# Filtering effect induced by rigid massless embedded foundations

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**Abstract**

It is well recognised that the dynamic interaction between structure, foundation and supporting soil can affect significantly the seismic behaviour of buildings. Among other effects, embedded and deep foundations can filter the seismic excitation, causing the foundation input motion (FIM) to differ substantially from the free-field motion. This paper presents a theoretical and numerical investigation on the filtering effect induced by rigid massless embedded foundations. Based on the results of dimensional analysis and numerical simulations, it is shown that the problem can be reasonably described by two sole dimensionless groups, namely: (i) $\omega H/V_S$, relating the wave length of the signal to the embedment depth of the foundation, and (ii) the aspect ratio of the foundation, $B/H$, where $B$ is the foundation width in the polarization plane. New simplified and physically sound expressions are derived for the kinematic interaction factors, $I_u = u_{FIM}/u_{ff0}$ and $I_\theta = \theta_{FIM}H/u_{ff0}$, which are frequency-dependent transfer functions relating the harmonic steady-state motion experienced by the foundation to the amplitude of the corresponding free-field surface motion. Standard methods for using these functions in the evaluation of the FIM are critically reviewed, with reference to both static and dynamic procedures for the seismic design of structures.

**Keywords:**

Soil-structure interaction; foundation input motion; free-field motion; kinematic response; embedded foundation

## INTRODUCTION

The seismic performance of structures is usually evaluated under a fixed-base assumption by applying a base slab input motion equal to the free-field motion, *i.e.* neglecting the dynamic interaction between the structure, the foundation and the supporting soil (Soil-Structure Interaction, SSI). Nevertheless, the presence of a deformable soil-foundation system affects the dynamic behaviour of buildings in at least three different ways, making the seismic response of the flexibly-supported structure possibly different from that of the rigidly-supported counterpart (Bielak, 1975; Mylonakis & Gazetas, 2000): (i) it lengthens the fundamental period of the structure; (ii) it allows additional dissipation of energy into the soil by radiation and hysteresis; (iii) it filters the signal transmitted to the structure by incident waves, as a results of both base slab averaging (Veletsos *et al*., 1997) and embedment effects (Elsabee & Morray, 1977).

SSI can be thought conveniently, both from a conceptual and computational point of view, as the contribution of two concurrent phenomena (Mylonakis *et al*., 2006): (i) kinematic interaction, in which a massless foundation modifies the motion of the surrounding soil by means of its sole stiffness and (ii) inertial interaction, in which the motion of the foundation itself is further modified by the D'Alembert forces acting in the structure-foundation system. The distinction between kinematic and inertial effects, which also underlies the substructure method, provides a powerful key to interpretation of SSI problems, as observed in experimental works (Rayhani & El Naggar, 2008), numerical works (Mahsuli & Ghannad, 2009; Politopoulos, 2010; Vega *et al*., 2013) and field measurements (Stewart, 2000; Kim & Stewart, 2003), where many factors can affect the overall dynamic response of the structure.

By focusing on the filtering effect, kinematic interaction has been recognised to play a significant role in the case of both embedded (Avilés *et al*., 2002; Politopoulos, 2010) and deep foundations (Di Laora & de Sanctis, 2013), for which the foundation input motion (FIM) can differ substantially from the free-field motion recorded at ground surface. Under the assumption of vertically propagating

plane shear waves, base slab averaging cannot occur and filtering effect is physically related to the inability of the foundation elements to follow soil deformations induced by travelling waves.

In the case of rigid embedded foundations, scattering effects reduce the horizontal displacement of the base slab, $u_{FIM}$, with respect to the free-field case, $u_{ff0}$, but can introduce a rotational component, $\theta_{FIM}$. This phenomenon can be described by two kinematic response factors, namely $I_u = u_{FIM}/u_{ff0}$ and $I_\theta = \theta_{FIM}H/u_{ff0}$, which are frequency-dependent transfer functions relating the harmonic steady-state motion experienced by the foundation to the amplitude of the corresponding free-field surface motion (see Figure 1).

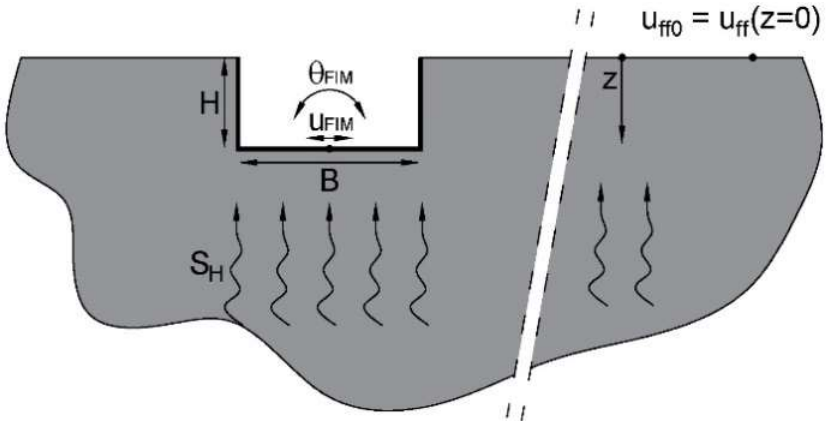


**Figure 1. Schematic representation of the soil-foundation kinematic interaction in the case of embedded foundations and vertically propagating SH waves**

Many works in the literature have been devoted to the problem of filtering effects induced by rigid embedded foundations, using different numerical techniques, most of them considering the case of a massless rigid foundation – with cylindrical or rectangular shape - embedded in a uniform elastic or viscoelastic half-space (Elsabee & Morray, 1977; Day, 1978; Dominguez, 1978; Karabalis & Beskos, 1986; Luco & Wong, 1987; Mita & Luco, 1989). More recently, Brandenberg *et al*. (2015) have proposed a Winkler-type simplified model, relating the kinematic response factors to the translational and rotational impendence functions for the soil.

Further studies, taking into account both the soil-foundation system and the superstructure (complete SSI), have shown that the filtering effect is usually beneficial for squat structures while it may increase the ductility demand in the case of slender buildings with deeply embedded foundations or

basements (Mahsuli & Ghannad, 2009). Moreover, in the case of base isolated structures, such as nuclear plants, where standard devices have isolation capacity only in the horizontal plane, the dynamic response of non-isolated modes can be significantly amplified by rocking oscillations of the foundation (Politopoulos, 2010; Politopoulos *et al*., 2015).

The filtering effect induced by embedded foundations is usually described using the formulas proposed by Elsabee & Morray (1977):

$$I_u = \frac{u_{FIM}}{u_{ff0}} = \begin{cases} \cos\left(\frac{\omega H}{V_S}\right) & \omega \le 0.7\frac{\pi}{2}\frac{V_S}{H} \\ 0.453 & \omega > 0.7\frac{\pi}{2}\frac{V_S}{H} \end{cases} \quad (1)$$

$$I_\theta = \frac{\theta_{FIM} \cdot H}{u_{ff0}} = \begin{cases} \frac{2H}{B}0.257\left[1-\cos\left(\frac{\omega H}{V_S}\right)\right] & \omega \le \frac{\pi}{2}\frac{V_S}{H} \\ \frac{2H}{B}0.257 & \omega > \frac{\pi}{2}\frac{V_S}{H} \end{cases} \quad (2)$$

where $B$ is the foundation width, $\omega$ is the angular frequency of the excitation and $V_S$ is the shear wave velocity of the supporting soil (FEMA 440, 2005; Mylonakis *et al*., 2006).

