Liquidity Traps, Learning and Stagnation[*]

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Abstract

We examine global economic dynamics under learning in a New Keynesian model in which the interest-rate rule is subject to the zero lower bound. Under normal monetary and fiscal policy, the intended steady state is locally but not globally stable. Large pessimistic shocks to expectations can lead to deflationary spirals with falling prices and falling output. To avoid this outcome we recommend augmenting normal policies with aggressive monetary and fiscal policy that guarantee a lower bound on inflation. In contrast, policies geared toward ensuring an output lower bound are insufficient for avoiding deflationary spirals.

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1 Introduction

There is now widespread agreement that the zero lower bound on nominal interest rates has the potential to generate a "liquidity trap" with major implications for economic performance. A substantial literature has discussed the plausibility of the economy becoming trapped in a deflationary state and the macroeconomic policies that might be able to avoid a liquidity trap or extricate the economy from one.[1] Our own view, reflected in the current pa-

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[1]See Krugman (1998) for a recent seminal discussion and Adam and Billi (2007), Coenen, Orphanides, and Wieland (2004), Eggertsson and Woodford (2003), Eggertsson and Woodford (2004), and Goodfriend (2000) for representative recent analyses and further references. Braun and Waki (2006) provide a calibrated model for Japan that incorporates the zero interest-rate lower bound.

per as well as in the earlier paper Evans and Honkapohja (2005), is that the evolution of expectations plays a key role in the dynamics of the economy and that the tools from learning theory are needed for a realistic analysis of these issues.

The importance of expectations in the liquidity trap is now widely accepted. For example, Benhabib, Schmitt-Grohe, and Uribe (2001b), Benhabib, Schmitt-Grohe, and Uribe (2001a) show the possibility of multiple equilibria under perfect foresight, with a continuum of paths to an unintended low or negative inflation steady state. Similarly, Eggertsson and Woodford (2003) emphasize the importance of policy commitment for influencing expectations under the rational expectations (RE) assumption. Evans and Honkapohja (2005) emphasize how the learning perspective alters both the assessment of the plausibility of particular dynamics and the impact of policy.

Under learning private agents are assumed to form expectations using an adaptive forecasting rule, which they update over time in accordance with standard statistical procedures. In many standard set-ups least-squares learning is known to converge asymptotically to rational expectations, but cases of instability can also arise. Evans and Honkapohja (2005) examine a flexible price model with a global Taylor-rule, which, because of the zero lower bound, generates a low-inflation steady state below the one intended by policy. There it was found that while the intended steady state was locally stable under learning, the lower one was not[2] and there was also the possibility of inflation slipping below the low-inflation steady state. It was also shown that switching to a sufficiently aggressive monetary policy at low inflation rates could avoid these unstable trajectories. Fiscal policy in these circumstances was ineffective.

The analysis of Evans and Honkapohja (2005) was, however, conducted in a flexible-price model with exogenous output. In the current paper we employ a New Keynesian model to reexamine these issues in a framework that allows for a serious analysis of monetary and fiscal policy for an economy in which recessions or slumps can arise due to failures of aggregate demand.[3] We obtain a number of striking results.

The possibility of liquidity traps taking the form of a deflationary spiral, under a global Taylor rule, emerges as a serious concern. Although the targeted steady state is locally stable under learning, a large pessimistic shock to expectations can result, under learning, in a self-reinforcing deflationary process in which inflation and output decline over time. We consider a number of policies to insulate the economy from this possibility. Each of these policies maintains the Taylor rule over most of the range, but augments it by switching

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[2]See also McCallum (2002) for an argument that the low-inflation steady-state is not stable under learning.The instability under learning of the low inflation steady state is sensitive to the form of the interest-rate rule, as shown by Eusepi (2007).

[3]Our analysis provides a theoretical framework for the potential role of fiscal policy in combatting liquidity traps, which has been a contraversial topic in the empirical literature on Japan's slump. See Ball (2005), Kuttner and Posen (2002) and Perri (2001).

to aggressive policies if inflation or output falls below some threshold.

We first consider an inflation threshold policy in which aggressive monetary policy is used whenever inflation falls below, or threatens to fall below, some specified threshold. It turns out that this policy, although it does offer some protection, is not sufficient if the negative expectations shock is very large. Next, we augment the preceding policy by adding aggressive fiscal policy if monetary policy alone is inadequate to keep inflation at or above the threshold. We demonstrate that this combination of aggressive policies, with a threshold chosen at a suitable level, can always eliminate the possibility of deflationary spirals and ensure global stability of the targeted steady state. This is the central policy finding that emerges from the adaptive learning perspective.

Our central policy result leads to several further questions. One natural question is whether an output threshold could be substituted for an inflation threshold. Surprisingly the answer is no: using an output threshold to trigger aggressive monetary and fiscal policies will not necessarily avoid deflationary spirals. Another question concerns the timing for implementing our recommended policy in which normal monetary and fiscal policy is augmented by inflation threshold policies. Using simulations we show that it is better to adopt inflation threshold policies earlier rather than later. Ideally, our inflation threshold policy is in place before substantial negative expectation shocks impact the economy.

2 The Model

We adopt a fairly standard representative agent model along the lines of Benhabib, Schmitt-Grohe, and Uribe (2001b), Section 3, except that we allow for stochastic shocks and conduct the analysis in discrete time.[4] There is a continuum of household-firms, which produce a differentiated consumption good under conditions of monopolistic competition and price-adjustment costs. We allow for both fiscal and monetary policy and for the government to issue debt.

2.1 Private Sector

The objective for agent j is to maximize expected, discounted utility subject to a standard flow budget constraint:

$$Max \ E_0 \sum_{t=0}^{\infty} \beta^t U_{t,j} \left(c_{t,j}, \frac{M_{t-1,j}}{P_t}, h_{t,j}, \frac{P_{t,j}}{P_{t-1,j}} - 1 \right) \qquad (1)$$

$$st. \ c_{t,j} + m_{t,j} + b_{t,j} + \Upsilon_{t,j} = m_{t-1,j}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1,j} + \frac{P_{t,j}}{P_t}y_{t,j}, \qquad (2)$$

where $c_{t,j}$ is the Dixit-Stiglitz consumption aggregator, $M_{t,j}$ and $m_{t,j}$ denote nominal and real money balances, $h_{t,j}$ is the labor input into production,

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[4]We develop our analysis within a closed-economy model. For discussions of liquidity traps in open economies, see for example McCallum (2000) and Svensson (2003).

$b_{t,j}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t, $\Upsilon_{t,j}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t, $P_{t,j}$ is the price of consumption good j, $y_{t,j}$ is output of good j, P_t is the aggregate price level and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β. The utility function has the parametric form

$$U_{t,j} = \frac{c_{t,j}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2}\left(\frac{M_{t-1,j}}{P_t}\right)^{1-\sigma_2} - \frac{h_{t,j}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2}\left(\frac{P_{t,j}}{P_{t-1,j}} - 1\right)^2,$$

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. The final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982).

The production function for good j is

$$y_{t,j} = h_{t,j}^{\alpha}$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward sloping demand curve given by

$$P_{t,j} = \left(\frac{y_{t,j}}{Y_t}\right)^{-1/\nu} P_t. \tag{3}$$

Here $P_{t,j}$ is the profit maximizing price set by firm j consistent with its production $y_{t,j}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. Y_t is aggregate output, which is exogenous to the firm.

2.2   Fiscal and Monetary Policy

The government's flow budget constraint is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \tag{4}$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1} + \eta_t, \tag{5}$$

where η_t is a white noise shock and where $\beta^{-1} - 1 < \kappa < 1$. The restriction on κ means that fiscal policy is "passive" in the terminology of Leeper (1991), and implies that an increase in real government debt leads to an increase in taxes sufficient to cover the increased interest and at least some fraction of the increased principal.

We assume that g_t is stochastic

$$g_t = \bar{g} + u_t, \tag{6}$$

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where u_t is an observable stationary AR(1) mean zero shock. From market clearing we have

$$c_t = h_t^\alpha - g_t. \tag{7}$$

Monetary policy is assumed to follow a global interest rate rule

$$R_t - 1 = \theta_t f(\pi_t). \tag{8}$$

The function $f(\pi)$ is taken to be positive and non-decreasing, while θ_t is an exogenous, observable stationary AR(1) positive random shock with mean 1 representing random shifts in the behavior of the monetary policy-maker. We assume the existence of π^*, R^* such that $R^* = \beta^{-1}\pi^*$ and $f(\pi^*) = R^* - 1$. π^* can be viewed as the inflation target of the Central Bank, and we will assume that $\pi^* \geq 1$. In the numerical analysis we will use the functional form

$$f(\pi) = (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{AR^*/(R^*-1)},$$

which implies the existence of a nonstochastic steady state at π^*. Note that $f'(\pi^*) = AR^*$, which we assume is bigger than β^{-1}.

Equations (4), (5) and (8) constitute "normal policy". In the first part of the paper we examine the system under normal policy. Later we consider modifications to these policy rules in exceptional circumstances.

2.3 Key Equations

In Appendix A.1 it is shown that private sector optimization yields the key equations

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t = h_t\left(h_t^\varepsilon - \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha-1}c_t^{-\sigma_1}\right) + \beta\frac{\alpha\gamma}{\nu}E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\right], \tag{9}$$

$$c_t^{-\sigma_1} = \beta R_t E_t\left(\pi_{t+1}^{-1}c_{t+1}^{-\sigma_1}\right), \tag{10}$$

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_t^{-\sigma_1}}{E_t\pi_{t+1}^{\sigma_2-1}}\right)^{-1/\sigma_2}, \tag{11}$$

to which we add the equations (4) - (8). Equation (9) is the nonlinear New Keynesian Phillips curve, which describes the optimal price-setting by firms. To interpret this equation, note that the bracketed expression in the first term on the right-hand side is the difference between the marginal disutility of labor and the product of the marginal revenue from an extra unit of labor with the marginal utility of consumption. The terms involving current and future inflation arise from the price-adjustment costs resulting from marginal variations in labor supply. Equation (10) is the standard Euler equation giving

the intertemporal first-order condition for the consumption path.[5] Equation (11) is the money demand function resulting from the presence of real balances in the utility function. Note that for our parameterization, the demand for real balances becomes infinite as $R_t \to 1$.

Consider first the nonstochastic steady states in the absence of random shocks. For any steady state π, equation (10) implies that the nominal interest rate factor satisfies the Fisher equation

$$R = \beta^{-1}\pi. \tag{12}$$

As emphasized by Benhabib, Schmitt-Grohe, and Uribe (2001b), because $f(.)$ is nonnegative, continuous (and differentiable) and has a steady state π^* with $f'(\pi^*) > \beta^{-1}$, there must be a second steady state $\pi_L < \pi^*$ with $f'(\pi_L) < \beta^{-1}$. For our parametrization of $f(\cdot)$, there are no steady states other than the intended steady state π^* and the unintended low-inflation steady state π_L.

The other steady state equations are given by

$$c = h^\alpha - \bar{g}, \tag{13}$$

$$-h^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(1-\beta)(\pi-1)\pi + \alpha\left(1 - \frac{1}{\nu}\right)h^\alpha c^{-\sigma_1} = 0 \tag{14}$$

and a steady state version of (11). For a given steady state $\pi \geq 1$, it is shown in Appendix A.2 that there is a corresponding unique interior steady state $c > 0$ and $h > 0$. For steady states $\pi < 1$, there continue to be unique values for c and h provided π is close to one and $\bar{g} > 0$.[6]

When there is a nonstochastic steady state, it can be shown that stochastic steady states exist when the support of the exogenous shocks is sufficiently small. Furthermore, in this case the steady state is locally determinate, provided the corresponding linearization is determinate. Throughout the paper we assume that the shocks are small in the sense of having small support. We now consider determinacy of the linearized system.

