# Title: Fundamental basis of single-point liquid limit measurement approaches

## Technical Paper

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# ABSTRACT

The liquid limit is defined as the point at which a clay's behaviour changes from liquid to plastic. This transition is in reality gradual, rather than sudden. The definition of when this transition has been crossed must therefore be determined based on some arbitrary criterion. The percussion cup method of determining liquid limit in the manner suggested by Atterberg and subsequently standardised by Casagrande determines liquid limit as the water content at which 25 standard blows are required to cause closure of a standard groove. In order to speed up the determination of the liquid limit, a single-point method is defined in ASTM D4318, and in many other codes, to interpret liquid limit from groove closure at a different numbers of blows by assuming a relationship between water content and the number of blows required for groove closure. These methods differ considerably between different codes of practice currently in use worldwide. This paper examines the procedures for single-point determination of the liquid limit and offers some fundamental explanations that underpin the applicability of these procedures. This paper demonstrates that the variation in single-point liquid limit procedures suggested by various codes of practice can be attributed to the variability of liquid limit devices, rather than to variation in the soils being tested.

# NOTATION

*Roman*

| | |
|---|---|
| $a$ | curve fitting parameter |
| $b$ | curve fitting parameter |
| $c_u$ | undrained shear strength |
| $c_u/\rho$ | specific soil strength |
| $(c_u/\ \rho)_{\mathrm{LL}}$ | specific soil strength at liquid limit |
| $FI$ | flow index |
| $G_s$ | specific gravity |
| $I_L$ | liquidity index |
| $I_P$ | plasticity index |
| $n$ | curve fitting parameter |
| $N$ | number of blows during the Casagrande Liquid Limit Test |
| $PI_{cone}$ | plasticity index determined using the fall cone liquid limit and the thread rolling test |
| $w$ | water content |
| $w_L$ | liquid limit (Casagrande) |
| $w_{L\_cone}$ | liquid limit (Fall Cone) |

*Greek*

| | |
|---|---|
| $\alpha$ | fitting parameter |
| $tan\beta$ | slope of the flow line |
| $\rho$ | density of soil |
| $\rho_w$ | density of water |

*Statistical terms*

| | |
|---|---|
| $R^2$ | coefficient of determination |
| $n$ | number of data points used to generate a regression |
| $SE$ | standard error of a regression |

# INTRODUCTION

The consistency limits first defined by Atterberg (1911a and 1911b) describe the behaviour of clays with varying water content, and as such play a vital role in the use of clays in both geotechnical and industrial applications (e.g. Andrade et al. 2011). The plastic limit of clays is the water content at which the transition from ductile to brittle behaviour suddenly occurs, as discussed by Haigh et al. (2013), Haigh et al. (2014) and Barnes (2013). The liquid limit is defined as the water content when a clay's behaviour changes from liquid to plastic this transitions is gradual rather than sudden. The definition of the transition boundary is thus inherently arbitrary. Warkentin (1961) postulated that the liquid limit (tested using the Casagrande apparatus) was controlled by interparticle forces – this thesis was further examined by Nagaraj and Jayadeva (1981) who suggested that liquid limit was associated with a certain spacing between clay platelets and thus with surface areas of particles.

A clay's liquid limit can be determined using either fall-cone or percussion methods. The percussion cup method of determining the liquid limit has its origins in the work of Atterberg (1911a, 1911b) and was standardised by Casagrande (1932). The standard test involves manipulating the water-content of a soil specimen such that 25 blows are required for closure of a standard groove over a length of 13 mm. As it is difficult to achieve groove-closure at exactly 25 blows, data from several tests are plotted on axes of water content versus the logarithm of the number of blows and a straight-line, termed the flow-line, is fitted to the data. The liquid limit is taken to be the water-content at which this line crosses 25 blows.

The fall-cone test for liquid limit involves manipulating the water-content of a clay specimen such that an 80g, 30° cone placed with its tip on the surface of the clay will fall 20 mm before coming to rest – this test can also be used to estimate undrained shear strength of clays (e.g. Hansbo, 1957 and Yukselen-Aksoy, 2010). In this procedure data from several tests is typically plotted on axes of water content versus the logarithm of penetration and a straight line is fitted to the data.

In order to improve the speed at which these tests can be carried out, single point methods have been proposed for both percussive (Waterways Experiment Station, 1949) and fall-cone tests (Clayton and Jukes 1978) to allow the liquid limit to be inferred from a test in which the clay sample was not at the liquid limit water content.

This paper will demonstrate that these single-point methods, while originally determined empirically, can be derived from fundamental mechanical principles. Further it will be shown that the variations in these methods prescribed by international design codes relate to differences in the equipment in use worldwide, rather than to differences between the soils tested.

## DEVELOPMENT OF SINGLE POINT LIQUID LIMIT PROCEDURES

The single point method for percussive liquid limit was first proposed by the US Army Waterways Experiment Station (1949). This test allowed the liquid limit to be inferred from a test in which the number of blows for closure of the groove was between 10 and 35 (this range is assumed to be the extent to which one can safely rely on the interpolation function used to determine the water content at 25 blows). The method utilised the observation that the slope of the flow-line on log-log axes (*tan* $\beta$) for a sample of 767 soils from the Mississippi valley was approximately constant. This observation could then be used to project from a measured data-point to the water content at which 25 blows would be needed for groove closure; the liquid limit. This procedure was defined such that:

$$\log\left(\frac{w}{w_L}\right) = -\tan\beta\ \log\left(\frac{N}{25}\right) \qquad (1)$$

which can be rearranged to show that:

$$w_L = w\ \left(\frac{N}{25}\right)^{\tan\beta} \qquad (2)$$

The average value of $tan\ \beta$ for the soils tested was found to be 0.121, (Waterways Experiment Station, 1949). This relationship subsequently became the single-point liquid limit method implemented as ‘Method B’ in ASTM D4318 (2010). Equation (2) was also reported to be an acceptable match for 676 soils from the Buenos Aires region of Argentina (Trevisán, 1960). Table 1 shows various reported values of $tan\beta$ based on seven databases of liquid limit tests for which this analysis has been reported.

