

# Quantum Reality via Late Time Photodetection

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We further investigate postulates for realist versions of relativistic quantum theory and quantum field theory in Minkowski space and other background space-times. According to these postulates, quantum theory is supplemented by local variables that depend on possible outcomes of hypothetical measurements on the late time electromagnetic field in spacelike separated regions. We illustrate the implications in simple examples using photon wave mechanics, and discuss possible extensions to quantum field theory.

## INTRODUCTION

In previous papers [1–3], we described ideas aimed at defining realist and Lorentz covariant versions of relativistic quantum theory or quantum field theory in Minkowski space. The definitions also extend to other background spacetimes with appropriate asymptotic properties in the far future. These papers set out a variety of possible ways of defining variables that extend quantum theory and may allow a description of physical reality consistent with our observations, including the observation that measurements have single definite outcomes. These variables are defined at each point in space-time, but do not follow local equations of motion. Following Bell’s terminology, we refer to them as *beables*.

In the main part of this paper, we focus on the ideas proposed in Ref. [3], according to which the beables at any point  $x$  in space-time depend on the initial quantum state and also on the outcomes of fictitious measurements made on the electromagnetic field on asymptotically late time surfaces that are space-like separated from  $x$ . We begin to flesh out this proposal and its implications, using somewhat more realistic models than the toy models discussed in Ref. [3], including models of photon dynamics and of photon measurement. We represent matter by finitely many massive charged particles, coupled to the electromagnetic field via interactions that generate or absorb finitely many photons. We suppose the initial state of the electromagnetic field also contains finitely many photons, so that, given the assumption that the photon number changes only finitely, the final state also does. This allows us to describe the electromagnetic field by photon wave mechanics [4–10], which gives a physically intuitive picture of the relationship between events in space-time. We also comment on possible extensions of these models into the full quantum field theory regime.

## INFERRING LOCAL EVENTS FROM LATE TIME SPACE-LIKE PHOTODETECTIONS

We are interested, initially, in understanding quantum theory in Minkowski space-time. We now set out some temporary simplifying assumptions, which should be dropped eventually, but which hold in this paper except where explicitly relaxed.

First, we will neglect gravity. We take the background Minkowski space-time to be fixed. Our quantum theory does not include even a semiclassical treatment of gravity. Suppose now that in some finite region  $R$  of space-time we have a system modelled by a finite number  $N_m > 0$  of massive particles, interacting via forces that we may model by potentials. These particles may, for example, initially be in spatial or energy superpositions. Suppose also that a finite number  $N_p^i \geq 0$  of photons arrive at this region and interact with the massive particles within the region. We allow that the particles may absorb and/or emit photons, so the photon number may be variable. For simplicity, we assume that no massive particles are created or destroyed, so  $N_m$  is constant throughout. The state after the region may contain components with various numbers of photons leaving the region. We will want a significant amplitude (in some examples close to modulus one) for states with one or more photons leaving the region. We also assume an upper bound  $N_p^o$  on the number of outgoing photons in any component. We allow the possibility  $N_p^i = 0$ , since we can illustrate our postulates in simple models in which the state on surfaces prior to the region contains no incoming photons. Realistic models of familiar physical systems would include both incoming and outgoing photons, of course.

We suppose further that the above description includes all the matter (massive particles and photons) in the space-time, and that all the interactions between photons and massive particles take place within the region  $R$ . We can

model this by taking the Hamiltonian to be local and space-time dependent, with a particle-photon interaction that is turned on only within  $R$ . [25] Suppose then that we are given an initial state  $|\psi_0\rangle$  on some spacelike hypersurface  $S_0$  prior to  $R$ , which describes  $N_m$  massive particles and  $N_p^i$  photons. We suppose we have some relativistic (albeit spacetime-dependent) unitary evolution law that allows us to use the Tomonaga-Schwinger formalism. We can thus define the evolved state  $|\psi_S\rangle$  on any hypersurface  $S$  in the future of  $S_0$  via a unitary operator  $U_{S_0S}$ .

### Intuitive picture

At an informal intuitive level, then, we have the following picture. Photons may scatter from, or be absorbed or emitted by, the massive particles within the region  $R$ . Scattered and emitted photons then travel away from  $R$ , along roughly lightlike paths. They do not interact further with massive particles or with one another, so their direction of propagation outward from  $R$  is roughly constant. On spacelike hypersurfaces  $S$  in the future of  $R$ , the outward propagating electromagnetic field degrees of freedom will generally be highly entangled with the massive particle degrees of freedom. The outcomes of some measurements of the electromagnetic field will thus generally be highly correlated with the outcomes of some measurements of the massive particle states.

We will work in the centre-of-mass frame coordinates  $\underline{x}, t$ , which we will take for the moment to define a preferred frame. Consider now a spacelike hypersurface  $S$  comprising three segments,  $S_1, S_2, S_3$ . Here  $S_1$  runs near to, and in the future of, the future boundary of  $R$ ,  $S_2$  runs close to outward propagating lightlike directions from that boundary, and  $S_3$  runs outward along the constant time hyperplane  $t = T$ , for some large value of  $T$ . (In particular,  $T \gg T_{\max}^R$ , where  $T_{\max}^R$  is the largest value of the coordinate  $t$  of points in  $R$ .) Suppose that we have an infinite array of efficient photodetectors, each of finite volume  $V$ , set up along  $S_3$ . The volume  $V$  is chosen to be large enough to allow efficient detection of photons of frequencies characteristically (i.e. with non-negligible amplitude) arising in our model, but no larger. If we take the detectors to be cubic, we can take the array to be regular and to fill  $S_3$  (except for a small volume around the boundary with  $S_2$ ). We assume the detectors are all stationary in the centre-of-mass frame defining  $S_3$ , and produce a measurement result within some characteristic time  $\delta$  in that frame.