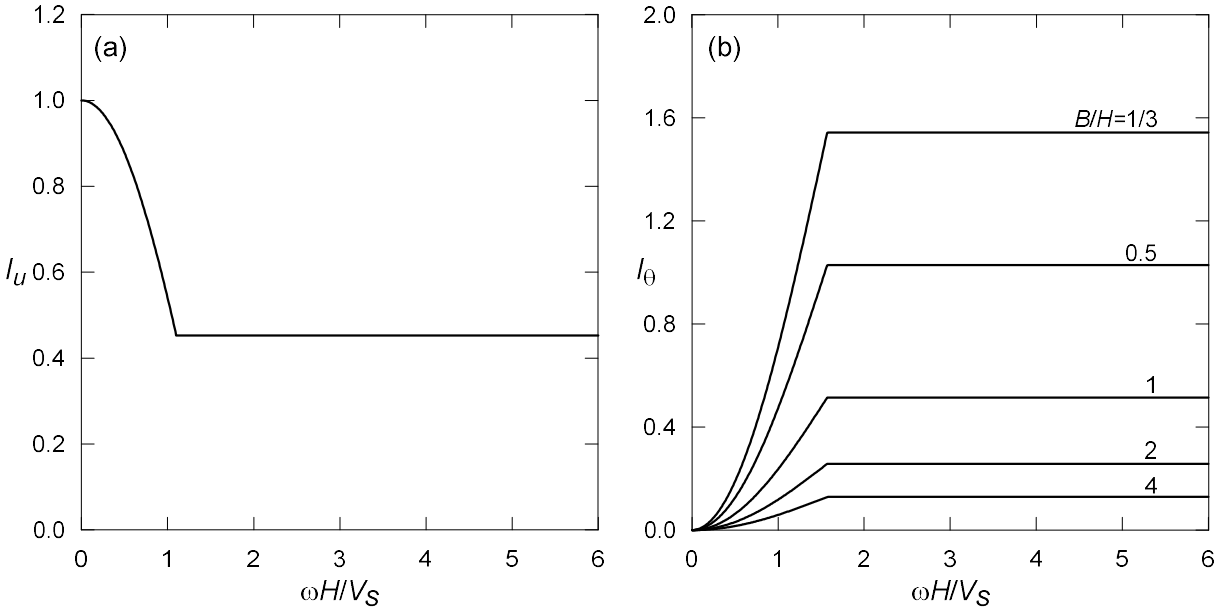


**Figure 2. Formulas proposed by Elsabee & Morray (1977) for the kinematic response factors: (a) Iu and (b) Iθ**

As shown in Figure 2, despite their simplicity these equations provide a quite crude approximation of the actual physical phenomenon, essentially relating the motion of the rigid foundation to the translation of the free-field at the foundation level (Eq. (1)) and to the so-called free-field "pseudo-rotation", resulting from the differential displacement of the soil in the embedment region (Eq. (2)). This paper presents a numerical and theoretical investigation of the filtering effect induced by rigid massless embedded foundations. The goal of this study is threefold: (i) to offer insight into the relevant factors affecting the problem; (ii) to extend the numerical observations available in the literature; (iii) to define new simplified, but physically sound, solutions to be incorporated in recommendations for the seismic design of structures with embedded foundations. Results of this work will be useful not only to improve the understanding of the mechanisms underlying filtering effects, such as the little relevance of the three-dimensional features of the foundation, but also for the design practice.

## PROBLEM DEFINITION AND DIMENSIONAL ANALYSIS

In this section we derive the dimensionless groups governing the filtering effect induced by embedded foundations, under the assumption of vertically propagating plane shear waves. We refer to the general case of a rectangular foundation (embedment depth $H$, width $B$, length $L$, mass density $\rho^*$, shear modulus $G^*$, damping ratio $\xi$) embedded in a homogeneous isotropic visco-elastic soil layer (depth $H_d$, mass density $\rho$, shear modulus $G$, Poisson's ratio $\nu$, damping ratio $\xi$). Table 1 summarises the 14 physical variables relevant for the problem at hand, which can be formulated as:

$$\begin{aligned} u_{FIM} &= f\left(u_{ff0}, \omega, H, B, L, \rho^*, G^*, \rho, G, \xi, \nu, H_d\right) \\ \theta_{FIM} &= g\left(u_{ff0}, \omega, H, B, L, \rho^*, G^*, \rho, G, \xi, \nu, H_d\right) \end{aligned} \tag{3}$$

**Table 1. Variables governing the dynamic interaction between soil and embedded foundations**

| | variable | dim. | description |
|---|---|---|---|
| foundation input motion | $u_{FIM}$ | L | base horizontal disp. |
| | $\theta_{FIM}$ | $L^0$ | rotation |

| | | | |
|---|---|---|---|
| free-field motion | $u_{ff0}$ | L | surface horizontal disp. |
| | $\omega$ | $T^{-1}$ | angular frequency |
| foundation properties | $\rho^*$ | $ML^{-3}$ | mass density |
| | $G^*$ | $MT^{-2}L^{-1}$ | shear modulus |
| | H | L | embedment |
| | B | L | width |
| | L | L | length |
| soil properties | $\rho$ | $ML^{-3}$ | mass density |
| | G | $MT^{-2}L^{-1}$ | shear modulus |
| | $\nu$ | $L^0$ | Poisson's ratio |
| | $\xi$ | $L^0$ | damping ratio |
| | $H_d$ | L | depth of soil deposit |

Applying the Buckingham theorem, it is possible to rescale Equations (3) in dimensionless form, using $H$, $G$ and $\rho$ as dimensionally independent variables:

$$I_u := \frac{u_{FIM}}{u_{ff}} = F\left(\frac{\omega H}{V_S}, \frac{\rho^*}{\rho}, \frac{G^*}{G}, \frac{B}{H}, \frac{L}{H}, \frac{H_d}{H}, \xi, \nu\right)$$
$$I_\theta := \frac{\theta_{FIM} H}{u_{ff0}} = G\left(\frac{\omega H}{V_S}, \frac{\rho^*}{\rho}, \frac{G^*}{G}, \frac{B}{H}, \frac{L}{H}, \frac{H_d}{H}, \xi, \nu\right) \quad (4)$$

The dimensionless ratios in Equations (4) take into account the physical, mechanical and geometrical properties of the problem. Among these,

(i) $\omega H/V_S$ relates the embedment depth of the foundation to the wavelength of the excitation ($\lambda = V_S/f = 2\pi V_S/\omega$), *i.e.* the deeper the foundation the longer the wavelengths which can be filtered by scattering effects;

(ii) $G^*/G$ and $\rho^*/\rho$ are the relative shear stiffness and mass density, respectively, between the foundation and the soil: the stiffer and the denser the foundation, the stronger its filtering capacity;

(iii) $H_d/H$ is the relative depth between the soil deposit and the foundation embedment, which, for soil layers of finite depth, can introduce spurious oscillations in the kinematic interaction factors (Elsabee & Morray, 1977);

In order to reduce the problem at hand and/or in the light of a kinematic-inertial decomposition method, some simplifying assumptions are usually introduced in the literature, *i.e.*: the embedded foundation is rigid ($G^*/G \gg 1$) and massless ($\rho^*/\rho \ll 1$); the soil deposit is assimilated to a homogeneous half-space ($H_d/H \gg 1$). As far as $\xi$ and $\nu$ are concerned, numerical works have shown that, while affecting the dynamic response of both the foundation and the soil, they have a minor influence on the kinematic response factors (Mita & Luco, 1989; Di Laora & de Sanctis, 2013). Moreover, it will be shown in the following that, as far as the aspect ratios of the foundation are concerned, only the foundation width $B$ in the polarization plane of the shear wave affects significantly the filtering phenomenon. Under these assumptions, the interaction factors can be expressed as functions of two sole parameters:

$$\begin{aligned} I_u &= F\left(\frac{\omega H}{V_S}, \frac{B}{H}\right) \\ I_\theta &= G\left(\frac{\omega H}{V_S}, \frac{B}{H}\right) \end{aligned} \tag{5}$$