These geographically diverse observations confirm the general trend of flow lines having slopes of approximately 0.1 with a standard deviation of the order of 0.03, but do show substantial differences in different regions. This has since resulted in ASTM D4318 (2010) using a value of $tan\ \beta$ of 0.121 following Waterways Experiment Station (1949), BS 1377-1990 using a value of 0.092 following Norman (1959) and AS1289 (2009) using a value of 0.091. The Indian standard IS2720 (1985) uses a slightly different formula that was proposed by Nagaraj and Jayadeva (1981):

$$w_L = \frac{w}{1.3215 - 0.23\log(N)} \quad (3)$$

The effect of these different corrections on the liquid limit measured can be seen in Figure 1. It can be seen that while the lines differ marginally in shape, there is little significant difference between the formulae suggested by BS1377-1990 and IS2720-1985. The IS2720-1985 formula can be shown to be functionally equivalent to the use of a value of $tan\ \beta$ of 0.101.

There are two plausible explanations for the difference in value of $tan\ \beta$ between the United States and the United Kingdom (or in a comparison of data from any other two countries); differences in soils or differences in testing equipment (the type of Casagrande device used). If the single-point method is to be used in countries whose soils have not been so thoroughly analysed, the influence of these two aspects is important in determining an appropriate value of $tan\ \beta$.

This paper draws on recently published work on both the mechanics of the Casagrande liquid limit test (Haigh, 2012) and the variation of soil strength between the liquid and plastic limits (O'Kelly, 2013 and Vardanega and Haigh, 2014) to demonstrate the origins of the relationships used in the single-point liquid limit method and to show that it is the characteristics of the equipment used that should determine which value of *tan* $\beta$ is appropriate, rather than the origin of the soil samples. The *single point method* remains a viable method for liquid limit determination, permitted in many codes of practice worldwide, and is therefore worthy of further examination.

## ANALYSIS

Haigh (2012) carried out a Newmarkian sliding block analysis (Newmark, 1965) of the percussion cup test, using the vertical acceleration pulse measured on the cup during its impact with the base of the liquid limit device to drive a slope-failure within the soil. He demonstrated that the liquid limit of soil, as measured with ASTM standard percussion cup apparatus, corresponds to a ratio of undrained shear strength to soil density of approximately 1 $m^2s^{-2}$. Haigh (2012) also demonstrated, by utilisation of this analysis, the dependence of the number of blows required to cause groove closure on the specific strength of the soil, as shown in Figure 2.

In order to use this analytically calculated curve in the analysis presented here, a power-law relationship of the form:

$$N = a + b\left(\frac{c_u}{\rho}\right)^n \tag{4}$$

is fitted to the specific strength curve between 10 and 35 blows, as shown in Figure 2. Giving the experimental curve for an ASTM device (Haigh, 2012):

$$N = 6.22 + 21.43\left(\frac{c_u}{\rho}\right)^{1.893} \tag{5}$$

Casagrande (1958) recognised the variability between different liquid limit devices and made efforts to further standardise construction of the devices. Two categories of device still exist, however, those with hard plastic bases as specified by ASTM D4318 (2010), and those with softer rubber bases as specified by BS1377 (1990), IS9259 (1979) and AS1289 (2009). The reasons for this distinction appear to be historical rather than based on any scientific decision.

The base characteristics prescribed by the aforementioned codes are shown in Table 1. The hardness of the base alters the characteristics of the shock loading on the clay slopes during the liquid limit test and hence the movement of the soil that will occur during one blow for any given soil specific strength. By measuring the vertical acceleration measured on impact with the base and following the analysis procedure outlined by Haigh (2012), the relationship between number of blows for groove closure and specific strength can be derived for each particular Casagrande apparatus. Table 2 shows the best-fit parameters for new apparatus manufactured by ELE International conforming to the ASTM and British Standards and for Indian Standard apparatus tested at the *Indian Institute of Science (Bangalore)* - the parameters derived for the three devices tested differ considerably.

The single-point liquid limit method (defined by equation 2) implies that a unique relationship exists between the water content of a soil normalised by that at its liquid limit and the number of blows required to cause the groove to close in the liquid limit test. Following Haigh (2012), this implies relationships between the normalised water content and both the strength and density of the soil.

The relationship between the water content of the soil and its density in a saturated state can be found from:

$$\rho = \frac{G_s(1+w)}{1+wG_s}\rho_w \qquad (6)$$

As specific gravity is reasonably constant for the majority of soils at around 2.65 ± 0.2, the relationship between water content and density is also approximately identical for all soils.

The variation of undrained strength with water content has been estimated using a variety of relationships, usually assuming either a linear relationship between the logarithm of undrained strength and liquidity index, (e.g. Wroth and Wood, 1978) or a power law relationship between undrained strength and liquidity index (e.g. Feng, 2001, Yılmaz, 2009 and Zentar et al. 2009).

Vardanega and Haigh (2014) have shown through the analysis of a large database collected fall-cone data on a diverse database of over 100 soils that a log-linear relationship between strength and liquidity index provides an acceptable match to the data for liquidity indices between 0.2 and 1.1 (in the same paper a power law is shown to be also a plausible fit to the dataset and the following analysis could be repeated assuming such a relation that would make use of the logarithmic liquidity index proposed by Koumoto and Houlsby, 2001 and used in Vardanega and Haigh, 2014). The slope of the relationship is, however, shown to be significantly less than that suggested by (Wroth and Wood, 1978); the strength variation with water content being shown to be approximated by:

$$c_u = 1700 \times 35^{\left[\left[\frac{w_{L\_cone} - w}{PI_{cone}}\right]\right]} \qquad c_u \text{ in Pa} \qquad 0.2 < I_L < 1.1 \qquad (7)$$

Similar values for the variation of strength with water content around liquid limit can be derived from the fall-cone single-point liquid limit procedure outlined by Clayton and Jukes (1978).

