# An exploratory factor analysis of visual performance in a large population

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A factor analysis was performed on 25 visual and auditory performance measures from 1060 participants. The results revealed evidence both for a factor relating to general perceptual performance, and for eight independent factors that relate to particular perceptual skills. In an unrotated PCA, the general factor for perceptual performance accounted for 19.9% of the total variance in the 25 performance measures. Following varimax rotation, 8 consistent factors were identified, which appear to relate to (1) sensitivity to medium and high spatial frequencies, (2) auditory perceptual ability (3) oculomotor speed, (4) oculomotor control, (5) contrast sensitivity at low spatial frequencies, (6) stereo acuity, (7) letter recognition, and (8) flicker sensitivity. The results of a hierarchical cluster analysis were consistent with our rotated factor solution. We also report correlations between the eight performance factors and other (nonperformance) measures of perception, demographic and anatomical measures, and questionnaire items probing other psychological variables.

Individual differences, factor analysis, vision, audition, psychophysics, cluster analysis, contrast sensitivity, eye movements, stereopsis, personality

# Introduction

Different individuals perceive the world differently from one another. These differences may arise from inherited variations in the structure of the visual and auditory systems or from variations in experience during an individual's lifetime. Variations in human perception have been perhaps less studied than variations in cognitive skills. However, not only do they have significant impact on our behaviour but they also offer a powerful method of analysing perceptual mechanisms (Kanai & Rees, 2011; König & Dieterici, 1892; Peterzell, 2016; Wilmer, 2008).

Whenever visual or auditory performance is measured in a sample of participants, there is variance evident in the data. This variance comes from three sources: (i) instrumental and other measurement error, (ii) within-individual variation (e.g. temporal fluctuations in motivation and arousal), and (iii) between-individual variation, i.e. persisting differences between participants caused by between-individual variation in processes underlying perceptual functions. To demonstrate true between-individual differences it is necessary to show one or both of two types of result. First, one may show a significant test-retest reliability for the trait of interest (Spearman, 1904b; Wilmer, 2008). Second, one can demonstrate a significant correlation between the trait of interest and another, independent, phenotypic or genotypic measure (Kanai & Rees, 2011; Wilmer, 2008).

Correlational methods can be successfully used to analyse the mechanisms that underlie traits of interest. A classical example in vision was the identification of the genetic polymorphisms that underlie colour vision deficiency and that also contribute to the normal variation in Rayleigh matches (Nathans, Piantanida, Eddy, Shows, & Hogness, 1986; Winderickx et al., 1992). More recently, genome-wide association has been applied to variation in visual performance in the PERGENIC cohort, whose data are the basis of the present paper: Correlations have been found between genetic polymorphisms and hetereochromatic flicker photometric settings (Lawrance-Owen et al., 2014), phorias (Bosten et al., 2014), face detection (Verhallen et al., 2014) and sensitivity to 'frequency-doubled' gratings (Goodbourn et al., 2014). In another fruitful use of the correlational method, relationships have been discovered between the size of cortical structures and visual performance on a range of tasks, including visual acuity (Duncan & Boynton, 2003), orientation sensitivity (Song, Schwarzkopf, & Rees, 2013), susceptibility to geometric illusions (Schwarzkopf, Song, & Rees, 2011) and rate of perceptual rivalry (Kanai, Bahrami, & Rees, 2010).

A celebrated approach to the analysis of correlations is factor analysis (Mulaik, 2009; Spearman, 1904a, 1927; Thurstone, 1931). Factor analysis aims to discover whether a smaller set of underlying unobserved variables – known as *factors* – are responsible for the intercorrelations between a set of observed variables. The meanings of any factors revealed by factor analysis are open to interpretation, but in psychology they have often been thought to relate to the psychological processes that determine the variation in the observed data.

Factor analysis has been comprehensively applied in the field of cognition, to address the question of whether cognitive ability is determined by a set of independent factors or whether it is determined by a single underlying factor, Spearman's g (Mackintosh, 2011; Spearman, 1904a; Thurstone, 1944). For vision, an equivalent question is whether the observed variation in performance on a battery of visual tasks is determined for each task separately, or whether it is determined by a single underlying factor or small set of factors,

Several studies have applied factor analysis to perception. An early example was the analysis of 22 auditory tests by Karlin (1942). Particularly celebrated is the study of Thurstone (1944), who administered 40 tests to 194 subjects. Most of the tests were visual, including some measures of "low-level" visual processes such as dark adaptation, peripheral span and flicker fusion, and many tests of "high-level" processes such as Necker cube rivalry, various geometric illusions, Gestalt figure completion, colour-form memory, block design and the Gottschaldt figures. Also included were some non-visual tests, including reaction time to an auditory tone, social judgement and a test of social influence. Thurstone cautioned that his study was exploratory and required confirmation by future studies, but he found 11 factors, the first seven of which he interpreted as perceptual closure, susceptibility to geometric illusions, reaction time, perceptual alternation, ability to manipulate two alternative mental processes, perceptual speed and general intelligence.

Since Thurstone's study, researchers have applied factor analysis to a range of visual tasks, but they generally have been concerned with particular aspects of visual perception. A good example is provided by Peterzell and Teller's studies of spatial, temporal and chromatic contrast sensitivity (Dobkins, Gunther, & Peterzell, 2000; Peterzell & Teller, 1996; Peterzell & Teller, 2000; Peterzell, 2016), which have supported the idea that sets of distinct visual factors underlie contrast sensitivity functions. In the domain of colour, factor analysis has been used to investigate

sensitivity as a function of wavelength (Jones, 1948; Jones & Jones, 1950), to test the independence of the cardinal colour mechanisms (Gunther & Dobkins, 2003), to explore the sources of individual variation in colour matching (MacLeod & Webster, 1983), to investigate wavelength discrimination (Diener, 1986; Pickford, 1962), and to explore sources of variation in tests for colour vision deficiency (Aspinall, 1974). Other authors have examined measures of perceptual closure (Beard, 1965; Keehn, 1956; Mooney, 1954; Thurstone, 1950; Wasserstein, Barr, Zappulla, & Rock, 2004).

A smaller number of studies have followed Thurstone (Thurstone, 1950; Thurstone, 1944) in using factor analysis (or the related method of principal components analysis; PCA) to study the factors underlying variation in a large range of visual abilities. For a group of 20 participants, Halpern et al. (1999) made measurements of orientation discrimination, wavelength discrimination, contrast sensitivity, vernier acuity, motion direction discrimination, velocity discrimination and identification of complex forms. They observed many significant intercorrelations between the tests, and concluded, using PCA, that a single factor (accounting for 30% of the total variance) predicts a portion of the variance on each test apart from discrimination of motion direction.

For a group of 40 participants, Cappe et al. (2014) applied PCA to measurements of visual acuity, vernier acuity, backward masking, contrast sensitivity and bisection discrimination. They emphasised the low correlations between pairs of tasks, with only four significant correlations, for which shared variance ranged between 10 and 30% (Test-retest reliabilities for individual tasks were not reported, however). Using PCA, they found that one factor explained 34% of the total variance. However, they applied a different criterion to that of Halpern et al. (1999) in deciding how much variance a common factor must explain, concluding that 34% shared variance was *not* evidence for a single factor underlying the intercorrelations between visual tests.

