

Stock market participation, portfolio choice and pensions over the life-cycle*

Steffan Ball[†]

University of Cambridge

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Abstract

In this paper we present a calibrated life-cycle model which is able to simultaneously match asset allocations and stock market participation profiles over the life-cycle. The inclusion of per period fixed costs and a public pension scheme eradicates the need to assume heterogeneity in preferences, or implausible parameter values, in order to explain observed patterns. We find a per period fixed cost of less than two percent of the permanent component of annual labour income can explain the limited stock market participation. More generous public pensions are seen to crowd out private savings and significantly reduce the estimates of these fixed costs. This is the first time that concurrent matching of participation and shares has been achieved within the standard preference framework.

Keywords: precautionary saving, portfolio choice, stock market participation and uninsurable labour income risk

JEL Classification: G11, H31

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[†]Corresponding address at: Faculty of Economics, University of Cambridge, Sidgwick Avenue, Cambridge, CB3 9DD. *Email:* sgb24@econ.cam.ac.uk

1 Introduction

This paper presents a life-cycle portfolio choice model, with realistically calibrated stochastic labour income and reasonable risk aversion, explaining some of the stylised facts of household asset allocations. Empirical studies consistently find that approximately fifty percent of US households do not invest in the stock market (whether directly or indirectly), and those that do participate hold only a small proportion of their wealth in risky assets. Despite recent developments in financial markets, which have led to greater levels of participation and higher shares of risky assets in household portfolios, the empirical evidence still presents a significant challenge to the life-cycle model. This paper demonstrates that the addition of a per period fixed cost to stock market participation, and public pension provision, enables us to explain the observed limited participation and substantial portfolio diversification.

A wide literature presents models of intertemporal choice incorporating precautionary and retirement motives for saving; many of the empirical patterns of wealth accumulation and consumption have been accounted for within this framework (Hubbard, Skinner and Zeldes, 1995; Carroll, 1997; Attanasio et al., 1999; Gourinchas and Parker, 2002). More recently, a literature has emerged which allows for the simultaneous determination of consumption and portfolio allocation within a life-cycle framework. However, it has proved difficult to explain asset allocations without assuming unrealistic wealth accumulation, extreme parameter values or heterogeneity in preferences. Cocco, Gomes and Maenhout (2005) present a thorough analysis of the standard household portfolio choice model without any fixed cost considerations. They are able to force portfolio shares down to reasonable levels, in addition to obtaining significant age effects. However, this is achieved by accepting unrealistically high levels of saving¹. Further, their model predicts one hundred percent participation at all ages, a result which is clearly in conflict with reality.

¹A combination of using a very high coefficient of risk aversion ($\gamma = 10$) and assuming a small probability of a zero income event, which triggers the precautionary response too much.

The literature has come to accept that some form of information cost is required to move away from the complete participation prediction of the standard model. This is corroborated by the empirical work of Paiella (2001) and Vissing-Jorgensen (2002), both of whom have shown that fixed costs to stock market participation can empirically rationalise the observed limited participation. These fixed costs can be one-off entry costs or per period costs; however, the estimates of these costs are very low². There has been significant progress with incorporating one-off fixed costs of stock market entry into household portfolio choice models; Alan (2006) calibrates the level of this entry cost to match moments from the PSID, producing a value of two percent of annual labour income. With this one-off fixed cost, Alan is able to match the participation rate very precisely. As a buffer stock saving model, Alan's framework cannot attempt to match the wealth data; the resulting low levels of saving means that her model cannot address the issue of portfolio diversification³. Gomes and Michaelides (2003) set an exogenous fixed entry cost of two and a half percent of annual income and are able to attain reasonable age profiles of both participation and shares. However, they only achieve this by assuming preference heterogeneity, Epstein Zin utility functions, and very high levels of savings.

Hitherto, it appears that the life-cycle model is restricted to matching one key statistic at a time (wealth, participation or shares) within the standard preference framework. This paper illustrates how this inopportune prediction can be overcome by assuming two relatively simple extensions: per period fixed costs to stock market participation and public pension provision. Firstly, while entry costs are a convincing way to lower participation early in life, they cannot be the causal factor behind low levels of participation for older households. Our innovation of considering per period fixed costs enables us to account for low participation at both early and late stages of the life-cycle. Furthermore, per period fixed costs ensure that stock market participants endogenously hold greater levels

²Paiella (2001) finds a per period fixed costs of US\$ 95-175, while Vissing-Jorgensen (2002) finds that transaction costs of US\$ 260 can explain the behaviour of 75% of nonparticipants.

³Alan's model predicts complete specialisation in stocks for all participants.

of savings; therefore, they are more able to diversify away from full specialisation in risky assets. In addition to the afore mentioned empirical significance, per period fixed costs are also intuitively appealing: it is clear that information plays an important role in investment decisions, and these decisions must be made at each and every period over the life-cycle. Gathering and processing such information is costly: these costs may be the material cost of paying for the assistance of a financial advisor, or time costs of collecting and managing the data. These information requirements directly affect the cost of saving, by determining access to and utilisation of different investment technologies, and this has important implications of households' ability to accumulate wealth and diversify their portfolios.

Secondly, the retirement motive for saving is of great importance for wealth accumulation, especially when considering investment in stocks, and tends to be ignored or treated rather imprecisely in the literature. Previous work tends to either completely disregard participation and portfolio share decisions during retirement⁴ or impose exogenous retirement pension income, with little consideration of the effect of collecting the tax revenues required to fund such schemes. Taxation has a direct effect on the cost of saving and this has important implications for optimal participation rates and portfolio shares. In this paper we introduce a stylised pay-as-you-go public pension scheme, funded by a proportional tax on labour income. We demonstrate the effect of changing public pension generosity on optimal portfolio allocations and on the size of the fixed cost necessary to explain the observed participation.

This paper simulates a household model of portfolio choice and calibrates the model to match moments from the US data. By means of these two innovations, we advance the literature by demonstrating how the life-cycle framework is now able to account for at least two statistics concurrently, without resorting to heterogeneity in preferences or implausible parameter values, and can match all three moments if we are willing to accept relatively

⁴As in Alan (2006).

high risk aversion and discount rates. This paper is the first study to simultaneously match participation, shares and asset holdings with the data; we are able to achieve this with a per period fixed cost of less than two per cent of the permanent component of labour income and a proportional tax rate of twenty percent.

In the next section we provide some data on asset holdings, participation and portfolio shares by households in the US. Section 3 develops our household model of portfolio choice. Section 4 provides simulated life-cycle profiles of asset accumulation, stock market participation rates and risky asset shares; showing the effect of introducing per period fixed costs, varying the public pension provision, and changing the level of risk aversion. Section 5 concludes.

2 Data on asset allocations

This section details some stylised facts on asset accumulation and allocation which are addressed by our life-cycle model. Asset accumulation profiles are well documented, where mean asset holdings are seen to rise sharply when households are in their 30s and 40s and then diminish gradually during retirement⁵. Low (2005) shows that the mean asset holdings peak around retirement, at a magnitude of around seven times greater than mean income. Low calculates that the US median asset to income ratio across working life is 1.84⁶, and it is to this statistic that we calibrate our simulated asset holdings.

When addressing household investment allocations it is important to distinguish between two distinct decisions; participation and portfolio shares. First, participation represents the binary choice of whether to participate in the stock market or not. Stock market participation in the US is currently just above fifty percent, Bertaut and Starr-McCluer (2002) report that 56.9% of households hold risky assets⁷. There is a clear age effect on

⁵Although this decline is not always present in median asset holdings, see Burnheim (1987).