In a neighborhood of a nonstochastic steady state (c, π) we can derive a linear approximation

$$c_t = -\sigma_1\beta\pi^{-1}cR_t + c_{t+1}^e + \sigma_1 c\pi^{-1}\pi_{t+1}^e + k_c \tag{15}$$

$$R_t = a\pi_t + \delta\theta_t + k_R, \text{ where } a = f'(\pi), \delta = f(\pi) \tag{16}$$

$$\frac{\alpha\gamma}{\nu}(2\pi-1)\pi_t = \frac{\beta\alpha\gamma}{\nu}(2\pi-1)\pi_{t+1}^e - \frac{1+\varepsilon}{\alpha}(c+\bar{g})^{(1+\varepsilon)\alpha^{-1}-1}(c_t+u_t) \tag{17}$$

$$+\alpha(1-\nu^{-1})\left(-(c+\bar{g})\sigma_1 c^{-\sigma_1-1}c_t + c^{-\sigma_1}(c_t+u_t)\right) + k_\pi$$

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[5]If equation (9) is linearized around $\pi = 1$ we obtain $\tilde{\pi}_t = \lambda\tilde{h}_t + \beta E_t\tilde{\pi}_{t+1}$, where $\lambda > 0$, $\tilde{\pi}_t = \pi_t - 1$ and $\tilde{h}_t$ is the proportional deviation of h_t from the steady-state employment corresponding to $\pi = 1$. The linearization of (9) thus corresponds closely to the New Keynesian Phillips curve based on Calvo pricing. Similarly the linearization of (10) leads to the standard New Keynesian IS curve $\tilde{c}_t = -\sigma_1^{-1}(\tilde{R}_t - E_t\tilde{\pi}_{t+1}) + E_t\tilde{c}_{t+1}$, expressed in proportional deviation form.

[6]Cases of multiple values for c and h for given $\pi < 1$ do exist. Throughout the paper we rule out these exceptional cases.

together with the linearized evolution of bonds

$$b_t + m_t + \kappa_0 + \psi_t + \eta_t = g_t + \pi^{-1}m_{t-1} + (\beta^{-1} - \kappa)b_{t-1}$$
$$+b\pi^{-1}R_{t-1} - (m\pi^{-1} + \beta^{-1}b)\pi^{-1}\pi_t + k_b$$

and the linearized money equation

$$m_t = -\sigma_2^{-1}(\chi\beta)^{1/\sigma_2}(1 - R^{-1})^{-1/\sigma_2 - 1}\frac{c^{\sigma_1/\sigma_2}}{\pi^{1/\sigma_2}} \times$$
$$\left(-\sigma_1(1 - R^{-1})\pi c^{-1}c_t + (1 - \sigma_2)(1 - R^{-1})\pi_{t+1}^e + \beta R^{-1}R_t\right) + k_m,$$

where b and m are steady-state values.

The block of the first three equations determines the values of c_t, π_t and R_t solely in terms of c_{t+1}^e, π_{t+1}^e and the exogenous shocks. Determinacy of a steady state can therefore be assessed from this block plus stationarity of the bond dynamics. Substituting (16) into (15) yields a bivariate forward-looking system of the form

$$\begin{pmatrix} c_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} B_{cc} & B_{c\pi} \\ B_{\pi c} & B_{\pi\pi} \end{pmatrix}\begin{pmatrix} c_{t+1}^e \\ \pi_{t+1}^e \end{pmatrix} + \begin{pmatrix} G_{cu} & G_{c\theta} \\ G_{\pi u} & G_{\pi\theta} \end{pmatrix}\begin{pmatrix} u_t \\ \tilde{\theta}_t \end{pmatrix} + \begin{pmatrix} \tilde{k}_c \\ \tilde{k}_\pi \end{pmatrix}, \quad (18)$$

where the coefficients can be computed by solving the equations. Here $\tilde{\theta}_t = \theta_1 - 1$, so that the stochastic shocks have zero means. Denoting by B the 2×2 matrix multiplying $(c_{t+1}^e, \pi_{t+1}^e)'$, a necessary condition for determinacy is that both eigenvalues of B lie inside the unit circle. There is then a unique nonexplosive solution of the form

$$\begin{pmatrix} c_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} c \\ \pi \end{pmatrix} + \begin{pmatrix} G_{cu} & G_{c\theta} \\ G_{\pi u} & G_{\pi\theta} \end{pmatrix}\begin{pmatrix} u_t \\ \tilde{\theta}_t \end{pmatrix}. \quad (19)$$

The corresponding solution for m_t then takes the form of a constant plus white noise shocks. From the linearized bond equation it follows that the remaining condition for determinacy is $|\beta^{-1} - \kappa| < 1$, which holds since we have assumed that $\beta^{-1} - 1 < \kappa < 1$.

Determinacy needs to be assessed separately for the π^* and π_L steady states. We have the following result:

Proposition 1 *In the linearized model there are two steady states $\pi^* > \pi_L$. Provided $\gamma > 0$ is sufficiently small, the steady state $\pi = \pi^*$ is locally determinate and the steady state $\pi = \pi_L$ is locally indeterminate.*

Proof. As noted above, from the steady-state interest-rate equation $R - 1 = f(\pi)$ and the properties of f it follows that there are two steady state inflation rates $0 < \pi_L < \pi^*$. As $\gamma \to 0$ it is easily seen that $B_{cc}, B_{c\pi} \to 0$ and $B_{\pi\pi} \to (a\beta)^{-1}$. At π^* we have $a > \beta^{-1}$ while at π_L we have $a < \beta^{-1}$. Hence for $\gamma > 0$ sufficiently small the roots of B are inside the unit circle at π^*, while at π_L one root is larger than 1. The result follows. ∎

This result generalizes the corresponding results in Evans and Honkapohja (2005), which considered an endowment economy with flexible prices.

3   Learning and Expectational Stability

We now formally introduce learning to the model in place of the hypothesis that RE prevails in all periods. In the modeling of learning it is assumed that private agents make forecasts using a reduced form econometric model of the relevant variables and that the parameters of this model are estimated using past data. The forecasts are input to agent's decision rules and in each period the economy attains a temporary equilibrium, i.e. an equilibrium for the current period variables given the forecasts of the agents. The temporary equilibrium provides a new data point, which in the next period leads to re-estimation of the parameters and updating of the forecasts and, in turn, to a new temporary equilibrium. The sequence of temporary equilibria may generate parameter estimates that converge to a fixed point corresponding to a rational expectations equilibrium (REE) for the economy, provided the form of the econometric model that agents use for forecasts is consistent with the REE. When the convergence takes place, we say that the REE is stable under learning.

The literature on adaptive learning has shown that there is a close connection between the possible convergence of least squares learning to an REE and a stability condition, known as E-stability, based on a mapping from the perceived law of motion (that private agents are estimating) to the implied actual law of motion generating the data under these perceptions. E-stability is defined in terms of local stability, at an REE, of a differential equation based on this map. For a general discussion of adaptive learning and the E-stability principle see Evans and Honkapohja (2001).

For the case at hand, when the exogenous shocks u_t and g_t are stationary AR(1) processes, the appropriate forecast rule based on (19) is for private agents to estimate the linear projections of c_{t+1} and π_{t+1} onto an intercept and the exogenous shocks u_t and θ_t. To estimate the coefficients of this projection, agents use a version of Least Squares to estimate

$$
\begin{aligned}
c_s &= a_c + du_{s-1} + e\tilde{\theta}_{s-1} + \varepsilon_{c,s} \\
\pi_s &= a_\pi + fu_{s-1} + g\tilde{\theta}_{s-1} + \varepsilon_{\pi,s},
\end{aligned}
$$

using the data for periods $s = 1, \ldots, t-1$. Here the usual timing assumption in the learning literature is being made that at the end of period $t-1$, agents estimate the parameters using data on all variables through time $t-1$. This yields estimates $a_{c,t-1}, d_{t-1}, e_{t-1}, a_{\pi,t-1}, f_{t-1}, g_{t-1}$. Then, at the start of time t agents form forecasts using these estimates and exogenous data at t,

$$
\begin{aligned}
c_{t+1}^e &= a_{c,t-1} + d_{t-1}u_t + e_{t-1}\tilde{\theta}_t \\
\pi_{t+1}^e &= a_{\pi,t-1} + f_{t-1}u_t + g_{t-1}\tilde{\theta}_t.
\end{aligned}
$$

Based on these expectations households and firms determine actual current period values of c_t, π_t. Then, at the end of period t the parameters are updated using the extra data point, and the process continues.

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It is now convenient to make a simplification, which does not in any way affect our key theoretical results. It turns out that the stability under learning of the two different steady states is governed by stability of the intercepts, not by the coefficients of the exogenous shocks. We will therefore focus on the case in which the exogenous shocks u_t and $\tilde{\theta}_t$ are *iid* processes. In this case the RE solutions for π_t and c_t described above are simply noisy steady states, i.e. *iid* processes, and forecasts π_{t+1}^e and c_{t+1}^e do not depend on current values of the exogenous variables u_t and $\tilde{\theta}_t$.[7] This simplifies the presentation of the analysis of learning since it is now natural for private agents to omit these variables from their regressions and forecast by simply estimating the mean values of π_t and c_t. In the learning literature this is often called "steady-state learning."

Under steady state learning agents treat (19) as a Perceived Law of Motion and for each variable they estimate simply the intercept or mean. We can thus identify expectations of the variables with the estimates of their means, and this has a simple formulation as recursive algorithms:

$$\pi_{t+1}^e = \pi_t^e + \phi_t(\pi_{t-1} - \pi_t^e) \tag{20}$$
$$c_{t+1}^e = c_t^e + \phi_t(c_{t-1} - c_t^e), \tag{21}$$

where ϕ_t is known as the gain sequence. Under least-squares learning the gain-sequence is usually taken to be $\phi_t = t^{-1}$, often termed a "decreasing-gain" sequence, whereas under "discounted least-squares" or " constant gain" learning it is set to $\phi_t = \phi$, where $0 < \phi < 1$ is a small positive constant. Decreasing gains have the advantage that they can asymptotically converge to RE, while constant-gain learning rules are more robust to structural change.

In what follows, we analyze both theoretically and numerically the model under various specifications of monetary and fiscal policy. The theoretical results for learning are based on E-stability analysis of the system under the learning rules (20)-(21). When we say that an equilibrium is stable (or unstable) under learning this implies that it is stable (or not) under these learning rules with decreasing gain. In the simulations we instead use a small constant gain. Under constant gain, when an equilibrium is E-stable there is local convergence of learning in a weaker sense to a random variable that is centered near the equilibrium.[8]

We next investigate our system under learning under the "normal policy" rules that we have described. In studying the economy under learning we return to the nonlinear model so that we can examine the global dynamics of the system. In doing so it is convenient to make the assumption of point expectations, e.g. replacing the expectation of $\pi_{t+1}^{-1} c_{t+1}^{-\sigma_1}$ by $(\pi_{t+1}^e)^{-1}(c_{t+1}^e)^{-\sigma_1}$. For stochastic shocks u_t and θ_t with small bounded support this is a reasonable approximation and it allows us to deal directly with expectations of future consumption and inflation rather than with nonlinear functions of them. Making

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[7]That is, in the RE stochastic steady state, the coefficients $d = e = f = g = 0$.