In a study of 101 normal participants, Ward et al. (2016, published in the current special issue of *Vision Research*) obtained data for seven visual tasks: detection of gabors, contrast sensitivity, detection of Glass patterns, detection of coherent motion, visual search, detection of curvature and judgement of temporal order. They applied a factor analysis to test the hypothesis that there are two visual factors that reflect the activity of the parvocellular and magnocellular systems. They found two components, which accounted for 19% and 18% of the total variance. Tasks involving high spatial frequencies generally loaded on the first component, and tasks involving low spatial frequencies on the second. The authors concluded that this was compatible with a magnocellular–parvocellular distinction.

The present study continues the tradition of Thurstone (1944) and those who have followed, in applying factor analysis to a range of visual tasks to explore the underlying causes of individual variation in visual ability. We do this on a much larger sample (n = 1060) than has been used previously, and we include 25 visual, oculomotor and auditory measures. Our primary analysis is an exploratory factor analysis; and we demonstrate the reliability of the analysis by showing that very similar factors emerge if the total cohort is randomly divided into two subsets of participants. We also show that a comparable structure is recovered when the data are entered into a hierarchical cluster analysis. In a further analysis, we correlate factor scores with additional measures gathered from questionnaires (e.g. personality and Autism Quotient), with subjective (non-performance) measures of visual function, and with demographic and anatomical measures (e.g. sex, iris colour and digit ratio).

### Methods

#### **Participants**

1060 participants (647 female) took part in the PERGENIC study (e.g. Goodbourn et al., 2012; Lawrance-Owen et al., 2013). Their ages ranged from 16-40 (mean 22.1; s.d. 4.1). They were recruited from the Cambridge area, and many were students at the University of Cambridge. Participants were paid £25 for taking part. A subset of 105 participants were selected at random to return for a second testing session on a different day, an average of 26.4 days (s.d. 23.3 days) after their first, allowing us to measure test-retest reliabilities. All participants in our sample were of self-reported European origin.

The study was approved by the Cambridge Psychology Research Ethics Committee and was carried out in accordance with the tenets of the Declaration of Helsinki. All participants gave written informed consent before taking part.

#### Questionnaires

All participants completed a 75-item online questionnaire before they attended the lab for testing. The questionnaire included items to gather demographic information (including age, sex, educational level and ancestry) and the mini International Personality Item Pool-Five-Factor Model (mini-IPIP) to measure the 'Big 5' personality traits of extraversion, imagination, agreeableness, conscientiousness and neuroticism (Donnellan, Oswalk, Baird, & Lucas, 2006). We included various items intended to assess visual and auditory ability, physical ability, memory and handedness – abilities that we considered may impact performance on visual and auditory psychophysical tests. We included three items intended to measure synesthetic experience. In our analysis, data from the three items were collapsed into one total *synaesthesia score*.

The items that will be discussed in this paper are listed in Table 1. For each item, participants were required to rate their agreement with a statement on a Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). Additional items were included to record the extent of a participant's computer and video game use, and amount of musical experience.

A subset of 555 participants completed a second online questionnaire about 6 months after completing the psychophysical tests. The second questionnaire included a set of 50 items to measure the Autism Quotient (Baron-Cohen, Wheelwright, Skinner, Martin, & Clubley, 2001), items to measure history of smoking and alcohol use, and items to measure GCSE score. The latter is a number that quantifies performance in nationally moderated exams that UK school children sit at age 16. Our motivation for including the GCSE measure is that it has been found to correlate highly with general intelligence (Deary, Strand, Smith, & Fernandes, 2007).

I often forget things that have happened in the past that others remember

I have a photographic memory

I can remember the exact shade of a colour several days after seeing it

I find it difficult to hear and produce sounds in foreign languages that are different from my native language

I am good at dancing

I am generally good at sport

I am good at ball sports I always use my right hand when writing I always throw a ball with my right hand I make decisions quickly and easily In my mind's eye, particular numbers or letters or words always have a particular colour When I think of the days of the week or the months of the year, I see them laid out in a pattern in space When I hear, smell, taste or touch something, it evokes an impression in a different sense (e.g. musical notes evoke specific colours) I have a natural talent for music I have *absolute* or *perfect* pitch

# Table 1 list of questionnaire items considered in this paper

#### Psychophysical and optometric tests, and anatomical measurements

Participants attended the lab for optometric and psychophysical assessment lasting approximately 2.5 hours. The sessions were run as a steeplechase in 6 different rooms, with each participant following the next at 40-minute intervals. There was a break of about 20 minutes in the middle of the session. Tests in the first testing room were those requiring administration by an experimenter. In subsequent rooms participants were tested automatically with instructions appearing on the computer screens, but an experimenter was always on call if the participant had questions or if the participant failed to perform adequately on initial practice trials. A list showing the order of tests in the battery is provided in Table 2. For many tests, methods have already been published elsewhere, and in these cases the citation is given in the table. Methods for tests that have not already been published are included in the present method section.

Participants were corrected to best optical acuity at the beginning of the testing session, and 234 participants were given lenses to wear because their acuity was improved by at least 0.1 LogMAR with the addition of the correction. Visual acuity was recorded before and after the additional correction was given.

Room 1 Visual acuity (corrected* and with usual correction only) Pelli-Robson contrast sensitivity* Ishihara Plates (5 plates) OSCAR test TNO stereo acuity* Horizontal and vertical near and far phorias Pupil size	Pelli, Robson and Wilkins, (1988); Bosten et al. (2015) Lawrence-Owen et al. (2014) Lawrence-Owen et al. (2014) Bosten et al. (2015) Bosten et al. (2014) Bosten et al. (2015)
Iris colour Digit ratio MPOD macular pigment density MPOD critical flicker fusion*	Lawrence-Owen et al. (2013)
Room 2 Coherent motion* Ambiguous motion Coherent form (Glass patterns* and sine wave*) Contrast sensitivity for 'frequency-doubled' gratings*	Goodbourn et al. (2012) Bosten et al. (2015) Goodbourn et al. (2012)
Room 3 Contrast sensitivity on pulsed* and steady* pedestals Sensitivity to S-cone increments and decrements* Simultaneous lightness contrast Simultaneous colour contrast Rivalry for ambiguous figures: Necker cube Rivalry for ambiguous figures: Duck-rabbit	Goodbourn et al. (2012); Bosten et al. (2015) Goodbourn et al. (2012); Bosten et al. (2014)
Room 4 Binocular rivalry Dichoptically masked* and unmasked* contrast sensitivity for gratings of 3 c.p.d.	Bosten et al. (2015) Bosten et al. (2015)

Crossed and uncrossed stereo acuity* Vernier acuity*	Bosten et al. (2015)
Room 5	
Main sequence*	Bargary et al. (n.d.)
Pro-saccadic latency*	Bargary et al. (n.d.)
Latency variability*	Bargary et al. (n.d.)
Express saccades*	Bargary et al. (n.d.)
Anti-saccade error rate*	Bargary et al. (n.d.)
Anti-saccade latency*	Bargary et al. (n.d.)
Smooth pursuit RMSE*	Bargary et al. (n.d.)
Room 6	
Auditory frequency sensitivity*	Bosten et al. (2015)
Auditory duration sensitivity*	Bosten et al. (2015)
Sensitivity to auditory temporal order*	Goodbourn et al. (2012)
Follow up tests online	
Glasgow face matching test	Verhallen et al. (2017)
Cambridge face memory test	Verhallen et al. (2017)
Composite face test	Verhallen et al. (2017)
Mooney face test	Verhallen et al. (2017)

Table 2. Psychophysical tests in the order that they were run. If methods have been published previously, the citation is provided. If methods have not been previously published, details can be found in the present Methods section. Asterisks indicate the measures that were included in our primary analyses.