What should we expect, in this scenario? The detectors should produce a number of clicks, bounded by  $N_p^o$ , giving an approximate spatial distribution of detected photons that (may have) interacted with the massive particles in the region  $R$ . Because both  $S$  and  $S_0$  are spacelike hypersurfaces, with  $S$  in the future of  $S_0$ , we can calculate the probability distribution for this distribution from the initial state  $|\psi\rangle_{S_0}$  and the unitary evolution  $U_{S_0S}$ . Similarly, we can calculate the joint probability distribution of possible configurations of clicks and of a measurement of any operator  $A$  defined locally on the surface  $S_1$ . In particular, we can do this for operators  $A$  defined solely on the massive particle degrees of freedom, such as the local mass density or energy density of these particles.

### *Informal description of the interpretation*

How might we interpret these calculations? The idea we pursue here is that the possible distributions of clicks correspond, roughly speaking, to “possible worlds”, or “possible ways the world might turn out”, or “possible descriptions of reality”. Each possible distribution of clicks defines an expectation value  $\langle A \rangle$  for the local operator  $A$ , and these expectation values, for suitably chosen local operators, give us variables that augment the quantum state in describing physical reality on  $S_1$ . By applying this procedure for each small region and taking an appropriate limit, we obtain variables – call them beables, following Bell – associated with each point  $x$  in space-time. These expectation values are functions of points (or small volumes) in space-time and are to be understood as a description of physical reality at these points or in their local regions. However, they do not follow local equations of motion.

To see how this can work, note that, while so far we focussed on the physics of region  $R$ , and on detectors on the subset  $S_3$  of the plane  $P_T$  defined by  $t = T$ , we can imagine similar photodetectors set up throughout  $P_T$ . We consider just a single “run” of the “experiment” defined by physics between  $S_0$  and  $P_T$ , and one distribution of clicks from the detectors distributed over all of  $P_T$  defining the “outcome”. Then to define the conditional expectation value of a local operator  $A_x$  defined on the massive particle Hilbert space at point  $x$ , we condition only on the distribution of clicks from detectors outside the future light cone of  $x$ , ignoring any information (clicks or lack of clicks) about the distribution within the future light cone. In this way, by using partial information from a single full distribution  $D$  of clicks, we define a distribution of local variables  $\langle A_x \rangle_D$  throughout space-time, for whichever local operators  $A$  we choose. We interpret this as meaning that each full distribution of clicks  $D$  gives us data corresponding to a “possible world”,  $W_D$ , which defines a corresponding “description of reality” via the beables  $\langle A_x \rangle_D$ . Quantum theory, via the Born rule, gives us a probability distribution  $P(D)$  on the full distributions of clicks  $D$ , and hence on the

configurations of beables  $\langle A_x \rangle_D$  in spacetime. These distributions depend on the initial state  $|\psi\rangle_{S_0}$  and the unitary evolution  $U_{S_0 S}$ , as they must for the picture that emerges to have any plausible physical meaning.

To make this concrete, we need to specify the set of operators  $A$  from which beables are defined. In realistic theories based on QFT, we suggest the stress-energy tensor components  $T_{\mu\nu}(x)$  as natural candidates.[26] The charge 4-current components  $J_\mu(x)$  are another natural possibility.

The intuition then is that, in realistic models, we expect the beables  $\langle T_{\mu\nu}(x) \rangle$  to give a description of reality that corresponds well with the quasiclassical world we observe. In particular, they describe high matter densities in the region occupied by a pointer corresponding to one measurement outcome in a lab experiment, and very low matter densities in the regions that would be occupied by pointers corresponding to the other possible outcomes (assuming these various pointer regions are disjoint). Moreover, they do this with the correct Born rule probabilities. In Bell experiments, or other experiments on entangled quantum states, they produced correlated pointer outcomes (in the sense just given) following standard Bell correlations.

The reason for this intuition is that the information carried away by outgoing photon states, in their position degrees of freedom, is correlated with the positions of pointers built from massive particles with which the photons have interacted. So long as large numbers of photons radiate away to future infinity without reinterfering, this information can be extracted – using the conditional expectation value rules given above – from the approximate position measurements made by the photodetector array.

At this intuitive level, this picture has some limitations. It is hard to justify rigorously in realistic models using QED or any other non-trivial relativistic quantum field theory, partly because infinite particle number and photon number states come into play, but also because we have no mathematically rigorous definition of non-trivial quantum field theories and no completely satisfactory way of modelling the dynamics of macroscopic physical systems and their interaction with microscopic systems within these theories.

At first sight it also breaks Lorentz invariance, by requiring a frame in which the final photodetector array is stationary. For the moment, we take this to be the centre of mass frame for the full unitarily evolving quantum state (photons plus massive particles). Another issue that arises for a full QFT treatment is the choice of photodetector size. If we assume that our ideal photodetectors respond only to photons of wavelengths somewhat smaller than their size, we effectively impose an infra-red cutoff. This need not matter in simple models which contain only finitely many photons and have a lower frequency bound. However, in realistic models, there is no natural cutoff, and the beable configuration probability distribution seems likely to depend (at least slightly) on the cutoff choice. A further issue is that the beable configuration probability distribution depends (if only very slightly) on the precise configuration of photo-detectors. Perhaps this dependence disappears in the limit  $T \rightarrow \infty$ , but that is not clear at this level of discussion.

### Implications so far

It is certainly worth addressing these concerns as far as possible, and we do consider them further in the rest of the paper. However, there is also a case to be made for accepting the picture at the informal level just given, and developing its scientific implications. Two points argue in its favour.

First, as shown below, in simple but natural models, using versions of single- or multi-photon quantum theory derived from quantum field theory, our postulates give a well-defined construction that gives probability distributions for natural choices of expectation value beables.

Second, although QED and other relativistic QFTs are not mathematically fully rigorously defined, and do not have a complete theory of measurement, much empirical evidence supports the partial intuitive understanding of these theories that has been developed. Calculations fit data with very high precision; photodetectors can in fact detect single photons to good approximation. Empirical evidence also strongly suggests that our postulates would extend to work in the observed universe, in the sense that expectation values for  $\langle T_{\mu\nu}(x) \rangle$ , fitting our actual observations, could be reconstructed given an array of ideal photodetectors with some lower frequency cutoff on a late time hypersurface, stationary with respect to cosmological centre-of-mass, using our rules. That is, the expectation value for  $\langle T_{\mu\nu}(x) \rangle$  could be reconstructed by conditioning on photodetector data from outside the causal future of  $x$ .

