Influence of the phase difference between kinematic and inertial loads on seismic behaviour of pile foundations in layered soils

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Abstract

A series of dynamic centrifuge experiments was conducted on model pile foundations embedded in a two-layered soil profile consisted of soft-clay layer underlain by dense sand. These experiments were specifically designed to investigate the individual effect of kinematic and inertial loads on a single pile and a 3×1 row pile group during model earthquakes. It was observed that the ratio of free-field soil natural frequency to the natural frequency of structure might not govern the phase relationship between the kinematic and inertial loads for pile foundations as reported in some previous research. However, the phase relationship obtained in this study agrees well with the conventional phase variation between the force and displacement of a viscously damped simple oscillator subjected to a harmonic force. Further, as expected, the pile accelerations and bending moments can be smaller when the kinematic and inertial loads act against each other compared to the case when they act together on the pile foundations. This study also revealed that the peak kinematic pile bending moment will be at the interface of soil layers for both single pile and pile group. However, in the presence of both kinematic and inertial loads, the peak pile bending moment can occur either at the shallower depths or at the interface of soil layers depending on the pile cap rotational constraint.

Key words: centrifuge, earthquakes, inertial load, kinematic load, phase difference, pile foundations.

Background

The seismic behaviour of a structure is influenced by the response of its foundation and the foundation response depends on the ground response and its interaction with the superstructure. This dependency of structural response on foundation and vice versa is termed as soil-structure interaction (SSI). SSI is usually evaluated in terms of kinematic and inertial interactions. The foundation input motion will be different from the free-field soil surface acceleration due to the stiffness contrast between the foundation and its surrounding soil. This phenomenon is referred to as kinematic interaction. The forces caused by the vibrations of superstructure on the foundation is referred to as inertial effects. For the pile-foundation supported structures, the lateral vibrations of surrounding soil during earthquakes induce kinematic loads on to the pile foundations in addition to the inertial loads. Except for the case of lateral spreading or other large permanent ground displacements, the pile foundations are usually designed to resist static superstructure loads and inertial loads alone during earthquakes. However, even in the absence of large ground deformations, the kinematic effects can induce considerable force on piles, especially when there is a significant stiffness contrast between (i) the pile and surrounding soil (as in case of piles in soft clays) or (ii) the layers of soil when the pile foundation is embedded in a layered soil profile. In addition to the lateral spreading effects, Eurocode-8 (CEN, 2004) highlights the importance and recommends the consideration of kinematic loads in the design of piles and piers embedded in horizontal, non-liquefiable soil layers under certain conditions. However, there are no specific recommendations on the procedure to be adopted for computing the kinematic loads.

The simple pseudo static methods for the analysis of SSI usually neglect the kinematic effects by considering only the inertial force of superstructure in the analysis. The continuum-based or finite element methods can consider both the kinematic and inertial effects, however, the accuracy of results from these methods depend on several parameters, including the constitutive behaviour of soil and boundary conditions adopted in such analyses. The conventional sub-structure method deals with the kinematic and inertial effects separately for the SSI investigation though these two phenomena occur simultaneously in the field scenario. In such sub-structure analysis, it is usually assumed that the kinematic and inertial loads act together on the pile foundations to obtain a conservative design.

However, considering the phase relationship between the kinematic and inertial loads will replicate the true pile behaviour and emphasise the consequences of any phase difference on the seismic response of pile foundations. However, the existing limited literature on phase relationship between the kinematic and inertial loads for pile foundations offer a minimal support in the analysis of complex interaction problem.

The results of shaking table tests on a 2×2 steel pile group embedded in dry and liquefiable soils from Tokimatsu et al. (2005) concludes that the kinematic and inertial loads will act in-phase with each other thereby increasing the stresses in the piles when the natural period of the superstructure is less than that of the ground and vice-versa. Contrarily, Adachi et al. (2004) study reveals that the maximum inertial force and kinematic displacement may not act simultaneously on the pile foundations when the natural period of the superstructure is less than that of the ground by performing the shaking table tests on a 3×3 pile group embedded in liquefiable soil. The recent study of Yoo et al. (2017) suggests that the inertial and kinematic forces will be always out-of-phase irrespective of the ground conditions by performing a series of dynamic centrifuge experiments on single piles with different diameters, embedded in dry and liquefiable saturated sands. These contradictory conclusions highlight the ambiguity in understanding the phase relationship between the kinematic and inertial loads and its effects on the seismic behaviour of pile foundations. Further, most of the studies discussed above are for piles in dry and liquefiable soils, thereby providing limited information for the pile foundations in non-liquefiable and stratified soils.

In this research, a series of dynamic centrifuge experiments was carried out at 60g on a model single pile and a 3×1 row pile group embedded in a two-layer soil model to investigate the effects of earthquake induced kinematic and inertial loads. The soil profile consisted of a soft kaolin clay overlying a dense, fraction-B Leighton Buzzard (LB) sand. The individual effect of kinematic and inertial loads on pile foundations was evaluated by performing each centrifuge test in two-flights, with negligible mass acrylic plexiglass as pile caps in flight-01 and pile caps made from brass in flight-02. This study initially discusses the strength and stiffness of soil layers, followed by the dynamic behaviour of soil strata and pile foundations in both the flights. Later, the phase difference between the kinematic

and inertial loads is evaluated for different earthquakes and its effect on pile accelerations and bending moments are discussed in detail for both single pile and pile group.

