(\theta_{ij})$. To see that everything then goes through unchanged consider letting $\bar{c}_j = c_j(M_j)$ and $f_j(\theta_{ij}) = c_j(M_j) - c_j(\theta_{ij})$.*

Firm i's overall profitability is then given by the sum of their profits across all markets, and so when firms' capabilities are fixed, firm i solves the problem:

$$\max_{\boldsymbol{q}_i \in \mathbb{R}_+^m} \sum_{j=1}^{m} \pi_{ij}(q_{ij}, Q_{-ij}).$$

Standard results apply to our setting. Fixing the capabilities of firms, there is a unique Nash equilibrium output choice for all firms in all markets. We let $\boldsymbol{q}_i^*$ denote the unique Nash equilibrium output choices of firm i.

Remark A.2. *There is a unique Nash equilibrium. In equilibrium, firms who are more capable in a given market produce more: If $\theta_{ij} \succeq \theta_{kj}$ then $q_{ij}^* \geq q_{kj}^*$, with strict inequality if $q_{ij}^* > 0$.*

Given our assumptions, when $m = 1$ it follows directly from Gaudet and Salant (1991) that there is a unique equilibrium. This result is easily extended to our setting because each firm's profitability in each market is independent of its output choices in other markets. It is also intuitive that more capable firms in a given market have higher output because they have lower marginal cost. An immediate implication is that the firms entering a given market are the most capable (i.e., those with the lowest marginal costs for that market).

All our primitive conditions on profits are satisfied by this Cournot model. As $P_j(0) \leq c(\emptyset)$ firms with no capabilities makes zero profits (condition (i)). Conditions (ii)-(iii) require that only firms operating in a market affect the profits obtained in that market and that firm labels do not matter, both of which follow immediately from the Cournot formulation. Condition (iv), which in the Cournot setting requires that profits increase when others' constant marginal costs increase, is also a standard property of the model.

In Section 6.2 we introduced a consumer surplus function. The primitives we imposed on consumer surplus also follow from the Cournot formulation. Condition (i), that only firms operating in market j affect consumer surplus in market j is immediate. Condition (ii) requires of the Cournot model that if the marginal costs of all firms competing in a given market weakly decrease, with at least one strictly decreasing, then consumer surplus increases. This is also a well known property of the Cournot model. Further note that although this decrease in marginal cost for one firm can drive some others out of the market, this can only occur if the market price falls below the marginal cost of these firms. But if the market price decreases, consumer surplus necessarily increases.

Assumption 1, which places a lower bound on the value of additional capabilities, is also consistent with the Cournot model. This is equivalent to placing a lower bound on the amount by which a firm's marginal cost decreases in a given market it competes in when it gains access to an additional capability valued by that market. In such a scenario a firm i responds to its lower marginal cost in market j by increasing its output (while other firms reduce their output because these choices are strategic substitutes), and firm i's profits in market j strictly increases. As i's profit in market j is strictly increasing in the size of the marginal cost reduction in market j, the primitives $c_j(\cdot)$ and $\kappa(1)$ can be chosen to guarantee that the profit increase from gaining access to additional relevant capability is at least $\kappa(1)$, as required.

Assumption 2 can be interpreted as an assumption on how capabilities lower marginal costs. After combining the non-overlapping capabilities of different firms to create a single firm l, firm l will have a weakly lower marginal cost in all markets than that of any of its constituent firms. Consider a market j. If one or fewer of the constituent firms competed in market j prior to the merger, then profits must weakly increase. However, if two or more of the constituent firms competed in market j prior to the merger, then the analysis is a little more subtle. As these firms have disjoint capabilities, firm l has a strictly lower marginal cost than any of them i.e. there are strictly positive synergies from the merger. All else equal this serves to increase profits. However, there is also a reduction in competition in market j which can increase or decrease the merging firms' profits. Indeed, the possible reduction in profits from such mergers is known as the Cournot paradox (see, for example, Szidarovszky and Yakowitz, 1982; Salant et al., 1983). In the Cournot context, Assumption 2 just requires that any such effects are outweighed by the reduction in marginal cost obtained through the synergies.

Assumption 3 requires that when two firms i and k merge to create a firm l with the union of their capabilities, and spin off a competitor h containing the intersection of their capabilities, the merger is profitable. We can decompose this change into two adjustments in marginal costs. First, we decrease the marginal cost of production for firm i from $c_j(\theta_{ij})$ to $c_j(\theta_{lj})$ where $c_j(\theta_{lj}) < c_j(\theta_{ij})$. Next, we increase the marginal cost of firm k from $c_k(\theta_{ij})$ to $c_h(\theta_{lj})$ where $c_h(\theta_{lj}) > c_k(\theta_{ij})$. As long as the marginal cost reduction achieved by firm i is sufficiently large relative to the marginal cost increase for firm k, these adjustments will lead to higher joint profits for i and k. Moreover, such changes will also increase overall output Q_j, leading to higher consumer surplus, hence satisfying Assumption 4.

A.1 Linear Cournot (Special Case)

We now show that our assumptions are all satisfied in a special case of the Cournot Model.

Proposition A.1. *If firms compete à la Cournot with:*

(i) $P_j(Q_j) = |M_j| - Q_j$

(ii) $c_j(\theta_{ij}) = |M_j| - |\theta_{ij}|$

(iii) $\kappa(1) \leq 1/4$

then Assumptions 1 - 4 are satisfied.

The proof of Proposition A.1 is in the Supplementary Appendix I.2.

B A Counterexample

In this appendix we present an overview of a counterexample in which markets value more capabilities but firms end up holding fewer capabilities.

