Wormhole throats in R^m gravity

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Abstract

We consider wormhole geometries subject to a gravitational action consisting of non-linear powers of the Ricci scalar. Specifically, wormhole throats are studied in the case where Einstein gravity is supplemented with a Ricci-squared and inverse Ricci term. In this modified theory it is found that static wormhole throats respecting the weak energy condition can exist. The analysis is done locally in the vicinity of the throat, which eliminates certain restrictions on the models introduced by considering the global topology.

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1   Introduction

Wormhole type solutions in Einstein gravity have long been studied. Flamm [1] seems to have been the first to study such objects while considering the then newly discovered solution of Schwarzschild. In 1935, Einstein and Rosen considered wormhole type bridges as potential models for elementary particles [2]: the famous Einstein-Rosen bridge. The field lay dormant for approximately two decades until the consideration by Wheeler of the possibility of a space-time foam. This foam, due to violent fluctuations

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in the metric when considering quantum gravity, could be viewed as wormhole-like structures permeating through space-time which would potentially be manifest at energies near the Planck scale [3]. As well, wormholes have relevance to issues such as chronology protection [4], topology change [5] and in studies of horizons and singularities [6], [7].

There has been a sizeable amount of literature produced over the past 15 years on the issue of Lorentzian as well as Euclidean wormholes. This re-newed interest in the subject was mainly sparked by the work of Morris and Thorne [8] where they considered static, traversable wormholes for pedagogical purposes. In their work, and subsequent papers by others, it was found that a static wormhole throat could not be supported unless the weak energy condition (WEC) was violated. The violation could be confined to a small region in the vicinity of the throat but it must occur to prevent the throat from collapsing (see [9], [10], [11], [12], [13] and references therein). Non-spherical geometries have also been studied (some examples are [14], [15], [16], [17]). For a more complete list of pre-1996 references, as well as an excellent exposition on the subject, the reader is referred to the book by Visser [18].

There have been many studies of static wormhole solutions employing various exotic matter models within Einstein gravity. However, it is possible that the action governing gravitational dynamics is not the Einstein-Hilbert one, but consists of a more complicated Lagrangian, which reduces to the usual Einstein-Hilbert action in some limit. In this vein theories with generalized actions of the form [1]:

$$ S = \frac{1}{16\pi} \int \mathcal{L}(R) \sqrt{g} \, d^4x + S_{matter} \, , \tag{1} $$

with $\mathcal{L}(R)$ being some function of the Ricci scalar, have been employed to explain various phenomena [20]. The most popular of these supplements the Einstein-Hilbert Lagrangian with an R^2 term so that $\mathcal{L}(R) = R + \alpha \, R^2$ (for recent work the reader is referred to [21]) . This leads to modifications which could be utilized to drive inflation purely from the gravitational sector (the Starobinsky inflationary theory [22]). A thorough study of solutions in R^m cosmology may be found in the recent papers [23] as well as [24].

A more recent modification which has received attention is the addition of a $1/R$ term in the Lagrangian. The major motivation for this modification is in the realm of matter dominated era cosmology. For example, it is thought that perhaps such a term may contribute to the present day acceleration of the universe without the need for an exotic dark energy [25], [26], [27], [28], [29]. Inverse Ricci terms may also appear in certain sectors of string/M theory (see [30], [31] for example). These are interesting ideas although it has been shown that adding inverse powers of the Ricci scalar to the gravitational Lagrangian is accompanied by some potential problems [32], [33], [35], [36].

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[1]Notations and conventions here follow those of [19]

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Several issues are those of stability and the Newtonian limit. It has been argued that, in both the metric variation and the Palatini formulation of the $1/R$ theory, instabilities or potentially unphysical weak-field limits may exist. The Newtonian limit for the metric variation theory has been carefully studied by Dick [37] who derived criteria for well defined Newtonian limit. (A similar analysis has been done by Domínguez and Barraco [35] in the Palatini formulation). We address this, and the issue of stability, later in the paper. It has also been shown by Chiba [33] that if the $1/R$ sector is to drive the present day cosmological acceleration, the theory will yield results incompatible with solar system experiments. However, this does not preclude singular non-linear gravity theories which do not dominate present day cosmological evolution yet whose effects may be important under conditions as those potentially found near wormhole throats. This is also the motivation in a recent paper [34] where the future evolution of the universe is considered with inverse curvature Lagrangians.