⁶He uses the 1995 PSID wealth supplement, defining wealth to include housing wealth and using PSID weights.

⁷Using 1998 SCF weighted data. Risky assets are defined as including: directly held stock; stock held through mutual funds, retirement accounts, trusts and other managed assets; corporate, foreign and

participation and a hump shape in the decision of whether or not to hold risky assets has been much discussed (Alan, 2005; Ameriks and Zeldes, 2001; Poterba and Samwick, 1999; Gomes and Michaelides, 2005). The top panel in Figure 1 uses data from the US Survey of Consumer Finances to show how stock market participation follows such a hump shape, with limited participation at early and late stage of life. Participation is also increasing in wealth, with richer households much more likely to hold risky assets than poorer households. Using multivariate probit techniques Bertaut and Starr-McCluer (2002) find that both age and wealth effects are significant for the participation decision in the US.

The second important statistic is the portfolio share; this represents the share of risky assets to total assets, conditional on participating in the stock market. These portfolio shares are typically well below 100% (Ameriks and Zeldes, 2001; Bertaut and Starr-McCluer, 2002; Gomes and Michaelides, 2005; and others). Bertaut and Starr-McCluer (2002) find that the average portfolio share in the US is 54.4%⁸. Gomes and Michaelides (2005) find a similar results, with an average share of 54.8%. The bottom panel of Figure 1 demonstrates that the age effect on portfolio shares is fairly weak. In reality, it is common practice for financial advisors to tell their clients to shift their portfolios away from risky assets as they age, especially as they enter retirement⁹. This is evident in Figure 1(b), where portfolio shares fall from just over 60% for households aged 55-64 to around 55% for the retired. However, Bertaut and Starr-McCluer (2002) find that these age effects are not robust to more sophisticated econometric analysis.

The model in this paper is calibrated to match these average statistics and it aspires to explain both the limited participation and strong portfolio diversification.

mortgage-backed bonds; business equity; and investment real estate.

⁸Using 1998 SCF weighted data. Defining risky assets as before.

⁹So called "life-cycle" funds do this automatically for investors.

3 Life-cycle model

Individuals are assumed to maximise the sum of expected discounted lifetime utility in light of uncertain and uninsurable labour income and rate of return risk. Utility is defined over a single nondurable consumption good, and is additively separable across time but non-separable within period. Let there be T periods in the life-cycle of which a given K periods are spent in retirement and $T - K$ give the working life. This standard set-up gives the following constrained maximisation problem:

$$\max_{\omega_s, C_s} E_t \left[\sum_{s=t}^T \beta^{s-t} u(C_s) \right] \quad (1)$$

subject to,

$$A_{s+1} = (A_s + Y_s - C_s - I(\cdot)F)[\omega_s(1 + r_s^{eq}) + (1 - \omega_s)(1 + r)] \quad (2)$$

and a terminal condition, $A_{T+1} = 0$. In each period β is the discount factor; C_t is consumption; A_t is the amount of assets held at the start of period t ; Y_t is exogenous real disposable labour income net of taxes; F is the absolute per period fixed costs of stock market participation; ω_t is the portfolio share; r and r_t^{eq} are the risk free and risky returns respectively; and $I(\cdot)$ is an indicator function taking a value of unity when the individual participates in the stock market, and zero otherwise.

In each period t the agent has to decide how much to consume and how much to save out of cash-on-hand, which is comprised of assets held at the start of the period and realised income ($A_t + Y_t$). Any savings, given by $(A_t + Y_t - C_t)$, are invested and earn a composite portfolio return denoted by the terms in square brackets in equation (2) above. We assume the portfolio investment is comprised of two distinct saving tools, a riskless and a risky asset. The riskless asset has a constant real return of r and the risky asset has a stochastic real return of r_t^{eq} , which is assumed to be i.i.d. over time and distributed lognormally. We do not impose any correlation between stocks and labour income as the

empirical evidence on the size and magnitude is mixed. Heaton and Lucas (2000) find insignificant estimates for all but the highest educational group, and Davis and Willen (2001) surprisingly find negative correlations for low educational groups. Further, the effect of such a correlation has been shown to make very little difference to participation and portfolio shares in practice (Gomes and Michaelides, 2005), unless unrealistically high correlation coefficients are chosen.

The portfolio share, ω_t , represents the proportion of assets held in risky forms. In order to invest in risky assets in any given period the individual has to pay a fixed cost of F . These per period fixed costs can be thought of as representing brokerage or membership fees. Alternatively, the F represents time costs of information gathering and corresponds to the opportunity costs of acquiring data on financial markets, monitoring brokers, keeping up to date with trends etc.. Let us define a parameter f representing this absolute fixed cost, F , as a proportion of permanent income. This follows from the motivation of the fixed cost being an opportunity cost of time; also, such a specification significantly simplifies the solution to the model as it removes the necessity to have an additional state variable.

Some limited borrowing is allowed, up to the discounted sum of the minimum income individuals will receive in each remaining period, with no borrowing permitted against pension income. A short sales constraint is imposed on equity such that the portfolio shares must always lie between zero and one (inclusive). Therefore, agents can only borrow at the risk free rate and can only invest in risky assets if they hold positive balances¹⁰.

During working life, labour income is uncertain and non-diversifiable. Following the standard specification in the literature,¹¹ we shall assume that the stochastic labour income process can be broken down into a deterministic component, which can be calibrated to match the hump shape of income over the life-cycle; and two stochastic components, a permanent and a transitory shock:

¹⁰While it is true that we do see households simultaneously borrowing and holding stocks in the data, this restriction is imposed on the model for simplification.

¹¹See MaCurdy (1982), Abowd and Card (1989), Moffitt and Gottschalk (2002) for examples of papers modelling the properties of individual earnings.

$$Y_{t+1} = (1 - \tau)G_{t+1}P_{t+1}v_{t+1} \quad (3)$$

where v_t is transitory income, P_t is permanent income, τ is a proportional pension-tax (set exogenously) and G_t is a deterministic growth trend. The log of permanent income is assumed to follow a random walk.

$$P_{t+1} = P_t \varepsilon_{t+1} \quad (4)$$

Both transitory and permanent shocks are assumed to be independently and identically distributed lognormally such that $\ln(v_t) \sim N(-0.5\sigma_v^2, \sigma_v^2)$ and $\ln(\varepsilon_t) \sim N(-0.5\sigma_\varepsilon^2, \sigma_\varepsilon^2)$. The income process is truncated such that a zero level of income cannot be realised, otherwise individuals would optimally choose never to borrow.

