Modelling the cleaning of viscoplastic layers by impinging coherent turbulent water jets

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Abstract

Impinging liquid jets are widely used to clean unwanted soil layers from the walls of structures and vessels. When the soil is a thin layer of an immiscible viscoplastic material, removal involves the growth of a cleared area (which is circular for a jet impinging normally) bounded by a berm of displaced material. Glover *et al.* [1] presented a semi-empirical model relating the rate of removal to the momentum flow rate in the liquid film. We present a first-order model for cleaning thin layers of these materials based on the rate of rate of viscous dissipation in a shallow wedge of material at the cleaning front. This yields a result of the form of the model in [1], with expressions linking the kinetic parameters to measurable quantities including the rheology of the soil. The fully coupled problem is not solved: the wedge angle and residual layer thickness need to be specified and were obtained here by fitting to the data. The model is compared with experimental results obtained for three soft solids – two petroleum jellies and a soft paraffin – which exhibited Bingham plastic behaviour and creep, for jet Reynolds numbers between 10,000-37,000, and $0.1 < \bar{h}/\delta_0 < 1.5$, where $\bar{h}$ is the average film depth and δ_0 the layer thickness. The data show reasonable agreement with the model.

Keywords: Cleaning; Impinging jet; Modelling; Viscoplastic fluid; Viscous dissipation

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1. Introduction

Cleaning is an important step in the manufacturing of foods, pharmaceuticals and fast moving consumer goods (FMCG), as it is essential to ensure product hygiene, product quality and to prevent cross-contamination between batches. Cleaning-in-place (CIP) operations using impinging water jets are regularly used to remove unwanted layers of material - hereafter referred as the soil - from the internal walls and fittings of processing tanks and vessels. Bhagat *et al.* [2] characterized different cleaning mechanisms in terms of (i) the mobility of the soil, i.e. the response of the material to the forces imposed by the liquid, and (ii) the interactions between the soil and the substrate. The first is intrinsically associated to the soil's rheology, as the cohesive interactions within the material influence the rate of removal, whereas the second is related to tribology and adhesion between the soil and the substrate. This paper focuses on mobile, i.e. fluid, soil layers. The cleaning of immobile (solid) layers has been studied by Oevermann *et al.* [3] and Wilson *et al.* [4]: both these studies involved peeling mechanisms, where the force imposed by the liquid film caused adhesive breakdown at the interface between the soil and the substrate.

The hydrodynamics of a liquid jet impinging on a clean, flat surface is a classic fluid mechanics problem, and has been extensively studied [5–9] since the work by Watson [10]. An understanding of the hydrodynamics of spreading liquid films generated by circular nozzles is required for the study of cleaning soil layers using such jets. Figure 1 shows the main features of the flow of a horizontal coherent liquid jet generated by a nozzle with diameter d_N with mass flow rate $\dot{m}$, impinging at point O on a clean, flat vertical surface. The liquid spreads radially outwards as a thin film in the radial flow zone (RFZ) with local thickness h . At the film jump, the flow of momentum in the liquid film, M , is counter-balanced by surface tension generating a sudden increase in thickness [9]. The liquid then drains down the wall, either in the form of a falling film for liquid impinging below BB' in Figure 1, or as circumferential ropes which bound the falling film [11].

Figure 1 – Schematic of the flow behaviour of a horizontal coherent liquid jet impinging on a vertical flat surface. (a) Side view through section AA' and (b) front view.

The displacement of immiscible Newtonian liquids, such as oils, by water jets has been investigated previously [12,13]. The removal of non-Newtonian soils show more complex behaviour and has been investigated for shear thinning [14], viscoelastic [15,16] and viscoplastic [1,17] layers. Cleaning viscoplastic materials that are insoluble in the cleaning solution is particularly challenging. The removal of these soils is driven by hydraulic forces and the soil is not displaced unless a minimum force (or stress) is applied. Many fast moving consumer goods (FMCG) products, such as moisturizing creams [18], yoghurts [19,20], toothpastes [21,22] and hair gels [23,24] are viscoplastic, meaning that a critical stress τ_c (often termed the yield stress [25]) marks the transition from solid-like to liquid-like behaviour. Recent advances indicate that the yielding transition is not sharp and takes place over a range of shear stresses and strains [26–30], and a number of different methods to characterize the solid-fluid transition have been reported [27,31,32].

The ratio between the thickness of the liquid film, h , and of the soil layer, δ_o , has been found to determine the cleaning dynamics. Figure 2 presents three distinct cases:

- (a) Very thin soil layers, for which $\delta_o \ll h$, where the film flows over the layer and removal is primarily by viscous drag, as modelled by Yeckel and Middleman [12].
- (b) Thin soil layers, for which $\delta_o \sim h$, and where the film is deflected away from the wall by the displaced soil and removal involves a moving front. This is the scenario considered here.
- (c) Thick soil layers, when $\delta_o \gg h$. Tuck *et al.* [33] reported blister formation and bursting with thick layers of FMCG products, including Carbopol®. The jet liquid initially burrows into the soil layer and accumulates in a blister which eventually ruptures.

For a jet impinging normally the area cleaned is approximately circular. The cleaned radius, a , is the radial position where the residual layer thickness is negligible. Cases (a) and (b) were investigated experimentally by Fernandes *et al.* [17] and are commonly observed on the side walls of processing tanks and vessels. Case (c) is more likely to be observed at the base of such vessels.

Figure 2 – Schematic of jet cleaning for different soil layers scenarios. (a) Very thin soil layer, (b) thin soil layer and (c) thick soil layer. Inset in (b) shows the wedge construction employed by Glover *et al.* [1]

Wilson and co-workers [1,3,17,34] investigated the removal of viscoplastic thin soil layers from flat surfaces by coherent, turbulent impinging water jets. They presented an empirical model in which the rate at which the layer is visibly removed, i.e. the velocity of the visible cleaning front, V , depends on M , the local rate of flow of momentum in the liquid film per unit length of the cleaning front, with:

$$\frac{da}{dt} = V = k'(M - M_y) \quad M \geq M_y \quad (1)$$

$$V = 0 \quad M < M_y. \quad (2)$$

Here, M_y is a limiting value which results in no removal, and k' is a rate constant. This kinetic scheme differs from film erosion models such as those presented by Yeckel and Middleman [12] and Yeckel *et al.* [13], where the flow exerts a shear stress $\tau_{w,l}$ on the layer and causes it to flow downstream. The local thinning rate is then determined by the shear stress exerted on the layer. We use the terms ‘film’ to describe the cleaning liquid and ‘layer’ for the soil material. Data such as those in Figure 3, reproduced from Fernandes *et al.* [17], showed that the local cleaning rate for an immiscible viscoplastic soil is not determined by a shear driven mechanism (as in [12,13]) since the shear stress imposed by the liquid film is smaller than the critical stress of the soil.

Wilson *et al.* [4] investigated the removal of starch layers from flat surfaces and found that their data fitted Equation (1) with $M_y = 0$. Glover *et al.* [1] investigated the removal of layers of petroleum jelly (a viscoplastic material) with thicknesses ranging from about 0.05 to 1.3 mm, and observed that an asymptotic cleaned radius $a_{\max}$ was reached which corresponds to a non-zero value of M_y . They approximated the shape of the cleaning front at $a_{\max}$ to a planar wedge with angle of inclination χ (see inset in Figure 2 (b)). A force balance on the wedge gave

$$M_y = \frac{\tau_c \delta_o}{(\tan \chi - \sin \chi)} \quad (3)$$

where δ_o is the initial thickness of the soil layer. They calculated χ from their experimental results and obtained values in the range $8^\circ < \chi < 26^\circ$.

