On Economic Equivalence of Economic MPC and Hierarchical Control Schemes

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Abstract

The traditional approach to optimal economic operation of industrial processes has been the use of a hierarchically structured control system. This hierarchical structure comprises a steady-state economic optimization layer that computes optimal operating set-points of the system and a dynamic control layer using model predictive control to track these set-points. One limitation of this structure is that the economics of the process is not being dynamically optimized. To overcome this limitation, economic model predictive control, which optimises the economics directly in the dynamic layer in order to give a superior economic performance, has been proposed to replace the hierarchical approach. This paper investigates this notion of superior economic performance and proposes a redesign of the traditional hierarchical structure that gives an economic performance that is as good as that of economic model predictive. Specifically, a new approach to selecting the objective function in the model predictive control layer is presented.

1 Introduction

The development of optimal operation and control strategies for dynamic systems and processes has been a subject of active research for many years. In the process industry and in most industrial system operation, one of the goals these strategies try to achieve is the best way to maximize economic efficiency in the presence of constraints, disturbances and uncertainties. This notion of economic efficiency varies across fields and can be in terms of monetary profit, fuel usage, time, or a combination of these. Achieving this objective is usually not easy and straightforward especially when the system is multivariable and complex. Over the years, an established way of achieving this objective is to apply a hierarchically structured control system.¹⁻⁵ The main idea in this hierarchical control structure is to decompose the overall objective into a sequence of different, simpler and hierarchically structured sub-objectives. This leads to a technically well-defined, reliable, transparent and simpler overall design.

The general hierarchical structure used in the process industry is shown in Figure 1,^{1,2} which consists of a planning and scheduling layer, real time optimization layer (RTO) and a process control layer. The planning and scheduling system provides production goals (such as what to produce), parameters of the cost function (which can be the prices of variables) and the constraints to enforce, taking into account information from the market and the plant. In the real time optimization (RTO) layer, optimal values for critical state variables (setpoints or steady-states) are generated using a rigorous steady state model of the process, taking into account the profit considerations of the process. These setpoints are then passed down to the lower layer, where the process control ensures that the process tracks the setpoints in the presence of disturbances and uncertainties using the available inputs to the process. The control scheme implemented in this control layer is often model predictive control (MPC) because of its ability to handle multi-variable constrained systems and satisfy optimal performance considerations.^{4,6-8}

Most commercial implementation of model predictive control technology is based on this

Figure 1: A typical hierarchical control structure

hierarchical architecture^{4,8} where economic optimization and control are split into different tasks and executed on different layers. One property of this hierarchical architecture is the time-scale separation between the RTO and MPC layers which allows the effective decomposition of the overall objective into different tasks on different layers. However, two points are worthy of note in this structure. First is the limitation the time-scale separation might cause in achieving an overall optimal operation of the process. For instance, the RTO is usually executed only when the plant is at steady-state which could be at an hourly or daily rate while the base control computes optimal inputs for the process at a faster rate, usually seconds or minutes. It may thus happen that before the next RTO execution, events which would have shifted the optimal steady-states have occurred while the controller still tries to enforce the old optimal, now sub-optimal, steady-state. One way of overcoming this limitation is the inclusion of a steady-state target optimization (SSTO) layer in the MPC

layer.^{4,9,10} The SSTO is executed at the same frequency as the MPC and its objective is to update the pre-computed optimal setpoints from the RTO layer, using an estimate of the effect of the ‘event’ on the process, and move these setpoints closer to what would have been obtained assuming a new real time optimization is carried out. The second point worthy of note in this structure is that two different objective functions are employed in the overall architecture– an economic objective function in the RTO layer which determines the optimal static equilibrium and a tracking control objective function in the MPC layer which governs the dynamic operation of the system. This implies that the economics or profit is only considered at the static equilibrium and not in the dynamics or transient of the system. If the process spends its entire life cycle at this equilibrium then there should be no problem. However, different kinds of disturbances and events often affect the system that moves it from the static equilibrium. It is therefore possible that the system is not being optimally dynamically operated as the economic objective has no influence on the dynamic operation of the system in this traditional setup. Hence it will be of importance to consider the economics in the dynamics. Economic MPC^{11–22} is an attempt aimed at overcoming this limitation. The main difference between economic MPC (e-MPC) and the conventional MPC in the hierarchical control structure (Figure 1) is the cost function (and usually model) employed in generating the optimal inputs to the system. While conventional MPC uses a quadratic, usually positive-definite, cost function designed to give some desired control performance, economic MPC uses the economic objective function of the system operation in the dynamic layer. Also, economic MPC setups favour the use of nonlinear models as against use of linear models in conventional MPC as it is assumed that a nonlinear model will be of higher fidelity which should lead to a more correct optimal input. As reported in most economic MPC literature however, the main advantage of economic MPC over conventional MPC is seen in the transient response of the process where simulation evidence has shown that economic MPC gives a better transient economic performance.

The goal of this paper is to investigate this notion of better economic performance of eco-

economic MPC when compared with the traditional hierarchical control structure. Specifically, we study the possibility of equivalent economic performance between economic MPC and the hierarchical control structure where a tracking objective function is employed in the MPC layer. We propose that when properly designed, the hierarchical control structure, which uses a linear model and a quadratic cost function that attempts to track a pre-computed optimal static equilibrium in the MPC layer, can give an equivalent (and sometimes better) economic performance when compared with economic MPC.

