Influence of fuel plenum on thermoacoustic oscillations inside a swirl combustor

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Abstract

Large Eddy Simulation (LES) of a lean methane/air flame exhibiting self-excited thermoacoustic oscillations inside a swirl combustor is performed. The influence of fuel plenum on the combustor dynamics is studied. The velocity, temperature and mixture fraction statistics computed from the two LES cases are compared with the measurements. The results show good overall agreement for both cases with slight improvements by including the fuel plenum. In particular, the near-field mixing and flame lift-off height are captured more accurately. The dominant unstable Helmholtz frequency (≈ 270 Hz) computed by including the plenum is closer to the measured value (≈ 290 Hz) compared to that (≈ 250 Hz) obtained without the plenum. Mode shape analysis shows that including the fuel plenum is necessary for obtaining the phase difference between the pressure oscillations in the combustion chamber and air plenum. It is also found that self-excited oscillations are amplified with an increase by 6% in the mean Rayleigh index and 50% in the gain of the flame transfer function due to the equivalence ratio fluctuations when the fuel plenum is included.

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1. Introduction

Combustion instability is a central topic for the development of modern gas turbine (GT) operating under fuel lean conditions to reduce emissions [1]. These instabilities arise when unsteady heat release and pressure fluctuations are coupled with the combustor acoustic modes leading to self-excited oscillations with large amplitudes, which are undesirable for GT combustion systems [2]. Thus, estimation of these oscillations has been the subject of major interest over the last few decades resulting in a significant progress in its modelling approaches (see Refs. [3–5] for a review).

In a classical system acoustic analysis, also referred to as the low-order modelling approach, the combustor geometry is simplified as a network of low-dimensional (mostly 1D) elements, within which the linearised Navier-Stokes equations are solved [6, 7]. The low computational cost of this approach makes it attractive for stability analysis of practical GT combustors. However, the success of lower-order modelling depends on the specification of flame transfer/describing function (FTF/FDF) which describes the heat release rate response to velocity perturbations. The FTF/FDF is obtained *a priori* using analytical models [8, 9], experiments [10, 11] or numerical simulations [12, 13]. Further to the requirement of FTF/FDF, it could be quite challenging to use network models for combustors having complex and multiple air and fuel passages. These passages can have different impedances and thus their incoming velocities respond differently to the acoustic waves leading to substantial variations in mixing (i.e., equivalence ratio)

and flame (heat release rate) behaviours [14–17]. Therefore, the combustor system stability can change significantly due to the geometrical complexities of the flow passages, which are difficult to consider for network models.

Computational Fluid dynamics (CFD) has emerged as a powerful alternative approach to understanding and estimating combustion instabilities [18]. The unsteady Reynolds-averaged Navier-Stokes (URANS) [12, 19–21] and *incompressible* large eddy simulation (LES) [13, 22, 23] approaches have been applied for a wide range of problems and successfully served as a tool to obtain the FTF or FDF required for the network models. In these approaches, a low-Mach number assumption is often employed which implies that the density variation in the flow depends only on the changes in temperature and mixture composition but not the pressure. It is also implied that the flame response is driven by the hydrodynamic (or velocity) perturbations, which are typically imposed using external forcing. It is quite straightforward to characterise these forced oscillations for perfectly premixed systems with a single inflow passage [10, 11] but not so for systems with multiple inlet passages [24, 25], which are common in practical combustors. Also, partially premixed systems require more than one transfer function to describe the flame response to the velocity and equivalence ratio perturbations [21], adding further complexities to the network models.

The rapid advances of massively parallel computing enables the use of fully compressible LES for studying combustion instabilities in complex real GT engine geometries [26]. This approach solves the 3D compressible Navier-Stokes equations governing the turbulent reacting flows with acoustic wave motions. A good example is the successful estimation of self-excited azimuthal instability in a real-engine annular combustor [27, 28]. Although such a LES remains too costly

for routine use in a practical combustor design cycle, it offers very useful insight on important processes and design factors influencing combustion instability.

In this LES study, it is of particular interest to study the influence of fuel plenum on the LES of self-excited oscillations in a model GT combustor experimentally investigated by Meier *et al.* [24, 29] at the German Aerospace Center (DLR). This combustor features a dual-swirler configuration for the two-stream air supply and methane is injected between the two air streams, which is a common design for real aero engine combustors to achieve enhanced fuel–air mixing. An unstable flame exhibiting pronounced thermoacoustic oscillations was well characterised in the experiments, i.e., highly repeatable for both frequency and amplitude with the same operating conditions, and this flame is of interest for this study.