For the above reasons we believe it is of interest to study wormhole geometries in a theory whose action is of the form (1). That is, can the modified gravitational sector of the theory eliminate the need for exotic matter in supporting a static wormhole throat? The most complete case for which the analysis is tractable seems to be the choice $\mathcal{L}(R) = c_{-1}R^{-1} + R + c_2R^2$, the c_n's being the "coupling" constants of the theory. The R^2 theory was considered by Ghoroku and Soma [38]. In their meticulous study it was concluded that, under the assumption that an asymptotically flat global solution exists, no WEC respecting wormhole can exist in such a theory. The $1/R$ theory has not been considered in the context of wormholes.

Here we study wormholes from a local geometric perspective, without constraining the asymptotics. As pointed out in [39], geometric information is less limiting than topology to the issue of defining and locating wormhole throats. We find that in the local analysis, R^2 terms permit the existence of a WEC respecting throat. The near throat solution can be patched, via appropriate junction conditions, to other solutions with desired far throat geometry and topology. This approach is often taken in, for example, studies of stellar structure or gravitational collapse. We briefly discuss the junction conditions in a later section.

There are compelling reasons to believe that if gravitational fields are dictated by an action such as (1), then the non-linear contributions could be important in the case of wormholes. The R^2 terms would certainly be important in the high curvature regime which may be present in the vicinity of wormhole throats. As well, higher derivative contributions, introduced by both the R^2 and $1/R$ terms, may be important since the wormhole structure only places restrictions on the spatial components of the metric and its first derivatives. We show below that, in the WEC, the dominant terms near the throat are in fact those introduced by these non-linear terms, regardless of how small the coupling constants may be. It is found that the near throat region can respect the WEC in the higher derivative theory.

2 R^m gravity and the wormhole geometry

The general R^m action is given by:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{g} \sum_m c_m R^m + S_{matter}, \tag{2}$$

which, in the metric variation theory, gives rise to the equation of motion:

$$R^\mu_\nu \sum_m mc_m R^{m-1} - \frac{1}{2}\delta^\mu_\nu \sum_m c_m R^m + \delta^\mu_\nu \sum_m mc_m \left(R^{m-1}\right)^{;\sigma}_{;\sigma} - \sum_m mc_m \left(R^{m-1}\right)^{;\mu}_{;\nu}$$

$$=: G^\mu_\nu + H^\mu_\nu = 8\pi T^\mu_\nu \ , \tag{3a}$$

with

$$G^\mu_\nu := R^\mu_\nu - \frac{1}{2}R\,\delta^\mu_\nu \ . \tag{3b}$$

In the subsequent text, we consider only $m = -1,\ 1$ and 2. In such a theory, the vacua $(T^\mu_\nu = 0)$ are the de Sitter and anti-de Sitter solutions with effective cosmological constant, Λ, given by:

$$\Lambda = \pm\frac{\sqrt{3}}{4}\sqrt{|c_{-1}|} = \frac{R_0}{4} \quad \text{for}\ \ c_{-1} < 0. \tag{4}$$

Here, R_0 is the Ricci scalar of the vacuum solution.

In the curvature coordinates often employed in wormhole studies, the static line element may be written as:

$$ds^2 = -\,e^{\gamma(r)}\,dt^2 + e^{\alpha(r)}\,dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2, \tag{5}$$

$$-\infty < t < \infty, \quad 0 < r_0 \le r < r_1, \quad 0 < \theta < \pi, \quad 0 \le \phi < 2\pi \ .$$

As we are interested in the region near the throat, we consider here only the local geometry in this vicinity. The $t =$constant, $\theta = \pi/2$ profile curve is displayed in figure 1 where it is embedded in a higher dimensional space, whose extra coordinate is denoted x. Geometric structure is given to the wormhole manifold by imposing the constraint $x = P_\pm(r)$, with $P_\pm(r)$ being the shape function of the profile curve. (The $+$ corresponds to the upper portion of the curve while the $-$ corresponds to the lower portion). The wormhole is obtained by considering the surface of revolution generated when rotating the profile curve about the x-axis (inset). The lower half of the wormhole is obtained via a similar construction with a potentially different profile function. The line element (5) may be written is terms of the shape function as:

$$ds^2 = -e^{\gamma(r)\pm}\,dt^2 + \left\{1 + [P_{\pm,r}(r)]^2\right\}\,dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2\ . \tag{6}$$

