# $|V_{cb}|$ using lattice QCD

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Lattice QCD calculations of hadronic matrix elements allow one to draw inferences about quark flavor interactions from measurements of hadron decays. Within the context of the Standard Model, the magnitude of the charm-bottom quark coupling $V_{cb}$ can be determined from semileptonic decays such as $B \to D^{(*)} \ell \nu$. This brief review summarizes the present status and short-term outlook for determining $|V_{cb}|$ using lattice QCD.

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## 1. Introduction

At this workshop the CKM matrix element $V_{cb}$ needs no introduction. Its present estimates are summarized in Fig. 1. The most precise determination of $|V_{cb}|$ using an exclusive decay mode comes from combining experimental results for $B \to D^* \ell \nu$ with the relevant form factor at zero recoil [1, 2].[1] With similar precision, one can infer $|V_{cb}|$ from inclusive semileptonic $b \to c$ decays using an operator product expansion [4, 5]. These two values disagree at the $3\sigma$ level.

The determination of $|V_{cb}|$ from $B \to D\ell\nu$ decay has been less precise due to larger experimental uncertainties. Figure 1 shows the published results from two collaborations [6, 7]. The FLAG combined fit of the lattice results [8] including new experimental results [9] is also shown. The fact that the $B \to D\ell\nu$ $|V_{cb}|$ determinations lie between those from $B \to D^*\ell\nu$ and $B \to X_c\ell\nu$ implies that the explanation for the discrepancy is not due to new physics manifesting itself as a new right-handed interaction. Such an effect would cause the $B \to D\ell\nu$ determination to be the outlier, since only the vector current contributes to this decay.



**Figure 1:** $|V_{cb}|$ from different inputs.

Below I discuss a few more details about published work on this topic, and I review the status of calculations in progress. First I wish to give a brief survey of some lattice QCD details.

## 2. Survey of methods

The choice of discretization of the Dirac Lagrangian (or action) is a crucial decision one makes when carrying out a lattice QCD (LQCD) calculation. Figure 2 summarizes (in part) the unquenched lattice configurations relevant for recent and present calculations of $|V_{cb}|$; each graph corresponds to a different lattice action. The plots show the pion mass, a proxy for the light quark mass input into the calculation, against the lattice spacing inferred by requiring some dimensionless lattice output be equal to its physical value. Both quantities enter the plot squared, making extrapolation to the physical limit roughly linear. For $B$ decays, effects of discretization and unphysical quark mass have been the most sensitive to get under control. Of course one needs to ensure other systematic errors, such as finite volume effects, are also quantified.

The LQCD results used in Fig. 1 were all obtained using improved versions of the staggered quark action, using the MILC AsqTad configurations. Staggered quarks are computationally inexpensive and can be improved to have small discretization errors; however, an additional assumption is required to include a number of sea quarks which is not a multiple of 4. There is a body of literature discussing this approach, and the empirical evidence supports its soundness.

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[1]Note a new $B \to D^*\ell\nu$ result, $(37.4 \pm 1.3) \times 10^{-3}$, from Belle recently appeared [3].

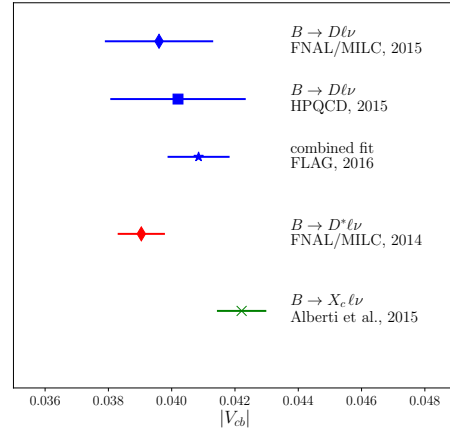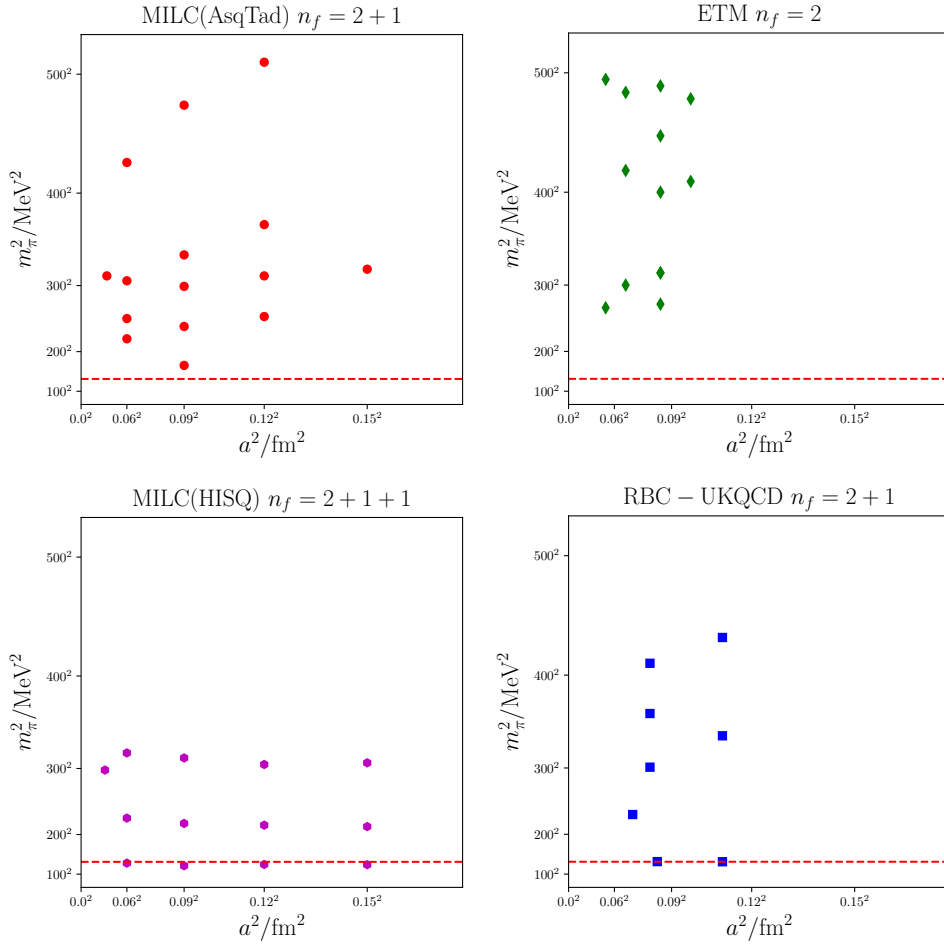**Figure 2:** Plots showing the pion masses and lattice spacings for unquenched gauge field configurations with $n_f$ flavours of sea quarks. Each plot corresponds to a separate ensemble of configurations, differing most significantly in the choice of fermion discretization (see text). The red dashed line corresponds to the physical pion mass.