### Iris colour

To measure objectively the lightnesses, chromaticities and chromatic variances of participants' irises we imaged participants' eyes using a digital camera under controlled lighting conditions. We used a Canon EOS 1000D digital camera fitted with a Tamron A15 telephoto lens and a Sigma EF-530 DG ST flash, positioned 1.2 m from participants' eyes. The aperture value was F5, the exposure time was 5 ms, the ISO speed was 100, and the focal length was 171 mm. Participants viewed a CRT monitor displaying a blank grey field of luminance 35 cd.m<sup>-2</sup> and chromaticity CIE x=0.29, y = 0.32. Images of the eyes were captured through a rectangular aperture in a piece of white card perpendicular to the camera's line of view. Displayed on the white card was an array of Kodak colour patches, which allowed us to colour calibrate the images using measurements of the same patches made by a SpectraScan PR650 spectroradiometer. The average colour, average lightness and average variance of each participant's iris were extracted using the (linear) raw images, colour calibrated by accounting for the camera's spectral sensitivity functions (which we measured using a set of 31 interference filters peaking at 10 nm intervals between 400 and 700 nm), and the RGB values of the Kodak colour patches in each image (See Lawrance-Owen, 2012 for more detailed methods).

Macular pigment density and peripheral and central critical flicker frequencies Macular pigment density was estimated using the MPOD (Tinsley Precision Instruments Ltd., Braintree, UK). The MPOD measures the point of minimum flicker sensitivity as a function of ratio of blue (460 nm) to green (540 nm) light (Murray & Carden, 2008; Van Der Veen et al., 2009). It makes separate estimates for central and peripheral viewing, and macular pigment density is estimated using the difference between the two minima. For extracting minima, we replaced the MPODs default fits with our own custom fits to the raw data. As part of the MPODs pre-testing routine measures of central and peripheral CFF (critical flicker fusion frequency) are made, on the basis of the mean of five settings (Van Der Veen et al., 2009).

## Simultaneous contrast (colour and lightness)

Colour. On each trial, a grey reference disc of diameter 1° embedded in a coloured annular surround of 4° was presented on one side of a central  $(0.1^{\circ} \times 0.1^{\circ})$  grey fixation cross, at an eccentricity of 2.5°. The disc was metameric with equal energy white, and had a luminance of 28 cd.m<sup>-2</sup>. The surround was isoluminant with the reference disc and had a chromaticity of L/(L+M) = 0.629 and S/(L+M) = 0.016 in the MacLeod–Boynton (1979) chromaticity diagram. Between the reference disc and the surround was a black line of thickness 0.05°. On the other side of the fixation cross, also at an eccentricity of 2.5°, was presented a test disc of diameter 1°, isoluminant with the reference stimulus and with a chromaticity that varied according to the participant's responses. The surround to reference and test stimuli was dark.

The side of presentation of the test and reference stimuli was decided at random on each trial. The participant's task was to identify which of the two small discs appeared redder. The S/(L+M) value of the test disc was 0.016, and the L/(L+M) value was decided on each trial by two randomly interleaved ZEST staircases (King-Smith et al. 1994; Watson & Pelli, 1983) with starting L/(L+M) values of 0.641 and 0.689. There were 72 trials, 36 for each staircase.

Luminance. The spatial characteristics of the stimuli for simultaneous luminance contrast were the same as those for simultaneous colour contrast. The luminance of the reference disc was 69.2 cd.m<sup>-2</sup>, and the luminance of the surround was 138.4 cd.m<sup>-2</sup>. The luminance of the test disc was decided on each trial by two randomly interleaved ZEST staircases with starting luminances of 89.1 cd.m<sup>-2</sup> and 49.3 cd.m<sup>-2</sup>. Test and reference stimuli were themselves presented on a matrix of random luminance noise, with square  $0.06^{\circ} \times 0.06^{\circ}$  pixels. The luminance noise was binary at 13.8 cd.m<sup>-2</sup> and 124.6 cd.m<sup>-2</sup>. For simultaneous luminance contrast the procedure was analogous to that for simultaneous colour contrast, but the participant had to choose on each trial the lighter disc.

For both simultaneous colour contrast and simultaneous luminance contrast, a participant's point of subjective equality was defined as the 50% point on the psychometric function, where the test and reference discs were equally likely to be judged as redder (or lighter).

## Rivalry for ambiguous figures

We measured the rate of percept alternation for two ambiguous figures: the Necker cube (Necker, 1832) and a version of the duck-rabbit ambiguous figure (Kihlstrom, 2012). The Necker cube was white, presented on a black background. The two square faces of the Necker cube were approximately  $2^{\circ} \times 2^{\circ}$ . The duck-rabbit stimulus was a greyscale image of  $6^{\circ}$  (horizontal)  $\times 5^{\circ}$  (vertical).

For the Necker cube, participants were instructed to maintain fixation on a central  $(0.1^{\circ} \times 0.1^{\circ})$  white cross, and for the duck-rabbit stimulus, they were instructed to maintain fixation on the eye. Participants were instructed to press a button whenever their percept changed, either from duck to rabbit in the case of the duck-rabbit figure, or from one conformation to the other for the Necker cube. Responses were gathered over a period of 2 minutes for each stimulus using a CT3 response box (Cambridge Research Systems, Rochester, UK).

For the duck-rabbit stimulus, pilot testing revealed that a minority of participants did not spontaneously see both interpretations of the stimulus. We therefore presented participants with the stimulus before the testing period started, and instructed them to alert the experimenter if they could not see both animals.

#### Vernier acuity

The stimuli for measuring vernier acuity were three white anti-aliased vertical lines (1  $\times$  8 minutes of arc), offset vertically by 4 minutes of arc. The central line was offset horizontally from the upper and lower lines by a variable distance decided according to a staircase procedure. The luminance of the lines was 100 cd.m<sup>-2</sup>, and that of the 2.5°  $\times$  1.8° background was 11 cd.m<sup>-2</sup>. The stimuli were presented on a Trinitron Multiscan17seII CRT monitor (Sony, Tokyo, Japan), and the image was reflected in a mirror to give a total viewing distance of 7.3 m. Responses were gathered using a CT3 response box.

On each trial the stimuli were displayed for 150 ms, followed by the background alone until participants made a response. Participants were instructed to judge the direction of displacement of the central line relative to the upper and lower lines, and to press a button accordingly. Thresholds were measured using four randomly interleaved 3-up 1-down staircases. The starting position for two of the staircases placed the central line 100 seconds of arc to the left of the upper and lower lines (and left was defined as a correct response). The starting position for the other two staircases placed it 100 seconds of arc to the right of the upper and lower lines (and right was defined as a correct response). The initial step size for each staircase was 40 seconds of arc. This reduced to 20 seconds of arc following 4 reversals, and then to 10 seconds after 8 reversals. Each staircase terminated following 10 reversals.

Psychometric functions were fitted to the trial-by-trial data from each pair of staircases using the local linear fit functions from the Matlab toolbox *Modelfree* (Zychaluk & Foster, 2009). The 81% points on each of the two psychometric functions ( $P_{L(81)}$  and  $P_{R(81)}$  from the left-starting and right-starting staircases, respectively) were extracted. Bias was defined as

$$b = \frac{P_{L(81)} + P_{R(81)}}{2} ,$$

yielding a measure of lateral bias in response independent of threshold. Threshold was defined as

$$T = P_{R(81)} - b$$

yielding a measure of vernier threshold independent of bias.

