CARBON DIFFUSION ACROSS DISSIMILAR STEEL WELDS

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For Grandpa

If you can dream - and not make dreams your master
If you can think - and not make thoughts your aim:

Rudyard Kipling
PREFACE

This dissertation is submitted for the Degree of Doctor of Philosophy in Natural Sciences at the University of Cambridge. The investigation described in this thesis was carried out under the supervision of Dr. H. K. D. H. Bhadeshia at the Department of Materials Science and Metallurgy, Cambridge, between October 1989 and October 1992. Except where acknowledgement and reference to previous work is made, this work is original and carried out without collaboration. Neither this, nor any substantially similar dissertation has been or is being submitted for any degree, diploma or other qualification at any other university.

This dissertation contains less than 60,000 words.

Julia Race
October 1992
ACKNOWLEDGEMENTS

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The project was supervised enthusiastically by Dr. Harry Bhadeshia, to whom I am extremely grateful for his encouragement and advice. My thanks and best wishes go to the rest of the PT group with whom it has been a pleasure to work.

Finally, I would like to thank my parents and family for their constant love and support.
ABSTRACT

Dissimilar steel welds are used extensively in the power generation industry in boiler tubing to accommodate changing temperature, pressure and corrosive conditions in different locations. Joints like these are prone to failure and can cause steam leakage resulting in expensive shutdowns. A principle cause of failure is the migration of carbon, in general, from the low alloy side to the high alloy side of the weld. This leaves an area on one side of the weld denuded of carbon and therefore weaker in terms of stress rupture. The other side is correspondingly enriched and there may be further detrimental effects on the mechanical properties as a consequence. The partitioning of carbon occurs even when both sides of the weld have identical carbon concentrations because the driving force for its migration is the difference in carbon chemical potential as caused by the variation in the substitutional solute content across the junction.

Mathematical models have been produced in this work to model the diffusion of carbon across dissimilar metal welds in both the austenite, and ferrite plus carbide phase fields. Carbon diffusion profiles across joints in the austenite phase field can be accurately predicted by the combination of the equations to describe diffusion. The only limitation to this is the accurate prediction of the partition coefficient in concentrated solutions. However, this method could not be extended to the ferrite phase field. Experimental results from dissimilar metal joints heat treated in the ferrite phase field indicated that the dissolution and precipitation of carbides is instrumental in providing a 'source' and 'sink' for the diffusion carbon, a fact which could not be accounted for in any previous model. The present model is based on a series of discrete carbide particles on either side of the weld interface which dissolve and grow, thereby simulating the dissolution and precipitation of carbides. The real situation is further complicated by the fact that the experimental measurements of the decarburised zone widths do not exhibit parabolic kinetics as is normally assumed but tend to level off on a plot against root time. Furthermore, the influx of carbon into the high alloy side of the weld changes the precipitation sequences that occur on that side of the weld. It has been shown that these changes in precipitation sequences are significant in causing a reduction in the rate of decarburisation. This has been successfully incorporated into the model allowing the prediction of decarburised zone widths and relative comparisons of the extent of decarburisation to be made between different alloy systems.
### Nomenclature List.

- $a_c$: Activity of carbon
- $a_\alpha^0$: Activity of carbon at $\alpha/\theta$ interface
- $a_\theta^\alpha$: Activity of carbon at $\theta/\alpha$ interface
- $A_1$: A constant
- $A_2$: A constant
- $A_3$: A constant
- $A_4$: A constant
- $B$: A constant
- $B_1$: A constant
- $B_2$: A constant
- $C_1$: A constant
- $C_{Bi}$: Concentration of B atoms on plane $i$
- $d_{0g}$: Original particle size on the high carbon activity side of the interface
- $D$: Average integrated diffusivity
- $D_B$: Diffusion coefficient of B atoms
- $D_{ik}^{O-T-O}$: Diffusion coefficient of element $i$ under the influence of a gradient in element $k$
- $D_{i-O-T}$: Diffusion coefficient for carbon atoms jumping from an octahedral to a tetrahedral site via a tetrahedral site
- $D_{T-O-T}$: Diffusion coefficient for carbon atoms jumping from a tetrahedral to a tetrahedral site via an octahedral site
- $D_{T-T}$: Diffusion coefficient for carbon atoms jumping from a tetrahedral to a tetrahedral site
- $D_a$: Diffusion coefficient of carbon in $\alpha$
- $D_\beta$: Diffusion coefficient of carbon in $\beta$
- $D_\theta$: Diffusion coefficient of carbon in $\theta$
- $D_1$: Slab size on the high carbon activity side of the interface
- $D_2$: Particle size on the high carbon activity side
- $D_3$: Interparticle spacing on the high carbon activity side
- $D_4$: Particle size on the low carbon activity side of the interface
- $D_5$: Slab size on the low carbon activity side
- $D_6$: Interparticle spacing on the low carbon activity side
- $E_T^u$: Change in Gibbs free energy of pure solvent when a solute atom is inserted in a tetrahedral site
- $E_Q^u$: Change in Gibbs free energy of pure solvent when a solute atom is inserted in an octahedral site
- $f$: Fraction of remaining atoms which jump via the T-T route
- $G_1$: Free energy of the B-rich alloy
- $G_2$: Free energy of the A-rich alloy
- $G_3$: Initial free energy of the alloy block
- $G_4$: Free energy of the homogeneous A-B alloy
- $h$: Planck’s constant
- $J_B$: Flux of B atoms
- $J_i$: Flux of element $i$
- $\vec{J}$: Flux of B atoms from left to right
- $\vec{J}$: Flux of B atoms from right to left
- $k$: Partition coefficient
- $k_B$: Boltzmann’s constant
- $n_i$: Number of B atoms on plane $i$ /m$^2$
\( R \)  \quad \text{Universal gas constant}

\( S_u^T \)  \quad \text{Change in vibrational entropy of pure solvent when a solute atom is inserted in a tetrahedral site}

\( S_u^O \)  \quad \text{Change in vibrational entropy of pure solvent when a solute atom is inserted in an octahedral site}

\( T \)  \quad \text{Temperature}

\( t \)  \quad \text{Time in seconds}

\( V_i^A \)  \quad \text{Rate of change of} \ x_i^A

\( \nu_\alpha \)  \quad \text{Volume fraction of carbide A}

\( \nu_\beta \)  \quad \text{Volume fraction of carbide B}

\( x_k \)  \quad \text{Concentration of element} \ k

\( x_{A_o} \)  \quad \text{Carbon concentration in the carbide A on the high carbon activity side in equilibrium with the ferrite}

\( x_{B_\beta} \)  \quad \text{Carbon concentration in the carbide B on the low carbon activity side}

\( x_{B_2\beta} \)  \quad \text{Carbon concentration in the carbide B2 on the low carbon activity side in the second slice in equilibrium with the ferrite}

\( x^\alpha \)  \quad \text{Carbon concentration in} \ \alpha

\( x^\theta \)  \quad \text{Carbon concentration in} \ \theta

\( \overline{x} \)  \quad \text{Average carbon concentration of the alloy}

\( \overline{x}_\alpha \)  \quad \text{Average carbon concentration on the} \ \alpha \ \text{side of the interface}

\( \overline{x}_\beta \)  \quad \text{Average carbon concentration on the} \ \beta \ \text{side of the interface}

\( x^{\alpha\alpha}_\text{Fe_3C}/\alpha \)  \quad \text{Carbon equilibrium between ferrite and cementite}

\( x^{\alpha\alpha}_\text{C}/\alpha \)  \quad \text{Maximum permissible carbon content in the austenite at the transformation interface}

\( X \)  \quad \text{Atom fraction of carbon atoms}

\( z_o \)  \quad \text{Co-ordination number of an octahedral site in the austenite lattice}

\( z \)  \quad \text{Coordinate normal to the diffusion boundary}

\( z_\alpha \)  \quad \text{Distance between particles on the low carbon activity side of the interface}

\( z_i^A \)  \quad \text{Distance from weld interface to carbide particle after time} \ t \ \text{on the high carbon activity side}

\( z_{i}^A \)  \quad \text{Initial distance from the weld interface to the carbide on the high carbon activity side}

\( z_j^B \)  \quad \text{Distance from weld interface to carbide particle after time} \ t \ \text{on the low carbon activity side}

\( z_{j}^B \)  \quad \text{Initial distance from the weld interface to the carbide on the low carbon activity side}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Carbon concentration in solution on the high carbon activity side of the interface</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>High carbon activity side of the interface</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Low carbon activity side of the interface for ferritic joints</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>Activity coefficient of the activated complex</td>
</tr>
<tr>
<td>$\Delta c$</td>
<td>Amount of carbon transferred from slice 1 to slice 2 on the low carbon activity side of the weld</td>
</tr>
<tr>
<td>$\Delta^0 G_i^{\gamma\rightarrow\theta}$</td>
<td>Molar Gibbs free energy of pure carbon for the change in crystal structure $\gamma \rightarrow \text{graphite}$</td>
</tr>
<tr>
<td>$\Delta F^*$</td>
<td>Activation free energy</td>
</tr>
<tr>
<td>$\Delta x_1$</td>
<td>Difference in carbon concentration across the interface</td>
</tr>
<tr>
<td>$\Delta x_2$</td>
<td>Difference in alloy concentration across the interface</td>
</tr>
<tr>
<td>$e_{ij}$</td>
<td>Wagner interaction coefficient between element $i$ and $j$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Distance between {002} austenite planes</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Separation of planes of atoms</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Chemical potential</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>Chemical potential of the standard state</td>
</tr>
<tr>
<td>$\mu_{\alpha\theta}$</td>
<td>Chemical potential of carbon at $\alpha/\theta$ interface</td>
</tr>
<tr>
<td>$\mu_{\theta\alpha}$</td>
<td>Chemical potential of carbon at $\theta/\alpha$ interface</td>
</tr>
<tr>
<td>$\Gamma_B$</td>
<td>Jump rate of interstitial atoms</td>
</tr>
<tr>
<td>$\Gamma_i$</td>
<td>Activity coefficient of element $i$</td>
</tr>
<tr>
<td>$\Gamma_\alpha$</td>
<td>Activity coefficient of carbon on the $\alpha$ side of the weld</td>
</tr>
<tr>
<td>$\Gamma_\theta$</td>
<td>Activity coefficient of carbon on the $\theta$ side of the weld</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Low carbon activity side of the interface for austenitic joints</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Atom fraction of carbon atoms</td>
</tr>
<tr>
<td>$\eta^\alpha$</td>
<td>Decarburised zone width</td>
</tr>
<tr>
<td>$\eta^\theta$</td>
<td>Carburised zone width</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Dimensionless parameter</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Atom fraction of carbon atoms</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Fraction of the total interstitial atoms in the octahedral sites</td>
</tr>
</tbody>
</table>
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CHAPTER 1

Introduction

1.1 The Power Station

The operation of a turbine-generator plant is independent of the fuel source used to create the steam and works along the following basic principle. Water is converted to steam in the evaporator and is passed to the steam drum. The steam from the drum is collected in headers and passed through between one and three stages of superheat before being passed to the high pressure (HP) turbine. From the HP turbine, the steam returns to the reheater to be raised to the original steam temperature and pressure before entering the intermediate pressure (IP) turbine. The steam leaving the IP turbine is then passed through up to three LP turbines depending on the size of the plant. The steam exhausted from the last low pressure stage is condensed and circulated through a feed heater and economiser stage on its way back to the boiler. The basic layout of the steam generation plant is illustrated in Figure 1.1.

![Figure 1.1: Schematic representation of the power plant layout.](image)

1.2 Material considerations

1.2.1 Mechanical properties

The first requirement of any potential material is that it should exhibit adequate mechanical properties when exposed under design conditions for the specified life to the operating...
environment. Any material must therefore have the mechanical properties adequate enough to sustain steady, instantaneous or cyclic stresses when operating at the design temperature and pressure. Fossil fuelled boilers and gas and liquid metal cooled reactors can generate steam at up to 600°C, but more recently temperatures have been standardised at 540°C to produce lower capital costs and greater reliability. In the majority of cases the final steam temperature of both superheat and reheat cycles is 535-570°C. Water tubes rarely exceed 350-400°C, but final stage superheater and reheater components may reach temperatures as high as 650°C.

