Canadian Geotechnical Journal

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Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2020-0538.R1
Manuscript Type:	Article
Date Submitted by the Author:	23-Nov-2020
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Keyword:	Centrifuge Modeling, Earthquake, Earth Pressure Coefficient, Numerical Modeling, Retaining Walls
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

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Dual Row Retaining Walls in Dry Sand: The Influence of Wall Stiffness on the Seismic Response

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Abstract

Dual row retaining walls can form efficient port and embankment structures, or even be used as coastal defence against Tsunamis. The system of parallel sheet pile walls can have a large lateral capacity within serviceability limit states due to the combined strength and stiffness of the walls and confined soil. Optimising the design by reducing the wall section and ensuring greater utilisation of the soil capacity has economic and environmental benefits but requires a deeper understanding of the dynamic soil-structure interaction. Centrifuge and numerical modelling is used to elucidate the mechanics of two systems with relatively flexible and stiff walls. Considering the fraction of the walls plastic moment capacity mobilised alongside the peak deflections illustrates the advantages of using relatively flexible retaining walls in these systems. More fundamentally, the importance of vertical variations of both the stress and strain during the horizontal dynamic loading is shown. The limiting horizontal stresses and phasing of the stress components around the walls are better understood by considering the mobilisation of earth pressure coefficients, reinforcing previous work which recommends a move away from conventionally defined limiting dynamic earth pressure coefficients.

1 Introduction

Dual row retaining walls, formed by two parallel rows of sheet pile walls connected by tie rods at the wall heads and with a sand infill, are novel civil engineering systems that can make efficient use of the combined strength and stiffness of the wall and infill soil. Such systems, having a large lateral capacity within a limited horizontal footprint, can be used as embankment or port structures. Following the devastation of combined earthquake and Tsunami events, such as the 2011 Tōhoku incident, similar structures have been deployed in Kochi, Japan. The resilience of infrastructure to multi-hazard loading is an increasingly important design consideration and the geotechnical profession must balance performance and safety with economic and en-

26 vironmental concerns. Dual row retaining walls may be used as coastal defence systems if they can remain
27 resilient to both the preceding earthquake and subsequent Tsunami. Whilst ultimately used for protection
28 against a single major earthquake and Tsunami event, a deeper understanding of the deformation mecha-
29 nisms at both the serviceability and ultimate limit states can allow more efficient performance-based design
30 of these systems.

31 Exploiting the interaction between the walls and soil, particularly during dynamic events, to enhance the
32 systems resistance to deformations requires a fundamental understanding of the system mechanics. Though
33 potentially used as coastal structures, it is instructive to establish the mechanics of the structures founded in
34 dry sand to contextualise their seismic performance in saturated, and even liquefiable soils, and post-seismic
35 resilience to lateral loading (Madabhushi, 2018).

36 A combined centrifuge and finite element modelling investigation was conducted at the Schofield Cen-
37 tre at the University of Cambridge (Madabhushi, 2018). Madabhushi and Haigh (2018a) introduced the
38 experimental and numerical methods used to understand seismic performance of a dual row retaining wall
39 system when the walls were relatively flexible. A key finding of these works was the significant variation of
40 the vertical effective stresses during the dynamic loading. The complexity of the horizontal and vertical soil
41 stress distributions around the walls could not be captured, or even bounded by, assumed linearly increasing
42 distributions of stress with depth. More fundamentally, plotting the time varying earth pressure coefficient
43 K (defined as the ratio of the horizontal to vertical effective stresses) showed they were limited by the static
44 earth pressure coefficients depending only on the soil friction angle. A move away from traditionally used
45 dynamic earth pressure coefficients, derived using pseudo-static (Mononobe and Matsuo, 1929; Okabe, 1924)
46 or pseudo-dynamic (Steedman and Zeng, 1990) assumptions with constant linear vertical effective stresses,
47 was thus recommended.

48 In this paper the dynamic response of dual row retaining walls founded in dry sand is further exam-
49 ined, with the contrasting cases of systems with relatively flexible and stiff wall elements being presented.
50 Madabhushi and Haigh (2019) previously showed experimental results from these cases, focussing on the
51 effect of the earthquake sequence and multiple centrifuge flights. Measured temporal variations of the wall
52 displacement, bending moment, tie force and soil stresses illustrated how the wall stiffness influences both
53 the dynamic loads attracted and the resulting dynamic response. The progressive cyclic response of the
54 systems inferred from cyclic shear stress-strain loops, that showed plastic straining of the soil leading to
55 locked-in stresses and densification, were also shown to depend on the wall stiffness.

56 The measured distributions of the flexible and stiff wall bending and horizontal stresses will be presented
57 alongside the results from finite element modelling of the same moments. In addition to elucidating the
58 mechanical response of the dual row walls and it's dependency on the walls flexural stiffness, the phasing

of the horizontal and vertical earth pressures with respect to the wall displacements and soil strains will be explored.