For the final K periods of life individuals are retired, during which time they receive a constant and certain pension income, Γ . We construct a pay-as-you-go pension scheme, somewhat akin to the arrangements in the US, with pension payments being funded out of contemporaneous tax revenues. At any particular point in time, we assume that there exists an equal mass of individuals at each age; with working population share being $\frac{T-K}{T}$, and the fraction of households in retirement given by $\frac{K}{T}$. Let tax revenue collected from individual i at age a be given by $\rho_a^i = Y_a^i \cdot \frac{\tau}{(1-\tau)}$. Summing pension contributions over all individuals, H , and all working ages, $T - K$, and normalising by the number of retirees, $H.K$, gives the following constant annual pension income for each retired household:

$$\Gamma = \sum_{a=22}^{T-K} \frac{\sum_{i=1}^H \rho_a^i}{H.K}$$

This set-up gives us a system with three state variables (A_t, Y_t, P_t) and two control variables (C_t, ω_t). The first order condition of the value function with respect to consumption is given by:

$$u'(C_t) = \beta E_t \left\{ [\omega_t(1 + r_t^{eq}) + (1 - \omega_t)(1 + r)] u'(C_{t+1}) \right\} \quad (5)$$

and the first order condition with respect to portfolio share is given by:

$$0 = E_t \left[(r - r_t^{eq}) u'(C_{t+1}) \right] \quad (6)$$

These two conditions can be solved for the following policy functions in each period:

$$C_t(A_t, Y_t, P_t) \quad (7)$$

$$\omega_t(A_t, Y_t, P_t) \quad (8)$$

These two optimal decision rules are solved recursively from the final period for a discrete number of points in the state space. The details of the solution method are given in Appendix A.

4 Simulation results

Once the optimal policy functions have been determined, we can simulate the model to imitate the behaviour of a population of households and report average allocations to show the effect of introducing fixed costs and changing the public pension scheme. Initially, we shall detail the functional forms, parameter values and calibration statistics used. Subsequently, we shall show the results of simulating the model for 10,000 ex-ante identical households who differ in the realisation of shocks.

We then put forward the results in four subsections: first, we analyse effect of introducing per period fixed costs to our baseline life-cycle model and show this modification changes the participation results substantially. Second, we show the effects of altering the generosity of the public pension scheme. Third, we increase asset holdings and demonstrate

how our model is able to match both participation and shares. Finally, we demonstrate how we are able to match participation, share and asset holdings. This structure permits us to disentangle the different effects of varying fixed costs, public insurance and asset holdings in order to better fit the data.

Parameters and Calibration The utility function is assumed to take the typical constant relative risk aversion (CRRA) form

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}. \quad (9)$$

with a baseline value of 2 for the coefficient of risk aversion. This is in line with estimates based on consumption data: Gourinchas and Parker (2002) find estimates of risk aversion ranging from 0.28 to 2.29 and Alan and Browning (2003) find ranges from 1.99 to 4.79.

The riskless rate of return is set at a constant value of 1.6%, which represents the average annualised real rate of US 3-month treasury bills from 1960-2000. The risky return is drawn from a distribution with a mean of 5.6%, corresponding to a 4% equity premium¹², and a standard deviation of 0.2¹³. The deterministic trend in the income process is taken directly from Low (2005) with the parameter values emerging from a regression of log wages on a quadratic in age, controlling for demographics and cohort effects. This gives the well documented concave income profile over the life-cycle, where income rises from the start of working life to a peak around an age of 50, and then declines to retirement. The values for the variance of permanent and transitory shocks to income are taken from Meghir and Pistaferri (2004). These parameter values are shown in Table I.

[INSERT TABLE I ABOUT HERE]

A baseline tax rate of 20% is chosen giving a retirement replacement ratio of 0.57 of

¹²As is common in this literature; see Gomes and Michaelides (2005), Haliassos and Michaelides (2002), Yao and Zhang (2005), Cocco (2001), Campbell et al. (2001)

¹³See Alan (2005). Gomes and Michaelides (2005) and Haliassos and Michaelides (2002) use a similar value of 0.18

last working period permanent annual labour income. In reality the pension tax system in the US is more complicated than the set-up assumed here, but this proportionate rate is chosen as a reasonable proxy. The implied replacement ratio is in line with those used in the literature, for example, Low (2005) used a replacement rate of 0.55.

The values for the fixed cost, discount rate and (in the final two scenarios) risk aversion are chosen by calibration. The value of the fixed cost is chosen in order to match simulated mean participation with the observed mean in the data (0.57). We choose the discount rate such that simulated ratios of median asset holdings to income match the PSID for the working households (1.84)¹⁴. In addition, in the final two scenarios, we choose the risk aversion parameter to match the mean portfolio share statistic of 0.54. Calibration parameters and statistics are shown in Table II.

[INSERT TABLE II ABOUT HERE]

4.1 Introducing per period fixed costs

4.1.1 Baseline case without fixed costs

In our baseline parameterisation we set the per period fixed costs to zero and calibrate the model to match the PSID median asset holding to income ratio (1.84). The dashed-line graphs in Figure 2 show the cross-sectional mean profiles of asset holdings, stock market participation and conditional portfolio shares.

[INSERT FIGURE 2 ABOUT HERE]

Under these baseline parameters the discount rate necessary to match the level of assets is 8.6%. This is significantly higher than the discount rate Low (2005) obtains using a similar set-up without endogenising the return to savings¹⁵ but well within the range estimated by Alan and Browning (2003)¹⁶. This relatively high impatience is required to keep asset accumulation at the required level, since saving is now much more desirable and

¹⁴Except for the scenarios where we explicitly fix the discount rate.

¹⁵He finds a discount rate of 3% is able to match the US data.

¹⁶They find discount rates ranging from 5.3% to 11.6%, depending on the level of education.

the higher average returns make assets accumulate at a faster rate. The dashed line in the top panel of Figure 2, agents are seen to accumulate assets up until retirement, and then run their balances down during the last 15 years of non-working life, reaching levels that are in the same order as those seen in the data. Average assets are always positive, but some individuals do borrow, up to the endogenous borrowing limit, in the early stages of life.

The dashed line in Figure 2(b) shows participation in the stock market is 100% for all but a few years in early and late life; only those individuals with negative savings do not participate. This is in conflict with the data, clearly illustrating the famous ‘portfolio participation puzzle’, discussed at length in the finance literature. Moreover, in this model framework, without any transactions costs, there is no explicit participation decision; nonparticipants are simply those agents in debt who are constrained not to hold risky assets by the bounds on ω_t .

Figure 2(c) details portfolio shares conditional on participation in the stock market. The mean portfolio share of stock market participants is 0.98 which is well above the true value of 0.54. In contrast to the complete market models there is a modest age effect in the portfolio share profile. However, the shares remain very high; they equal unity for the initial stages of life and are subsequently seen to fall slightly. This depicts what has been termed the ‘portfolio specialisation puzzle’, whereby only wealthy agents, which in our model are the older agents, are able to diversify away from complete specialisation in risky assets. Households with low levels of savings invest all their assets in the risky investment tool, as their consumption path is driven mainly by the stochastic income process rather than the return to savings. As households build up larger savings balances, their consumption paths are determined more by the return to the risky investment; once a certain threshold of wealth is reached, it is no longer optimal for households to hold all of their assets in the risky saving tool (see policy functions in Appendix B). In this baseline model, it takes thirty years of asset accumulation for individuals to be able to hold both

risky and riskless assets in their portfolio.

The baseline model exemplifies the inability of the standard framework to explain household portfolio choice, neither participation nor shares are close to being matched. Some progress has been made at explaining portfolio shares (see for example Gomes and Michaelides, 2005) by allowing unrealistically high levels of asset holding. Reconciling shares with high levels of saving means that participation is necessarily unrealistically high, given few households will choose to borrow. This demonstrates the inopportune trade off between participation and shares that is inherent in these household models as they stand. We shall argue that the introduction of per period fixed costs provides a significant contribution to helping us resolve these much discussed problems.