Creep is therefore an important design consideration for steam tubes operating at temperatures in excess of 400°C. Steam pressures for conventional reheat plant have been standardised at $1.68 \times 10^7$ Pa because this is the highest pressure at which reasonable steam/water separation can be obtained. The large volume of working liquid ($1.26 \times 10^4$ kg/s for a 660 MW, 540°C unit) is conveyed between the boiler and the turbines by about 60m of pipework. These pipes must be large enough to pass this quantity of steam without excessive pressure loss and the wall thickness must be large enough to withstand the bursting forces generated by the pressure. The typical working conditions of a 550MW turbo-generator plant are given in Table 1.1, from which the variety of temperatures and pressures experienced by the plant can be gauged. Ductility is also a very important property which must be considered especially when new, as this affects the fabricability, but also when aged in service as loss of ductility can lead to crack initiation.

<table>
<thead>
<tr>
<th>Evaporation</th>
<th>$1.58 \times 10^6$ kghr$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drum conditions</td>
<td>18.3MPa, 680°C</td>
</tr>
<tr>
<td>Superheater outlet temperature</td>
<td>541°C</td>
</tr>
<tr>
<td>Superheater outlet pressure</td>
<td>16.6MPa</td>
</tr>
<tr>
<td>Reheater inlet temperature</td>
<td>348°C</td>
</tr>
<tr>
<td>Reheater inlet pressure</td>
<td>4.3MPa</td>
</tr>
<tr>
<td>Reheater outlet temperature</td>
<td>541°C</td>
</tr>
<tr>
<td>Reheater outlet pressure</td>
<td>4.1MPa</td>
</tr>
<tr>
<td>Initial feedwater inlet temperature</td>
<td>254°C</td>
</tr>
</tbody>
</table>

Table 1.1: Typical operating conditions for a 550MW power station.

1.2.2 Fabrication

The material must be able to be fabricated by the available equipment to the shape and
properties required. In the area of fabrication and especially in steam plant pipe lines, the weldability of a material is very important. Weldability is a widely used term, but is poorly defined and can probably be best described as:

‘the capacity of a metal or a combination of metals to be welded under the fabrication conditions imposed into a specific, suitably designed structure and to perform satisfactorily in the intended service. The better the weldability, the easier these requirements may be met.’

Gorton (1975)

Weldability is therefore a relative term which depends upon the application of the welded component and the severity of the conditions under which it operates.

1.2.3 Environmental resistance

The environment of a boiler is very harsh. The fireside conditions require material which is resistant to corrosion. In oil fired boilers, the gas is relatively clean but the particles entrained in it contain significant amounts of vanadium and sulphate which are corrosive at high temperatures. All the coal fired boilers use pulverised coal which is mainly low grade containing upto 20% ash. The ash fusion temperature is often relatively low and it contains appreciable amounts of \( \text{SO}_4^{2-}, \text{Cl}^- \) and \( \text{Na}^+ \). The small particles of ash are generally in the fused state in the combustion chamber and can cause corrosion.

In addition to these major requirements, any candidate materials must be readily available and economical.

1.3 Candidate materials

Ferritic steels provide the bulk of construction materials used in power plant because of their low cost, good availability, relatively easy fabrication and mechanical properties at both high and low temperatures. Bainitic steels are, in general, used in the more highly stressed or higher temperature components. Chromium is the main alloying element in these steels with strengthening being provided by the addition of Mo, V, Nb and W. These additions slow down the rate of transformation and combine with carbon to form fine dispersions of carbides which are more stable and therefore precipitate and coarsen more slowly than cementite in bainite. These dispersions increase the resistance of the matrix to deformation at high and low temperatures and can be used to develop steels with high yield and creep strengths.

Austenitic stainless steels are used in conditions of high temperature and severe corrosion. Corrosion resistance is achieved by the higher chromium contents which stabilise the austenite phase field and also combine with oxygen to form a protective oxide layer on the surface.
As has been mentioned previously, typical service conditions in a boiler might involve temperatures in the range 540–680°C, and no particular alloy is suitable to serve over this entire range. Consequently, the steel offering the most economic advantage in a narrower temperature range is used, necessitating a gradual change in alloy chemistry towards more concentrated alloys as the service temperature rises.

1.3.1 High strength low alloy steels

The most commonly used high strength low alloy steels used are the 1CrMo, 2\(\frac{1}{4}\)CrMo and \(\frac{1}{2}\)Cr\(\frac{1}{2}\)Mo\(\frac{1}{4}\)V types. Typical compositions are given in Table 1.2. These are the preferred materials for heavy section tube plates, pressure vessels and structural components, because they enable an economic wall thickness to be used which can withstand the design pressure. 1CrMo is used where the creep strength of C–Mn steel is insufficient but it offers little improvement in oxidation resistance. 2\(\frac{1}{4}\)CrMo is used extensively as a final superheat for power stations operating at 540°C main steam temperature, however, the creep strength is too low for applications operating at temperatures much above this and the stress–to–rupture values begin to fall off rapidly. It is ductile and readily fabricated and welded using electrodes of the same composition. The \(\frac{1}{2}\)Cr\(\frac{1}{2}\)Mo\(\frac{1}{4}\)V steel is one of the family of Cr–Mo–V steels whose applications range from small items such as bolting to larger components such as steam piping and turbine rotors. Vanadium carbide, VC, is the carbide which is precipitating to provide creep strength.

<table>
<thead>
<tr>
<th>Steel</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Mo</th>
<th>Ni</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2\frac{1}{4})CrMo</td>
<td>0.08–0.15</td>
<td>0.15–0.50</td>
<td>0.40–0.70</td>
<td>0.040</td>
<td>0.040</td>
<td>2.00–2.50</td>
<td>0.90–1.20</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1CrMo</td>
<td>0.056</td>
<td>0.40</td>
<td>0.77</td>
<td>0.010</td>
<td>0.012</td>
<td>1.11</td>
<td>0.44</td>
<td>0.035</td>
<td>0.015</td>
</tr>
<tr>
<td>(\frac{1}{2})Cr(\frac{1}{2})Mo(\frac{1}{4})V</td>
<td>0.13</td>
<td>0.23</td>
<td>0.50</td>
<td>0.033</td>
<td>0.055</td>
<td>0.41</td>
<td>0.58</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 1.2: Typical compositions of the commonly used high strength low alloy steels.

1.3.2 High Cr ferritic steels

Increasing the chromium content improves the creep strength and oxidation resistance. The main steel compositions used are 9CrMo and 12CrMo, typical compositions of which are given in Table 1.3. Both of these steels have compositions compatible with a fully transformable microstructure. In order to obtain maximum high temperature properties they are transformed to a fully martensitic structure and then tempered to produce chromium rich carbides.
In Switzerland, high Cr ferritics have been used since 1957 as superheaters and as steam headers since 1975. Their favourable strength in the 500–600°C temperature range permits the use of considerably smaller wall thicknesses than in 21/4CrMo. 9CrMo is marginally superior to 21/4CrMo in resistance to oxidation by combustion products and can be easily welded using 9CrMo or 21/4CrMo electrodes. 12CrMo has the higher creep strength and oxidation resistance of the ferritic steels, however, it does have welding problems in that it requires a high pre-heat and immediate post weld heat treatment.

<table>
<thead>
<tr>
<th>Steel</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Mo</th>
<th>Ni</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>12CrMo</td>
<td>0.20</td>
<td>0.37</td>
<td>0.58</td>
<td>0.035</td>
<td>0.003</td>
<td>11.5</td>
<td>0.92</td>
<td>0.74</td>
<td>0.31</td>
</tr>
<tr>
<td>9CrMo</td>
<td>0.14</td>
<td>0.44</td>
<td>0.45</td>
<td>0.012</td>
<td>0.016</td>
<td>8.25</td>
<td>0.84</td>
<td>–</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Table 1.3: Nominal chemical compositions of 12CrMo and 9CrMo steels.

### 1.4 The use of dissimilar metal welds

If the varying conditions in a power station require the most suitable and economic choice of materials then at some stage joints have to be made between steels of different phase (austenitic–ferritic joints) and/or steels of differing substitutional alloy content (primarily ferritic–ferritic joints). These types of joint will be referred to hereafter as dissimilar metal joints or transition welds. The major problems associated with dissimilar metal welds are the diffusion of carbon across the weld interface and the thermal stresses induced in the joint, both during welding and service, due to the differences in thermal expansion coefficients. These two effects are common to all transition joints (*i.e.* ferritic–ferritic or ferritic–austenitic welds) and are contributing factors in the failure of these joints.

#### 1.4.1 Carbon diffusion in dissimilar metal welds

Carbon diffusion across the weld interface is a significant factor in all dissimilar metal joint failures. The difference in substitutional alloy content on either side of the welded joint results in a gradient of chemical potential† of carbon being set up across the interface. Carbon will therefore diffuse down this chemical potential gradient resulting in one side of the interface being denuded of carbon whilst the other side is correspondingly enriched in carbon. The direction of carbon diffusion will, in general, be from the low chromium to the high chromium side of the interface.

---

† The chemical potential of an atom is its mean free energy in a solution of specified composition.
interface even if the average carbon concentrations on both sides of the interface are equal. The result of this diffusion of carbon is that the low alloy side of the weld adjacent to the interface becomes denuded of carbon, whilst the other side becomes correspondingly enriched leading to the formation of a carburised and decarburised zone on either side of the weld interface.

In some cases, the decarburised zone is characterised by large grains of recrystallised ferrite. The major consequence of carbon diffusion is a loss of strength in the carbon depleted region and an increase in hardness in the carburised region. These zones are immediately adjacent to one another and provide a significant change in properties across a narrow region. Attempts have been made to minimise carbon migration by the use of nickel based filler metals such as Inconel, the composition of which is given in Table 1.4. These have the advantage that they have a thermal expansion coefficient intermediate between that of ferrite and austenite thereby reducing the thermal stresses across the interface, and they also reduce the carbon activity gradient between ferrite and austenite. The diffusivity of carbon in Ni-based filler is lower than in austenitic stainless steel so the extent of carbon migration is reduced. Ryder et al. (1986) suggest that the use of Ni-based fillers can effect an improvement in the life of dissimilar metal joints of up to five times over that of austenitic fillers.

<table>
<thead>
<tr>
<th></th>
<th>Composition/wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>C</td>
</tr>
<tr>
<td>Inconel 132</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 1.4: Composition of Inconel 132

1.4.2 Stresses acting on the joints

By their very nature, dissimilar metal welds are under stress as soon as they are put into service. The welding together of two different metals with different coefficients of thermal expansion results in stresses being set up in the joint which cannot be adequately removed during post weld heat treatment. Further problems arise during service where cyclic conditions of temperature during start up and shut down procedures mean that tangential, longitudinal and radial components of stress are created in the joint due to this difference in thermal expansion coefficients. The pipework in general is also subject to external bending stresses, internal pressures and vibrational loadings which serve to enhance the stressed state of the weld.

1.5 Mechanism of dissimilar metal joint failure

Most of the failure analysis on dissimilar metal welds has been done between austenitic—
ferritic welds or between ferritic/Ni-based filler welds. Although there is some debate in the literature as to whether a universal mode of fracture can be determined for all dissimilar weld joints, three characteristics of failure emerge.

1.5.1 Formation of an oxide notch

In many joints between ferritic and austenitic welds an oxide notch is formed in the ferritic material close to the weld interface. Various authors postulate as to why these notches are formed. Viswanathan et al. (1985) have suggested that notch formation could be due to a stress concentration arising from the strength discontinuity at the interface under cyclic load conditions. Alternatively, they propose that general oxidation of the ferritic steel occurs because of the difference in bulk chromium concentration between the ferritic and austenitic steels. Close to the interface, local stresses cause the oxide to crack exposing fresh metal and allowing further oxidation to occur. Once the notch has formed, by either initiation method, it fills with oxide which has a greater volume than the metal consumed. This causes a stress concentration at the tip of the notch which leads to crack propagation. Klueh and King (1982), however, suggest that it is the loss of chromium from the ferritic matrix by the precipitation of chromium carbides, and not the difference in chromium content between the ferritic and austenitic sides of the weld that causes an oxide notch to form. They found that high chromium particles form in the grain boundaries parallel to the fusion line, thereby depleting the surrounding matrix of chromium, and rendering these areas more susceptible to oxidation. This leads to the formation of an oxide notch at the external surface which promotes crack nucleation and propagation. Crack propagation is therefore in the ferritic side of the weld interface and is associated with the carbon depleted zone.