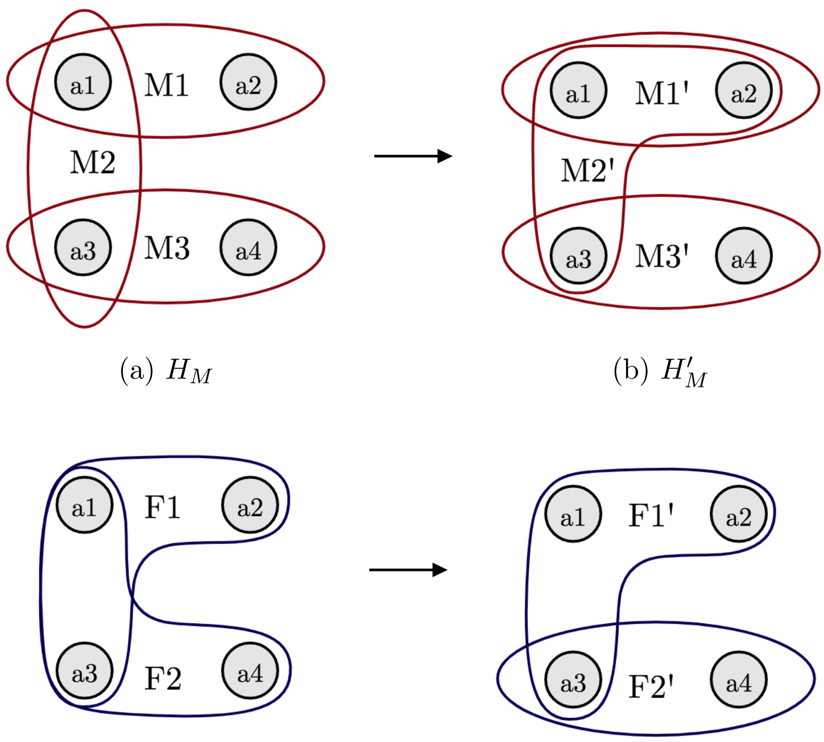
Example 2. *Panel (a) depicts the initial market hypergraph H_M with 3 markets, each valuing two capabilities. Panel (c) depicts the corresponding firm hypergraph H_F, where firm 1 holds capabilities a_1, a_2, a_3 and firm 2 holds capabilities a_1, a_3, a_4. For simplicity, assume $S = \emptyset$ and there are two competencies corresponding to a_1 and a_3, and a single competency corresponding to a_2 and a_4. For similar reasons as before, we defer the details to Supplementary Appendix II where claims are formalised.*

Begin by noting that the industry structure (H_M, H_F) is stable. To informally see why, first note $S = \emptyset$ and so there is no possible entry or procurement. Now as long as firms 1 and 2 make sufficient gross profits in markets 1, 2, and 2, 3 respectively, there are no profitable disposals or demergers. To see the latter, observe that any partition of capabilities necessarily destroys synergies. For instance, the partition of 1's capabilities into $\{a_1, a_2\}$ and $\{a_3\}$ destroys firm 1's synergies in market 2. Finally, notice that firm 1 and 2 would have no profitable merger as long as the cost of maintaining capabilities is sufficiently convex as to outweigh the gains in profits from increased market concentration.

Suppose now that market 2 additionally values a_2. The resultant market hypergraph, H'_M is shown on Panel (b). We now claim (H'_M, H_F) is no longer stable, and firm 2 has a unique profitable deviation to dispose of its capability a_1. The firm hypergraph after it does so H'_F is shown on Panel (d). The intuition is as follows. Under (H_M, H_F), firms 1 and 2 were equally competitive in market 2 since each held all the capabilities market 2 valued. But with the expansion of market 2, this has tilted the scales in favour of firm 1 and correspondingly depressed firm 2's profits in market 2. As such, a_1 cannot generate enough gross profits in markets $1'$ and $2'$ as to justify its marginal maintenance costs, and so firm 2 now finds it uniquely profitable to dispose of a_1 and focus its efforts in market $3'$.

We now claim (H'_M, H'_F) is stable. We will focus on ruling out profitable mergers between firms $1'$ and $2'$ and leave other deviations to Supplementary Appendix II. Once again, there are no synergies between firms $1'$ and $2'$: firm $1'$ holds call the capabilities valued by markets $1'$ and $2'$, while firm $2'$ holds all the capabilities valued by market $3'$. As such, a merger into firms with capabilities $\{a_1, a_2, a_3, a_4\}$ and $\{a_3\}$ would be as gross profitable as before, but by the convexity of κ, incur strictly more maintenance costs and hence be strictly unprofitable. Alternatively, the merger might generate a single firm with capabilities $\{a_1, a_2, a_3, a_4\}$ and so the resultant firm enjoys diminished competition in markets $2'$ and $3'$ and hence higher gross profits. However, as long as κ is sufficiently convex, this is outweighed by the increase in capability maintenance costs.

Figure 16: Firms contracting in response to markets valuing more capabilities



(a) H_M (b) H'_M

(c) H_F (d) H'_F

C Omitted Proofs

C.1 Proof of Lemmas 1 - 2

C.1.1 Proof of Lemma 1

Proof. If either $\mu(F_i) \not\subseteq \hat{A}_i$ or $|\mu(F_i)| < |F_i|$, then there exists a competency $b \in F_i$ that firm i does not use in any market. Disposing of one such competency therefore leaves i's relevant capabilities unaffected for all markets $j \in J_{i,\pi>0}$ and hence firm i's gross profit are unaffected. We now consider the case where $\mu(F_i) \subseteq \hat{A}_i$ and $|\mu(F_i)| = |F_i|$, but $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$ contains more than one component (condition (iii)). Denote the components of $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$ by $\{C_1, C_2, \ldots, C_p\}$ where $p > 1$. Now consider a demerger of firm i along the boundary of these components into the firms $\mathcal{F}_i$ such that $\{\mu(F_k) : k \in \mathcal{F}_i\} = \{C_1, C_2, \ldots, C_p\}$. This demerger is always feasible since $C \cap C' = \emptyset$ for all $C, C' \in \{C_1, C_2, \ldots, C_p\}$. In particular, for each market $j \in J_{i,\pi>0}$, there exists some firm $k \in \mathcal{F}_i$ such that $\theta_{kj} = \theta_{ij}$. By construction, all other firms $k' \in \mathcal{F}_i \setminus \{k\}$ cannot hold capabilities relevant to j, so $\theta_{k'j} = \emptyset$ and hence firm k makes identical gross profits as firm i in market j. Hence the demerged firms $\mathcal{F}_i$ make the same gross profits across all the markets as firm i. Therefore, there exists a demerger that firm i can undertake that weakly increases its gross profits. $\square$