As previously mentioned, the variation of the number of blows to cause the groove to close in the liquid limit test with water content is often characterised by the slope of the flow line *tan β*. The relationship between this slope and the variations of both the soil specific strength with water content and the number of blows for groove closure with specific strength can be determined by multiplication of the derivatives, as shown in equation 8.

$$\tan\beta = -\frac{\partial \log_{10} w}{\partial \log_{10} N} = -\frac{\partial \log_{10} w}{\partial \mathrm{w}} \frac{\partial \mathrm{N}}{\partial \log_{10} N} \frac{\partial \mathrm{w}}{\partial \frac{c_u}{\rho}} \frac{\partial \frac{c_u}{\rho}}{\partial \mathrm{N}} \qquad (8)$$

The derivatives required by equation 8 can be calculated by differentiating equations 6 and 7 to yield:

$$\frac{\partial \frac{c_u}{\rho}}{\partial w} = \frac{\rho\frac{\partial c_u}{\partial w} - c_u\frac{\partial \rho}{\partial w}}{\rho^2} = -\frac{c_u}{\rho}\left[\frac{\ln{(35)}}{PI_{cone}} + \frac{1-G_S}{(1+wG_S)(1+w)}\right] \tag{9}$$

And by differentiating equation 4 to yield:

$$\frac{\partial \mathrm{N}}{\partial \frac{c_u}{\rho}} = bn\left(\frac{c_u}{\rho}\right)^{n-1} \tag{10}$$

Evaluating *tan β* at Casagrande’s liquid limit $w_L$ thus yields:

$$\tan\beta = \frac{25}{(25-a)\, n\, w_L\left[\frac{\ln(35)}{PI_{cone}} + \frac{(1-G_S)}{(1+w_L G_S)(1+w_L)}\right]} \tag{11}$$

The first additive term in the denominator will always dominate, hence:

$$\tan\beta \approx \frac{25}{b\, n\left(\frac{c_u}{\rho}\right)_{LL}^{n} \ln(35)}\frac{PI_{cone}}{w_L} = \zeta\,\frac{PI_{cone}}{w_L} \tag{12}$$

Substituting in the values from table 2:

$$\tan\beta_{ASTM} \approx 0.198\,\frac{PI_{cone}}{w_L} \qquad \tan\beta_{BS} \approx 0.161\,\frac{PI_{cone}}{w_L} \qquad tan\beta_{IS} \approx 0.102\,\frac{PI_{cone}}{w_L} \tag{13}$$

Equation 13 is inconsistent, in that it combines the plasticity index found using the cone method for liquid limit determination with the liquid limit water content for the Casagrande cup method. In order to remove this inconsistency and to eliminate the need for the simplification of equation 11 to equation 12, *tan β* was evaluated numerically for plastic limits between 10% and 100% and cone plasticity indices between 10% and 300%. For each combination of parameters, the strength variation around liquid limit was assumed to be given by equation 7 and the density variation by equation 6. The number of blows to failure for a variety of water contents could then be determined based on equation 4 with appropriate parameters from table 1, and the flow index, Casagrande liquid limit and plasticity index could then be determined. Equation 12 can thus be modified to be consistent in only using Casagrande values of plasticity index and liquid limit to yield:

$$\tan\beta = \alpha\frac{PI}{w_L} \tag{14}$$

Figure 3 shows the calculated values of $\alpha$ (as defined by equation 14) for different Casagrande-style equipment. It can be seen that $\alpha$ is equal to 0.22 ± 0.02 for ASTM equipment, 0.14 ± 0.01 for British Standards equipment and 0.09 ± 0.01 for Indian Standard equipment. These numbers compare favourably with the values given in equation 13. As $\alpha$ is a function of plasticity index as well as liquid limit, the relationship between $\alpha$ and liquid limit was evaluated for soils lying on both the Casagrande A-Line (equation 15) and the U-Line (ASTM, 2006) (equation 16), considered the upper limit of the relation developed by Casagrande (1947). The origin of the U-line defined by equation 16 is discussed by Howard (1984). The resultant lines (on Figure 3) are essentially coincident. Plasticity index therefore has only a minor influence on the results for a sensible range of $I_p$ values.

$$I_P = 0.73(w_L - 0.20) \tag{15}$$

$$I_P = 0.90(w_L - 0.08) \tag{16}$$

Equation 14 with ASTM parameters was applied to the soils in the database of Vardanega and Haigh (2014) resulting in a prediction of $\tan\beta$ having an average value of 0.127 and a standard deviation of 0.026. This value is similar to the results from Eden (1959) and the Waterways Experimental Station (1949). Using the parameters found for British Standard equipment, a prediction of $\tan\beta$ having an average value of 0.081 and a standard deviation of 0.021 results. This is similar to the reported observations of Mohan and Goel (1958), Norman (1959) and Jain and Patwardhan (1960). These values are calculated assuming a constant value of $G_s$ for all soils of 2.65. The analysis can be shown to be insensitive to specific gravity, $\tan\beta$ only changing by 0.002 if the extremes of plausible values of $G_s$ for clays are used.

It can be seen from equation 12 that a relationship exists between the slope of the flow line and the ratio between plasticity index and liquid limit. The plastic limit could hence conceivably be estimated from the measured liquid limit data by extrapolation. Sridharan et al. (1999) defined the slope of results from a Casagrande liquid limit test using a flow index *FI* defined by:

$$FI = -\frac{\partial \mathrm{w}}{\partial \log_{10} N} \tag{17}$$

They then showed that a regression to a dataset gave a good correlation ($R^2$=0.88, *n*=55, *SE*=1.8%) between flow index and plasticity index of the form:

$$PI = 4.12FI \tag{18}$$

Data from Jain and Patwardhan (1960) can also be analysed within this framework to give a relationship between flow and plasticity indices with all test carried out by the same laboratory. This gives a substantially different but still significant ($R^2$=0.52, *n*=32, *SE*=0.69%) correlation:

$$PI = 1.96FI \tag{19}$$

The substantial difference between equations 18 and 19 calls into question the validity of any unique correlation between plasticity index and flow index, despite each of the correlations being significant for the data used to derive it. Soil characteristics are unlikely to be the key source of variability, rather it appears that the precise characteristics of the equipment used to carry out the testing have a large impact on the ratio of plasticity and flow indices.