[8]For formal details see Section 7.4 of Evans and Honkapohja (2001).

this assumption, and also using the production function to substitute out h_t, leads to the system

$$\beta \frac{\alpha\gamma}{\nu} \left(\pi_{t+1}^e - 1\right) \pi_{t+1}^e = -(c_t + g_t)^{(1+\varepsilon)/\alpha} + \frac{\alpha\gamma}{\nu} \left(\pi_t - 1\right) \pi_t \qquad (22)$$

$$+\alpha \left(1 - \frac{1}{\nu}\right) (c_t + g_t) c_t^{-\sigma_1}$$

$$c_t = c_{t+1}^e (\pi_{t+1}^e / \beta R_t)^{\sigma_1}, \qquad (23)$$

where $g_t = \bar{g} + u_t$. These equations, together with the interest-rate rule (8), implicitly define the temporary equilibrium values for c_t and π_t given values for expectations c_{t+1}^e, π_{t+1}^e and given the exogenous shocks u_t, θ_t. Formally we write the temporary equilibrium map as

$$\pi_t = F_\pi(\pi_{t+1}^e, c_{t+1}^e, u_t, \theta_t)$$
$$c_t = F_c(\pi_{t+1}^e, c_{t+1}^e, u_t, \theta_t),$$

where it follows from the implicit function theorem that such a map exists in a neighborhood of each steady state (the linearization was given above as (19)).[9]

The dynamic system for c_t and π_t under learning is then given by (22)-(23) and (8) together with (20)-(21). The full dynamic system under learning augments these equations with the money equation

$$m_t = (\chi\beta)^{1/\sigma_2} \left(\frac{\left(1 - R_t^{-1}\right) c_t^{-\sigma_1}}{(\pi_{t+1}^e)^{\sigma_2 - 1}} \right)^{-1/\sigma_2}$$

and the bond equation (4).

The stability of a steady-state REE under learning is determined by E-stability. The REE is said be E-stable if the differential equation (in notional time τ)

$$\begin{pmatrix} d\pi^e/d\tau \\ dc^e/d\tau \end{pmatrix} = \begin{pmatrix} T_\pi(\pi^e, c^e) \\ T_c(\pi^e, c^e) \end{pmatrix} - \begin{pmatrix} \pi^e \\ c^e \end{pmatrix}$$

is locally asymptotically stable at a steady state (π, c), where

$$T_\pi(\pi^e, c^e) = EF_\pi(\pi^e, c^e, u_t, \theta_t)$$
$$T_c(\pi^e, c^e) = EF_c(\pi^e, c^e, u_t, \theta_t)$$

is the mapping from the Perceived Law of Motion to the corresponding Actual Law of Motion. $T(\cdot)$ gives the actual means for π_t and c_t when private agents have expectations (π^e, c^e). E-stability is determined by the Jacobian matrix DT of $T = (T_\pi, T_c)'$ at the steady state, which for small noise, is approximately equal to the matrix B of (18) for the steady state in question. It follows that the E-stability conditions are that both eigenvalues of $B - I$ have real parts less than zero. We have the following result for low levels of price stickiness.[10]

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[9]Numerically it appears that this function is well-defined globally.

[10]Some further results for general $\gamma > 0$ are available. For example local stability of π^* obtains for all $\gamma > 0$ and $g \geq 0$ if $\sigma_1 \geq 1$.

Proposition 2 *For $\gamma > 0$ sufficiently small, the steady state $\pi = \pi^*$ is locally stable under learning and the steady state $\pi = \pi_L$ is locally unstable under learning taking the form of a saddle point.*

Proof. In the limit $\gamma \to 0$ it is straightforward to show that the eigenvalues of $B - I$ are -1 and $(\beta f'(\pi))^{-1} - 1$. Since $f'(\pi^*) = A/\beta$ and $A > 1$ we have that $(\beta f'(\pi^*))^{-1} - 1 = A^{-1} - 1 < 0$. At the π_L steady state we instead have $f'(\pi_L) < \beta^{-1}$ implying $(\beta f'(\pi_L))^{-1} - 1 > 0$, so that the roots are real and of different signs. ∎

The saddle point property of π_L creates a region in which there can be deflationary spirals. We illustrate this by numerically constructed phase diagrams. This also allows to examine larger $\gamma > 0$ and conduct a global analysis. Parameters are set at $A = 2.5$, $\pi^* = 1.05$, $\beta = 0.96$, $\sigma_1 = 0.95$, $\alpha = 0.75$, $\gamma = 5$, $\nu = 1.5$, $\varepsilon = 1$ and $\bar{g} = 0.1$. Figure 1 shows the E-stability dynamics under normal monetary and fiscal policy. These indicate how expectations will adjust over time under learning when the economy is perturbed from its steady state equilibrium.[11]
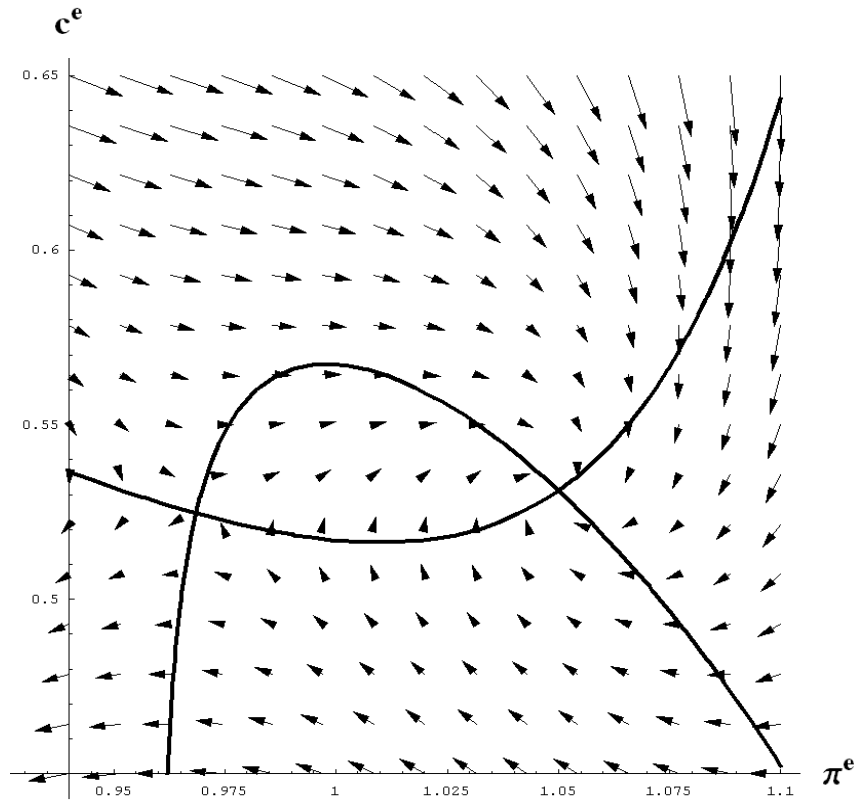

Figure 1: π^e and c^e dynamics under standard policy

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[11] The role of an unstable steady state can also be seen in a rather different setting, the monetary inflation model under learning. See Evans, Honkapohja, and Marimon (2001) and Marcet and Nicolini (2003).

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It can be seen that while π^* is locally stable, the low steady state $\pi_L \approx 0.969$ is a saddle. Under learning, normal policy works satisfactorily for moderate-sized perturbations from the targeted steady state. However, there are also starting points that lead to instability. In particular, if an exogenous shock leads to a strong downward revision of expectations, relative to the normal steady state, these pessimistic expectations generate paths leading to a deflationary spiral.

The intuition for the instability of the low steady state π_L is as follows. Near π_L we are close enough to the zero lower bound so that a reduction in π_t can only result in a small lowering of R_t. If π_{t+1}^e is slightly below π_L this must therefore lead to an increase in the real interest rate, to lower c_t through the household Euler equation and to lower π_t thought the new Keynesian Phillips curve. This set in motions downward movements in both c_t and π_t, which are reinforced as they feed into expectations. Of course, along these paths it is likely that eventually something would change, i.e. private agents or policymakers would alter their reactions. We think that the most plausible scenario is that policymakers would respond to the deteriorating situation by major changes in policy. The goal of this paper is, first, to exhibit that normal policies, while locally stable, have potential for instability after major shocks and, second, to propose policies that move the economy out of a deflationary spiral as well as insulate the economy against these unstable outcomes.

Proposition 2 indicates the need for more aggressive policies when expectations are pessimistic. We begin by considering changing to an aggressive monetary policy when inflation threatens to become too low. As we will see, it may be important also to alter fiscal policy in certain circumstances.

4   Adding Aggressive Monetary Policy

We first consider modifying monetary policy so that it follows the normal interest rate rule as long as $\pi_t \geq \tilde{\pi}$, but cuts interest rates to a low level floor $\hat{R}$ if inflation threatens to get below the threshold $\tilde{\pi}$. Thus

$$R_t = \left\{ \begin{array}{c} 1 + \theta_t f\left(\pi_t\right) \text{ if } \pi_t > \tilde{\pi} \\ \hat{R} \text{ if } \pi_t < \tilde{\pi}, \end{array} \right.$$

and

$$\hat{R} \leq R_t \leq 1 + \theta_t f\left(\pi_t\right) \text{ if } \pi_t = \tilde{\pi}.$$

We will assume throughout that $\pi^* > \tilde{\pi} > \hat{\pi} \equiv \beta\hat{R}$ and $1 < \hat{R} < 1 + f\left(\tilde{\pi}\right)$. We think of $\hat{R}$ as a value very close to one.[12] We therefore also assume that $\hat{R} < \beta^{-1}\pi_L$, so that our modified interest-rate rule reduces interest rates for all $\pi \geq \pi_L$.

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[12] In our numerical examples we set $\hat{R} = 1.0001$. We set $\hat{R}$ above one to keep money demand finite. For technical reasons we also assume that $\beta\hat{R} > 1/2$.

We remark that if $\pi_t < \tilde{\pi}$ the temporary equilibrium is given by (22)-(23), which yields

$$\beta\frac{\alpha\gamma}{\nu}\left(\pi_{t+1}^e - 1\right)\pi_{t+1}^e = -(c_t + g_t)^{(1+\varepsilon)/\alpha} + \frac{\alpha\gamma}{\nu}\left(\pi_t - 1\right)\pi_t$$

$$+\alpha\left(1 - \frac{1}{\nu}\right)(c_t + g_t)c_t^{-\sigma_1} \tag{24}$$

$$c_t = c_{t+1}^e(\pi_{t+1}^e/\beta\hat{R})^{\sigma_1}. \tag{25}$$

In the $\pi_t < \tilde{\pi}$ regime, where $R_t = \hat{R}$, expectations determine c_t through the Euler equation. Fiscal policy and the market-clearing condition $h_t^\alpha = c_t + g_t$ then determine h_t, and the Phillips Curve gives π_t.

A policy question of major importance is whether an aggressive monetary policy of this form is sufficient to eliminate deflationary spirals from arising when expectations are pessimistic. We now show that aggressive monetary policy will not always be adequate to avoid these outcomes (see Appendix B for a proof):

Proposition 3 *There is a steady state at $\hat{\pi} = \beta\hat{R}$ and there no steady state value for π_t below $\hat{\pi}$. For all $\gamma > 0$ sufficiently small the steady state at $\hat{\pi} = \beta\hat{R}$ is a saddle point under learning.*
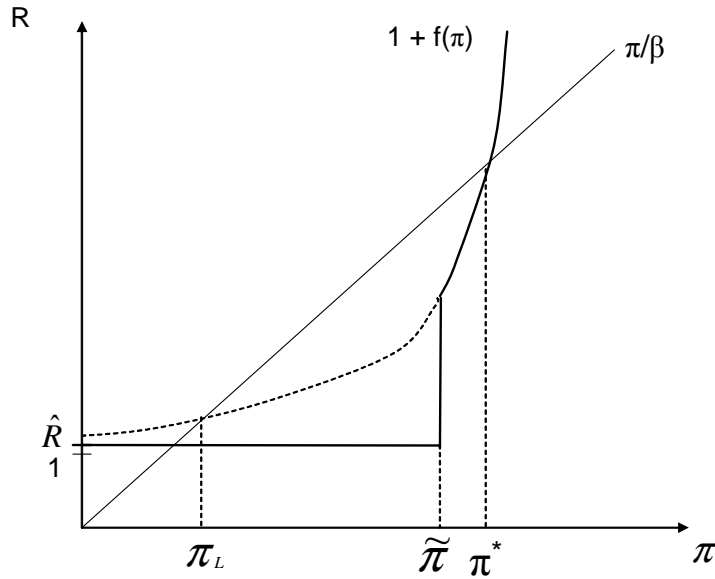

Figure 2: Aggressive interest-rate policy.