C.1.2 Proof of Lemma 2

Proof. Consider a demerger of firm i into a set of firms $\mathcal{F}_i$. If all three conditions hold then $\mu(F_i) \subseteq \hat{A}_i$ and $|\mu(F_i)| = |F_i|$. Hence all possible disposals involve competencies that are used by firm i in at least one market and any disposal will strictly reduce i's gross profits. Further, as $\hat{H}_M(\mu(F_i) \cap \hat{A}_i)$ must contain a single component, there is a path on this market subhypergraph between any two capabilities. Hence all possible partitions of the capabilities into more than one non-empty partition element, will partition the capabilities associated with at least one market firm i originally operates in into more than one non-empty partition element. Thus for a demerger into firms $\mathcal{F}_i$ there will be at least one market $j \in J_{i,\pi>0}$ such that $\{\mu(F_k) \cap M_j\} \subset \{\mu(F_i) \cap M_j\}$ for all firms $k \in \mathcal{F}_i$. For all such markets j, as $|\mu(F_i)| = |F_i|$, the firms $\mathcal{F}_i$ created by the demerger have disjoint capabilities in these markets, and so by Assumption 2, the demerger is strictly unprofitable. For all other markets j', either there exists a firm $k \in \mathcal{F}_i$ such that $\mu(F_k) \cap M_{j'} = \mu(F_i) \cap M_{j'}$ or a similar argument as before applies. Hence, gross profits strictly decrease as claimed. □

C.2 Proof of Theorem 1

C.2.1 Notation and Lemmas for Theorem 1

We set out some notation in advance. Denote the last iteration of Algorithm 1 with u. Use $J_l = \{j \in \mathcal{M} : M_j \cap \mu(F_l) \neq \emptyset\}$ to denote the set of markets the firm l has relevant capabilities for. We can bipartition the markets J_l into the disjoint sets $J^{(q)}_{l,\pi>0}$ and $J^{(q)}_{l,\pi=0}$ which are respectively, the set of markets firm l operates in after iteration q, and the set of markets firm l has relevant capabilities for, but does not operate in after iteration q.

The following lemmas are helpful for proving Theorem 1.

Lemma 4 (Firms in the same iteration operate in disjoint markets)**.** *For any iteration q of Algorithm 1 and any two firms $i, k \in \mathcal{F}^{(q)}$, for sufficiently low convexity of κ, firms i and k operate in disjoint sets of markets in the industry structure $(H_M, H^{(q)}_F)$.*

Lemma 5 (Firms hold maximal feasible capabilities)**.** *For any firm $i \in \mathcal{F}^{(q)}$, if firm i is operating in market j in the industry structure $(H_M, H^{(q)}_F)$, then for sufficiently low convexity of κ, $\{\mu(S^{(q)}) \cap M_j\} \subseteq \mu(F_i)$.*

Lemma 6 (Fixed set of operating markets)**.** *For any q and any firm $i \in \mathcal{F}^{(q)}$, $J^{(q)}_{i,\pi>0} = J^{(u)}_{i,\pi>0}$.*

Lemmas 4 - 6 are subsequently proven in Appendix I.1.

The proof of Theorem 1 relies on Lemmas 3 - 6. While we take these lemmas as given for now, it is worth noting that they rely on each other in a consistent (non-tautological) way. Lemma 4 relies on none of the others; Lemma 5 uses Lemma 4; Lemma 3 uses Lemmas 4 and 5; Lemma 6 uses Lemma 3.

C.2.2 Proof of Theorem 1

Proof. Our proof is constructive, and proceeds by showing all possible deviations – disposals, demergers, procurements, mergers, and entries—undertaken by firms in the industry structure generated by Algorithm 1 cannot be strictly net profitable. The proof considers these deviations sequentially.

(i) No firm has a weakly net profitable disposal.

The essential underlying idea is as follows. In each iteration of Algorithm 1, a firm $i \in \mathcal{F}^{(q)}$ constructed in that iteration undertakes its most profitable demerger. This generates a set of markets that i is operational in. In subsequent iterations, new firms are created. If such firms enter these markets, they must, by construction, find it profitable to do so—otherwise an alternate demerger would have been undertaken. However, by Lemma 5, they must hold fewer relevant capabilities for such markets than firm i. Then by Assumption 1 each additional capability firm i deploys in such markets generates at least $\kappa(1)$ of gross profits, and so for κ sufficiently low convexity, firm i continues to find all disposals net unprofitable.

We now make this argument precise and proceed by induction. By construction, all firms in the set $\mathcal{F}^{(u)}$ have no net profitable disposal since they are created in the last iteration of Algorithm 1.[45] We now establish for any iteration $q < u$, if all firms $\mathcal{F}^{(q+1)}$ find it strictly net unprofitable to undertake any disposal in the industry structure $(H_M, H_F^{(u)})$, then all firms $\mathcal{F}^{(q)}$ must also find it strictly net unprofitable to undertake any disposal in the industry structure $(H_M, H_F^{(u)})$.

For a given firm $i \in \mathcal{F}^{(q)}$, denote the set of all firms generated in iteration $q + 1$ which are nested in i with $\mathcal{F}_{\subseteq i}^{(q+1)} := \{k \in \mathcal{F}^{(q+1)} : \mu(F_k) \subseteq \mu(F_i)\}$. It will be sufficient to show that if all firms in the set $\mathcal{F}_{\subseteq i}^{(q+1)} \subseteq \mathcal{F}^{(q+1)}$ have no net profitable disposal, then neither does firm i. First observe from Lemmas 4 and 6, for all other firms $i' \in \mathcal{F}^{(q)} \setminus \{i\}$ created in iteration q, $J_{i,\pi>0}^{(u)} \cap J_{i',\pi>0}^{(u)} = \emptyset$. The concentric property of Lemma 3 also implies no other firm in the set $\mathcal{F}^{(q+1)} \setminus \mathcal{F}_{\subseteq i}^{(q+1)}$ competes with firm i in overlapping markets. As such, the only firms which compete in the same markets as firm i are the firms $\mathcal{F}_{\subseteq i}^{(q+1)}$ and their nested firms.

