Comparison of scalar resonances using two unitarisation procedures in $\pi\pi$ and πK scattering

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We review two previous approaches to studying pseudoscalar meson-meson scattering amplitudes to beyond 1 GeV using non-linear and linear chiral Lagrangians. In these approaches we use two different unitarisation techniques - a generalised Breit Wigner prescription and K-matrix unitarization respectively. We report some new findings on K-matrix unitarisation of $\pi\pi$ and πK scattering in the non-linear chiral Lagrangian approach and make some related remarks about the light scalar mesons.

1. Introduction

Pseudoscalar meson-meson scattering up to the 1-2 GeV energy range is of interest for several related reasons. On the one hand this region is beyond that where chiral perturbation theory has traditionally been applied and below that where we can use perturbative QCD, so it is a challenge to develop a framework to calculate these amplitudes from first principles. At the same time there are many resonances in this region, some of which are controversial from the point of view of establishing their properties experimentally and their quark substructure. In particular, the scalar mesons are a long-standing puzzle in meson spectroscopy because, for example, there are too many states to fit into a single SU(3) nonet and the masses and decay patterns of some of the scalar resonances are not what one would expect for quark-antiquark scalar states. This talk is based on approaches developed by the Syracuse group. Many other interesting approaches are given in the proceedings of this conference and also cited in the references given in the bibliography.

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2. Non-linear chiral Lagrangian approach to meson-meson scattering

We begin [1,2] with the conventional chiral Lagrangian, including only pseudoscalars:

$$\mathcal{L}_1 = \frac{F_\pi^2}{8}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \mathrm{Tr}\left[\mathcal{B}\left(U+U^\dagger\right)\right], \quad (1)$$

in which $U = e^{2i\frac{\phi}{F_\pi}}$, with ϕ the 3×3 matrix of pseudoscalar fields and $F_\pi = 132$ MeV the pion decay constant. $\mathcal{B}$ is a diagonal matrix (B_1, B_1, B_3) with $B_1 = m_\pi^2 F_\pi^2/8 = B_2$ and $B_3 = F_\pi^2(m_K^2 - m_\pi^2/2)/4$.

We add a nonet of scalar mesons, which transform like external fields under chiral transformations. The trilinear scalar-pseudoscalar-pseudoscalar interaction that follows from the general chiral invariant extension of $\mathcal{L}_1$ to include a scalar meson nonet is given by [3]

$$\begin{aligned}\mathcal{L}_{N\phi\phi} &= A\epsilon^{abc}\epsilon_{def}N_a^d\partial_\mu\phi_b^e\partial^\mu\phi_c^f \\ &+ B\mathrm{Tr}\left(N\right)\mathrm{Tr}\left(\partial_\mu\phi\partial^\mu\phi\right) \\ &+ C\mathrm{Tr}\left(N\partial_\mu\phi\right)\mathrm{Tr}\left(\partial^\mu\phi\right) \\ &+ D\mathrm{Tr}\left(N\right)\mathrm{Tr}\left(\partial_\mu\phi\right)\mathrm{Tr}\left(\partial^\mu\phi\right) \quad (2)\end{aligned}$$

The first term of (2) may be eliminated in favor of the more standard form $\mathrm{Tr}\left(N\partial_\mu\phi\partial_\mu\phi\right)$, but is interesting because it is the OZI rule conserving term for a dual diquark-antidiquark type nonet mentioned below.

The scalar particles with non-trivial quantum numbers are given by:

$$N = \begin{bmatrix} N_1^1 & a_0^+ & \kappa^+ \\ a_0^- & N_2^2 & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & N_3^3 \end{bmatrix} \tag{3}$$

with $a_0^0 = (N_1^1 - N_2^2)/\sqrt{2}$. There are two iso-singlet states: the combination $(N_1^1 + N_2^2 + N_3^3)/\sqrt{3}$ is an $SU(3)$ singlet while $(N_1^1 + N_2^2 - 2N_3^3)/\sqrt{6}$ belongs to an $SU(3)$ octet. These will in general mix with each other when $SU(3)$ is broken. We take a convention where the physical particles, σ and f_0, which diagonalize the mass matrix [3] are related to the basis states N_3^3 and $(N_1^1 + N_2^2)/\sqrt{2}$ by