The Proposition can be readily understood from Figure 2, which illustrates the normal and aggressive monetary policy rules. Normal monetary policy is represented by the convex curve $1 + f(\pi)$, while the straight line specifies the Fisher equation, which holds in the steady-state. When the inflation threshold

$\tilde{\pi}$ is introduced the interest-rate rule changes as shown, with $R = \hat{R}$ for $\pi < \tilde{\pi}$. In the Figure, we have set $\tilde{\pi} > \pi_L$, and this eliminates the π_L steady state. However, the intersection of the Fisher equation with $R = \hat{R}$ creates a new steady state with the even lower inflation rate of $\hat{\pi}$. Not surprisingly, the qualitative properties of $\hat{\pi}$ are the same as of π_L under normal policy, so that $\hat{\pi}$ is a saddle point under learning.[13]

We also illustrate this point numerically using a phase diagram showing expectational dynamics. We here set $\hat{R} = 1.0001$, so that net nominal interest rates are cut almost to zero. Figure 3 shows the impact of setting a value $\tilde{\pi} = 1.01 > \pi_L$. Other parameter values are as in Figure 1. The aggressive monetary policy triggered at $\tilde{\pi}$ eliminates the unstable steady state at π_L and increases the basin of attraction of the π^* steady state. However, the deflationary spiral continues to exist for sufficiently pessimistic expectations.
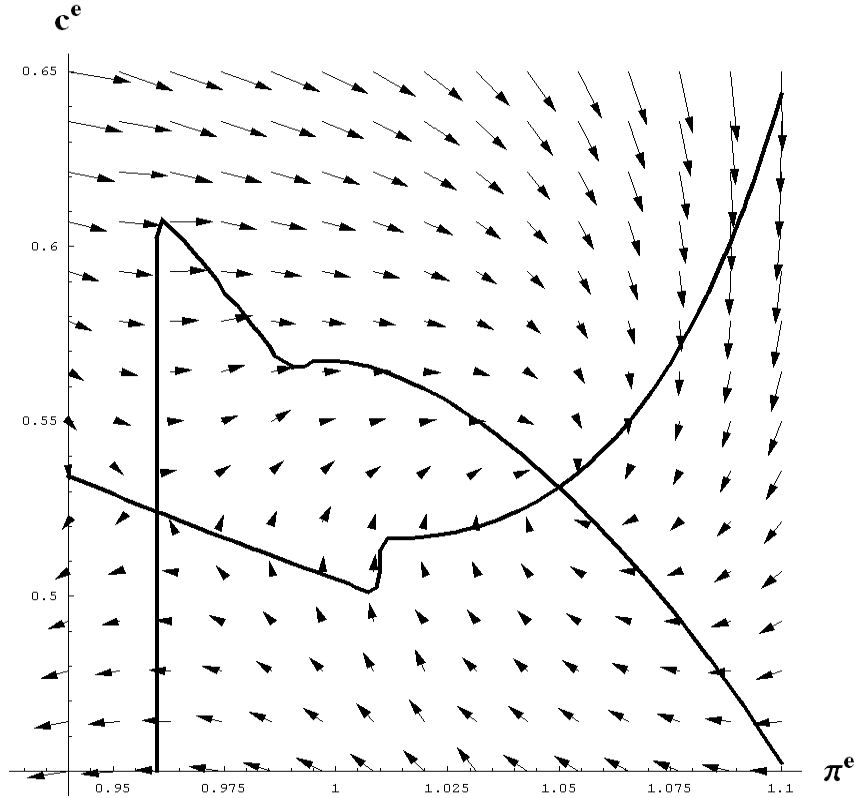

Figure 3: Two steady states with standard fiscal policy and $\pi_L < \tilde{\pi} < \pi^*$

The conclusion from this analysis is that aggressive monetary policy will not always be sufficient to eliminate deflationary spirals and stagnation. We therefore now take up fiscal policy as a possible additional measure.

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[13]If $\hat{\pi} < \tilde{\pi} < \pi_L$ there will be four steady states, and both $\hat{\pi}$ and π_L are locally saddle points under learning.

5 Combined Monetary and Fiscal Policy

We now introduce our recommended policy to combat liquidity traps and deflationary spirals. Normal monetary and fiscal policy is supplemented by a threshold for inflation that policy is designed to achieve:

$$\pi_t \geq \tilde{\pi}, \tag{26}$$

where $\tilde{\pi}$ satisfies the assumptions given in the beginning of Section 4. If this threshold were not to be achieved under normal policy, then monetary and/or fiscal policy is as follows.

First, monetary policy is relaxed as required to achieve these targets, subject to the requirement that the interest rate does not fall below a minimum value $\hat{R}$. The value of $\hat{R}$ can be set just above one, in accordance with the beginning of Section 4. If this is not sufficient to achieve (26), then we set $R_t = \hat{R}$ and fiscal policy is used, increasing g_t as required to meet the threshold. The following Lemma shows that this is indeed feasible:

Lemma 1 *Given expectations c_{t+1}^e and π_{t+1}^e and setting $R_t = \hat{R}$, any value of $\pi_t > 1/2$ can be achieved by setting g_t sufficiently high.*

Proof. First, note that $c_t = c_{t+1}^e (\pi_{t+1}^e / \beta \hat{R})^{\sigma_1}$ is fixed when $R_t = \hat{R}$. Implicitly differentiating (22) we obtain

$$\frac{d\pi_t}{dg_t} = \frac{\nu}{2\alpha\gamma} \frac{(1+\varepsilon)\alpha h_t^{1+\varepsilon-\alpha} - \alpha^2 (1-\nu^{-1}) c_t^{-\sigma_1}}{\pi_t - 1/2}.$$

Since $\frac{dh_t}{dg_t} = \alpha h_t^{1-\alpha}$ is bounded above zero and $c_t > 0$ is fixed in the temporary equilibrium, there exists g' such that $\frac{d\pi_t}{dg_t} > 0$ for $g_t > g'$. It follows that $\pi_t \to \infty$ as $g_t \to \infty$: if π_t were bounded then $\frac{d\pi_t}{dg_t}$ would be unbounded, which would be a contradiction. ∎

The Lemma shows that policy can be designed to guarantee an inflation floor. We now specify a policy based on this result. If the inflation bound $\tilde{\pi}$ is not achieved under normal policy, then we (i) compute the interest rate $\breve{R}_t$ consistent with equations (22), (23) and $\pi_t = \tilde{\pi}$, (ii) set $R_t = \max[\breve{R}_t, \hat{R}]$. If $R_t = \hat{R} > \breve{R}_t$ then g_t is adjusted upward and is set equal to the minimum value such that the bound is met. By the Lemma this is feasible.

Intuitively, if the target would not be met under normal policy the first priority is to relax monetary policy to the extent required to achieve it. If the zero net interest rate lower bound renders monetary policy inadequate to the task, then aggressive fiscal policy is deployed. In terms of Figure 2, monetary policy is unchanged, but fiscal policy ensures that the region of interest is $\pi_t \geq \tilde{\pi}$. As we will see below, the choice of the threshold inflation rate $\tilde{\pi}$ is crucial for the success of this policy, and our recommended policy sets $\pi_L < \tilde{\pi} < \pi^*$. We have the result (see Appendix B for the proof):

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Proposition 4 *(i) If $\pi_L < \tilde{\pi} < \pi^*$ then π^* is the unique steady state, and it is locally stable under learning for all $\gamma > 0$ sufficiently small.*
(ii) If $\hat{\pi} < \tilde{\pi} < \pi_L$ then there are three steady states π^*, π_L and $\tilde{\pi}$. π^* and $\tilde{\pi}$ are locally stable under learning for all $\gamma > 0$ sufficiently small.

The following figures illustrate two of the possibilities. In both figures we set $\hat{R} = 1.0001$, so that when aggressive monetary policy is triggered the nominal interest rate is cut almost all the way to the zero lower bound. (Other parameters are as before.) Figure 4 sets $\tilde{\pi} = 1 > \pi_L$. This illustrates our recommended policy in which we set $\pi_L < \tilde{\pi} < \pi^*$. There is now a unique steady state at π^* and it is evident in the figure that it is globally stable.
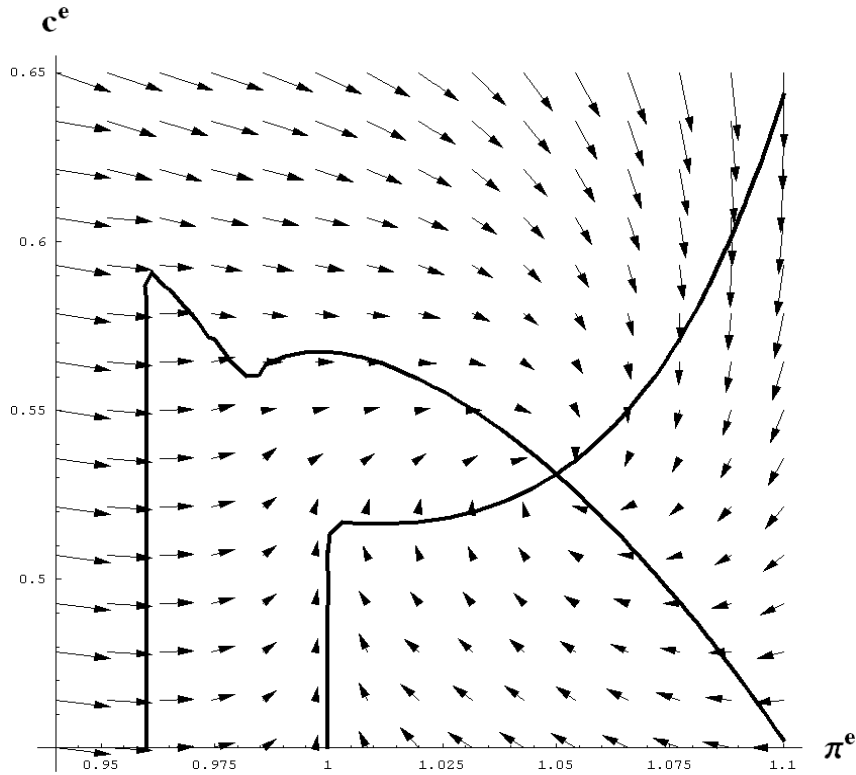

Figure 4: Inflation threshold $\pi_L < \tilde{\pi} < \pi^*$ for aggressive monetary and fiscal policies.

It is crucial to set $\tilde{\pi} > \pi_L$ to get the desired properties of our recommended policy. As indicated in part (ii) of Proposition 4, setting $\tilde{\pi}$ too low can result in multiple equilibria. An illustration is provided in Figure 4. To make the figure clearer we have changed $A = 1.5$ to make $\pi_L \approx 0.995$ and set $\tilde{\pi} = 0.975$. This illustrates possibility (ii) of the Proposition, in which there are three steady states, with the lowest steady state $\tilde{\pi}$ now also locally stable. This particular case might possibly have empirical relevance. As we will see later

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in the simulations, if aggressive fiscal policy is used late in a deflationary situation, policymakers may choose an inflation threshold too low, i.e. $\tilde{\pi} < \pi_L$, with the result that the economy converges to a steady state below the targeted inflation rate π^*.