We cannot expect to give a completely rigorous demonstration that this will work within QFT as presently understood: an absolutely precise and completely rigorous solution of the quantum reality problem compatible with relativistic quantum field theory will have to await an absolutely precise and completely rigorous definition of relativistic QFT. Even then, to reach our ultimate goal – namely, a perfectly and precisely specified realist version of relativistic quantum theory that also applies to observable cosmology – would require a quantum theory of gravity

that also (1) incorporates a new and rigorous understanding of field theory, (2) comes with a complete theory of initial conditions *and* (3) contains a solution to the quantum reality problem.

So, the two points made above may actually go a fair way towards what is presently achievable in the way of evidence. (Yet we still hope to do better!)

Finally, we would like to underline that one of the major aims of this project[1–3, 11] is to use it as a springboard to define testable generalizations [11, 12] of quantum theory that make distinctive experimental and cosmological predictions. Since such generalizations may well anyway at best be no more than broadly correct phenomenological models, we shouldn't necessarily let the finest details of precise specification hold us back in pursuing them. A good and well-motivated model that could point towards new data is all we need for this purpose, at least. It may be most realistic to use well-founded ideas on quantum reality to motivate steps forward, without requiring as a precondition solving every problem in physics at once.

## PHOTON WAVE MECHANICS MODELS

In a previous paper [3], we illustrated the intuitions underlying our proposals with very simple toy models. In these models, massive particles were represented by position space wave functions, and photons by pointlike quantum objects propagating along lightlike trajectories. Photon-particle interactions were represented by instantaneous reflections, which change the direction of photon propagation.

Here we give somewhat more realistic models. We still assume that there are finite (in our models, small) bounds on the numbers of photons and massive particles. However, we move away from a simplistic picture in which photons are taken to be pointlike and to follow precisely lightlike paths. Instead, we treat them quantum mechanically on an equal footing with the massive particles, represented by quantum mechanical wave functions. To do this, we use the formalism of photon wave mechanics, following in particular results and insights developed by Bialynicki-Birula [4, 6], Sipe [5], Muthukrishnan et al [8] and Smith and Raymer [7, 9, 10]. For models with a single photon, we consider a photon wave function that represents a local probability distribution for the photon energy. For models with several photons, we consider multi-photon wave functions [8–10] which correspond to multi-photon detection amplitudes and represent states in Fock subspaces of definite photon number within QED. These models allow reasonably realistic descriptions of particle-photon interactions and of photon emission and absorption by massive particles.

We illustrate our proposal by calculating expectation values for energy state and position beables, which are the natural choices in the simple models below. These models have relativistic features – in particular, the photon wave functions propagate causally with respect to the Minkowski metric – but are not fully Lorentz invariant. Fully Lorentz invariant models would require a more fundamentally satisfactory treatment of photon measurements – a task for the future.

Examples of photon wave function calculations suitable for our purpose were given by Muthukrishnan et al. (MSZ) [8]. MSZ define their photon wave functions in terms of the quantized electric field alone. This allows the wave function to be directly related to amplitudes for detection in standard photodetectors, which respond to the electric field. Another option [10] is to define photon wave functions in terms of both the quantized electric and magnetic fields. This would be appropriate for (perhaps hypothetical) photodetectors that are sensitive to both fields.

We follow MSZ's conventions and closely paraphrase their discussion in this section. MSZ consider the electric field in a mode volume  $V$ :

$$\begin{aligned}\hat{E}(\underline{r}, t) &= \hat{E}^+(\underline{r}, t) + \hat{E}^-(\underline{r}, t) \\ &= \sum_k \sqrt{(\hbar\nu_k/\epsilon_0 V)} (\hat{a}_k u_k(\underline{r}) e^{-i\nu_k t} + \hat{a}_k^\dagger u_k^*(\underline{r}) e^{i\nu_k t}).\end{aligned}\tag{1}$$

Here the  $u_k(\underline{r})$  are normalised spatial mode functions.

For a single photon state  $|\gamma\rangle$ , the probability density of detecting the photon at  $(\underline{r}, t)$  is

$$\kappa |\langle 0 | \hat{E}^+(\underline{r}, t) | \gamma \rangle|^2,\tag{2}$$

where  $\kappa$  is a normalisation constant chosen with dimensions such that the photon wave function

$$\gamma(\underline{r}, t) = \sqrt{\kappa} \langle 0 | \hat{E}^+(\underline{r}, t) | \gamma \rangle\tag{3}$$

has dimension  $L^{-3/2}$ .

### Example 1

MSZ proceed to calculate the wave function  $\gamma(\underline{r}, t)$  describing a photon that is spontaneously emitted by an atom located at the origin which is introduced at time  $t = 0$  in an excited state and decays at rate  $\Gamma$  to its ground state. They obtain

$$\gamma(\underline{r}, t) \approx K(\sin \theta/r)\theta(t - (r/c))e^{-i(\omega - i\Gamma/2)(t - (r/c))}. \quad (4)$$

Here  $c$  is the speed of light,  $K$  is a normalisation constant,  $r = |\underline{r}|$ ,  $\theta$  is the azimuthal angle in coordinates defined by the axis of the atom's dipole moment,  $\omega$  is the frequency of the atomic transition, and  $\theta(t - (r/c))$  is the Heaviside step function, so that  $\gamma = 0$  outside the future light cone of the spacetime point  $(0, 0)$ . At this point we take the mode volume to tend to infinity, and suppose that Eqn. (4) applies throughout space after  $t = 0$ .