Previous LES investigations of this burner [30, 31] excluded the fuel plenum (see Fig. 1) for the computational geometry by specifying the fuel inlet plane only. The results showed some discrepancies in both the oscillation frequency and amplitude, although the computed time-averaged flow and flame statistics agreed quite well with the measurements. Also, the *fuel jet flapping* was identified [30] to be the reason for the flame shape to change from a stable V-shape to a flat flame with self-excited oscillations. The pressure fluctuation in the combustor could also modulate the fuel flow rate, which could influence the fuel jet flapping and thereby produce different flame shape and behaviour. To explore these potential effects of the fuel plenum, the specific objectives of this paper are (1) to perform LES of this unstable flame with the fuel plenum included and compare with previous results without the plenum to identify the effects on the acoustics, flow and flame statistics; and (2) to quantify the contribution of the fuel stream perturbation on

the overall equivalence ratio and heat release fluctuations, so as to examine the relative importance of including the fuel plenum in the LES.

In the remainder of this paper, the target combustor configuration is described in the next section with a focus on the fuel supply arrangement. The following section briefly introduces the numerical approach including the combustion model and LES setup. After that the LES results are presented, followed by the discussion on the effects of the fuel plenum. The conclusions are summarised in the final section.

2. Burner setup and target flame

The target configuration of this study is the dual-swirl gas turbine model combustor (GTMC) experimentally investigated by Meier *et al.* [24, 29]. This GTMC comprises a single nozzle, slightly modified from an aero engine nozzle [32], where gaseous methane fuel was injected between two co-swirling air streams. A schematic of the burner setup with the dimensions is shown in Fig. 1 along with the time-averaged photographic image of the flame. Enlarged view shows the fuel plenum geometry along with the 72 square (0.5×0.5 mm) nozzles. The ambient air with a mass flow rate of 4.68 g/s was supplied through a circular pipe at the bottom of the air plenum. This air is divided into two separate passages with co-rotating radial swirlers before entering the combustion chamber. Pure methane was fed at a mass flow rate of 0.205 g/s through three inlets mounted on the air plenum wall and an annular fuel plenum is connected with these inlets using six circular tubes as shown in the enlarged view in Fig. 1. A conical injector is located on the top of this plenum and fuel is injected into the combustion chamber through a ring of 72 nozzles, which are mounted on the annular wall between the

two air streams. The exit plane of the outer air stream is marked as zero for the streamwise distance, i.e., $x = 0$. The fuel and inner air streams exit the nozzles at $x = -4.5$ mm. These above air and fuel flow rates resulted in a global equivalence ratio of 0.75 and a nominal thermal power of 10.3 kW for the unstable flame considered here, which was operated at ambient conditions.

Fig. 1: Schematic of the target GTMC configuration [24, 29]. P1, P2 and P3 are the pressure transducers.

Three microphones with a sampling rate of 20 kHz, denoted as P1, P2 and P3 in Fig. 1, were used for pressure measurement in the air plenum and opposite corners of the combustion chamber [33, 34]. The power spectral densities (PSDs) of the measured signals at these positions are presented in Fig. 2. As can be seen, all three signals show a pronounced peak at the frequency of about

290 Hz with the magnitude between 120 and 140 dB/Hz. The magnitude for P1 in the air plenum is about 10 dB/Hz larger than that measured at P2 and P3 in the combustion chamber (see inset of Fig. 2). In general, P2 and P3 exhibit very similar behaviour despite their different locations. Hence, only P2 is chosen to represent the acoustic behaviour for the combustion chamber along with P1 for the air plenum for the analyses discussed in later sections. This thermoacoustic oscillation frequency of about 290 Hz was highly repeatable in the experiments with an uncertainty of approximately 5% (± 15 Hz) due to the variations in ambient conditions (temperature, pressure, etc.) and small changes in the inlet mass flow rates [33].

Fig. 2: Power spectral density of the measured pressure signals (vertical dashed line denotes the thermoacoustic instability frequency of 290 Hz).

To characterise this unstable flame for CFD model validations, Meier and co-workers applied advanced laser diagnostics to obtain quantitative measurements for the velocity (PIV/LDV) and various scalar (Raman scattering) fields [24, 29,

33, 35] covering the entire combustion chamber. The measurement uncertainty was reported to be within 4% for the mean values of temperature, mixture fraction and major species concentrations, and about ± 0.3 m/s for the velocity. These measurements are compared with LES results to understand the fuel plenum effects. The numerical approach used for these simulations is described next.