By introducing a further hypothesis on the foundation geometry, two limiting 1D conditions can be identified, *i.e.*:

(i) $B/H = 0$ (infinitely thin foundation): the difference between the FIM and the corresponding free-field surface motion is related only to the variation of ground motion with depth. As a consequence, the two kinematic interaction factors can be computed with reference to the free-field motion in the embedment region, assuming elastic behaviour for the soil ($\xi = 0$), as:

$$\begin{aligned} I_u|_{(B/H=0)} &= \frac{u_{ff}|_{z=H}}{u_{ff0}} = \cos\left(\frac{\omega H}{V_S}\right) \\ I_\theta|_{(B/H=0)} &= \frac{\theta_{ff} \cdot H}{u_{ff0}} = 1 - \cos\left(\frac{\omega H}{V_S}\right) \end{aligned} \tag{6}$$

where $\theta_{ff} = \left(u_{ff0} - u_{ff}|_{z=H}\right)/H$ is the free-field pseudo-rotation of the soil.

(ii) $B/H = \infty$ (infinitely extended foundation): the foundation cannot rotate ($\theta_{FIM} = 0$) and the half-space condition results in $u_{FIM} = u_{ff0}$. As a consequence, the two kinematic interaction factors reduce to:

$$I_u|_{(B/H=\infty)} = 1 \qquad I_\theta|_{(B/H=\infty)} = 0 \tag{7}$$

These asymptotic conditions will be used in the following both to interpret numerical results and to provide simplified solutions for design.

## NUMERICAL ANALYSES

A total of 17 plane-strain analyses of a rectangular foundation of width $B$ and depth $H$, embedded in an homogeneous half-space, were carried out in the time domain using the finite difference code FLAC 2D v7 (Itasca, 2011). Moreover, two three-dimensional analyses were carried out with the code FLAC 3D. The complete set of analyses is reported in Table 2, with the ratio $B/H$ ranging from 0.25 to 20.

### Model definition

Figure 2 shows the typical mesh adopted in this study. The soil was modelled as a linear visco-elastic isotropic material, with mass density $\rho = 1.835$ t/m$^3$, shear wave velocity $V_S = 100$ m/s and Poisson ratio $\nu = 0.3$. A Rayleigh viscous damping was used, with a given value of 2 % at the reference frequencies of 1 Hz and 10 Hz.

**Table 2. Summary of the numerical analyses**

| # | $H$ [m] | $B$ [m] | $B/H$ | $L/B$ | $B_m$ [m] | $H_m$ [m] |
|---|---|---|---|---|---|---|
| 1 | 12 | 3 | 0.25 | ∞ | 300 | 27 |
| 2 | 12 | 6 | 0.5 | ∞ | 300 | 27 |
| 3 | 3 | 3 | 1 | ∞ | 300 | 27 |
| 4 | 6 | 6 | 1 | ∞ | 300 | 27 |
| 5 | 12 | 12 | 1 | ∞ | 300 | 27 |
| 6 | 3 | 6 | 2 | ∞ | 300 | 27 |
| 7 | 6 | 12 | 2 | ∞ | 300 | 27 |
| 8 | 12 | 24 | 2 | ∞ | 300 | 27 |
| 9 | 3 | 12 | 4 | ∞ | 300 | 27 |
| 10 | 6 | 24 | 4 | ∞ | 300 | 27 |
| 11 | 12 | 48 | 4 | ∞ | 450 | 27 |
| 12 | 3 | 18 | 6 | ∞ | 300 | 27 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 6 | 36 | 6 | ∞ | 450 | 27 |
| 14 | 12 | 72 | 6 | ∞ | 450 | 27 |
| 15 | 3 | 30 | 10 | ∞ | 450 | 27 |
| 16 | 6 | 60 | 10 | ∞ | 450 | 27 |
| 17 | 3 | 60 | 20 | ∞ | 450 | 27 |
| 18 | 6 | 6 | 1 | 1 | 100 | 27 |
| 19 | 6 | 12 | 2 | 1 | 100 | 27 |

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The rigid massless foundation was modelled as an open excavation with rigid boundaries and supports, in order to enforce rigid body displacements. To this purpose, elastic beams with reduced mass density were introduced (E = 5.0 GPa, A = 7.0 $m^2$, I = 6.0 $m^4$, $\rho_b$ = 0.008 t/$m^3$), both along the boundaries of the foundation and as an internal frame, thus increasing the overall shear stiffness of the foundation without affecting significantly its total mass. Perfect contact was assumed between the sidewalls and the soil elements.

Free-field boundary conditions were applied along the lateral sides of the mesh, involving the coupling of the main grid with a one-dimensional free-field column through viscous dashpots, in such a way that outward waves originating from the interior of the model can be properly absorbed.

As far as the boundary condition at the base of the mesh is concerned, both viscous dashpots and the dynamic input were applied in order to reproduce the upward propagation of shear waves within a semi-infinite domain (Joyner & Chen, 1975). The input was a constant amplitude sinusoidal sweep, defined in terms of a horizontal displacement time history, with a duration of 60 s and a frequency increasing linearly with time from 0.5 to 10 Hz. This range was chosen to include the typical frequency content of real earthquakes.

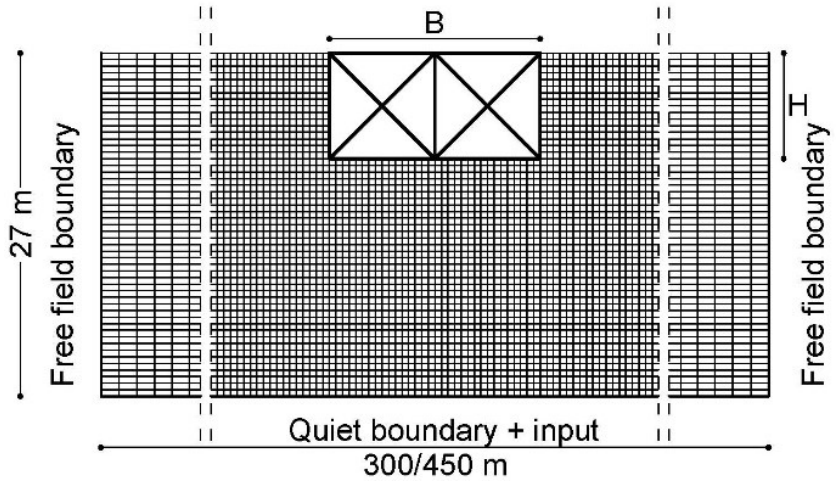


**Figure 3. 2D analyses: finite difference model**

The dimensions of the mesh were chosen after a preliminary parametric study so as to minimise possible side effects due to spurious wave reflections at boundaries, and hence to recover free field conditions. The elements of the mesh have a maximum size of 0.75 m close to the foundation, in order to describe correctly the minimum wavelength of the applied signal ($\lambda_{min}=V_S/f_{max}$ = 10 m). Moving to 3D analyses, the square foundation ($B/L$=1) was modelled as an empty excavation, with shell elements of increased stiffness and reduced mass density attached to the internal boundaries. The numerical model is consistent with that adopted in the 2D analyses, in terms of boundary conditions, dynamic input, mesh discretisation, and mechanical properties of both soil and foundation elements.