Overview of experimental setup

Physical modelling using the centrifuge

Centrifuge based physical modelling facilitates the stress-strain similarity between the prototype and a scaled model of a geotechnical structure by subjecting the model to an increased g -field. The principles, mechanics, scaling laws, advantages and disadvantages of the centrifuge modelling are well discussed in Schofield (1980) and Madabhushi (2014). In this research, the 10 m diameter Turner beam centrifuge (Schofield 1980) at Schofield Centre, University of Cambridge, was used to test the scaled centrifuge models at 60g. An equivalent shear beam (ESB) box made of rectangular aluminium sections stacked together with the rubber in between the sections (Brennan and Madabhushi 2002) was used as the container to prepare the centrifuge models. The centrifuge models were subjected to in-flight scaled earthquake motions using the servo-hydraulic shaker developed by Madabhushi et al. (2012).

Materials

The pile foundations embedded in a layered soil profile with large stiffness contrast between the soil layers can be subjected to significant kinematic effects during earthquakes. To replicate such a phenomenon, the soil model for centrifuge tests was prepared with a dense fraction-B Leighton Buzzard (LB) sand underlying the soft speswhite kaolin clay. Figure 1a shows the particle size distribution curve of fraction-B LB sand and as it shows, the sand can be characterised as poorly graded. Figures 1b and 1c illustrate the variation of deviatoric stress (q) and volumetric strain (ϵ_v) with the axial strain (ϵ_a), respectively for 85% relative density fraction-B LB sand samples under different confining stresses (σ_c) during consolidated drained triaxial compression tests. The dilatancy behaviour of dense sand with the increase in shear deformations can be seen in Fig. 1c. Table 1 lists the properties of fraction-B LB sand (Garala et al. 2020) and the properties of speswhite kaolin clay (Lau 2015) are given in Table 2.

An aluminium (Alloy 6061 T6) tubular model pile of outer diameter (d) 11.1 mm and thickness (t) 0.9 mm was used to fabricate a single pile (SP) and a 3×1 row pile group (PG) with a pile spacing

of 3d centre to centre. The bottom of the tubular piles is closed with an aluminium plug to restrict the entry of soil into the piles during pile installation. Table 3 shows the equivalent prototype characteristics of the model single pile. The single pile and the end piles of the pile group were strain gauged at discrete locations to measure the bending moments during earthquakes. Figure 2 shows the schematic view of the pile foundations tested along with the location of strain gauges.

Fig. 1. (a) Particle size distribution curve of fraction-B LB sand, (b) and (c) response of dense fraction-B LB sand under different confining stresses during consolidated drained triaxial tests.

Fig. 2. Schematic view of the pile foundations tested (a) single pile and (b) pile group.

Table 1. Properties of fraction-B Leighton Buzzard (LB) sand (Garala et al. 2020).

Property	Value
Specific gravity, G_s	2.65
Maximum void ratio, e_{max}	0.767
Minimum void ratio, e_{min}	0.49
Effective particle size, D_{10} (mm)	0.68
Average particle size, D_{50} (mm)	0.80
Coefficient of uniformity, C_u	1.221
Coefficient of curvature, C_c	0.97
Peak friction angle*, ϕ_p	37.2°

* determined at 85% relative density

Table 2. Properties of speswhite kaolin clay (Lau 2015).

Property	Value
Plastic limit, PL (%)	30
Liquid limit, LL (%)	63
Plasticity index, PI (%)	33
Specific gravity, G_s	2.60
Slope of critical state line (CSL) in q-p' plane	0.90
Slope of an unload-reload line, (κ)	0.039
Intercept of CSL at p'=1 kPa (I)	3.31
Slope of normal consolidation line (λ)	0.22

Table 3. Equivalent prototype characteristics of the model single pile.

Property	Model scale	Prototype scale	Equivalent field case
Material	Hollow aluminium tube	-	Reinforced concrete pile
Outer diameter	11.1 mm	0.666 m	0.695 m
Thickness	0.9 mm	0.054 m	Solid section
Length	300 mm	18 m	18 m
Young's modulus	70 GPa	70 GPa	30 GPa
Flexural rigidity	2.65×10^{-05} MN-m ²	344 MN-m ²	344 MN-m ²
Plastic yield moment capacity	27.8×10^{-03} kN-m	6005 kN-m	-