It is sufficient to study only the upper portion of the wormhole $(P_+(r))$ and we therefore suppress the subscript in the following.
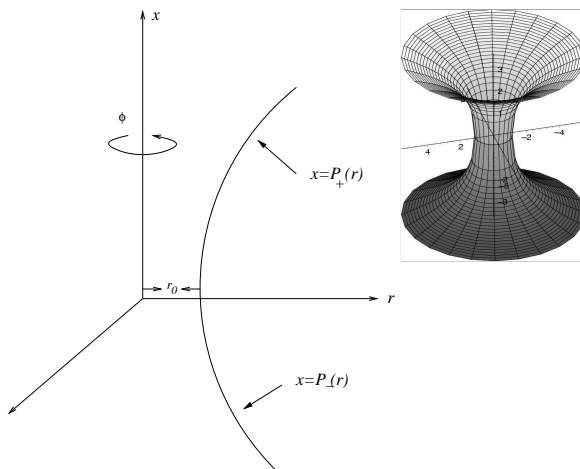
The restrictions on $P(r)$ are evident from the figure:

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Figure 1: Profile curve, $P_\pm(r)$, in the $t =$constant and $\theta = \pi/2$ submanifold. The wormhole is generated via rotation about the x-axis (inset).

- $P(r)$ must possess positive first derivative, at least in the vicinity of the throat.

- $P(r)$ must possess negative second derivative near the vicinity of the throat.

- $P_{,r}(r) \to +\infty$ as $r \to r_0$ (the throat).

It is this last condition which is problematic in the study of wormhole throats. Although the infinities should cancel out in calculations of physical quantities involving non-singular throats, the analysis (both numerical and analytic) is hampered by the presence of these infinities. A resolution is to switch to a different coordinate system where such infinities are not present.

Another coordinate system, sometimes utilized in wormhole analysis, is the the proper length gauge where the line element takes on the form:

$$ds^2 = -e^{\mu(\rho)}\, dt^2 + d\rho^2 + r^2(\rho)\, d\theta^2 + r^2(\rho) \sin^2 \theta\, d\phi^2. \tag{7}$$

The coordinate ρ represents the (signed) proper length coordinate in the radial direction:

$$\rho(r) = \pm \int_{r_0}^{r} e^{\alpha_\pm(r')/2}\, dr' \, . \tag{8}$$

The throat in this chart is located at $\rho = 0$. Although continuous at the throat, the equations of motion are significantly more complicated in this gauge. There is also the problem that the profile function is no longer explicit in the metric which makes the mathematical wormhole engineering a more difficult task.

In order to remedy these issues, we instead use a set of coordinates utilized in [40]. The construction is briefly summarized here: One begins with the standard curvature coordinates giving rise to figure 1. The profile function, $x = P(r)$ is inverted

and instead we consider the new function given by $r = Q(x) = P^{-1}(x)$. Essentially, we take the wormhole and rotate it as illustrated in figure 2. In this new gauge, the line element takes the form:

$$ds^2 = -e^{\lambda(x)}\,dt^2 + \left\{1 + [Q_{,x}(x)]^2\right\}dx^2 + Q^2(x)\,d\theta^2 + Q^2(x)\sin^2\theta\,d\phi^2. \qquad (9)$$

Notice that this has both the advantage of requiring a single coordinate chart as well as retaining the profile function in the metric. The infinite derivative at the throat is replaced with a vanishing derivative and therefore the metric singularity is eliminated, causing no trouble for analysis. Further, given the complexity of the equations of motion, we consider here the zero-tidal force class of wormholes as studied in [8], [41], [42], [43] and others. This amounts to setting $\lambda(x)$ constant, at least near the vicinity of the throat (actually, for the local analysis presented here, $\lambda(x)$ need not be constant so long as the first few derivatives of $\lambda(x)$ vanish at the wormhole throat as not to affect the analytic expansions presented below).
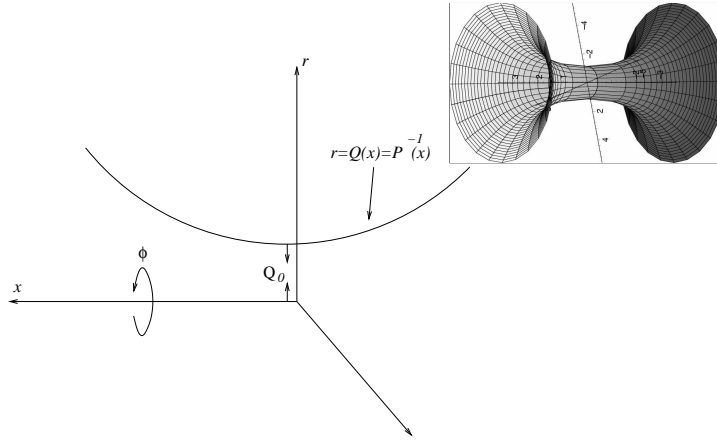