More than one school of thought regarding the interpretation of single photon wave functions can be found in the literature. It is generally agreed [13] that  $|\gamma(\underline{r}, t)|^2$  is approximately proportional to the low rate detection probability of the photon in a suitable photodetector centred at  $(\underline{r}, t)$  with dimensions much larger than the typical photon wavelength (given here by  $\lambda = c/\omega$ ). Some authors have considered stronger interpretations. For example, Sipe [5] suggests that “in attempting to write down a position-representation wave function, we should be seeking a probability amplitude  $\psi(\underline{r}, t)$  for the photon energy to be detected about  $d\underline{r}$  of  $\underline{r}$ .” Bialynicki-Birula [6] similarly suggests “one may introduce a tentative notion of the “average photon energy in a region of space” and try to associate a probabilistic interpretation of the photon wave function with this quantity.” MSZ [8] “maintain that a physically meaningful photon wave function  $\gamma(\underline{r}, t)$  can indeed be constructed, that is measurably localized in space, everywhere meaningfully defined in both phase and amplitude, and provides a valuable tool for understanding photon interference and correlation experiments.”

We pragmatically adopt the spirit of these suggestions here in order to define precise rules, without committing to a final view on the correct theory of measurement for photon wave functions or, more generally, for quantum electrodynamics or relativistic quantum field theories. Our discussion is thus meant as an illustrative placeholder, pending a more fundamentally satisfactory treatment of photon measurements, which we hope to tackle in future work.

We will adopt as a mathematical hypothesis that an appropriately normalised  $|\gamma(\underline{r}, t)|^2$  defines precisely the detection probability of the photon at  $(\underline{r}, t)$ , at asymptotically late times  $t$ , in the following sense. We introduce a hypothetical idealized photodetector that is infinitely thin and wide, so that it can occupy a hyperplane. We will assume it defines its own preferred frame: we think of it as composed of idealized particles which have a stationary frame. In our models, we will take it to occupy some plane  $t = T$  (for some  $T > 0$ ) in the centre-of-mass frame, and assume this coincides with its own preferred frame. We suppose it produces as measurement output a location  $\underline{r}$  at which the photon was detected at time  $T$ . We assume that the photon is absorbed in this process, so it has no post-detection state or wave function. Moreover, since our real interest is in the asymptotic properties of the photon wave function at late times, we take the idealized photodetector to be a construct defined only for the purpose of taking an asymptotic limit. This means we need not define any rules at all for physics after it acts. We will also assume that the idealized photodetector is perfectly efficient. As the squared amplitude for photon emission tends to 1 for large times, this implies that the probability of it detecting the photon at some point  $\underline{r}$  in space should tend to 1 as  $T \rightarrow \infty$ .

This idealized photodetector is meant as a useful mathematical fiction. We do *not* claim that it can be arbitrarily well approximated by real photodetectors, nor that it implements standard projective quantum measurements. Readers who prefer not to use this construct may revert to the standard interpretation of the photon wavefunction as defining approximate average low rate detection probabilities in suitably large regions, and consider detectors set up throughout space at a late time  $T$ . This requires approximations and epsilons when our postulates are used to define beables, but otherwise gives a similar picture.

We have

$$|\gamma(\underline{r}, t)|^2 \approx K^2(\sin \theta/r)^2\theta(t - (r/c))e^{-\Gamma(t - (r/c))}, \quad (5)$$

and our assumptions require

$$\lim_{t \rightarrow \infty} \int d^3r |\gamma(\underline{r}, t)|^2 = 1. \quad (6)$$

We neglect the approximation in the derivation of  $\gamma(\underline{r}, t)$ , and assume the limit  $L \rightarrow \infty$ , taking equation (5) to be exact, from now on. This gives us

$$K^2 2\pi = c^{-1}\Gamma \quad \text{i.e. } K = \sqrt{(\Gamma/2\pi c)}. \quad (7)$$

Suppose that the photodetector is in the plane of constant time  $T$ . Our rules imply that the probability of detection at some position with radial distance  $r$  is

$$K^2(2\pi)\theta(T - (r/c))e^{-\Gamma(T-(r/c))} = c^{-1}\Gamma\theta(T - (r/c))e^{-\Gamma(T-(r/c))}. \quad (8)$$

This gives us the probability density for the possible measurement outcomes at late time  $T$ . So the probability density for the radial distance of the detection  $r$  satisfying  $r = c(T - t)$  is  $c^{-1}\Gamma\theta(t)e^{-\Gamma t}$ , and the probability of  $r$  lying in the range  $[cT - c(t + dt), cT - ct]$  is  $\Gamma\theta(t)e^{-\Gamma t}dt$ .

We ignore recoil and spreading of the atom position space wave function, taking an idealised model in which the atom remains fixed at  $\underline{r} = \underline{0}$  for all  $t$ . The quantum state of the atom plus photon at time  $t > 0$  is then given by

$$|e\rangle e^{-\Gamma t/2} + |g\rangle \gamma(\underline{r}, t), \quad (9)$$

where  $|e\rangle$  is the excited state of the atom and  $|g\rangle$  the ground state. Consider now the hypersurface defined by the light cone from  $(\underline{0}, t)$  to late time  $T$ , joined with the outward part of the time  $T$  hyperplane. The effective quantum state on this hypersurface, defined as the limit of the quantum states on spacelike hypersurfaces that tend to it, is

$$|e\rangle e^{-\Gamma t/2} + |g\rangle \gamma(\underline{r}, T)\theta(r - (T - t)c). \quad (10)$$

In summary, a detection on the final hyperplane at radius  $r = cT - ct_0$  occurs with probability density  $\Gamma\theta(t_0)e^{-\Gamma t_0}$ . The detection takes place on the relevant limiting hypersurface for defining the beables if  $t \geq t_0$ . Our postulates imply that, given such a detection, the atom energy state beable is  $|e\rangle\langle e|$  for  $t < t_0$  and  $|g\rangle\langle g|$  for  $t \geq t_0$ . In other words, the atom energy state beable undergoes a transition from excited state to ground state at a definite time  $t_0$ , which is randomly selected, with probability density  $\Gamma\theta(t_0)e^{-\Gamma t_0}$ .

