UNDRAINED SLIDING RESISTANCE OF SHALLOW FOUNDATIONS SUBJECT TO TORSION

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ABSTRACT

While the behavior of shallow foundations under vertical load combinations has been the subject of numerous studies, the response of shallow foundations subjected to combined horizontal and torsional loading has received considerably less attention. New offshore applications of shallow foundations for LNG facilities and other subsea structures have underscored the importance of the behavior of shallow embedded foundations subjected to combined in-plane translation and torsion. This study investigates the undrained bearing capacity of rectangular and square shallow foundations under eccentric horizontal loads through comparisons of various limit equilibrium and plastic limit analysis solutions to 3-D finite element solutions. In general, the plastic limit approach considered in this paper agrees well with the finite element solutions, although it has some tendency to over-predict capacity at greater embedment depths. The studies revealed a general insensitivity in the shape of the yield envelope to variations in embedment depth, which permits a simplified analysis suitable for first order estimates of load capacity. The variables considered in this study include footing aspect ratio, embedment depth, and load direction in addition to eccentricity.

KEY WORDS: shallow foundations, marine foundations, torsional resistance, sliding resistance, limit equilibrium, plastic limit analysis, finite element analysis, failure envelope.

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26 INTRODUCTION

27 Conventional methods of analyzing the bearing capacity of shallow foundations are mostly
28 focused on the effect of moment and horizontal forces on vertical bearing capacity of the
29 foundation, while the response of the foundation under torsion and combined sliding-torsion
30 has received less attention. The increasing use of shallow foundations for offshore structures
31 has underscored the significance of developing a better insight on the effect of torsion on slid-
32 ing bearing capacity of foundations. Gravity based shallow foundations are widely used in
33 offshore environments as they are commercially attractive options to support subsea infra-
34 structure (Fisher and Cathie, 2002). In the offshore industry, Liquid Natural Gas (LNG) facili-
35 ties, protection structures, pipeline end manifolds (PLEMs) and terminations (PLETs), pipe-
36 line sleepers, and riser bases are frequently supported with small shallow foundations.
37 Dimmock et al. (2013) have studied the effect of adding short piles to the corners of the shal-
38 low foundation to improve the total capacity and reduce foundation footprints through numer-
39 ical and physical modeling. The sliding bearing capacity of these foundations could also be
40 significantly improved by placing short ribs or peripheral (and internal) foundation skirts.
41 Typically rectangular foundations with lengths from $L=2$ to 10m and width to length (W/L)
42 aspect ratios of 2:1 are used. The embedment (or skirt) depth (d) to length (L) ratios are up to
43 $d/L= 0.5$, but more typically 0.05-0.2 (Randolph et al. 2011). Although design codes (e.g.
44 DNV, 1992; API, 2000 and ISO, 2003) have general comments on considering the effect of
45 fishing gear and over-trawling in the design which are the sources of combined horizontal and
46 torsional loading, they do not offer explicit methods for taking the in-plane horizontal loads
47 and torsion into account for foundation design purposes.

48 This study considers an eccentrically loaded shallow foundation (Figure 1) of length L , width
49 W and embedment depth d . In this paper L will be taken as the shorter dimension and will be
50 aligned with the x -axis. In general, virtually any eccentricity angle ψ or load angle γ may oc-

cur. This paper will first establish a number of baseline solutions for special eccentric/non-eccentric loading cases where $\psi = 0$ or 90° and $\gamma = 0^\circ$, for which relatively simple analytical or semi-analytical solutions may be derived from limit equilibrium or virtual work principles. For general conditions of eccentricity and load orientation, more powerful methods are required. Toward this end, the plastic limit analysis (PLA) proposed by Murff et al. (2010) is adopted. The accuracy of the baseline solutions and PLA approach are evaluated through comparisons to three-dimensional finite element solutions.

BACKGROUND

Murff and Miller (1977) used upper bound limit analysis to study the bearing capacity of multiple-footing offshore structures where they also studied the effect of torsion on collapse loads of foundations. Using limit equilibrium analysis Finnie and Morgan (2004) showed that the sliding resistance of the surface foundation could be decreased significantly if combined with torsion moments. They developed a normalized failure surface based on the following interaction equation:

$$f(H, T) = \left[\left(\frac{H}{H_{\max}} \right)^n + \left(\frac{T}{T_{\max}} \right)^k \right] - 1 = 0 \quad (1)$$

Where H and T are horizontal and torsion loads and $H_{\max}$ and $T_{\max}$ are the ultimate values. The interaction powers of n and k for surface foundations are recommended as $n=k=1.75$ for square and ranges of $n=1-2$ and $k=2-2.5$ for rectangular foundations. However, they did not conduct a rigorous analysis to validate their recommendations. They also presented some equations to estimate the ultimate torsional bearing capacity of surface foundation.