Our main finding is that a combination of aggressive monetary and fiscal policy to maintain a sufficiently high lower bound on inflation will eliminate the possibility of a deflationary spiral. Choosing $\pi_L < \tilde{\pi} < \pi^*$ eliminates the π_L steady state and does not create any new ones. The key reason for this is that the inflation threshold $\pi_t \geq \tilde{\pi}$ is achievable by bringing in aggressive fiscal policy, if necessary, to supplement aggressive monetary policy. Having set the policy to ensure this inflation threshold, we simultaneously ensure that the system is restricted to a region in which there are stable learning dynamics.
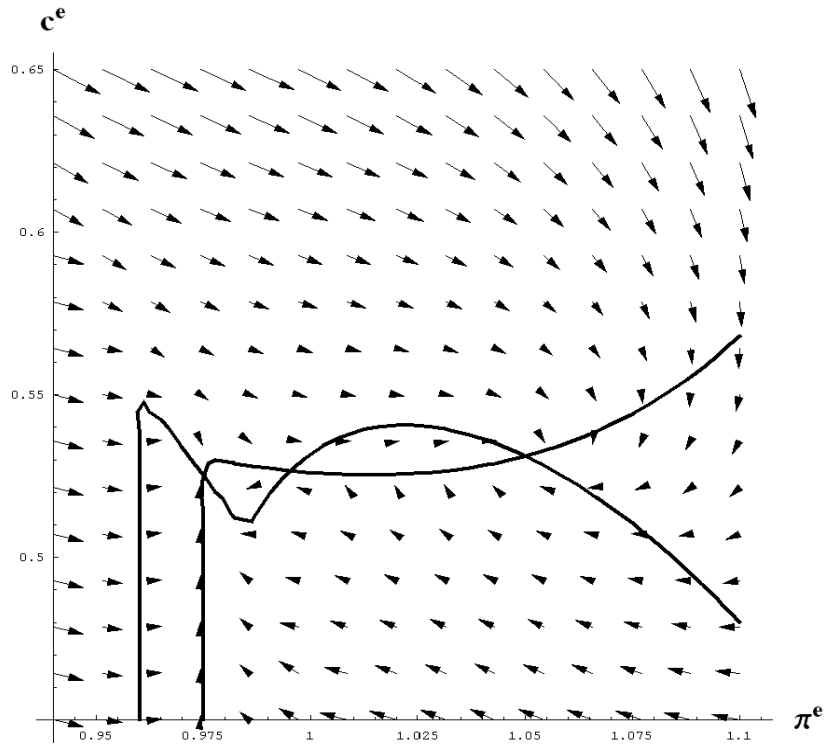

Figure 5: Inflation threshold $\tilde{\pi} < \pi_L$ for aggressive monetary and fiscal policies.

6 An Output Threshold for Policy?

The preceding discussion naturally raises the question of whether another type of threshold might be used for triggering aggressive policies. Consider in particular the possibility that the policy authorities choose an output lower bound, so that policies ensure $c_t + g_t \geq \tilde{y}$ by first dropping interest rates as needed to

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ensure this threshold, subject to their not falling below the floor $\hat{R}$. If setting $R = \hat{R}$ is not sufficient to meet the output threshold then also g_t is raised as required to ensure $y_t = \tilde{y}$. Thus this policy is analogous to the one recommended in Section 5, except that we now have an output lower bound instead of an inflation threshold.

Surprisingly, it turns out that this form of policy does not eliminate the possibility of the economy getting stuck in a deflationary spiral. The details of the analysis depend on the steady-state relationship between output and inflation. Combining (13), (14) and the production function $y = h^\alpha$ yields

$$-y^{(1+\varepsilon)/\alpha} + \frac{\alpha\gamma}{\nu}(1-\beta)(\pi-1)\pi + \alpha\left(1-\frac{1}{\nu}\right)y(y-g)^{-\sigma_1} = 0. \qquad (27)$$

We restrict attention to values $\pi > 1/2$, and so given y and g this equation determines a unique π whenever a solution exists. Also, for a given g it can be shown that a sufficient condition for $\partial\pi/\partial y > 0$ is that $\sigma_1 > 1$. (See Appendix A.3 for details on these points.) The condition $\sigma_1 > 1$ is sufficient but not necessary for an upward sloping long-run Phillips curve. For convenience, we here restrict attention to the case in which $\partial\pi/\partial y > 0$ throughout the range of admissible y. This in particular implies that $y_L < y^*$, where y_L denotes the output level in the π_L steady state and y^* denotes output in the π^* steady state.

Let $\pi_{\tilde{y}}$ denote the steady-state inflation at $y = \tilde{y}$ when $g = \bar{g}$. As usual we make an assumption that $\hat{R}$ is above, but close to one. We have the following result (see Appendix B for the proof):

Proposition 5 *Assume that $\partial\pi/\partial y > 0$.*
(i) If $\tilde{y} < y_L$ with $\pi_L > \pi_{\tilde{y}} > \hat{\pi}$ there are four steady states. These include the π_L and π^*steady states, a constrained steady state with $\pi = \pi_{\tilde{y}}$, $g = \bar{g}$ and $R > \hat{R}$, and a steady state at $\hat{\pi} = \beta\hat{R}$, with $R = \hat{R}$, $y = \tilde{y}$, $g > \bar{g}$. If instead $\tilde{y} < y_L$ with $\pi_{\tilde{y}} < \hat{\pi}$ then there are only the two unconstrained steady states at π_L and π^*.
(ii) if $y_L < \tilde{y} < y^*$ the steady states consist of the normal π^* steady state and a second steady state at $\hat{\pi} = \beta\hat{R}$, with $R = \hat{R}$, $y = \tilde{y}$, $g > \bar{g}$.
(iii) if $\tilde{y} > y^*$ then there is one steady state at $\hat{\pi}$ as in (ii) and a second steady state at $\pi_{\tilde{y}} > \pi^*$ with $R = \beta^{-1}\pi_{\tilde{y}}$.
In each case, the steady state with the lowest inflation rate is a saddle point under learning.

This Proposition shows that however an output target is set, the lowest inflation steady state is a saddle under learning and therefore there are nearby paths taking the form of a deflationary spiral.

To illustrate the results, suppose we set the output threshold so that $y_L < \tilde{y} < y^*$. In particular we set $\tilde{y}$ at 99.5 percent of the high steady state output (the other parameters are unchanged). In this case a constrained steady state at $\hat{\pi} = \beta\hat{R}$ exists, which again is locally a saddle point under learning. Figure

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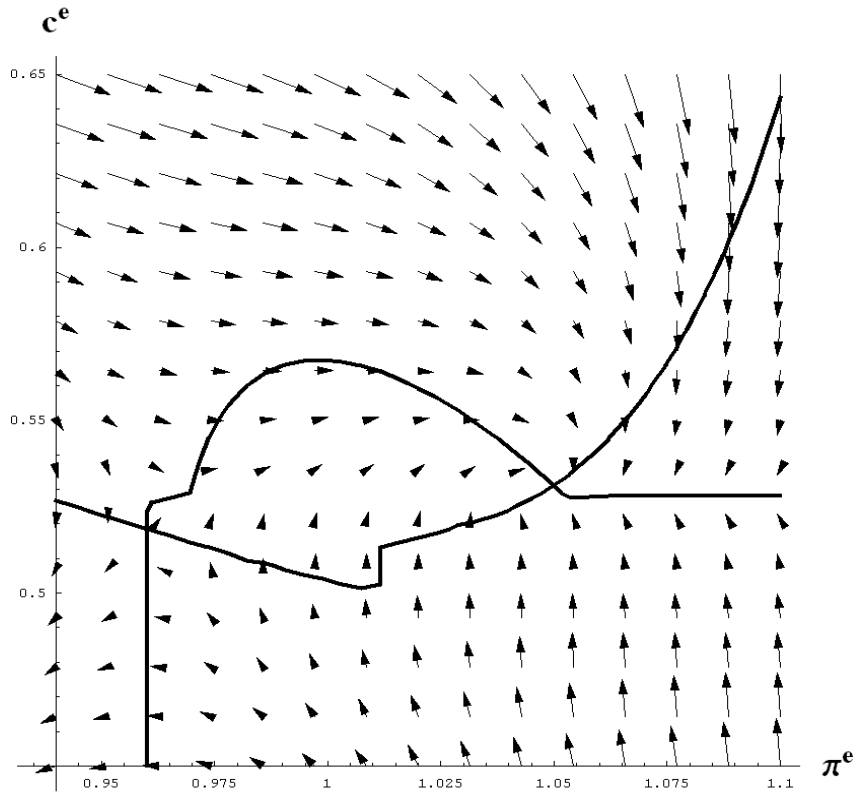6 shows that deflationary spirals exist at the bottom-left corner of the phase diagram.

Figure 6: Learning dynamics with output threshold $\tilde{y} > y_L$.

On these deflationary spiral paths consumption falls steadily after a certain point. Output is then sustained by ever increasing government spending. The intuition is that in a deflationary spiral, even at a near-zero net nominal interest rate $R_t = \hat{R}$, the ex-ante real interest rate increases, which depresses private consumption. Simply maintaining output is not enough. In order to put a floor on consumption it is critical to put an upper bound on real interest rates, and this can only be done by stabilizing inflation. One might think that stabilizing output at a high enough level is enough to stabilize π_t, but this is not the case. In the temporary equilibrium Phillips Curve (22), π_t depends separately on output $y_t = c_t + g_t$ and on consumption, c_t. In particular π_t depends negatively on the marginal utility of consumption. Consequently, if $y_t = \tilde{y}$ is maintained by increasing g_t in the face of falling c_t, inflation will continue to fall because household/firms become more willing to reduce prices as the marginal utility of consumption rises.

7 Stochastic Simulations

We now illustrate our recommended policy using real-time stochastic simulations. We here assume a constant gain form of the learning rule with a small gain. Simulations confirm local convergence to the stable targeted steady state under normal policy and global convergence under our recommended policy in which when normal policy is augmented by aggressive monetary and fiscal policy if π_t threatens to fall below a threshold $\tilde{\pi} > \pi_L$.

It is beneficial to have our recommended policies in place before a collapse in expectations. We illustrate how our policies work in real time, in the face of pessimistic expectations, if initially normal policies are used, and then our recommended policies are implemented after some point t_1. For the simulations we have chosen $\pi^* = 1.02$, corresponding to an inflation target of 2% p.a. With an interest-rate rule parameter of $A = 1.8$ the low inflation steady state π_L is approximately $\pi_L = 0.975$, a deflation rate of 2.5% p.a.. Other parameters are close to those used earlier.[14] For the inflation threshold that triggers aggressive monetary and fiscal policy we choose $\tilde{\pi} = 1$, i.e. zero net inflation or price stability.

We consider the impact, under real-time learning, of a negative expectations shock. We start in the targeted steady-state, with $\pi^* = 1.02$ and $c^* = 0.52864$. Then at $t = 1$ there is a negative shock to expectations, in which π^e falls to 1.01 and c^e falls to 0.486. This is a substantial fall in consumption expectations, of just over 8%, combined with a drop in inflation expectations. The magnitude of these expectation shocks, which we treat as an exogenous pessimistic shift that is not rooted in fundamentals, turns out to be just sufficient to put the economy on a path toward a deflationary spiral under normal policy. We consider the impact if our recommended policy is not implemented until $t_1 = 150$ vs. implementation at $t_1 = 80$, and we compare both to the outcomes if recommended policy is initially in place. Figures 7 - 9 give the results in the form of time paths of π, π^e, c, c^e, R, g and b.