3. Numerical approach

The LES framework and subgrid scale (SGS) closure models used for this work have been described and validated in a number of previous studies, e.g., Refs. [31, 36, 37]. In this approach, the 3D compressible Favre-filtered conservation equations for mass, momentum and energy are solved. The flamelet modelling approach is revised carefully to include partially premixed combustion at subgrid scales. The flamelet library which describes the SGS turbulence-chemistry interaction is characterised using four control variables: filtered mixture fraction, progress variable and their SGS variances. Bilger's formula is used for mixture fraction and progress variable is defined using the sum of CO and CO₂ mass fractions normalised by its burnt value for the local mixture. The transport equations for these variables in a compact form with $\tilde{\boldsymbol{\varphi}} = \{\tilde{Z}, \widetilde{Z'^2}, \tilde{c}, \widetilde{c'^2}\}^T$ are written as

$$\bar{\rho} \frac{D\tilde{\boldsymbol{\varphi}}}{Dt} = \nabla \cdot (\overline{\rho \mathcal{D}_{\text{eff}}} \nabla \tilde{\boldsymbol{\varphi}}) + \overline{S_{\boldsymbol{\varphi}}^+} - \overline{S_{\boldsymbol{\varphi}}^-}, \quad (1)$$

where the effective diffusivity is modelled as $\overline{\rho \mathcal{D}_{\text{eff}}} = \bar{\rho}(\tilde{\nu} + \nu_t / \text{Sc}_t)$ with a turbulent Schmidt number of $\text{Sc}_t = 0.4$ for all scalars [31, 38]. The molecular and turbulent viscosities, $\tilde{\nu}$ and ν_t , are obtained using Sutherland's law and the constant Smagorinsky model [39], respectively. The source and sink terms on the RHS of

Eqn. (1) are

$$\overline{\mathbf{S}}_{\phi}^+ = \left\{ 0, \quad 2\overline{\rho} \frac{\nu_t}{\text{Sc}_t} |\nabla \widetilde{Z}|^2, \quad \overline{\dot{\omega}}_c^*, \quad 2\overline{\rho} \frac{\nu_t}{\text{Sc}_t} |\nabla \widetilde{c}|^2 + 2(\overline{c\dot{\omega}}_c^* - \widetilde{c}\overline{\dot{\omega}}_c^*) \right\}^T, \quad (2)$$

$$\text{and} \quad \overline{\mathbf{S}}_{\phi}^- = \left\{ 0, \quad 2\overline{\rho} \widetilde{\chi}_{Z,\text{sgs}}, \quad 0, \quad 2\overline{\rho} \widetilde{\chi}_{c,\text{sgs}} \right\}^T, \quad (3)$$

where the reaction rate, $\overline{\dot{\omega}}_c^*$ accounts for both premixed and non-premixed mode contributions as in [31, 36, 40]. The premixed part is modelled using a collection of laminar premixed flamelets in conjunction with a SGS joint probability density function; and an algebraic model is employed for the non-premixed contribution. For the unclosed SGS scalar dissipation rate terms, $\widetilde{\chi}_{Z,\text{sgs}}$ and $\widetilde{\chi}_{c,\text{sgs}}$, two different closure models are used for the mixture fraction and progress variable, respectively, in order to retain physical consistencies in the micro-mixing processes of the conserved and reactive scalar fields as discussed in [36]. A linear relaxation model is used for $\widetilde{\chi}_{Z,\text{sgs}}$. The algebraic model developed in [41] and tested in a number of studies [31, 36, 40] is used for $\widetilde{\chi}_{c,\text{sgs}}$. This revised flamelet formulation for partially premixed combustion was referred to as *FlaRe* (Flamelets Revised for consistencies) in earlier studies [42, 43].

The computational grid used for simulation with the fuel plenum, denoted as case A hereafter, is presented in Fig. 3. The same mesh distribution was used in previous studies without the fuel plenum [30, 31] and this case is denoted as case B here. Instead of including the fuel passage geometry, the fuel inlet boundary was placed on the annular wall between the two air stream exits for case B (see Fig. 1). It is worth noting that the mesh distribution in case B is kept identical to the corresponding parts in case A so that a direct comparison of the results can be made. A mesh sensitivity study was conducted in [31] and the chosen grid here

was shown to be sufficient to obtain mesh independent time-averaged velocity statistics. Figure 4 shows the Pope’s criterion [44] for resolved turbulent kinetic energy for the near field of the combustor up to 40 mm, which includes the entire flame region. It can be seen that for both the non-reacting and reacting flows more than 90% of the turbulent fluctuations are resolved by the LES grid and hence the criterion is satisfied. Note that the non-reacting flow case has a higher swirler number (~ 0.9) than the reacting case (~ 0.55) and thus stronger fluctuations are seen in the shear regions in Fig. 4a. Further details are available in Ref. [31].