The ETM collaboration use twisted mass quarks, an improved variant of Wilson's action. The RBC-UKQCD collaboration use domain wall quarks. The plots in Fig. 2 hint at some of the advantages and disadvantages of the different formulations. The (relative) efficiency of staggered fermions allows for calculations to be done with many different sets of input parameters. The high level of improvement possible with staggered actions, first the AsqTad variant then HISQ, mean that discretization errors are greatly reduced at larger values of the lattice spacing. Twisted mass fermions have sizable discretization errors, so calculations with smaller values of *a* have been necessary. Domain wall quarks are computationally expensive; however, there is strong motivation to invest heavily in this approach since they have the continuum-like chiral and flavor symmetries.

Heavy quarks on the lattice provide another challenge. The energy scale at which discretization errors typically become large is given by the inverse lattice spacing. The charm quark mass is at or below this scale, and the bottom quark mass is significantly higher. Effective field theories have been used to make progress, especially for *b* quark physics. Until recently, the Fermilab approach

to heavy quarks has been only one used to pursue high precision lattice QCD calculations of $b \to c$ matrix elements. With the increase in computational power and more highly improved actions, other approaches are now able to provide independent checks of Fermilab/MILC results.

On the MILC AsqTad lattices, the Fermilab/MILC collaboration constructed correlation functions using the same action for the light valence quarks and the Fermilab approach for the $c$ and $b$ quarks. The HPQCD collaboration used the more highly improved, HISQ version of the staggered action for the valence quarks (both light and charm), using nonrelativistic QCD (NRQCD) for the $b$ quark. A subset of the MILC AsqTad configurations were used by HPQCD, while Fermilab/MILC used the whole set. Both groups have used their data to refine the Standard Model prediction for $R(D)$ the ratio of $B \to D\tau\nu$ to $B \to D\ell\nu$ branching fractions (where $\ell = e, \mu$) [10, 6, 7]. Fermilab/MILC have also used their form factors to estimate the neutral $B$ meson fragmentation functions, of use in extractions of the rate for $B_s^0 \to \mu^+\mu^-$ [11].

The Paris group have used the ETM twisted-mass configurations (Fig. 2) to calculate the $B_s \to D_s\ell\nu$ form factors near zero recoil [12]. They interpolate a well-chosen ratio between a known result in the static limit and lattice calculations performed with charm-like masses for the $b$. Their quoted results, e.g. for the form factor extrapolated to zero recoil, are consistent with those obtained on MILC lattices, albeit with much larger uncertainties.

Baryonic decays also play a role in over-constraining $|V_{cb}|$. LHCb has measured the ratio of branching fractions $\mathscr{B}(\Lambda_b \to p\ell\nu)/\mathscr{B}(\Lambda_b \to \Lambda_c\ell\nu)$ [13] which, when combined with LQCD determinations of the corresponding form factors [14], constrains $|V_{ub}/V_{cb}|$ to be $0.083(4)_{\text{stat}}(4)_{\text{sys}}$. Combining this with $|V_{ub}|$, as determined from $B \to \pi\ell\nu$ decay [8] gives $|V_{cb}| = 0.044(3)$, in somewhat better agreement with the higher values plotted in Fig. 1. Using the inclusive determination of $|V_{ub}|$ would imply an even larger value for $|V_{cb}|$.