The data from one participant was excluded because a poor fit to the psychometric functions caused *T* to be negative.

#### Online tests of face perception

About two years after participants had been tested psychophysically in the laboratory we administered four online tests of face perception. These tests, completed by 397 participants, are described in Verhallen et al. (2017).

## Analysis and results

We selected for primary analysis a set of measures that satisfied the following criteria;

- (i) They provided continuous (rather than categorical) data,
- (ii) They were performance measures rather than phenomenological measures. Thus measures such as magnitude of simultaneous colour contrast and rate of binocular rivalry were not included since they are not performance measures.
- (iii) They were part of the primary PERGENIC test battery and were therefore completed by all participants. The online tests of face perception were not included because data were available only for 397 participants.

24 measures met these criteria and are listed in Table 3, with summary statistics and Spearman's rho test-retest reliabilities. We added one further measure: CFF (flicker fusion frequency). This is not strictly a performance measure (the participant is asked to report the moment when flicker becomes apparent), but is a traditional and fundamental measure of the temporal resolution of the visual system.

		Units	Mean	Median	s.d.	Min	Max	n	6
1.	Main sequence		114.5	114.1	13.6	69.9	164.2	1040	.86
2.	Pro saccadic latency	ms	177.1	174.0	18.5	142.0	322.0	1040	.83
3.	Latency variability	$(\times 10^{-4})$	10	9.9	2.4	5.2	21	1040	.78
4.	Express saccades	$(\times 10^{-2})$	4.4	2.6	5.7	0	42	1040	.70
5.	Antisaccadic error rate		0.38	0.35	0.22	0	1	1040	.82
6.	Antisaccadic latency	ms	305.5	301	43.1	113	539	1040	.73
7.	Smooth pursuit RMSE		3.10	2.50	1.83	0.87	13.5	1040	.79
8.	Contrast sensitivity (3 cpd)	contrast <sup>-1</sup>	33.3	32.7	13.0	6.4	72.2	1000	.73
9.	Coherent form (Glass patterns)	coherence <sup>-1</sup>	6.0	6.3	1.3	0.3	8.7	1057	.56
10.	Coherent motion	coherence <sup>-1</sup>	17.7	17.4	7.6	0.2	51.0	1055	.62
11.	Corrected acuity	logMAR	-0.14	-0.14	0.075	-0.3	0.1	1059	.69
12.	Coherent form (sine wave)	coherence <sup>-1</sup>	2.4	2.5	0.5	0.3	4.1	1057	.53
13.	Duration discrimination	$(\Delta \text{ duration})^{-1}$	6.2	6.0	2.4	1.3	17.4	1049	.70
14.	Frequency discrimination	$(\Delta \text{ frequency})^{-1}$	100.1	94.8	54.0	1.1	379.6	1052	.56
15.	Frequency doubling	contrast <sup>-1</sup>	34.2	33.8	10.2	2.5	74.7	1057	.73
16.	Order discrimination	$s^{-1}$	4.8	4.6	1.9	0.7	17.1	1049	.77
17.	Pelli–Robson	contrast <sup>-1</sup>	43.7	44.7	1.26	19.1	89.1	1057	.51
18.	Binocular masking	contrast <sup>-1</sup>	17.0	13.9	11.5	1.2	63.4	1050	.80
19.	Pulsed pedestals	contrast <sup>-1</sup>	19.7	18.7	6.45	2.4	71.5	1059	.58
20.	Steady pedestals	contrast <sup>-1</sup>	112.0	110.6	31.1	16.2	314.2	1059	.52
21.	TNO	arc seconds	112.0	60.0	124.9	15.0	480.0	1059	.57
22.	Vernier acuity	arc seconds	43.9	37.5	24.9	5.8	193.4	1059	.63
23.	Sensitivity to s-cone stimuli	contrast <sup>-1</sup>	26.3	25.8	5.3	9.8	46.6	1058	.73
24.	Stereo acuity	arc seconds	125.1	87.9	94.3	0.92	350.0	1060	.78
25.	CFF	Hz	40.0	40.1	3.0	27.2	50	1046	.58

Table 3. Summary statistics and test-retest reliabilities (Q) for the variables included in our primary analysis. Note that n varies between the 25 measures between 1000 and 1060. The reasons for this variation are several: in some cases data were missing owing to equipment failure, in others, for example, data were excluded if thresholds could not be extracted for particular participants. The individual reasons for exclusions and for missing data are described in published methods (Table 2).

## Intercorrelations

Table 4 is a matrix showing the Spearman intercorrelations between our 25 primary measures. Of 300 pairs of measures, 159 (53%) are significantly correlated (P < .05, following a Bonferroni correction for 300 tests), though mostly with only modest effect sizes. The mean correlation coefficient over the whole matrix is .175, the standard deviation .20, and the range -.47 to .54. For all measures, performance was ordered from worst to best, meaning the directions of some of the variables listed in Table 3 were reversed.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

2	.18																							
3	.07	.14																						
4	07	47	.53																					
5	.02	17	.23	.33																				
6	.08	.33	.18	12	.08																			
7	08	.26	.27	06	.34	.14																		
8	.00	07	19	12	30	15	29																	
9	07	07	13	08	22	.10	19	27																
10	-02	.06	18	.00	28	.08	28	25	23															
11	- 01	.00	05	- 01	10	07	08	16	07	10														
12	01	03	16	13	26	.07	21	31	54	29	14													
13	- 01	.05	10	.15	20	.00	.21	17	27	.2)	.17	25												
14	.01	.07	.19	.05	.29	_ 01	.27	.17	.27	.21	.00	.23	34											
14	.00	.07	.09	- 02	.10	.01	.17	20	.11	20	.05	.10	12	11										
16	.00	.14	.04	.05	.14	.09	.17	.20	.10	.20	.00	.23	.12	.11	07									
17	.02	.00	.15	.07	.23	.04	.20	.10	.14	.1/	.04	.21	.34	.30	.07	05								
1/	.02	02	.04	.04	.07	01	.04	.15	.09	.10	.25	.10	.08	.05	.11	.05	11							
18	.04	.07	.20	.14	.29	.11	.31	.53	.32	.29	.17	.37	.24	.14	.21	.20	.11	•						
19	.01	.09	.10	.01	.20	.16	.22	.25	.28	.19	.07	.22	.23	.09	.18	.13	.16	.26						
20	01	.10	.10	.03	.22	.05	.24	.26	.22	.18	.08	.24	.19	.16	.39	.20	.19	.24	.31					
21	.01	.05	.06	.01	.04	.04	.14	.05	.04	.14	.16	.11	.05	.05	.07	01	.10	.09	.00	.04				
22	.07	.21	.17	.02	.27	.16	.34	.36	.37	.24	.19	.32	.27	.15	.27	.22	.15	.36	.25	.24	.09			
23	03	.09	.12	.05	.18	.04	.20	.19	.26	.20	.08	.24	.21	.16	.28	.17	.13	.25	.23	.29	.07	.23		
24	01	.08	.14	.05	.17	.06	.16	.16	.20	.19	.10	.23	.19	.13	.11	.10	.02	.25	.09	.09	.32	.20	.14	
25	.04	.08	03	05	.13	.08	.13	.08	.07	.06	.07	.11	.05	.08	.27	.06	.06	.13	.07	.15	.05	.10	.06	.00

Table 4 Spearman's correlations between the 25 performance tests. Significant correlations (following a Bonferroni correction for 300 tests) are indicated in bold. The 25 measures are numbered 1–25, by the same convention as in Table 3.