Of course, from the perspective of observers within the system, who are unaware of the outcome of the final measurement (both because it occurs at late times and because it is anyway fictional!), the beable transition time is not predictable. Such an observer can only assign the beable the mixed state

$$(1 - e^{-\Gamma t_0})|g\rangle\langle g| + e^{-\Gamma t_0}|e\rangle\langle e|. \quad (11)$$

Nonetheless, on the view we are advocating, this subjective mixed state describes the observer's incomplete knowledge of an objective fact. At any given time  $t_0$ , the atom energy beable is either in the state  $|e\rangle\langle e|$ , or has transitioned to state  $|g\rangle\langle g|$ . In the latter case, in this simple model, it remains in this state at all later times.

### Example 2

Suppose now that at time  $t = 0$  the atom is in a superposition of two position states, which we call  $|\pm d/2\rangle$ . These have centre of mass at  $\underline{r}_\pm = (0, 0, \pm d/2)$ , where the atomic dipole is aligned with the  $z$  axis. As before, it is introduced at  $t = 0$  in the excited state  $|e\rangle$ . We thus take the initial atom state to be

$$\alpha| -d/2\rangle|e\rangle + \beta|d/2\rangle|e\rangle. \quad (12)$$

As before, we will ignore recoil and the spread of the atom position space wavefunction. Thus the time  $t$  atom-photon state is

$$\alpha(| -d/2\rangle|e\rangle e^{-\Gamma t/2} + | -d/2\rangle|g\rangle\gamma(\underline{r} - \underline{r}_-, t)) + \beta(|d/2\rangle|e\rangle e^{-\Gamma t/2} + |d/2\rangle|g\rangle\gamma(\underline{r} - \underline{r}_+, t)). \quad (13)$$

We take  $d \gg \lambda$ , where as above  $\lambda = c/\omega$  is the characteristic wavelength, and  $\Gamma \ll \omega$ . For  $\Gamma \ll \omega$ , the overlap is [27]

$$\int d\underline{r} \gamma^*(\underline{r} - \underline{r}_-, t)\gamma(\underline{r} - \underline{r}_+, t) = \frac{3(\sin(2\pi d/\lambda) - (2\pi d/\lambda)\cos(2\pi d/\lambda))}{(2\pi d/\lambda)^3}, \quad (14)$$

which is small for  $d \gg \lambda$ .

At large time  $T$ , our idealized photo-detector will register a photon either in the effective support of  $\gamma(\underline{r} - \underline{r}_-, T)$  or in that of  $\gamma(\underline{r} - \underline{r}_+, T)$ , and these supports are almost disjoint. Neglecting small contributions from detections at points where  $|\gamma(\underline{r} - \underline{r}_+, T)|^2 \approx |\gamma(\underline{r} - \underline{r}_-, T)|^2$ , we find a detection in the effective support of the former with probability  $|\alpha|^2$ , and the latter with probability  $|\beta|^2$ .

In the first case, a detection at a point  $\underline{r}$ , with  $|\underline{r} - \underline{r}_+| = c(T - t)$ , implies that the energy beable at  $(\underline{r}_+, t)$  transitions from  $|e\rangle\langle e|$  to  $|g\rangle\langle g|$ , as above. However, the position beable also transitions at that point, so that the

total local contribution to the energy at  $\underline{r}_+$  transitions from  $|\alpha|^2|e\rangle\langle e|$  to  $|g\rangle\langle g|$ . At  $\underline{r}_-$ , the transition from  $|\beta|^2|e\rangle\langle e|$  to (approximately) a zero density matrix takes place at  $t'$ , where  $(\underline{r}_-, t')$  is lightlike separated from  $(\underline{r}, T)$ . This gives us

$$t' = t - \frac{1}{c}\hat{n}\cdot(\underline{r}_+ - \underline{r}_-) + O\left(\frac{1}{T}\right), \quad (15)$$

where  $\hat{n}$  is the unit vector in the spatial direction separating the detection point from  $(\underline{r}_+, t)$ . So, in the asymptotic limit  $T \rightarrow \infty$ ,  $t'$  depends only on  $t$  and on  $\hat{n}$ .

Similarly, if the detection is in the effective support of  $\gamma(\underline{r} - \underline{r}_-, T)$ , the energy and position beables at  $\underline{r}_-$  transition to approximately  $|g\rangle\langle g|$ , and those at  $\underline{r}_+$  to approximately zero.

In other words, the beables describe an effective transition from position superposition to (approximately) one component of the superposition. This takes place in a frame that depends on the photo-detection result in a way that has a well-defined asymptotic limit.

### Example 3

Again following MSZ, consider a cascade decay of an atom introduced at the origin at  $t = 0$  in a level two excited state, which decays via a level one excited state to the ground state, emitting two photons with characteristic frequencies  $\omega_1$  and  $\omega_2$  at rates  $\Gamma_1$  and  $\Gamma_2$ . The two-photon wave function [8–10] can be defined from the field state  $|\Psi\rangle$  by

$$\Psi(\underline{r}_1, t_1; \underline{r}_2, t_2) = \sqrt{\kappa'} \langle 0 | \hat{E}^+(\underline{r}_2, t_2) \hat{E}^+(\underline{r}_1, t_1) | \Psi \rangle. \quad (16)$$

We suppose that, in a photon number eigenstate with  $N_\gamma = 2$ , our idealized photodetector lying in the hyperplane  $t = T$  will always detect two photons, producing as measurement outputs the pair of positions  $\underline{r}_1$  and  $\underline{r}_2$  with probability  $|\Psi(\underline{r}_1, T; \underline{r}_2, T)|^2$ , which gives us a normalisation condition by integrating over  $\underline{r}_1$  and  $\underline{r}_2$ . Again, we stress that our measurement rules for the idealized photodetector are mathematical fictions, not arbitrarily well approximated by real photodetectors.