The remainder of this paper is organized as follows. Section 2 presents the preliminaries which include notations used and definitions. The proposed redesign of the hierarchical control structure is presented in Section 3 while Section 4 presents some examples to show the economic properties of this new design. Section 5 concludes the paper.

2 Preliminaries

In this paper, we consider the continuous-time, time-invariant nonlinear system of the form

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathbb{X} \subseteq \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{U} \subset \mathbb{R}^{n_u}$ and the state transition map, $f : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{X}$. The sets $\mathbb{X}$ and $\mathbb{U}$ are assumed compact and the constraints $(x, u) \subseteq \mathbb{X} \times \mathbb{U}$ are assumed to be point-wise in time. The economic objective of the system is defined by the generic continuous function $l_e(x, u)$ where $l_e : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{R}$. The optimal static equilibrium of the system with respect to the given economic objective is defined as the pair (x_s, u_s) that satisfies

$$l_e(x_s, u_s) = \min_{x, u} \{l_e(x, u) \mid f(x, u) = 0, x \in \mathbb{X}, u \in \mathbb{U}\} \tag{2}$$

Associated with the steady-state optimization problem (2) is the Lagrangian $L(x, u, \lambda)$, defined as

$$L(x, u, \lambda) = l_e(x, u) + \lambda^T(f(x, u)) \tag{3}$$

where λ is the vector of Lagrange multipliers associated with the equality constraints in (2). Here we have assumed that no inequality constraint is active at steady-state. If some are active, they can be appropriately handled by slack variables.

The steady-state optimization problem (2) is said to be strongly dual if the pair (x_s, u_s) that solves (2) also uniquely solves

$$\min_{x,u} L(x, u, \lambda), \quad x \in \mathbb{X}, \quad u \in \mathbb{U} \quad (4)$$

where λ is the vector of optimal Lagrange multipliers from (3). In general and for a generic $l_e(x, u)$, strong duality is usually not guaranteed to hold unless $l_e(x, u)$ is known to be convex. It is however worthy of note that solving (2) is equivalent to solving

$$\min_{x,u} \{L(x, u, \lambda) \mid f(x, u) = 0, \quad x \in \mathbb{X}, \quad u \in \mathbb{U}\} \quad (5)$$

which implies that the objective function $l_e(x, u)$ can be replaced by the Lagrangian $L(x, u, \lambda)$. Hence when applicable, $l_e(x, u)$ can be interchanged with $L(x, u, \lambda)$.

Central to economic MPC approach to maximising the economic efficiency of the nonlinear system (1) is the solution to the continuous-time optimization problem

$$\begin{aligned} \min_{\mathbf{u}} J(x, \mathbf{u}) &\triangleq \int_0^T l_e(x(t), u(t)) \, dt \\ \text{subject to } &\begin{cases} \dot{x}(t) = f(x(t), u(t)) \quad \forall t \in [0, T] \\ x(t) \in \mathbb{X}, \quad u(t) \in \mathbb{U}, \quad \forall t \in [0, T] \\ x(0) = x_0, \quad x(T) = x_s \end{cases} \end{aligned} \quad (6)$$

where x_0 is the initial condition. Assuming feasibility, optimization problem (6) is repeatedly minimized in a receding horizon manner: the first control action from the returned input trajectory, $\mathbf{u}$, is implemented over one sampling period, measurements from the system are then taken as new initial condition and the optimization problem re-solved to generate a

new input trajectory.

Remark 1. *The class of systems for which it is known that a periodic operation or forced cyclic operation outperforms steady state operation is explicitly excluded from our consideration in this paper.*

Definition 1. *The nonlinear system (1) is said to be dissipative around the static equilibrium (x_s, u_s) with respect to the objective function $l_e(x, u)$ if there exists a differentiable storage function $V(x)$ such that*

$$\frac{dV(x)}{dt} \leq l_e(x, u) - l_e(x_s, u_s). \quad (7)$$

If (7) holds with strict-inequality, then the system is strictly-dissipative around the static equilibrium.

The concept of dissipativity has played a prominent role in the analyses of economic MPC over the last few years. It was shown in ^{18,19} that if a system is strictly-dissipative with respect to its economic objective function, then the economic controller obtained by minimizing this objective function subject to the dynamics of the system is an asymptotically stabilizing controller for the system. The authors in ^{19,21,23} further showed, under some controllability assumption, that dissipativity is also a necessary and sufficient condition for the system to be optimally operated at the static equilibrium pair, (x_s, u_s) .

Though this work covers setups for which the dissipativity condition holds, in general, we do not assume in this work that the nonlinear system (1) is (strictly)dissipative with respect to the economic objective function $l_e(x, u)$. Hence, the focus is not on asymptotic stability of the closed-loop system obtained using economic MPC. Rather, what is assumed is the existence of an optimal static equilibrium, (x_s, u_s) , around which the system can be operated. If (strict)dissipativity holds around this equilibrium, then operating the system around this equilibrium is the optimal operating regime of the system and the closed-loop system is expected to be asymptotically stable.

3 Proposed Redesign of the Hierarchical Control Structure

In this section, the continuous-time optimal control problem defined in (6) is considered. The goal is to show that the economic MPC problem can be well defined by a reference tracking MPC problem with a linear model. This will be achieved by showing that, for regions close enough to the static equilibrium (x_s, u_s) , the system dynamics and objective function in the optimization problem (6) can be well approximated by a linear system with an appropriate quadratic cost, assuming functions $f(x, u)$ and $l_e(x, u)$ are sufficiently smooth.