The mesh details for these two cases are given in Table 1 and it is noted that an additional 8 million cells are needed for the fuel plenum geometry leading to a considerable increase in the computational cost. As shown in Fig. 3, a large hemispherical domain is added to the downstream of the combustor outlet in order to avoid the reflection of acoustic waves from the outlet. The hemispherical surface is set to be at atmospheric pressure in both LES cases. Alternative approaches such as the time-domain impedance boundary conditions (TDIBC) [45, 46] can also be used for the combustor exhaust tube but would require the geometry impedance to be specified, which could be challenging. All inlets are specified with fully reflective mass flow conditions using the experimental values discussed in the previous section.

The CFD toolbox OpenFOAM 2.3.0 is used to solve the LES equations for this study. The pressure-based compressible PIMPLE solver is adopted with an in-house implementation of the *FlaRe* combustion model described earlier in this section. A time-step size of $\Delta t = 3 \times 10^{-7}$ s was used in [30] for case B and it is halved for case A due to the halved smallest grid spacing for the fuel nozzle region as shown in Table 1. For both cases, a “fully excited” state showing limit

Fig. 3: Computational setup and mesh distribution for case A (red lines mark the fuel fluid path).

Fig. 4: Typical mid-plane variation of Pope's criterion for resolved turbulent kinetic energy, $k_{\text{resolved}}/k_{\text{total}}$, computed for (a) non-reacting and (b) reacting flows.

cycle oscillations is reached first and subsequently the LES data are collected for 100 ms, which corresponds to about 12 flow-through-times for the combustion chamber and 25 thermoacoustic cycles. The computations for this work are performed using 1280 processors on the UK National Supercomputer (ARCHER, 2.7 GHz 12-core processor with 32GB RAM) and the Cambridge University Cluster

Table 1: Numerical grid spacing [mm] details for the simulated cases.

Mesh region	Case A	Case B
Fuel plenum		
included or not	Yes	No
Fuel nozzle	0.05	0.1
Swirlers	0.3	0.3
Chamber	0.5	0.5
Total No. of cells	29M	21M

(CSD3, 2.6 GHz 16-core processor with 96GB RAM). The typical time required for the computation of 1 ms of physical time is about 70 minutes on the wall-clock for case A, which is nearly twice the time required for case B. This is because of the finer grid spacing and the necessity to use a smaller time-step to resolve the flow in the fuel injector passages in case A.

4. Flame structure and statistics

The computed mean axial velocity with streamlines and temperature are shown in Fig. 5 for the two cases listed in Table 1. These results are shown for one half of the mid-plane. Note that these contours and streamlines are for time-averaged quantities with a spatial averaging in the azimuthal direction. It can be seen in Fig. 5a that the difference in the mean flow field is quite small between the two cases despite a slightly shorter inner recirculation zone (IRZ) (see streamlines) for case A which the fuel plenum. This is consistent with the mean temperature field in Fig. 5b showing a weaker penetration of the fresh gases (denoted by darker

colours) in both the axial and radial directions for case A. Another noticeable difference is that in case A the flame base is further lifted along the centreline as shown in the dashed box. The lift-off height is marked here using the temperature of 1500 K following earlier studies [30, 37, 40], which will be discussed later in this section.

Fig. 5: Comparison of mid-plane contours (plotted region marked in Fig. 1a by red dashed lines): (a) mean axial velocity with streamlines and (b) mean temperature (white arrows indicate the flame lift-off height along the centreline).

For a quantitative assessment of the fuel plenum effects on the flow and flame structures, the time-averaged statistics obtained from the two LES cases are compared with the measurements [24, 29]. Figure 6 shows the radial variations of the mean and resolved root mean square (RMS) axial velocities at six streamwise locations: $x = 1, 5, 10, 20, 30$ and 50 mm. An overall good agreement is observed. Similar to Fig. 5a, little difference is seen between the two cases suggesting that including the fuel plenum in the simulation does not influence the axial velocity statistics unduly. A small increase in the RMS value is observed for $x = 1$ mm

Fig. 6: Radial variation of (a) mean and (b) resolved RMS axial velocity at five streamwise distances. Dashed lines indicate the subgrid rms.

location for case A and this is due to the velocity oscillation inside the fuel nozzles introduced by the acoustic pressure fluctuations, which is absent in case B.

This difference diminishes in the downstream. There is some under-estimation of the RMS values for locations at $x = 20$ and 30 mm in both LES cases because the grid resolution becomes coarser (see Fig. 3) in the downstream leading to less resolved turbulent kinetic energy. However, this does not influence the flame as it is mostly contained in the region of $x < 20$ mm as seen in Fig. 5b. The subgrid RMS for locations $x < 20$ mm is also plotted in Fig. 5b. This RMS is computed using a scale similarity model [44], $u'_\Delta = |\widehat{\tilde{U}} - \tilde{U}|$, where $\tilde{U}$ is the filtered velocity vector and $\widehat{\cdot}$ denotes a test-filtering operation. It is clear that the subgrid RMS is much smaller than the resolved value suggesting that the numerical resolution used is satisfactory for the regions of interest.