Yun et al. (2009) conducted three dimensional finite element (3D-FE) analysis to more rigorously study the interaction of vertical, horizontal, and torsion loads ($V-H-T$) acting on surface circular, square, and rectangular footings. The 3D-FE results of ultimate horizontal and tor-

sional bearing capacity overestimated Finnie and Morgan (2004) limit equilibrium values by 4% and 11 % respectively. They also developed normalized failure envelopes ($H/H_{\max}$ - $T/T_{\max}$) for square and rectangular foundations of various aspect ratios.

Murff et al. (2010) developed an upper bound plastic limit analysis (PLA) formulation for shallow embedded square and rectangular foundations under combination of co-planar sliding and torsion. Figure 2 shows a schematic of the sliding-torsion failure mechanism. The mechanism comprises a rigid block DEFG of depth d rotating about a vertical axis passing through the center of footing rotation (x_o, y_o). The motion of the block results in formation of triangular wedges along each side of the foundation. Depending on the position of rotation center (x_o, y_o), these wedges undergo passive or active failure as indicated by letters “p” and “a”. Figure 2(b) shows a section view (CC') of the rigid block and wedges of w_i . Examination of Section CC' shows that energy dissipation will occur at the discontinuities bounding the wedges (e.g., slip planes A_1B_1 and B_1C_1) and within a zone of continuous deformation (e.g., volume $A_1B_1C_1$). Energy dissipation also occurs in the vertical planes at both ends of each wedge due to velocity jump from the wedge to the surrounding rigid soil. Finally, energy dissipation occurs due to slip at the base of the rigid block DEFG relative to the underlying rigid soil. Owing to the inclusion of both active and passive zones in the wedge the work rate done by the unit weight of the soil is zero. The equations for evaluation of the energy dissipation D_i from the sources described above in terms of a virtual rotation rate $\dot{\beta}$ given by Murff et al. (2010) are contained in Appendix A.

The magnitude of force H passing through load application point (x_f, y_f) is calculated by equating external work to internal energy dissipation:

$$H = \frac{\sum_{i=1}^5 \dot{D}_i}{\left[(x_f - x_o) \sin \theta + (y_f - y_o) \cos \theta \right] \dot{\beta}} \quad (2)$$

where $\sum \dot{D}_i$ is the total rate of energy dissipation, θ is the angle between line of action of the external force and x -axis. A least upper bound is obtained by minimizing H with respect to the rotation center coordinates (x_o, y_o) , the width of the deforming wedges (w_1, w_2, w_3 , and w_4), and the depth of the failure plane (d) as optimization variables. Absent an underlying weak soil layer, the optimal depth of the failure plane generally coincides with the skirt penetration depth, in which case d need not be considered as an optimization variable.

BASELINE SOLUTIONS

For limiting load conditions of eccentricity angle $\psi = 0$ or 90° and load angle $\gamma = 0$ or 90° , a number of analytical and semi-analytical solutions are possible which can provide convenient reference solutions for evaluating PLA and FEA solutions.

Pure sliding and rotation

For pure sliding in a homogeneous soil with an adhesion factor α at the soil-skirt interface and foundation embedment d , the bearing factors for sliding in the x and y directions derived from the assumed stress distributions in Figure 3 and the failure mechanism in Figure 2(c) are:

$$N_{x \max} = \frac{H_{x \max}}{s_u L W} = \alpha_{base} + 2 \left[(2 + 0.8\alpha) + \alpha \frac{L}{W} \right] \frac{d}{L} \quad (3)$$

$$N_{y \max} = \frac{H_{y \max}}{s_u L W} = \alpha_{base} + 2 \left[(2 + 0.8\alpha) \frac{L}{W} + \alpha \right] \frac{d}{L} \quad (4)$$

where s_u is the soil undrained shear strength, L and W are the smaller and larger sides of the foundation. A different adhesion factor α_{base} is introduced here to allow for the case of a skirted foundation, in which case $\alpha_{base} = 1$. Otherwise, it would equal the adhesion factor α applicable to the sides of the foundation.

