

The Shape of Bouncing Universes

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What happens to the most general closed oscillating universes in general relativity? We sketch the development of interest in cyclic universes from the early work of Friedmann and Tolman to modern variations introduced by the presence of a cosmological constant. Then we show what happens in the cyclic evolution of the most general closed anisotropic universes provided by the Mixmaster universe. We show that in the presence of entropy increase its cycles grow in size and age, increasingly approaching flatness. But these cycles also grow increasingly anisotropic at their expansion maxima. If there is a positive cosmological constant, or dark energy, present then these oscillations always end and the last cycle evolves from an anisotropic inflexion point towards a de Sitter future of everlasting expansion.

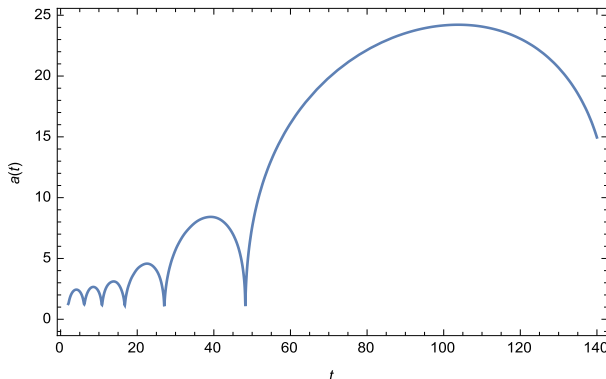
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Cyclic universes have a long history of ups and downs. Hurdling myths about the death and rebirth of a universe, the inception of the general theory of relativity led quickly to precise possibilities. Alexander Friedmann noticed the scope for periodically expanding and contracting closed universes in his first cosmological solutions of Einstein's equations [1]. The next important consideration of the physical reality of such eternal, cyclic universes was made by Richard Tolman in 1928 [2]. Tolman, a chemist as well as a physicist, wanted to introduce the thermodynamic arrow of time into these cosmological models by including the second law of thermodynamics. He made a striking discovery. The entropy increase from cycle to cycle in an oscillating closed Friedmann universe increased the maximum size and timespan of successive cycles (Figure 1). Today, we would interpret that as driving successive cycles towards 'flatness'. Successive cycles undergo longer and longer periods of their evolution in proximity to the expansion maxima. The greater the number of past cycles, the closer are dynamics to spatial flatness. This time asymmetry can be interpreted physically if the steady entropy increase from cycle to cycle result from the transfer of ordered energy, in pressureless dust, to disordered blackbody radiation. This entropy-increasing transfer of energy from pressureless matter to high-pressure radiation creates an asymmetry in cycles and steadily increases their height and length.

FIG. 1: Time evolution of the scale factor, $a(t)$, of a closed Friedmann universe with increasing radiation entropy.



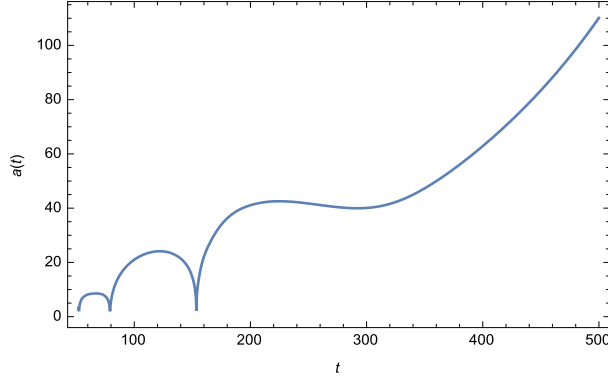
All of these insights apply to a universe in which there is no cosmological constant, Λ . If we introduce a positive cosmological constant then there is a big change: the sequence of growing oscillations always ends (Figure 2). What goes up, need not come down [3]. No matter how small the value of $|\Lambda|$, the sequence of growing oscillations will eventually attain a size that brings the Λ term into dynamical play. When that happens, it quickly accelerates the expansion towards asymptotically de Sitter expansion with a scale factor that expands exponentially in time. No further expansion maxima occur; entropy production becomes insignificant; and the dynamics are left close to flatness and slightly dominated by the energy density in the Λ , or 'dark energy', field – a situation not dissimilar to what we observe today [4] although the value of Λ remains unexplained.

This naïve cyclic universe scenario leaves several questions unanswered. How is it possible to 'bounce' through a spacetime singularity at the end of each cycle with no greater damage to the Friedmann dynamics than an injection of entropy? And what happens to the shape of more realistic closed universes over many cycles?