To begin, consider the general equality constrained optimization problem

$$\min_z l(z) \quad \text{subject to} \quad h(z) = 0. \quad (8)$$

where $l(z)$ is assumed to be twice-differentiable and $h(z)$, differentiable. Given a point z^* , and a perturbation $\delta z = z - z^*$ from z^* , the function $l(z)$ can be expanded around z^* using the Taylor's series expansion as :

$$l(z^* + \delta z) \approx l(z^*) + \nabla l(z^*)\delta z + \frac{1}{2}\delta z^T \nabla^2 l(z^*)\delta z \quad (9)$$

where $\nabla l(z^*)$ is the gradient of $l(z)$ evaluated at z^* and $\nabla^2 l(z^*)$ is the Hessian of $l(z)$ evaluated at z^* . Similarly, the constraint function ($h(z)$) can be expressed as

$$h(z^* + \delta z) \approx h(z^*) + \nabla h(z^*)\delta z. \quad (10)$$

Assuming z^* is a solution to (8), the first order necessary optimality condition associated

with (8) is that

$$\nabla l(z^*) + \lambda^T \nabla h(z^*) = 0 \quad (11)$$

$$h(z^*) = 0. \quad (12)$$

Note that by definition, (11) can be rewritten as $\nabla L(z^*, \lambda) = 0$ where $L(z^*, \lambda)$ is the Lagrangian associated with (8). Now consider the Lagrangian $L(z^*, \lambda)$. As discussed in Section 2, the objective function $l(z)$ in (8) can be replaced with $L(z^*, \lambda)$ which on expansion around z^* yields

$$L(z, \lambda) \approx L(z^*, \lambda) + \frac{1}{2}(z - z^*)^T \nabla^2 L(z^*, \lambda)(z - z^*) \quad (13)$$

where we have taken into account the fact that $\nabla L(z^*, \lambda) = 0$.

The Hessian, $\nabla^2 L(z^*, \lambda)$, describes the local curvature of the function $l(z)$ around the point at which it is evaluated, in this case the solution to (8). Though, generally, the Hessian can vary as a function of z , it is assumed fixed here since it is being evaluated at a fixed point, z^* . Hence, as shown in (13), given the point z^* as a fixed point of the system, the Hessian captures the behaviour of the objective function around this point. Thus, when the system is perturbed away from this point, the part of the objective function that governs its transient behaviour is the Hessian of the objective function, on the assumption that such perturbations are not big enough to render the Taylor's series expansion around z^* invalid. Positive (semi)definiteness of the Hessian evaluated at (z^*) is normally used as a (necessary) sufficient condition to guarantee that the point (z^*) is actually an optimal static equilibrium of the system. However, using only $L(z, \lambda)$ or $\nabla^2 L(z^*, \lambda)$ without considering the constraint $h(z)$ is basically assuming that the optimization problem (8) is strongly dual, which is not always the case. When the Hessian matrix does not fulfil the definiteness condition or the strong duality of the optimization problem cannot be verified, z^* being an optimal point can be verified by combining the behaviour of the Hessian with that of the system around this point i.e., by checking the dissipativity of the system around this point.

3.1 Re-writing the optimal control problem

Now consider the nonlinear system (1) with the stage cost $l_e(x, u)$ and the optimal static equilibrium (x_s, u_s) . Let the Lagrangian of the steady-state optimization problem be $L(x, u, \lambda)$ where λ is the vector of optimal Lagrange multipliers associated with the steady-state optimization (2). Expanding the Lagrangian and nonlinear system around this static equilibrium using Taylor's series expansion yields

$$L(x, u, \lambda) \approx L(x_s, u_s, \lambda) + \underbrace{\nabla^T L(x_s, u_s, \lambda)}_G \underbrace{\begin{bmatrix} x - x_s \\ u - u_s \end{bmatrix}}_{\delta z} + \frac{1}{2} \begin{bmatrix} x - x_s \\ u - u_s \end{bmatrix}^T \underbrace{\nabla^2 L(x_s, u_s, \lambda)}_H \begin{bmatrix} x - x_s \\ u - u_s \end{bmatrix} \quad (14)$$

and the nonlinear system as

$$f(x, u) \approx f(x_s, u_s) + \underbrace{\frac{\partial f(x, u)}{\partial x} \Big|_{x_s}}_{A_c} \delta x + \underbrace{\frac{\partial f(x, u)}{\partial u} \Big|_{u_s}}_{B_c} \delta u \quad (15)$$

linear dynamics

where $G = 0$, $f(x_s, u_s) = 0$ and $L(x_s, u_s, \lambda)$ is a constant by virtue of (x_s, u_s) being an optimal static equilibrium. Now, if x_s is indeed an optimal static equilibrium of the system, then for any perturbation of the system from x_s to x , optimal economic operation of the system implies that the system returns to x_s . This is what positive-definiteness of H (when the steady-state optimization problem is strongly dual) or dissipativity of the system with respect to the cost function usually guarantees. Moreover, using (14) and (15), the dissipativity of $f(x, u)$ with respect to $l_e(x, u)$ around (x_s, u_s) can be verified by checking if the local dynamics $A_c \delta x + B_c \delta u$ is dissipative with respect to the local cost function $\frac{1}{2} \delta z^T H \delta z$ (since other terms are either constant or zero when evaluated at (x_s, u_s)).