#### Factor analysis

The 25 primary measures were normalized (as far as possible) using an inverse rank normal transform before they were entered into a factor analysis. Inverse rank normal transformation normalizes the distribution of ranks, so that—except in cases of tied ranks—it achieves perfectly normal distributions, while preserving the ranks of the data. Of the 25 measures, only the Pelli–Robson test and the TNO test had large numbers of tied ranks.

The factor analysis was performed using SPSS version 22 (IBM, Armonk, USA), with principal components analysis as the method of extraction, and varimax as the method of rotation. Varimax rotation was chosen to favour a Thurstonian simple structure, maximising the number of zero or near-zero factor loadings (Browne, 2001). Since the rotation is orthogonal rather than oblique, it assumes that the factors do not intercorrelate. Of the  $1060 \times 25$  matrix that was the input to the analysis, data were missing in 279 cells, and these were eliminated pairwise. For all variables, performance was ordered in the same direction, from worst to best. For the oculomotor measures, we considered fast speed, low latency variability, and low numbers of express saccades to be 'good'.

For deciding the number of factors to retain, we applied the criterion that eigenvalues should be greater than 1. This criterion is widely used but has been criticized (Courtney, 2013). One alternative strategy is to inspect a 'scree plot' – a plot of eigenvalue against factor number – for a sudden change in the gradient. Figure 1 shows a scree plot for our data. The clearest "elbow" in the plot is at 3 components, suggesting three factors, but our results (below) do show interpretable and consistent factors beyond 3. In a third strategy, some researchers have chosen to retain factors so long as they are interpretable (e.g. Dobkins et al., 2000; MacLeod & Webster, 1983). When we applied this strategy to our own results, we found that additional factors with eigenvalues <1 loaded strongly only on single variables and therefore did not contribute usefully to the results. We therefore retained only factors with eigenvalues >1, which, independently, we do consider interpretable.

An eight-factor solution accounted for 57.4% of the variance. Eigenvalues and percentage of variance explained for each factor in both rotated and unrotated solutions are provided in Table 5, and factor loadings for the rotated solution are presented in Table 6.

- Factor 1 receives strong loadings (> .5) from contrast sensitivity at 3 cpd, • coherent form (Glass patterns), coherent form (sine wave), binocularly masked contrast sensitivity (also at 3 cpd), and vernier acuity. All of these tests require sensitivity to medium or high spatial frequencies.
- Factor 2 receives strong loadings from the three auditory tasks: duration • discrimination, frequency discrimination and order discrimination. This factor appears to correspond to auditory perceptual ability.
- Factor 3 receives strong loadings from pro-saccadic latency and anti-saccadic • latency. It appears to relate to oculomotor speed.
- Factor 4 receives strong loadings from express saccades and latency • variability. This appears to be a factor for oculomotor control.
- Factor 5 receives strong loadings from sensitivity to "frequency-doubled" gratings, sensitivity to pulsed and steady pedestals and sensitivity to s-cone stimuli. This appears to be a factor relating to contrast sensitivity at low spatial frequencies.
- Factor 6 receives strong loadings from the TNO test and our custom test of stereo acuity---it relates to stereo acuity.
- Factor 7 receives strong loadings from corrected visual acuity and Pelli-Robson contrast sensitivity. This factor is difficult to interpret. We considered the idea that it could be a visual acuity factor, but though the letters of the Pelli–Robson chart have sharp edges, the dominant spatial frequencies are lower than those that are required for measurements of visual acuity. Since both tests have in common letters, we provisionally call this factor sensitivity for letter recognition.
- Factor 8 receives loadings from CFF and, to a lesser extent, from sensitivity to frequency-doubled gratings. This appears to be a factor for flicker sensitivity.



Unrotated solution Varimax-rotated solution

Component	Eigenvalue	% Variance	Eigenvalue	% Variance
1	4.99	19.9	2.97	11.89
2	1.95	7.8	1.96	7.86
3	1.46	5.8	1.84	7.35
4	1.40	5.6	1.82	7.29
5	1.3	5.2	1.74	6.98
6	1.18	4.7	1.44	5.74
7	1.06	4.2	1.32	5.28
8	1.01	4.0	1.24	4.97

Table 5 Eigenvalues for and percentage variance explained by each factor, before and after rotation.

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Main sequence	.150	.027	.349	012	366	178	.075	.207
Pro-saccadic latency	039	.112	.817	293	.105	.099	.016	012
Latency variability	.056	.096	.362	.779	.092	.104	.001	140
Express saccades	.078	.015	350	.848	.013	.001	010	039
Anti-saccadic error rate	.378	.306	113	.466	.002	.004	.046	.291
Anti-saccadic latency	.129	089	.669	.105	.045	.011	025	.013
Smooth pursuit RMSE	.258	.330	.389	.240	.109	.138	.015	.231
Contrast sensitivity (3cpd)	.595	.076	.149	.212	.048	053	.257	.143
Coherent form (Glass patterns)	.748	.089	003	060	.166	.037	078	106
Coherent motion	.380	.168	.060	.127	.127	.306	.032	.051
Corrected acuity	.123	.032	.073	030	065	.209	.721	.038
Coherent form (sine wave)	.721	.074	064	007	.162	.149	031	.024
Duration discrimination	.282	.645	.043	.077	.157	.077	024	081
Frequency discrimination	.001	.766	039	.023	.081	.081	.052	.081
Frequency doubling	.169	020	.073	027	.601	.121	.016	.453
Order discrimination	.154	.743	.049	.046	.055	045	.026	.033
Pelli Robson	.063	.021	073	.032	.236	049	.770	026
Binocular masking	.630	.105	.102	.186	.087	.092	.148	.168
Pulsed pedestals	.372	.082	.182	.029	.490	188	.089	076
Steady pedestals	.192	.166	.070	.072	.639	094	.154	.230
TNO	020	019	.038	.016	011	.776	.187	.095
Vernier acuity	.540	.197	.281	.001	.145	.071	.165	.058
Sensitivity to s-cone stimuli	.205	.171	.026	.035	.601	.120	.037	039
Stereo acuity	.275	.103	.035	.034	.045	.703	044	072
CFF	.046	.048	.045	062	.090	.018	.003	.824

 Table 6. Results of factor analysis on combined data from 1060 participants. Factor

 loadings larger than .25 are highlighted in bold.

To test the reliability of the results of the factor analysis, we ran the same analysis but separately on two, randomly divided and independent, halves of the data, each containing results from 530 participants. We label the two subsets of participants Group A and Group B. Both analyses generated eight-factor solutions, and the factor loadings from both are provided in Table 7.