Figure 3 – Rate of increase of the cleaned radius of layers of petroleum jelly as a function of (a) M and (b) the wall shear stress imposed by the liquid film, calculated using Equations (S.4), (S.8) and (S.12) in the Supplementary Information, for experiments reported by Fernandes *et al.* [17]. Vertical dashed line in (b) indicates the critical stress of the soil layer.

Fernandes *et al.* [17] measured the shape of the berm of soil generated at the cleaning front for a different petroleum jelly. The petroleum jelly was hydrophobic and cohesive, and so it collected in a berm of dislodged material which moved steadily downstream with the cleaning front [35]. The shape of the berm was characterised by two angles, ϕ_1 and ϕ_2 , measured at the cleaning front (at a , where $l = 0$, see Figure 4 (a-i)), and at the level of the initial layer, where $\delta = \delta_o$, shown schematically in Figure 4 (a-ii). Figure 4 (b) shows their data alongside new data sets obtained with two other viscoplastic materials employed in this work. The angles are

plotted against the dimensionless film thickness, h/δ_o , at the location where the measurements were made. Two regimes are evident. When $h/\delta_o < 0.5$, the angle increased with distance from the cleaning front, from ϕ_1 at the leading edge with values similar to the χ values reported by Glover *et al*, to $\phi_2 \sim \pi/4$ at δ_o . The latter is associated with yielding under simple shear caused by the shear stress imposed by the flow of the liquid film. For thicker liquid films, $h \sim \delta_o$, the angle was almost constant, with $\phi_1 \sim \phi_2$.

Figure 4 – (a-i) Coordinates used to describe the shape of the cleaning front in the soil layer. (a-ii) Schematic representation of the shape of the rim, indicating the locations where angles ϕ_1 and ϕ_2 were measured. (b) Effect of h/δ_o on angles ϕ_1 and ϕ_2 measured for cleaning experiments conducted with different flow rates, nozzle diameters, layer thicknesses and

materials. $d_N = 2, 3$ and 4 mm; $6,300 \leq Re \leq 42,300$; $0.1 \leq \delta_o \leq 2$ mm. Properties of layer materials PJA, PJS and WSP are given in Table 2.

Studies such as those in Table 1 have demonstrated the applicability of the momentum-driven model (Equations (1) and (2)), and have combined it with hydraulic models of the flow in the film generated by an impinging jet to predict the performance of a moving jet and soils subject to soaking. To date, a rationalisation of this model which would enable prediction of k' and M_y , or interpretation of experimental results for different soil layer thicknesses and rheology, has not been available.

Table 1 – Works that have used Eq. (1) and (2) to describe the removal of soil layers by impinging water jets

Work	Materials removed	Water jet characteristics	Soil layer thicknesses [μm]
Wilson <i>et al.</i> [4]	Polyvinyl acetate (PVA), xanthan gum and petroleum jelly	Coherent perpendicular stationary jets	70-300
Wilson <i>et al.</i> [38]	Xanthan gum	Coherent, perpendicular, stationary and moving jets	Not reported
Wang <i>et al.</i> [39]	Paint	Coherent, perpendicular and angled jets	60
Bhagat <i>et al.</i> [2]	Petroleum jelly, PVA, xanthan gum	Coherent, stationary and moving, perpendicular and angled jets	322-409
Feldung Damkjær <i>et al.</i> [40]	Petroleum jelly	Coherent and non-coherent perpendicular jets	250-1,490
Glover <i>et al.</i> [1]	PVA and petroleum jelly.	Coherent, perpendicular, stationary and moving jets.	100-1,900
Chee <i>et al.</i> [41]	Petroleum jelly and Carbopol®	Coherent and non-coherent, perpendicular stationary jets – flat and curved surfaces.	0.2 and 1,300
Oevermann <i>et al.</i> [3]	Dried suspensions	Coherent, perpendicular and stationary jets.	175
Yang <i>et al.</i> [34]	Toothpaste	Coherent, perpendicular, stationary and moving jets.	500
Fernandes <i>et al.</i> [17]	Petroleum jelly	Coherent, perpendicular, stationary and moving jets.	194 – 1,050
Tuck <i>et al.</i> [33]	Thick layers of Carbopol®	Coherent, stationary, perpendicular and angled jets.	2,000 – 8,000
Lu <i>et al.</i> [42]	Numerical study: removal of Newtonian fluid layer	Submerged perpendicular laminar jet	100-1000

An argument based on viscous dissipation, which has been used to describe flows involving the translation of a contact line, such as the spread of droplets [36] and film draining [37], is presented here which yields similar forms to Equation (1) and provides predictive insight into the parameters k' and M_y . Simplifying assumptions are made in order to obtain a tractable result. In particular, the coupling of the liquid flow and soil deformation is not included, so that the shape of the cleaning front is not predicted *a priori*. The result provides insights which we

anticipate will promote detailed modelling investigations capable of predicting the shape and rate for a given soil rheological behaviour.

2. Models

2.1. Hydrodynamic description

Bhagat and Wilson [8] presented a model describing the flow in the thin film generated by a coherent turbulent liquid jet impinging on a flat surface. This model is used in the current work to compute the local values of the film thickness (h), momentum (M) and average velocity (U). The relationships are summarized in the [Supplementary Information](#). The flow in the RFZ is divided into three regions:

- (i) A boundary layer formation zone, that extends from $r_o = d_N/2$ to r_b

$$\frac{r_b}{d_N} = 0.24 Re_j^{1/3}. \quad (4)$$

- (ii) A laminar film zone, $r_b < r \leq r_t$, where

$$\frac{r_t}{d_N} = 0.2964 Re_j^{1/3}. \quad (5)$$

- (i) A turbulent region, $r_t < r < r_j$, where r_j is the film jump that can be found from the solution of

$$M = \frac{\rho \frac{64}{632\pi} Q U_o}{r_j \left[\frac{0.167}{Re_j^{0.25} \left(\frac{r}{d} \right)^{9/4} + (2.37 - 0.0108 Re_j^{1/2})} \right]} = \gamma_s. \quad (6)$$

Here, Re_j is the jet Reynolds number ($Re_j = \rho U_o d_N / \mu$), with U_o the average jet velocity and ρ , μ and γ_s the density, viscosity and surface tension of the liquid, respectively.

Figure 5 shows (a) M , (b) h and (c) U as a function of radial position for three different conditions employed in the tests reported here. The thickness of the liquid film varies, reaching a minimum at r_t . This is accompanied by a step in the M profile, owing to the assumption of a well developed velocity profile in the turbulent region. This discontinuity does not play a major role in the cleaning calculations. In this work, we focus on cases where $h/\delta_o < 1/2$.

Figure 5 – Predicted (a) M , (b) film thickness and (c) average velocity calculated using the Bhagat and Wilson [8] model (see supplementary information) for three jet conditions employed in this work.

2.2. Cleaning models

Figure 6 presents the geometry of the model. An impinging cylindrical liquid jet generates a liquid film of thickness h and momentum flow rate M (at radial distance a from the point of impingement). Point F is the location of the cleaning front, where the thickness of the soil layer is some small value δ_i : for hydrophobic substances such as the petroleum jelly studied by Fernandes *et al.* [17], their measurements showed that a thin layer of thickness of order microns

remained on the substrate. The physics of a moving contact line are complex [43] and the model focuses on the bulk of the layer.