The mean radial and swirl velocity profiles are presented in Fig. 7. In general, the computed results for both cases A and B are in very good agreement with the measurements. A notable improvement of about 10–15% is seen in Fig. 7a for $|r| > 20$ mm at $x = 5$ and 10 mm from case B to case A. The swirl velocity computed in both cases agree very well with the measured values as seen in Fig. 7b.

The radial profiles of mean and resolved RMS mixture fraction values computed in cases A and B are compared with the measured values in Fig. 8 for $x = 5, 10, 20, 30$ and 50 mm. In the near field, i.e., $x = 5$ mm, both simulations over-estimate the mean mixture fraction by about 10–20% for $|r| < 15$ mm, while some under-estimation is observed for $|r| > 20$ mm. These discrepancies becomes smaller as one moves to the downstream and, they eventually vanishes after $x = 30$ mm. This suggests that the radial transport of mixture fraction (or fuel) is slower in the computations compared to the experiments.

It was identified in [30] that this near-field fuel transport in the radial direction

Fig. 7: Radial variation of mean (a) radial and (b) swirl velocity (see Fig. 6 for legend).

is mainly driven by the flapping motion of the fuel jet close to the combustor dump plane, which is a consequence of the self-excited pressure oscillations influencing the incoming air flows. Thus, the thermoacoustic instability has a direct influence

Fig. 8: Radial variations of (a) mean and (b) resolved RMS mixture fraction (see Fig. 6 for legend). Dashed lines indicate the subgrid rms (multiplied by 10).

on the mixture fraction distribution in the near field and the discrepancies seen at $x = 5$ and 10 mm could be caused by the differences in the computed frequency and amplitude of the pressure oscillations compared to the measurements. This is supported by the fact that for the streamwise location of $x = 5$ mm, case A gives improved results near the centreline and also for the far radial region ($r > 20$ mm).

The pressure oscillations are indeed captured better in case A compared to case B as one shall see in the next section. The mixture fraction RMS values are estimated quite well in both cases except for $x = 5$ mm, where the peak is significantly over-estimated by about 50%. It should be noted that the probe size used for the Raman measurements is about 0.6 mm [29], which is larger than the LES filter size in the near field, and thus the measured RMS values tend to be smaller due to the averaging effect within the probe volume. The subgrid RMS multiplied by 10 is also shown for locations of $x = 5, 10$ and 20 mm. The subgrid RMS values are about an order of magnitude smaller than the resolved value at $x = 5$, where the lifted flame base experiences the strongest interaction with the mixing field. This suggests that the fuel and air are well mixed upstream of this location due to the dual swirl configuration and the flame burns in a predominantly stratified mode.

Figure 9 compares the radial variations of mean and RMS temperature values. The results from both simulations agree very well with the measurements. The centreline mean temperature at $x = 5$ mm is captured accurately in case A, which is improved from case B by about 15% (see the difference in Fig. 5b), suggesting that the flame lift-off height is better estimated when the fuel plenum is included in the LES.

5. Self-excited thermoacoustic instability

The pressure spectra computed using the results of the two LES cases and the measured signals are compared in Fig. 10 for the probes P1 and P2 located in the air plenum and combustion chamber, respectively (see Fig. 1). The frequency of the dominant unstable mode observed in the experiments is around 290 Hz with an uncertainty of ± 15 Hz marked using dashed lines in the figure. The estimated

Fig. 9: Radial variations of (a) mean and (b) resolved RMS temperature (see Fig. 6 for legend).

frequency in case A is about 270 Hz, which is shifted towards the experimental range compared to that about 250 Hz in case B. However, the amplitude is over-estimated in both LES cases especially in the air plenum as seen in Fig. 10a. Such over-estimations seem to be common for LES of self-excited combustion instabilities [45–47] and it is partly attributed to the fact that unlike in the simulations, the combustor walls are not perfectly rigid in the experiments and thus

can absorb a good amount of acoustic energy. Nonetheless, the self-excited oscillations observed in the experiments [33–35] are reproduced quite well in both LES cases with a closer estimation in case A, suggesting that the fuel plenum does play a role in the thermoacoustics of this burner. The accuracy for the peak frequency increases considerably when the fuel plenum is included in the simulation. Although system level acoustic calculations using low-order models show that acoustic oscillation frequencies can change with the air-volume in the system, this is the first LES study demonstrating this clearly in the context of thermoacoustic oscillations.