Analytical models of retaining walls under dynamic loading (Veletsos and Younan, 1994, 1997, 2000) can explore the sensitivity of the dynamic response of cantilever and propped walls to the wall flexibility. The displacements for the propped cases, which are comparable to half of the dual row retaining walls discussed here, were relatively insensitive to the wall flexure when the wall toes were clamped. In general, permitting wall rotation at the toe increases the sensitivity of the dynamic response to the wall flexibility (Veletsos and Younan, 1997; Psarropoulos et al., 2005) as it allows greater variation of the wall displacements and thus soil pressures mobilised. The demonstrated reduction of bending moments and prop forces with increasing wall flexibility during dynamic loading follows trends seen for the static equilibrium of similar structures (Diakoumi, 2007). Conti et al. (2012) presents experimental data on the dynamic response of propped retaining walls in dry sand, which highlights the link between wall displacement - and particularly toe displacements - on the bending moments developed. In this work the differing deformation modes of the dual row walls depending on the wall stiffness is shown to alter the dynamic response, with the phasing of the stresses mobilised also playing a significant role.

The phasing of the dynamic earth pressures around retaining walls, and the dependence of this phasing on the wall stiffness, is not straightforwardly predicted. Conventionally, horizontal wall displacements away from the retained soil are associated with dynamic active soil pressures with dynamic passive pressures developing in the soil that the wall displaces into. Elastic or visco-elastic Winkler spring models typically link horizontal displacements to the dynamic horizontal stress changes (Scott, 1973; Veletsos and Younan, 1994). More fundamentally, it is the strain state induced by the wall deformation that will drive the mobilisation of soil stress, which may have relevant horizontal and vertical components that are not readily incorporated into one dimensional spring models. In this paper the latter point will be explored using a combination of the centrifuge and numerical modelling results.

2 Centrifuge and Numerical modelling

Using centrifuge and numerical modelling to study dynamic geotechnical problems is often complementary; the centrifuge data revealing the deformation and failure mechanisms whilst a well calibrated numerical model can provide further insight into the variations of quantities that are difficult to experimentally measure, such as stresses and strains.

Figure 1 presents simplified schematics and prototype dimensions at 60 *g* of the centrifuge models discussed in this work. The test identifiers refer to the use of dry sand (D), the wall flexibility - relatively

Table 1: Test configuration details at prototype scale

Variable	Test Id	
	DF666	DS666
Retained Height (m)	6	
Embedment Depth (m)	6	
Wall Spacing (m)	6	
Wall Thickness (m)	0.18	0.36
Bending Stiffness ($MNm2/m$)	34	272

flexible (F) or stiff (S) - and the prototype height:embedment:wall spacing which are all 6 m for both cases explored in this paper. The instrument locations are detailed in the supplementary information. The wall systems were installed at 1 g . S28 Hostun Sand (Azeiteiro et al., 2017) was poured at $I_d \approx 80 - 85$ % using an automatic spot pluviator (Madabhushi et al., 2006). The walls were placed once the pouring reached the base level, after which the pouring was continued inside and around them. Relevant quantities for the wall systems at the prototype scale are given in Table 1. Schofield (1980) provides details of the centrifuge testing facilities used at the University of Cambridge and the scaling laws used to present the results.

Instrumentation included piezoelectric and electromechanical accelerometers to measure the soil and wall accelerations, load cells to measure the forces in the tie rod and full bridge strain gauges to measure the wall bending. Additionally, tactile pressure sensing sheets manufactured by Tekscan were used to measure the horizontal effective stresses around the right wall. The sheets were laminated and the remaining portions of the wall covered in acetate to ensure a uniformly smooth interface between the wall and soil. Paikowsky and Hajduk (1997) and Dashti et al. (2012) emphasise the need for careful calibration and interpretation of the Tekscan output when used with granular materials and in dynamic tests. The processing scheme suggested by Madabhushi and Haigh (2018b), which facilitates individual calibration factors for each sensing element to be expediently obtained from swing up data, was followed in this work.

It should be noted that the models were constructed in different centrifuge containers. The stiffer wall system was constructed within a laminar box whilst test DF666 used a rigid box with a perspex front to allow tracking of the wall and soil displacements using Particle Image Velocimetry (PIV) (White et al., 2003). Brennan et al. (2006) discuss the ability of a laminar box to minimise boundary effects. For the rigid box, the use of duxseal inserts at the boundaries to absorb some of the stress waves is discussed by Campbell et al. (1991) and Coe et al. (1985). Compliance of the soil and duxseal permits lateral deflections within the region of interest in the model, and did not lead to observably different deformation mechanisms between the centrifuge tests and previous numerical analyses which simulated laminar box boundary conditions (Madabhushi, 2018).