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} N_3^3 \\ \frac{N_1^1 + N_2^2}{\sqrt{2}} \end{pmatrix}, \tag{4}$$

We note that there are different possibilities, in addition to quark-antiquark configurations, for the underlying quark substructure of N which all give rise to the same SU(3) transformation properties. For example, forming diquark objects $T_a = \epsilon_{abc}\bar{q}^b\bar{q}^c$ and $\bar{T}^a = \epsilon^{abc}q_b q_c$, where the antisymmeterisation of the quark fields is implicit, we can form a pure tetraquark scalar nonet $N_a^b \sim T_a\bar{T}^b$ or construct linear combinations of $q\bar{q}$ and $qq\bar{q}\bar{q}$ nonets.

We studied s-wave pseudoscalar meson scattering in a framework beginning with Eqs.(1) and (2). If we begin with the tree-level scattering amplitudes, which due to chiral symmetry give good agreement with experiment close to the scattering threshold, we find that they soon deviate from the experimental data. They also violate unitarity. The approach that we took originally [2] was to add an imaginary piece by hand to the tree-level propogator of the s-channel resonance. For example, for πK scattering we called the lightest strange scalar resonance κ and made the substitution

$$m_\kappa^2 - s \longrightarrow m_\kappa^2 - s - im_\kappa G'_\kappa \tag{5}$$

in the denominator of the s-channel s-wave amplitude. In order to fit to experiment, the quantity G'_κ was left as a free parameter, not necessarily equal to the perturbative width, G_κ say. This is one kind of generalisation of the Breit-Wigner description of the resonance. Our fit [2] gave $\dfrac{G_\kappa}{G'_\kappa} = 0.13$ showing a substantial deviation from a Breit-Wigner resonance for which this ratio would be exactly equal to 1. Good agreement with experiment was also found [1] with this generalised Breit-Wigner prescription for the case of $\pi\pi$ scattering. The fitting parameters are the scalar-pseudoscalar-pseudoscalar coupling constants, which can all be written in terms of the four coefficients in the interaction terms in Eq. (2), the scalar meson masses and the mixing angle, θ_s. Our best fit for θ_s was about -20^o which we note, in our mixing convention, would be close to ideal mixing for a "dual" diquark-antidiquark nonet.

3. Pseudoscalar meson-meson scattering in SU(3) Linear Sigma Models

In the three flavor linear sigma model the pseudoscalar and scalar meson multiplets both appear from the beginning since the model is constructed from the 3×3 matrix field

$$M = S + i\phi, \tag{6}$$

where $S = S^\dagger$ represents a scalar nonet and $\phi = \phi^\dagger$ a pseudoscalar nonet. Under a chiral transformation $q_L \rightarrow U_L q_L$, $q_R \rightarrow U_R q_R$ of the fundamental left and right handed light quark fields, M is defined to transform as

$$M \longrightarrow U_L M U_R^\dagger. \tag{7}$$

We considered a general non-renormalizable Lagrangian[2] of the form

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left(\partial_\mu\phi\partial^\mu\phi\right) + \frac{1}{2}\mathrm{Tr}\left(\partial_\mu S\partial^\mu S\right) - V_0 - V_{SB} \tag{8}$$

where V_0 is an arbitrary function of the independent $SU(3)_L \times SU(3)_R \times U(1)_V$ invariants $\mathrm{Tr}\left(MM^\dagger\right)$, $\mathrm{Tr}\left(MM^\dagger MM^\dagger\right)$, $\mathrm{Tr}\left((MM^\dagger)^3\right)$ $6\left(\det M + \det M^\dagger\right)$. Of these, only I_4 is not invariant under $U(1)_A$. In this model there are many constraints among the parameters. For example, many of the trilinear scalar-pseudoscalar-pseudoscalar coupling constants are predicted