Factor	1	1	2	2	3	4	4	3	5	5	6	6	7	7	8	8
Group	Α	B	Α	B	Α	B	Α	B	Α	В	Α	B	Α	В	Α	B
Main sequence	.13	.09	.00	.08	02	02	.13	.19	.02	.07	12	16	.08	01	.74	55
Pro-saccadic latency	01	.02	.07	.19	37	22	.69	.84	.16	.00	.15	01	.02	.03	.35	.04
Latency variability	.05	.09	.09	.14	.68	.86	.41	.28	.05	12	.12	.03	01	.07	.21	.05
Express saccades	.06	.06	.02	.01	.88	.82	24	42	05	02	03	.02	01	.02	08	.02
Anti-saccadic error rate	.30	.38	.30	.29	.51	.41	.00	21	.10	.32	.04	.09	.02	01	18	25
Anti-saccadic latency	.12	.13	03	15	.08	.13	.74	.71	08	.14	02	.10	07	.02	09	15
Smooth pursuit RMSE	.20	.31	.26	.41	.32	.13	.37	.32	.28	.23	.20	.13	.06	08	.09	19
Contrast sensitivity (3cpd)	.57	.61	.00	.14	.31	.05	.23	.00	.18	.01	02	04	.23	.25	01	28
Coherent form (Glass patterns)	.78	.73	.14	.04	03	.01	02	.02	01	01	01	.09	02	06	.02	.14
Coherent motion	.37	.41	.16	.20	.07	.15	.11	01	.11	.13	.23	.38	.11	07	01	.00
Corrected acuity	.11	.08	03	.08	02	01	.03	.08	.01	01	.33	.15	.63	.74	.09	05
Coherent form (sine wave)	.77	.67	.09	.05	.02	.07	11	03	.16	.06	.13	.17	.02	04	.03	.12
Duration discrimination	.30	.27	.66	.63	.07	.10	.16	.03	.00	05	.07	.08	04	.04	22	.10
Frequency discrimination	.00	.02	.76	.76	.06	.01	08	04	.16	.04	.03	.12	.09	.03	.07	.03
Frequency-doubled gratings	.21	.26	01	.05	.00	02	.07	.11	.75	.61	.07	.04	.11	.00	09	.39
Order discrimination	.18	.13	.74	.74	.08	.04	.04	.04	.02	.07	06	02	.00	.05	.02	04
Pelli Robson	.10	.08	.08	.00	.02	.08	06	05	.06	.15	06	06	.83	.74	04	.15

Binocular masking	.62	.62	.11	.09	.29	.04	.17	.00	.21	.07	.10	.12	.06	.29	04	16
Pulsed pedestals	.33	.55	.09	.12	04	.06	.36	.15	.17	.15	18	25	.23	03	44	.11
Steady pedestals	.18	.40	.26	.17	.08	.04	.14	.07	.54	.45	06	25	.24	.10	28	.25
ΓΝΟ	02	06	09	.06	.04	02	.05	.04	.11	.09	.78	.77	.17	.14	09	02
Vernier acuity	.53	.56	.16	.23	.05	.01	.31	.24	.17	.02	.10	.04	.18	.13	.06	.07
Sensitivity to s-cone stimuli	.30	.30	.22	.21	.10	.00	.12	.08	.32	.14	.11	07	.09	.12	25	.56
Stereo acuity	.31	.28	.12	.12	.02	.05	.01	.07	04	11	.72	.62	05	02	.00	.19
CFF	.08	05	.07	.01	.00	09	10	.06	.72	.76	.03	.07	13	.12	.18	17

# Table 7. Results of two independent factor analyses on two random halves of the data (Groups A and B; n = 530 for each). Factor loadings greater than .25 are highlighted in bold.

For both groups a similar factor structure emerges, one which is also very similar to that for the full cohort (Table 6). Factors 3 and 4 appear to be in opposite order for the two groups, and are listed as such in Table 7. The only notable difference between the solutions for the two groups is for factor 8, which appears to run in opposite directions for each of the two groups, and which, apart from main sequence, loads differently on measures of contrast sensitivity for each group. Factor 8 is also different from the factor 8 that emerges from the full cohort: For the two groups factor 8 loads on main sequence, but for the full cohort, it loads on CFF and sensitivity to frequency-doubled gratings.

In summary, the high consistency of the factor solutions for the two randomly selected groups gives confidence that the factor solution is stable.

## Cluster analysis

We performed a hierarchical cluster analysis (by task) on the results of the same tasks that were entered into the factor analysis. Since any missing data exclude participants from the cluster analysis, our sample comprised the 941 participants with complete data. We used SPSS statistics version 22 (IBM, Armonk, USA) for the analysis.

The resulting dendrogram is shown in Figure 2. Its features tell a similar story to the factor analysis, as might be expected. Contrast sensitivity at 3 cpd, binocular masking, vernier acuity, coherent form (Glass patterns) and coherent form (sine wave) cluster: All require sensitivity to medium or high spatial frequencies. Similarly, the three auditory tasks form a cluster, as do the two measures of stereo acuity. Another cluster contains sensitivity to frequency-doubled gratings, sensitivity to steady and pulsed pedestals, and sensitivity to S-cone stimuli, all measures of contrast sensitivity at low spatial frequencies. Somewhat unexpected is how the oculomotor measures cluster. Pro- and anti-saccadic latencies cluster as measures of oculomotor speed, as do express saccades and latency variability as putative measures of oculomotor control. However, smooth pursuit RMSE and anti-saccadic error rate cluster with measures of contrast sensitivity rather than with the other oculomotor measures (on the relationship between smooth pursuit RMSE and anti-saccadic error rate see Zanelli et al., 2005). This was not obvious in the results of the factor analysis, but Table 6 shows that both of these measures load on factors 1 and 2 as well as factor 3 (with other oculomotor measures).



Figure 2. Dendrogram showing the results of a hierarchical cluster analysis by variable.

# Correlates of the performance factors

In a second-stage analysis, we correlated scores on each of the eight factors that emerged from the factor analysis with results on the tasks that did not meet our criteria for inclusion in the factor analysis, and also with the questionnaire items listed in Table 1. Significant correlations are shown in Table 8. We applied a Bonferroni correction for the 456 correlations.

Some items did not correlate even nominally significantly (i.e., P < .05, uncorrected) with any factor, and are not included in Table 8. These were the personality factor 'Intellect/Imagination', autism quotient, BMI, iris colour and lightness, variance of iris colour, performance on the Ishihara test for colour vision deficiency (for methods see Lawrance-Owen et al., 2014), rate of rivalry for the Necker cube, rate of rivalry for the duck-rabbit figure, and vertical near phoria.

Of the correlations that are significant, most have modest effect sizes, and many are not unexpected. Agreement with statements "I have a natural talent for music", "I have *absolute* or *perfect* pitch" and self-reported years spent practicing a musical instrument correlated significantly with factor 2, which relates to auditory ability. Similarly, agreement with the statement "I find it difficult to hear and produce sounds in foreign languages that are different from my native language" correlates negatively with the same factor. Time spent playing computer games correlates with factor 3, which relates to oculomotor speed. Factor 7 (on which visual acuity and Pelli-Robson contrast sensitivity load) correlates with uncorrected visual acuity, usual refraction (strength of lenses normally worn), and total refraction (strength of usual lenses plus any lenses that were added following assessment of visual acuity). Our vales for refraction are signed positive for lenses for hypermetropia and negative for lenses for myopia, though we note that while 537 participants wore lenses (total refraction) for myopia, only 28 wore lenses for hypermetropia.

Factor 6, relating to stereo acuity, correlates with far horizontal phoria and uncorrected visual acuity. These correlations, but with the two measures of stereo acuity individually, were already reported in Bosten et al. (2015).

Factor 8 (flicker sensitivity) correlates inversely with age and positively with average pupil size and macular pigment density. All three of these are not unexpected: CFF is known to reduce with age (e.g. McFarland, Warren, & Karis, 1958), increase with increasing pupil size following application of pharmacological pupil dilators (Lawrance, McEwen, Stonier, & Pidgen, 1982), and increase with macular pigment density (Hammond & Wooten, 2005).