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[14]Parameters are $A = 1.8$, $\pi^* = 1.02$, $\beta = 0.96$, $\sigma_1 = 0.95$, $\alpha = 0.75$, $\gamma = 5$, $\nu = 1.5$, $\varepsilon = 1$, $g = 0.1$, $\hat{R} = 1.002$. Other parameters are $\phi = 1/30$, $\sigma_\theta = 0.02$, $\sigma_u = 0.000001$, $\sigma_\psi = 0.000001$, $\sigma_\eta = 0.001$, $\kappa_0 = -0.005$, $\kappa = \beta^{-1} - 1 + 0.15$, and $\chi = 0.0005$.
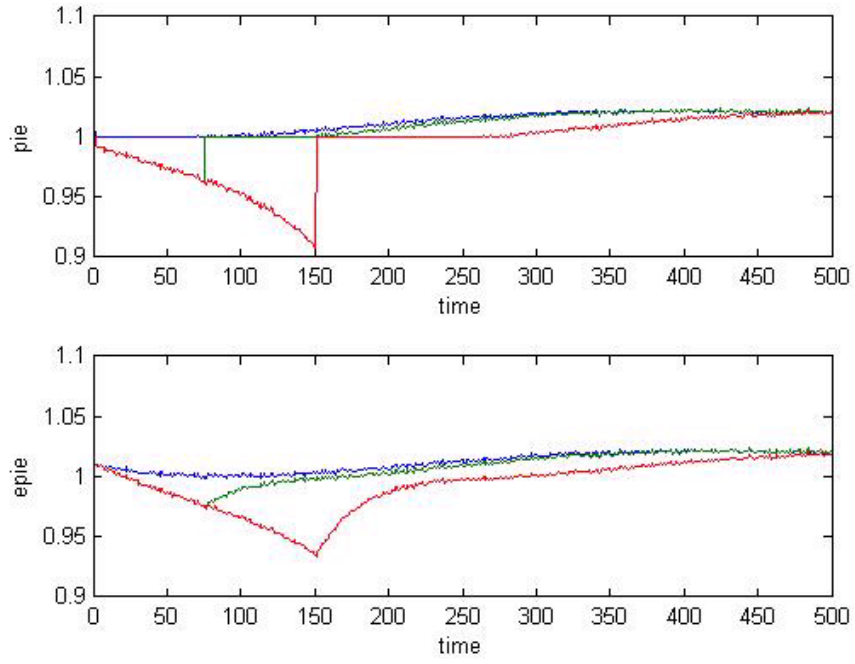
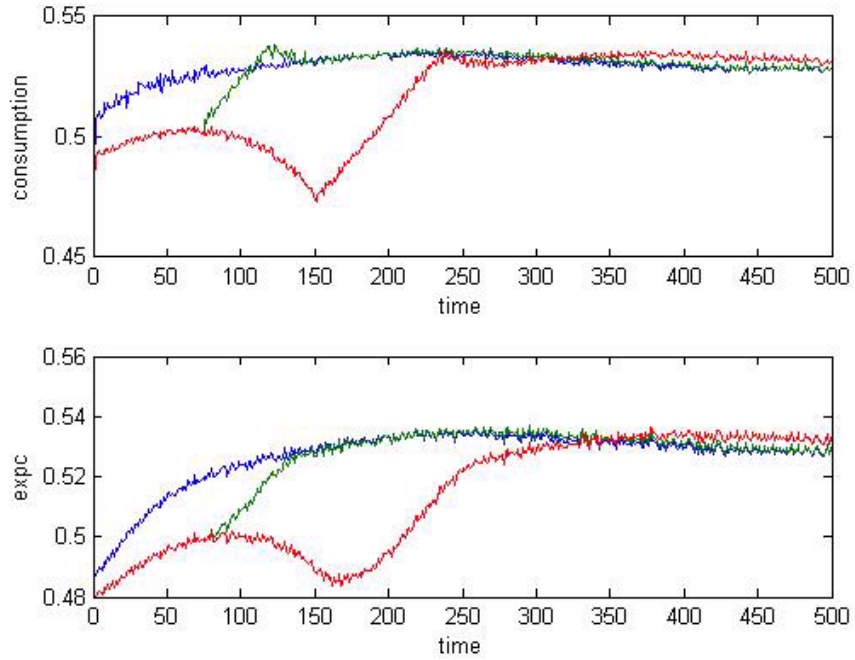Figure 7: Dynamics of π and π^e after pessimistic expectations shock.

Figure 8: Dynamics of c and c^e after pessimistic expectations shock.
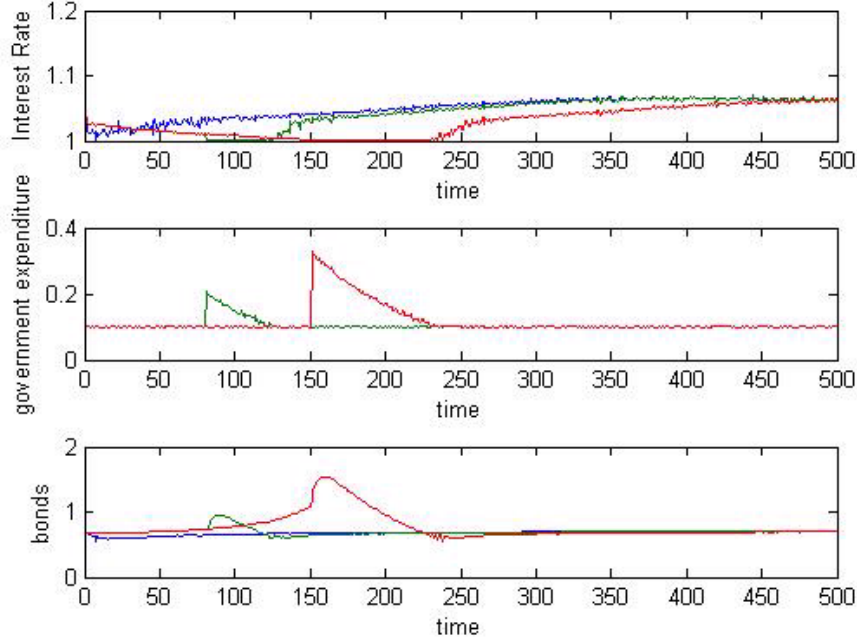
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Figure 9: Dynamics of R, g and b after pessimistic expectations shock.

For $t_1 = 150$ the Figures show consumption diverging to low values before the augmented policies are introduced. Inflation is on a steady downward trajectory when only normal policy rules are in place. Introduction of the aggressive policies at t_1 leads to a recovery of inflation and consumption to the targeted steady-state values. It is seen that interest rates fall to the floor level $\hat{R}$ and debt gradually rises under the normal policy regime in which government spending is constant. At time 150, when the augmented policies are introduced, this leads to an increase in government spending and consequently a further substantial increase in debt in a short interval in time. With the new policy government spending is gradually reduced as expectations of inflation and consumption recover. This also allows debt to return gradually to the steady state. Interest rates also return to normal levels and inflation converges towards π^*.

The results for $t_1 = 80$ show that introduction of our policies at an earlier time avoids the worst part of stagnation. Consumption does not fall as much and returns to normal levels much earlier, and the debt level does not rise nearly as much. Finally, if our policies are in place at the time of the expectations shocks, the impact of the shocks is much less severe. In fact, in this case aggressive monetary policy is enough to maintain inflation at $\tilde{\pi} = 1$ in the face of the shocks, and consequently aggressive fiscal policy is never required. These results clearly show that incorporation of an inflation threshold policy can prevent the economy from sliding into a deflationary spiral and can greatly

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attenuate the impact of pessimistic expectations shocks.

However, monetary policy alone is not *always* sufficient. Consider the economy with everything the same except that the initial drop in c^e is larger. Figure 10 shows the simulation results if π^e again falls to 1.01 but c^e now initially falls to 0.47, about 11%. Even with the inflation threshold policy in place, these shocks are sufficiently large that they cannot be offset by monetary policy even though interest rates are dropped immediately to the floor. Some use of fiscal policy is needed to stabilize prices and achieve $\pi_t \geq \tilde{\pi}$. However, it can again be seen that only a modest use of fiscal policy is needed if the threshold policy is in place when the shocks occur. Waiting to implement our recommended policies leads to lower consumption, and greater use of fiscal policy with a larger (though temporary) build-up of debt.
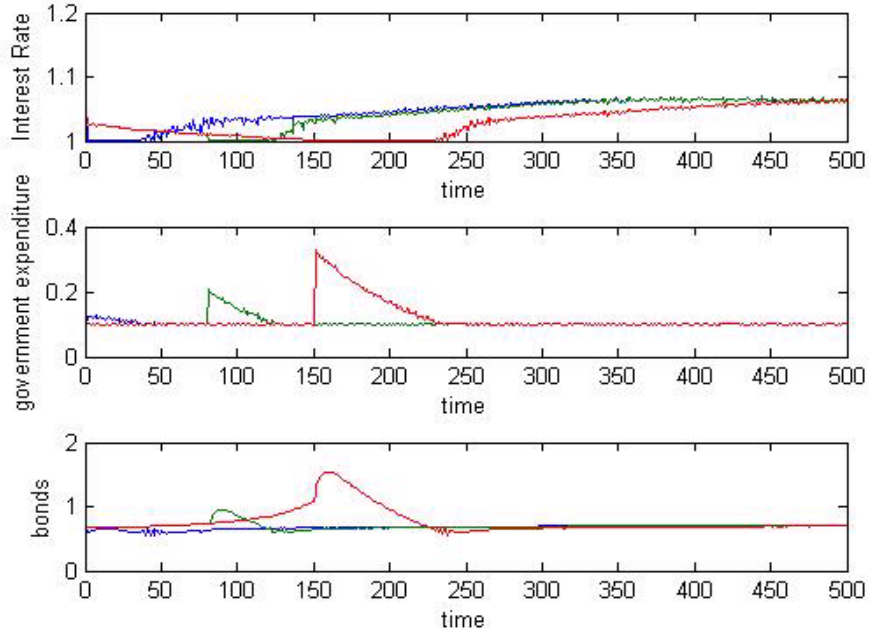

Figure 10: R, g and b after larger pessimistic shock to expectations.

Since the impact of aggressive monetary policy is limited by the zero lower bound, one might expect that a higher inflation target π^* would lead to a lower likelihood of needing countercyclical fiscal policy. Figure 11 reports the results for R, g and b when all the parameters and the sequence of random shocks are the same, except that $\pi^* = 1.05$ and that π^e falls from 1.05 to 1.04 instead of from 1.02 to 1.01. c^* is about the same and the initial drop in c^e is to the same level as before. We keep the inflation threshold at $\tilde{\pi} = 1$. As anticipated, there is now no need for fiscal policy because there is greater room for aggressive monetary policy. Although a higher π^* provides additional flexibility for monetary policy, this must be set against the greater inefficiency of having a
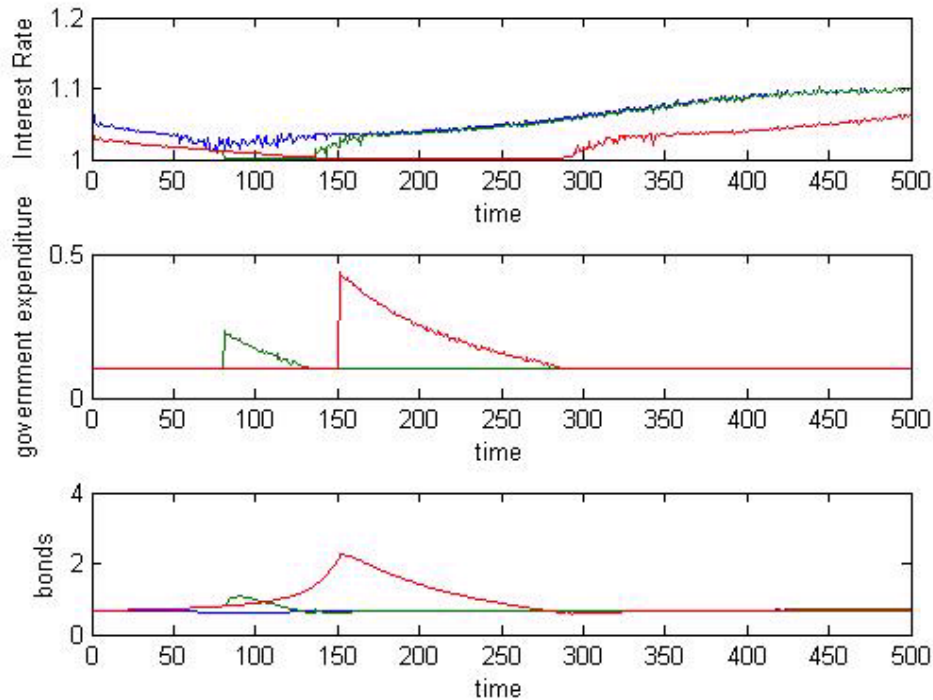
higher steady state inflation rate.

Figure 11: Larger expectations shocks but higher inflation target π^*.