The stress distribution normal to the foundation skirts in Figure (3) is equivalent to lateral earth pressure which is obtained using the upper bound limit analysis for a Coulomb wedge forming behind the foundation skirt. Lateral pressure (i.e. bearing factor) is equal to $2s_u$ and $2.8s_u$ for adhesion factor, α , equal to 0 and 1, respectively. For intermediate α , a linear interpolation between the two values resulted in lateral pressure of $(2+0.8\alpha)s_u$ (Figure (3)). For the special case of pure torsion with zero embedment, $d = 0$, the following closed form solution (Murff et al., 2010) for maximum torsional capacity T_{max0} (the subscript '0' referring to zero embedment) provides a useful check for numerical calculations:

$$N_{tmax0} = T_{max0} / s_u WL^2 = \frac{(W/L)^2}{6} \left\{ \frac{\sin \theta_o}{2 \cos^2 \theta_o} + \frac{\ln[\tan(\pi/4 + \theta_o/2)]}{2} \right\} + \frac{L/W}{6} \left\{ \frac{\cos \theta_o}{2 \sin^2 \theta_o} - \frac{\ln[\tan(\theta_o/2)]}{2} \right\} \quad (5)$$

where $\theta_o = \tan^{-1}(L/W)$.

A limit equilibrium analysis obtained by summing the torsion resistance on the edges of a rotating footing (Figure 3) assuming no interaction effects between bearing and tangential resistance will produce the following expression for the additional torsional resistance derived from footing embedment:

$$\Delta N_{tmaxe} = C_f \left[(1 + 0.4\alpha) \left(\frac{W}{L} + \frac{L}{W} \right) + 2\alpha \right] \frac{d}{L} \quad (6)$$

A correction factor C_f is included in Eq. 6 to emphasize that some adjustment for interaction effects is needed. The total torsional resistance is the sum of Eqs 5 and 6, $N_{tmax0} + \Delta N_{tmaxe}$. This closed-form expression provides a useful glimpse into the variables affecting torsional capacity and, as will be seen, a simple calibration is possible to provide a match to finite element solutions.

Combined loading

For the case no embedment ($d/L = 0$) and eccentricity and loading directions aligned with either the major or minor axis of the footing, a virtual work analysis is possible as shown in Figure 4. A horizontal load H is applied at a distance e from the center of the footing with an associated motion about a center of rotation located a distance ρ from the center. Equating external virtual work $\dot{W}$ to internal energy dissipation leads to:

$$H = \dot{D} / (\rho + e) \dot{\beta} \quad (7)$$

where $\dot{\beta}$ is a virtual angular velocity. The rate of internal energy dissipation $\dot{D}$ is the soil resistance times the local velocity integrated over the footing area:

$$\dot{D} = s_u \dot{\beta} \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} \sqrt{(\rho + x)^2 + y^2} dx dy \quad (8)$$

A least upper bound collapse load is obtained by minimizing H with respect to ρ and setting to zero, which leads to:

$$H = \frac{1}{\dot{\beta}} \frac{\partial \dot{D}}{\partial \rho} \quad (9)$$

For the case of zero foundation embedment the collapse load then becomes:

$$H = s_u \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} \frac{\rho_{opt} + x}{\sqrt{(\rho_{opt} + x)^2 + y^2}} dx dy \quad (10)$$

where ρ_{opt} is the distance to the optimal center of rotation corresponding to a least upper bound. Evaluation of the integral yields the following closed-form expression for H :

$$\begin{aligned} H &= s_u \left[a_1^2 \ln \left| \frac{b_1 + W/2}{b_1 - W/2} \right| - a_2^2 \ln \left| \frac{b_2 + W/2}{b_2 - W/2} \right| + W(b_1 - b_2) \right] \\ a_1 &= \rho_{opt} + L/2 \\ a_2 &= \rho_{opt} - L/2 \\ b_1 &= \sqrt{a_1^2 + W^2/4} \\ b_2 &= \sqrt{a_2^2 + W^2/4} \end{aligned} \quad (11)$$

With $\dot{D}$ and H evaluated from Eqs. 8 and 11, respectively, the eccentricity e associated with any arbitrarily selected ρ is evaluated from Eq. 7. Analytical integration of Eq. 8 is unwieldy; however, reduction to single integration is possible (Appendix B) to facilitate simple design spreadsheet calculations. As this method explicitly relates eccentricity to the distance to the optimal center of rotation, it requires no search or optimization procedure. Aside from the simple numerical integration required to evaluate the rate of energy dissipation, the method is formulated in terms of closed-form expressions. Thus, it can provide a simple robust tool for routine design calculations.