Figure 1: Simplified centrifuge model schematics, instrumentation locations and 60 *g* prototype dimensions following from Madabhushi and Haigh (2019)

Figure 2 shows the finite element mesh and boundary conditions used for both the flexible and stiff wall analyses. The mesh comprises of 8 noded quadrilateral elements created by Abaqus/CAE and analysed using the 2D FE Code Swandyné (Zienkiewicz et al., 2001; Chan, 1988). Madabhushi and Haigh (2018a) describes the numerical modelling procedure for case DF666, which is also adopted for modelling DS666 in this work, and thus will be briefly summarised.

The sequence of earthquake loadings applied to the centrifuge models is described by Madabhushi and Haigh (2019). The benefits of an initial, small ($< 0.05 g$) sine motion that may remove unstable soil arches present from the model construction process were shown. The performance of the dual row systems DF666 and DS666 are examined during the early cycles of a $\approx 0.35 g$, $1 Hz$ sinusoidal motion. This followed the non-destructive small sine shake for both models and a larger sine sweep (PGA of $\approx 0.4 g$ at $1.7 Hz$) in test DS666. The implications for and validity of comparisons to the numerical analyses which were only subjected to the large sine motion will be discussed in the results section. To facilitate consistent comparisons between cases, the numerical simulations of DF666 and DS666 were both subjected to the input motion recorded during DF666.

A ‘floating tie’ was used to model the pinned connections to the walls with a reduced axial stiffness back-calculated from the experiments (Madabhushi, 2018). The remaining elastic structural properties were derived directly from the experiments and are shown in Table 2. Between the walls and soil interface elements of finite thickness and constant stiffness and friction angle are defined. The relatively simple slip elements were sufficient for predicting the dynamic response under a small number of cycles (Desai et al., 1985), with the frictional strength guided by the acetate-soil interface in the experiments. The soil constitutive model ‘Mohr Coulomb V’ is described in Chan (1988) and follows the formulations in Hinton and Owen (1979). The model features non-associative flow and hardening based on plastic strains.

The soil parameters and damping parameters shown in Table 2 follow directly from Madabhushi and Haigh (2018a). As the numerical modelling is used in conjunction with the dynamic centrifuge data, previous work focussed on ensuring similarity of the mechanical response. In checking the numerical predictions against measured wall bending moments and wall displacements confidence is gained in the mobilised horizontal stress and stiffness predictions. Madabhushi and Haigh (2018a) also showed the measured and predicted soil accelerations could be usefully compared by abstracting cyclic shear-stress strain loops to confirm the damping and cyclic stress and strains mobilised are similar.

Figure 2: Numerical mesh and details of boundary conditions applied to the soil and between the walls and tie in the dual row wall analyses

Table 2: Parameters used in Numerical Analyses

	Property / Parameter	Value	Units
Structural Elements	Flexible and Stiff Bending Stiffness	34 / 272	MNm ² / m
	Axial Stiffness - Tie	50	MN/m
Interface Elements	Young's Modulus (Initial)	320	MPa
	Young's Modulus (Dynamic)	40	MPa
	Friction Angle	5.7	°
Soil Model - Mohr Coulomb V	Young's Modulus (Initial)	320	MPa
	Young's Modulus (Dynamic)	40	MPa
	Poisson Ratio	0.3	-
	Cohesion	1	Pa
	Friction Angle	33	°
	Dilation Angle	3	°
	Reference Stress	100	kPa
	Stiffness depth variation	0.5	-
	Voids Ratio	0.65	-
	Hardening Parameter	0	Pa
Damping	α	1.48	-
	β	0.00125	-

3 Dynamic response of flexible and stiff walls

Figure 3 presents the dynamic response of tests DF666 and DS666. The distributions of the measured wall bending and horizontal effective stresses along the right wall are shown before and at two instants during the applied motion. In the case of DS666 the distribution following the ‘settling’ 0.05 *g* shake and the larger sine sweep which preceded the presented response are shown. Madabhushi and Haigh (2019) justified the cross-comparison of these dynamic centrifuge tests with different earthquake sequences. Whilst horizontal stresses and bending moments are ‘locked-in’ with successive shakes of increasing PGA, the magnitudes of the elastic dynamic oscillations of a succeeding shake with the same PGA are less affected. The co-seismic instants reveal the peak inward bending of one wall, which corresponds to the peak outward bending developed in the other, and the distributions half a cycle later when the situation is reversed. The tensile forces measured in the ties connecting the two walls are also indicated.

The intra-cyclic behaviour for case DF666 shows a dynamic peak of horizontal stress near the external soil surface resulting in greater inward bending of the lower portion of the wall. As the accelerations reverse the reduction of the external stress peak and increased inward horizontal stresses, particularly mid-depth of the infill, yield the largest outward bending of the wall.