[2] See [4] and references therein for more detail

in terms of the pseudoscalar and scalar meson masses. Another difference is that [compare with Eq. (2)] this trilinear interaction does not involve derivatives. Both models give the current algebra results in the limit where the scalar mesons are integrated out. If we calculate the tree level s-wave amplitudes in the Linear Sigma Model they deviate from experiment and also violate unitarity as we go beyond the threshold region. We used [4] the well-known K-matrix procedure to unitarise the linear sigma model amplitudes and then checked if the resulting unitary amplitudes can give a good fit to data. In the standard parameterization [5] of a given partial wave S-matrix:

$$S = \frac{1+iK}{1-iK} \equiv 1 + 2iT, \tag{9}$$

we identify $K = T_{\rm tree}$ where $T_{\rm tree}$ is the given partial wave T-matrix computed at tree level in the Linear Sigma Model and so is purely real. This scheme gives exact unitarity for T but violates the crossing symmetry which $T_{\rm tree}$ itself obeys.

We show our best fits to the I=J=0 $\pi\pi$ scattering data in the linear sigma model with K-matrix unitarisation in [4]. The parameters in this fit are the "bare" masses of the two I=0 scalar mesons in M and their mixing angle. Using these parameters we can solve for the poles in the unitarised amplitude in the complex s plane. Labelling these poles z_σ and $z_{\sigma'}$ we can identify the physical masses and widths as usual from the Real and Imaginary parts, for example $z_\sigma = m_\sigma^2 - im_\sigma\Gamma_\sigma$. In [4] we also fit the I=1/2, J=0 πK scattering data. We note that since this model only contains one strange scalar resonance, it of course cannot fit the data over the full experimental energy range since a second strange scalar state, the well-known $K_0^*(1430)$, which is important in this channel is not present in this model.

4. Summary and comparison between models

We have found good agreement with scattering data in the approaches based on the non-linear chiral Lagrangians outlined in Sections 2 and 3. The best-fit values of the parameters - scalar meson masses and coupling constants - were somewhat different in the two approaches, as can be seen for example in Table 1. We are currently studying scattering using the non-linear chiral Lagrangian approach outlined of Section 2, but employing the K-matrix unitarisation as described in Section 3. This should make it easier to compare the linear and non-linear chiral Lagrangian models more directly and to understand the effects of the unitarisation prescriptions in themselves. This was partly motivated by our work on extending the non-linear chiral Lagrangian approach to include vector mesons [8]. This enabled us to study the interesting rare radiative decay processes $\phi \to \pi\pi\gamma$ and $\phi \to \pi\eta\gamma$. We found that the shape of the partial branching fraction depends quite sensitively on whether we use derivative or non-derivative scalar-pseudoscalar-pseudoscalar coupling as in Section 2 or 3 respectively.

A summary of our results for the masses of the light scalar mesons is shown in Table 1 for $\pi\pi$ and πK scattering. In the third and fourth columns we show the results of the analyses described in sections 3 and 2 respectively. In the fourth column we give the results with and without the inclusion of the ρ vector meson for the case of $\pi\pi$ scattering. The $K^*(892)$ is included in the πK scattering result in column 4. In column 2 we show the results of our current analysis, which are preliminary. However we can see some trends, namely that the $f_0(980)$ parameters are quite stable, whereas the σ and κ parameters seem to depend more on the model and, even more, on the unitarisation procedure. For both $\pi\pi$ and πK scattering the K-matrix unitarisation seems to yield lower values for the lightest scalar meson masses. These results are preliminary because we have only done a fit of $\pi\pi$ scattering data over a limited energy range. Also we have not included the inelastic channel and so the important $K\bar{K}$ threshold region. Similarly in πK scattering we have included the κ, but not the $K_0^*(1430)$ state. These and a similar study of related scattering channels are interesting directions for future work.

Table 1
Comparison between models and unitarisation procedures

MODEL	Non-linear chiral Lag.	Linear Sigma Model	Non-Linear chiral Lag.
UNITARISATION	K-matrix (without vectors)	K-matrix (without vectors)	Generalised BW (with/without ρ)
$m_{\sigma,\,\mathrm{phys}}$(MeV)	444	457	559/378
$m_{f_0,\,\mathrm{phys}}$(MeV)	986	993	990
$m_{\kappa,\,\mathrm{phys}}$(MeV)	720	798	897

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