We now rule out two trivial cases. First consider the case in which $\mathcal{F}_{\subseteq i}^{(q+1)} = \emptyset$ i.e. firm i has no nested firms. This might obtain because, having generated the firm i in iteration q, all interim firms generated in iteration $q + 1$ which intersect firm i face strong competition from the firms generated before iteration q, and so find it profitable to dispose of all competencies. If so, then by the above observation, there are no firms generated after iteration q which operate in the markets $J_{i,\pi>0}^{(u)}$. By our regularity assumption (iv), this implies firm i's profits in all such markets must remain unchanged between iteration q and iteration u. Since firm i found it net unprofitable to undertake any disposal in iteration q, then this must also be the case in industry structure $(H_M, H_F^{(u)})$. Next consider the case in which there exists some $k \in \mathcal{F}_{\subseteq i}^{(q+1)}$ such that $\mu(F_i) = \mu(F_k)$. We know Algorithm 1 never creates firms with multiple competencies corresponding to the same capability and since firms i and k have identical capabilities, they

[45] By the tie breaking rule, if they did have a net profitable disposal, they would have undertaken an alternate demerger rather than the one in fact performed.

must make the same profits in all markets, and bear the same maintenance costs. Then since firm k, by the induction hypothesis, finds all disposals strictly net unprofitable, so too must firm i. The remaining non-trivial case to consider is therefore when $\mu(F_k) \subset \mu(F_i)$ for all firms $k \in \mathcal{F}^{(q+1)}_{\subseteq i}$.

To rule out the remaining case, we will consider potential disposals undertaken by firm i, and show that i cannot find it net profitable to dispose of capabilities relevant to markets subsequent firms operate in, or capabilities only relevant to markets no subsequent firms operate in.

Consider some disposal of the capabilities $D \subseteq \mu(F_i)$ undertaken by i. We first rule out the possibility that $D \cap \{\bigcup_{j \in J^{(u)}_{k,\pi>0}} M_j\} \neq \emptyset$ for some $k \in \mathcal{F}^{(q+1)}_{\subseteq i}$. First observe $D = \mu(F_i) \cap \{\bigcup_{j \in J^{(u)}_{k,\pi>0}} M_j\}$ i.e. disposing all capabilities relevant to markets k operates in must be strictly dominated by an alternate disposal retaining the capabilities $\mu(F_k)$ for sufficiently low convexity of κ, since by the induction hypothesis, firm k does not find it net profitable to dispose of any capability and so it must be making strictly net positive profits. Further, for sufficiently low convexity of κ, this alternate disposal retaining $\mu(F_k)$ must also dominate one which retains only a strictly subset of the capabilities $\mu(F_k)$ since by the induction hypothesis, firm k does not find it net profitable to dispose of any capability. Next, observe each capability in the set $\left\{\mu(F_i) \cap \{\bigcup_{j \in J^{(u)}_{k,\pi>0}} M_j\}\right\} \setminus \mu(F_k)$ is valuable on the margin by Assumption 1, and so for sufficiently low convexity of κ, the disposal retaining capabilities $\mu(F_k)$ is in turn dominated by a disposal which retains all capabilities $\mu(F_i) \cap \{\bigcup_{j \in J^{(u)}_{k,\pi>0}} M_j\}$. We can therefore rule out any disposal D which disposes of any capability relevant to any market in the set $\bigcup_{k \in \mathcal{F}^{(q+1)}_{\subseteq i}} J^{(u)}_{k,\pi>0}$.

Finally, we consider disposals of capabilities relevant only to the markets $J_i \setminus \left\{\bigcup_{k \in \mathcal{F}^{(q+1)}_{\subseteq i}} J^{(u)}_{k,\pi>0}\right\}$, i.e. markets in which no firm in the set $\mathcal{F}^{(q+1)}_{\subseteq i}$ operates. We have already observed that the only firms created during or after iteration q competing in the same markets as firm i are the firms $\mathcal{F}^{(q+1)}_{\subseteq i}$ and their nested firms. By definition, such firms do not operate in the markets under consideration. Further, by regularity assumption (iv), firms which do not operate in a market do not influence the profitability of those which do. As such, for any such disposal of capabilities D by firm i, the decrease in gross profits in markets $J_i \setminus \left\{\bigcup_{k \in \mathcal{F}^{(q+1)}_{\subseteq i}} J^{(u)}_{k,\pi>0}\right\}$ as well as the decrease in capability maintenance costs must be identical in iteration q and u. Then since this disposal was strictly net unprofitable in iteration q, it must continue to be so in the industry structure $(H_M, H^{(u)}_F)$.

(ii) No firm has a weakly net profitable demerger.

We first show for κ not too convex, any demerger which disposes of any capabilities must be strictly dominated by a demerger which does not. We then show for κ not too convex, any partition (demerger without disposals) of a firm's capabilities must in turn be strictly dominated by no demerger at all.

For any iteration q, and any firm $i \in \mathcal{F}^{(q)}$, consider the demerger of firm i into the firms $\mathcal{F}_i$ such that

$$\mu(F_i) = \left\{\bigcup_{i' \in \mathcal{F}_i} \mu(F_{i'})\right\} \cup D,$$

where D is the set of capabilities which are disposed. We have already established in the special case where $|\mathcal{F}_i| = 1$ and $D \neq \emptyset$ these demergers are strictly net unprofitable (since they are simply disposals).

We now show any demerger with $|\mathcal{F}_i| > 1$ and $D \neq \emptyset$ is strictly unprofitable. Consider an alternative demerger of $\mathcal{F}_i$ into the single firm l where $\mu(F_l) = \{\bigcup_{i' \in \mathcal{F}_i} \mu(F_{i'})\}$. This is a pure disposal of capabilities, and so by the above argument, it must be strictly unprofitable and satisfies

$$\sum_{j \in \mathcal{M}} \Big(\pi_{ij}(H_M, H_F^{(u)}) - \pi_{lj}(H_M, H_F'^{(u)}) \Big) > \Big(\kappa(|F_i|) - \kappa(|F_l|) \Big), \tag{1}$$

where $H_F'^{(u)}$ is the firm hypergraph after iteration u but with firm i replaced with firm l. By Assumption 2, firm l must make at least as much gross profits in every market as the firms $\mathcal{F}_i$, so

$$\sum_{j \in \mathcal{M}} \Big(\pi_{lj}(H_M, H_F'^{(u)}) - \sum_{k \in \mathcal{F}_i} \pi_{kj}(H_M, H_F''^{(u)}) \Big) \geq 0, \tag{2}$$

where $H_F''^{(u)}$ is the firm hypergraph after iteration u but with firm i replaced with the firms $\mathcal{F}_i$. Combining inequalities (1) and (2),

$$\sum_{j \in \mathcal{M}} \Big(\pi_{ij}(H_M, H_F^{(u)}) - \sum_{k \in \mathcal{F}_i} \pi_{kj}(H_M, H_F''^{(u)}) \Big) > \Big(\kappa(|F_i|) - \kappa(|F_l|) \Big).$$

Since for any $\epsilon > 0$, we can reduce the convexity of κ until

$$\Big(\kappa(|F_i|) - \sum_{k \in \mathcal{F}_i} \kappa(|F_k|) \Big) - \Big(\kappa(|F_i|) - \kappa(|F_l|) \Big) < \epsilon,$$

for all κ not too convex,

$$\sum_{j \in \mathcal{M}} \Big(\pi_{ij}(H_M, H_F^{(u)}) - \sum_{k \in \mathcal{F}_i} \pi_{kj}(H_M, H_F''^{(u)}) \Big) > \Big(\kappa(|F_i|) - \sum_{k \in \mathcal{F}_i} \kappa(|F_k|) \Big).$$

Thus, the demerger under consideration is strictly net unprofitable.