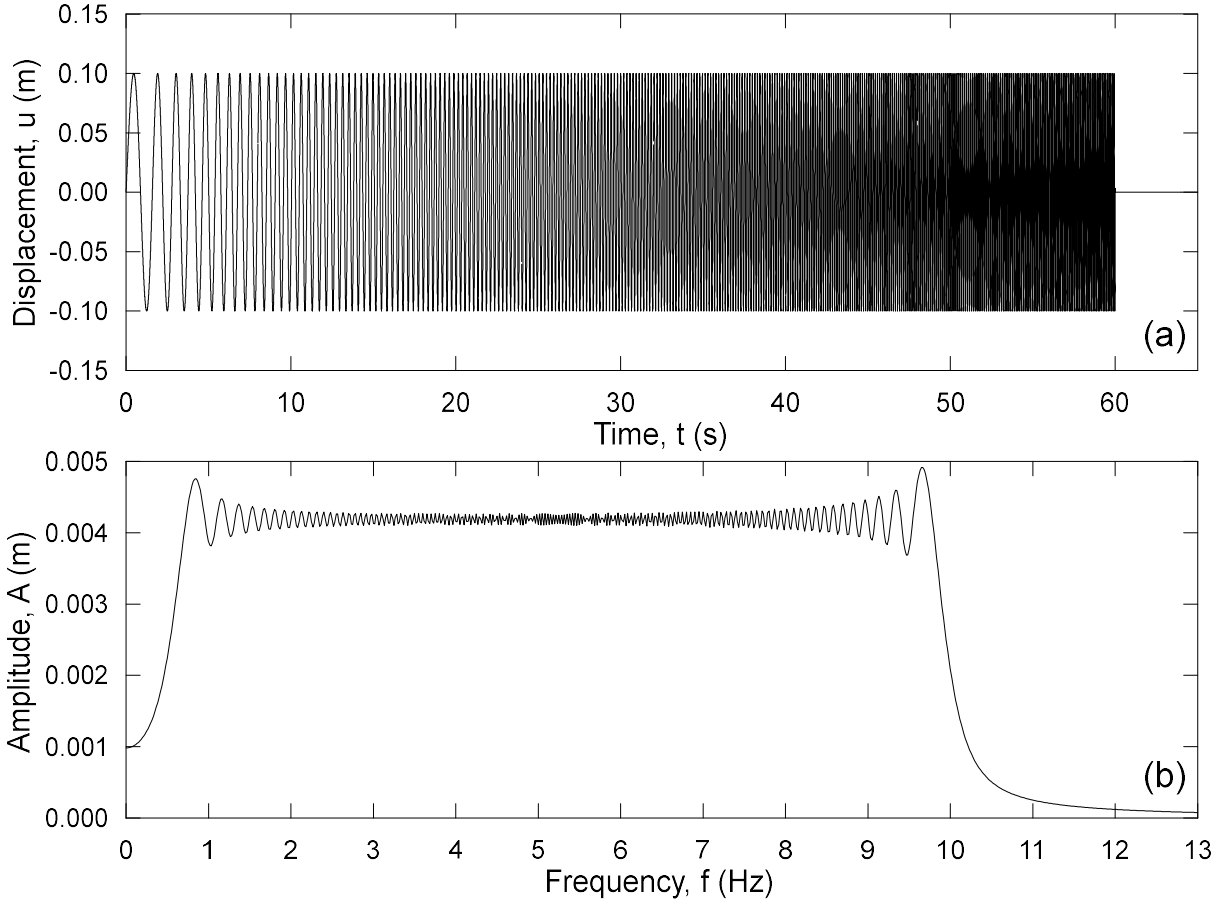


**Figure 4. Sweep input signal: (a) displacement time history and (b) Fourier amplitude spectrum**

## Results

The complex valued functions $I_u$ and $I_\theta$ were obtained from the Fourier transform of $u_{ff0}(t)$, $u_{FIM}(t)$ and $\theta_{FIM}(t)$, where $u_{FIM}$ corresponds to the horizontal displacement of the bottom centre of the foundation and $\theta_{FIM}$ is computed as $(v_1-v_2)/B$, where $v_1$ and $v_2$ are the vertical displacements of the two corners at the base of the foundation. As an example, Figure 3 shows, for analysis No. 11, the numerical values of the real part, imaginary part and amplitude of (a) $I_u$ and (b) $I_\theta$.

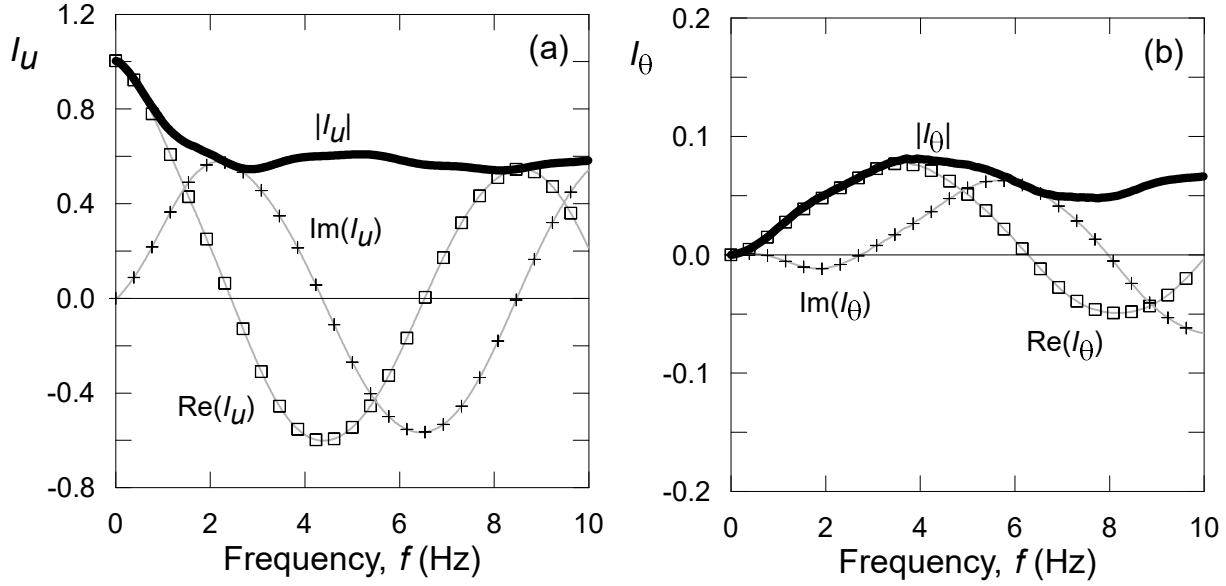


**Figure 5. Analysis No. 11: Real and imaginary part of the kinematic interaction factors: (a) $I_u$ and (b) $I_\theta$**

In order to ascertain three-dimensional effects on the filtering problem, Figure 4 compares the results from this study ($B/H$ = 1, 2 and $L/B$ = 1, ∞), in terms of (a) $|I_u|$ and (b) $|I_\theta|$, with some of those available in the literature, obtained with BEM, FEM or hybrid BEM-FEM approaches. All literature results refer to the case of cylindrical (Day, 1978; Luco & Wong, 1987) and square (Mita & Luco, 1989) foundations ($L/B$=1) embedded in a uniform elastic half-space, with the only exception of Elsabee & Morray (1977), who considered a cylindrical foundation embedded in an elastic soil layer of finite depth overlying a rigid bedrock. In spite of showing some scatter, numerical data are in substantial agreement, both qualitatively and quantitatively, with a maximum difference of ±15% on the average in terms of $I_\theta$ ($B/H$ = 2). Moreover, FDM results are in perfect agreement with each other and with