From the discussions so far, for sufficiently smooth nonlinear system dynamics and economic objective functions, behaviours around the static equilibrium can be well captured by the linear dynamics of the system and the Hessian of the Lagrangian. Hence the nonlinear economic optimal control problem (6) can be represented using the Hessian of the Lagrangian evaluated at steady-state as the objective function and the linear dynamics of the system i.e.,

$$\begin{aligned} \min_{u - u_s} \quad & \frac{1}{2} \int_0^T \begin{bmatrix} x(t) - x_s \\ u(t) - u_s \end{bmatrix}^T H \begin{bmatrix} x(t) - x_s \\ u(t) - u_s \end{bmatrix} dt \\ \text{subject to} \quad & \begin{cases} \dot{x} = A_c(x(t) - x_s) + B_c(u(t) - u_s) \\ x(t) \in \mathbb{X}, u(t) \in \mathbb{U}, \forall t \in [0, T] \\ x(0) = x_0, \quad x(T) = x_s. \end{cases} \end{aligned} \quad (16)$$

Thus if the system is being operated around a static equilibrium, the economic MPC problem (6) can be expressed as a reference tracking control problem (16). This fits into the hierarchical control scheme of tracking a pre-computed static equilibrium using a conventional MPC layer. The main difference is that the conventional scheme employs a positive-definite quadratic cost designed to achieve asymptotic stability while the design here uses the Hessian of the Lagrangian as the quadratic cost with no emphasis on asymptotic stability, perfect tracking or convergence to the static equilibrium. From examples cited in economic MPC literature, the economic advantage of economic MPC over the conventional tracking MPC is usually seen in the transient response. Hence we note that by choosing the tracking cost function as the Hessian (which governs the transient behaviour around steady-state) of the static economic optimization problem, the transient performance of both schemes should be approximately equivalent, once again provided the perturbations are not large enough to invalidate the Taylor's series approximation of the system.

If the system is strictly-dissipative with respect to the objective function around the steady-state, asymptotic stability of the static equilibrium of the closed-loop system using

the optimal controller from (16) is guaranteed. However, stability is not the focus here. Rather we propose that the economic MPC problem can be adequately represented as a tracking problem.

Remark 2. *Ideas similar to these are used in neighbouring extremal (NE) path approach to solving unconstrained dynamic optimization problems.²⁴ However, while NE assumes the existence of an optimal trajectory and studies effects of perturbations away from this optimal trajectory, we assume knowledge of the optimal static equilibrium. Hence, our expansions are done about a static point. The sequential quadratic programming (SQP) approach to the solving of constrained nonlinear optimization problems is also similar to this. The major difference is that while we assume knowledge of the optimal variable (z^*) (hence the gradient is zero and Hessian is fixed), this is not the case in SQP approach since the gradient is usually taken about a current estimate of the optimal solution to the optimization problem and hence non-zero unless the estimate is the required solution.*

Remark 3. *Constraints such as average constraints are not considered in this paper. Average constraints are usually included to enforce a cyclic, periodic or more complex behaviour in systems.^{25–27} However, the focus here is on systems that are being operated around a static equilibrium.*

In concluding this section, we note that though the Lagrangian defined in (3) is a standard Lagrangian definition, it has found prior use in economic MPC under the context of closed-loop stability analysis. It was established in¹⁷ that replacing $l_e(x, u)$ with the Lagrangian, $L(x, u, \lambda)$ in (6) results in the same optimal trajectory. Under further assumptions, such as strong duality of the steady-state optimization problem, asymptotic stability of the closed-loop system was established by replacing the Lagrangian with the real economic objective. We note that this similarity between the use of $l_e(x, u)$ and $L(x, u, \lambda)$ in (6) for stability analysis further strengthens the proposed formulation since the quadratic cost proposed here is an approximation of the Lagrangian.

Remark 4. *The possibility of a linear-quadratic formulation giving similar characteristics with economic MPC was proposed in²⁸ where a linear control policy, based on the economic MPC formulation, was first developed and then a quadratic cost function designed using the principles of inverse optimality. This approach is however different from the formulation presented in this paper.*

4 Illustrative Examples

Three examples are presented in this section to illustrate the performance of the proposed design approach. In each example, the economic optimal control problem setup (6) is compared with the tracking optimal control problem (16) in an MPC framework where both optimization problems are solved in a receding horizon manner and the first element of the returned optimal input sequence applied to the nonlinear continuous-time system at each time instant. As noted by the authors in,²⁹ stability properties (if present) of the continuous-time optimal control problem can be lost if the discrete approximation of the optimal control problems is not properly done. Hence proper care has been taken in setting up the discrete approximations of the examples being considered. Once again, we note that the difference between the tracking MPC setup here and the conventional tracking MPC is that the quadratic cost is selected as the Hessian of the Lagrangian of the steady-state optimization problem.