Figure 2: Wormhole profile curve in the t =constant and $\theta = \pi/2$ submanifold using the rotated system. The profile function is given by $r = Q(x) = P^{-1}(x)$ and the radius of the throat is Q_0. As before, the wormhole is generated via rotation about the x-axis (inset).

Here the restrictions for a throat are:

- The derivative of $Q(x)$ must change sign at the throat. $Q_{,x}(x) < 0$ for $x < 0$ and $Q_{,x}(x) > 0$ for $x > 0$.

- $Q(x)$ must possess positive second derivative near the vicinity of the throat.

- $Q_{,x}(x)_{|x=0} = 0$.

Save for the assumption of static spherical symmetry, the only other assumption we make regarding the spatial metric is that it is analytic in the throat region. A

general form of profile curve, suitable for studying any near throat geometry, is given by [40]:

$$Q(x) = Q_0 + A^2 x^{2n} e^{h(x)}, \tag{10a}$$

$$Q_{,x}(x) = A^2 x^{2n} e^{h(x)} \left[2nx^{-1} + h_{,x}(x) \right], \tag{10b}$$

$$Q_{,x,x} = A^2 x^{2n} e^{h(x)} \left\{ \left[2nx^{-1} + h_{,x}(x) \right]^2 + h_{,x,x}(x) - 2nx^{-2} \right\}. \tag{10c}$$

Here, A and $Q_0 > 0$ are constants, n is a positive integer sufficiently large to preclude singularities and $h(x)$ is an arbitrary analytic function of x. Higher derivatives are required for the field equations (3a) but no restriction is placed on them by the requirement that the metric describe a wormhole throat. The presence of arbitrary functions allows the modeling of infintely many near-throat geometries. It is worth noting at this point that the above restrictions may be relaxed. One could, for example, leave $Q(x)$ as completely arbitrary, save for the analyticity requirement and the assumptions on the signs of the first two derivatives. This yields recondite analysis but does not change the qualitative results of this paper.

3 Weak energy condition

The weak energy condition is the statement that no time-like observer measures a local energy density which is negative. It is usually formulated as:

$$T_{\mu\nu} u^\mu u^\nu \geq 0 \ \forall \ \text{time-like} \ u^\alpha, \tag{11}$$

the u^α being the observer's four-velocity. The solution for the general WEC in spherical symmetry has been solved in [44]. For the static model presented here, it is sufficient to study the limiting trajectories in each of the three principal spatial directions. If the WEC is satisfied for these trajectories, it will be satisfied for all trajectories. Enforcing $u^\alpha u_\alpha = -1$, the limiting cases of (11) reduce to the four conditions:

$$-T^0_0 \geq 0, \tag{12a}$$

$$-T^0_0 + T^i_i \geq 0, \quad \text{no summation over } i. \tag{12b}$$

Using the metric ansatz (10a) in (3a), we expand the components of the stress-energy tensor about the throat ($x = 0$). The results are summarized here:

$$-T^0_0 \approx \frac{1}{4Q_0^4} \left[4Q_0^2 + \left(1 + 96 Q_0^3 A^2 e^{h(0)} \right) \left(Q_0^6 c_{-1} + 8c_2 \right) \right]$$
$$+ \frac{1}{Q_0} 120 A^2 e^{h(0)} (h_{,x})_{x=0} \left(Q_0^6 c_{-1} + 8c_2 \right) x + \dots \quad \text{for } n = 2, \tag{13a}$$

$$-T^0_0 \approx \frac{1}{Q_0^2} \left(1 + \frac{c_{-1}}{4} Q_0^4 + \frac{2c_2}{Q_0^2} \right) + \frac{(2n)!}{(2n-4)!} A^2 e^{h(0)} \left(Q_0^5 c_{-1} + \frac{8c_2}{Q_0} \right) x^{2n-4}$$
$$+ \dots \quad \text{for } n > 2, \tag{13b}$$