Figure 3b shows that for system DS666 that the dynamic bending moments also cycle about the static distribution, with greater inward and outward bending induced every half cycle. The contrasting static distributions between the stiff and flexible walls is touched upon in Madabhushi and Haigh (2019), in the context of justifying the greater tie force for the stiffer systems, and is attributed to the tendency of the flexible walls to bow out between the tie and wall toe versus a stiffer wall which more rigidly rotates about the tie. Figure 3b substantiates this theory. The static equilibrium for DF666 has a peak external horizontal stress near the surface which leads to inward flexure of the wall. By contrast, DS666 shows a general increase of effective stress in the external soil which lowers the point of action of the restoring soil force and necessitates a large tie force to resist the internal soil’s overturning moment.

During the dynamic loading these trends persist, with no discernible peak in the horizontal stress outside the stiffer walls. This leads to greater outward bending of the top portion of the dual row walls throughout the dynamic loading. Between DF666 and DS666 the measured internal horizontal stresses within the stiffer walls are larger and also show a less pronounced peak at the infill mid-depth. This peak may arise when the flexible dual row wall is displacing into and shearing the top of the internal soil whilst the wall toes are relatively fixed. The stiffer walls on the other hand remain more rigid and thus need to shear the entire height of the internal soil. Practically, that the choice of wall section influences the static and dynamic loading attracted by the wall, by altering the soil stresses mobilised, is clearly relevant though seldom incorporated

in design codes.

The contrasting mobilised soil stresses and resulting bending moments are intrinsically linked to the wall deformations. Figure 4 presents the numerically predicted wall and soil displacements for both dual row wall systems subjected to the input motion recorded in test DF666. In fig. 4a the measured wall displacements from PIV and in fig. 4b the inferred wall displacements from accelerometer and LVDT data are overlain. In both cases the wall displacements appear to have been predicted accurately. In the case of fig. 4b, where the experiment featured a slightly larger input motion (PGA of 0.4 g vs. 0.35 g) which was applied after a sine sweep (PGA of ≈ 0.4 g at 1.7Hz), the match suggests the peak dynamic displacement was insensitive to the small variation in the input motion and motion order. The latter observation is in agreement with Madabhushi and Haigh (2019), where the effect of multiple shaking events could be understood through permanent plastic deformations accrued proportional to the PGA superposed with elastic vibrations during prolonged shaking.

Overall, the resulting displacement of the walls at the tie level is similar for both cases, though the wall displacement mode and compatible soil displacements are very different. Figure 4a confirms that for the flexible walls the embedded portion of the walls do not cause significant soil displacement. The contours of maximum shear strain, calculated using the small displacement theory, highlight the flexure of the walls induces more shear strain in the internal soil above the ground level. In the case of DS666, fig. 4b shows a more rigid rotation of the two stiff walls. In response, the soil deformation mechanism extends deeper into the internal soil, with larger bands of shear strain extending from the left and right wall toes. Similarly, there is deeper shearing of the soil external to the two walls in DS666.

A key finding from Madabhushi (2018); Madabhushi and Haigh (2018a) was that during the applied horizontal shaking there were significant variations of the vertical effective stress, especially in the soil below and within the two walls. The dependence of this phenomenon on the wall-soil interface friction was explored, and it was found that for relatively smooth walled systems vertical complementary shear stresses between the wall and infill soil cannot develop despite vertically propagating horizontal shear stresses from an applied base motion. It was further theorised that the lack of complementary shear stress at the wall soil interface leads to an overturning moment from the soil between the walls which is equilibrated by a redistribution of vertical stresses across the infill width. The occurrence of this phenomena should not depend on the wall stiffness, and fig. 5 confirms that for both DF666 and DS666 there is significant vertical effective stress variation in response to the applied base motion. Comparing fig. 5a and fig. 5b shows that the magnitude of these variations are not identical and may be phased differently. The importance of the wall stiffness and mass on the dynamic earth pressure phasing is further explored in section 4.

The walls in DS666 are eight times as stiff and have four times the plastic moment capacity compared

(a) Peak bending moment distributions and associated earth pressures for the flexible walls

(b) Peak bending moment distributions and associated earth pressures for the stiff walls

Figure 3: Oscillation of the measured dynamic response of the flexible and stiff dual row wall systems about the pre-seismic values

(a) Flexible wall system showing greater flexure of the walls above the ground level

(b) Stiff wall system showing relative rigid rotation of the two walls below the ground level

Figure 4: Contrasting the predicted peak dynamic displacements and maximum shear strains for the dual row wall systems

Figure 5: Significant intra-cyclical dynamic variations of the vertical effective stresses predicted for both dual row wall systems

to DF666. In fig. 6a the fraction of the walls plastic moment capacity utilised during the peak dynamic outward or inward bending in the early cycles of loading is plotted as a function of the walls flexural stiffness. In addition to the experimental and numerical models of DF666 and DS666, numerical predictions for intermediate flexural stiffness's are also presented. A greater utilisation of plastic moment capacity may be associated with more efficient structural design. However, another design criterion is acceptable dynamic displacements of the wall-soil system. Figure 6a also shows the peak displacement predicted at the top of the systems modelled. Overall, fig. 6a shows for the geometries studied a fairly large reduction in flexural stiffness does not have a significant displacement or bending moment penalty, demonstrating a good scope to optimise the wall thickness for practical design.