1.5.2 Prior austenite grain boundary cracking

This mode of failure is common when the filler metal is stainless steel. Creep cavitation occurs adjacent to carbides at the prior austenite grain boundaries of the ferrite. This mode of cracking is associated with a carbon depleted zone, but does not occur in joints were gross decarburisation has occurred resulting in the formation of a soft ferrite zone. The creep cavities are orientated relative to the fusion line in a manner indicative of shear stresses resulting from differences in thermal expansion coefficients.

1.5.3 Weld interface cracking

This mode of failure is commonly observed with Ni-based fillers. During service a planar array of globular carbides develops along the fusion line and cracking occurs along this array. Again, this failure is associated with a decarburised zone in the ferrite.
1.6 Techniques to minimise failure

Although there is some contention in the literature as to the exact cause and mode of failure, the various authors all recognise the effects of carbon migration and stress in the failure of transition joints. Consequently, most of the effects in minimising failures have been concentrated around these two areas.

1.6.1 Reduction of carbon migration

The use of Ni fillers has already been discussed earlier in this chapter. Another technique is to use a buttering layer. This is where a layer of a stabilising alloy, such as a Ni based alloy is ‘buttered’ onto the weld preparation before the weld metal is laid down to provide a barrier to diffusion. Stabilisation techniques are also used on the ferritic side of a ferritic–austenitic weld by the addition of strong carbide formers such as Cr to minimise carbon migration. Another method which has become of technological interest is that of using graded transition joints by hot isostatic pressing of powders. This results in a smoother transition from the high alloy to the low alloy side of the weld and therefore a reduction in the severity of the carbon activity gradient.

1.6.2 Reduction of stress

The reduction of thermal induced stresses can be achieved by a closer match of expansion coefficients in the transition joint components. This can be accomplished by the use of Ni based filler metals, or again by graded transition joints which spread-out the coefficient of thermal expansion mismatch. External stresses due to bending and vibrational loadings can also be minimised.

1.7 Conclusions

Although dissimilar metal welds are necessary in components such as boiler tubing which experience a range of temperatures and pressures, there are inherent problems in their use. Failure of these joints is largely due to the stresses acting on the joint, both internal and external, and to the migration of carbon across the interface. The risk of failure of dissimilar metal joints is reduced by minimising either one of these causes.
CHAPTER 2

Physical Metallurgy of Chromium Steels

2.1 Introduction

During welding, post weld heat treatment and subsequent service, the dissimilar metal weld is subjected to a variety of temperatures and temperature cycles which have an effect on the resultant microstructure. In general, the weld microstructure will consist of allotriomorphic ferrite, bainite and martensite in varying quantities depending on alloy chemistry. The microstructure will then be tempered during post weld heat treatment and during service at the operating temperature. It is therefore important to understand the physical metallurgy and specifically, the tempering characteristics of the steels under investigation.

2.2 Tempering Characteristics

The tempering of martensite in plain carbon and low alloy steels, in general, can be divided into three well-defined but overlapping stages:

1. Immediately after quenching, the microstructure of the steel will consist of untempered martensite and retained austenite. If the quenched steel is then heated at temperatures up to 200°C metastable $\epsilon$ is precipitated from the martensite. During the precipitation there is a corresponding reduction in the carbon concentration of the martensite whose lattice contracts and becomes less tetragonal.

2. At temperatures between 200 and 315 °C, the retained austenite decomposes to ferrite and carbides. This transformation, which is only important in medium or high carbon steels where there is an appreciable amount of retained austenite present, tends to decrease the volume of the steel.

3. The third stage is marked by the precipitation of Fe$_3$C at the expense of the $\epsilon$-carbide platelets.

In the presence of strong carbide formers, the early stages of tempering are identical to those of a plain carbon steel, but at temperatures above 500°C the cementite is replaced by more stable alloy carbides. It has been suggested that this could constitute a fourth tempering stage. The exact mechanism by which the alloy carbide precipitates is dependent on the particular solute element present. In low chromium steels (<3 wt%), the Fe$_3$C transforms to Cr$_7$C$_3$ by an ‘in-situ’ mechanism i.e. the Cr$_7$C$_3$ is regarded as precipitating within the boundary of the initial Fe$_3$C. In higher chromium steels the Cr$_7$C$_3$ is nucleated separately. It is these
carbides which give rise to secondary hardening and account for the good creep resistance of the steels. In alloys containing a number of carbide forming elements *e.g.* chromium, molybdenum, vanadium, tungsten and niobium, a sequence of alloy carbides may be formed, giving rise to the more complex $\text{M}_{23}\text{C}_6$ and $\text{M}_6\text{C}$ carbides at higher tempering temperatures. In this notation “M” indicates a mixture of metal atoms. Nutting (1969) summarises the sequence of carbides as follows:-

*Chromium steels*

$$\text{Fe}_3\text{C} \rightarrow \text{Cr}_7\text{C}_3 \rightarrow \text{M}_{23}\text{C}_6$$

*Molybdenum steels*

$$\text{Fe}_3\text{C} \rightarrow \text{Mo}_2\text{C} \rightarrow \text{M}_6\text{C}$$

*Vanadium steels*

$$\text{Fe}_3\text{C} \rightarrow \text{V}_4\text{C}_3 \rightarrow \text{M}_6\text{C}$$

In comparison with martensite, bainite grows at relatively higher temperatures so some recovery of the microstructure can occur during transformation. Most of the carbon in ordinary bainitic steels is tied up in the form of cementite precipitates which tend to be coarser than those carbides found in tempered martensite. Consequently, the kinetics of secondary hardening reactions for bainite are found to be slower than those for martensite because the cementite particles take longer to dissolve. However, the secondary hardening reaction in bainitic steels is no different from that produced in martensite in that the tempering resistance of the steels is increased in both cases.

### 2.3 Structure and morphology of carbides forming in Cr–Mo type steels

#### 2.3.1 Cementite

Cementite is an iron–rich carbide having the orthorhombic structure. Cementite can take a large amount of other elements into solution. Manganese in cementite is limited only by the manganese content of the steel and the partitioning coefficient of the element between the carbide and ferrite. Chromium is highly soluble in cementite and in fact upto one fifth of the iron atoms may be replaced by chromium. Hippsley (1981) has also shown that the chromium content in cementite increases with increasing tempering temperature. Molybdenum will dissolve in Fe$_3$C but only up to a metal-atom ratio of about 0.04.

#### 2.3.2 M$_7$C$_3$

This is a chromium rich carbide which has the trigonal structure of Cr$_7$C$_3$. M$_7$C$_3$ has a very high solubility for iron and manganese and can dissolve molybdenum and vanadium up to metal-
atom ratios of 0.055 and 0.16 respectively. The composition of $M_7C_3$ remains constant during tempering and, according to Pilling and Ridley (1982), is independent of carbon concentration.

2.3.3 $M_{23}C_6$

The composition of this carbide is dependent on the alloy content of the steel. In steels that are Mo and W free, it occurs as a Cr rich carbide having the face centred cubic structure $Cr_{23}C_6$ according to Woodhead and Quarrell (1965). In the total absence of Cr, in steels containing Mo the composition is $Fe_{21}Mo_2C_6$. Therefore, in steels containing chromium and molybdenum, the actual composition can vary between these two limits. Dyson and Andrews (1969) have shown the composition to be nearer $(FeCr)_{21}Mo_2C_6$ in chromium containing steels. Beech and Warrington (1966) have shown that the transformation to $M_{23}C_6$ occurs by separate nucleation and growth in the matrix.

2.3.4 $M_6C$

$M_6C$ is a ternary carbide of Fe and Mo. It has a complex cubic structure and compositions in simple ternary systems ranging from $Fe_4Mo_3C$ to $Fe_3Mo_3C$, but it shows appreciable solubility for other elements, taking into solution chromium and vanadium up to metal–atom ratios of 0.3 and 0.1 respectively. Pilling and Ridley (1982) found that $M_6C$ was enriched in Si which Yu (1989) also found partitioned to $M_6C$ thereby stabilising the carbide and increasing its nucleation rate.

2.3.5 $Mo_2C$

$Mo_2C$ has the face centered cubic structure and shows considerable solubility for chromium and vanadium. This carbide forms by separate nucleation in the ferrite as fine needle shaped clusters. Initially it is coherent with the matrix which confers maximum hardness but as it grows into needles it loses some of its coherency with the matrix. Baker and Nutting (1959) have shown that acicular $Mo_2C$ may also be formed at prior austenite grain boundaries and appears if austenite is retained on cooling. As with $M_6C$, Yu (1989) has shown that the precipitation of $Mo_2C$ is accelerated by the presence of Si.

2.4 Carbide precipitation in 1CrMo steels

The initial microstructure of a 1CrMo steel will depend to some extent on the steel composition and method of production but chiefly on the final heat treatment. Microstructural and precipitation studies have been made on 1CrMo steel which had an initial ferrite/pearlite structure (Toft and Marsden, 1961) and on initial ferrite/bainite or fully bainitic structures (Afrouz et al., 1983).
Toft and Marsden found that the microstructural changes that occur during prolonged ageing of a 1CrMo steel are carbide spherodisation and precipitation. These processes occur separately but proceed together. In the ferrite areas, precipitation and growth of $M_7C_3$ and $Mo_2C$ is evident, whilst in the pearlite areas there is a spherodisation of $Fe_3C$ and the precipitation of $M_7C_3$ and $Mo_2C$. From this work, Toft and Marsden propose the following precipitation sequence in 1CrMo steels containing ferrite and pearlite.

$$Fe_3C \rightarrow Fe_3C + Mo_2C \rightarrow M_7C_3 + Mo_2C \rightarrow M_6C$$

The $M_6C$ in this case is Mo rich but contains Cr and Fe.

In the ferrite/bainite structures after long term tempering, Afrouz et al. found spherodised $Fe_3C$ in the bainite regions and at the ferrite/bainite boundaries. $Mo_2C$ precipitates in the form of fine needles were present in the ferrite grains. In the bainite regions Afrouz et al. suggest a precipitation sequence similar to that found by Baker and Nutting (1959) for $2\frac{1}{4}$CrMo steels. In the ferrite regions the sequence was:

$$Mo_2C \rightarrow M_6C$$

The fully bainitic structure contained $M_7C_3$ and $M_6C$ which was not encountered in the predominantly ferritic specimen. An interesting extension of this work was that Afrouz et al. (1983) have shown that the chemical composition changes that occur in $Fe_3C$ carbides during ageing can be used to estimate the time/temperature exposure of a component.

### 2.5 Carbide precipitation in $2\frac{1}{4}$CrMo steels

The major metallurgical advantages of $2\frac{1}{4}$CrMo steel are that it has a high bainitic hardenability so it will transform to a fully bainitic microstructure over a wide range of temperatures, and that it forms stable alloy carbides conferring high temperature stability to the steel. During the tempering process the morphology, distribution and type of carbide will change depending on the time and temperature at which the steel is held.

The precipitation sequence in a quenched $2\frac{1}{4}$CrMo steel, put forward by Baker and Nutting (1959), can be summarised in the form of Figure 2.1 and the flow diagram:

$$
\begin{align*}
\text{c-carbide} & \rightarrow \text{cementite} \rightarrow + \rightarrow M_{23}C_6 \rightarrow M_6C_3 \\
\text{cementite} & \rightarrow \rightarrow \text{Cr}_7C_3 \\
\end{align*}
$$

Pilling and Ridley (1982) suggest a similar precipitation sequence although they found that the cementite dissolved and was replaced by $Mo_2C$, $M_7C_3$ and $M_{23}C_6$ which all nucleated
Baker and Nutting (1959) found that the Cr$_7$C$_3$ precipitated at or within existing cementite particles probably at the interface between the carbide and the ferrite. The precipitate is usually found to have either a rod-like or small parallelogram morphology within the matrix. The precipitation of M$_{23}$C$_6$ occurs at the grain boundaries separately from M$_7$C$_3$ at temperatures above 650°C. The M$_{23}$C$_6$ grows at the expense of cementite and Mo$_2$C leading to a decrease in the number of particles present. Baker and Nutting (1959) found no evidence that the Cr$_7$C$_3$ dissolved after the formation of M$_{23}$C$_6$ and therefore concluded that the M$_{23}$C$_6$ carbide was based on Fe and Mo as if it was Cr rich, it would be expected to grow at the expense of Cr$_7$C$_3$. By contrast, Pilling and Ridley found the M$_{23}$C$_6$ carbide to be high in Cr and low in Mo as shown by its characteristic X-ray spectra in Figure 2.2. The M$_6$C was initially low in Mo because most it was present in the form of Mo$_2$C, but as tempering progressed the Mo$_2$C dissolved and the carbide composition changed to one containing a large concentration of molybdenum.