along with:

$$-T_0^0 + T_1^1 \approx \frac{(2n)!}{(2n-4)!} A^2 e^{h(0)} \left(Q_0^5 c_{-1} + \frac{8c_2}{Q_0} \right) x^{2n-4}$$
$$+ \frac{(2n+1)}{(2n-3)} \frac{(2n)!}{(2n-4)!} A^2 e^{h(0)} (h_{,x})_{|x=0} \left(Q_0^5 c_{-1} + \frac{8c_2}{Q_0} \right) x^{2n-3} + \ldots, \quad (14a)$$

$$-T_0^0 + T_2^2 \approx \frac{1}{Q_0^2} - Q_0^2 c_{-1} + 4\frac{c_2}{Q_0^4} - \frac{n(2n-1)}{Q_0} A^2 e^{h(0)} \left(2 + \frac{3}{2} Q_0^4 c_{-1} + 24\frac{c_2}{Q_0^2} \right) x^{2n-2}$$
$$+ \ldots \approx -T_0^0 + T_3^3. \quad (14b)$$

It is easy to see from the above expressions that the WEC, in principle, may be satisfied in the throat vicinity. The WEC expression most sensitive to deviations from Einstein gravity is (14a) (this is the term whose near throat region must violate the WEC in Einstein gravity).

The condition for well defined Newtonian limit in singular non-linear gravity models provided by Dick [37] read that, given $R_0 > 0$:

$$|\mathcal{L}(R)\mathcal{L}''(R)|_{|R=R_0} \ll 1. \quad (15)$$

Here primes denote differentiation with respect to R and R_0 is the Ricci scalar of the supported vacuum solution. It was shown that these conditions yield an acceptable Newtonain limit on length scales $\ll (R_0)^{-1/2}$. The vacuum of (3a) possesses Ricci scalar given by (4) (for $m = -1, 1, 2$ terms). Therefore, sufficient conditions for (3a) to possess correct Newtonian limit are:

$$c_{-1} < 0, \quad R_0 = +\sqrt{3|c_{-1}|}, \quad c_2 \approx \frac{1}{3^{3/2}\sqrt{|c_{-1}|}}. \quad (16)$$

The first of these conditions guarantees the existence of a maximally symmetric gravitational vacuum solution (de Sitter) whereas the second and third conditions satisfy Dick's restrictions [37]. Therefore, if (16) holds, WEC violation will not occur as long as the condition

$$\frac{8}{\sqrt{27}\,|c_{-1}|^{3/2}} > Q_0^6 \quad (17)$$

is satisfied with $n > 1$. For a theory with only Ricci squared terms the sole restriction is $c_2 > 0$ whereas near throat WEC violation is more severe in the case where only $1/R$ modifications are present.

Regarding the issue of stability, it is known that the theory governed by the Lagrangian in (1) with $m = -1, 1$ and 2 is equivalent to, in the Einstein frame, gravitation coupled to a scalar field. The stability of such a theory has been studied in [45] where effective potential techniques were utilized to analyze the stability. The effective potential, $V(R)$, is given by [45]

$$c_2 V(R) = \frac{(2|c_{-1}| + c_2 R^3)\, c_2 R^3}{(R^2 + |c_{-1}| + 2c_2 R^3)^2}. \quad (18)$$

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If the last term in (16) is treated as an equality, the potential has real stationary points at $R = +1/(3c_2)$, $R = 0$ and $R = -1/(3c_2)$ with the product

