How far from equilibrium is active matter?

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NUMERICAL SIMULATIONS

We use Euler time-stepping to simulate the dynamics of AOUPs:

$$\dot{\mathbf{r}}_i = -\mu\nabla_i\Phi + \mathbf{v}_i; \quad \tau\dot{\mathbf{v}}_i = -\mathbf{v}_i + \sqrt{2D}\boldsymbol{\eta}_i \tag{1}$$

To observe the MIPS reported in Fig. 1 of the main text, we use periodic boundary conditions and the repulsive potential given by Eq.(2) of the main text. The figure is obtained by starting from a random homogenous configuration, integrating the dynamics (1) with a time-step $dt = 10^{-3}$ and taking a snapshot of the particle positions after a time $t = 10^4$.

To test the validity of our modified FDT, we consider AOUPs in $\mathbb{R}^2$, confined by a harmonic potential

$$\begin{aligned} V_W(x,y) = {} & \frac{\lambda}{2}\theta(x-L)(x-L)^2 + \frac{\lambda}{2}\theta(-x)x^2 \\ & + \frac{\lambda}{2}\theta(-y)y^2 + \frac{\lambda}{2}\theta(y-L)(y-L)^2, \end{aligned} \tag{2}$$

where $\theta(u)$ is the Heaviside function. In the simulations reported on Fig. 2 of the main text, we use $\lambda = 10$. We integrate the dynamics of AOUPs using $dt = 5.\,10^{-4}$. We first let the system relax to its steady-state by simulating its dynamics for a time 50.

To measure the correlation function $C_{\rm eff}(t)$, we choose a given value of t_0 and store $x_i(t_0)$ and $\dot{x}_i(t_0)$. We then compute $[x_i(t_0) - x_i(t_0+t)]x_i(t_0+t)$ and $[\dot{x}_i(t_0) - \dot{x}_i(t_0+t)]\dot{x}_i(t_0+t)$ for $t \in [0,2]$. We finally average over $20\,000$ values of t_0 to obtain the correlation function plotted in Fig. 2 of the main text.

To measure the susceptibility $\chi(t)$, we create a copy of the system at a given time t_0. This copy evolves with a perturbed dynamics in which $\Phi \to \Phi - f\epsilon_i x_i$ where the ϵ_i are chosen at random in $\{-1,1\}$. The original system evolves with the unperturbed dynamics and we use the *same noise realisations* $\boldsymbol{\eta}_i$ for the two systems [1]. We then deduce the susceptibility as

$$\chi(t) = \sum_i \varepsilon_i \frac{x_i^{\rm c}(t+t_0) - x_i(t+t_0)}{f}. \tag{3}$$

for $t \in [0,2]$, where $x_i^{\rm c}$ are the abscissa of the perturbed system. We finally average over $20\,000$ values of t_0 to obtain $\chi(t)$.

[1] D. Villamaina, A. Puglisi, and A. Vulpiani, J. Stat. Mech. **2008**, L10001 (2008).