Fig. 10: Comparison of pressure spectra in the (a) air plenum and (b) combustion chamber. Vertical dashed lines show the measured peak at 290 Hz with an uncertainty of ± 15 Hz.

It is important to understand the spatial evolution of unstable modes inside the combustor. Since these oscillations are purely axial behaving like an 1D planar wave (verified using pressure animations generated from LES results), the multi-dimensional post-processing methods such as proper orthogonal decomposition is not worthwhile for this study. Instead, specific line paths along the axial direction are used, which is sufficient for the mode shape analysis here. The computed mode shapes in cases A and B for the two pronounced modes peaking at about

250 and 90 Hz (marked in Fig. 10a) are depicted in Fig. 11 for both the amplitude and phase. These results are generated as follows. First, the pressure spectrum at every axial locations marked in Fig. 11 is computed to identify the two dominant modes and then their amplitude and phase differences are compared. These two modes are referred to as mode I and II as marked in Fig 10a. Note that the phase angle of mode I for the most downstream location ($x = 0.1$ mm) is defined as the zero in each case. The amplitude is normalised using the peak value observed in each case. Since the air path has two separate swirler passages from the air plenum inlet to the combustion chamber outlet as shown in Figs. 11a and 11b, this analysis is done for these two paths.

For the most dominant mode I, the mode shapes observed in both cases are typical Helmholtz modes showing flat profiles on either side of the “neck”, which corresponds to the swirler location. The amplitude inside the air plenum is about 3 times larger than that in the combustion chamber, but this ratio decreases slightly (by about 10%) when the fuel plenum is included. There is a notable increase in the amplitude from $x = -0.03$ to 0.01 m for the path through the inner swirler, whereas the variation is more gradual for the outer path. This is due to the existence of the precessing vortex core (PVC) inside the inner air nozzle introducing additional hydrodynamics-induced pressure oscillations locally near the root of the core. Details of this coherent flow structure and its interaction with the flame and thermoacoustics are elaborated in Ref. [31]. The amplitude variations are quite similar for the two cases. The phase lag for Mode I between the air plenum and combustion chamber increases significantly, from about 0.33π to 0.47π , when the fuel plenum is included. The computed phase shift for case A is in excellent agreement with the measured value of 85° , whereas it is largely under-estimated

Fig. 11: Amplitude and phase shapes of the two unstable modes I and II (marked in Fig. 10a) along the paths through (a) inner and (b) outer swirlers.

by about 25° in case B. This result indicates that even though the fuel plenum volume is small, the interplay between this small volume and the pressure oscillations in the main volume (i.e., combustion chamber, injector passages, air plenum) has a non-negligible impact on the overall behaviour of the dominant mode.

The structure of Mode II shown in Fig. 11 is quite different. Compared to the most dominant mode, mode II exhibits more or less uniform amplitude and

phase distributions throughout the air plenum and combustor except for some small variations near the swirlers. The reason for this behaviour is unclear at this time. However, it is unlikely to be related to the resonant acoustic modes associated with the combustor geometry because of the low frequency ($f_{II} \approx 90$ Hz) and phase uniformity. Interestingly, this frequency corresponds to the convective timescale for the mean flow, i.e., $1/f_{II} \approx l_{\text{cham}}/U_b$, where $l_{\text{cham}} = 114$ mm is the length of the combustion chamber and the bulk mean axial velocity U_b is about 10 m/s. Therefore, it is postulated that this low-frequency mode could be related to the entropy waves [6], corresponding to the so-called *hot-spots* generated by the unsteady combustion. These hot-spots, carried by the mean convective flow, can generate acoustic waves when accelerating through a converging nozzle such as the outlet duct of the present burner (see Fig. 1). Due to the low frequency at which these acoustic waves are generated, their wavelengths are of several orders of magnitude larger than the combustor length and hence no significant phase difference can be observed between the pressure signals measured in the air plenum and combustion chamber. Further investigation into this mechanism is beyond the scope of this study and will be explored in future. Nevertheless, the LES results shown in Fig. 11 provide useful additional insights into the detailed structures of the self-excited modes in this combustor, which could not be obtained from the experiments due to the limitations on pressure measurements. This analysis also highlights the necessity of including fuel plenum in the simulation to estimate these unstable modes accurately.

To elucidate the relative significance of the amplitudes for the two unstable modes, Fig. 12 compares the mode II to mode I amplitude ratio for cases A and B. The axial variations along both the inner and outer paths shown in Fig. 11 are

Fig. 12: Mode II to mode I amplitude ratio for cases A and B: axial variation along the path through the inner and outer swirlers (see Fig. 11).

presented. It can be clearly seen that this ratio is small in air plenum (i.e., mode I is much stronger than mode II) and becomes large in the combustion chamber. This increase is more evident in case A and the ratio reaches up to 0.8 suggesting that the two modes are of the same level of significance as one approaches the combustor exit when the fuel plenum is included in the simulation.