$$|c_{-1}|c_2^2 = \frac{1}{27}. \tag{19}$$

Under condition (19) the critical point at $R = +1/(3c_2)$ is a rising point of inflection whereas the critical point at $R = -1/(3c_2)$ is a minimum. However, the last criterion of (16) is an approximate equality and it is easy to see that even a slight perturbation in the value of (19) will shift the inflection point and turn it into a minimum. Thus yielding both weak field stability and consistent weak field limit. We plot this region of the potential in figure 3. It should be noted that in strong gravitational fields (such as those potentially present near the wormhole throat), R need not be restricted to an interval near this value. However, this analysis is useful in illustrating that the theory possesses acceptable weak field behaviour.
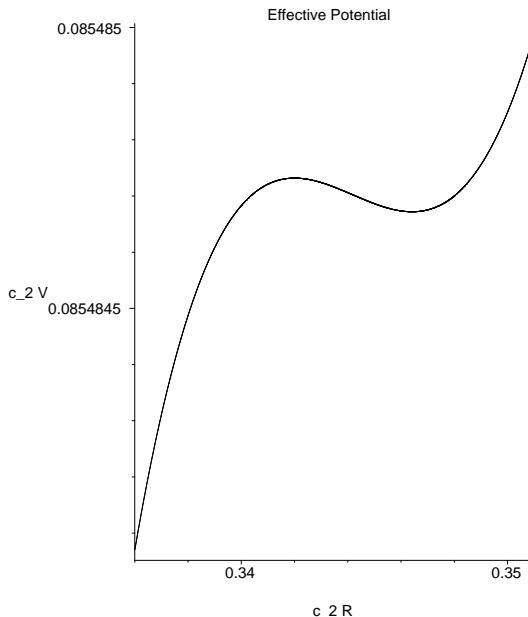

Figure 3: Effective potential for stability analysis with $|c_{-1}|c_2^2 \approx 0.038$.

4   Junction conditions

We briefly comment here on the patching at the junction away from the throat (located at $x = x_* > 0$). As mentioned above, a patching of the near throat solution to a WEC respecting space-time could yield a WEC respecting wormhole. There are several physically acceptable junction conditions one could employ at the boundary (see, for example, [12], [40], [46], [47], [48] for applications to wormholes in Einstein gravity).

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One condition is that of Synge [49] which reads:

$$\lim_{\epsilon \to 0} \left[T^\mu_\nu \, \hat{n}^\nu \right]_{|x=x_*-\epsilon} = \lim_{\epsilon \to 0} \left[T^\mu_\nu \, \hat{n}^\nu \right]_{|x=x_*+\epsilon} . \tag{20}$$

Here $\hat{n}^\nu$ is the outward pointing unit normal vector of the junction surface.

In Einstein gravity, another condition is the Israel-Sen-Lanczos-Darmois (ISLD) junction conditions [50], [51], [52], [53] which reads:

$$\lim_{\epsilon \to 0} \left[K_{\mu\nu}(x_* - \epsilon) \right] = \lim_{\epsilon \to 0} \left[K_{\mu\nu}(x_* + \epsilon) \right], \tag{21}$$

with $K_{\mu\nu}$ being the extrinsic curvature tensor of the junction hyper-surface. It has been shown that, on a time-like boundary in a spherically symmetric R-domain, the above two conditions are equivalent in Einstein gravity [44].

The Synge junction condition (20) carries over, in a straight forward manner, into higher derivative gravity. The ISLD condition (21), relying on first derivatives of the metric, needs to be modified to be valid in higher derivative theories. One way this could be accomplished is to consider the boundary terms which arise when applying the variational principle to (1). The continuity of such terms at the junctions will lead to an appropriate junction condition. Without detailed analysis, it is probably safe to state that continuity of up to fourth derivatives of the metric will satisfy such a junction condition (although this restriction can most likely be relaxed somewhat). This guarantees continuity of $G^\mu_\nu + H^\mu_\nu$ (or, equivalently here, T^μ_ν). The above solution may be patched, for example, to an anisotropic fluid which is characterized by:

$$T^\mu_\nu = \left(\rho + p_\perp \right) u^\mu u_\nu + p_\perp \delta^\mu_\nu + \left(p_\parallel - p_\perp \right) s^\mu s_\nu, \tag{22}$$

with u^μ the fluid four-velocity, which is perpendicular to the vector s^μ. The quantities ρ, $p_\perp$, $p_\parallel$ represent the energy density, transverse pressure and parallel pressure of the fluid respectively. Other boundaries further away from the throat, such as a matter-vacuum boundary, may similarly be considered.

5 Concluding remarks

In summary, wormhole throats were studied in a gravitational theory governed by the Einstein-Hilbert Lagrangian supplemented with $1/R$ and R^2 Ricci scalar terms. The resulting equations of motion was utilized to study energy conditions in the vicinity of the wormhole throat. It was found that the weak energy condition (WEC) may be respected in the throat region in the modified theory. The conditions for a WEC respecting throat are compatible with those required for stability and an acceptable Newtonian limit. Away from the throat, the system cold be joined to WEC respecting matter solutions and, at other junctions, even the vacuum. It would be of interest to study these junction conditions in detail in future work. Other curvature invariant contributions could also be investigated.

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