# Turbulence Parameters for Non-Reacting conditions of Cambridge Stratified Swirl Burner 

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## 1. Abstract

The turbulence parameters of non-reacting conditions of Cambridge Stratified Swirl Burner are of interest to the community studying the burner. The LDA data [2] was used to calculate key turbulence parameters in axial, radial and tangential directions as well as the total velocity component. This report presents the data processing routines and results for the turbulence parameters.

## 2. Data processing

The turbulence parameters in axial, radial and tangential directions as well as the total velocity component were processed using the same routines with subscript $u, v$ and $w$ denoting each direction. The axial direction was used as an example in the following subsections.

### 2.1 Integral scales

The integral time scale $T_{u u}$ is important for experimentalists and modelers alike. As long as two samples are delayed by more than $T_{u u}$, they can be considered to be statistically independent. There is no point in compiling averages over time series lasting only 1-2 $T_{u u}$, since the averages will simply not have converged. The current study presents a direct calculation of $T_{u u}$ whereas the integral length scale $L_{u u}$ is approximated under Taylor's hypothesis.

### 2.1.1 Integral time scale

The autocorrelation function (ACF) can be obtained from the spectral analysis of the LDA data [1]. Integral time scales are derived from the integration of the ACF coefficient. The ACF coefficient is integrated to the first zero crossing, or to the minimum of the ACF coefficient in the absence of a zero crossing.

### 2.1.2 Integral length scale

Taylor's hypothesis provides a link between temporal and spatial scales. In this case, the integral length scale is related to the integral time scale by:

$$
\begin{equation*}
L_{u u}=T_{u u} \cdot \bar{U} \tag{1}
\end{equation*}
$$

-where $\bar{U}$ denotes the mean axial velocity.

### 2.2 Kolmogorov scales

The smallest length, time and velocity scales in turbulence have been referred as Kolmogorov scales $\eta_{K}, \tau_{K}$ and $v_{K}$ respectively. They were calculated using Eq. (2) through Eq. (5). The components from the estimations can be considered as noise if their scales are smaller than the kolmogorovs scales.

$$
\begin{align*}
& \eta_{K}=L_{u u} \operatorname{Re}_{t}^{-3 / 4}  \tag{2}\\
& \tau_{K}=T_{t u r b}^{\prime} \operatorname{Re}_{t}^{-1 / 2}  \tag{3}\\
& v_{K}=u^{\prime} \operatorname{Re}_{t}^{-1 / 4}  \tag{4}\\
& \operatorname{Re}_{t}=\frac{u^{\prime} L_{u u}}{v} \tag{5}
\end{align*}
$$

where $R e_{t}$ and $v$ are turbulent Reynolds number and kinematic viscosity, respectively. The kinematic viscosity is held constant at the room temperature value for an equivalence ratio of $\phi=0.75\left(v=1.6069 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}\right) . T_{t u r b}^{\prime}=\frac{L_{u u}}{u^{\prime}}$ is the large-eddy time scale.

### 2.3 Damköhler number

The Damköhler number is derived from the axial velocity integral length scale, the laminar flame speed $s_{L}$, the RMS axial velocity, the laminar flame thickness $\delta_{L}$. The laminar flame speed and laminar flame thickness are derived from laminar flame calculations at an equivalence ratio of $\phi=0.75\left(s_{L}=0.2483 \mathrm{~m} / \mathrm{s}, \delta_{L}=5.8835 \times 10^{-4} \mathrm{~m}\right)$ :

$$
\begin{equation*}
D a_{u}=\frac{s_{L} L_{u u}}{u^{\prime} \delta_{L}} \tag{6}
\end{equation*}
$$

### 2.4 Karlovitz number

The Karlovitz number is derived from the laminar flame thickness $\delta_{L}$ and the Kolmogorov number. The laminar flame thickness is derived from laminar flame calculations at an equivalence ratio of $\phi=0.75 \quad\left(\delta_{L}=5.8835 \times 10^{-4} \mathrm{~m}\right)$ :

$$
\begin{equation*}
K a_{u}=\frac{\delta_{L}{ }^{2}}{\eta_{K_{-} u}^{2}} \tag{7}
\end{equation*}
$$

### 2.5 Total velocity component

The calculations of moments and integral length scales for the total velocity were based on the following routines. Other quantities were derived from these combined components:

$$
\begin{equation*}
\text { Mean: } \bar{U}_{\text {mag }}=\sqrt{\bar{U}^{2}+\bar{V}^{2}+\bar{W}^{2}} \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
\text { RMS: } u_{\text {mag }}^{\prime}=\sqrt{u^{\prime 2}+{v^{\prime 2}}^{2}+w^{\prime 2}} \tag{9}
\end{equation*}
$$

Integral length scale: $L_{m a g}=\sqrt{L_{u u}^{2}+L_{v v}^{2}+L_{w w}^{2}}$

## 3. Results

This section presents the key non-reacting turbulence parameters for three velocity components as well as the total velocity component, at the centre of each annular gap of the burner [2]. The results were provided near the burner exit $z=2 \mathrm{~mm}$ where there is no PIV measurement available.