M$_{23}$C$_6$ and M$_6$C are believed to be the stable carbides at 975K and the equilibrium microstructure is expected to be one of M$_6$C in a ferrite matrix. Klueh and Leitnaker (1975) have disputed this saying that in a multi-component system such as $2\frac{1}{4}$CrMo steel it is possible that two carbides could coexist at equilibrium, and that only very long term heat treatments would resolve this. It is therefore suggested that the equilibrium structure could be one of M$_{23}$C$_6$ and M$_6$C. Yu (1989) has shown that the nucleation and growth of carbides is sensitive to the presence of minor alloying additions. Thus, in Si-containing steels, the precipitation sequence
Figure 2.2: Characteristic X-ray spectra of carbides precipitating from $2\frac{1}{4}$CrMo steel.

is given as:–

$$\epsilon$$-carbide $\rightarrow$ Fe$_3$C $\rightarrow$ Mo$_2$C $\rightarrow$ M$_6$C $\rightarrow$ M$_7$C$_3$ $\rightarrow$ M$_{23}$C$_6$

Baker and Nutting also studied the precipitation sequences in initially martensitic or ferritic microstructures. With martensite, the sequence was similar to that presented for bainite. The ferrite contained M$_2$C particles formed during its growth which transformed to M$_6$C during ageing. Bhadeshia (1992) suggests that the reason for these differences is due to the nature of the transformation in both cases. Ferrite forms by a reconstructive mechanism where the atomic mobility is such that the precipitation of alloy carbides can occur. The displacive nature of the martensitic and bainitic transformations precludes the formation of alloy carbides during transformation.

2.5.1 Morphology

Pilling and Ridley (1982) suggest that there are four distinct carbide morphologies in specimens tempered at 700°C:–

i) globular precipitates at the prior austenite and lath grain boundaries,

ii) rod like precipitates in the matrix,
iii) clusters of needle shaped precipitates in the matrix, 
iv) parallelogram shaped particles in the matrix.

Mo$_2$C has the distinctive needle shaped morphology, and Pilling and Ridley suggest that the grain boundary carbides are M$_6$C or M$_{23}$C$_6$ and that the rod shaped precipitates are M$_7$C$_3$. However, Balluffi et al. (1951) have shown that M$_{23}$C$_6$ has a spherical shape and Beech and Warrington (1966) have shown that M$_7$C$_3$ and M$_{23}$C$_6$ have similar morphologies. Thus, it can be concluded that the shape of the particle is not a satisfactory guide to identification.

2.6 Tempering characteristics of 9CrMo steel

The composition of 9CrMo is such that it is close to the δ/γ + δ phase boundary during high temperature heat treatment. Therefore, the structure may contain varying proportions of austenite/δ-ferrite and possibly some undissolved carbides depending on the exact alloy composition and austenitising temperature. The equilibrium phase diagram for the Fe–Cr–0.1wt%C system is given in Figure 2.3.

The presence of carbides and δ-ferrite can cause variations in the transformation characteristics of the austenite and the tempering characteristics of the martensite formed. The microstructures obtained as a function of austenitising temperature are summarised by Pickering and Vassiliou (1980) in Table 2.1. This is particularly important in weldments which are subjected to elevated temperatures and rapid cooling and can therefore contain small amounts of δ-ferrite.

<table>
<thead>
<tr>
<th>Treatment temperature /°C</th>
<th>Initial structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>γ + undissolved α + undissolved M$_{23}$C$_6$</td>
</tr>
<tr>
<td>1000</td>
<td>coarser γ + fine undissolved M$_{23}$C$_6$</td>
</tr>
<tr>
<td>1100</td>
<td>coarse grained γ</td>
</tr>
<tr>
<td>1200</td>
<td>γ + 35% δ-ferrite</td>
</tr>
</tbody>
</table>

Table 2.1: Initial microstructures as a function of treatment temperature (Pickering and Vassiliou, 1980).

Sanderson (1977) has carried out studies into the tempering of 9CrMo steel, heat treated in the δ + γ phase field. The normalised microstructure contains lath martensite and auto-tempered M$_7$C precipitates. Auto-tempering is a process occurring during quenching when carbon atoms can redistribute to low energy sites associated with dislocations or vacancies and precipitate as fine carbides. This is a consequence of the relatively high martensite start (M$_s$)
temperature, about 320°C. During the early stages of tempering at 600°C, $M_2X$ needles precipitate at dislocations throughout the matrix, providing some dispersion strengthening and also maintaining a high dislocation density. Tempering proceeds by $M_2X$ precipitate coarsening, decreasing dislocation density and the precipitation of $M_{23}C_6$ at grain and tempered lath martensite boundaries. $M_3C$ is still present in the structure after 8 hours. Sanderson (1977) identified the precipitate $M_2X$ as $Cr_2N$ and $M_{23}C_6$ as $(Fe_5Cr_{18})C_6$. At 700°C no $M_3C$ is observed and the structure is one of coarsened $M_2X$ and $M_{23}C_6$. More severe heat treatments lead to $M_{23}C_6$ precipitate coarsening and the dissolution of $M_2X$.

The need for a ferritic steel of improved yield strength for use in power plant has led to the development of a modified 9CrMo steel, which differs from the standard 9CrMo by virtue of a tighter compositional specification, the addition of small amounts of niobium, vanadium and nitrogen and the use of a higher austenitising temperature. The compositional differences between the standard and modified 9CrMo steels are illustrated in Table 2.2.

![Equilibrium diagram for the Fe-Cr-0.1 wt%C. (Sanderson, 1977)](image-url)
Sikka *et al.* (1983) claim that this modified alloy has a creep strength which exceeds that of standard steels with compositions in the range 2.2-12 wt%CrMo for the temperature range 427-704°C, and can therefore carry higher design stresses. This means that for high temperature components, such as steam headers and pipework, reductions can be made in section size resulting in benefits in terms of material and fabrication requirements and improvement in plant performance.

In the modified 9Cr alloy, the high temperature strength advantage is due to the precipitation of NbC. This fine dispersion of precipitate seems to form in the matrix without regard for dislocations, producing a secondary hardening effect. Panton-Kent (1989) suggests that the higher creep properties arise from the formation of $M_{23}C_6$ containing vanadium.

### 2.7 Tempering characteristics of 12CrMo steel

It was shown in the previous section that in order to achieve a significantly higher creep rupture strength 9CrMo must be modified by employing other alloying additions or the chromium level has to be raised to about 12wt%. If reference is made to Figure 2.3, it can be seen that for 0.1 wt% C, (which is desirable for ease of welding and good ductility), the austenite loop
is closed at 12.5 wt% Cr. It is important to obtain a fully austenitic structure at the solution treatment temperature to ensure a fully martensitic transformation on cooling. The austenitising temperature is therefore very important. A compromise is sought between a temperature adequate enough to ensure dissolution of the carbides but not so high that δ-ferrite forms, which detracts from the strength of the alloy, or austenite grain growth is excessive, which reduces impact properties. The recommended temperature is 1050°C. This alloy is highly hardenable and can transform to martensite on air cooling even in large section size.

In order to maximise the strength on tempering it is necessary to increase the temper resistance by alloying. The most effective alloying additions for temper resistance are the ferrite stabilisers Mo and V. However, these will tend to produce δ-ferrite at the solution temperature so it is necessary to balance the constitution by the addition of austenite stabilisers Ni, Co, Mn and Cu. An additional concern in making alloying additions is not to depress the martensite start temperature ($M_s$) below room temperature. Mo and V increase the tempering resistance by the formation of fine $M_2X$ precipitates at the expense of $M_7C_3$ and the retardation of the transformation to $M_{23}C_6$. $M_2X$ has a close packed hexagonal structure based on $(CrMoV)_2(CN)$ in 12CrMoV steels and in fact, Irvine et al. (1960) have shown it to be $Cr_2C$ in 12Cr steels. The precipitation of $M_2X$ causes a secondary hardening peak in the tempering curve and it is also stable over a large temperature range, thereby increasing the tempering resistance. In the overaged condition, $M_{23}C_6$ coarsens and $M_2X$ is replaced by the more stable carbide.

In work on 12CrMoV steels Irvine et al. (1960) found that an initial tempered structure contained only $Fe_3C$ at 300°C. At higher temperatures $Fe_3C$ dissolved and $M_7C_3$ precipitated both 'in-situ' and by a separate nucleation process as a fine precipitate in the matrix. $M_2X$ precipitated as a fine precipitate within the martensite laths (Murphy and Branch, 1968 and Little et al., 1978). At 500°C, $M_{23}C_6$ was observed at the martensite lath and prior austenite grain boundaries as a coarse precipitate (Murphy and Branch, 1968 and Irvine et al., 1960) which grows at the expense of $M_7C_3$ on further tempering and accounts for the softening of the structure. Murphy and Branch (1968) propose that the equilibrium carbide will be $M_{23}C_6$, a view which is corroborated by Thomson and Bhadeshia (1992). Using thermodynamic calculations they showed that $M_{23}C_6$ is the equilibrium stable carbide between 400°C and 800°C. This carbide precipitates initially with a composition close to that predicted by thermodynamics and no significant change in composition was detected during tempering.

The precipitates found by Park et al. (1980) in a 12CrMoVNb steel can be summarised in Table 2.3. The effect of Nb is to extend the range over which $M_2X$ is stable and thereby increase the temper resistance further.
2.8 Equilibrium thermodynamic calculations

The previous sections have outlined the precipitation studies made experimentally on the steels to be used in this investigation. The precipitation sequences have been calculated, using thermodynamic calculations for the exact compositions of the experimental steels, together with the equilibrium carbide compositions. These calculations were made using MTDATA. This is a computer package developed by the NPL which is capable of performing detailed equilibrium calculations for a wide range of alloys. The program utilizes an optimisation routine for minimising the Gibbs free energy of the system. The stable equilibrium phases can be predicted using data obtained from simple systems and experimental evidence. Thus for a given temperature and pressure or volume, plus initial concentrations of the elements in the system, the amounts of elements within each stable phase can be calculated. A precipitation sequence can be determined by calculating the most stable phase, suppressing its formation and performing the calculation again. In all of these calculations, the components allowed for were Fe, Cr, C, Mn, Mo, V, Si, Ni, S and P and the possible phases were ferrite, Fe$_3$C, Mo$_2$C, M$_7$C$_3$, M$_{23}$C$_6$ and M$_6$C. MTDATA has currently no data for the M$_2$X phase, so this is missing from the precipitation sequences. The precipitation sequences calculated plus the compositions of the phases are presented using the chemical compositions of Table 5.2.