238 those reported by Mita & Luco (1989). Based on this comparison, it is apparent that the ratio $L/H$ has
239 a minor influence on the kinematic interaction factors, with respect to $\omega H/V_S$ and $B/H$, and, therefore,
240 that it can be ignored without any loss of relevant information. This implies that, when looking at
241 filtering effects, 2D analyses provide a reasonable representation of the 3D behaviour of rigid
242 embedded foundations.
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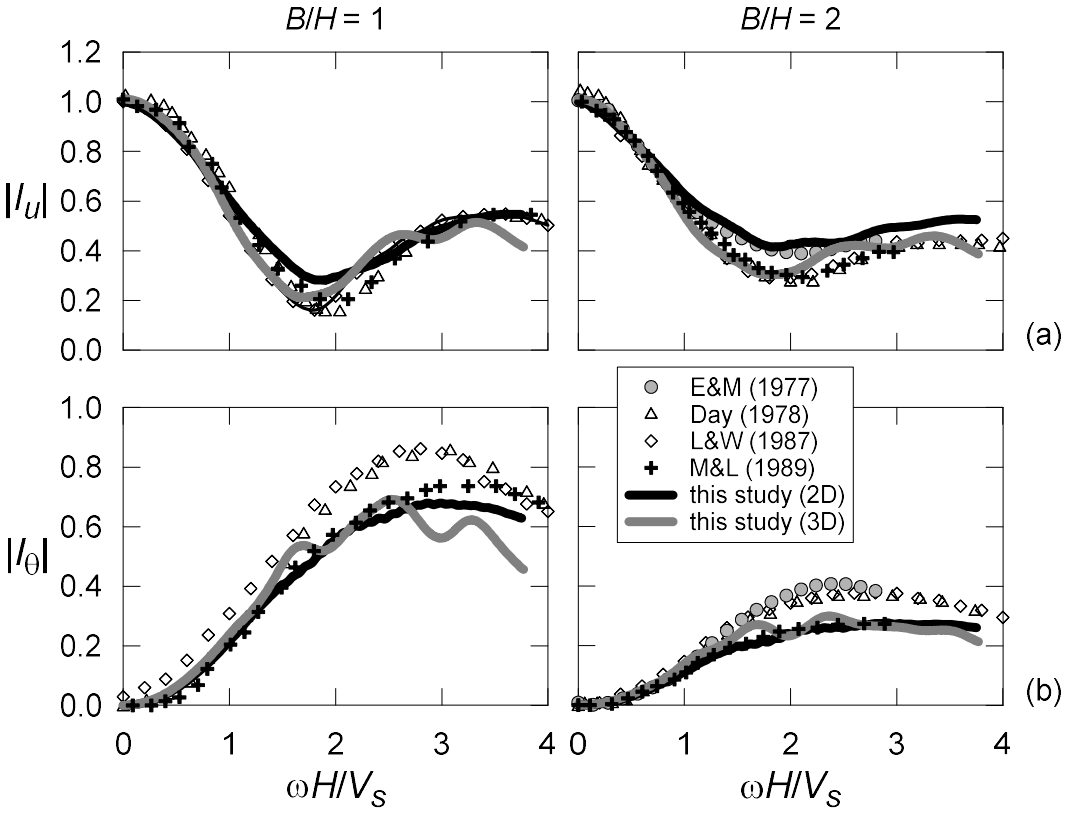


244
245 **Figure 6. Comparison between numerical FDM results ($B/H$ = 1, 2) and literature data, in terms of: (a) $|I_u|$ and**
246 **(b) $|I_\theta|$**
247

248 Figures 5 shows the numerical values of (a) $|I_u|$ and (b) $|I_\theta|$, as a function of the dimensionless
249 frequency $\omega H/V_S$ and for different values of $B/H$, together with the theoretical solutions for the 1D
250 limit conditions of $B/H = 0$ and $B/H = \infty$. For small $B/H$ (squat/slender foundations), the interaction
251 factors tend to the free-field 1D conditions, where filtering effects are mostly due to the embedment
252 of base slab. In this condition, $|I_u|$ shows an oscillating trend, with local minima and maxima clearly
253 related to the resonant frequencies of the corresponding free-field case, and a significant rocking

component emerges in the FIM, as reflected by $|I_\theta|$. On the other hand, for large $B/H$ values (spread foundations), both factors tend to stabilize, moving towards the asymptotic solution for $B/H = \infty$, without significant oscillations as $\omega H/V_S$ increases. As a result, filtering of the horizontal displacements reduces, but no rocking component is introduced. In other words, numerical analyses indicate that both $I_u$ and $I_\theta$ are strongly affected by the aspect ratio of the foundation and are not related merely to the embedment depth of the foundation. In particular, for a given value of $H$, the larger the foundation the smaller the overall filtering effect induced on the free-field ground motion.

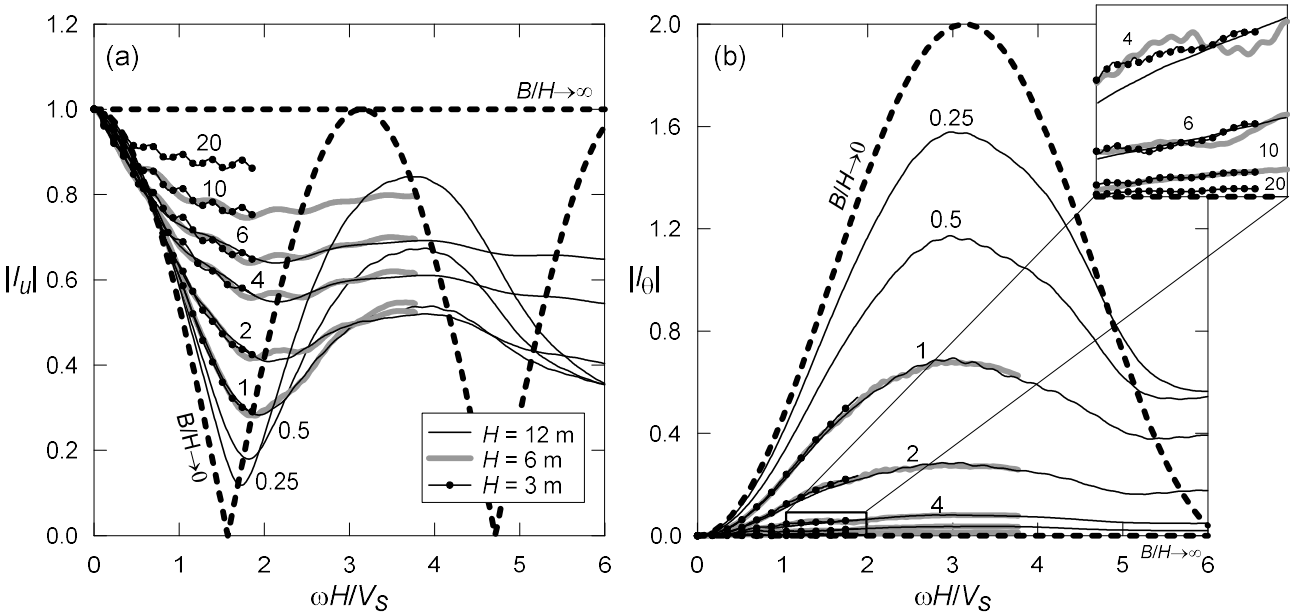


**Figure 7. Numerical results and limit conditions of: (a) $|I_u|$ and (b) $|I_\theta|$**

Based on the best fit of numerical data, simplified expressions for $|I_u|$ and $|I_\theta|$ were defined using *ad hoc* functions which allow to recover the 1D limiting conditions as $B/H \to 0$ or $B/H \to \infty$:

$$\left|I_u\left(\frac{\omega H}{v_s}, \frac{B}{H}\right)\right| = a_1 \frac{1+a_2\left(\frac{\omega H}{v_s}\right)^2}{1+\left(\frac{\omega H}{v_s}\right)^2} + (1-a_1)\frac{\left|\cos\left(a_3\frac{\omega H}{v_s}\right)\right|}{\left[1+\left(\frac{\omega H}{v_s}\right)^2\right]^{2a_1}} \quad (8)$$

$$\left|I_\theta\left(\frac{\omega H}{v_s}, \frac{B}{H}\right)\right| = a_4\left[1-\cos\left(\frac{\omega H}{v_s}\right)\right] \quad (9)$$

in which coefficients $a_1$, $a_2$, $a_3$, $a_4$ depend on the ratio $B/H$, as detailed in Appendix. As shown in Figure 6, Equations (8) and (9) provide a good description of the actual trend exhibited by $|I_u|$ and $|I_\theta|$, particularly for $\omega H/V_S<5$, which corresponds to the range of frequencies typical of real earthquakes.