4.1 Example 1

The system considered is that of a single first-order, irreversible chemical reaction in an isothermal continuous stirred tank reactor.^{17,18} The material balances are modelled with the equations

$$\begin{aligned}\dot{c}_a &= \frac{u}{V_R}(c_{af} - c_a) - k_r c_a \\ \dot{c}_b &= \frac{u}{V_R}(c_{bf} - c_b) + k_r c_a.\end{aligned}\tag{17}$$

The molar concentrations, c_a and c_b , are the states, x_1 and x_2 , of the system and are constrained to $x_1, x_2 \in [0, 1]$. The input u is the feed flow-rate with the input constraint set $\mathbb{U} = \{u \in \mathbb{R} : 0 \leq u \leq 20\}$. The volume of the reactor, V_R , is 10 L with the feed concentrations, $c_{af} = 1 \text{ mol L}^{-1}$ and $c_{bf} = 0 \text{ mol L}^{-1}$ while the rate constant, $k_r = 1.2 \text{ L mol}^{-1} \text{ min}^{-1}$. The economic objective of the process, based on the price of product x_2 and a separation cost, is given as

$$l_e(x, u) = 30 - 2ux_2 + 0.5u. \quad (18)$$

The Lagrangian of the steady-state optimization is

$$L = 30 - 2ux_2 + 0.5u + \lambda_1(0.1u_1 - 0.1u_1x_1 - 1.2x_1) + \lambda_2(1.2x_1 - 0.1ux_2). \quad (19)$$

Solving the first-order necessary conditions that the optimal (x_s, u_s, λ) must satisfy yields

$$x_s = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \quad u_s = 12, \quad \lambda = \begin{bmatrix} -10 & -20 \end{bmatrix}. \quad (20)$$

The Hessian, H , evaluated at this steady-state is

$$H = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (21)$$

and the linear dynamics obtained by linearizing the nonlinear system around this steady-state is described by

$$A_c = \begin{bmatrix} -2.4 & 0 \\ 1.2 & -1.2 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0.05 \\ -0.05 \end{bmatrix}. \quad (22)$$

Though H is indefinite, hence optimality of (x_s, u_s) can not be guaranteed based on only the Hessian H , the linear dynamics can be shown to be dissipative with respect to the Hessian. This implies that this static equilibrium is actually optimal.

The optimization problems are solved using direct transcription methods. The MATLAB-based Imperial College London Optimal Control Software (ICLOCS)³⁰ is used to transcribe the optimal control problems and IPOPT,³¹ an interior point optimization algorithm, to solve the resulting nonlinear programming problem. A prediction horizon $T = 20$ is used with a sampling interval $t_s = 0.5$. While the tracking MPC problem could have been solved using an exact discretization of the continuous-time setup, the same setup has been chosen for both optimization problems in order to demonstrate a fair comparison between both approaches.

The behaviour of the closed-loop systems using the two control schemes is shown in Figure 2. The system is initialized at steady-state and assumed to be perturbed to extremes of the state constraint set by impulse disturbances $d = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}$ and $d = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}$ on the states at time instants $t = 5$ and $t = 15$ respectively. It is observed that both controllers yield equivalent trajectories. The economic performance is computed for both closed-loop trajectories using the economic objective (18) and also shown in Figure 2. The average economic performance (over the simulation period) of the economic MPC scheme is 23.5162 while that of the tracking MPC is 23.5169, which shows the equivalence of the economic performance of both setups.

Figure 2: Closed-loop state, input and instantaneous cost profiles for the economic MPC scheme (red solid line) and tracking MPC (blue dashed lines)

4.2 Example 2

Next, a nonlinear continuous flow stirred-tank reactor with parallel reactions^{26,27} treated under the context of economic MPC in²⁰ is considered. The nonlinear system is modelled as

$$\begin{aligned}
 \dot{x}_1 &= 1 - 10^4 x_1^2 e^{-1/x_3} - 400 x_1 e^{-0.55/x_3} - x_1 \\
 \dot{x}_2 &= 10^4 x_1^2 e^{-1/x_3} - x_2 \\
 \dot{x}_3 &= u - x_3
 \end{aligned} \tag{23}$$

where the input is constrained to lie between 0.049 and 0.449 and the states are non-negative. The economic objective of the process is to maximize x_2 which represents the concentration

of one of the products of the stirred-tank reactor i.e., $l_e(x, u) = -x_2$.

The optimal static equilibrium is obtained as $x_s = \begin{bmatrix} 0.0832 & 0.0846 & 0.1491 \end{bmatrix}$, $u_s = 0.1491$ with the Lagrange multipliers at steady-state as $\lambda = \begin{bmatrix} -0.1561 & -1 & 0 \end{bmatrix}$. The Hessian and linear dynamics evaluated at this static equilibrium are

$$H = \begin{bmatrix} -20.6323 & 0 & -38.5946 & 0 \\ 0 & 0 & 0 & 0 \\ -38.5946 & 0 & -65.004 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad (24)$$

$$A_c = \begin{bmatrix} -13.0349 & 0 & -24.3921 \\ 2.0341 & -1 & 3.8064 \\ 0 & 0 & -1 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (25)$$

respectively. The linear dynamics is not dissipative with respect to the Hessian hence it is not expected that the nonlinear system will be dissipative with respect to the economic objective around the optimal static equilibrium.