It can be seen from Figure 4 that for each of the three sets of equipment tested, the ratio of plasticity index to flow index is approximately constant for liquid limits between 20% and 400%, but that there are large differences between the three devices; ASTM equipment giving a ratio of approximately 2, British Standard equipment 3.1 and Indian Standard equipment 4.7. This latter value approximates the value of 4.12 given in equation 18 and derived by Sridharan et al. (1999) using the precise Indian Standard equipment tested for this research. As described earlier, Jain and Patwardhan (1960) observed a different correlation between plasticity and flow indices, but the ratio observed in their data (1.96) also falls within the range that would be predicted for the three sets of equipment tested here, being consistent with the use of ASTM equipment.

The plasticity and flow indices for the 55 soils for which Casagrande liquid limits were presented by Sridharan et al. (1999) are shown in Figure 5 together with the predicted relationships using the three Casagrande cups tested here. Using the derived parameters for the equipment used to measure the plasticity and flow indices, (i.e. that at IISc Bangalore), a good prediction of the data can be obtained, though there is significant scatter about the trend. Predictions of the value of flow index based on liquid limit were also made as part of a single-point liquid limit procedure developed by Fang (1960). The method involved predicting the slope of the flow line for a given soil and then extrapolating this flow line to the water content at 25 blows to give the liquid limit. In order to predict the slope of the flow-line, Fang correlated data on 469 soil tests carried out during the AASHO (American Association of State Highway Officials) road test (Burggraf and McKendrick, 1956) and by the Washington State Highway Department to predict that the flow index could be approximated by:

$$FI = 0.36(w_L - 0.08) \qquad (20)$$

Utilising the analysis presented here, (assuming ASTM apparatus) this can be compared to the predicted values of flow index for soils falling on the A and U lines, as shown in Figure 6. It can be seen that the average values of flow index measured by Fang fall in exactly the region expected, being appropriate for soils lying above the A-line and below the U-line. The power law expressions of the single-point liquid limit test (equation 2) can also be presented on this figure. ASTM method B, using a value of tan $\beta$ of 0.121 can be shown to give:

$$FI = \tan\beta \ln(10)\, w_L = 0.279 w_L \qquad (21)$$

This is broadly consistent with the analysis presented here, being applicable for soils lying close to the A-line for soils having liquid limits between 20 and 120%.

## SUMMARY

The following summary points can be made:

(a) The liquid limit of soils as measured by the Casagrande apparatus was shown by Haigh (2012) to be an assessment of specific strength. Utilising this analysis along with trends of changing soil strength with water content ($I_L$), as outlined by Vardanega and Haigh (2014) allows an understanding of the mechanics underpinning the single-point liquid limit tests proposed by several authors and implemented as part of ASTM D4318-10 and BS1377-1990 amongst other design codes.

(b) The difference between the implementation of the single point liquid limit method by the two codes (ASTM and BS1377) has been shown to be predominantly a function of the differences in equipment specified (i.e. hard or soft base Casagrande devices) rather than being due to the nature of the soils in the two countries. It is suggested that those countries utilising hard-based Casagrande equipment (e.g. USA) should use a value of tan $\beta$ of 0.121, and those using soft-base equipment (e.g. UK, India & Australia) a value of 0.092, regardless of the origin of the soils being tested.

(c) While the relationship between the plasticity index and flow index, as previously described by Sridharan et al. (1999), has a fundamental basis in the mechanics of the test, potentially allowing a liquid limit test to be used to estimate the plastic limit of a soil, this would however have substantial uncertainties, both due to the scatter seen in the data for a single set of equipment and due to the variable nature of liquid limit test devices in operation worldwide.

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## Figure Captions

**Figure 1: International variations in single-point liquid limit formulae**

**Figure 2: Relationship between number of blows for groove closure and specific strength**

**Figure 3: Variation of $\alpha$ with Casagrande Liquid Limit**

**Figure 4: Predicted ratio of plasticity index and flow index**

**Figure 5: Plasticity Index (*PI*) predicted from Flow Index (*FI*)**

**Figure 6: Predicted and measured relationships between the Flow Index (*FI*) and the Liquid Limit ($w_L$)**

Highlights:

1) Atterberg’s liquid limit can be measured rapidly using 1-point methods but these vary worldwide.
2) This paper demonstrates that different methods are a result purely of different equipment, not of soil types
3) The analysis presented shows why a value of tan β = 0.121 can be used for hard-base equipment and tan β = 0.092 for soft-base equipment.

## Table 1: Published databases with average tanβ values stated

| Publication | Soil Tested | Average tanβ value reported | Description of soils tested |
|---|---|---|---|
| Waterways Experiment Station (1949) | 767 US soils | 0.121 | Recent, Pleistocene, Tertiary and glacial till. |
| Olmstead and Johnston (1954) | 759 US soils | 0.135 | 15% $< w_L <$ 100%+ |
| Eden (1955, 1959) | 484 Canadian soils | 0.100 | - |
| Mohan and Goel (1958); Mohan (1959) | 250 Indian soils | 0.068 | Black Cotton soil |
| Jain and Patwardhan (1960) | 32 Indian soils | 0.085 | Gangetic alluvium |
| Norman (1959) | 455 British soils | 0.092 | 15% $< w_L <$ 170%+ |
| Kim (1973) | 1017 Korean soils | 0.118 | - |
| Roje-Bonacci (2004) | 88 Croatian soils | 0.063 | High-plasticity clays |

**Table 2**

## Table 2: Parameters for liquid limit apparatus tested

| | ASTM D4318 (2010) | BS1377:2 (1990) | IS9259 (1979)<br>Indian Institute of Science (IISc) (Bangalore) |
|---|---|---|---|
| *a* | 6.22 | 5.40 | 5.40 |
| *b* | 21.43 | 173.3 | 374.4 |
| *n* | 1.893 | 2.226 | 3.510 |
| $\frac{c_u}{\rho}$ at liquid limit | 0.932 | 0.376 | 0.432 |
| Prescribed base harness | 80-90 Shore D | 84-94 IRHD | 86-90 IRHD |
| Estimated Young's Modulus | 260-446 MPa | 11.5-28 MPa | 13-18.5 MPa |
| Prescribed Resilience | 77-90% | 20-35% | 30-40% |