Figure 6 – Schematic of idealised cleaning front

Impingement of the liquid film on the soil layer creates a wedge inclined at angle ϕ to the substrate which extends to point B (with horizontal coordinate x_L), corresponding to the height of the liquid film: beyond B, the slope is expected to steepen. A momentum balance in the horizontal direction over FABC gives the net force per unit width F_w acting on the soil layer, generated by the pressure acting on FB:

$$F_w = M(1 - \cos\phi). \quad (7)$$

This assumes that the magnitude of M does not change significantly between OA and AB. The work done in moving the soil will reduce the kinetic energy of the film and thereby M . This is assumed to be small and is checked in subsequent calculations. We consider thin layers, where δ/a and $|FC|/a$ are small so contributions from the change in circumferential length are small and the system can be described in Cartesian co-ordinates x and y .

Assume that the front FB advances at velocity V with approximately constant shape. The flow path by which material beyond BC is advected upwards and removed or builds up a berm of dislodged material is not currently understood. The material in region FBC is moved by force F_w , whereas that beyond BC is dragged by the shear stress $\tau_{w,l}$ acting on the top surface. The

223 rate at which work is done per unit width is equated with Q_D , the power consumed by viscous
 224 dissipation within region OBC, viz.

$$F_w \cdot V = \int_0^{x_L} \int_0^{\delta(x)} \tau \cdot \dot{\gamma} dy dx = Q_D. \quad (8)$$

225 Equation (8) is evaluated for different rheological relationships, i.e. $\tau(\dot{\gamma})$. The models for
 226 Newtonian and Bingham soil layers are described in Sections 2.2.1 and 2.2.3, respectively. [The](#)
 227 [results for a power law fluid and a Herschel-Bulkley material are reported and their derivation](#)
 228 [given in the Supporting Information.](#)

229

230 **2.2.1. Newtonian soil**

231 This is the simplest case, with constant viscosity, μ , and is used to illustrate the concepts.
 232 Consider the vertical plane at X, height δ , located at distance x downstream from O. The mean
 233 velocity across the plane is V . The local stress distribution is not known and is modelled as a
 234 pressure driven flow with the pressure gradient dP/dx being uniform over the plane, and
 235 varying with x . The shear stress profile is assumed to be linear, viz.

$$\tau = \tau_w \left(1 - \frac{y}{\delta}\right). \quad (9)$$

236 The constitutive relation for Newtonian fluids is:

$$\tau = \mu \frac{du}{dy} \quad (10)$$

237 where u is the local horizontal velocity. With no slip at the substrate (which may not be correct
 238 for complex soils and/or surfaces), $u = 0$ when $y = 0$. From Eq. (9) and (10):

$$u = \frac{\tau_w}{\mu} \left(y - \frac{y^2}{2\delta}\right). \quad (11)$$

239 Continuity requires

$$V\delta = \int_0^\delta u dy \quad (12)$$

240 which combined with Eq. (11) gives the velocity profile for a viscous flow,

$$u = \frac{3V}{\delta^2} \left(\delta y - \frac{1}{2}y^2\right) \quad (13)$$

241 and the shear rate distribution,

$$\dot{\gamma} = \frac{du}{dy} = \frac{3V}{\delta^2}(\delta - y). \quad (14)$$

242 The rate of viscous dissipation is obtained by substituting Eq. (9) and (14) into Eq. (8),

$$F_w \cdot V \approx 9\mu V^2 \int_0^{x_L} \left\{ \int_0^{\delta(x)} \frac{1}{\delta^4} (\delta - y)^2 dy \right\} dx \quad (15)$$

$$F_w \approx 3\mu V \int_0^{x_L} \frac{dx}{\delta}. \quad (16)$$

243 For a simple wedge, $\delta = x \tan \phi$, and thus

$$M(1 - \cos \phi) = 3\mu V \cot \phi \ln(h/\delta_i). \quad (17)$$

244 The cleaning velocity is then given by

$$V = M \frac{(\tan \phi - \sin \phi)}{3\mu} \frac{1}{\ln(h/\delta_i)} \quad h \leq \delta_o. \quad (18)$$

245 This expression is of the form of Equation (1), with a linear dependency on M . The film
246 thickness h varies gradually with r (see Figure 5-b). The rate of cleaning decreases with the
247 viscosity of the soil, which is expected. The dependency of the rate on the soil layer thickness
248 is not straightforward, with two scenarios anticipated:

- 249 (i) $h > \delta_o$, the ‘very thin layer’: in Eq. (18), $\ln(h/\delta_i)$ would be replaced by $\ln(\delta_o/\delta_i)$
250 (assuming that little work is done in translating the removed material). M is defined
251 as the total momentum flow rate in the liquid film and in this scenario a fraction of
252 this quantity will impinge directly on the soil layer (See Figure 2 (a)).
- 253 (ii) $h \leq \delta_o$, the ‘thin layer’. The work done in dislodging the material beyond BC needs
254 to be estimated. In the absence of knowledge of the flow pattern in the soil, this is
255 approximated as either (a) the wedge with angle ϕ extending to D (Figure 6), so
256 that $\ln(h/\delta_i)$ is again replaced by $\ln(\delta_o/\delta_i)$, or including a change in the wedge
257 angle to $\phi_2 \sim \frac{\pi}{4}$ as indicated in Figure 4 (b) when $h/\delta_o > 1/2$, so that
258 $\cot \phi \ln(h/\delta_i)$ in (17) is replaced by $\cot \phi \ln(h/\delta_i) + \ln(\delta_o/h)$, giving

$$V = M \frac{1}{3\mu} \frac{(\tan \phi - \sin \phi)}{\ln(h/\delta_i) + \tan \phi \ln(\delta_o/h)}. \quad (19)$$

259 Since $h/\delta_i > \delta_o/h$ and $\tan \phi < 1$, the first term in the denominator is expected to dominate.

260

261

2.2.2. Power law soil

For a power law fluid with constitutive equation:

$$\tau = k_{\text{PL}} \dot{\gamma}^n \quad (20)$$

where k_{PL} is the constitutive index and n is the power-law exponent, the corresponding results are:

$$(i) \quad h > \delta_o,$$

$$V = \left\{ M(\tan \phi - \sin \phi) \frac{m-1}{m} \frac{1}{k_{\text{PL}}(m+2)^{1/m} \left[\delta_o^{\left(\frac{m-1}{m}\right)} - \delta_i^{\left(\frac{m-1}{m}\right)} \right]} \right\}^m \quad (21)$$

$$(ii) \quad h \leq \delta_o, V$$

$$V = \left\{ M(\tan \phi - \sin \phi) \frac{m-1}{m} \frac{1}{k_{\text{PL}}(m+2)^{1/m} \left[h^{\left(\frac{m-1}{m}\right)} - \delta_i^{\left(\frac{m-1}{m}\right)} \right]} \right\}^m \quad (22)$$

where $m = n^{-1}$. Eq. (22) approaches the Newtonian case, Eq. (19), when $n_{\text{PL}} \rightarrow 1$ and $k = \mu$.