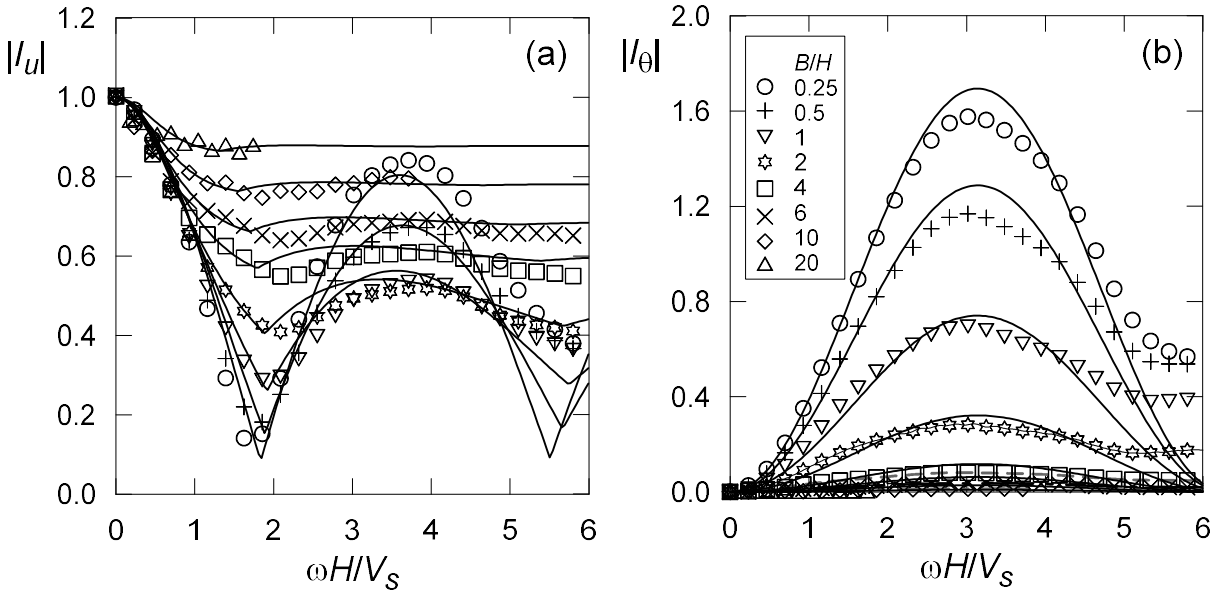


**Figure 8. Comparison between numerical results (symbols) and interpolating functions (lines): (a) $|I_u|$ and (b) $|I_\theta|$**

## GUIDELINES FOR DESIGN

In the light of the results presented so far, and bearing in mind that the accuracy of any simplified method should be consistent with the uncertainties involved in the characterization of the free-field ground motion, this section aims to review critically standard guidelines for including filtering effects in the evaluation of the FIM (FEMA 440, 2005; Mylonakis *et al*., 2006). We will refer to both dynamic and static procedures for the seismic design of structures, where the design earthquake is defined in terms of a time history or a response spectrum, respectively.

Since the interaction factors are complex valued transfer functions, the mathematical rigorous procedure for computing the FIM consists in applying $I_u$ and $I_\theta$ to the Fourier spectrum of the free-field motion, when the latter is defined as a time history (Method M1). However, taking into account that simplified formulas are available only for real-valued functions $|I_u|$ and $|I_\theta|$, the latter are adopted

in the design practice, ignoring any possible phase shift between the translation and rocking components of the foundation motion (Method M2).

Moving to static procedures, Mylonakis *et al*. (2006) suggested to apply $|I_u|$ and $|I_\theta|$ directly to the free-field acceleration spectrum ($S_{a,ff0}(\omega)$), if the design earthquake is specified in this form (Method M3). Accordingly, for a structural mass located at a vertical distance $H_c$ from the base, foundation rocking and translation result in:

$$S_{a,FIM}(\omega) = [|I_u(\omega)| + |I_\theta(\omega)| \cdot H_c/H] S_{a,ff0}(\omega) \,. \quad (10)$$

Clearly, methods M2 and M3 involve quite crude simplifications, whose effects must be ascertained. To this end, we considered the ideal case of an embedded foundation ($H$ = 6 m, $B/H$ = 1, 4 and 6, $V_S$ = 100 m/s), supporting a structural mass located at $H_c$ = 20 m and subjected to ten acceleration time histories, all registered during real earthquakes. For sake of simplicity, we considered the above accelerations as free field surface accelerations ($\ddot{u}_{ff0}$), that is we ignored any stratigraphic amplification effects. Table 3 summarizes the values of peak acceleration, *PGA*, duration, $T_{5–95}$ and mean frequency, $f_m$. Figure 7 shows (a) the acceleration time histories and (b) the Fourier amplitude spectra, while Figure 8 shows the elastic response spectra at 5 % damping of the ten signals.

**Table 3. Ground motion parameters of input earthquakes**

| # | Earthquake | PGA [g] | $T_{5-95}$ [s] | $f_m$ [Hz] |
|---|---|---|---|---|
| 1 | Kocaeli - Turkey (1999) | 0.34 | 17.6 | 2.67 |
| 2 | Loma Prieta - USA (1989) | 0.37 | 15.7 | 4.11 |
| 3 | Friuli - Italy (1976) | 0.32 | 4.8 | 3.98 |
| 4 | Imperial Valley - USA (1979) | 0.33 | 10.3 | 4.34 |
| 5 | Hollister - USA (1961) | 0.19 | 9.2 | 2.84 |
| 6 | Kobe - Japan (1995) | 0.33 | 18.6 | 4.11 |
| 7 | Trinidad - USA (1983) | 0.17 | 3.2 | 4.30 |
| 8 | Northridge - USA (1983) | 0.58 | 19.0 | 3.38 |
| 9 | Chi Chi – Taiwan (1999) | 0.21 | 9.4 | 4.23 |
| 10 | Landers - USA (1992) | 0.44 | 22.3 | 5.72 |