Performing the analysis for a sweep of ρ values yields the predicted reduction in load capacity (H/H_{max}) versus eccentricity e as shown in Figure 5, which shows the significant reduction in sliding resistance for loading in the x and y directions. Square footings, $W/L = 1$, experience the greatest reductions in capacity H , with eccentricity as low as $e/L = 0.1$ reducing load capacity by more than 5% and eccentricity $e/L = 0.5$ (load application at the edge of the footing) reducing capacity by more than 40%. With increasing aspect ratio W/L the foundation becomes progressively more resistant to eccentric loading. The predictions also show that the reduction in capacity is always greatest for loading in the x direction; i.e., for load orientations normal to the long axis of the foundation.

FINITE ELEMENT ANALYSES

A series of finite element analyses (FEA) were performed to provide a basis for evaluating the various analytical procedures described above. The commercial software *ABAQUS* (HKS, 2008) is used in this study.

Geometry and material parameters

Clay under undrained conditions is modeled as linear elastic perfectly plastic material, obeying a von Mises failure criterion and an associated flow law. Undrained shear strength in a

simple shear mode, denoted by s_u , is used in order to allow comparison with Tresca based analytical solutions, since Von Mises criterion with the simple shear undrained strength provides a reasonable approximation of the Tresca criterion. Furthermore, Gourvenec et al. (2006) showed a very close agreement of the vertical bearing capacity of the surface footing for the Von Mises with simple shear strength and Tresca criteria. The Young's modulus of the soil is taken as a multiple of undrained shear strength $E/s_u = 500$. The ultimate capacity of the foundation is not affected by the pre-failure elastic behavior of the soil (Chen and Liu 1990), so elastic behavior is introduced purely for the purpose of numerical implementation of the finite element analysis. Poisson's ratio is taken as 0.49 to simulate no volume change for undrained clay in a total stress analysis. The foundation is modeled as a rigid body with Young's modulus 10^{10} times that of the soil. Strictly speaking, the solutions presented apply to a solid block foundation, although the predictions can be reasonably applied to skirted foundations provided no weak layers are encompassed by the skirts. No separation is assumed is assumed to occur at the foundation-soil interface. The interaction between soil and foundation is modeled using surface to surface contact pair in which footing outer surface is chosen as a "master surface" and the soil surface in contact with the foundation as a "slave surface".

Finite element model

Figure 6 shows an example 3D mesh and various zones of the model for a rectangular foundation. The length (L) and width (W) of the footing are parallel to x and y coordinates respectively. The model dimensions are $9L \times 9L \times 5L$ and $9L \times 12L \times 5L$ for the square and rectangular foundations respectively. The 3D mesh consists of 8-node full-integration 3D hybrid first order elements. The shallow foundation is simulated as square and rectangular ($W/L=2$) shapes with various embedment depths of $d = L/40, L/28, L/20$, and $L/14$. In order to create transitions in mesh density within the model the whole mesh is divided to 6 zones. The finite element mesh takes advantage of the "tie" constraint in ABAQUS which accommodates the rapid

transition of mesh density. This constraint ties active degree of freedom of the common surfaces along the transition interface and therefore the two surfaces at the transition zone do not necessarily have common nodes. The finest zone extending to $2d$ around the foundation embedment consists of two way bias grading of elements across the footing area as well as the height of the footing skirt. The size of the elements under the footing is very small, less than $0.006L$. The bias grading of elements also provides adequately fine mesh of soil with dimension of $0.0004L$ to $0.0008L$ in the vicinity of the sharp edges of the rigid foundation. The final FE mesh was obtained after checking various mesh densities especially for the finest zone, which has the main impact on the accuracy of the ultimate results as well as computational expense of the numerical simulation.

Boundary conditions and loading

Full fixity in all directions is enforced at the base boundary condition, while lateral fixity is enforced at the far-field vertical boundaries. The analysis is conducted using displacement control until ultimate sliding and torsional resistance is mobilized. This typically required translation in excess of $u/L = 0.003$ and rotations greater than 0.02 radians as shown in Figures 7 and 8. All the FEA simulations are conducted for the foundation fully bonded to the soil.