We conclude by showing any demerger with $|\mathcal{F}_i| > 1$ and $D = \emptyset$ must also be strictly net unprofitable. First observe that, by the construction of firm i, $|\mu(F_i)| = |F_i|$. Next, recall that firm i was created in iteration q. By construction of the algorithm, firm i cannot have had a strictly profitable demerger at the end of iteration q. By Lemma 1, it must therefore have been the case that: (i) $\mu(F_i) \subseteq \hat{A}_i^{(q)}$ where $\hat{A}_i^{(q)} = \bigcup_{j \in J_{i,\pi>0}^{(q)}} M_j$; and (ii) the market subhypergraph $\hat{H}_M(\mu(F_i) \cap \hat{A}_i^{(q)})$ must contain a single component. By Lemma 6, $J_{i,\pi>0}^{(q)} = J_{i,\pi>0}^{(u)}$, we thus conclude: (i) $\mu(F_i) \subseteq \hat{A}_i^{(u)}$; and (ii) the market subhypergraph $\hat{H}_M(\mu(F_i) \cap \hat{A}_i^{(u)})$ must contain a single component. We have shown that the three conditions of Lemma 2 are satisfied. This implies that the demerger of firm i into firms $\mathcal{F}_i$ must strictly reduce gross profits. For any $\epsilon > 0$, we can reduce the convexity of κ until $\kappa(F_i) - \sum_{k \in \mathcal{F}_i} \kappa(|F_k|) < \epsilon$. Hence for all κ not too convex, the proposed demerger is strictly net unprofitable.

(iii) No firm has a strictly net profitable merger.

Without loss of generality, suppose $i \in \mathcal{F}^{(q)}$ and $k \in \mathcal{F}^{(p)}$ where $q \leq p \leq u$. We want to show i and k have no strictly net profitable mergers. Observe by the concentric property of Lemma 3, there must exist some firm $k' \in \mathcal{F}^{(q)}$ such that $\mu(F_k) \subseteq \mu(F_{k'})$. In the special case $q = p$, $k = k'$. Use $\{\hat{C}_r^{(q)}\}_{r=1}^{p}$ to denote the set of all components of the subhypergraph $\hat{H}_M(S^{(q)})$. By Lemma 1, if $i \in \mathcal{F}^{(q)}$, then there must exist $r \in \{1...p\}$ such that $\mu(F_i) \subseteq C_r^{(q)}$ and so let $\mu(F_i) \subseteq C_r^{(q)}$ and $\mu(F_{k'}) \subseteq C_{r'}^{(q)}$.

First suppose $k' = i$, then $\mu(F_k) \subseteq \mu(F_i)$ so k is nested within i. Then i and k have no feasible non-trivial merger.

Now suppose $k' \neq i$ and $r \neq r'$. This implies i and k' are on different components of $\hat{H}_M(S^{(q)})$ and so by Lemma 2, there cannot be a strictly net profitable merger between i and k'. Since $\mu(F_k) \subseteq \mu(F_{k'}) \subseteq C_{r'}^{(q)}$, this is also the case between i and k.

Finally, suppose $k' \neq i$ and $r = r'$, i.e. i and k' are subsets of the same component $C_r^{(q)}$. This implies i and k' were demerged from the same interim firm with capabilities corresponding to a component of $\hat{H}_M(S^{(q)})$. We first show there cannot be synergies between i and k', and then show this implies there cannot be synergies between i and k. Observe by construction $\mu(F_i) \cap \mu(F_{k'}) = \emptyset$ and recall $J_{i,\pi>0}^{(u)}$ is the set of markets in which firm i makes strictly positive profits under $(H_M, H_F^{(u)})$. Now by Lemma 4, $J_{i,\pi>0}^{(u)} \cap J_{k',\pi>0}^{(u)} = \emptyset$ and so there are no synergies among markets either firm operates in. We also need to rule out markets which neither i nor k' operates in, but the resultant firm generated by a merger between i and k' does. Suppose, towards a contradiction, there exists some $j \in \mathcal{M} \setminus \{J_{i,\pi>0}^{(u)} \cup J_{k',\pi>0}^{(u)}\}$ such that the firm with capabilities $\mu(F_l) = \mu(F_i) \cup \mu(F_{k'})$ can make strictly positive gross profits. By Lemma 5, there does not exist a firm $l \in \mathcal{F}^{(q)} \setminus \{i, k'\}$ such that $j \in J_{l,\pi>0}$. This then implies for sufficiently low convexity of κ, the demerger could not have been optimal in iteration q of the algorithm. This is because an alternate demerger generating a single firm with capabilities $\mu(F_i) \cup \mu(F_{k'})$ in lieu of firms i and k' would make strictly greater gross profits across markets, while leaving the gross profits of all firms $l \in \mathcal{F}^{(q)} \setminus \{i, k'\}$ unchanged. Then for sufficiently low convexity of κ, this alternate demerger must be strictly more profitable than the one in fact undertaken, a contradiction.