2.8.1 1CrMo

The predicted precipitation sequence for this alloy at 700°C is shown below:

$$
Fe_3C \rightarrow Mo_2C \rightarrow Mo_2C + M_7C_3 \rightarrow M_{23}C_6 \rightarrow M_{23}C_6 + M_6C
$$

In comparison with the precipitation sequence of Toft and Marsden (1961), it can be seen that the equilibrium carbides predicted for this alloy are $M_{23}C_6$ and $M_6C$. However, the composition of the $M_6C$ is as found by Toft and Marsden in that it is Mo rich but contains Fe and Cr. The chemical compositions of the phases are given in Table 2.4.
Table 2.4: Weight fractions of the components within the carbides found in 1CrMo steel.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Weight fraction of component within phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fe</td>
</tr>
<tr>
<td>M₆C</td>
<td>0.288</td>
</tr>
<tr>
<td>M₂₃C₆</td>
<td>0.517</td>
</tr>
<tr>
<td>M₇C₃</td>
<td>0.270</td>
</tr>
<tr>
<td>Mo₂C</td>
<td>0.009</td>
</tr>
<tr>
<td>Fe₃C</td>
<td>0.590</td>
</tr>
</tbody>
</table>

The predicted precipitation sequence at 700°C for this alloy is shown below. This sequence seems to provide evidence for the view of Klueh and Leitnaker (1975) that it is possible for two carbides to exist at equilibrium. However, the calculation seems to provide anomalous results concerning the position of Mo₂C in the precipitation sequence as no other workers have found that M₇C₃ dissolves before Mo₂C precipitates. The chemical compositions of the carbides are given in Table 2.5.

\[
\text{Fe}_3\text{C} \rightarrow \text{M}_7\text{C}_3 \rightarrow \text{Mo}_2\text{C} \rightarrow \text{Mo}_2\text{C} + \text{M}_{23}\text{C}_6 \rightarrow \text{M}_{23}\text{C}_6 + \text{M}_6\text{C}
\]

Table 2.5: Weight fractions of the components within the carbides found in 2\frac{1}{4}CrMo steel.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Weight fraction of component within phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fe</td>
</tr>
<tr>
<td>M₆C</td>
<td>0.279</td>
</tr>
<tr>
<td>M₂₃C₆</td>
<td>0.364</td>
</tr>
<tr>
<td>M₇C₃</td>
<td>0.172</td>
</tr>
<tr>
<td>Mo₂C</td>
<td>0.004</td>
</tr>
<tr>
<td>Fe₃C</td>
<td>0.429</td>
</tr>
</tbody>
</table>

2.8.3 9CrMo

Unfortunately, MTDATA does not allow for the formation of the phase M₂X and therefore the precipitation sequence omits this phase. The calculations show that the carbides formed
are very dependent on the bulk carbon concentration and in this alloy the equilibrium carbide is \( M_7C_3 \) at 730°C and not \( M_{23}C_6 \) as found by Sanderson (1977).

\[
\text{Fe}_3\text{C} \rightarrow \text{Mo}_2\text{C} \rightarrow \text{Mo}_2\text{C} + M_7\text{C}_3
\]

Table 2.6 shows the chemical compositions of carbides formed.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Weight fraction of component within phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fe</td>
</tr>
<tr>
<td>( M_7\text{C}_3 )</td>
<td>0.069</td>
</tr>
<tr>
<td>( \text{Mo}_2\text{C} )</td>
<td>0.001</td>
</tr>
<tr>
<td>( \text{Fe}_3\text{C} )</td>
<td>0.165</td>
</tr>
</tbody>
</table>

**Table 2.6:** Weight fractions of the components within the carbides found in 9CrMo steel.

### 2.8.4 12CrMo

This precipitation sequence shows the prediction of \( M_{23}C_6 \) as the stable carbide at 730°C, a result which is verified by Thomson and Bhadeshia (1992) and Murphy and Branch (1968).

\[
\text{Fe}_3\text{C} \rightarrow M_7\text{C}_3 \rightarrow M_{23}\text{C}_6
\]

Table 2.7 gives the chemical compositions of all the carbides calculated.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Weight fraction of component within phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fe</td>
</tr>
<tr>
<td>( M_{23}\text{C}_6 )</td>
<td>0.119</td>
</tr>
<tr>
<td>( M_7\text{C}_3 )</td>
<td>0.059</td>
</tr>
<tr>
<td>( \text{Fe}_3\text{C} )</td>
<td>0.139</td>
</tr>
</tbody>
</table>

**Table 2.7:** Weight fractions of the components within the carbides found in 12CrMo steel.

### 2.9 Conclusions

The search of the literature has revealed that the tempering characteristics for the chromium steels under investigation is very dependent on the exact composition of the steel,
the treatment prior to tempering, the initial microstructure and the temperature at which tempering occurs. The MTDATA calculations together with the available literature provide a reference base for assessment of the experimental results obtained in the present work.
3.1 Introduction

The diffusion of atoms will always occur in order to produce a decrease in the Gibbs free energy of the system. This concept is illustrated in Figure 3.1. Consider welding two blocks of the same A-B solid solution together and holding them at some temperature in order to allow diffusion to occur. The Gibbs free energy of the system is also illustrated in Figure 3.1. Thus, the B-rich side of the joint has a free energy $G_1$ and the A-rich side a corresponding energy $G_2$. The initial free energy of the block is given by $G_3$, but if diffusion is allowed to occur between the two sides then the free energy will be reduced to $G_4$, the free energy of the homogeneous alloy. Thus in this case, a decrease in free energy is achieved by the diffusion of A and B atoms down the resultant concentration gradient which is also parallel to the chemical potential gradient.

![Figure 3.1: Free energy diagram for 'downhill' diffusion (after Porter and Easterling (1981))](image)

Consider now a homogeneous solid solution of composition $x$ made up of components A and B metals whose free energy diagram is as illustrated in Figure 3.2. The initial Gibbs free energy is given by $G_3$. However, a reduction in free energy in this case is achieved by the development of A-rich and B-rich regions thereby causing an overall decrease in the free energy of the system to $G_4$. Thus, the system is achieving a lowering of its free energy by diffusion up...
a concentration gradient and therefore this type of diffusion is sometimes termed 'uphill' diffusion, even though diffusion is still occurring down a chemical potential gradient. The situation just described is one in which an initially homogeneous solid solution is unstable to infinitesimal composition perturbations. It decomposes by uphill diffusion into an inhomogeneous solution, i.e. it spinodally decomposes. This classical description of uphill diffusion proves that diffusion is driven by chemical potential gradients rather than mere concentration gradients. Thus when a transition joint is made between two steels of equal carbon concentration but different substitutional solute content, the chemical potential of carbon on either side may differ. This is because the substitutional solute content alters the chemical potential of the carbon. Consequently, initially homogeneously distributed carbon will partition across the weld junction along this chemical potential gradient, by uphill diffusion to lead to an inhomogeneous distribution. An alternative but equivalent physical explanation is that the attraction for carbon atoms by the substitutional solute on one side of the weld is larger than on the other.

![Figure 3.2: Free energy diagram for 'uphill' diffusion (after Porter and East-erling (1981)).](image)

**3.2 Mechanisms of diffusion**

There are two methods of diffusion operative in crystal lattices dependent on the site that the diffusing atom occupies in the lattice i.e. whether it is a substitutional or interstitial atom. In the modelling of carbon diffusion in either the ferritic or austenitic phase field, it is the interstitial mechanism which is of interest as the free carbon available for diffusion is found in the interstices of both lattices.
3.2.1 Interstitial diffusion

3.2.1.1 Derivation of Ficks first law

Consider a model consisting of a simple cubic lattice of A atoms in which the interstices between atoms are filled with the interstitial atom B. The B atoms are arranged such that every A atom is surrounded by 6 interstitial sites into which the B atoms can fit without causing any distortion in the A lattice. If a concentration gradient is set up as shown in Figure 3.3 then the B atoms will tend to diffuse by a random walk process (i.e. the direction of each new jump is independent of the direction of the previous jump) to eliminate the gradient.

Figure 3.3: Planes of A atoms and a concentration gradient of interstitial B atoms. (after Porter and Easterling (1981))

If the diffusion of a B atom from plane 1 to plane 2 is considered, assuming that, on average, an interstitial atom jumps $\Gamma_B$ times per second and there are $n_1$ B atoms/m$^2$ that will jump from plane 1 to plane 2, then the flux of B atoms will be given by:

$$J = \frac{1}{6} \Gamma_B n_1$$  \hspace{1cm} (3.1)

However, if each atom can jump in any direction and $\Gamma_B$ is independent of concentration, then there is also a corresponding flux from plane 2 to plane 1 of:

$$J = \frac{1}{6} \Gamma_B n_2$$  \hspace{1cm} (3.2)

Now, because of the concentration gradient, $n_1 > n_2$, and therefore, a net flux from left to right can be defined as:

$$J_B = \frac{1}{6} \Gamma_B (n_1 - n_2)$$  \hspace{1cm} (3.3)
\( n_1 \) and \( n_2 \) can be translated into concentrations if the separation of the planes is taken to be \( v \), then the concentration of B atoms on plane 1 is given by:

\[
C_{B_1} = \frac{n_1}{v} \tag{3.4}
\]

and similarly for plane 2:

\[
C_{B_2} = \frac{n_2}{v} \tag{3.5}
\]

The difference between \( n_1 \) and \( n_2 \) is therefore given by:

\[
n_1 - n_2 = v(C_{B_1} - C_{B_2}) \tag{3.6}
\]

However, with reference to Figure 3.4, it can be seen that \( (C_{B_1} - C_{B_2}) \) can be expressed as:

\[
(C_{B_1} - C_{B_2}) = -v \frac{\partial C_B}{\partial z} \tag{3.7}
\]

Therefore, substitution into (3.3) yields the following expression for the net flux of atoms:

\[
J_B = -\left(\frac{1}{6} \Gamma B v^2\right) \frac{\partial C_B}{\partial z} \tag{3.8}
\]

If a diffusion coefficient is defined such that:

\[
D_B = \frac{1}{6} \Gamma B v^2 \tag{3.9}
\]

then (3.8) reduces to:

\[
J_B = -D_B \frac{\partial C_B}{\partial z} \tag{3.10}
\]

This equation is known as Ficks First Law of Diffusion. The diffusion coefficient defined in equation (3.9) is only strictly applicable when the jumping of B atoms is truly random with a frequency independent of concentration. However, in many crystal systems this is found not to be the case and in fact the diffusion coefficient is dependent on crystal direction (e.g. atoms in hexagonal lattices diffuse at different rates parallel and perpendicular to the basal plane) or concentration. However, Ficks first law is still found to hold if appropriate concentration dependent expressions are substituted for the diffusion coefficient.

### 3.2.2 Ficks Second Law - non steady state diffusion

Ficks first law is only applicable for steady state diffusion conditions, i.e. where the concentration at any point does not change with time. This cannot, therefore, be applied to a system like a transition joint where the concentration at any point is constantly changing with both time and distance i.e. under non-steady state conditions. In order to deal with these conditions
consider the situation illustrated in Figure 3.5 where a concentration gradient exists in the z direction only.

As in the derivation of Ficks first law, the B atoms are still diffusing interstitially down this concentration gradient. In order to calculate how the concentration of B varies with time, consider a thin slice of material of area A and thickness δz as shown in Figure 3.6. In a time δt the number of B atoms that will have diffused into this slice is given as $J_1 A δt$. However,
under non-steady state conditions, the number of atoms that will have diffused out of the slice is given by $J_2 A \delta t$ where $J_1 > J_2$ causing a net increase in the number of B atoms in the slice of:-

$$\delta n_B = (J_1 - J_2) A \delta t$$  (3.11)

and therefore, a net increase in the concentration of B atoms in the slice of:-

$$\delta C_B = \frac{(J_1 - J_2) A \delta t}{A \delta z}$$  (3.12)

Now, since $\delta z$ is small then $J_2$ can be written as:-

$$J_2 = J_1 + \frac{\partial J}{\partial z} \delta z$$  (3.13)

which on substitution into (3.12) and in the limit as $t \to 0$:-

$$\frac{\partial C_B}{\partial t} = -\frac{\partial J_B}{\partial z}$$  (3.14)

Substituting for Ficks first law in the above expression gives:-

$$\frac{\partial C_B}{\partial t} = -\frac{\partial}{\partial z} \left( D_B \frac{\partial C_B}{\partial z} \right)$$  (3.15)

If $D_B$ does not depend on $C_B$, this equation simplifies to:-

$$\frac{\partial C_B}{\partial t} = D_B \frac{\partial^2 C_B}{\partial z^2}$$  (3.16)

which is known as Ficks Second Law and is the basis for many of the models concerning the diffusion of carbon in dissimilar metal welds.