Model preparation and centrifuge testing

The sand layer for the centrifuge models was prepared at 1g by pouring the dry sand into the ESB box with a relative density of $85\% \pm 2\%$ using an automatic sand pouring machine (Madabhushi et al. 2006). The sand layer was saturated by allowing the de-aired water to seep slowly from the bottom to top of sand layer through the drain valves at the bottom of ESB box. The clay layer was prepared from slurry stage by mixing the speswhite kaolin clay powder and de-aired water in 1:1.25 ratio under the vacuum

and poured on top of the sand layer. The ESB box with clay slurry and the sand layer was placed under a computer-controlled hydraulic press to consolidate the clay slurry under the vertical stresses, targeting an average undrained shear strength of 10-15 kPa for the clay layer. The depth of the clay layer after consolidation and trimming for the levelled surface was 150 mm with a saturated unit weight of 16.2 kN/m^3 . During the unloading phase of consolidation process, no water was allowed to enter into the model. This will keep the clay under the suction, in turn, creating higher effective vertical stresses in the clay layer. However, some suction in the clay layer can continue to drop at a slower rate until spinning of the model in centrifuge due to possible intake of water from the saturated sand layer.

The pile foundations were installed manually at 1g with the piles being gently pushed into the clay at an approximate rate of 2-4 mm/sec. The piles were embedded into the sand layer at an approximate rate of 0.5-1 mm/sec using a hydraulic jack. The pile caps for both the single pile and pile group were placed at 20 mm above the soil surface. Figure 3 shows the plan view of the model and Fig. 4 illustrates the elevation of the model along with the instrument locations. The dimensions in Figs. 2-4 are at model scale, and the values within the parentheses represent the prototype dimensions. For the soil-pile configuration shown in Fig. 4, the axial load carrying capacity of single pile is computed as 336 N at model scale and 1.21 MN at prototype scale. Further, the single pile and pile group were sited on different vertical planes in the direction of shaking as shown in Fig. 3. Therefore, the dynamic interaction between the two will be very minimal. To measure the response of soil strata and pile foundations during model earthquakes, a range of miniature sensors were used. Piezoelectric accelerometers (PAs) were used to measure the accelerations in the soil model and micro-electro-mechanical systems accelerometers (MEMSs) were used to measure both the horizontal and vertical accelerations at the pile-cap level in pile foundations. The response from the piezoelectric accelerometers can be considered as a far-field soil response as they were placed reasonably far from the pile foundations and in a different vertical plane as shown in Fig. 3. The pore pressures in the soil model were measured using pore pressure transducers (PPTs), and the linear variable displacement transducers (LVDTs) were used to measure the displacements of soil and pile foundations during the swing up of model and earthquakes.

To separate the kinematic effects from inertial loads, each centrifuge experiment was carried out in two-flights. In flight-01, the pile caps made from acrylic plexiglass were used for both the single pile and pile group. The mass of the plexiglass pile caps is negligible in comparison with the self-weight of the pile material and hence the measurements (accelerations and bending moments) during the flight-01 can be considered as the effect of soil (kinematic effects) alone on the pile foundations. In flight-02, the caps made from brass were used for both the single pile and pile group to simulate the inertial effects. Therefore, the accelerations and bending moments measured in the flight-02 are due to the combined effect of both kinematic and inertial loads. The brass caps will induce a static vertical force

of 167.75 N and 503.25 N at model scale (0.604 MN and 1.812 MN at prototype scale) for the single pile and pile group, respectively, thereby vertical load acting per pile is same for both the single pile and pile group. The applied vertical load is half the axial load carrying capacity of the pile foundations.

In each flight, the model was swung up to 60g in increments of 10g with the bottom drain closed in the centrifuge. Once the model attains equilibrium at 60g, an in-flight T-bar test and shear wave velocity (v_s) profiling were conducted, and then the model will be subjected to planned base excitations (BEs) using the servo-hydraulic shaker. Figure 5 shows the acceleration time-histories and corresponding continuous wavelet transforms (CWTs) of the base excitations considered in both the flights. The base excitations shown in Fig. 5 are measured using the piezo-electric accelerometer (PA8) at the base of ESB container (see Fig. 4). As shown in Fig. 5, the sinusoidal base excitations of different frequencies and intensities, scaled 1995 Kobe earthquake motion and a sine-sweep excitation with frequencies ranging in between 0.3 Hz to 2.5 Hz were considered as base excitations in this study. The excitations considered enable to understand the kinematic and inertial loads on the pile foundations for a variety of shaking intensities and frequencies. The important research outcomes from this series of centrifuge experiments are discussed in the following sections. Hereafter, the results from flight-01 and flight-02 are referred as ‘K’ and ‘K+I’ flights, respectively, and all the experimental data is presented at prototype scale unless stated otherwise.

Strength and stiffness of the soil layers

A 40 mm wide and 4 mm diameter T-bar was used to perform an in-flight T-bar test (Garala 2020) to determine the undrained shear strength (c_u) of the clay layer in K flight before subjecting the model to base excitations. The rate of penetration of the T-bar was approximately 1.7-1.9 mm/sec. The penetration resistance (q) measured by the T-bar is converted into c_u using a T-bar bearing factor, N_T ($=q/c_u$), of 10.5 as recommended by Randolph and Houlsby (1984). Figure 6a shows the c_u profile of the clay layer tested. Further, the shear wave velocity (v_s) at different depths was evaluated in both the flights from the air hammer device (Ghosh and Madabhushi 2002) before firing the base excitations.