4.1.2 Including calibrated per period fixed costs

Introducing per period fixed costs, and calibrating the model to match the asset to income ratio of 1.84, leads to a slightly lower discount rate (from 8.6% to 8.2%)¹⁷. Introducing the fixed cost, increases the cost of saving; and so the discount rate must fall to keep the level of savings constant. In addition, the per period fixed costs are calibrated to match the average participation rate of 0.57, giving a per period fixed cost of 5.0% of permanent annual labour income. These results are detailed in the solid lines in Figure 2; we can see that the increased cost of saving depresses asset accumulation early in life; this reduced precautionary saving is compensated by more retirement saving in mid-life, in order to keep the median level calibrated to the PSID.

The introduction of per period fixed costs has a marked effect on the stock market participation profile, with a clear hump shape now evident in Figure 2(b). Few individuals accumulate sufficient wealth in the first ten years of working life to make it viable to pay the fixed costs of participation, and they hold only riskless assets in their portfolio. As individuals age they accumulate assets, risky investments become profitable and participa-

¹⁷The results from keeping the discount rate the same and only calibrating the fixed cost are not significantly different.

tion increases; peaking at age 57 with 97% participation. As assets are drawn down later in life, the number of individuals participating is seen to fall gradually back to very low levels. Figure 2(c) shows that the inclusion of fixed costs generates a more pronounced age profile in the conditional portfolio shares, with shares declining into retirement as expected. However, the model continues to predict complete specialisation in risky assets early in life and a mean portfolio share of 0.97¹⁸, which is considerably higher than observed in the data.

A per period fixed cost of 5% of the permanent component of labour income is able to explain the observed limited participation, zero portfolio shares of small savers and conditional shares declining with age. However, predicted shares remain high and the calibrated value of the per period fixed cost is considerably higher than those reported in the data studies of Paiella (2001) and Vissing-Jorgensen (2002), the next subsection offers a potential explanation for these low empirical estimates.

4.2 Changing the generosity of the public pension scheme

In this section we analyse the effect of changing the public pension generosity in a model where fixed costs are calibrated to match the US average participation in the stock market. Varying the tax rate changes the effective impatience through altering the income profile; lowering the tax leads to less pronounced income growth and lower effective impatience. This gives us the choice of either allowing this lower effective impatience to manifest in the form of higher savings or increasing the discount rate in order to keep savings at the same level as before the tax change. Thus, we consider two different scenarios; first, we shall analyse the effect of varying the tax rate while keeping the level of asset holdings constant, and allowing the discount rate to change in order to achieve this. Second, we consider keeping the discount rate constant, and allow savings to vary. We shall reduce the tax rate, and by so doing decrease the generosity of the state pension, in order to determine

¹⁸This is marginally smaller than the mean share predicted by the model without fixed costs, as savings behaviour has altered little.

the effect on the calibrated level of fixed costs and on the profiles of asset accumulation, participation and portfolio shares.

4.2.1 Keeping savings constant

In this scenario we calibrate the model separately under each tax rate to match two empirical statistics, asset holdings and participation. This explores the effect of changing the generosity of the pension scheme assuming that average savings and participation remain the same. The discount rate is calibrated to match median asset holdings (1.84) and the per period fixed cost is calibrated to match mean participation of 0.57. The results of these simulations are shown in Figure 3.

[INSERT FIGURE 3 ABOUT HERE]

Reducing the tax rate from 20% to zero, leads to the discount rate increasing from 8.3% to 9.9%. Changing the tax rate directly affects the growth rate of income and, hence, the effective patience of the individuals. The lower the tax rate, the less steep is income growth over an individual's life, and the more patient they become. In order to keep the median asset to income ratio consistent with the US data, the calibrated discount rate must rise as we reduce the tax. If we had kept the discount rate the same across the scenarios, assets would be higher under lower tax rates due to a lower effective impatience. Ensuring that asset holdings remain the same, leads to a redistribution of savings from earlier years in the working life to mid-age; representing a shift from precautionary savings to retirement savings, which is evident in the top panel in Figure 3.

In order to explain the observed limited average participation, the (calibrated) per period fixed cost increases from 5.0% to 6.4% of permanent annual income, as the tax rate is brought down from 20% to zero. The tax change has a modest effect on the participation profile; because of lower precautionary balances, fewer individuals participate during the early years of working life when taxes are low, as can be seen in Figure 3. Individuals hold ever more retirement savings, increasing the peak level of assets and the mid-age

participation rate. This large mid-age participation necessitates lower participation during later life, in order to keep participation at the required level.

Figure 3(c) shows how lowering the level of public insurance leads to more diversified portfolios. Mean shares fall from 0.97 to 0.82 (as shown in Table II), as higher asset levels during mid-life push households onto the non-linear portions of their portfolio share policy functions (see Appendix B). As taxes are reduced, the age effects become more pronounced; reaching the complete markets outcome at retirement, in the case when taxes are zero and there is no pension income¹⁹.

It can be seen that lowering the generosity of the public pensions improves the fit of our model in terms of portfolio shares (for both mean levels and life-cycle profiles); although, they remain considerably greater than the observed shares. However, more generous public pensions moderate the calibrated discount rate and per period fixed costs, enabling asset holdings and participation to be matched at more reasonable parameter values.

4.2.2 Keeping the discount rate constant

In this scenario we hold the discount rate constant, at the level calibrated in Section 4.1.2, while varying the tax rate. This scenario examines the effect of allowing average savings to increase in response to lower taxes. As before, we calibrate the per period fixed costs to match average participation. We find that reducing the generosity of the public pensions scheme has a significant effect on the level of fixed costs; increasing from 5.0% (when the tax rate is 0.2), to 7.3% (when the tax rate is 0.1), to 8.5% (when the tax rate is zero).

[INSERT FIGURE 4 ABOUT HERE]

In Figure 4(a) the crowding out effect of public pensions is clearly evident. Reduced public pension provision is compensated for by greater private retirement savings; leading to a welfare loss being associated with public pension provision²⁰. The increased saving over

¹⁹The seminal work by Samuelson (1969) analysed portfolio choice in a complete markets situation with no background risk. He showed that the portfolio share should be independent of wealth and age and be given by $\omega = \frac{\mu}{\gamma\sigma^2}$, giving a value of 0.5 using our parameter values.

²⁰This loss stems from the higher average returns to self-insurance.

the life-cycle which less generous pension provision creates, allows earlier initial entry and higher participation during mid-life. Again, smaller tax rates lead to more diversification of assets; however, shares remain unrealistically high at early stages of life, and mean shares are substantially above their true value (see Table II).

These few scenarios show that fixed costs are negatively related to the generosity of the public insurance mechanism. When taxes are high, the per period fixed costs required to match participation rates are substantially less than when taxes are low, or nonexistent. With a fairly well developed public pension set-up, individuals are well catered for in retirement and so have less incentive to save; given low retirement saving only a small fixed cost is required to explain the observed limited participation in the stock market. However, in the situation where there is no public provision for old age, individuals hold large amounts of assets in mid-life and are highly motivated to invest in risky investment tools; high fixed costs are then required to keep participation at the observed level. Our findings demonstrate that it is not surprising to find empirical estimates of these fixed costs remain very low, given the well developed social security system prevalent in the US (see Paiella, 2001, and Vissing-Jorgensen, 2002).

In all parameterisation of the model we are able to obtain realistic hump shapes in the participation decision, if slightly more pronounced than the data. However, portfolio shares remain considerable; mean values are too high and the prediction of 100% risky holdings for young participants is unconvincing. This is a common problem with household models of portfolio choice, a trade-off between matching participation and shares seems inevitable. The problem stems from the nature of power utility and the ensuing portfolio share policy functions; which dictate that large diversification can only take place if savings are substantial, but with high savings households are highly motivated to participate. Models with no fixed cost considerations, or with one-off entry costs, cannot aspire to restrain participation when savings are high under a standard preference framework. However, the innovation of this paper to include fixed costs for each period of participa-

tion, enables us to overcome this predicament and simultaneously match participation and shares; it is to this we turn to next.