Figure 3.6: Derivation of Ficks Second Law (after Porter and Easterling (1981))
3.3 Carbon diffusion across dissimilar metal joints in the austenite phase field
- review and critique of published work

There have been various analytical models proposed for the modelling of ‘uphill’ diffusion of carbon in austenitic joints using the classical data of Darken (1949). In these experiments steels of virtually the same carbon content, but differing in substitutional solute concentration, were welded together and held at 1050°C for about two weeks. The compositions of the steels welded are shown in Table 3.1.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel A</td>
<td>0.49</td>
<td>0.25</td>
<td>3.80</td>
<td>0.011</td>
<td>0.006</td>
<td>0.31</td>
</tr>
<tr>
<td>Steel B</td>
<td>0.04</td>
<td>0.28</td>
<td>4.78</td>
<td>0.006</td>
<td>0.011</td>
<td>-</td>
</tr>
<tr>
<td>Steel C</td>
<td>0.45</td>
<td>0.88</td>
<td>0.05</td>
<td>0.02</td>
<td>0.008</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.1: Composition of the steels used in the Darken experiment. The carbon and silicon concentrations are highlighted to outline the differences between the steels and the main contributors to the diffusion.

After heat treatment, the specimens were sliced at various distances from the fusion line and analysed for carbon. The profiles thus obtained are given in Figure 3.7.

The aim of any model is to predict carbon concentration profiles such as those above in which the carbon is diffusing down a gradient of chemical potential and up a gradient of concentration. On reviewing the literature, it was apparent that the types of models fell into two categories, those that would predict Darken’s data and those that required the data for fitting equations describing the variation in solute concentration as a function of distance from the weld junction, but assuming that the concentrations at the junction are known from experiments. These two categories of model will now be discussed.

3.3.1 Category 1: Prediction of Darken’s profiles

There are three published models that propose equations for the prediction of carbon concentration profiles under the influence of a chemical potential gradient. In chronological order, these are due to Stark (1980), Agren (1983) and Bhadeshia (1988). However, on studying the methods by which the solutions were obtained it was discovered that all the models start with the equation for the continuity of matter (Fick’s second law) and apply certain boundary conditions to solve this equation. Thus all three models arrive at the same set of equations. This general method will now be explained.
3.3.1.1 Solution of Fick’s second law

The situation of diffusion in a system where two media are present can be treated as a combination of two semi-infinite media in which the surface concentration remains constant. This is shown diagramatically in Figure 3.8 where a diffusion couple is formed between two alloys which vary in substitutional alloy content such that redistribution of carbon will tend to occur from $a$ to $b$.

Fick’s second law can be written as:

$$\frac{\partial x}{\partial t} = D \frac{\partial^2 x}{\partial z^2} \quad (3.17)$$

where $x$ is the concentration of carbon, $z$ is the position, $t$ is time and $D$ is the diffusion coefficient. Onsager (1945-46) proposed a logical extension of Fick’s Law, that the flux of a given element in a multi-component system can be represented as a linear function of the concentration gradients of all other elements (and, of course, of itself), viz.,

$$J_i = -\sum_{k=1}^{n} D_{ik} \frac{\partial x_k}{\partial z}$$

where the $D_{ik}$ are functions of all the concentrations $x_k$. It should be noted here that this applies to concentration independent diffusion coefficients, although the diffusion coefficient of
carbon in austenite is concentration dependent. However, if it is assumed that the composition of Si is time independent, consistent with its relatively slow diffusion rate, then the flux can again be expressed as equation (3.17).

The Laplace transform solutions to this equation are given as:-

\[ \tilde{x}^\alpha = A_1 + B_1 \text{erf} \left\{ \frac{z}{2\sqrt{D_\alpha t}} \right\} \]  
\[ \tilde{x}^\theta = A_2 + B_2 \text{erf} \left\{ \frac{|z|}{2\sqrt{D_\theta t}} \right\} \]  

In this equation the term \( \alpha \) refers to the \( \alpha \) side of the weld and the term \( \theta \) to the \( \theta \) side. \( A \) and \( B \) are constants whose values can be found from the boundary conditions.

3.3.1.2 The boundary conditions

1. Continuity of chemical potential across the interface.

The chemical potential must be continuous across the weld, therefore:-

\[ \mu_{\alpha\theta} = \mu_{\theta\alpha} \]  
\[ \mu = \mu_\omega + RT \ln a_c \]  
so

\[ \mu_\omega + RT \ln a_c^{\omega\theta} = \mu_\omega + RT \ln a_c^{\theta\omega} \]
\(a_c\) is the activity of carbon in austenite, and the superscript \(a\beta\) refers to the \(\alpha/\beta\) boundary. Similarly, the superscript \(\theta\alpha\) refers to the \(\theta/\alpha\) boundary, i.e. when \(z=0\).

\(\mu_\alpha\) is the partial molar free energy of the pure substance in the standard state.

Since the phase on either side of the interface is austenite then the value of the partial molar free energy will be the same, therefore:

\[
RT \ln a_c^{a\beta} = RT \ln a_c^{\theta\alpha}
\]

(3.23)

therefore

\[
a_c^{a\beta} = a_c^{\theta\alpha}
\]

(3.24)

so

\[
x^{a\beta} \Gamma_\alpha = x^{\theta\alpha} \Gamma_\beta
\]

(3.25)

since

\[
a_c = x \Gamma
\]

(3.26)

where \(\Gamma\) is the activity coefficient of carbon. If the partition coefficient is defined as:

\[
k = \frac{\Gamma_\alpha}{\Gamma_\beta}
\]

(3.27)

then

\[
x^{\theta\alpha} = k x^{a\beta}
\]

(3.28)

So applying this condition to (3.18) and (3.19) when \(z=0\) \(A_1 = x^{a\beta}\) and \(A_2 = x^{\theta\alpha}\) but,

\[
k = \frac{x^{\theta\alpha}}{x^{a\beta}} = \frac{A_2}{A_1}
\]

(3.29)

which rearranges as:-

\[
A_2 = k A_1
\]

(3.30)

2. Continuity of flux across the interface.

The flux across the interface must be continuous therefore, there must be no build up of solute at the \(\alpha/\beta\) interface, i.e. the flux leaving \(\alpha\) must be equal to the flux entering \(\beta\) i.e. \(J_\alpha = J_\beta\).

It follows that

\[
D_\alpha \frac{\partial x^{a\beta}}{\partial z} \bigg|_{z=0} = -D_\beta \frac{\partial x^{\theta\alpha}}{\partial z} \bigg|_{z=0}
\]

from Fick’s first law.
Applying this condition to equations (3.18) and (3.19) and differentiating with respect to z:

\[ \frac{2}{\sqrt{\pi}} D_\alpha B_1 \exp\left\{ \frac{-z^2}{4D_\alpha t} \right\} \frac{1}{2\sqrt{D_\alpha t}} = -\frac{2}{\sqrt{\pi}} D_\theta B_2 \exp\left\{ \frac{-z^2}{4D_\theta t} \right\} \frac{1}{2\sqrt{D_\theta t}} \]

(The method for differentiation of the error function is given in Appendix I) Therefore when \( z=0 \)

\[ \frac{D_\alpha B_1}{2\sqrt{D_\alpha t}} = -\frac{D_\theta B_2}{2\sqrt{D_\theta t}} \]  
\[ B_2 = -\sqrt{\frac{D_\alpha}{D_\theta}} B_1 \]  

(3.31)  
(3.32)

If the carbon concentration is not the same on both sides of the weld:-

when \( z=\infty \) \( \text{erf}(\infty)=1 \), \( x^\alpha = x^{\alpha \infty} \) and \( x^\theta = x^{\theta \infty} \) so, from (3.18) and (3.19)

\[ x^{\alpha \infty} = A_1 + B_1 \]  
\[ x^{\theta \infty} = A_2 + B_2 \]  

(3.33)  
(3.34)

substituting from (3.30) and (3.32) into (3.34)

\[ x^{\theta \infty} = k A_1 - \sqrt{\frac{D_\alpha}{D_\theta}} B_1 \]  

(3.35)

rearranging (3.33)

\[ x^{\alpha \infty} - A_1 = B_1 \]

and substituting into (3.35)

\[ x^{\theta \infty} = k A_1 - \sqrt{\frac{D_\alpha}{D_\theta}} (x^{\alpha \infty} - A_1) \]  
\[ x^{\theta \infty} = k A_1 - x^{\alpha \infty} \sqrt{\frac{D_\alpha}{D_\theta}} + \sqrt{\frac{D_\alpha}{D_\theta}} A_1 \]  
\[ A_1 = \frac{x^{\theta \infty} + x^{\alpha \infty} {\sqrt{\frac{D_\alpha}{D_\theta}}}}{k + \sqrt{\frac{D_\alpha}{D_\theta}}} \]  

(3.36)  
(3.37)  
(3.38)

and

\[ B_1 = x^{\alpha \infty} - A_1 \]

therefore (3.18) and (3.19) rearrange as

\[ x^\alpha = A_1 + (x^{\alpha \infty} - A_1) \text{erf}\left\{ \frac{z}{2\sqrt{D_\alpha t}} \right\} \]  
\[ x^\theta = k A_1 + \sqrt{\frac{D_\alpha}{D_\theta}} (x^{\alpha \infty} - A_1) \text{erf}\left\{ \frac{|z|}{2\sqrt{D_\theta t}} \right\} \]  

(3.39)  
(3.40)
From these equations the carbon concentration can be calculated at any position \( z \) if the value of the partition coefficient is known. As Stark points out in his paper the value of the partition coefficient is given simply by the ratio of the activity coefficients (equation (3.27)).

3.3.2 Category 2: Fitting of Darken’s profiles

There are two workers who have published models that will fit Darken’s profiles as long as fitting parameters taken from the data are used. These are Kirkaldy (1957) and Kucera and Stransky (1982). As in the previous case the models are very similar in their derivation of the equations and a general method can again be discussed. The starting point is again Fick’s second law.

Onsager (1945-46) proposed as a logical extension of Fick’s Law, that the fluxes in a multi-component system can be represented as linear functions of the concentration gradients, viz.,

\[
J_i = - \sum_{k=1}^{n} D_{ik} \frac{\partial x_k}{\partial z} \tag{3.41}
\]

where the \( D_{ik} \) are functions of all the \( x_k \).

In a one dimensional system such as the semi-infinite diffusion couple the above equation becomes, for the two independent components:

\[
\frac{\partial x_C}{\partial t} = \frac{\partial D_{11}}{\partial z} \frac{\partial x_C}{\partial z} + \frac{\partial D_{12}}{\partial z} \frac{\partial x_{Si}}{\partial z} \tag{3.42}
\]

and

\[
\frac{\partial x_{Si}}{\partial t} = \frac{\partial D_{21}}{\partial z} \frac{\partial x_C}{\partial z} + \frac{\partial D_{22}}{\partial z} \frac{\partial x_{Si}}{\partial z} \tag{3.43}
\]

where, \( x_C \) is the concentration of carbon and \( x_{Si} \) is the concentration of Si. For this situation the subscript 1 refers to carbon, subscripts greater than 1 refer to metal (in the case of the Darken experiment this metal is silicon), thus \( D_{12} \) for example represents the effect of the gradient of Si concentration on the flux of carbon. For the initial and boundary conditions corresponding to the semi-infinite diffusion couple, the solutions have the general form described by Fujita and Gosting (1956):

\[
x_i = x_i(\lambda) \tag{3.44}
\]

where \( \lambda = \frac{z}{\sqrt{t}} \) and more specifically,

\[
x_C = a \text{ erf} \left( \frac{\lambda}{2\sqrt{u}} \right) + b \text{ erf} \left( \frac{\lambda}{2\sqrt{v}} \right) + c \tag{3.45}
\]
where $a - f$ are functions of $D_{ik}$ and $u$ and $v$ are functions of $D_{ik}$ and the boundary conditions. These equations take a simple form when component 1 is more rapidly diffusing than component 2 as $D_{21} = 0$. Kirkaldy (1957) showed that the equation for the carbon profiles becomes:

$$
\begin{align*}
x_C &= \frac{x^{o} + x^{\infty}}{2} + A[\text{erf}(\frac{\lambda}{2\sqrt{D_{22}}}) - \frac{2A - x^{o} + x^{\infty}}{2A}\text{erf}(\frac{\lambda}{2\sqrt{D_{11}}})]
\end{align*}
$$

where $A = \frac{D_{12}}{D_{11} - D_{12}} \frac{x^{o} - x^{\infty}}{2}$ and is found by fitting the results to the equations. The result of fitting the equations to Darken’s data is shown in Figure 3.9.

![Figure 3.9: Darken’s diffusion profiles after Kirkaldy (1957).](image)

The problems with this model are that:

1.) There is the same carbon concentration in the austenite on both sides of the weld at the interface, therefore the activity of carbon cannot be the same, so this is incorrect.