There are two small but significant correlations with personality factors. Factors 1 and 2 correlate inversely with Extraversion, implying that there are inverse relationships between this personality factor and both contrast sensitivity and auditory ability.

There are two significant correlations with sex: factor 1 (contrast sensitivity) and factor 3 (oculomotor speed), in both cases in the direction of better performance by males.

There is a positive correlation between GCSE score (our surrogate measure for g) and factor 2 (auditory ability) with a p-value (0.00016) that is marginally greater than the Bonferroni-corrected alpha (0.00011). Interestingly, there are no significant correlations between GCSE score and any of the visual or oculomotor factors.

There is a surprising significant correlation between factor 2 (auditory ability) and susceptibility to simultaneous luminance contrast. There is a positive correlation between interpupillary distance and factor 3 (oculomotor speed) with a p-value that falls just above the Bonferroni-corrected alpha (p = 0.00013,  $\alpha = 0.00011$ ).

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Big 5 Agreeableness	114							
Big 5 Contentiousness				.069				
Big 5 Extraversion	146	140						.074
Big 5 Neuroticism	095			066				
Age	.065	.076		.092				191
Sex	138		165	.087			098	081
Height	.099		.095	091	071		.108	.101
Weight	.103		.113				.101	
GCSE points		.197			.117			
Preferred eye	102				.067			
I always use my right hand when writing					.084			
I always throw a ball with my right hand	.066							
I am good at dancing				.065				076
I am generally good at sport								.070
I am good at ball sports		067						.071
Years spent practicing a musical								
instrument		.393			.081		.068	

		······					
	.340						
071	.137						
	130						
.099		.192					
							071
		.111					
	073						.097
		.065					
081					110		
085		070					
		.078					
				.120	.088		.111
	085			.122		.186	.081
126					211	526	095
	073			.105		.210	
			095	.115			.192
.066		.125				.093	
		068			.128		
		064			.078		
	096	.065					
	.090					067	.143
171						120	098
081					120	103	
.068			.103	.070	.100		
078	174				096	.069	
	121			.118			
.176		.121					
.144		.146			107		
.141			.130				
			.130				
	071 .099 081 085 126 .066 .066 .066 .068 078 .176 .144 .141	071       .137        071       .137        099	.340        071       .137        130         .099       .192         .099       .192         .001       .111        073       .065        081       .065        085      070         .005       .0078         .006       .125         .066       .125         .066       .125         .066       .125         .066       .065         .090       .065         .090       .065         .090       .171         .081       .068         .078       .121         .176       .121         .176       .121         .144       .146			071      137	071      137

Table 8 Significant correlations between factor scores and results on questionnaire measures, and other visual measures that were not included in the factor analysis. All nominally significant correlations are shown. Those that survive a Bonferroni correction for 456 tests are indicated in bold.

## Discussion

#### Intercorrelation matrix

More than half of the intercorrelations between our 25 performance tests listed in Table 4 are significant. However, they are generally modest in size. Figure 3 compares our observed distribution of Spearman correlation coefficients to a simulated null distribution, created by conducting 10 000 correlations between randomly sampled pairs of tests, but randomising the ranks of participants separately for each member of the pair. The figure shows that the distribution of observed coefficients (solid black line) is shifted rightward by about .15 compared to the distribution expected under the null hypothesis (dotted line).

What can we conclude from the size of correlations between our measures? Unless the test-retest reliabilities are known for individual tests, nothing can be concluded from the *absence* or *weakness* of correlations between tests: an unknown part of the variance may be due to the first two of the three sources of variance that we identified in the Introduction: (i) instrumental or measurement error and (ii) intra-individual variability. Test-retest reliability allows us to estimate these sources of variance, and calculate the maximum expected correlation between two measures, given the noise that we know to be present in them. The maximum expected correlation is given by

$$r_o = r_t \sqrt{r_a r_b} \quad ,$$

where  $r_a$  and  $r_b$  are the reliabilities of the two measures, and  $r_t$  is their true correlation (i.e. the correlation between the "universe scores"—the means of an infinite number of measurements).

The dashed line in Figure 3 shows the distribution of maximum expected correlations given our set of test-retest reliabilities listed in Table 3, in the extreme case that the true correlation between pairs of variables ( $r_t$ ) is in all cases 1. This distribution has a mean of r = .68. It is not plausible, of course, that the 25 measures would be perfectly correlated. With known reliabilities, it is possible to estimate true correlation coefficients:

$$r_t = \frac{r_o}{\sqrt{r_a r_b}}$$

where  $r_o$  is the observed correlation (the solid black line in Figure 3). The estimated distribution of true correlation coefficients between our 25 measures is shown in Figure 3 by the solid grey line. This distribution peaks at r = .32, which is .17 rightward of the mean of the distribution of observed correlation coefficients.

Figure 3 shows that the distribution of observed correlation coefficients (solid black line) is much lower than the distribution of maximum expected correlation coefficients (dashed line): we can conclude that our 25 measures are not perfectly correlated. Instead, the true correlations between them are likely to range between about 0.15 and about 0.6 (solid grey line).



Figure 3. Distribution of correlation coefficients. The distribution of correlation coefficients in Table 4 is indicated by the solid black line. A permuted distribution is shown by the dotted line: This is the distribution of correlation coefficients that would be expected by chance. The dashed line shows the maximum observable distribution of correlation coefficients given the reliabilities of our measures. The solid grey line shows an estimate of the distribution of true correlation coefficients between our 25 measures, accounting for their reliabilities. All distributions are scaled so that the areas under the curves are unity.

Factor analysis

Our factor analysis revealed eight factors that together explained 57.4% of the total variance. Of these, five were visual factors, one was an auditory factor, and two were oculomotor factors. Visual performance (as measured by our battery of tests) segregates into factors that (on the basis of the strong factor loadings) seem to relate to: (i) sensitivity to medium and high spatial frequencies, (ii) contrast sensitivity at low spatial frequencies, (iii) stereo acuity, (iv) letter recognition and (v) flicker sensitivity.

We have interpreted the factors above on the basis of 'strong' factor loadings (> .45), but the factor analysis that we conducted on two random halves of our data reveals that weaker factor loadings are also very consistent. How might these weaker loadings nuance our interpretation of the meanings of the factors?

Factor 1 receives its strongest loadings (> .5) from contrast sensitivity at 3 cpd, coherent form (Glass patterns), coherent form (sine wave), binocular masking and vernier acuity, which all require sensitivity to medium to high spatial frequencies. However, there are smaller but consistent loadings from anti-saccadic error rate (.378), smooth pursuit RMSE (.258), coherent motion (.380), duration discrimination (.282), sensitivity to pulsed pedestals (.372), and stereo acuity (.275). Each of these additional measures loads on factor 1, but also (more strongly) on one of the other factors.

Similarly, factor 2, which we have interpreted as reflecting auditory perceptual ability, exhibits receives small but consistent loadings from antisaccadic error rate (.306), smooth pursuit RMSE (.330), and, to a lesser extent, contrast sensitivity to steady pedestals (.166), and sensitivity to s-cone stimuli (.171). Factor 3, which we have interpreted as related to oculomotor speed, also receives a small loading from vernier acuity (.281).

Though the split-cohort analysis shows that the smaller factor loadings are often consistent, many of them are not intuitively obvious. It is possible that future research may reveal some molecular or neural mechanisms that are shared by perceptual processes that seem quite unrelated.