MSZ obtain

$$\begin{aligned} \psi(\underline{r}_1, t_1; \underline{r}_2, t_2) = & K'(\sin \theta_1/r_1)(\sin \theta_2/r_2)\theta(t_1 - (r_1/c))e^{-(i\omega_1 + (\Gamma_1/2))(t_1 - (r_1/c))} \\ & \theta((t_2 - (r_2/c)) - (t_1 - (r_1/c)))e^{-(i\omega_2 + (\Gamma_2/2))((t_2 - (r_2/c)) - (t_1 - (r_1/c)))} \\ & + (1 \leftrightarrow 2). \end{aligned} \quad (17)$$

Taking  $t_1 = t_2 = T$  allows us to take  $r_2 < r_1$  without loss of generality. We obtain, as above, the probability of detection of at least one photon at time  $T$  and radial distance  $r_1 > (T - t)c$  to be  $(1 - e^{-\Gamma_1 t})$ , for  $0 < t < T$ . Conditioned on detecting at least one photon, and with  $r_1$  as the maximum radial distance of detection, the probability of detection of a second photon at radial distance  $r_2 < r_1$  is  $(1 - e^{-\Gamma_2((r_2 - r_1)/c)})$ . Again, these results are independent of  $T$ , so the asymptotic limit for  $T \rightarrow \infty$  is well defined. According to our postulates, the atom energy beables now transition from the second excited state to the first excited state, and then from the first excited state to the ground state. These transitions almost certainly occur by time  $t$  within a few multiples of  $(\Gamma_1^{-1} + \Gamma_2^{-1})$ .

### Example 4

Consider again the atom of Example 1, introduced in an excited state at  $(\underline{r}, t) = (\underline{0}, 0)$ . Suppose now that we have a larger massive object, which acts as an essentially perfect absorber of photons over a range of frequencies including the atom's characteristic transition frequency  $\omega$ . Suppose that the object is spherical, with radius  $R \gg 10\Gamma^{-1}c$ , and suppose it is initially in a superposition of two stationary states, which we call  $|0\rangle_{\text{obj}}$  and  $|100\rangle_{\text{obj}}$ , with centres of mass  $\underline{r}_0 = \underline{0}$  and  $\underline{r}_{100} = (100R, 0, 0)$  respectively. As with the atom, we neglect the spread or evolution of the object's position space wave function, and we also neglect any momentum transfer from photon absorption. We also assume the atom can coexist with the object in state  $|0\rangle_{\text{obj}}$  without any interaction other than the emission and subsequent absorption of a single photon, although they have the same centre-of-mass.

To make this more plausible, we may take the object to be a thick uniform spherical shell with outer radius  $R$  and inner radius  $R' < \Gamma^{-1}c$ . To make it even more plausible, we may allow a small hole in the shell, so that the atom

can be introduced at  $(0,0)$  without interacting with the  $|0\rangle_{\text{obj}}$  component of the object, and thus without initially affecting the superposition. We neglect the small amplitude of the emitted photon propagating through the hole in the calculations below.

The initial combined state is thus

$$|e\rangle_{\text{atom}}(\alpha|0\rangle_{\text{obj}} + \beta|100\rangle_{\text{obj}}). \quad (18)$$

At late time  $T$ , this evolves to approximately

$$|g\rangle_{\text{atom}}(\alpha|0^*\rangle_{\text{obj}} + \beta|100\rangle_{\text{obj}})\gamma(\underline{r}, T) \quad (19)$$

where  $|0^*\rangle$  is the state of the object at centre of mass  $\underline{r} = 0$  after having absorbed a photon,  $\gamma(\underline{r}, T)$  is the photon wave function calculated as in Example 1, and we neglect small components arising from the object in state  $|100\rangle_{\text{obj}}$  absorbing a photon propagating close to the positive  $x$  direction, and also neglect the exponentially small amplitude of the atom remaining in excited state at time  $T$ .

Our postulates now imply that at late time  $T$  the beable describing the atom energy has evolved to  $|g\rangle\langle g|$ , as before. Our ideal photo-detector will detect a photon with probability  $|\beta|^2$ , in which case our postulates imply beable  $|100\rangle_{\text{obj}}\langle 100|_{\text{obj}}$  for the object. Alternatively, with probability  $|\alpha|^2$ , there is no photo-detection, and our postulates imply beable  $|0^*\rangle_{\text{obj}}\langle 0^*|_{\text{obj}}$  for the object.

In other words, our postulates describe a transition for the object beables, transitioning to one of the superposition components, with the standard Born rule probabilities.

### QFT AND INFRARED CUTOFFS

Our models so far have been within a version of quantum theory in which the photon propagation respects the Minkowski causal structure, but which relies on approximations and which is valid only in a restricted regime. Modelling within full relativistic quantum field theory poses several problems. Among these is that realistic physical processes involving charged particles interacting with an electromagnetic field generally generate an indefinite number of low energy photons. Including these in a model directly would mean we could not restrict to states of finite photon number, nor assume that our late time idealized photodetector would produce only finitely many photodetections.

Standard treatments of QED solve the infrared divergence problem by noting that real world detectors cannot detect photons below some cutoff energy. The amplitudes corresponding to photon emissions below this energy can thus be combined with amplitudes involving low energy virtual photons, producing a finite sum from two contributions that would separately be infinite. This suggests possible ways to treat the infrared divergence in our model.

Perhaps the simplest option is to impose an infrared cutoff in the calculations for each late time  $T$  hypersurface, so that photon states of frequency less than  $\nu(T)$  are neglected, with  $\nu(T) \rightarrow 0$  as  $T \rightarrow \infty$ . This would imply a finite number of photodetections on any given late time hypersurface, although that number would generally be expected to tend to infinity as the hypersurface moves towards future infinity. Taking photodetections of photons with average frequency  $\nu(T)$  to give an outcome with position uncertainty at least some multiples of the wavelength  $\lambda(T) = c/\nu(T)$  suggests requiring  $\nu(T) \approx Nc/T$ , for not very large  $N$ , in the relevant frame. Intuition then suggests that (i) soft photon emissions, from bremsstrahlung or other processes, should correlate with the states of matter in a very similar way to the emissions and scatterings of hard photons, modelled above, (ii) the beable expectation values inferred from detections of soft and hard photons ought to be approximately consistent, in a way which tends to a stable limit as  $T \rightarrow \infty$  and  $\nu(T) \rightarrow 0$ . Testing this intuition in models that include bremsstrahlung is a task for the future.