4.3 Matching participation and shares

It is not possible to match shares, under a standard preference framework, using the exogenous parameter values detailed in Table II. The reason for this stems from the lower bound on shares implied by the Samuelson rule²¹. Under these parameters the lowest possible portfolio share, realised as savings go to infinity, is 0.5; with shares at reasonable levels of savings being substantially higher. Therefore, attaining an average share of just over a half is virtually impossible, suggesting different parameter values are required. The Samuelson rule is determined solely by three parameters; the coefficient of risk aversion, the variance of the risky return, and the equity premium. Given that the variance and return of risky assets are more precisely estimated, the most reasonable approach is to vary the risk aversion parameter.

If we desist from matching asset holdings, our model framework allows us to calibrate the coefficient of risk aversion to match the mean portfolio share observed in the data; giving a calibrated value of $\gamma=4$. This is somewhat higher than empirical estimates generally attained from analysing consumption data, but remains well within the bounds considered reasonable by Mehra and Prescott (1985) and Alan and Browning (2003); it is also in line with estimates attained from matching wealth data²². Furthermore, a value of 4 is lower than parameters commonly used in the literature²³.

In this scenario we fix the discount rate to the value calibrated in Section 4.1.2 and determine fixed costs by calibrating simulated participation averages with the data; producing a per period fixed cost of 6.5%. The simulated profiles are shown in the dashed line in Figure 5, with the profile under low risk aversion left in for comparison (represented

²¹See footnote 19.

²²For example, Cagetti (2003) estimates a risk aversion parameter of 4.26 and Gakidis (1998) of 3.

²³Gomes and Michaelides (2005) use a CRRA of 5 and Cocco, Gomes and Maenhout (2005) use 10.

by the solid line in Figure 5).

[INSERT FIGURE 5 ABOUT HERE]

This model parameterisation demonstrates that we are able to match both mean participation and mean asset allocation decisions, while still using reasonable parameter values and a standard preference framework. Figure 5(b) shows the age profile for participation displaying the expected hump shape; in addition, the hump is less pronounced than under low risk aversion, bringing it more in line with the data seen in Figure 1. Further, the portfolio share profile in Figure 5(b) is more realistic, with values well below unity for most of life and declining with age.

The simple adaptation of the standard model to include per period fixed costs has resolved the long standing dilemma of simultaneously matching shares and participation. In addition to keeping participation low at early and late stages of life, per period fixed costs ensure that stock market participants hold higher asset balances, enabling them to diversify away from full specialisation in stocks. However, this is only achieved by allowing households to accumulate greater amounts of wealth than they do in reality; with these parameters the median assets are more than five times greater than median income, whereas in reality they are less than double (see second column in Table II). In the next subsection, we show how the flexibility of our model specification enables simultaneous matching of participation, shares and asset holdings.

4.4 Matching participation, shares and wealth

In this scenario we demonstrate our model's ability to match all three of the key asset statistics; participation, shares and asset holdings. We calibrate the discount rate to match median asset holdings, risk aversion to match mean portfolio shares and per period fixed costs to match mean participation; the results are detailed in the dotted lines in Figure 5.

The age profile for asset accumulation now entails larger precautionary savings and less

asset holdings for retirement compared to the scenario with low risk aversion, but remains comparable to the cross section mean seen in the PSID. The participation profile is hump shaped and much less pronounced than the case with a risk aversion of 2. The shares are also reasonable; shares fall as households age and, in contrast to the low risk aversion parameterisation, are less than unity for all ages. The portfolio share profile displays a slight hump close to retirement, which is interestingly also evident in the data (see Figure 1). This derives from the steepness of the policy functions at this level of risk aversion; with steep policy functions, the effect of running down assets (which pushes portfolio shares up) outweighs the age effect (which shifts the policy functions in and pushes shares down) over some range of ages - see Appendix B.

The calibrated fixed costs now fall to 1.7% of the permanent component of labour income, which is close to the empirical findings discussed earlier. The calibrated coefficient of risk aversion is 6.4. This is significantly higher than the value commonly used in the consumption literature; however, it is perfectly in line with Woodford and Rotemberg's (1998) estimate of 6.25 and well below the upper bound of 10 instructed by Mehra and Prescott (1985)²⁴. In order to counter this risk aversion, and the entailing high precautionary motive, an uncomfortably large discount rate of 49% is required to match asset holdings. Even the high tax rate of 20% cannot moderate impatience at these parameter values.

To date, the literature has only been able to match one of these statistics at a time, under power utility and homogeneous preference; for example, Alan (2006) matches participation and Cocco, Gomes and Maenhout (2005) match shares, but neither match asset holdings. In the previous subsection we demonstrated the ability of our model to match participation and shares simultaneously, under realistic parameters. Here, we match all three statistics, with reasonable age profiles, without the need to resort to heterogeneous preferences or non-power utility functions. However, we achieve this at the cost of high

²⁴Which is used by Cocco, Gomes and Maenhout (2005).

calibrated risk aversion and discount rate parameters.

5 Conclusion

In this paper we analyse household consumption and saving decisions in the presence of two distinct saving tools and public pension insurance. We are able to match the life-cycle averages of savings and asset allocations, while keeping to a fairly simple framework with realistic parameter values.

We find that a standard portfolio choice life-cycle model, calibrated to match the level of savings in the PSID, gives extreme rates of stock market participation and very high portfolio shares, which is at odds with empirical findings. Introducing per period fixed costs to the baseline model generates the desired hump shape in participation and some age effects in portfolio shares, while still matching asset accumulation profiles. We have introduced a stylised pay-as-you-go pension scheme and shown how this acts as a moderator for impatience; in the absence of pensions we would need a higher discount rate to keep asset accumulation at the required level. More generous public pensions are seen to crowd out self-insurance and lead to higher average conditional shares. Further, the size of fixed costs are found to be conditional on the level of taxes, providing some explanation for the small empirical estimates.

The conditional portfolio shares remain implausibly high for all scenarios using low risk aversion coefficients. Disregarding the level of wealth, we can calibrate the model to match the mean of both participation and conditional shares, with very reasonable age-profiles. Furthermore, if we are willing to accept slightly higher risk aversion with a very high discount rate, it is possible to match all three statistics simultaneously.

It is important to stress that the literature has tended to neglect the matching of wealth, and tends to match one statistic at a time (participation or shares, but not both) under standard preference assumptions. This is the first time the life-cycle model has

achieved simultaneous matching of asset holdings and allocations, without resorting to heterogeneous preferences or moving away from power utility functions. We are able to achieve this through our innovation of augmenting the model to include per period fixed costs of stock market participation. We demonstrate that matching the observed participation and portfolio shares entails either allowing unrealistic levels of saving or accepting uncomfortably high discount rates. This suggests to us that we have not captured the full extent of background risk effectively. Such extensions are left for future research, but promising avenues include models that incorporate housing risk (Yao and Zhang, 2005) or heterogeneity in expected returns (Alan and Ball, 2007).