**Figure 1**
**Click here to download Figure: fig1.eps**

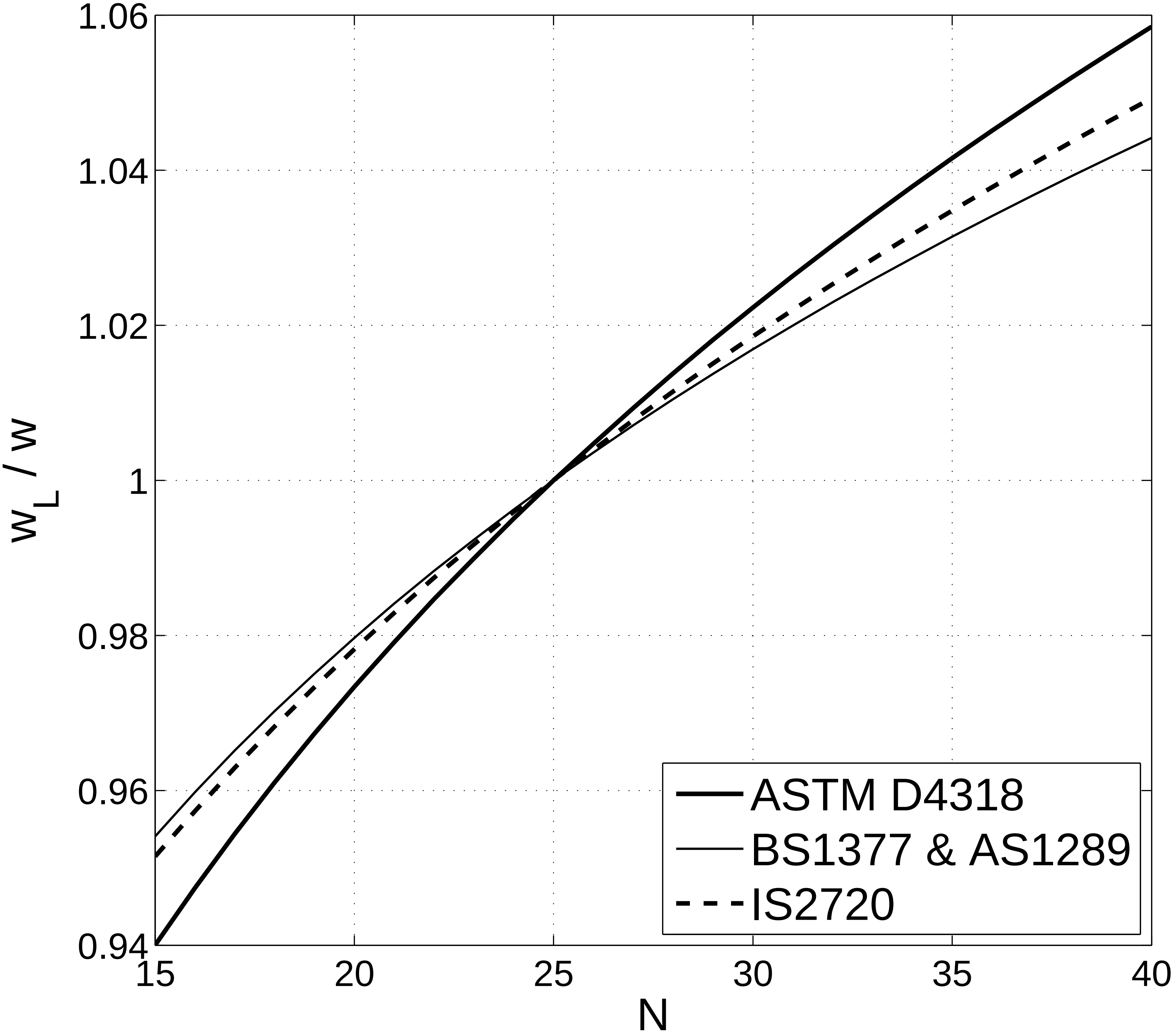


**Figure 2**