Figure 6b presents generalisations of the two distinct behaviours observed for the flexible and stiff dual row wall systems. The terms relatively flexible and relatively stiff refer to the wall stiffness with respect to the soil stiffness for the geometries investigated. The wall's flexural stiffness to soil stiffness ratio cannot be straightforwardly normalized by wall height as the soil stiffness is stress and strain dependent (Li, 1990; Madabhushi and Haigh, 2019).

In terms of the static equilibrium, fig. 6b illustrates how the displacement mode of the flexible wall mobilizes a peak horizontal stress externally and thus inward bending of the walls. For stiffer walls the more uniform mobilisation of soil strength with depth leads to greater tie forces and only outward bending of the walls. The compatible soil displacements within the walls are also different, with greater overlap of the mechanisms induced by the stiff wall leading to the overlapping shear bands within the infill soil.

These displacements serve as the starting point for the differing dynamic responses summarised in fig. 6b. Again, greater overlap of the soil deformations within the stiff walls leads to the overlapping of shear bands. Stiffer wall-soil-wall systems have more congruous displacements due to the rigid rotation of the walls which induce comparatively smaller shear strains in the soil. This is in contrast to the larger local curvature and corresponding shearing of the infill between the flexible walls. The general trend of the bending moments developed are comparable, with outward bending of the lagging wall and inward bending of the leading wall below the ground level. The outward and inward bending is proportionally more and less for the stiffer walls due to the initial outward bending from the static equilibrium. The specific use case of a dual row retaining wall will drive the overall design; i.e. maximising soil shearing to dissipate more energy under lateral loading or minimising wall displacement to satisfy serviceability limits. Ultimately however, that the relative wall stiffness significantly alters the static and dynamic loads and response should be borne in mind.

(a) Effect of stiffness on the wall bending and maximum displacement during 0.35 g sinusoidal shaking

(b) Simplified mechanics of flexible and stiff dual row wall systems

Figure 6: Influence of stiffness on the behaviour of dual row retaining walls

4 Phasing of Earth Pressures

The mobilisation of the horizontal dynamic earth pressures around the walls, and crucially the phasing of these during the dynamic loading, controls the resultant dynamic forces and bending the walls will experience. In this section, the role of the wall stiffness on the earth pressure phasing is explored. Following from the dynamic variation of vertical effective stresses for both cases, the phasing of the lateral earth pressure coefficients will be compared with the dynamic wall deflections and soil strains to further elucidate the role of the wall stiffness on the dynamic loading the dual row wall systems attract.

Figure 7 presents measurements from the two centrifuge tests which are used to explore the earth pressure phasing. In fig. 7a the applied acceleration and horizontal effective stresses within and outside the right flexible wall are shown. Additionally, the relative displacements of the wall and adjacent soil patches obtained from PIV are plotted. Remembering that the leftward cyclic displacements were defined as positive, above the ground level fig. 7a shows that the soil monotonically compresses against the inside of the right wall. The larger cyclic displacement of the wall suggests the soil is further compressed or unloaded, which could correspond to the small increases and drops of horizontal earth pressures. Below the ground level the cycling of the horizontal earth pressure is much larger whilst the soil and wall movements are smaller, consistent with the expected increase of soil stiffness with depth.

It is interesting to note that the internal and external horizontal earth pressures remain in phase during the applied motion. The measurements are contrary to the idea of soil on one side of a wall relaxing to the active state whilst the other is compressed and tends to a passive state, in which case the stresses would cycle in anti-phase. Figure 3a confirms that across the depth of soil below the ground level the dynamic earth pressures on both sides of the flexible right wall are simultaneously increased or decreased relative to the static distribution. The lack of measurable wall movement suggests that the soil deformation is driving the earth pressure changes. The discernible negative displacement peaks of the internal soil (i.e. rightwards) do correspond to increases in the internal soil stress. Examining fig. 4a, it is possible that there is simultaneous lateral compression of the infill soil and the external soil outside the wall that the dual row walls deflect towards. The flexure of the lagging wall compresses the infill, increasing the horizontal stress inside the leading wall, which is equilibrated and restrained by an increase in horizontal stress in front the leading wall.

In stark contrast to this behaviour, fig. 3b showed that below the ground level an increase of the horizontal stress inside the stiff right wall was met with a simultaneous decrease outside it and vice versa. Figure 7b confirms this phase difference. Figure 7b also shows the dynamic displacements inferred from the wall acceleration and accelerometers in the middle of the infill. The results suggest that for the stiffer wall there are larger wall movements at all depths, particularly below the ground level. Figure 4b shows these

displacements predominantly arise from the stiff wall rotation, and may lead to a relaxation of strain in the soil the wall rotates away from in phase with an increase on the opposite side.