8 Further Discussion

Our analysis raises a number of questions, some of which may lead to fruitful extensions. First, in principle one could dispense with aggressive monetary policy and simply resort to aggressive fiscal policy whenever π_t threatens to fall below $\tilde{\pi}$. However, we think our recommended policy is clearly preferable because there are good reasons to treat monetary policy as the primary tool for counter-cyclical macroeconomic policy. It is reasonable to assume that the mean levels of government spending have been set to balance costs and benefits. If extensive government spending is used guarantee the inflation threshold, then it is likely that much of the spending will be wasteful in the sense that private consumption would be more highly valued. We therefore prefer to use fiscal policy as a policy of last resort to ensure the inflation threshold.

Second, our fiscal policy takes the form of changes in government spending, since with lump-sum taxes Ricardian Equivalence holds under rational expectations. We remark that if variations in g_t are balanced by equal changes in lump-sum taxes, then the temporary debt build-up, that sometimes accompanied our recommended policy, could be avoided. Of course, lump-sum taxes

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are unrealistic and a useful extension would be to look at a model that includes tax distortions, to make sure that our recommended policy continues to guarantee global stability in this set-up. With distortionary taxes there is an efficiency advantage to tax-rate smoothing, so one would again expect a temporary build-up of debt whenever aggressive fiscal policy is required.

Another issue concerns the potential role of commitment or announcements of future policy changes. Commitment to an optimal policy rule can readily be handled within an adaptive learning approach, as discussed by Evans and Honkapohja (2003) and Evans and Honkapohja (2006) for monetary policy in normal times. The history dependence introduced into the economy by the Central Bank should be reflected in the form of the Perceived Law of Motion used by private agents: the list of explanatory variables should be augmented to include, for example, lagged GDP. The general orientation of the adaptive learning approach is that commitment to a specific policy rule will affect private-agent expectations, possibly gradually, as the parameters of the agents' forecasting model adapt statistically to observed outcomes. It would be of interest to examine in our setting more general interest-rate rules that incorporate some form of history dependence.

A related issue concerns the planning horizon assumed for our boundedly rational agents. The approach we have adopted here is based on "Euler equation learning," in which we treat the Euler equations (22)-(23) as the behavioral equations that determine π_t and c_t. This is a valid and convenient approach to modeling bounded rationality since the Euler equations express necessary first-order conditions for optimum decision-making. Euler equation learning converges to rational expectations equilibria in a variety of contexts, including Real Business Cycle models, simple Overlapping Generations Models and New Keynesian models with appropriate interest-rate rules. An alternative approach, stressed in Preston (2005), retains adaptive learning, but asks agents each to forecast infinitely far into the future and to re-solve their dynamic optimization problem each period. Frequently, these approaches do not come to significantly different qualitative conclusions concerning stability. One situation where the planning horizon is important is when private agents confidently anticipate unique future structural or policy changes that have not yet been implemented. How to treat this within an adaptive learning framework is analyzed in Evans, Honkapohja, and Mitra (2007) for a flexible price model and future fiscal policy changes.

9   Conclusions

The recent theoretical literature on the zero lower bound to nominal interest rates has emphasized the possibility of multiple equilibria and liquidity traps when monetary policy is conducted using a global Taylor rule. Most of this literature has focused on models with perfect foresight or fully rational expectations. We take these issues very seriously, but our findings for these

models under adaptive learning are quite different and in some ways much more alarming than suggested by the rational expectations viewpoint. We have shown that under standard monetary and fiscal policy, the steady state equilibrium targeted by policymakers is locally stable. In normal times, these policies will appropriately stabilize inflation, consumption and output. However, the desired steady state is not globally stable under normal policies. A sufficiently large pessimistic shock to expectations can send the economy along an unstable deflationary spiral.

To avoid the possibility of deflation and stagnation we recommend a combination of aggressive monetary and fiscal policy triggered whenever inflation threatens to fall below an appropriate threshold. Monetary policy should immediately reduce nominal interest rates, as required, even (almost) to the zero interest floor if needed, and this should be augmented by fiscal policy if necessary. Intriguingly, using an aggregate output threshold in the same way will not always successfully reverse a deflationary spiral.

When aggressive fiscal policy is necessary, this will lead to a temporary build-up of government debt. However, government spending and debt will gradually return to their steady state values. An earlier implementation of the recommended policies will mitigate the use of government spending, and if our recommended policy is already in place at the time of the shocks, the immediate use of aggressive monetary policy can in some (but not all) cases entirely avoid the need to use fiscal policy. Raising the inflation target π^* is an alternative way of reducing the likelihood of needing to employ fiscal policy, but this may be undesirable for other reasons.

A Derivations

A.1 Private sector optimization

One can show that

$$
\begin{aligned}
\frac{P_{t,j}}{P_t} y_{t,j} &= Y_t^{1/\nu} y_{t,j}^{1-1/\nu} \\
&= Y_t^{1/\nu} h_{t,j}^{\alpha(1-1/\nu)}
\end{aligned}
$$

and that firm $j's$ gross inflation can be expressed as

$$
\begin{aligned}
\frac{P_{t,j}}{P_{t-1,j}} &= \frac{\left(\frac{y_{t,j}}{Y_t}\right)^{-1/\nu} P_t}{\left(\frac{y_{t-1,j}}{Y_{t-1}}\right)^{-1/\nu} P_{t-1}} \\
&= \left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \left(\frac{h_{t,j}}{h_{t-1,j}}\right)^{-\alpha/\nu} \frac{P_t}{P_{t-1}}.
\end{aligned}
$$

These allow us to write the utility function in the form

$$
\begin{aligned}
U_{t,j} &= \frac{c_{t,j}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(m_{t-1,j} * \pi_t^{-1}\right)^{1-\sigma_2} - \frac{h_{t,j}^{1+\varepsilon}}{1+\varepsilon} \\
&\quad - \frac{\gamma}{2}\left(\left(\frac{Y_t}{Y_{t-1}}\right)^{1/\nu} \left(\frac{h_{t,j}}{h_{t-1,j}}\right)^{-\alpha/\nu} \frac{P_t}{P_{t-1}} - 1\right)^2 .
\end{aligned}
$$

Next, the Lagrangian can be expressed as

$$
\begin{aligned}
\mathcal{L} &= E_{0,j} \sum_{t=0}^{\infty} [\beta^t U_{t,j} - \beta^{t+1}\lambda_{t+1,j}[c_{t,j} + w_{t+1j} + \Upsilon_{t,j} - ml_{t,j}\pi_t^{-1} \\
&\quad - R_{t-1}\pi_t^{-1}\left(w_{t,j} - ml_{t,j}\right) - Y_t^{1/\nu} h_{t,j}^{\alpha(1-1/\nu)}] \\
&\quad - \beta^{t+1}\mu_{t+1,j}\left(ml_{t+1,j} - m_{t,j}\right) - \beta^{t+1}\eta_{t+1,j}\left(hl_{t+1,j} - h_{t,j}\right)]
\end{aligned}
$$

where the notation

$$
\begin{aligned}
w_{t+1,j} &= m_{t,j} + b_{t,j} \\
ml_{t+1,j} &= m_{t,j} \\
hl_{t+1,j} &= h_{t,j}
\end{aligned}
$$

is employed. Here $w_{t,j}$, $ml_{t,j}$ and $hl_{t,j}$ are the state variables and $c_{t,j}$, $m_{t,j}$ and $h_{t,j}$ are the control variables. In addition, π_t, Y_t and Y_{t-1} are state variables, which are viewed as exogenous by each agent. Here we also, for clarity, use the notation $E_{t,j}$ for the expectation of agent j conditional on information at time t.

Following Chow (1996), we write the Lagrangian into the general form

$$\mathcal{L} = E_0 \left[\sum_{t=0}^{\infty} \left\{ \beta^t r\left(x_t, u_t\right) - \beta^{t+1} \xi'_{t+1} \left[x_{t+1} - f\left(x_t, u_t\right) - \varepsilon_{t+1}\right] \right\} \right],$$

for which the FOCs are expressed compactly as

$$\frac{\partial}{\partial u_t} r\left(x_t, u_t\right) + \beta \frac{\partial}{\partial u_t} f'\left(x_t, u_t\right) E_t \xi_{t+1} = 0$$

$$\xi_t = \frac{\partial}{\partial x_t} r\left(x_t, u_t\right) + \beta \frac{\partial}{\partial x_t} f'\left(x_t, u_t\right) E_t \xi_{t+1}.$$

Using these, the FOCs for the problem at hand are obtained as follows:
wrt $c_{t,j}$:

$$c_{t,j}^{-\sigma_1} - \beta E_{t,j} \lambda_{t+1,j} = 0, \tag{28}$$

wrt $m_{t,j}$:

$$E_{t,j} \mu_{t+1,j} = 0, \tag{29}$$

wrt $w_{t,j}$:

$$\lambda_{t,j} = \beta \left(R_{t-1} \pi_t^{-1}\right) E_{t,j} \lambda_{t+1,j}, \tag{30}$$

wrt $ml_{t,j}$:

$$\mu_{t,j} = \chi \left(ml_{t,j} \pi_t^{-1}\right)^{-\sigma_2} \pi_t^{-1} + \beta \left(\pi_t^{-1} - R_{t-1} \pi_t^{-1}\right) E_{t,j} \lambda_{t+1,j}, \tag{31}$$

wrt $h_{t,j}$:

$$-h_{t,j}^{\varepsilon} + \gamma \left(\left(\frac{Y_t}{Y_{t-1}} \right)^{1/\nu} \frac{P_t}{P_{t-1}} \left(\frac{h_{t,j}}{hl_{t,j}} \right)^{-\alpha/\nu} - 1 \right)$$

$$* \frac{\alpha}{\nu} \left(\frac{h_{t,j}}{hl_{t,j}} \right)^{(-\alpha/\nu)-1} \frac{1}{hl_{t,j}} \left(\frac{Y_t}{Y_{t-1}} \right)^{1/\nu} \frac{P_t}{P_{t-1}}$$

$$+ \beta Y_t^{1/\nu} \alpha \left(1 - \frac{1}{\nu} \right) h_{t,j}^{\alpha(1-1/\nu)-1} E_{t,j} \lambda_{t+1,j} + \beta E_{t,j} \eta_{t+1,j} = 0, \tag{32}$$

wrt $hl_{t,j}$:

$$\eta_{t,j} = -\gamma \left(\left(\frac{Y_t}{Y_{t-1}} \right)^{1/\nu} \frac{P_t}{P_{t-1}} \left(\frac{h_{t,j}}{hl_{t,j}} \right)^{-\alpha/\nu} - 1 \right)$$

$$* \frac{\alpha}{\nu} \left(\frac{h_{t,j}}{hl_{t,j}} \right)^{(-\alpha/\nu)-1} \left(\frac{Y_t}{Y_{t-1}} \right)^{1/\nu} \frac{P_t}{P_{t-1}} \frac{h_{t,j}}{hl_{t,j}^2}. \tag{33}$$

First, we rewrite (33) in terms of $\pi_{t,j} = P_{t,j}/P_{t-1,j}$. From the demand function (3) we have

$$\left(\frac{h_{t,j}}{hl_{t,j}} \right)^{-\alpha/\nu} = \pi_{t,j} \pi_t^{-1} \left(\frac{Y_t}{Y_{t-1}} \right)^{-1/\nu}$$

and so
$$\eta_{t,j} = -\frac{\alpha\gamma}{\nu}(\pi_{t,j} - 1)\pi_{t,j}\frac{1}{h_{t-1,j}}. \tag{34}$$

Similarly we rewrite (32) as

$$
\begin{aligned}
0 = {} & -h_{t,j}^{\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_{t,j} - 1)\pi_{t,j}\frac{1}{h_{t,j}} \\
& + \beta\alpha\left(1 - \frac{1}{\nu}\right)Y_t^{1/\nu}\frac{y_{t,j}^{(1-1/\nu)}}{h_{t,j}}E_{t,j}\lambda_{t+1,j} + \beta E_{t,j}\eta_{t+1,j}.
\end{aligned}
$$

Using (28) and (34) we get

$$
\begin{aligned}
0 = {} & -h_{t,j}^{\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_{t,j} - 1)\pi_{t,j}\frac{1}{h_{t,j}} \\
& + \alpha\left(1 - \frac{1}{\nu}\right)Y_t^{1/\nu}\frac{y_{t,j}^{(1-1/\nu)}}{h_{t,j}}c_{t,j}^{-\sigma_1} + -\frac{\alpha\gamma\beta}{\nu}\frac{1}{h_{t,j}}E_{t,j}(\pi_{t+1,j} - 1)\pi_{t+1,j}.
\end{aligned}
$$

Since all agents are identical, we get the Phillips curve

$$-h_t^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t + \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha}c_t^{-\sigma_1} = \beta\frac{\alpha\gamma}{\nu}E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\right],$$

which is the same as equation (9) in the main text.