The optimization problem is set up using a prediction horizon $T = 10$ and a sampling interval $t_s = 0.1$. The system was first initialized at steady-state and the closed-loop trajectories compared. Since the linear dynamics is not dissipative with respect to the Hessian, the closed-loop system resulting from the use of tracking MPC will not be asymptotically stable.³² We note that the necessity of dissipativity for asymptotic stability in the linear-quadratic case has been established in literature. This behaviour is seen in Figure 3 where the closed-loop system under both controllers is unstable. Moreover, the behaviour in the linear-quadratic case perhaps gives an insight into the observed closed-loop behaviour from the use of economic MPC since the linear-quadratic formulation is an approximation of economic MPC's behaviour. As shown in Figure 3 however, there is a difference between the system's evolution under both controllers. This is perhaps due to the degree of nonlinearity of the system. However, the aim here is not to compare closed-loop trajectories but rather the eco-

Figure 3: Closed-loop state and input profiles for the economic MPC scheme (red solid line) and tracking MPC (blue dashed lines) when the system is initialized at steady-state

nomic performance over the simulation period. The economic performance is computed for both closed-loop trajectories using the economic objective $l_e(x, u) = -x_2$. The average economic performance (over the simulation period) using economic MPC scheme was -0.0851 while that of the tracking MPC scheme was -0.0850 which shows that despite the different closed-loop trajectories, the average economic performances are approximately equivalent. The system is then initialized off steady-state at $x(0) = \begin{bmatrix} 0.4 & 0.05 & 0.01 \end{bmatrix}$. The resulting closed-loop trajectories from the two control schemes are shown in Figure 4. Once again, the closed-loop trajectories from both schemes are different with those obtained from the tracking MPC oscillating around the optimal static equilibrium $x_s = \begin{bmatrix} 0.0832 & 0.0846 & 0.1491 \end{bmatrix}$.

Figure 4: Closed-loop state and input profiles for the economic MPC scheme (red solid line) and tracking MPC (blue dashed lines) when the system is initialized at $x(0)=[0.4 \ 0.05 \ 0.01]$

However, the average economic performance using the economic MPC scheme was -0.0855 while that of the tracking MPC was -0.0899 with the tracking MPC giving a better economic performance compared with the economic MPC. This shows that even for systems that are not dissipative with respect to the economic cost but operated around a static equilibrium, the tracking MPC can still give an equivalent or better economic performance when compared with the economic MPC scheme.

The simulation length of 10 time units was used in Figure 4 in order to observe the transient performance of the system since it is usually reported that economic MPC gives a better transient economic performance. When the system was simulated over a longer time

unit as shown in Figure 5 however, the difference in the economic performances evened out with the average economic performance of the system under economic MPC evaluated as -0.0865 while that of tracking MPC was -0.0866 . This agrees with the result in³³ where it was stated that while transient averages of economic MPC may take up any value, the asymptotic average is never worse than that of keeping the system at the best admissible steady-state.

Figure 5: Closed-loop state and input profiles for the economic MPC scheme (red solid line) and tracking MPC (blue dashed lines) when the system is initialized at $x(0)=[0.4 \ 0.05 \ 0.01]$

Perhaps one more observation from this system that is known to be complex²⁷ is the effect of initial condition on the appearance (or non-appearance) of limit cycles when using economic MPC. As shown in Figure 3 where the system was initialized at steady-state, the

closed-loop behaviour under economic MPC was chaotic whereas when initialized off steady state as seen in Figures 4 and 5, a somewhat periodic behaviour is observed. While one may not be able to generalize the dependence of limit cycles on initial condition based on this example alone, it will perhaps be of interest in future research to study the effects of initial condition on closed-loop behaviour under economic MPC.

Enforcing Convergence to the Steady-state

It is sometimes desired that the closed-loop trajectories of the system converge to the steady-state around which the system is being operated even when the system is not dissipative with respect to the objective function. Methods that have been proffered in the literature include adding a sufficient convex term to the economic objective that makes the system dissipative with respect to the new objective function or by using a ‘zero-variance’ constraint.^{19,20} However, since it is in general hard to verify dissipativity for a nonlinear system, it may also not be easy computing the convex term that is just sufficient to render the nonlinear system dissipative. A formal attempt of computing this term was proposed in³⁴ using a form of parametric optimization.

Based on our earlier discussions, if dissipativity around a steady-state is desirable, then rather than looking for a means to modify the full objective function, what should be modified is the local behaviour of the cost function around the static equilibrium i.e., the Hessian of the objective function at this equilibrium. This is the approach taken here. Using the local behaviour of the system around this equilibrium, a sufficient ‘regularizing’ term that renders the system dissipative around this equilibrium is computed. In the case of a linear system with quadratic cost, dissipativity is equivalent to the existence of a symmetric matrix P_c such that the linear matrix inequality (LMI)

$$\begin{bmatrix} A_c^T P_c + P_c A_c & P_c B_c \\ B_c^T P_c & 0 \end{bmatrix} + H \leq 0 \quad (26)$$

is feasible.^{35,36} Thus, finding a sufficiently regularizing term implies computing the minimum symmetric matrix H_c (using a suitable matrix norm) for which there exists a symmetric matrix P_c that ensures feasibility of the LMI

$$\begin{bmatrix} A_c^T P_c + P_c A_c & P_c B_c \\ B_c^T P_c & 0 \end{bmatrix} + H + H_c \leq 0. \quad (27)$$