The maximum shear modulus (G_0) was determined from the measured v_s using eq. (1), where ρ is the mass density of the corresponding soil layer.

$$(1) \quad G_0 = \rho v_s^2$$

Fig. 5. Acceleration time-histories and corresponding continuous wavelet transforms of the base excitations considered in this study.

Fig. 6. (a) Undrained shear strength of the clay layer and (b) maximum shear modulus of the soil layers.

G_0 values obtained at different depths in both the flights before subjecting the model to base excitations are shown in Fig. 6b. As can be seen, there is no considerable change in the stiffness of soil layers between the flights. Also, the G_0 values of the soil layers tested were computed from the well-known literature methods at the known pore water pressure locations (see Fig. 4) and shown in Fig. 6b. The equations used for computing the G_0 from literature methods can be found in Garala et al. (2020). For further analysis, an average G_0 of 23 MPa was considered for the clay layer. As not many data points are available from the centrifuge tests in the sand layer, an average G_0 of 184 MPa that fits in between the literature methods (Oztoprak and Bolton 2013 and Hardin and Drnevich 1972) was considered for the sand layer. The average G_0 considered for both the soil layers correspond to G_0 at a depth of $4d$ - $5d$ above and below the interface and results in a stiffness contrast of eight between the soil layers tested.

Natural frequencies of the soil strata and soil-pile systems

A sine-sweep excitation with the frequencies ranging between 0.3 to 2.5 Hz was fired as BE6 excitation in both the flights to determine the natural frequencies of soil strata and soil-pile systems. However, the

sine-sweep excitation was not successfully implemented by the servo-hydraulic shaker in the K flight due to some manual errors in executing the excitation. Therefore, the scaled Kobe motion (BE5) was used to determine the natural frequencies in the K flight as it consists of a larger range of frequencies in comparison to the simple sinusoidal excitations (see Fig. 5). Figure 7 shows the acceleration-time histories of the base excitation, soil surface, single pile and pile group accelerations measured in both the flights for selective base excitations. As Fig. 7 portrays, the pile foundations are driven by the soil movement in the absence of vertical loads at the pile cap level. In the K flight, a slightly larger acceleration amplitudes for the pile foundations in comparison with the soil surface is due to the higher mass density of the pile material and corresponding inertial effects. Further, as expected, the piles are responding differently from the soil vibrations in the presence of pile cap inertial loads as shown in Fig. 7.

To determine the natural frequencies, the amplitude of fast Fourier transforms (FFTs) of the soil surface, single pile and pile group accelerations are normalised with the amplitude of corresponding base excitation FFT in both the flights and plotted in Fig. 8 as the variation of amplification ratio against the frequency. The acceleration time-histories shown in Fig. 7 for BE5 and BE6 excitations during K flight and K+I flight, respectively are used for computing the amplification ratios shown in Fig. 8. Both the soil surface and pile foundations have a clear peak at 1.5 Hz and 1.7 Hz in Fig. 8a, indicating the natural frequency of soil strata can be in between 1.5-1.7 Hz in the K flight. Figure 8b depicts that the single pile and pile group have a natural frequency of 0.6 Hz and 1.7 Hz, respectively in the presence of inertial loads. The peak at 2 Hz for the soil surface in Fig. 8b indicates the natural frequency of soil strata in the K+I flight. The discrepancy in natural frequency of the soil strata in Figs. 8a and 8b is due to the difference in shear strains induced by the corresponding base excitations. The stronger intensity scaled Kobe motion (BE5) might have induced larger shear strains in the soil than the smaller intensity sine-sweep excitation (BE6) and hence a lower natural frequency for the soil strata during the scaled Kobe motion. Garala et al. (2020) and Garala (2020) show the experimentally derived stiffness degradation curves and shear strains mobilised in the soil layers during different base excitations considered in this study. The small-strain natural frequency of the soil strata can be computed

approximately using eq. (2) as 2.44 Hz, by using 23 MPa and 184 MPa as the average small-strain shear moduli for the top and bottom soil layers (see Fig. 6b), respectively.

$$(2) \quad f = \left(4 \sum_{i=1}^n \left(\frac{h_i}{\sqrt{G_{0i}/\rho_i}} \right) \right)^{-1}$$

where n is the number of soil-layers and h_i is the thickness of i^{th} soil layer in the stratified soil.

Fig. 7. Acceleration time histories of soil surface and pile foundations for a certain base excitation in K flight and K+I flight.

Fig. 8. Variation of amplification ratio against the frequency for the soil strata and pile foundations in (a) K flight using BE5 excitation and (b) K+I flight using BE6 excitation.