Appendix A: Solution and simulation methods

The results presented in Section 4 use standard techniques to solve the model by backwards induction; starting from a terminal condition, in order to obtain the optimal policy functions for each age, mapping the state variables into the controls. Using these functions the model is simulated forward from $t = 1$ with an initial asset level of zero. The model is simulated 10,000 times with ex-ante homogenous individuals who differ, ex-post, due to different shock realisations; and the mean accumulated wealth and asset allocations are computed.

Solving the Euler equations corresponds to the determination of a fixed point within an infinite dimension state space, involving expectations over a non-linear marginal utility function, where the unknown is a function over a continuum of points. Such complexity means that the model cannot be solved analytically, which entails the implementation of numerical techniques. The state space is discretised into a finite number of nodes and interpolated using local approximation methods²⁵.

The grids are defined so as to avoid the need for extrapolation outside the grid²⁶. The

²⁵Four hundred points are used for both the asset and savings grids. Linear splines are used for interpolation.

²⁶Extrapolation is much less reliable than interpolation, especially where the policy functions are non-

concavity of the consumption function leads us to choose a non-uniform spacing of the asset nodes. Extra points are positioned close to the lower bound, where the consumption policy function displays a significant amount of curvature. The nodes are more spread out at high asset levels, at which point the functions become approximately linear. The savings grid is also non-uniformly spaced as the portfolio share policy function is non-linear. More points are positioned around the kink in the policy function, where the short sales constraint ceases to bind, and fewer nodes at high levels of savings, where the policy function becomes horizontal as it approaches the complete markets outcome. The solution is found using NAG routines²⁷, except for when these methods fail to converge, in which case the non-linear system is solved using a bisection method²⁸.

We perform all numerical integration using Gaussian quadrature to approximate the distributions of labour income and the risky asset. The income shocks are discretised into six values and the risky return uses three point quadrature. In the simulations, the permanent shock to labour income is approximated as a continuous random variable. Each time period is taken as a year of life. T was taken to be 58 years and K as 15 years, giving a working life of 43 years (from age 22 to 65) and life coming to an end at 80.

A check on the accuracy of the solution method is undertaken by computing the realised values of the Euler equations. When averaged across individuals, these results do not deviate significantly from their expected values.

Appendix B: Portfolio share policy functions

Figure 6 shows the portfolio share policy functions for three different ages, under parameters outlined in Table I. At each age the policy function follows a highly nonlinear pattern. At low levels of saving, the short sales constraint is binding and agents hold all of their

linear.

²⁷Fortran code is available from the author on request.

²⁸Bisection is an iterative procedure that computes the root of a one-dimensional function on a bounded interval of the real line. It is one of the most robust procedures but it converges slowly, hence, it is only used when the NAG routine fails.

wealth in risky assets; represented by the horizontal section of the function. This derives from non-tradable labour (or pension) income acting as a substitute for the risk free asset²⁹. At low levels of saving the agent is highly endowed with this implicit risk free asset, driving portfolio shares to 100%. As savings are increased, income becomes a relatively less important fraction of wealth, giving low implicit risk free asset holdings and allowing more diversification. At high levels of saving, income becomes a relatively insignificant determinant of wealth and portfolio shares tend to the complete market solution³⁰.

[INSERT FIGURE 6 ABOUT HERE]

As agents age, the policy functions in Figure 6 shift inwards, enabling more portfolio diversification for a given level of saving. The young have a high present discounted value of lifetime income, and this represents a large implicit holding of riskless assets, resulting in high portfolio shares. As they grow older, the proportion of total lifetime wealth accounted for by the present discount value of income declines, leading to lower implicit holdings of riskless assets and a tilting of portfolios away from equities.

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²⁹Income is risky but bounded below, and hence reduces effective risk tolerance.

³⁰Under these parameter values the Samuelson rule gives a complete markets portfolio share of 0.5.

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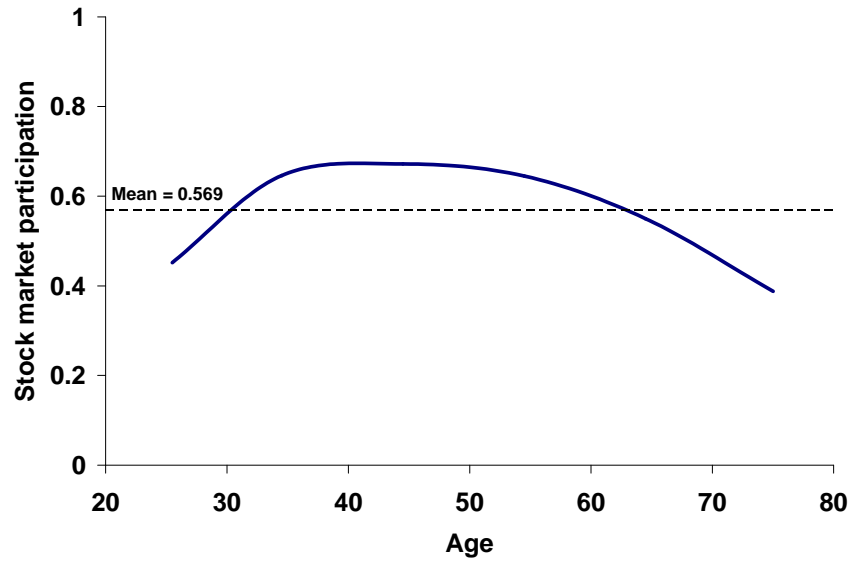
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Parameter	Value
CRRA (γ)	2
Riskless rate (r)	1.6
Mean risky return (r^{eq})	5.6
Variance of risky return	0.04
Variance of permanent shock (σ_n^2)	0.031
Variance of transitory shock (σ_u^2)	0.030
Proportional tax (τ)	0.2

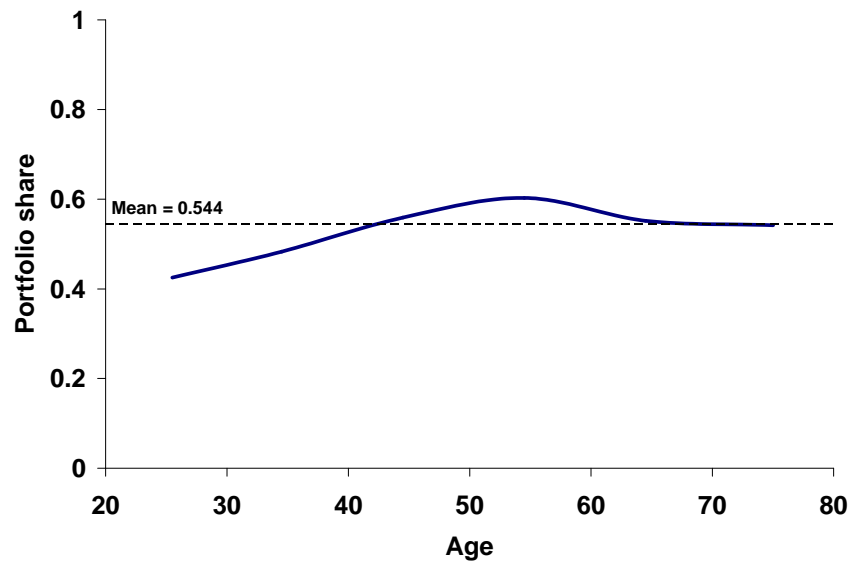
Table I: Exogenous parameters

Scenario	Simulated statistics			Parameters	
	$\frac{\text{Median}[A_{i,t}]}{\text{Median}[y_{i,t}]}$	Mean participation	Mean shares	$1+\delta$	Fixed costs
US Data	1.84	0.57	0.54	-	-
<i>No FC</i> tax = 0.2	1.84	0.95	0.98	1.086	-
<i>Calibrated FC</i> tax = 0.2	1.84	0.57	0.97	1.083	0.050
<i>Keeping</i> tax = 0.2	1.84	0.57	0.97	1.083	0.050
<i>savings</i> tax = 0.1	1.84	0.57	0.90	1.090	0.063
<i>constant</i> tax = 0.0	1.84	0.57	0.82	1.099	0.064
<i>Keeping</i> tax = 0.2	1.84	0.57	0.97	1.083	0.050
<i>discount rate</i> tax = 0.1	2.33	0.57	0.89	1.083	0.073
<i>constant</i> tax = 0.0	2.86	0.57	0.82	1.083	0.085
$\gamma = 4.0$ tax = 0.2	5.76	0.57	0.54	1.083	0.065
$\gamma = 6.4$ tax = 0.2	1.84	0.57	0.54	1.491	0.017