In order to construct the interaction curve or yield locus for combined shear-torsion loading, two types of displacement control methods are used: the swipe test and the probe test. The swipe test was used by Tan (1990) during his centrifuge tests to determine the shape of failure envelope. The advantage of this approach is that yield locus is determined by one single analysis in two separate steps. In the swipe test the foundation is firstly displaced in the direction of the degree of freedom (DoF) under examination from zero to ultimate capacity until the collapse load in that direction is reached. In the second step, the displacement is imposed in the second DoF until the ultimate capacity in the new direction is fully developed. In the

small strain finite element once the ultimate force is reached in the first step, there will be no more increase in the reaction force in that DoF, thus no further expansion in overall failure locus of foundation as the footing movement progresses. The disadvantage of this method is that, due to the elasto-plastic yielding occurring within the failure locus, the swipe test tracks a load path marginally inside the actual overall foundation failure envelope (Bransby and Randolph 1998). In the probe test or fixed displacement method, a single point on the failure envelope is identified for every fixed ratio of the prescribed combined displacement. Therefore, the yield locus could be created by conducting a number of finite element analyses with different displacement ratios. The prescribed fixed displacement ratio gives rise to load path beginning from the origin with initial gradient based on elastic stiffness. As approaching failure envelope the gradient diminishes to follow the interaction curve until it stops where there is no further increase in the forces developed in each intended degree of freedom (Bransby and Randolph 1998). This method gives accurate failure envelope, but requires several tests.

COMPARISON TO BASELINE AND PLA SOLUTIONS

The finite element studies are now compared to the baseline and PLA solutions from two perspectives. Firstly, the size of the yield envelope is evaluated through comparisons of predicted capacities under pure translational and rotational loading, H_{xmax} , H_{ymax} , and T_{max} . Secondly, the shape of the yield envelopes are evaluated through comparisons to a yield envelope in normalized space, $f(H_x/H_{xmax}, H_y/H_{ymax}, T/T_{max}) = 0$.

Pure translation and rotation

Figure 9 presents the computed bearing factors versus embedment depth. For both sliding and torsion, maximum capacity increases linearly with increasing embedment depth d/L as predicted from the LE solutions in Eqs. 3-6. The FEA sliding bearing factors are in excellent agreement with LE predictions from Eqs. 3 and 4. For the zero embedment case, the FEA tor-

sional resistance bearing factors N_t match Eq. 5 to within 0.1%. As embedment increases, the uncorrected ($C_f = 1$) LE solutions are seen to increasingly exceed the FE solutions, by up to 7.5% for $W/L = 1$, $d/L = 1/14$. It can be shown that an interaction factor $C_f = 0.87$ in Eq. 6 will bring the limit equilibrium predictions into close agreement with finite element solutions for both the square and rectangular footings.

For the case of zero embedment, $d/L = 0$; the FEA results are in essentially very close agreement with the PLA. As embedment increases the PLA tends to over-estimate capacity somewhat, which is in accordance with expectations for an upper bound solution. For translational resistance, N_{smax} and N_{symax} , the difference between FEA and PLA is, with one exception, less than 1%. For the case of pure torsion the difference between FEA and PLA become more severe with increasing embedment depth, and can exceed 6% for $d/L = 1/14$. The largest differences between FEA and PLA occur for the case of a square foundation. Overall, the situation for the PLA is similar to that of the LE analysis; PLA estimates of pure translational load capacity agree very well with FEA solutions, but the PLA tends to overestimate the increase in torsional capacity due to foundation embedment. Close examination indicates that the PLA overestimates the additional torsional resistance derived from embedment by about 10%.

Figure 10 shows contours of plastic displacements for the cases of pure sliding and pure rotation. Figure 10a clearly shows the wedge formation at the ends of the foundation that is assumed in the PLA analysis. Figure 10b shows a simple pattern at the base of the foundation comprising circular contours about the center of rotation. At the edges of the foundation more complex displacement patterns develop as the wedges rotate and distort. Interestingly negligible plastic deformation is observed in the soil elements next to the corners of the foundation. This is consistent with Murff et al. (2010) PLA mechanism depicted in Figure 2, which assumes rigid soil at the corners of the foundation.

Combined loading

Figures 11 and 12 show example FEA interaction diagrams for a rectangular ($W/L = 2$) foundation with embedment $d/L = 1/14$ under combined loading. Figure 11 shows the shear-torsion interaction, while Figure 12 shows the x -sliding and y -sliding interaction. These illustrate the trend of swipe tests tending to give an apparent yield locus marginally inside the actual locus. Accordingly, yield loci taken from the probe approach are adopted as more reliable.