2.2.3. Bingham soil

The Bingham model, Eq. (23), is the simplest description of yield stress fluid behaviour. Here, μ_{B} is the Bingham viscosity.

$$\tau = \tau_c + \mu_{\text{B}} \dot{\gamma} \quad (23)$$

The local wall shear stress within the soil layer rim, τ_w , is related to the local pressure gradient in the soil by $\tau_w = \delta \, dP/dx$ and the shear stress in the layer is again assumed to be given by Eq. (9). When $\tau < \tau_c$, corresponding to $y > y_c$ (with $\frac{y_c}{\delta} = 1 - \frac{\tau_c}{\tau_w}$), the fluid moves as a plug: shearing occurs in the lower part of the plane, and the shearing zone reaches the substrate when $\tau_c = \tau_w$. This yields the velocity profile

$$u = \left(\frac{\tau_w - \tau_c}{\mu_{\text{B}}} \right) y - \frac{\tau_w}{2\mu_{\text{B}}} \frac{y^2}{\delta}. \quad (24)$$

Conservation of volumetric flow rate requires

$$V\delta = \int_0^\delta u dy = \int_0^{y_c} u dy + \int_{y_c}^\delta u_c dy \quad (25)$$

279 where

$$u_c = \frac{\tau_w \delta}{2\mu_B} \left(1 - \frac{\tau_c}{\tau_w}\right)^2. \quad (26)$$

280 Writing $\tau_w^* = \tau_w/\tau_c$, integration yields, with some rearrangement,

$$V = \frac{\tau_c \delta}{6\mu_B} \left(1 - \frac{1}{\tau_w^*}\right)^2 (1 + 2\tau_w^*). \quad (27)$$

281 The RHS increases monotonically with τ_w^* . Since V is constant in the front and δ increases
 282 with x , this indicates that the local pressure gradient (and hence τ_w^*) decreases with distance
 283 from F and at some location flow horizontally will stop.

284 This result can also be written in the form of a dimensionless characteristic velocity, V^* :

$$V^* = \frac{6\mu_B V}{\tau_c \delta} = \left(1 - \frac{1}{\tau_w^*}\right)^2 (1 + 2\tau_w^*) \quad (28)$$

285 which approaches the linear result $V^* = 2\tau_w^* - 3$ at large τ_w^* (see Figure 7).

286 The rate of viscous dissipation Q_D can now be evaluated. For the element between x and $x +$
 287 dx ,

$$\frac{dQ_D}{dx} = \int_0^\delta \tau \cdot \dot{\gamma} dy = \int_0^{y_c} \tau \cdot \dot{\gamma} dy = \frac{\tau_w^2}{\mu_B} \int_0^{y_c} \left(1 - \frac{y}{\delta}\right) \left(\frac{y_c}{\delta} - \frac{y}{\delta}\right) dy \quad (29)$$

288 giving

$$\frac{dQ_D}{dx} = \frac{\delta \tau_w^2}{6\mu_B} \frac{1}{\tau_w^*} \left(1 - \frac{1}{\tau_w^*}\right)^2 (1 + 2\tau_w^*). \quad (30)$$

289 Substituting the terms in brackets from Eq. (28) yields

$$\frac{dQ_D}{dx} = \frac{\delta \tau_w^2}{6\mu_B} \frac{1}{\tau_w^*} \frac{6\mu_B V}{\tau_c \delta} = \tau_w V. \quad (31)$$

290 The total rate of viscous dissipation is estimated by integrating Eq. (31) over the wedge

$$Q_D = \int_0^{x_T} \tau_w V dx = \tau_c V \int_0^{x_T} \tau_w^* dx \quad (32)$$

291 where x_T is set by the location where $\tau_w^* = 1$ and there is no further horizontal motion. The
 292 viscous dissipation is thus limited to the region where $\tau_w^* \geq 1$, and the dissipation that occurs
 293 beyond BC is not included. Equating this with the rate of work done by the liquid film gives

$$M(1 - \cos\phi)V = \tau_c V \int_0^{x_T} \tau_w^* dx \quad (33)$$

294 or

$$\frac{M(1 - \cos\phi)}{\tau_c} = \int_0^{x_T} \tau_w^* dx. \quad (34)$$

295 Eq. (28) states that τ_w^* is related to a scaled velocity so Eq. (34) contains the relationship
 296 between the momentum in the film and the cleaning rate. The form of this relationship can be
 297 extracted by setting $x_T = x_L$ and approximating the locus by the asymptotic form
 298 $\tau_w^* = 1.5 + \frac{1}{2}V^*$, viz.

$$\frac{M(1 - \cos\phi)}{\tau_c} = \int_0^{x_L} \left(\frac{3}{2} + \frac{1}{2}V^* \right) dx \quad (35)$$

$$\frac{M(1 - \cos\phi)}{\tau_c} = \frac{3}{2}x_L + \frac{3\mu_B V}{\tau_c} \int_0^{x_L} \frac{1}{\delta} dx. \quad (36)$$

299 Setting $\delta = \delta_i + x \tan\phi$ and $x_L = h \cot\phi$, gives

$$\frac{M(1 - \cos\phi)}{\tau_c} = \frac{3}{2}h \cot\phi + \frac{3\mu_B V}{\tau_c \tan\phi} \ln \left(1 + \frac{h}{\delta_i} \right). \quad (37)$$

300 Rearranging gives an expression similar to Eq. (1),

$$V = \frac{(\tan\phi - \sin\phi)}{3\mu_B \ln \left(1 + \frac{h}{\delta_i} \right)} \left\{ M - \frac{3}{2} \frac{\tau_c h}{(\tan\phi - \sin\phi)} \right\} \quad (38)$$

301 yielding an expression for the cleaning rate constant

$$k'_B = \frac{(\tan\phi - \sin\phi)}{3\mu_B \ln \left(1 + \frac{h}{\delta_i} \right)} \quad (39)$$

302 and the limiting momentum flow rate

$$M_{y,B} = \frac{3}{2} \frac{\tau_c h}{(\tan\phi - \sin\phi)}. \quad (40)$$

303 The latter is identical in form to Glover *et al.*'s result, Equation (3), with $3h/2$ replacing δ_0 .
 304 The values of χ reported by Glover *et al.* correspond to ϕ values in the range 9-30° evaluated
 305 using Equation (40). These expressions encapsulate the expected physics, namely M_y

increasing with the soil critical stress and k' inversely proportional to μ_B . The dependency of k' on h is modest as viscous dissipation is concentrated at the front of the ramp: h/δ_i is expected to be $\gg 1$, so $\ln(1 + h/\delta_i) \approx \ln(h/\delta_i)$ and h varies slowly with r [8]. Furthermore, Eq. (38) collapses to the result for a Newtonian fluid, Eq. (18), when $\tau_c = 0$.

Figure 7 shows that other expressions relating V^* to τ_w^* can be identified and capture the behaviour at lower values of V^* . For instance, a simple power law fitting $\tau_w^* = 1 + aV^{*b}$ gives good agreement ($R^2 = 0.993$) for $V^* < 200$ where $a = 0.60$ and $b = 0.96$. Substituting this expression into Eq. (34) gives

$$\frac{M(1-\cos\phi)}{\tau_c} = \int_0^{x_L} (aV^{*b} + 1) dx. \quad (41)$$

Substituting $\delta = \delta_i + x \tan \phi$ and $x_L = h \cot \phi$, yields after some rearrangement

$$V = \left[\frac{\tau_c^{b-1}(1-b)(\tan \phi - \sin \phi)}{a(6\mu_B)^b [(\delta_i + h)^{1-b} - (\delta_i)^{1-b}]} \right]^{1/b} \left(M - \frac{h\tau_c}{\tan \phi - \sin \phi} \right)^{1/b} \quad (42)$$

which follows the form:

$$V = k'_{BP} (M - M_{y,BP})^{1/b} \quad (43)$$

with

$$k'_{BP} = \left[\frac{\tau_c^{b-1}(1-b)(\tan \phi - \sin \phi)}{a(6\mu_B)^b [(\delta_i + h)^{1-b} - (\delta_i)^{1-b}]} \right]^{1/b} \quad (44)$$

and

$$M_{y,BP} = \frac{h\tau_c}{\tan \phi - \sin \phi}. \quad (45)$$

Both versions of the model, Eq. (38) and (42), require the Bingham parameters, τ_c and μ_B , which can be obtained from the flow curve of the material. The performance of the Newtonian and linear Bingham model are compared to the Glover *et al.* [1] model in the Supplementary Information.