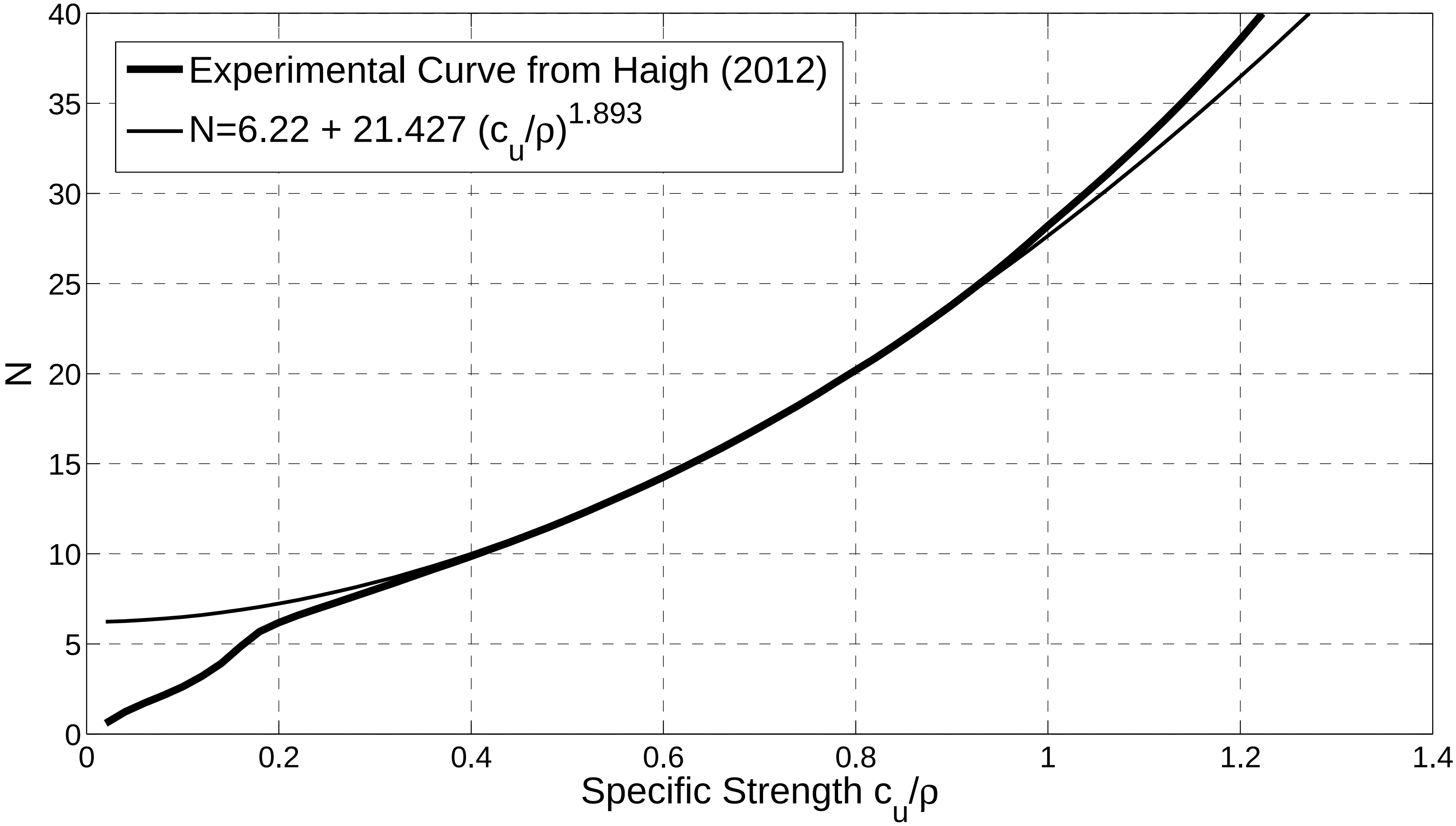

Experimental Curve from Haigh (2012)
N=6.22 + 21.427 (c_u/ρ)^1.893
N
Specific Strength c_u/ρ
0
5
10
15
20
25