Figures 13- 15 compare the FEA shear-torsion interaction predictions to PLA solutions and to the zero embedment ($d/L = 0$) virtual work analysis Eqs. 7-11. The figures present the computed load capacities in normalized form, H/H_{max} and T/T_{max} for the case of a square footing (Figure 13) and the x and y sliding for a rectangular footing (Figures 14 and 15, respectively.) The FEA predictions show the shape of the yield envelopes to be insensitive to embedment depth, with all predictions for the embedment range $d/L = 1/28$ to $1/14$ essentially lying on a single unique curve. The yield envelopes from the PLA method are in general agreement with the FEA solutions, although the PLA is slightly unconservative for the square footing (Figure 13). Remarkably, the virtual work solution for zero embedment also provides a realistic portrayal of the yield envelope for non-zero embedments, with some tendency for being on the conservative side, especially for the $W/L = 2$ footing loaded in the y direction (Figure 15).

Figure 16 shows the yield envelopes for the case of non-eccentric loading for varying load directions γ . Again the FEA solutions indicate that embedment d/L has minimal influence on the shape of the envelope. The PLA predictions agree well with the FEA solutions, albeit with a slight tendency for being on the unconservative side. For the case of zero embedment, the yield envelope obtained from the PLA approach (Eqs. 10 and 11) will be circular. Plastic Limit Analysis (PLA) approach produces the circular yield envelope for the foundation with zero embedment. This concept is also easy to explain theoretically: for foundation with area

of base = A , embedment = 0.0, and soil undrained shear strength = s_u the sliding resistance will be constant and equal to $H_{max} = H_{x,max} = H_{y,max} = As_u$, regardless of the direction of sliding. This value actually represents the radius of the circular yield curve in the H_x - H_y space as shown in Figure 16. The components of sliding resistance in x and y directions are $H_x = H_{max} \times \cos \eta$ and $H_y = H_{max} \times \sin \eta$ for the angle of foundation sliding with respect to x axis equal to η . This circular interaction relationship is correct for foundation with any geometric base shape when embedment is zero. Figure 16 indicates that embedment of the footing will induce a small but noticeable departure from a circular envelope. This is also the case for square footing. A simplifying assumption of a circular $H_x/H_{x,max}$ - $H_y/H_{y,max}$ yield envelope should therefore produce slightly conservative yet realistic predictions. Thus, Eq. 1 could be rewritten as below:

$$f(H_x, H_y, T) = \left\{ \left[\left(\frac{H_x}{H_{x,max}} \right)^p + \left(\frac{H_y}{H_{y,max}} \right)^q \right]^r + \left(\frac{T}{T_{max}} \right)^k \right\} - 1 = 0 \quad (12)$$

where for the assumed simplified circular relationship between H_x and H_y we have $p=q=2$. The interaction relationship in Eq. 12 represents the ellipsoidal surface in the H_x - H_y - T space.

LOAD CAPACITY REDUCTION DUE TO ECCENTRICITY

An alternative approach to view and evaluate the combined loading effects is to relate the load capacity to eccentricity as shown in Figures 17-20, for $W/L = 1, 2, 4$, and 6. Figures 18-20 also include the effects of eccentricity angle, showing the predictions for $\psi = 0$ and 90° when $\gamma = 0^\circ$ (sliding in y and x directions with various eccentricity). In these figures the load capacity is expressed in terms of bearing factor form ($N = H / s_u LW$) so the comparisons reflect the differences in the computation of both the magnitude of resistance as well as the interaction effects. Also in Figures 17 and 18, the FEA results are shown, which indicate gener-

ally good agreement between the FEA and PLA solutions, although the PLA predictions are somewhat high for the square foundation.

The insensitivity of the shape of the yield locus to variations in embedment depth d/L (Figures 13-15) introduce the possibility of developing a simplified approach to evaluating sliding-torsional capacity using the simple baseline solutions presented earlier. For instance, the relationships for translational load capacity (Eqs. 3 and 4), which proved to be fairly accurate, can be used in conjunction with the virtual work equations for zero embedment (Eqs. 7-11, Figure 5) to make first order predictions of load capacity. As an example, if the maximum horizontal load capacity values from Eqs. 3 and 4 are used with the corresponding capacity reduction curves for x and y directional loading from Figure 5, simple estimates of horizontal sliding resistance versus eccentricity are possible as shown in Figures 17-20 for aspect ratios $W/L = 1$ to 6. Comparisons to FEA and PLA solutions indicate that the simplified approach provides realistic, albeit conservative, estimates of load capacity reduction due to eccentricity. Although the PLA can be performed easily with minimal computation time, also performing the simplified analysis is advisable. The reason for this is that the PLA performs an optimization analysis to search for a least upper bound collapse load H . The possibility always exists that the search may identify a local minimum that is greater than the absolute minimum. The simplified analysis provides a useful benchmark that can aid in judging if the absolute lower bound has been found.