Figure 7 – Relationship between τ_w^* and V^* given by Eq. (28) alongside two simple fitted expressions.

2.2.4. Herschel-Bulkley soil

The Herschel-Bulkley model is written

$$\tau = \tau_c + k_{HB} \dot{\gamma}^n \quad (46)$$

where k_{HB} is the consistency index and n describes the non-linear behaviour for $\tau(\dot{\gamma}) > \tau_c$.

The velocity of the cleaning front for a Herschel-Bulkley soil is given by:

$$V = \left\{ \frac{\tau_c^{mb-1}(1-b)(\tan \phi - \sin \phi)}{a[(m+1)(m+2)k_{HB}^m]^b [((\delta_i + h)^{1-b} - \delta_i^{1-b})] \left[M - \frac{h\tau_y}{\tan \phi - \sin \phi} \right]} \right\}^{1/b} \quad (47)$$

where $m = n^{-1}$ and parameters a and b are obtained from fitting functions of the form $\tau_w^* = aV^{*b} + 1$ to the V^* and τ_w^* curves obtained for the value of n . This result approaches the Bingham model when $n \rightarrow 1$.

2.2.5. Energetic considerations

The energetic efficiency of the cleaning process can be calculated. The rate of viscous dissipation per unit width is given by

$$Q_D = F_w V = M(1 - \cos \phi)V. \quad (48)$$

The change in the kinetic energy flow in the film as a result of moving the cleaning front can be estimated. The rate of flow of kinetic energy in the liquid film per unit width, $\dot{E}_k$, is given by

$$\dot{E}_k = \frac{\dot{m}}{2\pi a} \frac{U^2}{2}. \quad (49)$$

The ratio of these two quantities yields the energetic efficiency of the cleaning process, η_e , viz.

$$\eta_e = \frac{4\pi a M(1 - \cos \phi)V}{\dot{m}U^2}. \quad (50)$$

The energetic efficiency observed in the experiments is found by calculating M for a given value of a using the film flow model, and setting V equal to the observed value of $\left. \frac{da}{dt} \right|_{r=a}$ or estimating V using Eq. (42).

The models presented in this section do not include slip or deformation beyond $\overline{BC}$, so some differences from the experimental data are expected. Additionally, all the models discussed here assume generalised Newtonian fluid formulations for the soils. Both the Newtonian and Bingham models yield almost-linear relationships between $V = da/dt$ and M , Eq. (19) which match experimental observations for a Newtonian soil layer. The assumption of a power-law relationship between τ_w^* and V^* for a Bingham soil generates slightly non-linear V vs M curves, Eq. (42).

Neither of these models capture creep effects that occur at shear stresses below the critical stress [44], which was cited for the strong curvature at lower M observed by Fernandes *et al.* [17]. Creep is expected to occur in cleaning experiments when the soil layer is exposed to the shear stresses and forces imposed by the liquid film for long periods. The models are therefore expected to deviate from experimental data at longer observation times.

3. Materials and methods

3.1. Impinging jet apparatus

The experimental rig used was that described by Glover *et al.* [1] and was also used in studies [17,34,41]. The targets were 150×150×5 mm vertical Perspex[®] transparent plates. Cleaning

was monitored by a camera located on the dry side of the target. De-ionized water was pumped from a large reservoir, through a rotameter, before entering a horizontal 150 mm long pipe of 9.5 mm i.d., ending in a 55° convergent entry brass nozzle with diameter $d_N = 2, 3$ or 4 mm. The flow rate was controlled by a ball valve and monitored using the rotameter. An interrupter plate, located between the nozzle and the target, was held in place for at least 30 s after the flow was initiated to ensure that a stable jet had formed. The flow rates used correspond to Reynolds numbers in the range 10,000-37,000 and Weber numbers ($We = \rho U_0^2 d_N / \gamma_s$) in the range 760-5,600. All the experiments were conducted at room temperature, approximately 20 °C, and were filmed at 1920×1080 pixels resolution at either (i) 60 frames per second (fps) using a Nikon D3300 D-SLR digital camera, or (ii) 50 fps using a Sony HX80 camera.

3.2. Materials, image analysis and crater profilometry

Three different materials, all insoluble in water were used. The first, a petroleum jelly (Atom Scientific GPS5220, provided by APC Pure, UK), was the material used by Fernandes *et al.* [17]. Most of the experiments were performed with this petroleum jelly, labelled PJA. The other two materials, a white soft paraffin (GlaxoSmithKline, UK), labelled WSP, and a pharmaceutical white petroleum jelly (Merkur 500, Sasol, South Africa), labelled PJS, were used to confirm the validity of the models. Layers of uniform thickness δ_0 were coated on the target plates using the spreader device described by Cuckston *et al.* [45]. The thickness was then calculated from the mass applied, the area coated and the density ρ_s of the materials (Table 2). The coating procedure lasted from 5 to 15 s, so a characteristic coating time of 10 s was assumed for this stage. The cleaned radius a was extracted from image analysis of the experiments using a MatlabTM script. The shapes of the craters were investigated using a LED confocal profilometer (ConfocalDT IFS 2405-3, Micro-Epsilon, Germany) mounted on a computer-controlled x-y stage. Details of the image analysis and profilometry are given in [17].

3.3. Rheology

The rheological behaviour of the materials was investigated using a stress-controlled rheometer (Kinexus Lab +, Malvern Instruments, UK), using rough 40 mm diameter parallel plates. The temperature was kept at 20 °C using a thermostatic bath, to match the temperature at which the materials were stored and the cleaning experiments performed. After loading the samples in

the rheometer, pre-shearing at 1 s^{-1} for 10 s was employed to impose a known strain history to the sample.

Table 2 – Soil material properties

Material	ρ_s [kg·m ⁻³]	τ_c [Pa]	μ_B [Pa·s]
PJA (Petroleum jelly, Atom Scientific)	812±13	140	1.0
WSP (White soft paraffin, GSK)	838±88	200	0.1
PJS (Petroleum jelly, Sasol)	826±14	570	0.3

4. Results and discussion

4.1. Rheological characterization

Many yield stress materials display time-dependent behaviour [46], meaning that their viscosity [47] and critical stress [48] can change over time when the material is left at rest. Since we are interested in performing cleaning experiments using soil layers at a known state, the protocol established by Fernandes *et al.* [17] was employed here. The structural recovery of the materials was quantified by an oscillatory sweep at 1 Hz over 1800 s, with constant shear stress amplitudes within the linear viscoelastic regime [47]. Figure 8 indicates that the storage modulus, G' , and the loss modulus, G'' , were constant for the three materials after 1800 s. Therefore, an ageing (waiting) time of at least 30 min was employed in these cleaning tests. The storage moduli were all considerably larger than the loss moduli, indicating predominantly elastic behaviour.