Table II: Calibrated parameters and statistics

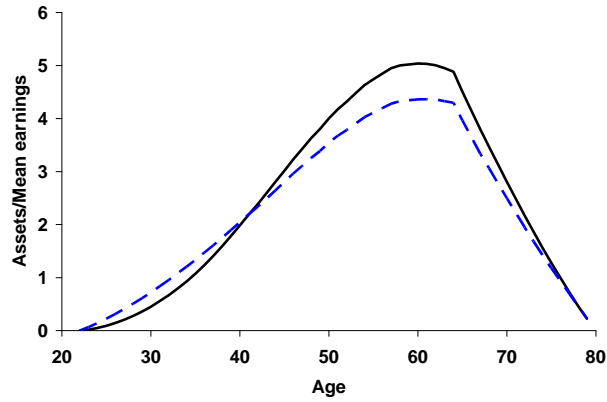


(a)

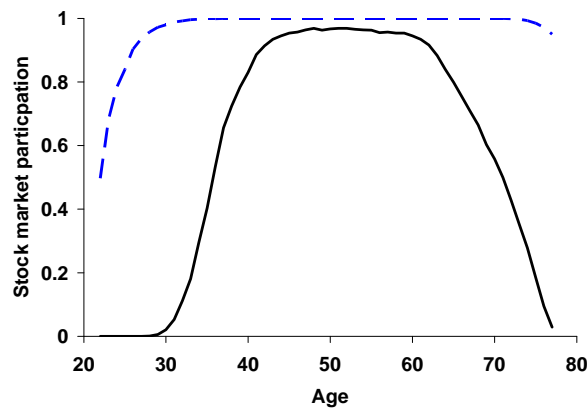


(b)

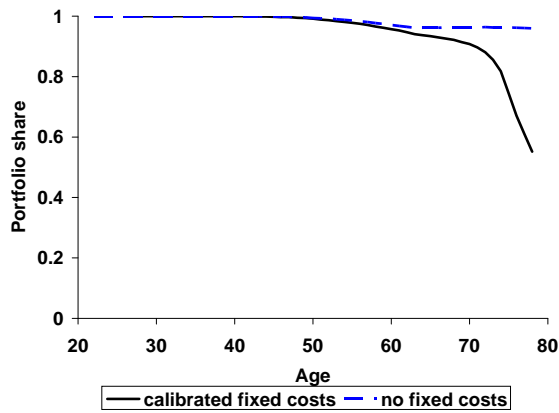
Figure 1. Asset allocation profiles in the Survey of Consumer Finances. Graph (a) shows the share of households holding risky assets. Graph (b) shows the portfolio shares conditional on stock market participation. The results for both graphs are taken from Bertaut and Starr-McCluer (2002), using weighted data from 1998 SCF. Risky assets include directly held stock; stock held through mutual funds, retirement accounts, trusts and other managed assets; corporate, foreign and mortgage-backed bonds; business equity; and investment real estate.



(a)

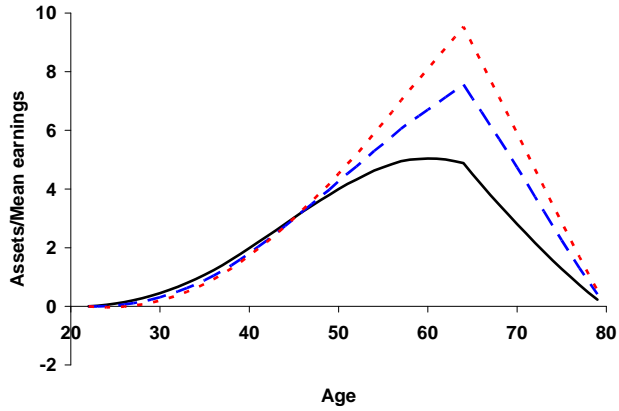


(b)

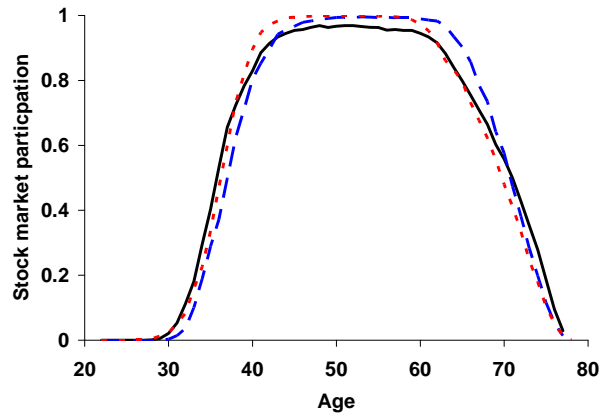


(c)

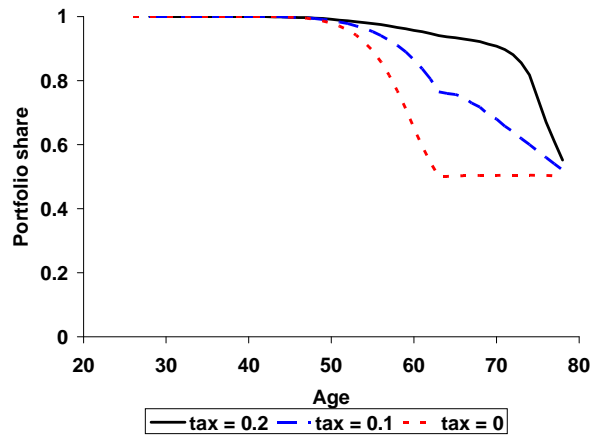
Figure 2. The effect of introducing per period fixed costs. Graph (a) shows mean asset holdings divided by mean earnings (averaged across individuals of working age). Graph (b) shows stock market participation by age. Graph (c) shows conditional portfolio shares at each age. The discount rate is calibrated separately in each scenario such that median assets match the data. For the scenario with fixed costs (represented by the solid line in the graphs), the costs are calibrated to match mean participation.