Up to this point the analyses under consideration have been restricted to conditions for which the load angle $\gamma = 0$. A question now arises as to how to deal with cases of arbitrary load angles. The issue is clearly significant, since increasing γ will decrease the torsional loading and for $\gamma = 90^\circ$ the torsion vanishes. The virtual work approach cannot handle the cases of $\psi \neq 0$ or 90° and $\gamma \neq 0$ or 90° . However, response of shallow foundation subjected to any combina-

tion of coplanar translation and torsion could be evaluated through PLA solution and appropriate factors of p , q , r , and s in Eq. 12 could be determined to define the ellipsoid interaction surface.

CONCLUSIONS

This study evaluates the ultimate sliding resistance of square and rectangular shallow embedded foundations under purely horizontal eccentric loads. 3D-FE simulations are performed to evaluate a number of simple relationships developed for special conditions of loading as well as an upper bound plastic limit analysis (PLA) developed by Murff et al. (2010) for general conditions of shear-torsional loading. The study indicates the following:

1. Eccentricity of lateral loading begins to significantly affect ultimate sliding resistance of square footings (>5% reduction in capacity) at eccentricity levels e/L greater than about 0.1 (Figure 5). For perspective, an eccentricity $e/L = 0.5$ – a load applied at the edge of the footing – reduces capacity by more than 40%.
2. Foundation performance with respect to eccentric loading improves with increasing aspect ratio W/L (Figure 5). The load capacity reduction due to eccentricity is always greater for loading in the direction normal to the long axis of the foundation.
3. The limit equilibrium equations for translational load capacity, Eqs. 3 and 4, give reliable predictions for pure translational loading (Figure 9). Eq. 5, which predicts pure torsional load capacity for the case of no footing embedment $d/L = 0$, is also accurate. The limit equilibrium solution for the increase in torsional capacity for $d/L > 0$ (Eq. 6) can provide reasonably accurate solutions when used in conjunction with a correction factor $C_f = 0.87$.
4. Predictions from the PLA method of Murff et al. (2010) generally agree well with FEA predictions (Figures 13-15 and 17-18). The method over-predicts the increase in

pure torsional resistance associated with increased embedment d/L ; however, this appears to have minimal effect on the quality of the predictions for eccentricity levels likely to be encountered in practice.

5. The general insensitivity of the shape of the shear-torsion yield envelope to embedment depth d/L (Figures 13-15) permits a simplified analysis in which analytical expressions for translational resistance (Eqs. 3 and 4) are used in conjunction with a virtual work analysis for the case of zero embedment (Eqs. 7-11) to predict the reduction in load capacity due to eccentricity. The method is fairly robust, as it requires no search for a least upper bound. Predictions from the simplified method provide reasonable conservative estimates of the reduction in sliding resistance due to eccentricity.
6. The effect of eccentricity angle ψ increases with increasing aspect ratio W/L (Figures 17-20). The simplified analysis does not accurately capture the effects of ψ at large W/L and large eccentricity for sliding in the y direction (parallel to the long axis of the foundation), where it can be overly conservative.