As an approximation, one could replace our idealized photodetector at late time  $T$  with a more realistic (though of course, still fictional) array of adjacent photodetectors, which have finite volumes  $V \approx L(T)^3$ , have a centre-of-mass frame, and which only detect photons above a certain cutoff frequency  $\nu(T)$  with respect to that frame. This requires  $L(T) = N\lambda(T)$  for some large, but not necessarily very large,  $N$ . We would expect that, even if  $\nu(T) \rightarrow \nu_{\text{min}} > 0$ , the beables should define quasiclassical physics in realistic models, if the cutoff  $\nu_{\text{min}}$  is small enough.

This picture has the advantage that it is empirically supported. The behaviour of real photodetectors is well understood, and we are pretty confident that, all else being equal, we can sensibly speak of the sort of readings we would expect from an array of them at late times, either in Minkowski space models or in the universe we observe. However, it seems hard to make it perfectly precise. Since realistic photodetectors produce an approximate measurement output corresponding to an unsharp measurement of the photon position, this introduces a corresponding uncertainty in the definition of the beable expectation values. Imposing a finite infrared cutoff also appears to break Lorentz invariance, and the photodetectors' volumes and other characteristics require further arbitrary parameter choices. Although the



picture of quasiclassical physics defined by the beables need not be very sensitive to the precise choices made, within sensible ranges, the precise beable values nonetheless will depend at least slightly on these choices.

An underlying issue here is that while in principle relativistic quantum field theory is a Lorentz covariant dynamical theory, and while the recipes that are used to extract experimental predictions from it are consistent with Lorentz symmetry, we do not presently have a well-defined Lorentz covariant theory of approximately localized measurements in quantum field theory. Our resorting here to idealizations or approximations in defining photodetectors reflects this. Some of the well-known no-go theorems [14–16] may perhaps be specific to algebraic quantum field theory. However the lack of rigorous treatments of approximately localized measurements within other formulations of QFT suggests that every formulation of QFT currently has significant gaps in its conceptual framework.

As already noted, for the purpose of defining testable generalizations of quantum theory that make distinctive experimental and cosmological predictions, but which at best are only roughly correct, a lack of perfect precision may not matter. And perhaps complete precision is not possible until we get beyond the current formulation of quantum field theory and/or improve the associated theory of localized measurements. That said, of course an absolutely precisely specified theory, without any arbitrary parameter choices, would be preferable if available.

## FINAL TIME PHOTON FREQUENCY MEASUREMENTS

Another possible solution to the infrared problem is to drop the specific proposal of Ref [3] and use instead ideas outlined in [1, 2], and consider late time measurements of the momentum distribution of photons rather than approximate or exact position measurements.

The photon wave functions and number operators are well-defined and local in momentum space. We can thus simply postulate that a standard (idealised) quantum measurement takes place on a late time hyperplane  $t = T$  in some frame, producing as outcome a set of photon momenta  $\underline{p}_1, \underline{p}_2, \dots$ . This set may be infinite, and the  $\underline{p}_i$  may not necessarily be distinct.

Instead of defining beables at a point  $x$  via the construction of Ref. [3], described above, we use the ABL rule [17] construction of Refs. [1, 2]. The beable expectation values at a point  $x$  are defined on hypersurfaces asymptotically tending to the past surface of  $x$ . To do this, we use the initial state on the initial hypersurface as pre-selected initial data, and the outcome of the final momentum measurement on the late time hyperplane as post-selected final data. Here, as above, we use the final hypersurface photon data to define expectation value beables for the massive particle degrees of freedom.

The intuition is that this renders the infrared problem innocuous, since (i) the ABL rule allows us to make inferences about the massive particle expectation value beables from the distribution of the photons above a lower cut-off frequency, (ii) these inferences are either essentially confirmed, or else unaffected, by including the distribution of soft photons in the final measurement outcome. That is, either the distribution of soft photons is essentially uncorrelated with the massive particle states, in which case including that distribution makes essentially no difference, or it is usefully correlated. Either way, in realistic models, we expect the beables inferred from photons above a low energy cut-off to already characterise the expectation value beables fairly precisely, and including the entire photon spectrum to make only small corrections.

As with our earlier examples, testing this intuition in models that include bremsstrahlung is a task for the future. However, putting infrared problems aside, we can illustrate the basic proposal with another toy example:

### Example 5

Consider again the atom-object system of Example 4. The initial combined state is

$$|e\rangle_{\text{atom}}(\alpha|0\rangle_{\text{obj}} + \beta|100\rangle_{\text{obj}}). \quad (20)$$

At late time  $T$ , this evolves to approximately

$$|g\rangle_{\text{atom}}(\alpha|0^*\rangle_{\text{obj}} + \beta|100\rangle_{\text{obj}})\gamma(\underline{x}, T) \quad (21)$$

if we represent the photon by a position space wave function. Representing the photon by a momentum space wave function instead, we have

$$|g\rangle_{\text{atom}}(\alpha|0^*\rangle_{\text{obj}} + \beta|100\rangle_{\text{obj}})\gamma(\underline{p}, T) \quad (22)$$

where  $\gamma(\underline{p}, T)$  is sharply peaked around momenta for which  $|\underline{p}| = h\nu/c$ . A late time hyperplane measurement of the photon momentum distribution, in this model, will either (with probability  $|\alpha|^2$ ) produce no photon, or (with probability  $|\beta|^2$ ) produce one photon with momentum  $\underline{p}$ , obtained randomly from the distribution  $|\gamma(\underline{p}, T)|^2$ . From the ABL rule, we infer object position expectation value beables  $|0^*\rangle\langle 0^*|$  and  $|100\rangle\langle 100|$  respectively, at any time when the object is in the quantum superposition above. (We also infer atom energy expectation value beables  $|g\rangle\langle g|(1 - e^{-\Gamma t}) + |e\rangle\langle e|e^{-\Gamma t}$ , regardless of the photon momentum measurement outcome.)