2.) The plot implies uphill diffusion within a single phase. There is no justification for this.

In some later work, Kirkaldy (1958) interpreted Darken’s results as showing a sharp discontinuity at the boundary, then:

$$
D_{11}D_{22} \approx D_{21}D_{12}
$$

must apply. The fit in this case was found to be much better (Figure 3.10).
An interesting extension of Kucera and Stransky's work is that they use the Darken data to obtain further thermodynamic data, specifically the interaction coefficients for Si, Cr, Mn and Mo.

3.4 Carbon diffusion across dissimilar metal joints in the ferrite phase field
- review and critique of published work
and is not as readily available for diffusion as was the case in austenite when all the carbon is in the interstitial sites. There have been two major models that address the problems of carbon diffusion in the ferritic phase field, that due to Buchmayer et al. (1989) and that of Christoffel and Curran (1956).

### 3.4.1 Model due to Buchmayer et al. (1989)

Consider that the initial situation is one of a conglomerate phase of cementite and ferrite and carbon diffuses out of this conglomerate phase to form pure ferrite (Figure 3.12). The analytical solution for this situation is given by Jost (1960) after Wagner.

![Figure 3.12: Model for diffusion in a two phase system.](image)

At the surface, \( z = 0 \), the conditions of continuous activity across the interface impose a surface concentration \( x^\alpha \). The diffusion will proceed from the surface and after a time \( t \) will have extended to a distance \( z = \eta^\alpha \), where \( \eta^\alpha \) can be described by the parabolic growth equation as:-

\[
\eta^\alpha = 2\xi \sqrt{Dt}
\]  

(3.49)

where \( \xi \) is a dimensionless parameter.

The diffusion process will be governed by Fick’s second law such that:-

\[
\frac{\partial x}{\partial t} = D \frac{\partial^2 x}{\partial x^2}
\]

(3.50)

Now the boundary conditions for this situation are:-
And when \( z = \eta^\alpha \), the concentration of carbon in pure ferrite is that corresponding to equilibrium between ferrite and cementite, so \( x = x^{Fe_3C/\alpha} \) at \( z = \eta^\alpha \).

When the interface between the ferrite/cementite ‘phase’ and the ferrite ‘phase’ is placed by a distance \( \partial \eta \) in time \( \partial t \), the amount of solute that must be supplied is

\[
[x^{Fe_3C/\alpha} - x^{\alpha \eta}] \partial \eta
\]

per unit area.

\[
x = [x^{Fe_3C/\alpha} - x^{\alpha \eta}] \partial \eta = -D \partial t \left( \frac{\partial x}{\partial z} \right)
\]  
(3.51)

A particular solution for this is:-

\[
x = x^{\alpha \beta} - Berf \left( \frac{z}{2 \sqrt{Dt}} \right)
\]
(3.52)

for \( 0 < z < \eta^\alpha \) and \( B \) is a constant. So when \( z = \eta^\alpha \), from equation (3.52),

\[
x^{Fe_3C/\alpha} = x^{\alpha \beta} - Berf (\xi)
\]
(3.53)

Integrating (3.52) gives

\[
\frac{\partial x}{\partial z} = -\frac{2}{\sqrt{\pi}} B \exp \left( \frac{z^2}{4Dt} \right) \frac{1}{2\sqrt{Dt}}
\]
(3.54)

So, when \( z = \eta^\alpha \)

\[
\frac{\partial x}{\partial z} = -\frac{2}{\sqrt{\pi}} B \exp (-\xi^2) \frac{1}{2\sqrt{Dt}}
\]
(3.55)

Also,

\[
\frac{\partial \eta}{\partial t} = \frac{\sqrt{D\xi}}{\sqrt{t}}
\]

So substitution into (3.51) gives:-

\[
[x^{Fe_3C/\alpha} - x^{\alpha \eta}] \frac{\sqrt{D\xi}}{\sqrt{t}} = \frac{2D}{\sqrt{\pi}} B \exp (-\xi^2) \frac{1}{2\sqrt{Dt}}
\]
(3.56)

\[
[x^{Fe_3C/\alpha} - x^{\alpha \eta}] = \frac{B}{\sqrt{\pi \xi}} \exp (-\xi^2)
\]
(3.57)

Eliminating \( B \) from (3.53) and (3.56)

\[
\frac{x^{\alpha \beta} - x^{Fe_3C/\alpha}}{x^{Fe_3C/\alpha} - x^{\alpha \eta}} = \sqrt{\pi \xi} \exp (-\xi^2) \text{erf}\xi
\]
(3.58)

Using an iterative procedure a value of \( \xi \) can be found so that the width of the decarburised zone can be calculated using equation (3.49). If the Onsager extension of this equation is considered:-

\[
1 = -\sum^n n \partial x_k
\]
Kirkaldy and Young (1987) have shown that for steady state conditions in a three component system this equation becomes:

\[-D_{11} \frac{\partial x_1}{\partial z} - D_{12} \frac{\partial x_2}{\partial z} = 0\]  \hspace{1cm} (3.59)

Therefore

\[\frac{\partial x_1}{\partial z} = -\frac{D_{12}}{D_{11}} \frac{\partial x_2}{\partial z}\]  \hspace{1cm} (3.60)

On integration, the result

\[\Delta x_1 = -\frac{D_{12}}{D_{11}} \Delta x_2\]  \hspace{1cm} (3.61)

is obtained, where \(\Delta x_1\) is the difference in carbon concentration at the interface i.e. \((x^\beta_0 - x^{\alpha}_0)\)

and \(\Delta x_2\) is the difference in concentration of the other metal at the interface i.e. \(\Delta Cr\) assuming no diffusion of substitutional elements. Now it was shown by Bhadeshia (1985), that

\[\frac{D_{12}}{D_{11}} = \frac{\epsilon_{12} x_1}{1 + \epsilon_{11} x_1}\]  \hspace{1cm} (3.62)

\(\epsilon_{ik}\) is the Wagner interaction coefficient between element \(i\) and element \(k\). Again, the subscript 1 refers to carbon and the subscript 2 refers to the alloying element. If \(x_1\), the carbon concentration in solution in the ferrite, is taken to be very small, then this reduces to:

\[\frac{D_{12}}{D_{11}} = \epsilon_{12} x_1\]  \hspace{1cm} (3.63)

so equation (3.61) becomes

\[\Delta x_1 = -\epsilon_{12} x_1 \Delta x_2\]  \hspace{1cm} (3.64)

Now a third condition is required in order to fulfill the mass balance in the interface region i.e. the area \(A_I = A_{II}\) in Figure 3.13 therefore

\[\left[x^{\beta_0} - x^{\beta_0}\right] \frac{1}{2} \eta_\beta = \left[x^{\alpha_0} - x^{\alpha_0}\right] \frac{1}{2} \eta^\alpha\]  \hspace{1cm} (3.65)

substituting for \(\eta\) from (3.48)

\[\left[x^{\beta_0} - x^{\beta_0}\right] \frac{1}{2} 2\sqrt{D_{\beta}} l = \left[x^{\alpha_0} - x^{\alpha_0}\right] \frac{1}{2} 2\sqrt{D_{\alpha}} l\]  \hspace{1cm} (3.66)

where \(\eta^\beta\) is the carburised zone width. Substituting for \(x^{\alpha_0}\) from (3.64)
Figure 3.13: Schematic representation of the carbon profiles after Buchmayer et al. (1989).

where in the Buchmayer experiments, $\Delta x_2$ is the difference in chromium concentration $\Delta Cr$.

Rearrangement gives

$$x_{\beta 0} = \frac{x^{\alpha 0} + \sqrt{\frac{D_{\alpha}}{D_{\beta}}} x^{\beta 0} + \epsilon_{12} x_1 \Delta x_2}{1 + \sqrt{\frac{D_{\alpha}}{D_{\beta}}}}$$

(3.68)

which allows the limits to be set.

The limitations of this model are that it is strictly only applicable for temperatures below which chromium carbides are precipitated i.e. below 500–550°C, which limits its applicability in terms of the transition joints used in power plant, which are generally operating above this temperature. No account is made for precipitation on the high alloy side of the weld. This is an important factor as precipitation reduces the amount of dissolved carbon, therefore changing the activity of carbon. Precipitation also reduces the chromium concentration in the matrix resulting in a decrease in the driving force for diffusion.

3.4.2 Model due to Christoffel and Curran (1956)

Another more empirical method of producing carbon profiles was proposed by Christoffel and Curran (1956) who describe the process of carbon migration as occurring in four distinct stages to produce a carbon profile as shown in Figure 3.14. In this case there are really three phases present, the low alloy material, the high alloy material and a decarburised band which
previously, is the activity gradient arising from the difference in substitutional alloy content. As a result, the carbon concentration at the interface increases on the high alloy side of the weld (precipitation may occur if the carbon concentration rises above the solubility limit) causing the development of a carbon concentration gradient and carbon will migrate away from the fusion line under the influence of the gradient. As a further consequence of carbon diffusion across the boundary, the carbon concentration decreases on the low alloy side of the weld and carbon diffuses to the diffusion line under the action of the resulting gradient. When the carbon concentration in the low alloy material has dropped below the solubility limit by virtue of the migration across the fusion line, some of the carbon which is in the form of carbides, breaks up and goes into solution. Therefore, Christoffel and Curran conclude that the solution of carbides accounts for the occurrence of the decarburised band.
where \( x^{\alpha A} \) = carbon concentration solubility limit at the diffusing temperature in the low alloy side of the joint and \( x^{\alpha C} \) = carbon concentration on the high alloy side of the joint.

This equation is derived as follows:-

Consider the carbon distribution after exposure at an elevated temperature for times \( t \) and \( dt \) as shown in Figure 3.15. After time \( t \), the width of the decarburised zone is given by \( \eta^\alpha \). From Ficks law:-

\[
J = -D \frac{\partial x}{\partial z}
\]

therefore from Figure 3.15

\[
J = -D \left( \frac{x^{\alpha A} - x^{\alpha \beta}}{\eta^\alpha} \right)
\]

The amount of carbon which has crossed the fusion line into the high alloy side of the weld in time \( dt \) is given by:-

\[
J dt = -D \left( \frac{x^{\alpha A} - x^{\alpha \beta}}{\eta^\alpha} \right) dt
\]

The amount of carbon which has left the low alloy side is also given by \( J dt \) where from Figure 3.15,

\[
D \left( \frac{x^{\alpha A} - x^{\alpha \beta}}{\eta^\alpha} \right) dt = (x^{\alpha \alpha} - \frac{x^{\alpha A}}{2} - \frac{x^{\alpha \beta}}{2}) d\eta
\]

\[
\eta^\alpha d\eta = \frac{D(x^{\alpha A} - x^{\alpha \beta})}{x^{\alpha \alpha} - \frac{x^{\alpha A}}{2} - \frac{x^{\alpha \beta}}{2}} dt
\]

Integrating this expression,

\[
(\eta^\alpha)^2 = \frac{2D(x^{\alpha A} - x^{\alpha \beta})}{x^{\alpha \alpha} - \frac{x^{\alpha A}}{2} - \frac{x^{\alpha \beta}}{2}} t + A
\]

where \( A \) is a constant of integration which can be evaluated by applying the boundary conditions that when \( \eta^\alpha = 0, \ t = 0 \), therefore \( A = 0 \). Also the terms in \( x^{\alpha A} \) and \( x^{\alpha \beta} \) in the denominator can be neglected as they are very small in ferritic joints, therefore:-

\[
(\eta^\alpha)^2 = \frac{(x^{\alpha A} - x^{\alpha \beta})}{x^{\alpha \alpha}} 2Dt
\]

### 3.5 Conclusions

Models for predicting carbon diffusion profiles in both austenite and ferrite have been...
Figure 3.15: Schematic diagram of the development of the decarburised band (Christoffel and Curran, 1956).