By minimizing its 2-norm, the minimum H_c needed for the system to be dissipative with respect to the objective function was computed as

$$H_c = \begin{bmatrix} -0.1276 & -0.0188 & -0.0776 & 0.0056 \\ -0.0188 & 0.0035 & 0.0153 & -0.0005 \\ -0.0776 & 0.0153 & 0.0817 & 0.0076 \\ 0.0056 & -0.0005 & 0.0076 & 0.1882 \end{bmatrix}. \quad (28)$$

The objective functions $l_e(x, u)$ and H in (6) and (16) are then replaced with $\hat{l}_e(x, u)$ and $\hat{H}$ respectively where

$$\hat{l}_e(x, u) = l_e(x, u) + \begin{bmatrix} x - x_s \\ u - h_s \end{bmatrix}^T H_c \begin{bmatrix} x - x_s \\ u - h_s \end{bmatrix}, \quad \hat{H} = H + H_c. \quad (29)$$

Shown in Figure 6 is the closed-loop behaviour when the system is initialized at $x(0) = \begin{bmatrix} 0.4 & 0.05 & 0.01 \end{bmatrix}$. As shown, the system now converges to the steady-state. The economic performance was also computed using the original economic objective with the economic MPC scheme yielding -0.0894 and tracking MPC yielding -0.0893 .

Thus as long as the system can be operated around a static equilibrium, the optimal control of the system can still be set up as a reference tracking problem with stability and convergence being a function of the Hessian of the Lagrangian around the static equilibrium. As shown, keeping things simple by using the local behaviour of the system around this

Figure 6: Closed-loop state and input profiles for the ‘regularized’ economic (red solid line) and tracking (blue dashed lines) MPC schemes when the system is initialized at $x(0)=[0.4 \ 0.05 \ 0.01]$

equilibrium can give approximately equal economic performance when compared with the use of the nonlinear system with the generic cost. Moreover using the local behaviour of the system and cost function makes for easier analysis of the expected behaviour of the closed-loop system.

4.3 Evaporator Benchmark Case study

To conclude this section, we consider the forced circulation evaporator benchmark described in³⁷ and shown in Figure 7. In this study, the output variables of interest are the product

Figure 7: Forced-circulation evaporator

concentration X_2 and evaporator pressure P_2 while the available manipulated variables are P_{100} and F_{200} . The nonlinear differential equations describing the operation of the process are given as

$$\begin{aligned}\dot{X}_2 &= \frac{F_1 X_1 - F_2 X_2}{M} \\ \dot{P}_2 &= \frac{F_4 - F_5}{C}\end{aligned}\tag{30}$$

where

$$\begin{aligned}
F_4 &= \frac{Q_{100} - F_1 C_\rho (T_2 - T_1)}{\lambda} \\
F_5 &= \frac{Q_{200}}{\lambda} \\
Q_{100} &= 0.16(F_1 + F_3)(T_{100} - T_2) \\
T_2 &= 0.5616P_2 + 0.3126X_2 + 48.43 \\
T_{100} &= 0.1538P_{100} + 90 \\
T_3 &= 0.507P_2 + 55 \\
F_{100} &= \frac{Q_{100}}{\lambda_s} \\
Q_{200} &= \frac{UA_2(T_3 - T_{200})}{1 + \frac{UA_2}{2C_\rho F_{200}}} \\
F_2 &= F_1 - F_4.
\end{aligned} \tag{31}$$

The nominal values of the parameters in (30) and (31) are given in Table 1 while Table 2 contains the nominal values of potential input disturbances to the system. The overall economic objective of the evaporator process is to minimize the operating cost (in \$/h) of the process which consists of the costs of electricity, steam and cooling water^{38,39}

$$l_e = 1.009(F_2 + F_3) + 0.6F_{200} + 600F_{100}. \tag{32}$$

The constraints on the variables of interest are

$$\begin{aligned}
X_2 &\geq 25\%, \quad 40 \text{ kPa} \leq P_2 \leq 80 \text{ kPa} \\
0 &\leq P_{100} \leq 400 \text{ kPa}, \quad 0 \leq F_{200} \leq 400 \text{ kg/min}.
\end{aligned} \tag{33}$$

The optimal static equilibrium of this process under nominal operating conditions is

$$\begin{aligned}
\begin{bmatrix} X_2 & P_2 \end{bmatrix} &= \begin{bmatrix} 25 & 49.743 \end{bmatrix} \\
\begin{bmatrix} P_{100} & F_{200} \end{bmatrix} &= \begin{bmatrix} 191.7132 & 215.8884 \end{bmatrix}.
\end{aligned} \tag{34}$$

Table 1: Parameters of the Evaporator Model

Parameter	Value
M	20kg
C	4kg/kPa
C_ρ	0.07 kW/K(kg/min)
λ	38.5 kW/(kg/min)
λ_s	36.6 kW/(kg/min)
UA_2	6.84 kW/K

Table 2: Nominal values of input disturbance variables of the system

Variable	Description	Equilibrium Value
F_3	Circulating flow rate	50 kg/min
F_1	Feed flow rate	10 kg/min
X_1	Feed composition	5 %
T_1	Feed temperature	40 °C
T_{200}	Feed temperature	25 °C

Linearizing the nonlinear system around this optimal equilibrium point gives the continuous-time linear system

$$A_c = \begin{bmatrix} -0.2045 & -0.1878 \\ -0.0209 & -0.0559 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0.0479 & 0 \\ 0.0096 & -0.0017 \end{bmatrix}. \quad (35)$$