The first difficulty can be ameliorated to some extent by enabling the bounce to occur at a non-singular finite radius. This can be achieved by adding a 'ghost' field, with negative density, to the Friedmann equations so that the universe evolves smoothly through a non-singular sequence of finite oscillations [5]. The unitarity of a theory having such a 'ghost' field, has often been called into question. It has recently been shown [6] that 'ghost' fields require the inclusion of operators arising from a higher energy scale. On imposing shift symmetry, including these operators allows for a bounce that does not violate tree-level unitarity, at least within the context of the low-energy effective field theory. If a positive cosmological constant is included, then the non-singular oscillations must come to an end and the dynamics evolve towards a future de Sitter state, just as we described above.

The second difficulty is the one we focus upon. The isotropic Friedmann universes are not generic solutions of Einstein's equations at late times if matter is gravitationally attractive and so we need to understand what happens to closed cyclic universes that are allowed to expand anisotropically. The most general such universe

FIG. 2: Adding a cosmological constant to an oscillating Friedmann universe with entropy increase always ends the oscillations.



that allows anisotropic expansion whilst retaining spatial homogeneity is the Mixmaster universe of Bianchi type IX, well-studied in connection with its chaotic dynamical behaviour near a singularity [7, 8]. This chaotic behaviour only occurs on time intervals that include the zero of time and so, if we effect a bounce at finite expansion radius, we will not need to worry about it. In fact, if the bounce occurred at the Planck scale, $10^{-43}s$, then less than ten oscillations would occur all the way up to the present time of $10^{17}s$, because they proceed on a slow logarithmic time scale compared to the increase in the overall volume of the universe.

The diagonal Mixmaster universe is the most general anisotropic closed universe with S^3 spatial topology. Its three orthogonal scale factors, $a(t)$, $b(t)$ and $c(t)$, obey the field equation ($8\pi G = c = 1$) [9]

$$2(\ln a)'' + a^4 - (b^2 - c^2)^2 = a^2 b^2 c^2 \sum_{i=r,g} (\rho_i - p_i),$$

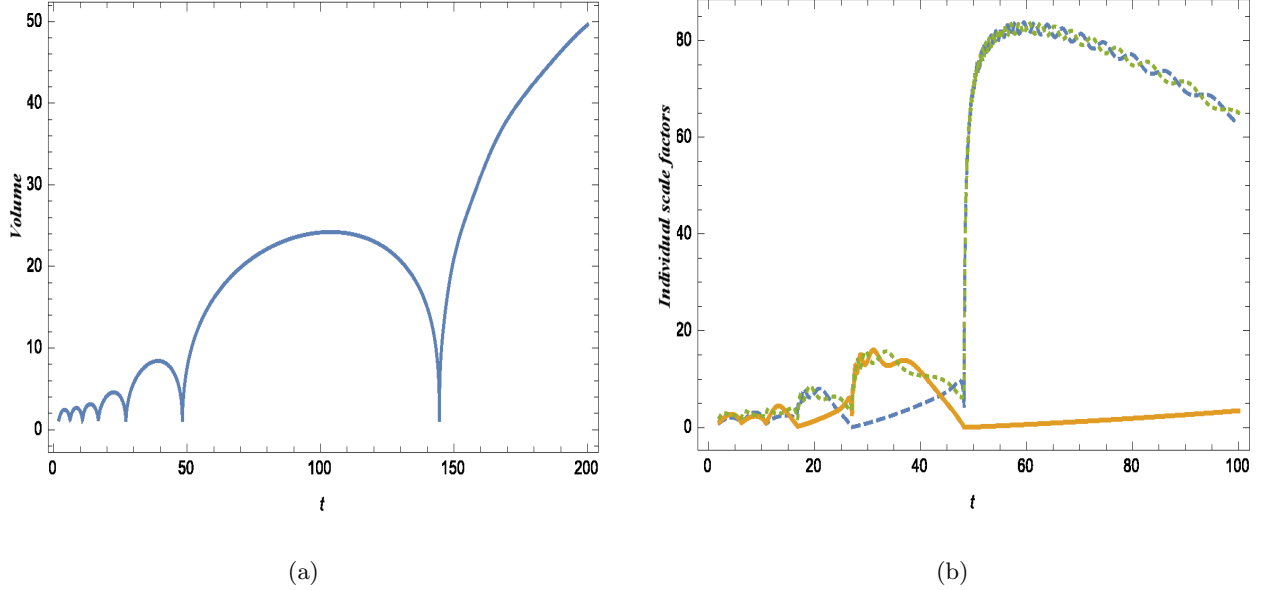
and the other two equations are obtained by permuting $a \rightarrow b \rightarrow c \rightarrow a$. The time derivatives are denoted by $' = d/d\tau \equiv abc d/dt$, where t is the comoving proper time and the sum on the right-hand side is over the different matter sources present ($r = \text{radiation}$, with $\rho_r = 3p_r \propto (abc)^{-4/3}$ and $g = \text{ghost field}$, with $0 > \rho_g = p_g \propto (abc)^{-2}$). The addition of a cosmological constant means adding a third $p_i = -\rho_i = -\Lambda$ constant field to this sum.