In realistic models, roughly localised collections of photons with different momentum distributions will be found in different regions. In our universe, this is so because of the uneven distribution of galaxies, stars, and inhomogeneities arising at early times after the Big Bang; on Earth, of course, the distribution of smaller energy sources is also uneven. This model illustrates that these distributions, combined with a late time photon momentum distribution measurement, can allow a quasiclassical description of massive particles with which the photons (may) interact. Note that we would get the same qualitative result if we allowed the object to re-emit the photon energy after absorption, so long as it emitted a different spectrum (for example, several photons of lower energy).

A striking feature of this example, in comparison to example 4, is that the object expectation value beables are assigned to one or other component of the position superposition as soon as that superposition is created, even if the interaction with the atom occurs much later. In examples 1-4, we used a construction [3] that makes inferences about the beables at  $x$  only from final time measurement results outside the future light cone of  $x$ . In this example, following the ideas of Refs. [1, 2], we use measurement outcomes on the electromagnetic field from the entire late time hypersurface to define beables for the massive particles, via the ABL rule [17]. In some sense, in this version, real (not just fictional late time) future measurement interactions between photons and massive particles cause past measurement outcomes for the massive particles. This feature may be counterintuitive or unaesthetic to some, but it is not clear that *per se* it is fundamentally problematic.

## CONCLUSIONS

It has long been well understood that photons very effectively decohere quantum systems (e.g. [18–20]). Our simple models could be made more realistic by drawing on these and other earlier analyses. The point of our models, however, is not to give another illustration of decoherence, but rather to illustrate that a coherent picture of quantum reality can be constructed from a single postulate [3]. This relies on the observation that a significant fraction of photons scattered from massive objects will propagate away to future infinity, with little or no further scattering. As our models illustrate, given the postulate and the right type of initial conditions, a single photon scattering to infinity can suffice. In our cosmology, we expect, the generic anisotropy of the photon environment supplies the right type of initial conditions and the relatively sparse distribution of matter allows some fraction of photons to propagate with little or no scattering. The apparently indefinite future expansion, with the corollary that interactions become less and less frequent, appears to allow the right asymptotic behaviour for the final time measurement postulate to be well defined.[28]

We have thus illustrated the proposal of Ref. [3] in some simple but (in their bare essence) qualitatively realistic quantum mechanical models, and shown it works satisfactorily in these models, giving natural realist beable picture for finite time physics from hypothetical asymptotically late time measurements on the electromagnetic field. We discussed the problem of infra-red effects, and proposed possible solutions. For comparison, we have also illustrated an alternative proposal [1, 2], which offers another possible way of addressing infrared problems and gives a qualitatively different, though also apparently consistent, beable picture.

These illustrations progress well beyond the toy models of Ref. [3]. More detailed models, and in particular models incorporating a more fundamentally satisfactory treatment of photon measurements, would still be desirable in developing this work further.

Even at the present level of development, though, we have set out a simple proposal for a rule that allows branches of the universal wave function to be approximately characterised by fictitious late time measurements of the electromagnetic field in realistically modelled photodetectors. Again we stress that, while this would not work in every cosmology, it should in an indefinitely expanding universe like ours in which interactions become less and less frequent at late times.

This gives us a natural one-world alternative to Everettian quantum theory, according to which only one branch is realised, chosen randomly via the Born rule applied to these late time measurements. The branching structure in our approach is most naturally described in terms of evolving expectation value beables, rather than (for example) sequences of orthogonal projective measurements. In our opinion, it compares well to other approaches, offering a

much simpler description of quasiclassical physics than achieved (at least to date) by any set selection rule in the consistent or decoherent histories approach to quantum theory [21–24] for example.

The models presented here have supposed that beables throughout space-time can be constructed from fictitious asymptotic late time measurements of photons. This is a natural choice when considering physics in Minkowski space or other fixed background space-times, assuming that photons are indeed massless, since if photons propagate outside the future light-cone  $L_x$  of a point  $x$  they do not return, absent rescattering that reflects them back into  $L_x$ . Assigning this role to gravitons might be an even more natural choice in a quantum theory of gravity. There are at least two reasons for this. First, postulating a fundamentally different role for matter and gravitational degrees of freedom is arguably more natural than one between massive and massless particles. Second, rescattering and reflection back of gravitons into the future light cone is less generic. In particular, good photon mirrors exist, but gravitational mirrors (in the standard sense) do not.

However, the nonlinearities of quantum gravity make it hard to set out a conceptually consistent discussion, since the future causal structure itself is affected by radiated gravitons. Without a full quantum gravity theory, it is in any case hard to give convincing illustrations in simple models. Nonetheless, it would be interesting to explore this possibility further.

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- [25] Or, as in the models below, we can let the dynamics effectively ensure that interactions are localized, with perhaps some exponentially decaying amplitude for late time interactions.
- [26] The relevant operator here would be the full stress-energy tensor, including the stress-energy of the electromagnetic field. Late time detections of photons play a special role in our postulates, but this does not require us to treat the electromagnetic stress-energy as any “less real” than that of massive particle fields. However, in the toy models below, we focus on operators describing the states of massive particles, since the models include only one or two photons, which ultimately escape to infinity.
- [27] This follows from MSZ’s given calculation method and agrees with their plot in their Figure 2(b). In particular, it gives overlap 1 at  $d = 0$ , as expected. MSZ’s stated result, given in their equation (14), appears to contain typos.
- [28] It should be acknowledged that there is theoretical and empirical uncertainty about the true asymptotic final cosmological state. For example, if, as some imagine, it reaches a true vacuum state, quasiclassical physics in earlier eras could not be reconstructed from our asymptotically late time measurement postulate.