30
35
40
0
0.2
0.4
0.6
0.8
1
1.2
1.4

**Figure 3**

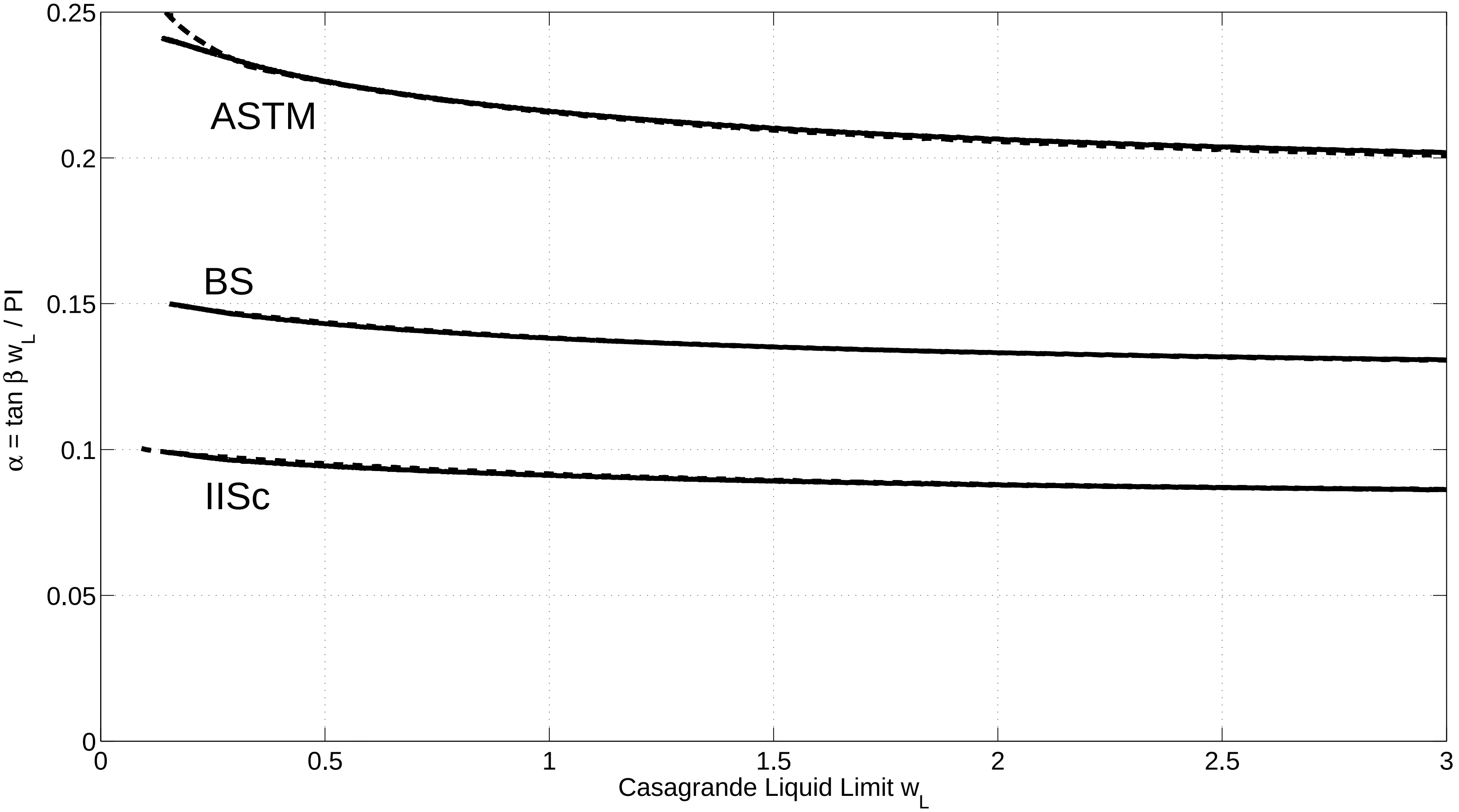
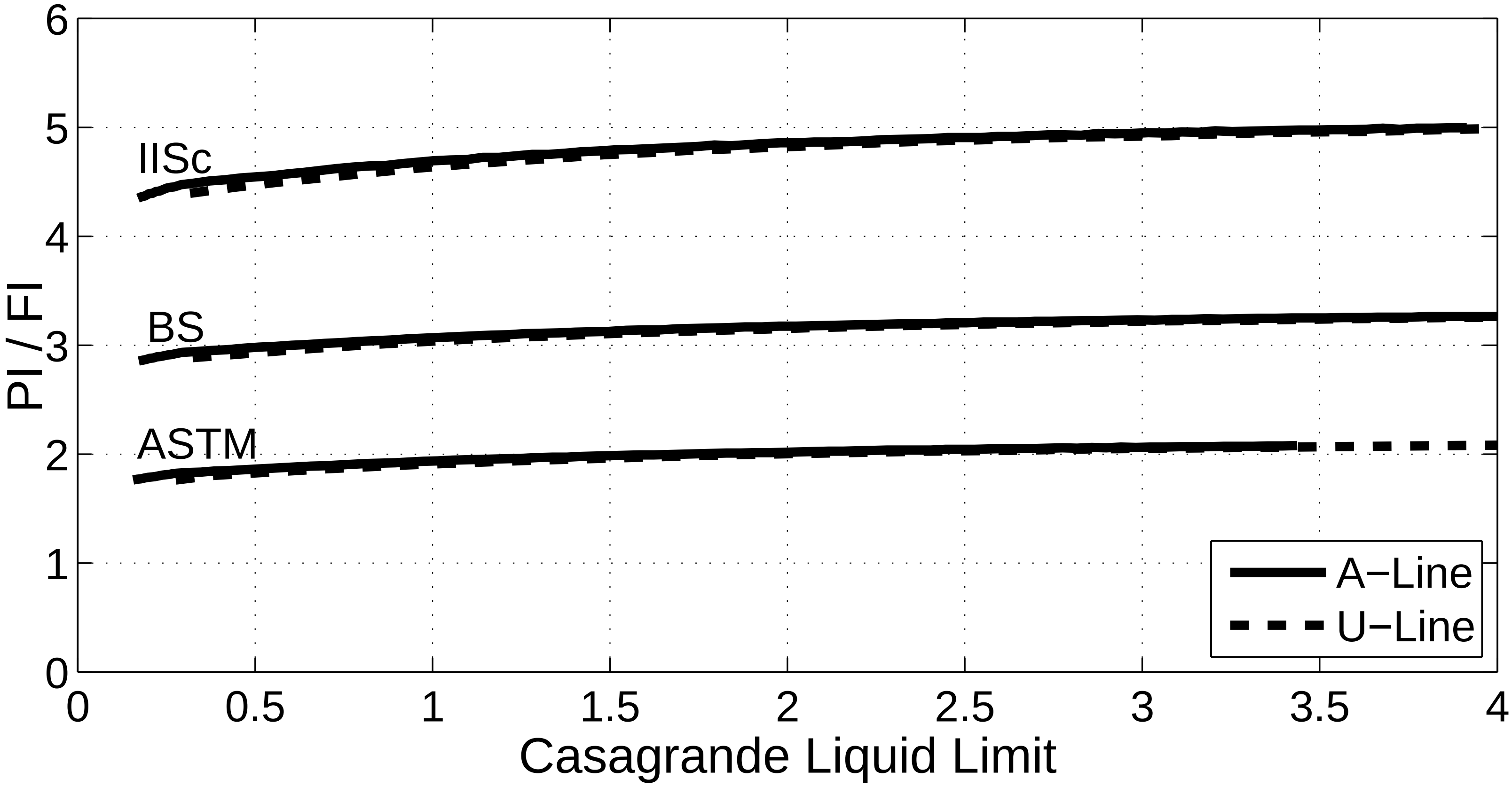