(a)

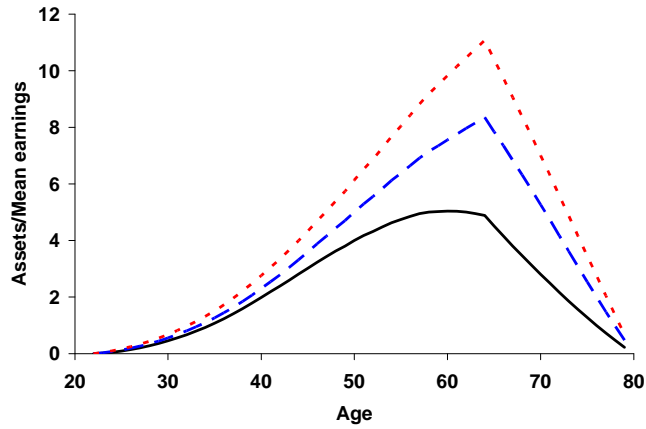


(b)

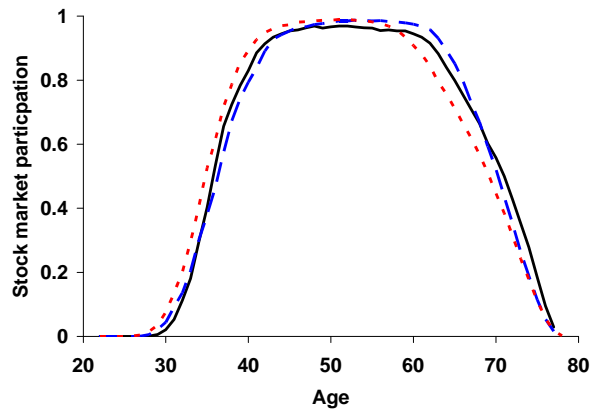


(c)

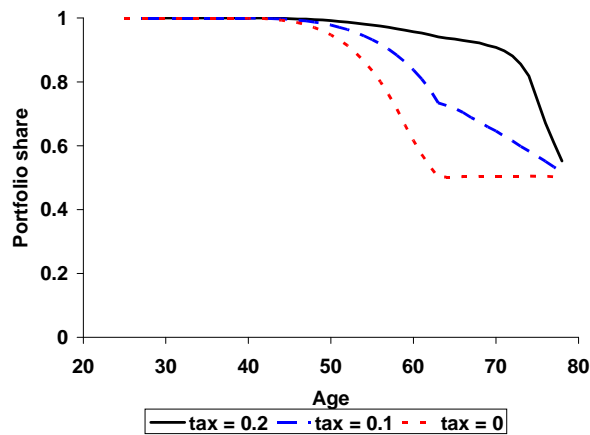
Figure 3. The effect of varying the generosity of public pension provision, keeping savings constant. Graph (a) shows mean asset holdings divided by mean earnings (averaged across individuals of working age). Graph (b) shows stock market participation by age. Graph (c) shows conditional portfolio shares at each age. The fixed costs are calibrated separately to match mean participation. Discount rates are calibrated separately to match asset holdings.



(a)

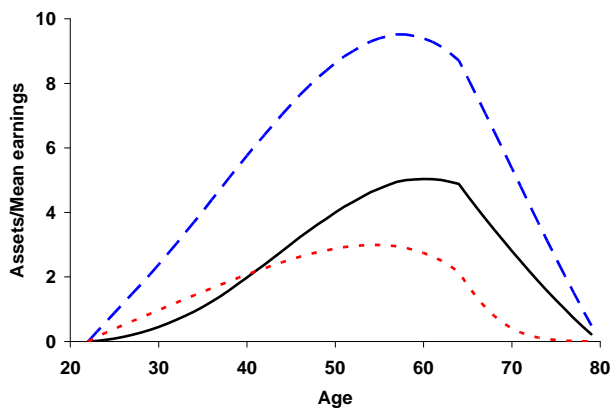


(b)

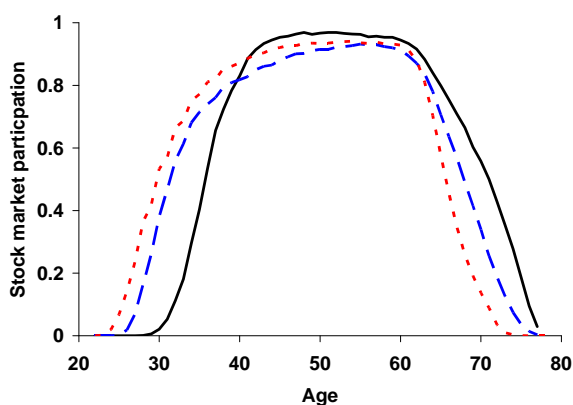


(c)

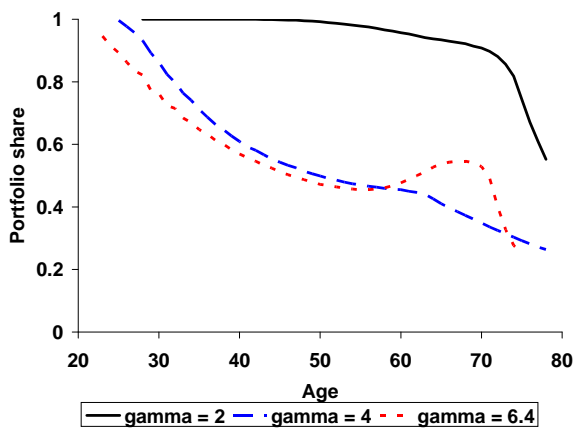
Figure 4. The effect of varying the generosity of public pension provision, keeping the discount rate constant (calibrated to the 20% tax rate). Graph (a) shows mean asset holdings divided by mean earnings (averaged across individuals of working age). Graph (b) shows stock market participation by age. Graph (c) shows conditional portfolio shares at each age. The fixed costs are calibrated separately to match mean participation.



(a)



(b)



(c)

Figure 5. The effect of varying the coefficient of risk aversion. Graph (a) shows mean asset holdings divided by mean earnings (averaged across individuals of working age). Graph (b) shows stock market participation by age. Graph (c) shows conditional portfolio shares at each age. The fixed costs are calibrated separately to match mean participation. In the scenarios represented by the solid line and the dotted line, the discount rate is calibrated in each to match asset holdings. In the scenarios represented by the dashed and dotted lines the risk aversion is calibrated to match mean portfolio shares.

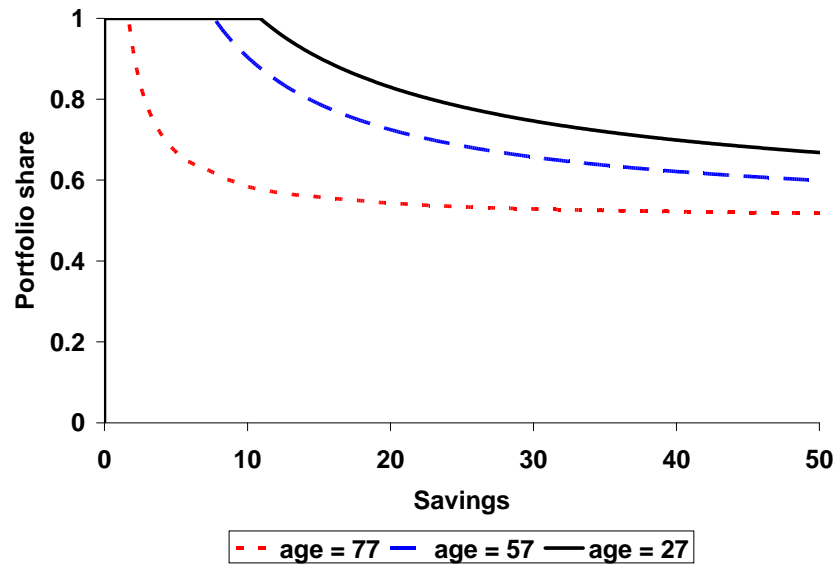


Figure. 6. Portfolio share policy functions for three different ages, under baseline parameters.