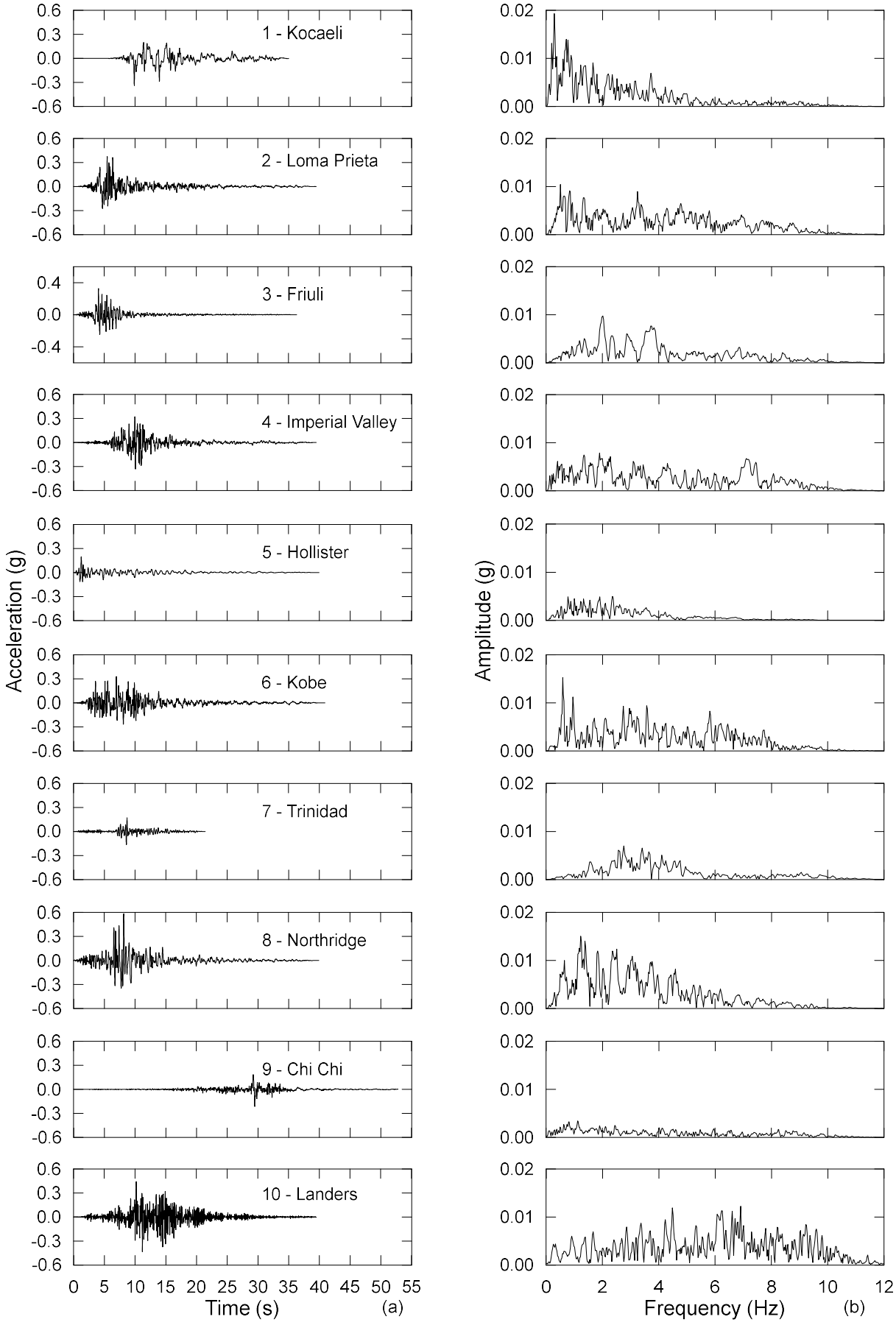


**Figure 9. Ten real earthquakes: (a) acceleration time histories and (b) Fourier amplitude spectra**

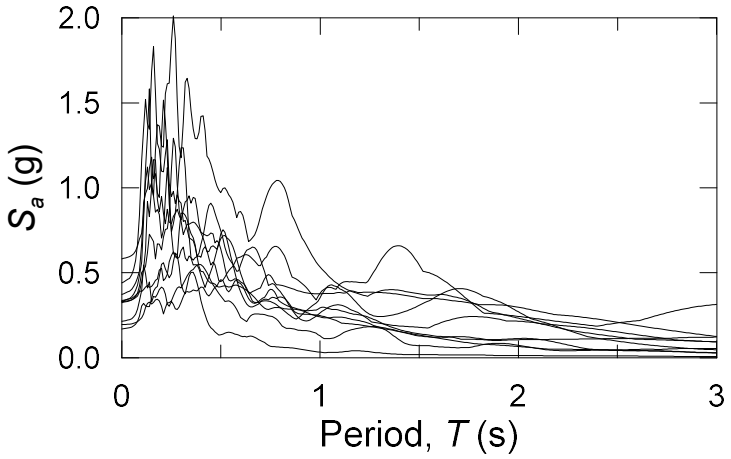


**Figure 10. Ten real earthquakes: elastic response spectra**

Figure 9 shows, for earthquake No.7 and for $B/H = 1$ and 4: (a, b) the acceleration time histories and (c) the elastic response spectrum at 5% damping of both free-field motion and FIM, the latter computed according to methods M1 and M2. As expected, filtering effects depend strongly on the ratio $B/H$, with a maximum reduction of about 50 % in the horizontal peak acceleration computed for $B/H = 1$, together with the appearance of a significant rocking component. Looking at the response spectra of Figure 9(c), the combined effect of translation and rotation of the foundation turns out to be either beneficial or detrimental for the structural system, depending on the ratio $B/H$, the major effects arising in the short period range. For the case at hand ($H_c = 20$m), procedure M1 provides a maximum amplification in the spectral ordinates of 65 % for $B/H = 1$ and a maximum reduction of 40 % for $B/H = 4$, with respect to the free-field condition. By comparing the results from methods M1 and M2, no substantial difference is observed in terms of maximum accelerations, the only difference being a phase shift in the time history. Moreover, neglecting phase angles in the computation of the FIM introduces only small errors in the spectral ordinates, with a maximum value of 25 % for structural periods larger than 0.2 s ($B/H = 4$), always on the conservative side. These observations apply to all the earthquakes considered in this work.