Table 1: Key non-reacting turbulence parameters for axial velocity component near the burner exit ( $z=2 \mathrm{~mm}$ at the centre of each annular gap).

| $S F R$ | $r$ | $\bar{U}$ | $u^{\prime}$ | $I$ | $R e_{t}$ | $T_{u u}$ | $L_{u u}$ | $\eta_{K_{-} u}$ | $\tau_{K_{-} u}$ | $v_{K_{-} u}$ | $D a_{u}$ | $K a_{u}$ | $u^{\prime} / s_{L}$ | $L_{u u} / \delta_{L}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(\mathrm{~mm})$ | $(\mathrm{m} / \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ | $(\%)$ |  | $(\mathrm{ms})$ | $(\mathrm{mm})$ | $(\mu \mathrm{m})$ | $(\mu \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ |  |  |  |  |  |
| 0 | 9 | 9.72 | 1.00 | 10.3 | 148 | 0.25 | 2.4 | 56 | 196 | 0.29 | 1.00 | 110 | 4.0 | 4.2 |  |
| 0 | 15 | 20.73 | 1.84 | 8.8 | 444 | 0.19 | 3.9 | 40 | 100 | 0.40 | 0.89 | 215 | 7.4 | 6.6 |  |
| 0.25 | 9 | 9.66 | 0.93 | 9.7 | 133 | 0.24 | 2.3 | 58 | 210 | 0.28 | 1.02 | 102 | 3.8 | 3.9 |  |
| 0.25 | 15 | 19.63 | 1.42 | 7.1 | 362 | 0.21 | 4.1 | 50 | 155 | 0.32 | 1.24 | 138 | 5.6 | 7.4 |  |
| 0.33 | 9 | 9.47 | 0.87 | 9.2 | 125 | 0.24 | 2.3 | 62 | 236 | 0.26 | 1.10 | 93 | 3.5 | 3.9 |  |
| 0.33 | 15 | 17.53 | 1.73 | 9.8 | 451 | 0.24 | 1.7 | 43 | 115 | 0.38 | 1.02 | 188 | 7.0 | 7.1 |  |

Table 2: Key non-reacting turbulence parameters for radial velocity component near the burner exit ( $z=2 \mathrm{~mm}$ at the centre of each annular gap).

| $S F R$ | $r$ | $\bar{V}$ | $v^{\prime}$ | $I$ | $R e_{t}$ | $T_{v v}$ | $L_{v v}$ | $\eta_{K_{-} v}$ | $\tau_{K_{-} v}$ | $v_{K_{-} v}$ | $D a_{v}$ | $K a_{v}$ | $v^{\prime} / s_{L}$ | $L_{v v} / \delta_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(\mathrm{~mm})$ | $(\mathrm{m} / \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ | $(\%)$ |  | $(\mathrm{ms})$ | $(\mathrm{mm})$ | $(\mu \mathrm{m})$ | $(\mu \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ |  |  |  |  |
| 0 | 9 | 0.52 | 0.60 | 116 | 4 | 0.21 | 0.11 | 38 | 88 | 0.43 | 0.07 | 243 | 2.4 | 0.18 |
| 0 | 15 | 0.41 | 0.77 | 186 | 1 | 0.07 | 0.02 | 22 | 31 | 0.72 | 0.02 | 695 | 3.1 | 0.05 |
| 0.25 | 9 | 0.14 | 0.59 | 424 | 1 | 0.18 | 0.03 | 27 | 45 | 0.60 | 0.02 | 479 | 2.4 | 0.04 |
| 0.25 | 15 | 0.56 | 0.81 | 143 | 5 | 0.15 | 0.08 | 29 | 51 | 0.56 | 0.04 | 421 | 3.3 | 0.15 |
| 0.33 | 9 | 0.47 | 0.55 | 118 | 3 | 0.18 | 0.09 | 38 | 91 | 0.42 | 0.06 | 237 | 2.2 | 0.14 |
| 0.33 | 15 | 1.02 | 1.19 | 117 | 15 | 0.21 | 0.21 | 27 | 45 | 0.60 | 0.07 | 481 | 4.8 | 0.35 |

Table 3: Key non-reacting turbulence parameters for tangential velocity component near the burner exit ( $z=2$ mm at the centre of each annular gap).