Increasing stress ramps starting from rest were performed [49]. The shear stress was increased at a constant rate of $\dot{\tau} = 10 \text{ Pa} \cdot \text{min}^{-1}$, and Figure 9 (a) shows the shear stress as a function of the shear rate for these experiments. Similar behaviour was observed for the three soil materials: the shear rate gradually increases when the material is below the critical stress, and increases rapidly as the critical stress is approached. Measurements of the normal stress differences showed noticeable changes around the critical stress (see Supplementary Information).

Figure 8 – Evolution of dynamic moduli for a constant shear stress amplitude oscillatory sweep at frequency 1 Hz for the materials used in this work (See Table 2). Filled symbols indicate G' and hollow symbols denote G'' .

The behaviour at shear stresses below 200 Pa is predominantly elastic, as shown in the stress-strain curves in Figure 9 (b). Close to the critical stress, the shear rate gradually increases, indicating that creep is taking place [50,51]. This creeping effect is not captured by the Bingham model, which assumes a perfectly rigid state below the critical stress. Above the critical stress, the shear rates sharply increases, indicating that the materials are quickly deforming under the levels of shear stress imposed. The PJS and WSP display predominantly plastic behaviour, indicated by the significant increase in strain without significant increase in the shear stress, Figure 9 (b). This results in the low values of the Bingham viscosity μ_B in Table 2. The value of τ_c for the PJA is lower than that reported by Fernandes *et al.* [17] as the material aged under storage.

Figure 9 – Shear stress as a function of (a) shear rate and (b) shear strain for shear stress ramps ($\dot{\tau} = 10 \text{ Pa} \cdot \text{min}^{-1}$). Lines in (a) indicate the Bingham fits that were used in the viscous dissipation model. The unit gradient within the elastic regime in (b) indicates good agreement with the Hookean behaviour at modest strains

4.2. Cleaning experiments – performance of the models

In this section, the performance of the models is discussed in detail for a selection of experimental conditions. The Bingham linear model, Eq. (38), was also fitted to a larger number of experiments, and these are discussed in Section 4.3. Figures 10-12 present the results

for some of the cleaning experiments, and the values of ϕ and δ_i used in each fitting are shown on the plot labels. Figures 10 (a) and 11 (a) show the evolution of the **non-dimensional** cleaned radius, a/δ_o , over time for experiments conducted with the PJA, along with the fit for the Bingham fluid model using the linear (Eq. (38)) and power law (Eq. (42)) approximations. The right-hand axis shows the values of h/δ_o . The **calculations** employed the hydrodynamic model of Bhagat and Wilson [8] to calculate h and M .

The early stages of the cleaning process are described well by **both models**, which is also reflected in the good gradient estimations in the da/dt vs M plots (Figure 10 (b) and 11 (b)). **Since h changes with a (see Figure 5), the gradient of the da/dt vs M curves changes gradually with a . The asymptotic behaviour**, where the cleaned radius approaches a maximum value, $a_{\max}$, **is also** captured by the models. At intermediate times, from approximately 1 to 30 s, the models overestimate the cleaned radius. Similar behaviour was reported by Glover *et al.* [1], and attributed to **creep effects that are not captured** by the generalised Newtonian fluid formulation adopted to describe the soil materials.

The measured shape of the berm can be compared with the values of ϕ obtained with the Bingham linear model. Figures 10 (c) and 11 (c) present the shapes of the rim for the cleaning experiments obtained with the confocal profilometer: the horizontal axis shows the **non-dimensional** distance from the **cleaning front** at $a_{\max}$, l/δ_o as indicated in Figure 10 (c-ii). The measurements were taken along four lines equally spaced at 90° from each other. The cardinal notation indicates the direction of measurement, where South points downwards and North upwards. The lines shown in the plots were constructed using Eq. (51), where ϕ is the angle obtained with the Bingham viscous dissipation model, Eq (51).

$$\begin{aligned} \delta &= l \tan \phi & \text{if } 0 < l \leq h \cot \phi \\ \delta &= l + h(1 - \cot \phi) & \text{if } h \cot \phi < l \leq \delta_o + h(\cot \phi - 1) \\ \delta &= \delta_o & \text{if } l > \delta_o + h(\cot \phi - 1) \end{aligned} \quad (51)$$

The Bingham linear model gives a good description of initial berm shape for the experiments performed with the 2 mm nozzle in Figure 10. The 45° angle for $h < \delta < \delta_o$ arises from the assumption of simple shear in the region where the liquid film is not causing any compression in the rim. A theory to explain the angle of the rim for $\delta \leq h$ is not yet available and is the topic of ongoing work.

Figure 10 – Results for cleaning experiments obtained with PJA, $d_N=2$ mm. (a) a/δ_0 (left y-axis) and h/δ_0 (right y-axis) as a function of time; (b) da/dt as a function of M and (c) profilometry of the crater, along with the shape of the rim given by the Bingham linear model. Column (i) $Q=2$ L·min $^{-1}$, $\delta_0=0.89$ mm. Column (ii) $Q=1$ L·min $^{-1}$, $\delta_0=0.37$ mm.

Figure 11 – Results for cleaning experiments obtained with PJA, $d_N=4$ mm, $Q=4$ L·min⁻¹. (a) a/δ_0 (left y-axis) and h/δ_0 (right y-axis) as a function of time; (b) da/dt as a function of M and (c) profilometry of the crater, along with the shape of the rim given by the Bingham linear model. (i) $\delta_0=0.51$ mm, (ii) $\delta_0=0.35$ mm.

For the experiments performed with PJA and a 4 mm nozzle in Figure 11, the description of the rim shape is not accurate. In these cases, $h/\delta_0 > 1/2$ and so belong to the very thin soil layer case. As a result, the model is not expected to give a reliable description of the shape of

the cleaning front. Figure 12 presents results for the evolution of the cleaned radius and berm shape for experiments performed with WSP and PJS. As with the PJA, the main features of the cleaning process are adequately described by the models. The Bingham model with the linear approximation for τ_w^* , Eq. (38), provides a good description of the experimental data.

Since the model was fitted to the experimental data by tuning ϕ and $\ln(\delta_i)$, it is important to establish the sensitivity of model to these two parameters. Figure 13 presents results for a sensitivity analysis performed with the Bingham linear model for another experiment performed with the PJA. The best fit was found with $\phi = 20^\circ$ and $\delta_i = 0.5 \mu\text{m}$. Figure 13 (a) shows the effect of changing ϕ by $\pm 20\%$, and Figure 13 (b) of changing $\ln(\delta_i)$ by $\pm 20\%$. The model is more sensitive to ϕ than to δ_i , as expected from the form of Eq. (38).

500 Figure 12 – **Cleaning of WSP and PJS layers.** (a) a/δ_0 (left y-axis) and h/δ_0 (right y-axis) as
 501 a function of time; (b) da/dt as a function of M and (c) measured shape of the berm, along
 502 with the **profile** given by the Bingham linear model. Conditions: $d_N=2$ mm, $Q=2$ L·min⁻¹ and
 503 (i): $\delta_0=0.86$ mm; (ii): $\delta_0=0.89$ mm.

Figure 13 – Sensitivity of the linear Bingham model, Eq. (38) to $\pm 20\%$ variation in (a) ϕ and (b) $\ln(\delta_i)$. (i) a/δ_o vs t plots and (ii) da/dt vs M plots. Experimental conditions: $d_N=2$ mm, $Q=2$ L·min $^{-1}$ and $\delta_o=0.38$ mm.