When $a = b = c$ these equations describe the isotropic Friedmann metric. However, when a, b and c are unequal they display remarkably complicated behaviour because of intrinsically general relativistic effects. Unlike the simple anisotropic cosmologies which have Newtonian analogues, Mixmaster universes have anisotropic spatial curvature fashioned by long-wavelength gravitational waves propagating on an expanding 3-geometry. Surprisingly, unlike in an isotropic universe, the sign of the 3-curvature can also change with time. When the dynamics are far from isotropy it is actually negative and so no expansion maximum can occur. Expansion continues until it is isotropic enough for the 3-curvature to become positive and only then can this type of closed universe recollapse, although it still does so anisotropically.

The Einstein equations for the Mixmaster universe cannot be solved exactly (in fact they form a non-integrable system) but we can study them approximately [10] and numerically to understand the detailed evolution over successive cycles if we introduce a ‘ghost’ field. The ‘ghost’ field produces smooth bounces at finite minima, so that we do not encounter chaotic Mixmaster oscillations there, and it has no significant effect on the expansion maxima. For simplicity, we use blackbody radiation as the matter source (although the results are qualitatively the same if dust is added) and we can increase the entropy of the universe, as Tolman envisaged, by injecting new radiation entropy at the start of each cycle. What happens?

When Λ is zero, oscillating Mixmaster universes with increasing entropy mimic their special Friedmann counterparts in two respects. They oscillate through a sequence of cycles that grow in size and total lifetime, and evolve closer and closer to flatness. Yet, here the resemblance with Tolman’s old scenario ends. In each cycle the anisotropy between the three scale factors grows and is larger at each successive maximum of the volume (Figures 3a and 3b). During the subsequent collapse phase of each cycle the anisotropy continues to amplify.

FIG. 3: Time evolution of the (a) volume, and (b) individual scale factors, of a Mixmaster universe containing radiation, ghost field and zero Λ , as it oscillates through many cycles with increasing radiation entropy. The blue dashed, green dotted and yellow solid lines trace the scale factors $a(t)$, $b(t)$ and $c(t)$. The cycles become increasingly anisotropic.

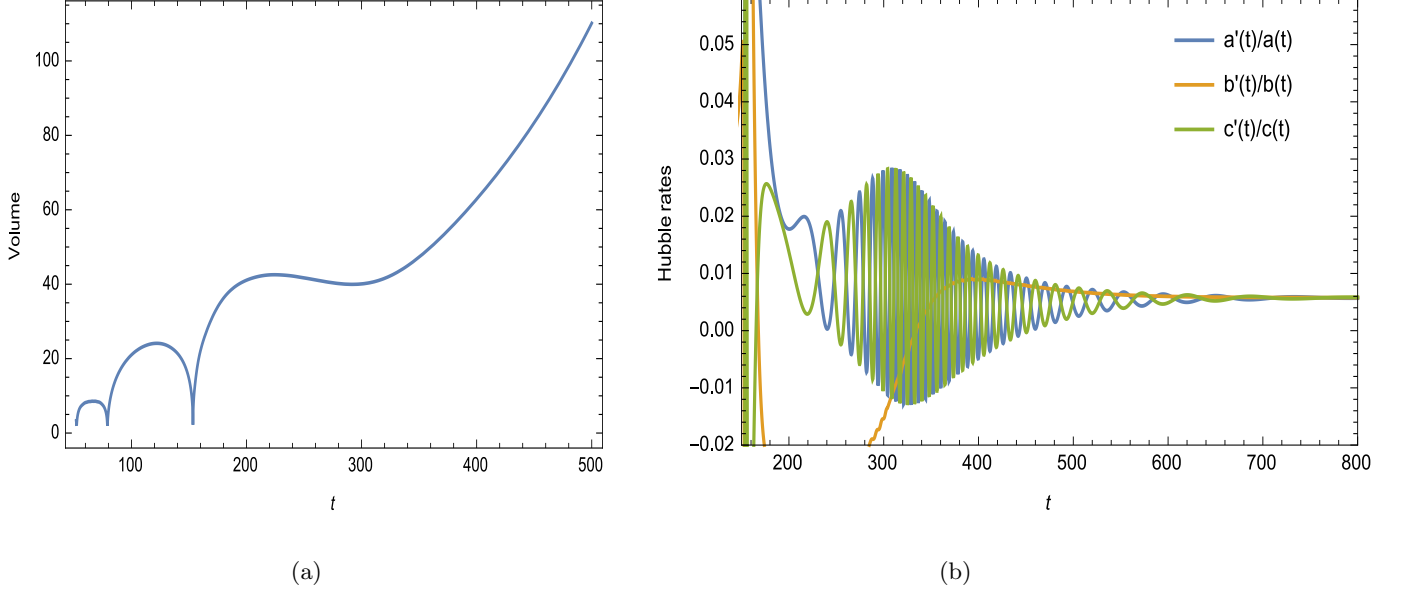


Even if we reduce its cumulative effects by allowing the isotropic ‘ghost’ field to dominate and produce a bounce at each minimum, the anisotropy level still grows larger at each successive larger maximum. This type of anisotropic universe is not like the universe we live in, despite its proximity to flatness. The appealing features of the isotropic cyclic universe have disappeared in a chaotic sequence of anisotropic oscillations.