The dynamic response of the dual row retaining walls may be understood in terms of the inertial and kinematic loads that the system attracts. Very simply, kinematic loads can arise if there is a mismatch between the structural and soil stiffness whilst inertial loads are generated if there is a mismatch of the masses. Drawing a clear distinction between the relative importance of changes to the structural stiffness and mass, and even more so quantifying these changes, is an area of research with many uncertainties (Brandenberg et al., 2015). The horizontal stress phasing for DF666 and DS666 presented in fig. 7 was largely explained by the difference in wall flexure. The numerical models will now be used to further explore the dynamic response and stress state phasing.

Figure 8 shows the dynamically varying horizontal and vertical effective stresses around the left and right wall for both DF666 and DS666. The numerical earth pressures plotted are the mean of the gauss points values found within 0.3 m of the interface elements. Additionally, the calculated horizontal and vertical normal strain components are plotted. Figure 8a confirms the experimental observations for DF666; the internal and external horizontal stresses cycle in phase. This is the case around both flexible walls, with the increases around the right wall coinciding with the drops around the left and vice versa - again in agreement with fig. 3a.

As previously mentioned, there are significant variations of the vertical effective stresses, particularly between the two walls below the ground level. Figure 7a shows that these are in phase with the horizontal stress variations inside the walls and in anti-phase with the variations outside both walls. Within the walls these variations act to equilibrate the 'rocking' infill soil (Madabhushi, 2018). When the walls are deflecting rightwards, the vertical stress is lowered below the left half of the infill and increases towards the right. Thus the vertical stress increase will be in phase with the horizontal stress increase arising from the soil being driven into the relatively stationary flexible wall. Outside the wall the vertical effective stress cycling is in part due to the dynamic vertical displacements compatible with the horizontal soil displacements. The tendency for diagonal soil movements from horizontal loading is shown by the horizontal and vertical strain cycling. When the system deflects rightwards the soil outside the right wall will accelerate upwards, decreasing the vertical effective stress, whilst conversely the soil outside the left wall accelerates downwards and thus increases the vertical effective stress. These stress variations are thus out of phase with the horizontal components.

In terms of the vertical stress variations, the predictions for test DS666 shown in fig. 8b have comparable phasing to DF666. Figure 8 shows that the peak vertical stress is smaller for DS666 than DF666. The differing boundary conditions that the flexible and stiff wall enforce on the rocking infill could justify this difference, but requires further research. Between the horizontal stress variations inside and outside a given

(a) Horizontal effective stresses cycling in phase around the flexible walls

(b) Horizontal effective stresses showing out of phase cycling around the stiff walls

Figure 7: Experimental observations of horizontal earth pressure phasing and relative wall displacements

304 wall the cycling is out of phase, though perhaps not to the extent seen experimentally.

305 The most apparent difference in the strain cycling between the cases is the larger accumulation of per-
 306 manent strains, particularly with depth, around the flexible walls. More subtly, though the phasing of the
 307 strain components is the same in fig. 8a and fig. 8b, the relative magnitudes are different. Specifically, there
 308 is proportionally more vertical to horizontal strain cycling seen within the flexible walls. It is interesting to
 309 note that instantaneous reductions in the horizontal strain are met with increases in the horizontal stress in
 310 this case.

311 To further understand the difference between the flexible and stiff wall cases fig. 9 presents the variation
 312 of the mobilised earth pressure coefficients K , defined as the ratio of the horizontal to vertical effective stress.
 313 Following the stress variations presented in fig. 8, the coefficients are the mean of the gauss points stresses
 314 found within 0.3 m of the interface elements. The specific elevations presented in fig. 9 are comparable to the
 315 experimental results in fig. 7 and were confirmed to be representative of the behaviour observed at all depths.
 316 As first described in Madabhushi (2018), the adherence of the dynamic mobilised earth pressure coefficients
 317 to the ‘static’ earth pressure limits K_A and K_P highlights the drawback of traditionally defined ‘dynamic
 318 earth pressure coefficients’. The predictions of the Mononobe-Okabe limits or subsequent pseudo-dynamic
 319 variations can be violated by horizontal stress measurements or predictions if geostatic vertical stresses are
 320 assumed.

321 Acknowledging the potentially non-linear and dynamically varying vertical stress distributions gives a
 322 much clearer insight into the limiting dynamic horizontal stresses that can develop. Figure 9 reinforces this
 323 notion for both DF666 and DS666. Further, despite the apparent differences in individual stress component
 324 phasing, fig. 9 seemingly unifies the cyclic behaviour of the flexible and stiff cases. For example, as either
 325 system deflects rightwards the soil to the left of the left wall relaxes - i.e. a drop in K and
 326 reduction in lateral compressive strain - whilst the soil to the right of the left and right wall shows increased
 327 compressive strain and mobilises a larger K .