Figure 13 depicts the temporal fluctuations of pressure (probe P2) and volume-integrated heat release rate from cases A and B. It can be seen that in both cases there is an evident dominant frequency for the heat release rate and chamber pressure oscillations, and this frequency corresponds to mode I in Fig. 10. The pressure and heat release fluctuations shown in Fig. 13 are generally in phase and thus the Rayleigh Criterion [48] is satisfied for the combustion instabilities to ensue. This is shown in Fig. 14 using the spatial variation of the time-averaged correlation of pressure and heat release rate fluctuations, also known as the Rayleigh Index,

$$\overline{p' \dot{q}'}(\mathbf{x}) = \frac{1}{\mathcal{T}} \int_{\mathcal{T}} p'(\mathbf{x}, t) \dot{q}'(\mathbf{x}, t) dt, \quad (4)$$

for the two LES cases and $\mathcal{T}$ is the time duration. It can be seen that this correlation is mainly positive in the flame region for both cases and some small negative values are only present in the outer shear layer. The absolute value in case A is generally larger than in case B particularly for the flame base region close to the nozzle where the influence of the fuel plenum is expected to be the most significant. For system stability analysis, the volume-integrated correlation quantity over the entire combustion chamber, denoted as $RI(t) = \int_V p'(\mathbf{x}, t) \dot{q}'(\mathbf{x}, t) dV$, is of interest and hence its temporal variation is also plotted in Fig. 14. The horizontal solid lines correspond to the mean values and the dashed lines denote zero. Despite the strong fluctuations on both the positive and negative sides, the mean values for both cases are positive and hence the system is unstable. As expected from the contour plot, the time-averaged value of $\int_V p' \dot{q}' dV$ for case A is about 6% larger than that for case B suggesting that the presence of the fuel plenum amplifies the self-excited oscillations in this burner.

Fig. 13: Normalised temporal variations of combustion chamber pressure and volume-integrated heat release rate fluctuations.

Since the fuel plenum has a much smaller volume compared to the air plenum, it is expected to have negligible effect on the combustor acoustics [49]. However, the pressure oscillations can modulate the fuel flow and thus have an impact on

Fig. 14: Normalised spatial variation of $\overline{p'q'}$ (left) and temporal variation of $RI(t) = \int_V p'q' dV$ (right, horizontal solid and dashed lines denote the mean and zero values respectively).

the combustion instability through the FTF/FDF, unless there is a very large pressure drop across the fuel injector. In that case the pressure field inside the fuel plenum would remain more or less constant despite the oscillations in the combustion chamber. The pressure spectrum at the probe P_F (marked Fig. 15a) located inside the annular fuel plenum is depicted in Fig. 15b. The spectral behaviour here is very similar to that for the combustion chamber seen in Fig. 10b, both showing a primary peak at around 270 Hz with similar amplitudes. This implies that although the cross-sectional area of the fuel nozzles is quite small (0.5×0.5 mm) the pressure fluctuations in the combustion chamber have a significant impact on the flow upstream of the fuel nozzles. There is also a phase difference of about 30° between the probes P_2 and P_F indicating a potentially influential time lag for the fuel stream perturbations. Therefore, these results suggest that it may be necessary to

include the fuel plenum in the LES to study self-excited instabilities.

To further demonstrate the effect of pressure oscillations on the inflows, Fig. 15c presents the mass flow rate fluctuations for both fuel and air (total value including inner and outer streams) at $x = -6$ mm (location marked in Fig. 15a). Note that the values shown are normalised by the total mass flow rates given at their respective inlet boundaries. It can be seen that the fluctuation level in the fuel flow rate is generally smaller than in the air flow. The maximum variation of the air mass flow rate reaches up to 40%, which is about twice of that for the fuel. As a result, the amplitude ratio of fuel to air mass flow rate fluctuations varies dramatically from one period to another. For instance, for the cycles in the period of $t = 0.03$ to 0.04 s (marked by solid box), this ratio is equal to one as shown by the overlapping curves; and then for $t = 0.05$ to 0.06 s (marked by dashed box), the peak amplitude for fuel is much smaller than that for the air indicating a low amplitude ratio. Consequently, this change in the amplitude ratio introduces equivalence ratio perturbations, additional to that caused by the fluctuation only in the air flow rates for case B, and further influences the thermoacoustic instability in case A with the fuel plenum.