We now show that there cannot be synergies between i and k. Since $\mu(F_k) \subseteq \mu(F_{k'})$, $J_{i,\pi>0}^{(u)} \cap J_{k,\pi>0}^{(u)} = \emptyset$. Now suppose, towards a contradiction, there exists some market $j' \in \mathcal{M} \setminus \{J_{i,\pi>0}^{(u)} \cup J_{k,\pi>0}^{(u)}\}$ in which the firm with capabilities $\mu(F_i) \cup \mu(F_k)$ can make strictly positive gross profits. Since $\mu(F_k) \subseteq \mu(F_{k'})$, then $J_{k,\pi>0}^{(u)} \subseteq J_{k',\pi>0}^{(u)}$. We have already observed $j' \notin \mathcal{M} \setminus \{J_{i,\pi>0}^{(u)} \cup J_{k',\pi>0}^{(u)}\}$ since there cannot be synergies between i and k'. This implies $j' \in \{J_{k',\pi>0}^{(u)} \setminus J_{k,\pi>0}^{(u)}\}$ i.e. j' must be a market firm k' but not k operates in. But since the firm with capabilities $\mu(F_i) \cup \mu(F_k)$ can operate in market j' although firms i and k cannot individually do so, this implies $\theta_{ij} \neq \emptyset$

and $\theta_{kj} \neq \emptyset$ and so i or k' cannot have held the maximal set of capabilities for j in iteration q which, by Lemma 5, is a contradiction. As such, a merger between i and k cannot be strictly net profitable.

(iv) No firm has a strictly net profitable procurement.

Consider firm $i \in \mathcal{F}^{(q)}$ generated in iteration q, and denote the market component i was demerged from with $\hat{C}_r^{(q)} \supseteq \mu(F_i)$, recalling we use $\mathcal{F}_r^{(q)} \subseteq \mathcal{F}^{(q)}$ to denote the set of firms generated in iteration q from the component $\hat{C}_r^{(q)}$. Now suppose, towards a contradiction, that firm i has a strictly net profitable procurement $A \subseteq \mu(S^{(u)})$ from the capabilities unassigned after Algorithm 1 terminates.

First observe firm i must find it strictly net unprofitable to undertake any procurement of capabilities outside $\hat{C}_r^{(q)}$. Call such capabilities $A' \subseteq A$. If a standalone firm with capabilities A' can make strictly net positive profits, we have a contradiction since $A' \subseteq A \subseteq \mu(S^{(u)})$ and so Algorithm 1 could not have terminated in iteration u, a contradiction. If it cannot, then since A' exists on another component on $\hat{H}_M(\mu(S^{(q)}))$ and $\mu(S^{(q)}) \supseteq \mu(S^{(u)})$, there are no synergies and such a procurement is strictly dominated by an alternate procurement of capabilities $A \setminus A'$.

We thus restrict our attention to possible procurements from the set $\hat{C}_r^{(q)} \cap \mu(S^{(u)})$. Denote the set of markets which intersect this component with $J_r := \{j \in \mathcal{M} \,:\, M_j \cap \hat{C}_r^{(q)} \neq \emptyset\}$.

Decompose the capabilities A such that $A = A_{other} \cup A_{not}$ and $A_{other} \cap A_{not} \neq \emptyset$ where (i) $A_{other} \subseteq \{\bigcup_{k \in \mathcal{F}_r^{(q)} \setminus \{i\}} \mu(F_k)\}$ is the set of capabilities held by *other* firms generated in iteration q; and (ii) $A_{not} \subseteq \{\hat{C}_r^{(q)} \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} \mu(F_k)\}$ is the set of capabilities which are not.

We can first rule out $A = A_{other}$ i.e. only capabilities held by other firms generated in the same iteration are procured. To see this, observe that if gross profits only strictly increase in markets $\bigcup_{k \in \mathcal{F}_r^{(q)} \setminus \{i\}} J_{k,\pi>0}^{(u)}$, then since by Lemma 5, firm i cannot possess capabilities relevant to these markets and κ is convex, the standalone firm with capabilities A must also be strictly net profitable. But if this is so, then since $A \subseteq \mu(S^{(u)})$, the algorithm could not have terminated in iteration u, a contradiction. Alternatively, if the procurement generates strictly positive gross profits in any market $j \in \{J_r \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}\}$, this implies there were synergies between the firms $\mathcal{F}_r^{(q)}$. Now consider an alternate demerger into the single firm l where $\mu(F_l) = \bigcup_{k \in \mathcal{F}_r^{(q)}} \mu(F_k)$. The difference in gross profits is

$$\sum_{j \in J_r} \left(\pi_{lj}(H_M, H_F^{(q)}) - \sum_{k \in \mathcal{F}_r^{(q)}} \pi_{kj}(H_M, H_F^{(q)}) \right)$$
$$= \underbrace{\sum_{j \in \left\{ J_r \setminus \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(q)} \right\}} \pi_{lj}(H_M, H_F'^{(q)})}_{>0} + \underbrace{\sum_{j \in \left\{ \bigcup_{k \in \mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(q)} \right\}} \left(\pi_{lj}(H_M, H_F'^{(q)}) - \sum_{k \in \mathcal{F}_r^{(q)}} \pi_{kj}(H_M, H_F^{(q)}) \right)}_{\geq 0}$$

where $H_F'^{(q)}$ is identical to $H_F^{(q)}$ with the exception that the firms $\mathcal{F}_r^{(q)}$ are replaced by the single firm l and we used the fact that the set of operating markets are unchanged between iterations q and u (Lemma 6). Hence for all κ not too convex, the demerger in fact undertaken in iteration q could not have been optimal, a contradiction.

We can also rule out $A = A_{not}$, recalling these are capabilities not held by other firms generated in the same iteration q. To see this, observe that this procurement could only have increased gross profits in markets $\{J_r \setminus \bigcup_{k\in\mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}\}$, otherwise by Lemma 5, at least one firm is not holding the maximum feasible capabilities, a contradiction. By our regularity assumption on profits (iv), competition across all markets could have only increased between iteration q and u, implying if firm i finds it strictly net profitable to procure A_{not} under $(H_M, H_F^{(u)})$, this must also be so under $(H_M, H_F^{(q)})$. And since $A_{not} \subseteq \mu(S^{(u)}) \subseteq \mu(S^{(q)})$, the demerger of the interim firm with capabilities $\hat{C}_r^{(q)}$ which was in fact undertaken in iteration q could not have been optimal, a contradiction.