328 Figure 9a shows that at instants of peak inward curvature of a given flexible wall the internal and external
 329 soil is at the active and passive limit respectively. When the flexible wall displaces towards the infill and
 330 experiences peak outward bending the mobilised K outside the flexible wall drops below the static value but
 331 remains above the active limit and the K mobilised internally.

332 Combining this information with fig. 8a potentially adds to the previous interpretation of the simultaneous
 333 increase of horizontal stress inside and outside the flexible wall - which was attributed to the soil below the
 334 ground being compressed against the relatively stationary flexible wall. Figure 9a could also suggest that the
 335 horizontal stress increase follows the vertical stress increase as the minor to major principal stress ratio is
 336 bounded by K_A . This can explain the previous observation where the horizontal strain component decreases

(a) Flexible wall case showing the horizontal and vertical stresses cycle in phase inside the walls and in anti-phase outside of them, with continued accumulation of permanent strains

(b) Stiff wall case showing out of phase cycling of horizontal and vertical stress inside the walls and in anti-phase outside of them, with limited accumulation of permanent strains

Figure 8: Numerically predicted effective stress and strain phasing for the dual row wall systems

whilst the horizontal stress component increases. Outside the wall the horizontal stress is driven against the passive limit K_P to equilibrate the wall displacement, but the stress magnitude is potentially capped by the increased acceleration of the external soil and reduced vertical effective stresses. As the same wall is driven into the infill all of the soil relaxes from the limit state.

For the stiff system fig. 9b shows a more dramatic reversal during the ‘relaxation’ phase, with the soil outside the wall cycling from a passive state to the active limit which mobilises greater resistance in the internal soil. The variation of the vertical stress magnitudes notwithstanding, the greater magnitude of horizontal stress variation inside the stiff walls may be related to the lack of accumulated strain when compared with the flexible walls. The larger displacement of the lagging stiff wall into the infill thus allows a larger reduction of the external horizontal stress. For the seismic design of retaining structures the critical acceleration direction for peak bending moments and displacements is not necessarily the same (Kramer, 1996). That this relationship could additionally depend on the wall stiffness and mass has been illustrated.

With the results presented for both relatively flexible and stiff retaining walls, there is a clear interaction between the dynamic vertical and horizontal soil deformations and resulting dynamic stress variations. Analytical modelling of dynamic retaining wall behaviours have typically concentrated on linking the relative horizontal wall-soil displacement with a mobilisation of soil stresses, i.e. dynamic Winkler spring models. Such 1-D dimensional models which cannot account for vertical displacements (Scott, 1973) or assume constant vertical displacements in the horizontal direction away from the walls (Veletsos and Younan, 2000) will struggle to capture the behaviours observed in this work. In addition to the physical inaccuracy, it is difficult to envisage a 1-D spring model with geostatic vertical stresses and ‘dynamic’ earth pressure coefficients consistently predicting the limiting horizontal stresses that can develop, when these are actually bounded by the vertical stress variations and limiting ratio of horizontal to vertical stress depending on the frictional strength of the material.

5 Conclusions

The combination of centrifuge and numerical modelling were used to explore the complex static equilibrium and dynamic response of two dual row retaining walls, where the two twelve meter tall wall systems modelled were relatively flexible and stiff with respect to the soil.

The wall stiffness was shown to alter both the static equilibrium and dynamic response of the dual row systems, with distinct displacement modes and soil deformations emerging. That a change to the wall stiffness alters the loads attracted by the system is an important practical consideration, and should be reflected in the stresses that are applied during the design. The flexible walls generated slightly smaller

(a) Cycling of K for the flexible wall systems in anti-phase between the soil inside and outside each wall and between the left and right walls

(b) Variations of K for the stiff wall system show the same phasing as the flexible walls though the K instantaneously mobilised can vary

Figure 9: Numerically predicted mobilised dynamic earth pressure coefficient phasing for both dual row wall systems are unified and bounded by the static earth pressure coefficients

horizontal stresses in terms of magnitude, but proportionally the distributions and phasing of the horizontal loads resulted in greater curvature of the walls. In turn, this led to greater shearing of the surrounding soil. Greater utilisation of the soil strength across the walls height by the flexible walls points to an efficient use of the structural material without necessarily increasing the total displacements during dynamic events.

Building on the conclusions drawn in Madabhushi (2018), interpretation of the limiting dynamic stresses that are mobilised is greatly simplified if the vertical stress variations are recognized. The vertical stress variations can arise from ‘rocking’ of the soil between smooth structural components due to a lack of complementary shear stresses or due to horizontal and vertical soil accelerations compatible with one boundary of the soil being displaced horizontally. Linked by the wall displacements, the magnitude of the vertical stress variations were found to have a dependence on the wall stiffness.