It has been shown in the past studies [5, 50] that equivalence ratio perturbations have a major impact on the heat release rate response of the flame. Unlike velocity perturbations (at low/moderate frequencies), for which the flame response is spatially uniform (for Mach number $\ll 1$), equivalence ratio perturbations can lead to strong spatial variations of the heat release rate in the flame region. This is because the velocity perturbations propagate at the speed of sound while the equivalence ratio perturbations are convected at the mean flow velocity. For the dominant unstable frequency of $f \approx 250$ Hz, the characteristic length scales for these two

Fig. 15: LES case A: (a) schematic of nozzle geometry and inflows, (b) pressure spectrum inside fuel plenum and (c) temporal variation of fuel and air mass flow rates.

perturbations can be estimated as $l_v = C/f \approx 3 \text{ m}$ and $l_\phi = U_b/f \approx 0.04 \text{ m}$, respectively. Here the speed of sound $C = 750 \text{ m/s}$ was evaluated at the mid-point of the flame with the filtered progress variable value of 0.5. Compared to the flame length l_F spanning over a streamwise distance of about 0.03 m (see Fig. 5b), l_v is two orders of magnitude larger and thus, the flame is acoustically compact and can be considered as a thin sheet as has been done in many past studies using network models, see Ref. [6]. For l_ϕ , however, it is of the same order of l_F , resulting in a more complex flame response to equivalence ratio perturbations. Only few past studies have investigated this in laminar flames using analytical and nu-

merical approaches [51, 52]; and characterising equivalence ratio perturbations is very challenging in the experiments. Thus, LES becomes a suitable approach for this issue as the relevant physical scales and their interactions are considered inherently by solving the LES equations described in Section 3.

Fig. 16: Power spectra of heat release rate and equivalence ratio fluctuations.

Figure 16 compares the power spectra of the heat release rate and equivalence ratio fluctuations (scaled by the mean) for the two LES cases. These two quantities are evaluated for the entire combustion chamber and at $x = -6$ mm respectively. For the equivalence ratio, case B shows a considerably larger peak value compared to case A. This is because in case A both fuel and air mass flow rates are modulated by the pressure oscillations, whereas in case B only the air flow rate varies resulting in a larger magnitude of equivalence ratio fluctuation. However, the peak magnitude is similar for the heat release rate fluctuations between the two cases. Following previous studies [50, 52], a flame transfer function for the equivalence ratio perturbations is defined as

$$F_{\phi}(\omega) = \frac{\dot{q}'/\bar{q}}{\phi'/\bar{\phi}} = G(\omega) e^{i\Phi(\omega)}, \quad (5)$$

where ω is the angular frequency, $G(\omega)$ is the gain and $\Phi(\omega)$ is the phase. The gain and phase are presented in Table 2 for cases A and B at their respective dominant frequencies. It is seen that the gain increases by nearly 50% when the fuel plenum is included but the phase value remains almost the same. This suggests that the fuel plenum has a significant influence on the magnitude of flame response to equivalence ratio perturbation.

Table 2: Equivalence ratio transfer function values at the dominant unstable frequency.

$F_\phi = (\widehat{\dot{q}}' / \overline{\dot{q}}) / (\widehat{\phi}' / \overline{\phi})$	Case A	Case B
Gain	0.52	0.36
Phase	-0.9π	-0.95π

6. Conclusions

The influence of the fuel plenum on self-excited thermoacoustic instability in a gas turbine model combustor is investigated. Two LES cases are considered: case A includes the fuel plenum in the computational domain; and the fuel is directly injected from the nozzle exit plane in case B. The results of these two cases are compared with the measurements of flow and flame statistics, and a good agreement is obtained for both simulations while case A gives improved near-field comparisons for the mixture fraction and flame lift-off height.

The self-excited thermoacoustic oscillation is captured in both cases. A considerable improvement by 20 Hz for the dominant frequency is obtained when the fuel plenum is included. The estimated frequency is around 270 Hz for case A, which is close to the measured range between 275 and 315 Hz, whereas the

dominant frequency is about 250 Hz for case B. The computed mode shapes show the typical Helmholtz mode features in both cases; and the phase difference between the air plenum and combustion chamber pressure oscillations is precisely estimated in case A, showing a substantial improvement from case B. The computed mean Rayleigh index is found to be higher in case A especially in the near field. Furthermore, it is found that both cases exhibit strong equivalence ratio fluctuations upstream of the flame, however, the heat release response is quite different. The gain of the flame transfer function at the dominant frequency is significantly higher in case A suggesting an important influence of fuel plenum through the equivalence ratio oscillations on the combustion dynamics of the dual-swirl combustor configuration investigated, which is similar to that used in aero engines. Since the self-excited oscillations ensue from heat release-sound-geometry interactions the quantitative influences of fuel plenum observed in this study are likely to be specific for the geometry considered here. However, the effects of fuel plenum observed here can be carried to other geometries in qualitative manners.

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