| $S F R$ | $r$ | $\bar{W}$ | $w^{\prime}$ | $I$ | $R e_{t}$ | $T_{w w}$ | $L_{w w}$ | $\eta_{K_{-} w}$ | $\tau_{K_{-} w}$ | $v_{K_{-} w}$ | $D a_{w}$ | $K a_{w}$ | $w^{\prime} / s_{L}$ | $L_{w w} / \delta_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(\mathrm{~mm})$ | $(\mathrm{m} / \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ | $(\%)$ |  | $(\mathrm{ms})$ | $(\mathrm{mm})$ | $(\mu \mathrm{m})$ | $(\mu \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ |  |  |  |  |
| 0 | 9 | 0.35 | 1.17 | 338 | 7 | 0.27 | 0.09 | 22 | 31 | 0.72 | 0.03 | 704 | 4.7 | 0.16 |
| 0 | 15 | 0.87 | 0.96 | 109 | 3 | 0.06 | 0.05 | 22 | 31 | 0.72 | 0.02 | 697 | 3.9 | 0.09 |
| 0.25 | 9 | 0.33 | 1.12 | 345 | 7 | 0.31 | 0.10 | 23 | 34 | 0.69 | 0.04 | 633 | 4.5 | 0.17 |
| 0.25 | 15 | 8.36 | 1.06 | 12.6 | 96 | 0.17 | 1.50 | 47 | 141 | 0.34 | 0.58 | 153 | 4.3 | 2.5 |
| 0.33 | 9 | 0.28 | 1.12 | 390 | 6 | 0.28 | 0.08 | 22 | 31 | 0.72 | 0.03 | 700 | 4.5 | 0.14 |
| 0.33 | 15 | 13.69 | 1.23 | 9.0 | 107 | 0.10 | 1.40 | 42 | 110 | 0.38 | 0.48 | 196 | 5.0 | 2.4 |

Table 4: Key non-reacting turbulence parameters for the total velocity component near the burner exit ( $z=2 \mathrm{~mm}$ at the centre of each annular gap):

| SFR |  | $\begin{aligned} & \bar{U}_{m a g} \\ & (\mathrm{~m} / \mathrm{s}) \end{aligned}$ | $u_{\text {mag }}^{\prime}$ <br> ( $\mathrm{m} / \mathrm{s}$ ) | I <br> (\%) | $R e_{t}$ | $L_{\text {mag }}$ (mm) | $\eta_{K_{-} \text {mag }}$ <br> ( $\mu \mathrm{m}$ ) | $\tau_{K_{-} \text {mag }}$ ( $\mu \mathrm{s}$ ) | $v_{K \_ \text {mag }}$ <br> ( $\mathrm{m} / \mathrm{s}$ ) |  | $K a_{\text {mag }}$ | $u_{\text {mag }}^{\prime} / s_{L}$ | $L_{\text {mag }} / \delta_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 9 | 9.74 | 1.65 | 17.0 | 247 | 2.4 | 38 | 93 | 0.42 | 0.62 | 233 | 6.6 | 4.1 |
| 0 | 15 | 20.75 | 2.21 | 10.7 | 537 | 3.9 | 35 | 76 | 0.46 | 0.74 | 284 | 8.9 | 6.6 |
| 0.25 | 9 | 9.68 | 1.58 | 16.3 | 225 | 2.3 | 39 | 98 | 0.41 | 0.62 | 220 | 6.3 | 3.9 |
| 0.25 | 15 | 21.32 | 1.95 | 9.2 | 529 | 4.4 | 40 | 98 | 0.41 | 0.95 | 221 | 7.8 | 7.4 |
| 0.33 | 9 | 9.50 | 1.52 | 16.1 | 218 | 2.3 | 41 | 103 | 0.40 | 0.64 | 210 | 6.1 | 3.9 |
| 0.33 | 15 | 22.04 | 2.433 | 11.0 | 335 | 2.2 | 29 | 50 | 0.57 | 0.39 | 433 | 9.8 | 3.7 |

## 4. Reference

[1] R. Zhou, S. Balusamy, S. Hochgreb. A Tool for the Spectral Analysis of the Laser Doppler Anemometer Data of the Cambridge Stratified Swirl Burner. Technical Report CUED/A-TURBO/TR.135.
URL: http://www.dspace.cam.ac.uk/handle/1810/243258
[2] R. Zhou, M.S. Sweeney, S. Hochgreb. Flow Field Results of the Cambridge Stratified Swirl Burner Using Laser Doppler Anemometer. Technical Report CUED/A-TURBO/TR.134.
URL: http://www.dspace.cam.ac.uk/handle/1810/243259