The ratio of the dynamic horizontal to vertical stresses mobilised were shown to cycle between the ‘static’ earth pressure coefficients K_A and K_P for both systems. The cyclic variation of the calculated earth pressure coefficients were always in anti-phase around a given wall, regardless of the trends of individual stress components. This is arguably a clearer route to interpret the dynamic loads that a structure in soil can attract than traditionally defined ‘dynamic earth pressure coefficients’ where geostatic vertical stresses are assumed. For example, the increase of horizontal stress when the flexible wall was driven away from the soil could be explained by the increase of vertical stress and minimum allowable ratio of stress components dictated by the soils frictional strength.

In closing, the efficiency of dual row retaining walls as port and embankment structures can be optimised by decreasing the wall stiffness without necessarily increasing the total displacements. As the Geotechnical profession looks towards performance based design for infrastructure subjected to multiple hazards, it can be favourable to design more structurally flexible systems that ultimately rely more on the strength and stiffness of the surrounding soil to limit the deformations. The general mechanical response of a relatively flexible and stiff system were described in this work. The phasing and magnitudes of the dynamic horizontal stresses developed were best understood by also recognising the dynamic changes in vertical stress. It is anticipated that future seismic design codes for soil-structure interaction problems that avoid dynamic earth pressure coefficients and focus on the dynamic vertical stress changes in addition to the mobilisation of soil strength between the active and passive limits will yield more accurate predictions of the serviceability limit state.

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(a) Flexible Wall Case, DF666

(b) Stiff Wall Case, DS666

Figure 1: Simplified centrifuge model schematics, instrumentation locations and 60 *g* prototype dimensions following from Madabhushi and Haigh (2019)

Figure 2: Numerical mesh and details of boundary conditions applied to the soil and between the walls and tie in the dual row wall analyses

(a) Peak bending moment distributions and associated earth pressures for the flexible walls

(b) Peak bending moment distributions and associated earth pressures for the stiff walls

Figure 3: Oscillation of the measured dynamic response of the flexible and stiff dual row wall systems about the pre-seismic values

(a) Flexible wall system showing greater flexure of the walls above the ground level

(b) Stiff wall system showing relative rigid rotation of the two walls below the ground level

Figure 4: Contrasting the predicted peak dynamic displacements and maximum shear strains for the dual row wall systems

(a) Flexible wall case

(b) Stiff wall case

Figure 5: Significant intra-cyclical dynamic variations of the vertical effective stresses predicted for both dual row wall systems

(a) Effect of stiffness on the wall bending and maximum displacement during 0.35 g sinusoidal shaking

(b) Simplified mechanics of flexible and stiff dual row wall systems

Figure 6: Influence of stiffness on the behaviour of dual row retaining walls

(a) Horizontal effective stresses cycling in phase around the flexible walls

(b) Horizontal effective stresses showing out of phase cycling around the stiff walls

Figure 7: Experimental observations of horizontal earth pressure phasing and relative wall displacements

(a) Flexible wall case showing the horizontal and vertical stresses cycle in phase inside the walls and in anti-phase outside of them, with continued accumulation of permanent strains

(b) Stiff wall case showing out of phase cycling of horizontal and vertical stress inside the walls and in anti-phase outside of them, with limited accumulation of permanent strains

Figure 8: Numerically predicted effective stress and strain phasing for the dual row wall systems

(a) Cycling of K for the flexible wall systems in anti-phase between the soil inside and outside each wall and between the left and right walls

(b) Variations of K for the stiff wall system show the same phasing as the flexible walls though the K instantaneously mobilised can vary

Figure 9: Numerically predicted mobilised dynamic earth pressure coefficient phasing for both dual row wall systems are unified and bounded by the static earth pressure coefficients

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Table 1: Test configuration details at prototype scale

Variable	Test Id	
	DF666	DS666
Retained Height (<i>m</i>)	6	
Embedment Depth (<i>m</i>)	6	
Wall Spacing (<i>m</i>)	6	
Wall Thickness (<i>m</i>)	0.18	0.36
Bending Stiffness (<i>MNm²/m</i>)	34	272

Table 2: Parameters used in Numerical Analyses

	Property / Parameter	Value	Units
Structural Elements	Flexible and Stiff Bending Stiffness	34 / 272	MNm ² / m
	Axial Stiffness - Tie	50	MN/m
Interface Elements	Young's Modulus (Initial)	320	MPa
	Young's Modulus (Dynamic)	40	MPa
	Friction Angle	5.7	°
Soil Model - Mohr Coulomb V	Young's Modulus (Initial)	320	MPa
	Young's Modulus (Dynamic)	40	MPa
	Poisson Ratio	0.3	-
	Cohesion	1	Pa
	Friction Angle	33	°
	Dilation Angle	3	°
	Reference Stress	100	kPa
	Stiffness depth variation	0.5	-
	Voids Ratio	0.65	-
	Hardening Parameter	0	Pa
Damping	α	1.48	-
	β	0.00125	-