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Mechanism	Location	Equation
Shear deformation within wedges	Sides 1 and 3	$Ww_1\dot{\beta}\sqrt{1+(d/w_1)^2}\int_{z=0}^{z=d}s_u(z)(1-z/d)dz$
	Sides 2 and 4	$Lw_2\dot{\beta}\sqrt{1+(d/w_2)^2}\int_{z=0}^{z=d}s_u(z)(1-z/d)dz$
Slip between rigid block DFEG and vertical faces of wedges	Sides 1 and 3	$\alpha\dot{\beta}\int_{z=0}^{z=d}\int_{y=-W/2}^{y=W/2}\left[\left(\frac{L}{2}-x_o\right)^2+\left(\frac{d}{w_1}\right)^2(y-y_o)^2\right]^{\frac{1}{2}}s_u(z)dydz$
	Sides 2 and 4	$\alpha\dot{\beta}\int_{z=0}^{z=d}\int_{x=-L/2}^{x=L/2}\left[\left(\frac{W}{2}-y_o\right)^2+\left(\frac{d}{w_2}\right)^2(x-x_o)^2\right]^{\frac{1}{2}}s_u(z)dx dz$
Slip along the wedge ramps	Sides 1 and 3	$\dot{\beta}\sqrt{(d/w_1)^2+1}\sqrt{(w_1/d)^2+1}\int_{z=0}^{z=d}\int_{y=-W/2}^{y=W/2} y-y_o s_u(z)dydz$
	Sides 2 and 4	$\dot{\beta}\sqrt{(d/w_2)^2+1}\sqrt{(w_2/d)^2+1}\int_{z=0}^{z=d}\int_{x=-L/2}^{x=L/2} x-x_o s_u(z)dx dz$
Slip at the two ends of each wedge	Sides 1 and 3	$w_1\dot{\beta} W/2-y_o \sqrt{(d/w_1)^2+1}\int_{z=0}^{z=d}(1-z/d)s_u(z)dz$
	Sides 2 and 4	$w_2\dot{\beta} L/2-x_o \sqrt{(d/w_2)^2+1}\int_{z=0}^{z=d}(1-z/d)s_u(z)dz$
Slip at the base of rigid block DEFG	Base	$\dot{\beta}\int_{y=-W/2}^{y=W/2}\int_{x=-L/2}^{x=L/2}s_u(d)\sqrt{(x-x_o)^2+(y-y_o)^2}dxdy$

Notes:

 w_1, w_2, w_3 , and w_4 : width of the failure wedges in soil (x_o, y_o) : coordinates of the center of rotation (x_f, y_f) : coordinates of the application point for the external force H θ : inclination angle of the external force H with respect to x axis d : depth of the failure/sliding plane (assumed to be equal to embedment depth of the shallow foundation) z : depth below seabed s_u : undrained shear strength of the soil L and W : Length and width of the foundation parallel to x and y axis respectively α : adhesion factor $\dot{\beta}$: virtual rate of rotation

442 **Appendix B. Integration of Dissipation Rate for $d=0$ analysis**

443 Analytical evaluation of the inner integral in Eq. 8 is possible to permit a single numerical in-
 444 tegration. The resulting expression is:

$$\dot{D} = \frac{1}{2} s_u \dot{\beta} \int_{-W/2}^{W/2} \left[(a_1 c_1 - a_2 c_2) + y^2 \ln \left| \frac{a_1 + c_1}{a_2 + c_2} \right| \right] dy$$

$$a_1 = \rho_{opt} + L / 2$$

445 $a_2 = \rho_{opt} - L / 2$ (Eq. B1)

$$c_1 = \sqrt{a_1^2 + y^2}$$

$$c_2 = \sqrt{a_2^2 + y^2}$$

446 Eq. B1 can be integrated between the limits $-W/2$ to $W/2$ using classical numerical integration
 447 formulas.

448

449 Figure 1. Schematic of eccentrically loaded foundation

450

(a) Plan view of the mechanism

(b) Elevation view of section CC'

(c) D-1: Elevation view of active wedge $A_1B_1C_1$

451

452 Figure 2. Assumed collapse mechanism in PLA analysis

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454

455 Figure 3. Assumptions for limit equilibrium equations

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458 Figure 4. Model for virtual work analysis of shear-torsion

459

Figure 5. Horizontal capacity reduction predicted from virtual work analysis for surface foundation ($d = 0$)

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465 Figure 6. Finite element model

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468 Figure 7. Typical load-displacement curve from FEA

469

470

471 Figure 8. Typical torsion-rotation curve from FEA

472

473

474 Figure 9. Normalized bearing capacity for pure translation and torsion

(a)

(b)

Figure 10. Contours of plastic displacement for cross section of the model under (a) pure translation and (b) pure torsion

Figure 11. FEA yield envelope for combined shear-torsion loading

Figure 12. FEA yield envelope for combined x-y translational loading

Figure 13. Comparisons of shear-torsion predictions for square footing

Figure 14. Comparisons of shear-torsion predictions for rectangular footing

Figure 15. Comparisons of sheary-torsion predictions for rectangular footing

Figure 16. Comparisons of shear stress predictions

Figure 17. Load capacity reduction versus eccentricity: square footing of $L/d=14$

Figure 18. Load capacity reduction versus eccentricity: $L/W = 2$ rectangular footing of $L/d=14$

Figure 19. Load capacity reduction versus eccentricity: $L/W = 4$ rectangular footing of $L/d=14$

Figure 20. Load capacity reduction versus eccentricity: $L/W = 6$ rectangular footing of $L/d=14$