Cyclic universes have recently become popular in theories of gravity that extend Einstein’s general relativity, as can be seen in the extensive review in reference [11]. Our findings here will place new pressures upon these and future scenarios as well. Ekpyrotic models incorporate a super-stiff field with $p \gg \rho > 0$ to drive the dynamics to isotropy at the end of each cycle [12]. But if anisotropic super-stiff pressures created by collisionless stresses are included in the momentum spectrum then super-stiff fields will fail to isotropise the collapse [13]. Anisotropies will accumulate and successive Mixmaster expansion maxima will become increasingly anisotropic. Our own model has various simplifying features: the metric is diagonal and the fluid flow lines of the radiation are comoving. We find that dropping these special features does not solve the growing anisotropy problem. It makes it worse.

If we now add a positive cosmological constant to the Mixmaster cycles of a radiation-filled universe with monotonic entropy increase, the oscillations still inevitably come to an end, just as in the special Friedmann case – and for the same reason. Ultimately, the Λ stress always dominates when the cycles grow to a certain size. The death throes of the cyclic universe yield a distorted period of anisotropic expansion as the dynamics are accelerated with equal antigravitating force in all directions (Figures 4a and 4b). We can confirm that the expansion indeed tends to de Sitter as the Hubble rates in the individual directions tend to a constant value, $\sqrt{\Lambda/3}$, when the cosmological constant starts to dominate. The volume evolves exponentially rapidly towards the asymptotic de Sitter state with $a(t)b(t)c(t) \simeq \exp(t\sqrt{3\Lambda})$. If we inhabited the last cycle where this changeover to Λ -domination had occurred we would see evidence of significant anisotropy in the angular intensities of the x-ray and microwave backgrounds, and probably also in the large-scale Hubble flow, together with anomalously large abundances of primordial helium-4 as a relic of the faster anisotropic expansion in the very early stages of the cycle. This completes our short story of the cyclic universe in general relativity.

FIG. 4: Time evolution of the (a) volume, and (b) the individual Hubble expansion rates, of a Mixmaster universe with positive Λ , radiation and a ghost field. The blue dashed, green dotted and solid yellow lines trace the Hubble rates $\dot{a}(t)/a(t)$, $\dot{b}(t)/b(t)$ and $\dot{c}(t)/c(t)$. Oscillations cease when Λ dominates. The Hubble rates then undergo an anisotropic transition phase before eventually approaching isotropic de Sitter-like expansion where the individual Hubble rates approach the same constant value



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- [1] A.A. Friedmann, Zeit. Physik **10**, 377 (1922) transl. Gen. Rel. & Gravitation **31**, 1991 (1999)
 - [2] R.C. Tolman, Phys. Rev. **38**, 1758 (1931) and in *Relativity, Thermodynamics and Cosmology*, Oxford U.P., Oxford, (1934)
 - [3] J.D. Barrow and M. Dąbrowski, Mon. Not. Roy. astron. Soc. **275**, 850 (1995)
 - [4] P. Ade et al (Planck Collaboration), A & A 594, A13 (2016)
 - [5] J.D. Barrow, D. Kimberly and J. Magueijo, Class. Quantum Grav. **21**, 4289 (2004)
 - [6] C. de Rham and S. Melville, arXiv:1703.00025v1
 - [7] C.W. Misner, Phys. Rev. Lett. **22**, 1071 (1969); V.A. Belinskii and I.M. Khalatnikov, Sov. Phys. JETP **29**, 911 (1969)
 - [8] J.D. Barrow, Phys. Rev. Lett. **46**, 963 (1981) and Phys. Reports **85**, 1 (1982)
 - [9] L. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, 4th rev. edn. (Pergamon, Oxford, 1975)
 - [10] A.D. Doroshkevich, V.N. Lukash and I.D. Novikov, Sov. Phys. JETP **37**, 739 (1973)
 - [11] R. Brandenberger and P. Peter, arXiv:1603.05834
 - [12] P.J. Steinhardt and N. Turok, Phys. Rev. D **65**, 126003 (2002)
 - [13] J.D. Barrow and K. Yamamoto, Phys. Rev. D **82**, 063516 (2010) J.D. Barrow and C. Ganguly, Class. Quantum Grav. **33**, 125004 (2016)