Dynamic behaviour of the soil strata and pile foundations

Figure 9 shows the accelerations measured at different depths in the soil strata along with the corresponding continuous wavelet transforms (CWTs) during BE2 excitation in the K flight. In addition to the driving frequency, the base excitations contain the harmonics of driving frequency as shown in CWT of BE2 excitation. As it shows in Fig. 9, the higher frequency harmonics propagated well in the sand layer but have been completely attenuated by the time they reach the clay surface as the soft clay layer is unable to transmit the higher frequency components. This smoothed acceleration response of soft soils for such higher frequencies is also a function of excitation intensity. More discussion related to this attenuation of higher frequency components in soft clays can be found in Garala and Madabhushi (2019). Further, the clay layer amplified the driving frequency component by almost two times, probably due to the driving frequency being close to the strain-dependent natural frequency of the soil strata. Figure 10 shows the acceleration response of soil strata and corresponding CWT plots for the scaled Kobe-motion (BE5) during the K flight. Similar to BE2 excitation, the higher frequency components of BE5 excitation are also filtered out here for the first few cycles of stronger intensity, but slightly amplified for the smaller intensity higher frequency components. Similar behaviour of the soil strata was observed even during other excitations in the K flight, and also for the base excitations in K+I flight.

Fig. 9. Acceleration response at different depths of the soil strata for BE2 excitation in the K flight.

Figure 11 shows the soil surface, single pile and pile group accelerations in both the flights for all the base excitations considered in this study (except BE6 as it was not implemented successfully in the K flight). The amplification of accelerations by the soil strata and pile foundations is primarily a function of the excitation intensity, natural frequency of the systems and excitation frequency components. As the base excitations considered possess a different range of frequencies and excitation intensities, the soil strata and soil-pile systems have peak amplifications at various frequencies. Table 4 lists the predominant frequencies of the soil strata and soil-pile systems at which the peak acceleration amplification was observed in both the flights, determined by following the same procedure

as natural frequencies estimation using the FFTs (see Fig. 8). The predominant frequencies listed in Table 4 can be fundamental natural frequencies or higher harmonics.

Fig. 10. Acceleration response at different depths of the soil strata for BE5 excitation in the K flight.

In the K flight, as discussed earlier, the response of pile foundations will be governed by the soil vibrations alone and hence only one predominant frequency for both the soil strata and pile foundations. However, in the K+I flight, the pile foundations can significantly amplify the accelerations either at the soil strata natural frequency or at soil-pile system natural frequencies. From Table 4, it can be inferred that the peak amplification for the pile group occurs at the soil strata natural frequency in most excitations, though the significant amplification response for the single pile is at its own soil-pile natural frequency during all base excitations. Further, a considerable drop in the soil strata natural frequency

for BE4 and BE5 excitations in K+I flight in comparison with K flight might be due to the significant clay softening for those excitations in the K+I flight.

Fig. 11. Acceleration response of soil surface, single pile and pile group in both the flights.

Table 4. Predominant frequencies of the soil strata and pile foundations.

Excitation	Frequency (Hz)	K flight	K+I flight		
		Predominant	Predominant frequency (Hz)		
		frequency (Hz)	Soil surface	Single pile	Pile group
BE1	0.667	1.95	1.92	1.02	1.95
BE2	1.167	1.56	1.53	0.79	1.53
BE3	0.667	1.52	1.31	1.25	1.31
BE4	0.83	1.35	0.75	0.54	0.76
BE5	~1.4	1.51	0.89	0.62	1.7

Phase difference between the kinematic and inertial loads

As Fig. 11 illustrates, there is a phase difference between the K flight and K+I flight pile accelerations, especially in the single pile, though the soil surface accelerations are in-phase in both the flights. In the K+I flight, the pile accelerations at the cap level are greatly influenced by the inertial loads due to the negligible kinematic effects at such pile levels. Therefore, the phase difference between the K flight and K+I flight pile accelerations can be considered as phase difference between the kinematic and inertial loads.

It can be seen in Fig. 11, the pile accelerations from both the flights are not always either in-phase or out-of-phase though the predominant excitation frequencies of the soil-pile systems are always less than or close to the soil strata natural frequency (see Table 4). This is in contrast with the conclusions of Adachi et al. (2004) and Tokimatsu et al. (2005). This implies that the ratio of soil strata natural frequency to the natural frequency of embedded structure may not govern the phase variation between the kinematic and inertial loads as stated in the previous studies. Also, the kinematic and inertial loads are not always out-of-phase in the pile foundations as suggested by Yoo et al. (2017). To investigate this aspect further, the phase difference between the pile accelerations from the two flights is computed for both the single pile and pile group using the cross-power spectral density (CPSD) and coherence functions. The CPSD, G_{xy} (see eq. 3), between the two signals, say $x(t)$ and $y(t)$, assists in identifying

the common range of frequencies in both the signals. Coherence, C_{xy} (see eq. 4), quantifies the similarity between the two signals at various frequencies. The C_{xy} between the two signals $x(t)$ and $y(t)$ will result in a value in between 0 and 1 for the chosen range of frequencies, with 0 being no dependency and 1 indicating a strong dependency between the two signals at a certain frequency.

$$(3) \quad G_{xy}(f) = X(f) \cdot Y^*(f)$$

$$(4) \quad C_{xy}(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)}$$

where $X(f)$ is the Fourier spectrum of $x(t)$, $Y^*(f)$ is the complex conjugate of the Fourier spectrum of $y(t)$, and $G_{xx}(f)$ and $G_{yy}(f)$ are the auto-spectral densities of the signals $x(t)$ and $y(t)$.