## 3. Ongoing work

At the Lattice 2016 symposium, there were several talks reporting on calculations underway. The HPQCD collaboration are completing an analysis of the $B_s \to D_s$ form factors [15], on the MILC AsqTad ensembles. In the future, HPQCD and Fermilab/MILC plan to extend their study of the $B_{(s)} \to D_{(s)}$ form factors on the MILC HISQ ensembles, reducing discretization, quark mass extrapolation, and other uncertainties.

HPQCD are in the final stages of calculating the $B \to D^*$ form factor $h_{A_1}(w)$ at zero recoil, $w = 1$ [16]. This calculation has been done using the MILC HISQ lattices, so it will be the first new result statistically independent from the Fermilab/MILC calculations, providing an important check of the discrepancy shown in Fig. 1.

By using two formulations for the $b$ quark, NRQCD and heavy HISQ, to compute the $B_c \to \eta_c$ and $B_c \to J/\psi$ form factors, the HPQCD collaboration have a nonperturbative means to determine the normalization of the NRQCD currents [17]. This provides an opportunity to quantify and possibly reduce a dominant uncertainty in NRQCD calculations.

The RBC-UKQCD collaboration presented preliminary results for the $B_s \to D_s$ form factors using domain wall fermions for light and charm quarks plus the RHQ variant of the Fermilab action for the bottom quark [18]. They plan to extend their work to include all $B_{(s)} \to D_{(s)}^{(*)}$ form factors. These will be useful since the methods used are different from Fermilab/MILC and HPQCD.

The calculation of the $B \to D^*$ form factors away from zero recoil is something several groups are pursuing. The main motivation is to have better control over fits to the shape of the differential branching fraction. To date, $|V_{cb}|$ has been obtained by fitting the experimental data and extrapolating to zero recoil, at which point the lattice calculation described above gives the normalization. Recent experience extracting $|V_{ub}|$ and $|V_{cb}|$ has underscored the efficacy of using *both* lattice and experimental information over a range of lepton invariant mass $q^2$. The challenges extending this method to the vector meson final state come from having to determine more form factors, and to overcome larger statistical errors in the lattice calculations. It is likely that progress will first come for the $B_s \to D_s^*$ form factors, making a measurement of the $B_s \to D_s^* \ell \nu$ differential decay rate very desirable. Eventually these calculations will also refine the Standard Model prediction for the $\tau/\ell$ final state ratio $R(D^*)$.

## Acknowledgments

## References

[1] Y. Amhis *et al.*, (2012), arXiv:1207.1158.

[2] J. A. Bailey *et al.*, Phys. Rev. D **89**, 114504 (2014), arXiv:1403.0635.

[3] A. Abdesselam *et al.*, (2017), arXiv:1702.01521.

[4] A. J. Bevan *et al.*, Eur. Phys. J. C **74**, 3026 (2014), arXiv:1406.6311.

[5] A. Alberti *et al.*, Phys. Rev. Lett. **114**, 061802 (2015), arXiv:1411.6560.

[6] J. A. Bailey *et al.*, Phys. Rev. D **92**, 034506 (2015), arXiv:1503.07237.

[7] H. Na *et al.*, Phys. Rev. D **92**, 054510 (2015), arXiv:1505.03925, [Erratum: Phys. Rev. D **93**, 119906 (2016)].

[8] S. Aoki *et al.*, Eur. Phys. J. C **77**, 112 (2017), arXiv:1607.00299.

[9] R. Glattauer *et al.*, Phys. Rev. D **93**, 032006 (2016), arXiv:1510.03657.

[10] J. A. Bailey *et al.*, Phys. Rev. Lett. **109**, 071802 (2012), arXiv:1206.4992.

[11] J. A. Bailey *et al.*, Phys. Rev. D **85**, 114502 (2012), arXiv:1202.6346, [Erratum: Phys. Rev. D **86**, 039904 (2012)].

[12] M. Atoui *et al.*, Eur. Phys. J. C **74**, 2861 (2014), arXiv:1310.5238.

[13] R. Aaij *et al.*, Nature Phys. **11**, 743 (2015), arXiv:1504.01568.

[14] W. Detmold, C. Lehner, and S. Meinel, Phys. Rev. D **92**, 034503 (2015), arXiv:1503.01421.

[15] C. J. Monahan *et al.*, Proc. Sci. **LATTICE2016**, 298 (2016), arXiv:1611.09667.

[16] J. Harrison, C. Davies, and M. Wingate, Proc. Sci. **LATTICE2016**, 287 (2016), arXiv:1612.06716.

[17] B. Colquhoun *et al.*, Proc. Sci. **LATTICE2016**, 281 (2016), arXiv:1611.01987.

[18] J. Flynn *et al.*, Proc. Sci. **LATTICE2016**, 296 (2016), arXiv:1612.05112.