0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
3
ASTM
BS
IISc
$\alpha = \tan\beta\, w_L / PI$
Casagrande Liquid Limit $w_L$

**Figure 4**



6
5
4
3
2
1
0
PI / FI
IISc
BS
ASTM
A−Line
U−Line
0
0.5
1
1.5
2
2.5
3
3.5
4
Casagrande Liquid Limit

**Figure 5**
**Click here to download Figure: fig5-new.eps**

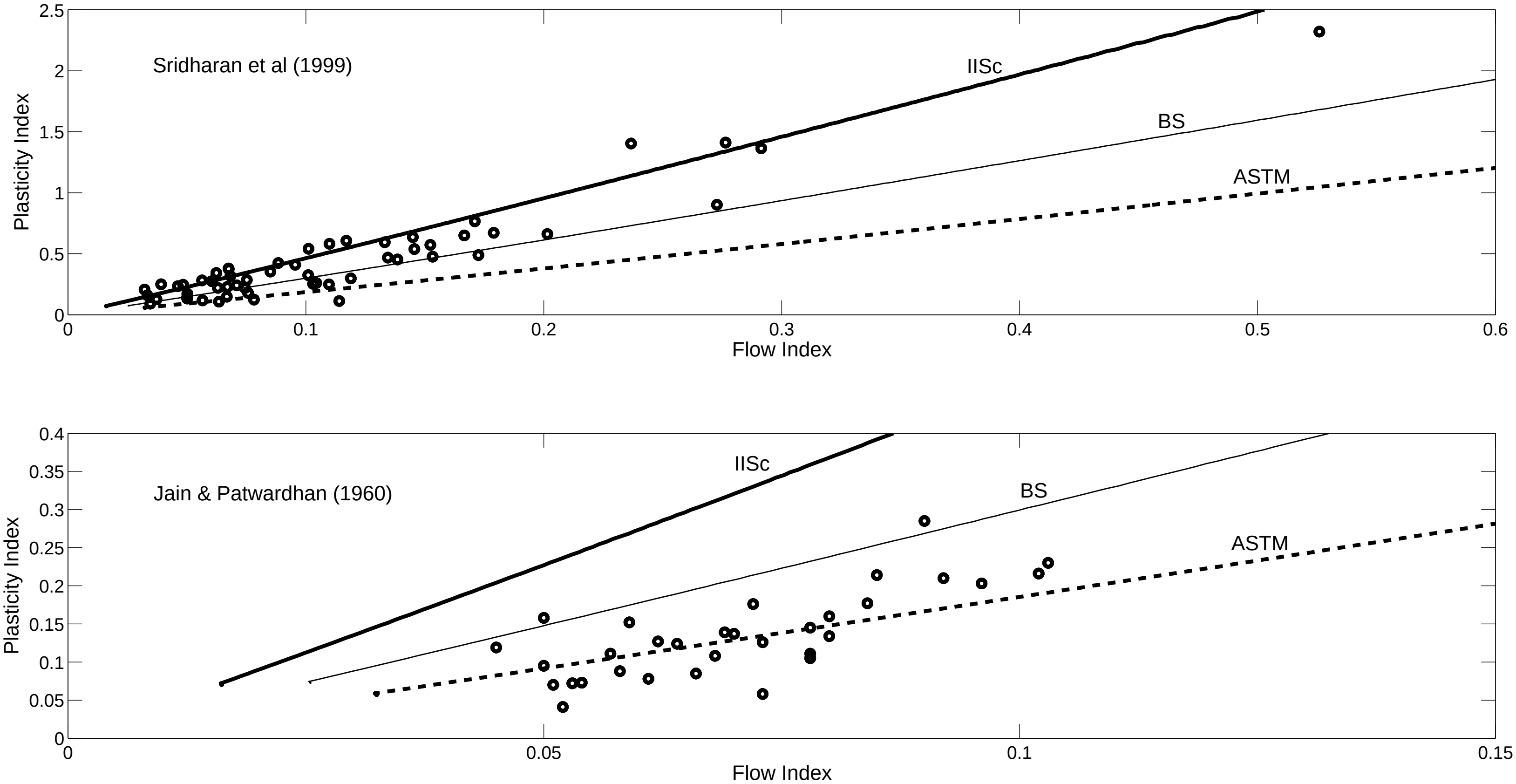
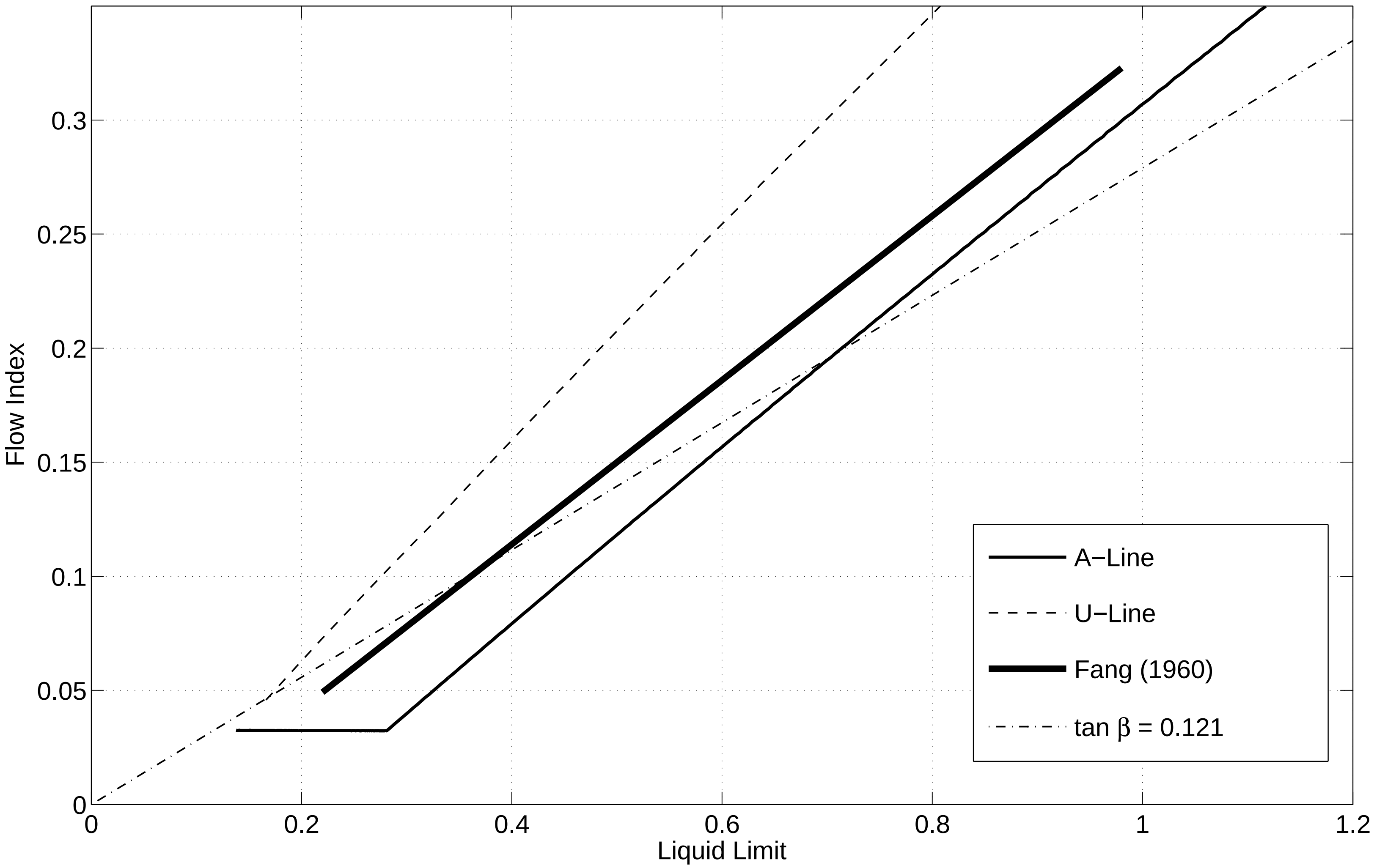


**Figure 6**


Flow Index
Liquid Limit
0
0.05
0.1
0.15
0.2
0.25
0.3
0
0.2
0.4
0.6
0.8
1
1.2
A−Line
U−Line
Fang (1960)
tan β = 0.121