Figure 12 shows the coherence and phase difference between the kinematic and inertial loads for the selective base excitations of single pile and pile group. In this analysis, the ordinate corresponding to higher coherence is considered as the phase difference between the accelerations from two flights for each base excitation. Further, the magnitude of phase difference is only considered, neglecting the sign as it is not significant for this problem.

Fig. 12. Coherence and phase difference as a function of frequency for the pile accelerations from both the flights.

The experimentally determined phase difference values are plotted against the corresponding normalised frequency ratios (f/f_n , f is the driving frequency of excitation and f_n is the natural frequency of soil-pile system) for both the single pile and pile group from all base excitations and shown in Fig. 13. In addition, the theoretical phase variation between the force and displacement of a viscously damped linear second-order system subjected to a harmonic response due to base acceleration or displacement is computed using eq. 5 and shown in Fig. 13 for various damping ratios.

$$(5) \quad \tan \phi = \frac{-2\zeta\left(\frac{f}{f_n}\right)}{1-\left(\frac{f}{f_n}\right)^2}$$

where ϕ is the phase difference and ζ is the damping ratio. The predominant frequencies (strain-dependent natural frequencies) shown in Table 4 for the K+I flight are considered as f_n while computing the f/f_n ratios for the experimentally determined phase difference values shown in Fig. 13.

Fig. 13. Experimentally determined phase difference values on the conventional phase variation between the force and displacement of a viscously damped single degree of freedom system.

As Fig. 13 depicts, the phase difference obtained between the kinematic and inertial loads for various f/f_n ratios agrees well with the conventional force-displacement phase variation of a simple oscillator excited by a harmonic force. This is valid only if the predominant soil-pile-structure frequency

is considered as f_n rather than the fixed base natural frequency of the structure causing inertial effects. Figure 13 also conveys that the typical damping exhibited by the soil-pile-structure system will be around 5 to 10% when both the kinematic and inertial loads act with smaller phase difference between them. A much higher damping is possible in the soil-pile-structure system when the two loads act against each other (see Fig. 13). The wide scattering of experimentally determined phase difference values over different damping curves in Fig. 13 might be due to the non-linearity induced by the stronger intensity excitations.

Moreover, the consequences of phase difference between the kinematic and inertial loads can be seen in Fig. 11 with the single pile having lower acceleration amplitudes for few loading cycles in the K+I flight than K flight during BE2, BE4 and BE5 excitations. To effectively quantify the decrease in pile acceleration due to the phase difference, the pile accelerations are normalised with the peak soil surface accelerations using eq. 6 in both the flights. Arias intensities (Arias, 1970) are computed with the normalised pile accelerations using eq. 7 and shown in Fig. 14 for all the excitations.

$$(6) \quad a_{Norm} = \frac{a_{pile}}{a_{sur_peak}}$$

$$(7) \quad I_A(t) = \frac{\pi}{2g} \int_0^t [a_{Norm}(t)]^2 g^2 dt$$

where a_{Norm} is the normalised pile acceleration, a_{pile} is the measured pile acceleration, a_{sur_peak} is the peak soil surface acceleration (see Fig. 11) and I_A is the Arias intensity.

As Fig. 14 portrays, due to insignificant phase difference between the kinematic and inertial loads for most excitations in the pile group, the final Arias intensity (I_A) is always higher in the K+I flight than the K flight. On the other hand, for the single pile, the difference in final I_A computed between the K+I flight and K flight is smaller for BE4 and BE5 excitations compared to BE1 and BE3 excitations. This is due to the significant phase difference between the kinematic and inertial loads during BE4 and BE5 excitations in comparison with BE1 and BE3 excitations, where the two loads act with lower phase difference (see Fig. 13). This emphasises that the total energy associated with the pile acceleration will be lower when the kinematic and inertial loads act with higher phase difference compared to the case

when they act together. Further, the peak pile acceleration (or final I_A) of single pile is larger during BE2 excitation than BE4 and BE5 excitations in the K+I flight though BE2 peak base excitation intensity is relatively smaller (see Figs. 5 and 11). This is probably due to the relatively lower phase difference and lower damping during BE2 excitation in comparison with BE4 and BE5 excitations, respectively (see Fig. 13).

Fig. 14. Arias intensities computed for the normalised single pile and pile group accelerations in both the flights.

To further investigate the influence of phase difference between the kinematic and inertial loads on the seismic behaviour of pile foundations, the rotations of single pile and pile group are computed during different base excitations in the K+I flight. The pile accelerations measured by the vertical MEMS accelerometers in the direction of shaking (see Figs. 3 and 4) are double integrated to get the displacements, from which the rotations of pile foundations are computed. However, the pile group vertical MEMS accelerometers did not work for all the base excitations due to broken cabling after the first few excitations. Therefore, the LVDTs placed at the two ends of the pile group in the shaking direction (see Fig. 3) are used to determine the rotations. LVDTs will measure the vertical displacements directly and hence the rotations computed from the LVDTs will be clearer and free from low-frequency noises induced by the double integration of accelerations to obtain the rotations from MEMS accelerometers. This can be seen in Fig. 15 for BE2 excitation. Also, as Fig. 15 shows, the rotation computed from the MEMS accelerometers is less than the rotation computed from the LVDTs, possibly due to the filtering techniques used to double integrate the vertical accelerations to compute the displacements from MEMS accelerometers.