Combining equations (30) and (28) yields

$$\lambda_{t,j} = \beta\left(R_{t-1}\pi_t^{-1}\right)E_{t,j}\lambda_{t+1,j}. \tag{35}$$

Using (28) we have $\lambda_{t,j} = \left(R_{t-1}\pi_t^{-1}\right)c_{t,j}^{-\sigma_1}$ and substituting into (35) yields the consumption Euler equation

$$c_{t,j}^{-\sigma_1} = \beta R_t E_{t,j}\left(\pi_{t+1}^{-1}c_{t+1,j}^{-\sigma_1}\right).$$

Since all agents are identical, we arrive at (10) in the main text. Similarly, by combining equations (29), (31) and (28), we get the money demand equation:

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_t^{-\sigma_1}}{E_t\pi_{t+1}^{\sigma_2-1}}\right)^{-1/\sigma_2},$$

which is equation (11) in the main text.

A.2   Further Properties of Steady States

We show existence of unique steady state values for c and h for a given steady state π under normal policy. Combining (13) and (14), we have the equation

$$-h^{1+\varepsilon} + (1 - \beta)\frac{\alpha\gamma}{\nu}(\pi - 1)\pi + \alpha(1 - \nu^{-1})h^{\alpha}(h^{\alpha} - g)^{-\sigma_1} = 0.$$

Let $\Lambda = (1-\beta)\frac{\alpha\gamma}{\nu}(\pi-1)\pi > 0$ and write the equation as

$$\Lambda + \alpha(1-\nu^{-1})h^\alpha(h^\alpha - g)^{-\sigma_1} = h^{1+\varepsilon}.$$

The RHS is increasing and convex. Consider first the case $\pi \geq 1$. For $g = 0$, the LHS is increasing and concave for $\sigma_1 \leq 1$ and at $h = 0$ it is positive (or zero if $\pi = 1$), so clearly there is a unique interior solution for h. For $g = 0$, when $\sigma_1 > 1$, the LHS is decreasing with limit Λ as $h \to \infty$ so again there is a unique solution. If $g > 0$, the LHS has an asymptote at plus infinity when $h \to g^{1/\alpha}$ from above. For $h > g^{1/\alpha}$ the LHS shifts up and also

$$\frac{\partial}{\partial g}\left(\frac{\partial}{\partial h}(h^\alpha(h^\alpha - g)^{-\sigma_1})\right) = \sigma_1(h^\alpha - g)^{-\sigma_1-1}\left(1 - \frac{1+\sigma_1}{1-g/h^\alpha}\right) < 0$$

for all values of σ_1, so that the preceding arguments can be extended accordingly and there is a unique interior solution.

Finally, if $\pi < 1$ (so that $\Lambda < 0$) various possibilities arise. There may be zero or two interior solutions when $g = 0$. However, for $g > 0$ and π sufficiently close to one, the argument above for the case $\pi = 1$ applies and there is a unique solution.

A.3   Details on Output-Threshold Policies

For an output target $\tilde{y}$ we can find an equivalent employment target $\tilde{h} = (\tilde{y})^{1/\alpha}$. An output-target constrained steady state must satisfy the equation

$$-h^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(1-\beta)(\pi-1)\pi + \alpha\left(1 - \frac{1}{\nu}\right)h^\alpha c^{-\sigma_1} = 0$$

or

$$(\pi-1)\pi = \frac{\nu}{\alpha\gamma(1-\beta)}\tilde{h}^{1+\varepsilon} - \frac{\nu-1}{\gamma(1-\beta)}\tilde{h}^\alpha(\tilde{h}^\alpha - g)^{-\sigma_1} \equiv L.$$

This equation is quadratic in π and solutions are

$$\pi = \frac{1 \pm \sqrt{1+4L}}{2}.$$

The positive root gives the economically sensible solution, i.e., there is a unique solution for π for any given $\tilde{h}$.

To show that for steady states $\partial\pi/\partial y > 0$ if $\sigma_1 > 1$ we consider the equation (27) for $\pi > 1/2$. It is sufficient to show that the expression $y(y-g)^{-\sigma_1}$ is decreasing in y. Clearly,

$$\frac{d}{dy}(y(y-g)^{-\sigma_1}) = (y-g)^{-\sigma_1}[1 - \sigma_1 y(y-g)^{-1}] < 0$$

when $\sigma_1 > 1$ and $g \geq 0$.

B   Proof of Propositions

Proposition 3: First, $\hat{\pi}$ is clearly a steady state and by equation (10) there is no other steady state value below $\hat{\pi}$. The corresponding value $\hat{c}$ for consumption can be computed from (24).

To prove the saddle-point property, we consider the temporary equilibrium defined by the system (24) and (25) that pertains to the region where $R_t = \hat{R}$. Then we show that the determinant of the Jacobian matrix of the E-stability differential equations evaluated at that steady state is always negative for γ sufficiently small.

The temporary equilibrium equation for c_t is simply (25). The corresponding equation for π_t is obtained by solving π_t in terms of π_{t+1}^e and c_t using (24), which is a quadratic equation in π_t (the relevant solution is the larger root), and substituting (25) into the solution of the quadratic.

Using Mathematica (routine available on request) it can be shown that the determinant of the Jacobian matrix of the E-stability differential equations at the steady state $(\hat{\pi}, \hat{c})$ is a ratio of two terms, of which the denominator is always positive. The numerator is proportional to

$$-[c^{1+\sigma_1}(c+g)^{\frac{1+\varepsilon}{\alpha}}(1+\varepsilon)\nu+c^2\alpha^2(\nu-1)(\sigma_1-1)+g^2\alpha^2(\nu-1)\sigma_1+cg\alpha^2(\nu-1)(2\sigma_1-1)].$$

This expression in the square brackets is increasing in σ_1, so that its minimal value obtains when $\sigma_1 = 0$. Imposing $\sigma_1 = 0$ and using (24) at the steady state the numerator can be simplified to

$$-(1+\varepsilon)[-\hat{R}(\beta-1)\beta(\beta\hat{R}-1)\gamma+(c+g)(\nu-1)]+\alpha(c+g)(\nu-1).$$

The final term $\alpha(c+g)(\nu-1)$ is dominated by the negative term $-(1+\varepsilon)(c+g)(\nu-1)$, while the first term in square brackets $-\hat{R}(\beta-1)\beta(\beta\hat{R}-1)\gamma$ can be made arbitrarily small by making γ sufficiently small. The result follows.

Proposition 4: We first remark that the Fisher equation $\beta R = \pi$ holds in all steady states. Furthermore, since π^* is unconstrained in both cases, local stability under learning has already been proved in Proposition 2.

Case (i) follows from the assumption that $\tilde{\pi} < \pi^*$. In a steady state we must have $\pi \geq \tilde{\pi}$. For $\tilde{\pi} \leq \pi < \pi^*$ the interest rate given by $\beta^{-1}\pi > 1 + f(\pi)$, which is impossible. For $\pi > \pi^*$ the floors are necessarily met but then the only solution is π^*.

To prove (ii), suppose $\tilde{\pi} < \pi_L$. Since $\hat{\pi} < \tilde{\pi}$, then clearly there are interior steady states at π^* and π_L in which normal policy is being followed. In the constrained region (where the constraint (26) is binding) we must have $\pi = \tilde{\pi}$ in a steady state. Clearly, $\pi = \tilde{\pi}$ is a steady state for $\breve{R} = \beta^{-1}\tilde{\pi}$. To prove local stability under learning of the steady state at $\tilde{\pi}$, note that the temporary equilibrium map takes a special form. First, $\pi_t = \tilde{\pi}$, which implies that the E-stability differential equation for π^e is $d\pi^e/d\tau = \tilde{\pi}-\pi^e$, which is independent of the c^e differential equation and which is stable. In this steady state we have normal fiscal policy. The temporary equilibrium value for c_t is determined

by (22) with $\pi_t = \tilde{\pi}$ and has the form $c_t = F_c(\pi_{t+1}^e, u_t)$, leading to $T_c(\pi^e) = EF_c(\pi^e, u_t)$.[15] Since $\pi^e \to \tilde{\pi}$, under the E-stability differential equation, c^e converges to the steady-state value of c.

Proposition 5: First, there are no steady states with $\pi_L < \pi < \pi^*$, since for such π the value of R given by the Fisher equation would be greater than the value given by normal monetary policy, which is not possible under output-constrained monetary policy.

(i) Clearly both π_L and π^* are steady states. There cannot be a steady state with $\pi > \pi^*$. In that case we would have $g > \bar{g}$ (otherwise the output target would be met), which implies $R = \hat{R}$, but by the Fisher equation this would be inconsistent. In the case $\pi_L > \pi_{\tilde{y}} > \hat{\pi}$ it is easy to verify that $\pi_{\tilde{y}}$ and $\hat{\pi}$ are additional steady states. If instead $\pi_{\tilde{y}} < \hat{\pi}$ then $\pi_{\tilde{y}}$ cannot be a steady state because the Fisher equation would imply $R < \hat{R}$. Moreover, no π with $\pi_{\tilde{y}} < \pi < \pi_L$ can be a steady state because the corresponding y would then satisfy $y > \tilde{y}$ and the steady state would be unconstrained. This is not possible because for unconstrained steady states both the Fisher equation and the normal interest-rate rule must hold.

(ii) It is clear that π^* is a steady state. Any other steady state is constrained, so that $y = \tilde{y}$. If $g = \bar{g}$, then $\pi_L < \pi_{\tilde{y}} < \pi^*$, which is impossible. Thus, for any steady state other than π^* we have $g > \bar{g}$, which implies that $R = \hat{R}$ and $\pi = \hat{\pi}$.

(iii) In this case $\pi_{\tilde{y}} > \pi^*$ and setting $R = \beta^{-1}\pi_{\tilde{y}}$ we have a steady state at $\pi = \pi_{\tilde{y}}$ and $y = \tilde{y}$ (with $g = \bar{g}$). Any other steady state is constrained at $y = \tilde{y}$ but with $g > \bar{g}$. Hence $R = \hat{R}$ and $\pi = \hat{\pi}$.

We now turn to the stability of the steady states with the lowest inflation. If the only steady states are at π_L and π^* then from earlier results we know that π_L is a saddle point under learning. Otherwise the steady state with the lowest inflation rate is at $\hat{\pi}$. In this case the constrained temporary equilibrium is given by

$$c_t = c_{t+1}^e (\pi_{t+1}^e / \beta \hat{R})^{\sigma_1}$$

and

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t = \frac{\alpha\gamma\beta}{\nu}(\pi_{t+1}^e - 1)\pi_{t+1}^e + (\tilde{y})^{(1+\varepsilon)/\alpha} - \alpha\left(1 - \frac{1}{\nu}\right)\tilde{y}(c_{t+1}^e)^{-\sigma_1}.$$

This system can be solved explicitly for (c_t, π_t) to obtain the constrained temporary equilibrium. Using Mathematica (routine available on request), it can be shown that the determinant of the linearized E-stability differential equation system is negative at the steady state. This implies that the steady state is a saddle point.

---

[15] We remark that the consumption Euler equation (23) determines the temporary equilibrium value of $R_t = \check{R}_t$, given c_{t+1}^e, π_{t+1}^e and the temporary equilibrium value of c_t.

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