Finally, consider the mixed procurement with $A_{not} \neq \emptyset, A_{other} \neq \emptyset$. As before, gross profits in markets $\bigcup_{k\in\mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}$ must remain unchanged, otherwise by Lemma 5, at least one firm is not holding the maximum feasible capabilities, a contradiction. Now recall we assumed the procurement was strictly net profitable, hence the inequality

$$\sum_{j\in\left\{J_r\setminus\bigcup_{k\in\mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}\right\}} \left(\pi_{i'j}(H_M, H_F'^{(u)}) - \pi_{ij}(H_M, H_F^{(u)})\right) > |A|\kappa(1)$$

must be fulfilled, where i' has capabilities $\mu(F_{i'}) = \mu(F_i) \cup A$, and $H_F'^{(u)}$ is identical to $H_F^{(u)}$ with the exception that firm i is replaced by firm i'. Then construct firm l such that $\mu(F_l) = \{\bigcup_{k\in\mathcal{F}_r^{(q)}} \mu(F_k)\} \cup \{A_{not}\}$. Since $\mu(F_l) \supseteq \mu(F_{i'})$, then profits across markets $\{J_r \setminus \bigcup_{k\in\mathcal{F}_r^{(q)}} J_{k,\pi>0}^{(u)}\}$ must increase by at least $|A|\kappa(1) \geq |A_{not}|\kappa(1)$ and for all $\epsilon > 0$, we can reduce the convexity of κ until $\kappa(|F_l|) - \sum_{k\in\mathcal{F}_r^{(q)}} \kappa(|F_k|) - |A_{not}|\kappa(1) < \epsilon$. Since $A_{not} \subseteq \mu(S^{(u)}) \subseteq \mu(S^{(q)})$, the demerger undertaken in iteration q cannot have been optimal, a contradiction.

(v) No firm has a strictly net profitable entry.

We want to show no firm i with capabilities $\mu(F_i) \subseteq \mu(S^{(u)})$ can be strictly net profitable for the industry structure $(H_M, H_F'^{(u)})$ where $H_F'^{(u)}$ is reached from $H_F^{(u)}$ by adding firm i. Suppose there exists such firm i with $\mu(F_i) \subseteq \mu(S^{(u)})$. Recall $\{\hat{C}_r^{(u)}\}_{r=1}^p$ is the set of components of the subhypergraph $\hat{H}_M(\mu(S^{(u)})$, and $\mathcal{P}^{(u)}$ is the index of its components. If firm i can make strictly net positive profits, then at least one firm i' with capabilities $\mu(F_{i'}) \in \{\mu(F_i) \cap \hat{C}_r^{(u)} : r \in \mathcal{P}^{(u)}\}$ must be making strictly net positive profits by Lemma 1. But if so, then since $\mu(F_{i'}) \subseteq \{\hat{C}_r^{(u)} \cap \mu(S^{(u)})\}$ for some $r \in \mathcal{P}^{(u)}$, Algorithm 1 could not have terminated in iteration u, a contradiction.

Finally, since we have ruled out strictly net profitable disposals, demergers, procurements, mergers, and entries for all firms in $(H_M, H_F^{(u)})$, the industry structure created by Algorithm 1 is stable.

$\square$

C.3 Proof of Lemma 3

C.3.1 Notation and a helpful lemma

Use $D_r^{(q)} := \hat{C}_r^{(q)} \backslash \{\bigcup_{i \in \mathcal{F}_r^{(q)}} \mu(F_i)\}$ to denote the set of capabilities disposed in the most profitable demerger undertaken in step q by the interim firm r with capabilities $\hat{C}_r^{(q)}$.

The following lemma, proved in Supplementary Appendix I.1, will be helpful to prove Lemma 3.

Lemma 7 (A firm never disposes of capabilities subsequent firms choose to retain)**.** *For any $q \geq 1$ and any firm $k \in \mathcal{F}_r^{(q+1)}$, if $\hat{C}_r^{(q+1)} \subseteq \hat{C}_{r'}^{(q)}$, then $\mu(F_k) \cap D_{r'}^{(q)} = \emptyset$.*

C.3.2 Proof of Lemma 3

Proof. Since the set of unassigned competencies $S^{(q+1)} \subseteq S^{(q)}$ can only shrink for any $q \geq 1$, then for all $r \in \mathcal{P}^{(q+1)}$, there exists $r' \in \mathcal{P}^{(q)}$ such that $\hat{C}_r^{(q+1)} \subseteq \hat{C}_{r'}^{(q)}$. This implies there must exist a component r from the subhypergraph $\hat{H}_M(\mu(S^{(q+1)})$ and component r' from the subhypergraph $\hat{H}_M(\mu(S^{(q)})$ such that $\mu(F_k) \subseteq \hat{C}_r^{(q+1)} \subseteq \hat{C}_{r'}^{(q)}$ for any $k \in \mathcal{F}^{(q+1)}$.

Now for any pair of firms $k \in \mathcal{F}^{(q+1)}$ and $i \in \mathcal{F}^{(q)}$, we want to show that if firms i and k have non-empty intersection, then $\mu(F_k) \subseteq \mu(F_i)$. Combined with Lemma 7 which guarantees for all $k \in \mathcal{F}^{(q+1)}$ there exists some such firm i, this is sufficient to imply Lemma 3. Denote the set of markets which value some capability held by both i and k, and which k operates in after iteration $q+1$ with $J_{ik} = \{j \in J_{k,\pi>0}^{(u)} : \{\mu(F_i) \cap \mu(F_k)\} \cap M_j \neq \emptyset\}$. Note that J_{ik} must be non-empty, otherwise the demerger generating firm k in iteration $q+1$ must have been strictly dominated by an alternate demerger which disposes $\mu(F_i) \cap \mu(F_k)$, a contradiction.

We now show for all markets $j \in J_{ik}$, $\{M_j \cap \mu(F_k)\} \subseteq \mu(F_i)$. For $j \in J_{ik}$, if firm i also operates in market j, then by Lemma 5, since $\mu(S^{(q)}) \supseteq \mu(S^{(q+1)})$, then $\{M_j \cap \mu(F_k)\} \subseteq \mu(F_i)$. Now suppose, towards a contradiction, that i does not also operate in market j. First observe if i does not operate in j, by Lemma 5, there cannot be some other firm $i' \in \mathcal{F}^{(q)}$ operating in market j since $\mu(F_i) \cap M_j \neq \emptyset$ and $\mu(F_i) \cap \mu(F_{i'}) = \emptyset$ by construction. Then consider the alternate demerger generating a single firm l in lieu of the firms $\mathcal{F}_{r'}^{(q)}$, where $\mu(F_l) = \bigcup_{i \in \mathcal{F}_{r'}^{(q)}} \mu(F_i)$. In all markets $j' \in \mathcal{M}$, since firms $\mathcal{F}^{(q)}$ operate in disjoint markets by Lemma 4

$$\pi_{lj'}(H_F'^{(q)}, H_M) \geq \sum_{i \in \mathcal{F}_{r'}^{(q)}} \pi_{ij'}(H_F^{(q)}, H_M).$$

By Assumption 1, this inequality is strict for market j since (i) by construction, firm k operates in J_{ik}; and (ii) $\mu(F_l) \supseteq \mu(F_k)$ by Lemma 7. Then for all $\epsilon > 0$, we can reduce the convexity of κ until $\kappa(|F_l|) - \sum_{i' \in \mathcal{F}_{r'}^{(q)}} \kappa(|F_{i'}|) < \epsilon$, and the demerger under consideration is strictly more profitable than the one in fact undertaken, a contradiction. Therefore, we have shown for all markets $j \in J_{ik}$, $\{M_j \cap \mu(F_k)\} \subseteq \mu(F_i)$.