Figure 16 shows the rotations computed for the single pile from MEMS accelerometers and pile group from the LVDTs during different base excitations. As can be seen in Fig. 16, the single pile has higher rotations than the pile group during all excitations, irrespective of phase difference between the kinematic and inertial loads. This is due to the lower rotational stiffness for the single pile in comparison with the pile group. In addition, the single pile has relatively higher rotations at those excitations where the kinematic and inertial loads act with higher phase difference (BE2, BE4 and BE5 excitations). Therefore, Figs. 11 and 16 indicate that the higher phase difference between the kinematic and inertial loads can result in lower pile acceleration amplitudes for few loading cycles, but will induce higher rotations in the pile foundations.

Fig. 15. Rotation of the pile group computed from LVDTs and MEMS accelerometers during BE2 excitation in the K+I flight.

Fig. 16. Rotations of the pile foundations during different base excitations in the K+I flight.

Bending moments in the pile foundations

The strain gauges placed at discrete locations along the length of single pile and end piles (P1 and P3) of the pile group (see Figs. 2 and 3) are used to compute the pile bending moments in both the flights. In this study, bending at the pile tip is assumed to be zero for both the single pile and end piles of the pile group. To compare the bending moments for different base excitations, the measured bending moments are normalised using eq. (8) by considering both the excitation intensity and dynamic behaviour of the soil strata. Further, depth (z) of the pile is also normalised with the pile diameter (d) and presented as normalised depth (z/d) in this section.

$$(8) \quad M_{Norm} = \frac{M}{\left(\frac{E_p I_p}{d}\right) \left(\frac{a_{sur_peak}}{a_{base_peak}}\right) \left(\frac{a_{base_peak}}{g}\right)} = \frac{M}{\left(\frac{E_p I_p}{d}\right) \left(\frac{a_{sur_peak}}{g}\right)}$$

where M_{Norm} is the normalised bending moment, M is the measured bending moment, $E_p I_p$ is the flexural rigidity of the pile, d is the pile diameter, a_{base_peak} and a_{sur_peak} are the peak base excitation and peak soil surface accelerations, respectively (see Figs. 5 and 11).

Figures 17 and 18 show the normalised bending moments of single pile and end piles (pile-1 and pile-3) of the pile group, respectively in both the flights, when the peak bending moment occurs in the corresponding pile foundation. It can be implied from Figs. 17 and 18 that the peak bending moment can occur at different instants for the single pile and end piles of the pile group in both the flights, depending on the frequency and intensity of the excitation. Figures 19 and 20 show the envelopes of maximum absolute normalised bending moments in both the flights for the single pile and end piles of the pile group, respectively. As can be seen in Figs. 19 and 20, the peak pile bending moment in the K flight always occurs at the interface of soil layers for both the single pile and end piles of the pile group, irrespective of intensity of the excitation. This is due to the strain discontinuity between the soil layers of significant stiffness contrast. More discussion related to the kinematic bending moments of pile foundations tested can be found in Garala (2020) and Garala et al. (2020).

In contrast to the kinematic pile bending moments, the bending moments along the pile in the presence of both kinematic and inertial loads are different for the single pile and end piles of the pile

group. In the K+I flight, the peak bending moment occurs at the shallower depths for the single pile, whereas for the piles in pile group, the bending moments at the shallower depths and at the interface of soil layers are significant (see Figs. 19 and 20). In some excitations, the bending moment at the interface of soil layers is larger than the bending moment at the shallower depths for the end piles of pile group in K+I flight as shown in Fig. 20. The probable reason for this might be the constrained rotation of pile group (see Fig. 16) due to the frame action in comparison to the single pile (Mylonakis 1995). Further, the influence of inertial loads on the single pile is only up to a depth of $12d$ and beyond that the kinematic loads dominate the pile response during the smaller intensity excitations, such as BE1 and BE2 excitations. However, as expected, the influence depth of inertial loads increases with the increase in intensity of the excitation due to larger pile accelerations and pile surrounding soil stiffness degradation.

Fig. 17. Normalised bending moments in the single pile at the maximum bending moment instant.

Fig. 18. Normalised bending moments in the end piles of pile group at the maximum bending moment instant.

Fig. 19. Normalised absolute maximum bending moment envelopes for the single pile.

Fig. 20. Normalised absolute maximum bending moment envelopes for the end piles of pile group.