The results presented in this section demonstrate that the momentum-driven viscous dissipation model can describe the cleaning of thin viscoplastic soil layers by these impinging water jets. The next section will thus explore the main features of this model when fitted to larger datasets.

4.3. Parametric analysis

Since h changes over the course of a cleaning test, Eq. (39) and Eq. (40) indicate that both k'_B and $M_{y,B}$ also change. We introduce the space averaged values of h , k'_B and $M_{y,B}$, defined:

$$\bar{h} = \frac{1}{a_{\max} - a_o} \int_{a_o}^{a_{\max}} h(r) dr \quad (52)$$

$$\bar{k}'_B = \frac{1}{a_{\max} - a_o} \int_{a_o}^{a_{\max}} k'_B(r) dr \quad (53)$$

$$\bar{M}_{y,B} = \frac{1}{a_{\max} - a_o} \int_{a_o}^{a_{\max}} M_{y,B}(r) dr \quad (54)$$

where a_o is the first cleaned radius detected. Results are grouped relative to the experimental conditions employed: (i) $d_N=2$ mm and $Q=1$ L·min⁻¹; (ii) $d_N=2$ mm and $Q=2$ L·min⁻¹ and (iii) $d_N=4$ mm and $Q=4$ L·min⁻¹. The remaining results refer to a selection of experimental conditions, with ranges $15,000 \leq Re \leq 37,000$, $2 \leq d_N \leq 4$ mm, and $0.4 \text{ mm} \leq \delta_o \leq 1.1$ mm. A summary of the experimental conditions investigated with the PJA is presented in Table 3, which also reports the values of ϕ and δ_i obtained when fitting Eq. (38) to the data.

Figure 14 shows the values of ϕ as a function of $\bar{h}/\delta_o$ for the experiments on PJA layers. The values of ϕ range between 15° and 32°, which is consistent with the χ values reported by Glover *et al.* ϕ tends to decrease with increasing $\bar{h}/\delta_o$, approaching 15-20° as $\bar{h}/\delta_o$ increases. The experimental data in Figure 4(b) indicate that a single value of ϕ in the range 5-15° described the berm shape at higher $\bar{h}/\delta_o$.

Figure 15 compares the parameters obtained by fitting Eq. (38) to the data obtained with PJA for the dependencies given by the model. Wilson *et al.* [4] hypothesised that the cleaning rate constant k' was a function of the soil layer thickness, while Glover *et al.* [1] reported a decrease of k' with increasing δ_o . Figure 15 (a) presents the values of $\bar{k}'_B$, given by Eq. (39), as a function of $\bar{h}/\delta_o$, showing a decreasing trend, somewhat mirroring the predicted dependency on $\ln(1 + h/\delta_i)$. The $\bar{M}_{y,B}$ values increase with $\tau_c \bar{h}$, as suggested by Eq. (40), Figure 15 (b). Eq. (39) and Eq. (40) state that k'_B and $M_{y,B}^{-1}$ are both related to $(\tan \phi - \sin \phi)$, but show different dependencies on h and δ_i . It follows from Eq. (39) and Eq. (40) that $\bar{k}'_B \propto \bar{h}/\ln(1 + \bar{h}/\delta_i)$, as demonstrated in Figure 15 (c). The trends show reasonable agreement with the model.

Table 3 – Experiments reported with the PJA

Q [L/min]	d_N [mm]	δ_o [mm]	Re	ϕ [°]	δ_i [μm]
1.0	2	0.10	10,600	19.5	1
		0.14		18.3	0.5
		0.34		18.4	1
		0.37		25.6	0.5
		0.37		26.2	0.1
		0.45		23.0	0.5
		0.90		20.8	0.01
		1.45		21.5	0.001
		2.02		25.0	0.005
2.0	2	0.11	21,200	21.5	5
		0.30		20.9	5
		0.39		24.0	5
		0.62		23.5	1
		0.86		25.7	0.3
		1.85		21.0	1
4.0	4	0.35	21,200	16.4	10
		0.36		19.6	1
		1.05		20.5	10
		1.49		20.5	8
		1.97		19.0	1
1.40	2	0.38	14,800	26.0	1
1.40	2	0.91	14,800	26.2	0.1
3.00	4	0.50	16,000	24.5	2
1.80	2	0.90	19,000	18.5	5
3.00	3	1.05	21,200	18.5	5
5.00	4	1.05	26,500	19.0	5
4.00	3	1.05	28,200	20.3	5
5.00	3	1.05	35,300	18.0	10
7.00	4	0.50	37,000	15.0	12

Figure 14 – Effect of $\bar{h}/\delta_0$ on fitted wedge angle ϕ .

Figure 16 compares the values of (a) $\bar{k}'_B$, (b) $\bar{M}_{y,B}$ and (c) ϕ obtained with the Bingham linear model with the values of k' , M_y and χ obtained by fitting the semi-empirical Glover *et al.* [1] model to the same data sets. Their model assumed a simple laminar velocity profile for the liquid film.

For the majority of these cleaning tests $a_{\max} > r_t$, where both hydraulic models give similar estimates of M , and the values of $\bar{M}_{y,B}$ and M_y are therefore similar (see Figure 16 (b)). The wedge angles ϕ and χ differ, with $\phi < \chi$ (Figure 16 (c)), and the ϕ values closer to the measured values presented in Figure 4(b). One reason why the Glover *et al.* angle χ is larger is that the static force balance result in Eq. (3) employs the layer thickness δ_0 rather than the thickness of the liquid film: if the liquid film thickness was used χ would be smaller. This is one of the insights arising from this work.

The values of $\bar{k}'_B$ are mostly larger than the values of k' , which gives the latter a better description of the approach to $a_{\max}$. The Glover *et al.* model, however, gives a poorer description of cleaning at early times (see Supplementary Information), and does not offer any predictive capacity. As discussed above, at later times the cleaning rate is expected to be subject to creep effects which are not implemented in the viscous dissipation model presented here

Figure 15 –Summary of the parameters obtained for PJA cleaning experiments. (a) Effect of $\bar{h}/\delta_o$ on $\bar{k}'_B$ (b) $\bar{M}_{y,B}$ as a function of $\tau_c \cdot \bar{h}$ and (c) $\bar{k}'_B$ as a function of $\bar{M}_{y,B}$. Range bars indicate the maximum and minimum values for each quantity along the test, whereas symbols indicate the average values given by Eq. (52), (53) and (54).

Figure 16 – Comparison of the parameters obtained for the linear Bingham model and the Glover *et al.* [1] model fitted to PJA tests. (a) $\bar{k}'_B$ and k' ; (b) $\bar{M}_{y,B}$ and M_y ; (c) ϕ and χ . Range bars indicate the maximum and minimum values for each quantity along the cleaning experiment, whereas symbols indicate the spatially averaged values given by Eq. (52), (53) and (54). The values of k' , χ and M_y were found by fitting the Glover *et al.* [1] model to the same data sets.

4.4. Energetic considerations

Figure 17 shows the variation of η_e with cleaned radius for the experimental conditions reported in Figure 10 (a). The data points denote where the measured value of da/dt is used to estimate V in Eq. (50), while the dashed lines indicate the result where the using the Bingham linear model is used to predict V . The values of η_e are small, of the order of 10^{-3} , indicating

that the change in the kinetic energy of the liquid in the film is small as it moves over the layer and the angled rim.