Finally, we show $\{M_j \cap \mu(F_k) \,:\, j \in J_{ik}\} = \mu(F_k)$, which in turn implies $\mu(F_k) \subseteq \mu(F_i)$. Let $\mathcal{F}' := \{i \in \mathcal{F}^{(q)}_{r'} \,:\, J_{ik} \neq \emptyset\}$. Suppose, towards a contradiction, that $|\mathcal{F}'| > 1$. If there exists $i', i'' \in \mathcal{F}'$ such that $J_{i'k} \cap J_{i''k} \neq \emptyset$, then there exists some market $j \in \{J_{i'k} \cap J_{i''k}\}$ such that $\{M_j \cap \mu(F_k)\} \subseteq \mu(F_{i'})$ and $\{M_j \cap \mu(F_k)\} \subseteq \mu(F_{i''})$ but since i' and i'' were generated in the same iteration, they must be disjoint, a contradiction. If for all $i', i'' \in \mathcal{F}'$, $J_{i'k} \cap J_{i''k} = \emptyset$, then consider the alternate demerger of firm k into the firms with the sets of capabilities $\{M_j \cap \mu(F_k) \,:\, j \in J_{ik}\}_{i \in \mathcal{F}'}$. Such a demerger must be strictly more net profitable than the one in fact undertaken, since the resultant firms collectively make identical gross profits as before, but bear strictly lower capability maintenance costs by the convexity of κ. As such, the demerger undertaken by firm k in iteration $q+1$ could not have been optimal, a contradiction. We have shown $|\mathcal{F}'| = 1$ and since we have established $J_{ik} \neq \emptyset$, $\mathcal{F}' = \{i\}$. This implies $\{M_j \cap \mu(F_k) \,:\, j \in J_{ik}\} = \mu(F_k) \subseteq \mu(F_i)$ as required.

□

C.4 Proof of Propositions 3 and 4

Recall that Proposition 4 is an analog of Proposition 3, but with (i) an additional criteria that consumer surplus across all markets must weakly increase for mergers to be permitted; and (ii) additional assumptions on consumer surplus (primitives and Assumption 4). We will now argue that Proposition 4 implies Proposition 3 i.e. if (H_M, H_F) is antitrust unstable under Assumptions 1-4, it must be unstable under Assumptions 1-3. To see this, first observe both Propositions 3 and 4 establish the same lower bound—that all industry structures with $|C_{max}| < |\{\bigcup_{j \in \mathcal{M}} M_j \,:\, A \cap M_j \neq \emptyset\}|$ are (antitrust) unstable. Next, note that Assumption 4 only imposes requirements on consumer surplus and not firm profitability. As such, if there exists some strictly profitable deviation under Assumptions 1-4, this deviation must continue to be strictly profitable under Assumptions 1-3 without consideration of antitrust. Finally, the space of profitable deviations under antitrust is a strict subset of that without, and so if a deviation is feasible under antitrust, it must continue to be so without antitrust.

As such, it will suffice to prove Proposition 4 which we now turn to.

C.4.1 Intermediate lemmas

The following two lemmas will be helpful to prove Proposition 4. They are subsequently proven in subsection I.1.

Lemma 8. *In all stable industry structures, for sufficiently low convexity of κ, for any market $j \in \mathcal{M}$, there must exist some firm $i \in \mathcal{N}$ such that $M_j \subseteq \mu(F_i)$.*

Lemma 9. *For a given market hypergraph H_M, if there exists some scarce capability $a \in \mathcal{A}$, then for sufficiently low convexity of κ, in all stable industry structures, there must exist some firm $i \in \mathcal{N}$ such that $\{\bigcup_{j \in \mathcal{M}} M_j \,:\, a \in M_j\} \subseteq \mu(F_i)$.*

C.4.2 Proof of Proposition 4

Proof. First observe if $a, a' \in \mathcal{A}$ are scarce and adjacent such that there exists $j \in \mathcal{M}$ such that $a, a' \in M_j$, then for sufficiently low convexity of κ, in all stable industry structures there must exist some firm $i \in \mathcal{N}$ such that $\{\bigcup_{j\in\mathcal{M}} M_j : \{a \cup a'\} \cap M_j \neq \emptyset\} \subseteq \mu(F_i)$. To see this, observe from Lemma 9 there must exist some firm $i \in \mathcal{N}$ such that $\{\bigcup_{j\in\mathcal{M}} M_j : a \in M_j\} \subseteq \mu(F_i)$ and some firm $i' \in \mathcal{N}$ such that $\{\bigcup_{j\in\mathcal{M}} M_j : a' \in M_j\} \subseteq \mu(F_{i'})$. Then since a and a' are scarce and adjacent, i and i' must be the same firm.

Now notice that if the scarce capabilities A are connected on the subhypergraph $\hat{H}_M(A)$, then for any pair of capabilities $a_1, a_n \in A$, there must exist some path on $\hat{H}_M(A)$ $\{a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n\}$ where for all $1 \leq i \leq n-1$, $a_i, a_{i+1} \in M_i$. The rest of the result follows immediately since there must exist some firm $i \in \mathcal{N}$ such that $\{\bigcup_{j\in\mathcal{M}} M_j : \{a_i \cup a_{i+1}\} \cap M_j \neq \emptyset\} \subseteq \mu(F_i)$ for all $1 \leq i \leq n-1$ implying $|F_i| \geq \mu(F_i) \geq |\{\bigcup_{j\in\mathcal{M}} M_j : A \cap M_j \neq \emptyset\}|$. $\square$