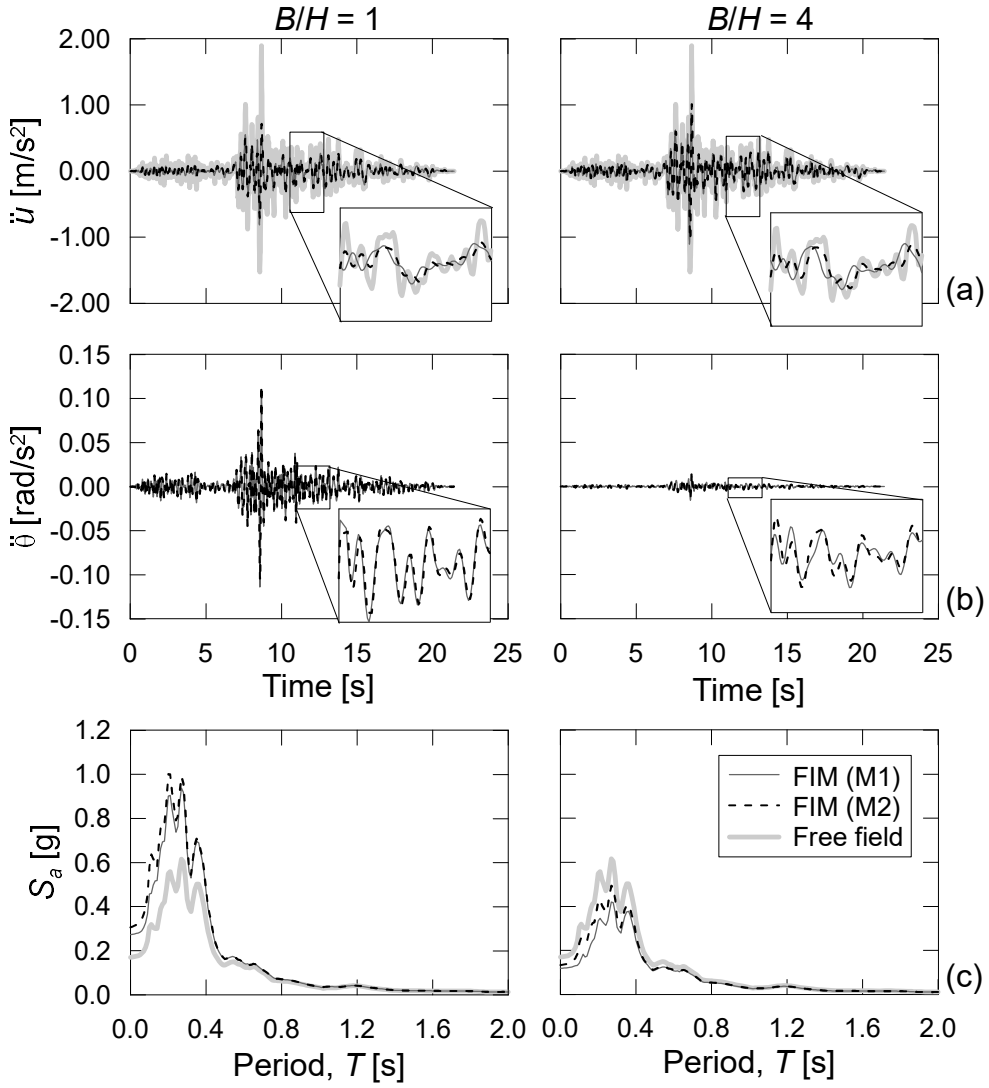


**Figure 11. Earthquake No. 7 - acceleration time histories of: (a) horizontal translation and (b) rocking of free-field motion and FIM, together with (c) their elastic response spectra.**

Figure 10 shows, for all earthquakes and for $B/H = 1$, 4 and 6, the ratio of FIM to free-field response spectral ordinates, the former computed according to methods M1 and M2. The figure shows also the values $|I_u|+|I_\theta|\cdot H_c/H$ (method M3), and those of $|I_u|$, to highlight errors in reducing the design spectrum of the free-field motion without taking into account $|I_\theta|$. Depending on the ratio $B/H$, *i.e.* on the interplay between the reduction in the horizontal motion and the introduction of rocking oscillations, kinematic interaction may lead to either an increase ($B/H = 1$) or a reduction ($B/H = 4$, 6) of the spectral ordinates, with respect to the free-field case. Both simplified procedures M2 and M3 allow to take into account these effects, providing always conservative results with respect to the rigorous procedure M1. However, the error introduced reduces with increasing periods, where the foundation

rocking tends to vanish and, for $T > 0.2$ s - i.e. for most of the real structures where considering the filtering effect induced by the foundation embedment makes sense - the maximum error associated to methods M2 and M3 is 35 % and 40 % respectively. On the other hand, in spite of being always unconservative, using solely $|I_u|$ provides a reasonable approximation of the actual trend for large values of $B/H$ ($B/H \geq 4$), where the foundation rocking is small.

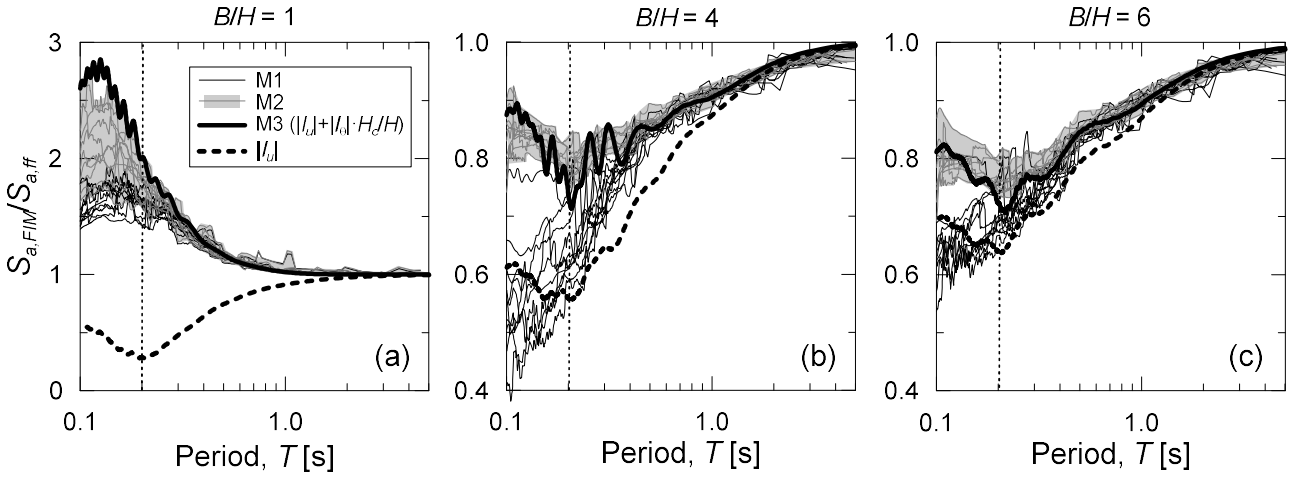


**Figure 12. All earthquakes: ratio of the response spectra between FIM and free-field motion for: (a) $B/H$ = 1, (b) $B/H$ = 4 and (c) $B/H$ = 6 ($H$ = 6 m, $H_c$ = 20 m, $V_S$ = 100 m/s).**

Based on the above observations, it can be concluded that both procedures M2 and M3 can be adopted effectively in the design practice, when computing the design motion either as a time history or as an elastic response spectrum, to take into account in a simplified manner filtering effects induced by embedded foundations.

## CONCLUSIONS

This work was devoted to the filtering effects induced by rigid massless embedded foundations subjected to vertically propagating shear waves. Based on results from dimensional analysis and numerical simulations, it was found that the problem can be reasonably described solely by two dimensionless groups, namely: (i) $\omega H/V_S$, relating the wave length of the signal to the embedment

depth of the foundation, and (ii) the aspect ratio of the foundation, $B/H$, where $B$ is the foundation width in the polarization plane.

New simplified and physically sound expressions were derived for the kinematic interaction factors, $|I_u|$ and $|I_\theta|$, and standard methods for using these functions in the evaluation of the FIM were critically reviewed, with reference to both static and dynamic procedures for the seismic design of structures. More in detail, it was pointed out that real-valued functions $|I_u|$ and $|I_\theta|$ can be used instead of complex-valued factors $I_u$ and $I_\theta$. Moreover, filtering functions can be applied either to the Fourier amplitude spectrum or to the response spectrum of the free-field signal, if the design motion must be specified as a time history or acceleration spectrum respectively.

In order to reduce the problem, usual approximations of linear viscoelastic material and uniform half-space were assumed for the soil. Possible nonlinearities or non-uniformities, leading to variability of shear wave velocity with induced strain level or depth, could be taken into account using conventional procedures, as summarized in Brandenberg *et al*. (2015).

Further studies are needed to explore the filtering behaviour of embedded foundations in subsoil conditions substantially different from those considered in this paper and to highlight the role of the stiffness of the foundation in cases where the rigidity assumption is no longer applicable.

## APPENDIX

Coefficients $a_1$, $a_2$, $a_3$, $a_4$ in Equations (8) and (9) are computed as a function $B/H$ from the best fit of numerical data:

$$a_1(B/H) = \frac{(B/H)^\alpha}{\beta+(B/H)^\alpha} \qquad \alpha = 1.04 \qquad \beta = 4.24 \qquad \text{(A1)}$$

$$a_2(B/H) = \frac{\alpha+(B/H)}{1+(B/H)} \qquad \alpha = 1.92 \qquad \text{(A2)}$$

$$a_3(B/H) = \frac{1}{1+\sqrt{B/H}} + \alpha(B/H)^\beta \qquad \alpha = 0.32 \qquad \beta = 0.38 \qquad \text{(A3)}$$

$$a_4(B/H) = \frac{1}{1+\alpha(B/H)^\beta} \qquad \alpha = 1.70 \qquad \beta = 1.62 \qquad \text{(A4)}$$

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