More of the kinetic energy is employed in cleaning at small values of a , i.e., when $V = da/dt$ is high. This indicates that the most efficient way to clean a viscoplastic soil with impinging water jets is by using traversing nozzles, when the liquid film is continuously exposed to fresh soil [2,3,17,38]. The discontinuity in the predicted efficiency loci in Figure 17 arises from the jump in M at r_t [8] (see Figure 5-a).

Figure 17 – Evolution of energetic efficiency of the cleaning experiments reported in Figure 10. Data points: V in Eq. (50) taken from experimental data. Red loci – V predicted using model. Dashed vertical lines indicate the location of r_t .

Critique

The cleaning model based on viscous dissipation at the cleaning front presented here is not able to predict the rate of cleaning *a priori*, as it requires two parameters, ϕ and δ_i , which have been obtained by fitting to the experimental data. However, unlike previous treatments such as those of Glover *et al.* [1] and Fernandes *et al.* [17], these tuning parameters are now expressed in terms of measurable quantities instead of empirical rate constants.

Further work remains to be able to predict the values of ϕ and δ_i . This would require identification of the shape of the cleaning front, which requires a rigorous solution of the coupled flow problem involving the liquid film and the soil layer.

Other topics requiring attention are the cleaning of very thin soil layers, when $h \gg \delta_o$, and thick soil layers, when $h \ll \delta_o$. The results presented by Fernandes *et al.* [17] and by Tuck *et al.* [33] indicate that the cleaning dynamics in these cases differ from the mechanisms discussed here.

5. Conclusions

A first order model describing the cleaning of a soil layers by impinging coherent turbulent water jets is proposed, in which the soil layers were described as generalised Newtonian fluids. The model assumes that the rate of cleaning is determined by viscous dissipation occurring in the soil at the liquid-soil contact line. Expressions for simple Newtonian, power law, Bingham and Herschel-Bulkley constitutive relationships are presented. This approach yields kinetic relationships of the form reported in earlier experimental investigations, providing (i) mechanistic insight into the phenomena controlling removal, and (ii) a basis for predicting cleaning behaviour.

Experimental data obtained from tests using three water-immiscible Bingham materials showed cleaning behaviour consistent with the form of the model developed for simple Bingham plastic soils. The model features two tuning parameters: the values obtained from fitting to the experimental data were consistent with physical observation.

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779

780 **NOMENCLATURE**

781 *Latin symbols*

Symbol	Unit	Description
a	[m]	Radius of the circular cleaned region
$a_{\max}$	[m]	Maximum cleaned radius
a_o	[m]	First detected cleaned radius
d_N	[m]	Diameter of the nozzle
$\dot{E}_k$	[W]	Rate of flow of kinetic energy in the liquid film per unit width
F_w	[N·m ⁻¹]	Net force per unit width acting on the soil layer
h	[m]	Thickness of the liquid film
$\bar{h}$	[m]	Space averaged liquid film thickness, Eq. (52)
k_{HB}	[Pa·s ^{<i>n</i>}]	Consistency index, Herschel-Bulkley fluid
k_{PL}	[Pa·s ^{<i>n</i>}]	Consistency index, power-law fluid
$\bar{k}'_B$	[m·s·kg ⁻¹]	Space averaged cleaning rate constant, Eq. (53)
k'	[m·s·kg ⁻¹]	Cleaning rate constant
k'_B	[m·s·kg ⁻¹]	Cleaning rate constant, Eq. (39)
k'_{BP}	[m·s·kg ⁻¹]	Cleaning rate constant, Eq. (44)
l	[m]	Distance from the leading edge of the berm
m	[-]	Inverse of the power-law and Herschel-Bulkley index: $m = n^{-1}$
M	[N·m ⁻¹]	Momentum flow rate per unit length
$\dot{m}$	[kg·s ⁻¹]	Mass flow rate
M_y	[N·m ⁻¹]	Momentum flow rate per unit length required to yield the soil layer
$M_{y,B}$	[N·m ⁻¹]	Momentum flow rate per unit length required to yield the soil layer, Eq. (40)
$\bar{M}_{y,B}$	[N·m ⁻¹]	Space averaged momentum flow rate per unit length required to yield the soil layer, Eq. (54)
$M_{y,BP}$	[N·m ⁻¹]	Momentum flow rate per unit length required to yield the soil layer, Eq. (45)
n	[-]	Power-law and Herschel-Bulkley index
P	[Pa]	Pressure
Q_D	[W·m ⁻¹]	Power per unit of width consumed by viscous dissipation
r	[m]	Radial co-ordinate
r_b	[m]	Radial position where the boundary layer reaches the free surface
Re_j	[-]	Reynolds number in the jet: $Re_j = \rho U_o d_N / \mu$
r_j	[m]	Radial location of the hydraulic jump
r_t	[m]	Radial position of the laminar-turbulent transition
U	[m·s ⁻¹]	Average velocity in the liquid film
U_o	[m·s ⁻¹]	Average jet velocity
u	[m·s ⁻¹]	Local horizontal velocity in the cleaning front
u_c	[m·s ⁻¹]	Plug velocity in the rim
V	[m·s ⁻¹]	Velocity of the cleaning front
V^*	[-]	Nondimensional velocity of the cleaning front: $V^* = \frac{6\mu_B V}{\tau_c \delta}$
We	[-]	Weber number, $We = \rho U_o^2 d_N / \gamma_s$
x	[-]	Cartesian coordinate parallel to the substrate
x_L	[m]	Horizontal position where $\delta = h$
x_T	[m]	Horizontal position where $\tau_w^* = 1$

y	[-]	Cartesian coordinate normal to the substrate
y_c	[m]	Position normal to the substrate where $\tau = \tau_c$

Greek symbols

Symbol	Unit	Description
δ_o	[m]	Thickness of the undisturbed soil layer
δ	[m]	Thickness of the soil layer
χ	[°]	Angle of inclination of the wedge to the substrate surface
δ_i	[m]	Thickness of the residual soil layer
γ	[-]	Shear strain
$\dot{\gamma}$	[s ⁻¹]	Shear rate
γ_s	[N·m ⁻¹]	Surface tension of the liquid
η_e	[-]	Energetic efficiency of the cleaning process
μ	[Pa·s]	Dynamic viscosity of the liquid
μ_B	[Pa·s]	Bingham viscosity
ϕ_1	[°]	Slope angle measured at the base of the rim
ϕ	[°]	Slope of the wedge to the substrate surface, viscous dissipation model
ϕ_2	[°]	Slope angle measured at the initial layer height
ρ	[kg·m ⁻³]	Density of the liquid
ρ_s	[kg·m ⁻³]	Density of the soil material
τ	[Pa]	Shear stress
τ_c	[Pa]	Critical shear stress
τ_w^*	[-]	Non-dimensional wall shear stress at the rim: $\tau_w^* = \tau_w/\tau_c$
τ_w	[Pa]	Local wall shear stress within the soil rim
$\tau_{w,l}$	[Pa]	Wall shear stress imposed by the liquid film

Acronyms

Symbol	Description
CIP	Cleaning-in-place
FMCG	Fast-moving consumer goods
FPS	Frames per second
PJA	Petroleum jelly provided by Atom Scientific
PJS	Petroleum jelly provided by Sasol
PVA	Polyvinyl acetate
RFZ	Radial flow zone
WSP	White soft paraffin

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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Supplementary Material

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