## A general perceptual factor?

Do we have evidence that, as well as independent perceptual factors, there is also a general factor underlying visual performance? We considered the possibility that factor 1 in our varimax-rotated factor analysis (Table 6) might represent a general performance factor, but this is inconsistent with very low loadings from pro-saccadic latency (-.039), frequency discrimination (.001), Pelli Robson contrast sensitivity (.063), the TNO test (-.020) and CFF (.046).

It is clear, however, from visual inspection of the intercorrelation matrix (Table 4) that there is a 'positive manifold': With the exception of a few of the eye-movement measures, intercorrelations between the 25 measures are almost uniformly positive. Is this an indication of the presence of a general factor for perceptual performance? In factor analysis, the choice of rotating the factorial solution, or not, can produce results that call for different interpretations. In the intelligence literature, this has been fuel for the long-running debate of how far intelligence can be summarised by a single factor g, or whether it is better described as a collection of different (but partially

correlated) abilities (see chapter 6 in Mackintosh, 2011). Not rotating the factorial solution produces a first factor that accounts for the maximum amount of correlated variance, and therefore tends to produce a factor that loads on most of the input variables. Orthogonally rotating the solution best identifies a set of independent factors that describe the pattern of correlations—it is this strategy we have pursued here.

If, however, we look at the unrotated results of the principal components analysis for our 25 measures, the first factor explains 19.9% of the total variance (Table 5), compared to 11.9% of the variance in the varimax-rotated solution. In the unrotated solution, all 25 measures load positively on the first factor, with loadings ranging between .07 and .65 (mean .42, s.d. .17). Considering both rotated and unrotated solutions, we conclude similarly to the current consensus over factors describing intelligence (Mackintosh, 2011): There is evidence both for a general factor underlying perceptual performance, and for independent perceptual abilities. The former is emphasised in the unrotated solution, and the latter in the rotated solution. We do not favour the results of either the rotated or the unrotated solution: we believe each provide a useful contribution to our understanding of the data.

### Relationship to previous work

How do our factors compare with those that have similarly aimed to discover factors underlying visual perception? None of our eight factors match those revealed by the work of Thurstone (1944), which is unsurprising since the task batteries in the two studies are very different. Thurstone's battery emphasised high-level perceptual tasks and included measures of reaction time, and so it is not surprising that the emerging factors related to perceptual closure, susceptibility to geometric illusions, reaction time and perceptual speed. In contrast, our own analysis included sensory threshold measures of different psychophysical abilities. The precise character of the factors that emerge from any factor analysis depends on the measures that are put in, and therefore comparable results are expected from different studies only if comparable task sets are used.

Our task set had more in common with those of Halpern et al. (1999), Cappe et al. (2014), and Ward et al. (2016) than that of Thurstone (1944). Halpern et al. provided loadings only for the first principal component resulting from their PCA. Their measures of orientation discrimination, wavelength discrimination, contrast sensitivity, vernier acuity, velocity discrimination and identification of complex form all correlated with the first principal component, while motion direction discrimination did not. This first principal component explained somewhat more of the variance (30%) than the first principal component in our own unrotated principal components analysis (19.9%). Halpern et al. (1999) did not provide information about subsequent factors, nor a rotated solution, so it is unclear whether or not their study produced, like ours, evidence for different factors associated with different perceptual domains.

Cappe et al. (2014) included in their test battery measures of visual acuity, vernier acuity, backward masking and contrast detection, which are related to our own. In their study, the first PCA accounted for 34% of the variance – again, a greater percentage than our own. Although Cappe et al. did not report loadings on each PCA for each of their tasks, they do report a correlation between vernier acuity and contrast

detection. Our own factor 8, loading on Pelli-Robson contrast detection and visual acuity is a similar result.

Four of the tasks of Ward et al. (2016) were related to our own: their detection of gabors, contrast sensitivity, detection of Glass patterns, and detection of coherent motion. Our results are in agreement with Ward et al.'s finding, and also the results of Peterzell and Teller (Peterzell & Teller, 1996; Peterzell, 2016) that sensitivity to low and high spatial frequencies load on separate factors. Ward et al. found that sensitivity to Glass patterns loaded on the same factor as tasks involving high spatial frequencies, while thresholds for coherent motion loaded on the same factor as tasks involving low spatial frequencies. Our own results do not show this distinction: thresholds for coherent motion and sensitivity to Glass patterns both load on factor 1, which is related to other tasks involving high spatial frequencies.

### Cluster analysis

The clusters of tasks revealed by the hierarchical cluster analysis reinforce those revealed by the factor analysis, as would be expected. One surprising result was the clustering of anti-saccadic error rate and smooth pursuit RMSE with factor 1 (sensitivity to medium and high spatial frequencies) rather than factors 2 and 3 (oculomotor speed and oculomotor control). However, the detailed factor loadings for anti-saccadic error rate and smooth pursuit RMSE (Tables 6 and 7) show that they more distributed over the factors than are the other oculomotor measures (see Zanelli et al., 2005). Both variables exhibit medium loadings on factors 1, 2 and 8, as well as on factor 4 (factor 3 for Group A in Table 7). It seems to be the case that performance on these oculomotor measures has more in common with psychophysical performance in other domains than do saccadic latency and variability.

#### Correlates of performance factors

Many of the correlations listed in Table 8, between the factors that emerged from the factor analysis and visual non-performance measures and questionnaire items, are not surprising. We found significant correlations between factor 2 (auditory ability) and self-ratings for musical and auditory abilities. It is interesting that time spent playing computer games correlates with factor 3 (oculomotor speed): Future work will be needed to disentangle the causal direction.

It is also interesting that GCSE score (our surrogate measure of g) shares variance with factor 2 (auditory perceptual ability), but not with any of the factors relating to visual perceptual ability. Since foreign language exams are taken by most GCSE students, a portion of the variance in GCSE score (perhaps about 10%) may reflect foreign language ability, which is likely to be related to auditory ability. However, it is unlikely that this fully accounts for the correlation: Our result is compatible with earlier reports of modest but significant correlations between auditory abilities and general cognitive ability (e.g. Burt, 1909; Carey, 1915; Kidd, Watson, & Gygi, 2007).

The shared variance between interpupillary distance and factor 3 (oculomotor speed) could be largely explained by sex, which also correlates with factor 3. When sex is included as a covariate in a partial correlation, rho falls to .06 (P = .06). It is, of course, also possible that the dependencies go in a different direction: that the correlation between sex and factor 3 is mediated by interpupillary distance.

Some relationships are curious by their absence. None of our factors is strongly correlated with Autism-spectrum Quotient, despite the large literature relating perceptual measures to autism (Simmons et al., 2009). It is also interesting that none of the factors relates to lightness of the iris, which is known to affect the level of stray light in the retina (van den Berg, Ijspeert, & de Waard, 1991). Though it is *a priori* very plausible that the personality variable Conscientiousness would contribute to psychophysical performance, we have found that no performance factor correlates significantly with it, and neither does it correlate with factor 1 of the unrotated factorial solution, our putative general factor for perceptual performance.

### Conclusions

Our factor analysis of perceptual performance has revealed evidence both for a general factor underlying perceptual ability (accounting, in the unrotated solution, for 19.9% of the total variance) and for independent factors that account for performance on particular groups of measures. The independent factors we identified (though they will depend on the component measures included in the analysis) relate to (1) sensitivity to medium and high spatial frequencies, (2) auditory perceptual ability (3) oculomotor speed, (4) oculomotor control, (5) contrast sensitivity at low spatial frequencies, (6) stereo acuity, (7) letter recognition, and (8) flicker sensitivity.

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