Furthermore, the bending moments in the pile foundations increase with the increase in intensity of the excitation as shown in Figs. 19 and 20 for most excitations in the K+I flight. However, the rate of increase in the bending moment with excitation intensity varies with the phase difference between the kinematic and inertial loads as shown in Fig. 19 for the single pile. Though the peak excitation intensities of BE4 and BE5 excitations are nearby peak excitation intensity of BE3 excitation (see Fig. 5), the normalised bending moments generated in the single pile during BE4 and BE5 excitations are much smaller than BE3 excitation in the K+I flight (see Fig. 19). This might be due to the smaller pile acceleration (see Fig. 11) because of higher phase difference between the kinematic and inertial loads for the single pile during BE4 and BE5 excitations in comparison to BE3 excitation (see Fig. 13). Further, BE2 excitation has caused larger normalised peak bending moment than BE4 and BE5 excitations in the single pile during K+I flight, though its excitation intensity is relatively smaller. This is due to the larger peak pile acceleration in BE2 excitation compared to BE4 and BE5 excitations (see Fig. 11). Further, when the inertial loads act against the kinematic loads, the bending moments occurred in the K+I flight are smaller than the soil alone induced bending moments (K flight) at deeper depths as shown for the single pile during BE4 and BE5 excitations in Fig. 19.

Moreover, the single pile has larger peak normalised bending moments than the end piles of pile group at all excitations in both the flights (see Figs. 19 and 20). It is interesting to observe the similar

response even during BE4 and BE5 excitations of the K+I flight, where the single pile has higher phase difference between the kinematic and inertial loads and the pile group response is at its near resonance conditions with f/f_n close to 1 (see Fig. 13). This is due to the larger intensity accelerations generated in the single pile during the first few cycles of excitations in K+I flight (see Figs. 11 and 17), though after which the pile accelerations in K+I flight are less than K flight due to the phase difference between kinematic and inertial loads. This implies that the bending moments in single pile can decrease after the first few loading cycles due to the lower pile accelerations induced by higher phase difference between the kinematic and inertial loads. Therefore, in the presence of both kinematic and inertial loads, a single pile can always be subjected to larger peak bending moments than the piles in a closely spaced pile group, irrespective of the phase difference and excitation intensity. However, the magnitude of peak bending moment and the variation of bending moment with the number of loading cycles will depend on the phase difference between kinematic and inertial loads.

Further, the end piles in the pile group have significantly larger peak bending moments during BE4 and BE5 excitations compared to BE3 excitation in the K+I flight (see Fig. 20). This can be linked to the near resonance conditions in the pile group as f/f_n is close to 1 during BE4 and BE5 excitations (see Fig. 13) and hence larger bending moments in the end piles of pile group. Also, again in the pile group, the pile-1 has larger bending moments than pile-3 during all base excitations and in both the flights, highlighting the group shadowing effects. The probable reason for the pile-1 always receiving larger bending moments than pile-3 in both the flights might be due to the bias in base excitations created by the first half-cycle of the excitation (see Fig. 5).

Conclusions

A series of dynamic centrifuge experiments was performed at 60g on a model single pile and closely spaced 3×1 row pile group in a two-layer soil model to investigate the influence of seismic kinematic and inertial loads on pile foundations. The following are the major conclusions of the study:

- The ratio of free-field soil natural frequency to the natural frequency of embedded structure might not necessarily govern the phase relationship between the kinematic and inertial loads on pile

foundations as reported in some of the previous studies. Instead, the phase difference between the two loads may follow the conventional phase variation between the force and displacement of a viscously damped simple oscillator excited by a harmonic force.

- The proposed phase shift theory holds good only if the strain-dependent natural frequency of the soil-pile-structure is considered rather than the fixed base natural frequency of the pile-pile cap-structure causing inertial effects in the analysis. This emphasises the importance of considering soil-structure interaction and soil non-linearity effects during medium to large intensity earthquakes.
- The higher phase difference between the kinematic and inertial loads can result in the reduction of pile accelerations and pile bending moments compared to a case when the two loads act together on the pile foundations. The influence of phase difference between the two loads on pile bending moments is significant at deeper locations. In addition, the pile foundations will be subjected to higher rotations when the kinematic and inertial loads act against each other.
- In the presence of both kinematic and inertial loads, the single pile always has larger peak bending moment than the piles of closely spaced pile group, irrespective of phase difference between the kinematic and inertial loads and excitation intensity. However, the phase difference between the two loads will govern the magnitude of peak bending moment and the bending moment variation with the number of loading cycles.
- The peak kinematic pile bending moment will always occur at the interface of soil layers for both the single pile and piles of the pile group, as there will be strain discontinuity at the interface due to significant stiffness contrast between the soil layers. However, in the presence of both kinematic and inertial loads and for the tested soil-pile configurations, the peak bending moment will be at the shallower depths for the single pile. In the case of closely spaced pile group, it can be either at the interface of soil layers or at the shallower depths, indicating that the pile cap rotational constraint critically governs the pile bending behaviour. Further, the piles in the pile group have different peak bending moments in both the flights, highlighting the group shadowing effects.

Conclusions of this study are only based on one pile stiffness and soil profile, which limits the applicability of conclusions for a wide range of pile stiffnesses and soil profiles with varying stiffness contrasts. A wider study looking at these variables can strengthen the observations made in this paper.

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