The Hessian of the Lagrangian of the steady-state optimization problem at this optimal static equilibrium is also estimated as

$$H = \begin{bmatrix} 40.7558 & 0.3555 & -0.1413 & -0.0114 \\ 0.3555 & 0.0139 & 0.0029 & -0.0031 \\ -0.1413 & 0.0029 & 0.0043 & 0.0021 \\ -0.0114 & -0.0031 & 0.0021 & 0.0055 \end{bmatrix}. \quad (36)$$

The economic MPC (e-MPC) control scheme is then set up as in (6) using the nonlinear system (30) with the economic objective (32) subject to the constraints on the system. Two linear tracking MPC schemes were set up using (16) with the linear dynamics (35): tracking

MPC1 and tracking MPC2. In tracking MPC1, the cost function is designed with H as in (36) while in tracking MPC2, an arbitrary cost function is used with

$$H = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}. \quad (37)$$

The resulting optimization problems are solved in a receding horizon manner. A sampling time $t_s = 1$ min is used with a prediction horizon $T = 200$ min and a stabilizing terminal constraint $x(T) = x_s$. Shown in Table 3 is the average economic performance (using the economic objective function (32)) of the closed-loop systems resulting from the use of the different control schemes when the system is simulated over a time period of 200 min. The initial conditions were chosen to reflect perturbations to either side of the optimal static equilibrium. As shown in Table 3, tracking MPC1 (MPC1), whose cost function is the Hessian of the Lagrangian of the steady-state optimization problem, gives an approximately equivalent average economic performance when compared with e-MPC and even marginally outperformed it in some of the initial conditions considered. Tracking MPC2 (MPC2) with an arbitrary cost function however gives a worse (though not much worse) economic performance compared with the other control schemes. This once again shows that the conventional tracking MPC in the traditional hierarchical structure when properly designed can give an equivalent performance compared with economic MPC for systems that are operated around a static equilibrium.

To simulate more transient behaviour, the system was initialized at the static equilibrium and the input disturbance variables in Table 2 are perturbed with random variations around their nominal values. Figure 8 shows trajectories of the closed-loop systems using the different control schemes where the economic cost is obtained using the economic objective (32). The average economic performance of economic MPC controlled system over

Table 3: Comparison of the average economic performance of the various control schemes from different initial conditions

Initial condition $x(0)$	e-MPC (\$/h)	MPC1 (\$/h)	MPC2 (\$/h)
[25, 49.743]	5738.2	5738.2	5738.2
[25, 46]	5726.5	5726.1	5775.4
[25, 43]	5717.2	5716.5	5804.3
[25, 52.743]	5747.5	5748.1	5747.5
[25, 55.743]	5756.8	5757.9	5756.8
[30, 49.743]	5713.0	5711.7	5730.3
[30, 46]	5701.4	5699.7	5767.0
[30, 43]	5692.0	5690.0	5795.4
[30, 52.743]	5722.4	5721.6	5726.7
[30, 55.743]	5731.7	5731.5	5734.0

the simulation period is \$5691.9, that of tracking MPC1 is \$5691.6 while that of tracking MPC2 is \$5867.2. Once again, due to the fact that the cost function in tracking MPC1 was designed to reflect the local behaviour of the economic cost around the static equilibrium, the economic performance of economic MPC and tracking MPC1 are quite similar. Tracking MPC2 however, due to the nature of the ‘non-economic oriented’ cost function (37), results in a more expensive economic operation as it pushes the system into a more expensive region while trying to keep the inputs close to their static equilibrium values.

5 Conclusion

This paper has investigated the possibility of economic equivalence between the traditional hierarchical control and economic MPC schemes. A possible redesign of the traditional hierarchical control structure which ensures dynamic optimization of the system’s economics while tracking the optimal static equilibrium was proposed. This design leads to the linear-quadratic form of conventional tracking MPC schemes with the main difference being the choice of the quadratic cost. Rather than using an arbitrary quadratic cost function, this paper proposes the use of the Hessian of the Lagrangian of the steady-state optimization problem as the cost function. Using three examples, we showed that the proposed design

Figure 8: Closed-loop trajectories and instantaneous economic cost profiles of the evaporator process under random input disturbances.

gives an economic performance that is as good as that of economic MPC for the class of systems and operations considered.

Though not considered in this paper, nominal closed-loop asymptotic stability proofs of economic MPC schemes are often based on the dissipativity condition. This dissipativity condition is however not easy to verify especially when considering nonlinear systems with generic costs. Since the proposed formulation uses linear dynamics with purely quadratic cost function, dissipativity can be easily tested for using the Linear Matrix Inequality (LMI) framework, which makes for ease of analysis. This paper has not focused on solution time comparison between the proposed formulation and the economic MPC formulation. Since the proposed formulation uses a linear prediction model with a quadratic cost function, it

is expected that it will have a faster solution time. However, the quadratic cost in this case is not necessarily convex since it is solely dependent on the Lagrangian of the steady-state optimization problem, hence there is no guarantee that this linear-quadratic case will always yield a faster solution time. However, provided dissipativity holds, the cost function can be rotated (by using an appropriate storage function)^{19,33} to give a positive-definite cost function which can then be used in place of the original cost function in setting up the optimization problem. This rotated cost function has been shown to yield equivalent solution with the original cost function^{19,33} and the proposed formulation is then guaranteed to give a faster solution time.

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