Diffusion of carbon in ferrite is complicated by the presence of carbon in the form of carbides. Two models for the diffusion of carbon in ferritic dissimilar metal welds have been described. It should be emphasised at this point that equations 3.49 and 3.75 are both of the form:

$$\eta^\alpha \propto \sqrt{t}$$

(3.76)

Therefore both the model of Buchmayer et al. and Christoffel and Curran predict that the growth of the decarburised zone will follow parabolic kinetics.
CHAPTER 4

Diffusion in austenite/austenite dissimilar metal joints

4.1 Introduction

The aim of this work was to produce a model for diffusion of carbon across dissimilar metal joints when both sides of the joint are held in the austenite phase field. Previous work, described in Chapter 3, did not provide a means by which the carbon profiles developed during diffusion in the austenite phase field could be predicted. The problem lies in the accurate determination of a value of the partition coefficient, a problem which none of the authors addressed in attempting to describe Darkens diffusion profiles. This chapter describes the attempts that have been made in this research to predict Darkens profiles and to extend this to other alloy systems. It was also intended that this work should provide a basis from which to tackle the problem of diffusion in the ferrite phase field.

To summarise the relevant points from Chapter 3, Stark (1980), Agren (1983) and Bhadeshia (1988) developed equations which can be described by the general formula:

\[ x^\alpha = A_1 + B_1 \text{erf}\left\{ \frac{z}{2\sqrt{D_\alpha t}} \right\} \]

\[ x^\theta = A_2 + B_2 \text{erf}\left\{ \frac{|z|}{2\sqrt{D_\theta t}} \right\} \]

By applying the boundary conditions of continuity of chemical potential and continuity of carbon flux across the interface, exact solutions could be calculated for these equations:

\[ x^\alpha = x^{\alpha\theta} + (x^{\alpha\theta} - x^{\alpha\theta}) \text{erf}\left\{ \frac{z}{2\sqrt{D_\alpha t}} \right\} \]

\[ x^\theta = x^{\alpha\theta} - \sqrt{\frac{D_\alpha}{D_\theta}}(x^{\alpha\theta} - x^{\alpha\theta}) \text{erf}\left\{ \frac{z}{2\sqrt{D_\theta t}} \right\} \]

These equations produce profiles represented schematically in Figure 4.1. To solve these equations values of the diffusion coefficient of carbon in austenite and the partition coefficient are required.

4.2 Calculation of the diffusion coefficient of carbon in austenite
function of position in space. Wells, Batz and Mehl (1950) noted also that the diffusion coefficient increased first slowly then more rapidly with increasing concentration. In fact Hillert (1959) empirically expressed the diffusion coefficient as an exponential function of solute concentration and therefore assumed that the best approximation to the diffusion coefficient could be obtained by considering a constant coefficient corresponding to the maximum solute concentration. However, this assumption is not justified and a more rigorous approach to the concentration dependence of diffusivity is given by Trivedi and Pound (1967) who proposed an integrated average diffusivity, which although only strictly valid for steady state conditions, has been shown to give a reasonable approximation (Bhadeshia, Svensson and Gretoft, 1986) even for conditions where the nature of the concentration profile changes with time.

\[
D = \int_{x_{\gamma}^\alpha}^{\bar{x}} \frac{D(x, T)\, dc}{(\bar{x} - x_{\gamma}^\alpha)}
\]  

where \(\bar{x}\) is the average carbon concentration of the alloy and \(x_{\gamma}^\alpha\) is the maximum permissible carbon content in the austenite at the transformation interface. It is therefore necessary to obtain an expression for \(D(x, T)\) over the specified carbon concentration range. Siller and McLellan (1969) have developed a model which takes account of the concentration dependence
The final equation becomes:

\[ D\{x, T\} = D'\zeta\{\omega\} \] (4.6)

where \( \omega \) is the atom fraction of carbon which is given by:

\[ \omega = \frac{X}{1 - X} \] (4.7)

where \( X \) is the atom fraction of carbon atoms. \( D' \) is a temperature dependent but carbon concentration independent term described by the equation:

\[ D' = \frac{k_B T}{h} \left[ \exp\left(\frac{\Delta F^*}{k_B T}\right) \left(\frac{\lambda^2}{3\gamma_m}\right) \right] \] (4.8)

In this expression:

- \( \Delta F^* \) is the activation free energy. This term is independent of composition and temperature and represents a difference in the free energy between the 'activated complex' and the 'reactants' when each is in its standard state at the temperature of the reaction.
- \( \gamma_m \) is the activity coefficient of the activated complex
- \( \lambda \) is the distance between \{002\} austenite planes
- \( k_B \) is Boltzmann's constant
- \( h \) is Planck's constant

The concentration dependent part of equation (4.6) is \( \zeta\{\omega\} \). This is defined by Siller and MacLellan as:

\[ \zeta\{\omega\} = a_c \left[ 1 + \frac{z_a(1 + \omega)}{1 - (\frac{2}{2} + 1)\omega + \frac{3}{2}(\frac{2}{2} + 1)(1 - \sigma)\omega^2} \right] + (1 + \omega) \frac{\partial a_c}{\partial \omega} \] (4.9)

In this expression, \( z_a \) is the co-ordination number of an octahedral site in the austenite lattice, \( a_c \) is the activity of carbon and \( \sigma \) is defined as:

\[ \sigma = 1 - \exp\left\{ -\Delta \epsilon \right\} \frac{k_B T}{k_B T} \] (4.10)

where \( \Delta \epsilon \) is the carbon-carbon interaction energy. The calculation of the carbon-carbon interaction energy and the activity of carbon in austenite are given by Bhadeshia (1981(b) and (c)).

Having calculated \( \zeta\{\theta\} \) and using experimental values of \( D\{x, T\} \), Bhadeshia (1981(a)) deduced that:
and

\[ \ln \left( \frac{3\gamma_m}{\lambda^2} \right) = 31.84 \]

Thus a value of \( D \) can be calculated for either side of the weld interface. Therefore, using the notation of Figure 4.1:

\[ D_{\alpha} = \int_{x_{\alpha}}^{x_{\alpha0}} \frac{D \{ x, T \} dc}{(x^{\alpha0} - x^{\alpha\theta})} \]  \hspace{1cm} (4.11)

\[ D_{\theta} = \int_{x_{\theta}}^{x_{\theta0}} \frac{D \{ x, T \} dc}{(x^{\theta0} - x^{\theta\alpha})} \]  \hspace{1cm} (4.12)

4.3 Calculation of the partition coefficient

There are basically three methods by which the partition coefficient can be calculated. The relative merits of each of these methods will be discussed later.

4.3.1 Wagner interaction parameters.

For dilute solutions, the activity coefficient for carbon is calculated using Wagner interaction parameters

\[ \ln \{ \Gamma_c \} = \sum_{i=1}^{n} \epsilon_i x_i \]  \hspace{1cm} (4.13)

where \( \epsilon_i \) is the Wagner interaction coefficient, in austenite, between carbon and carbon for \( i = 1 \) and carbon and a substitutional alloying element for \( i > 1 \). Baganis and Kirkaldy (1978) have deduced these as shown in Table 4.1.

The partition coefficient is then given by:

\[ k = \frac{\Gamma_{\alpha}}{\Gamma_{\theta}} \]

4.3.2 Uhrenius model.

This is a thermodynamic model developed from that due to Hillert and Staffansson (1970) to describe the Gibbs free energy of carbon in austenite. The basis of this model is the application of the regular solution model to phases which contain one interstitial and one substitutional solute element. Thus there are in effect two sublattices, one which is completely filled by the base metal and any substitutional elements and one which is partially filled with the interstitial elements and partially filled with vacancies. The molar ratios are therefore defined by:-
Element | $\epsilon_i$
--- | ---
C | 8910/T
Mn | -5070/T
Si | 4.84-(7370/T)
Ni | -2.2-(7600/T)
Cr | 24.4-(38400/T)
Mo | 3.855-(17870/T)
Cu | 4200/T
W | 23.4-(36214/T)
V | -24660/T
Nb | -28770/T
Cu | 2800/T

Table 4.1: Wagner interaction coefficients after Baganis and Kirkaldy, 1978.

(Where $T$ is the Absolute temperature.)

\[ y_c = \frac{a}{b} \frac{x_c}{1 - x_c} \]  \hspace{1cm} (4.15)

Where the subscript $m$ stands for iron and substitutional elements and $c$ stands for carbon in this case. The ratio $\left(\frac{a}{b}\right)$ (where $a$ represents the number of metal atoms per mole and $b$ the corresponding number of interstitial sites available) is 1 for austenite. This analogy was continued to derive an expression for the integral free energy of austenite i.e. the partial free energies summed over all the elements in the phase.

\[ RT\ln a_c + \Delta^0G_{c}^{\gamma \rightarrow \gamma'} = RT\ln \frac{y_c}{1 - y_c} + K_1 y_c + K_2 y_m + K_3 y_c y_m + K_4 y_m^2 \]  \hspace{1cm} (4.16)

In this expression $\Delta^0G_{c}^{\gamma \rightarrow \gamma'}$ is the molar Gibbs free energy of pure carbon for the change in crystal structure from $\gamma \rightarrow$ graphite. However, as the couple is comprised of austenite on both sides of the interface then this can be ignored, as can the parameters $K_3$ and $K_4$ since these are present in high order terms and will thus have little influence at low values of $y_m$ and $y_c$. The values $K_1$ and $K_2$ have been evaluated from experimental data for carbon activities and optimisation procedures to obtain detailed values of the parameters. These procedures are based on the minimisation of sum of squares between calculated and measured quantities (Uhrenius, 1977). Once these parameters are known then the activity can be calculated from equation (4.16) and thus the activity coefficient is given by equation (4.17):

\[ \Gamma_c = \frac{a_c}{x_c} \]  \hspace{1cm} (4.17)
and therefore $k$ can be calculated as before.

### 4.3.3 Model of Wada et al.

Wada et al. (1972) have calculated the activity coefficient for the Fe–Si–C system, for the composition range 0–3wt% Si and 0–1wt% C and give the equation:-

$$\ln \Gamma_c = \left[ \frac{(179 + 8.90 \text{wt}\% \text{Si})}{T} \right] \text{wt}\% \text{C} + [0.041 + \frac{62.5}{T}] \text{wt}\% \text{Si} \quad (4.18)$$

(the compositions in this equation are all in weight percent and $T$ is the Absolute temperature.)

### 4.4 Extension to the computer program

Once the basic theory had been established from the literature, a program could be written to determine carbon concentration profiles for any weld combination after a specified time at a holding temperature. The flow diagram for the program is illustrated in Figure 4.2 and a full listing given in Appendix II. After the initial values of the two steel compositions and the time and temperature have been input, the diffusion coefficient is calculated. With reference to equation 4.11 and 4.12, it can be seen that values of $x^{\alpha\theta}$ and $x^{\theta\alpha}$ are required. This is estimated using the following approximation:-

$$x^{\alpha\theta} = 0.66x^{\alpha\alpha} \quad (4.19)$$

$$x^{\theta\alpha} = 1.33x^{\theta\theta} \quad (4.20)$$

This enables an initial estimate of the diffusion coefficient to be made. The partition coefficient is then calculated using one of the four methods described or an experimental value which has to be input. This then enables a better value of the diffusion coefficient to be estimated. This process continues until the best estimate of the diffusion coefficient is obtained. The profiles on either side of the interface can then be calculated for the specified time using equations 4.1 and 4.2.

### 4.5 Prediction of the Darken profiles

The four methods for the prediction of the partition coefficient can be compared by using test data from the Darken weld A–C. The compositions for this weld are presented in Table 4.2.

The partition coefficients and diffusion coefficients calculated by each method are presented in Table 4.3. If these values are substituted into equations (4.3) and (4.4) then the profiles of Figure 4.3 are obtained.

It can be seen that the values calculated using the method of Wada et al. are the most
mentioned in Chapter 3 that an interesting extension of the work of Kucera and Stransky (1982) was that they obtained values of the interaction coefficients for Si, Cr, Mn and Mo. These results are compared with those of Baganis and Kirkaldy (1978) in Table 4.4. It can be seen that the values are vastly different in the case of silicon which is the substitutional element responsible for diffusion. If the values of Kucera and Stransky are used in Wagner's equation (4.13) then the profile in Figure 4.4 is produced.
Table 4.2: Composition of the steels used. The carbon and silicon concentrations are highlighted to outline the differences between the steels and the main contributors to the diffusion.

<table>
<thead>
<tr>
<th>Steel</th>
<th>C wt%</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.49</td>
<td>0.25</td>
<td>3.80</td>
<td>0.011</td>
<td>0.006</td>
<td>0.31</td>
</tr>
<tr>
<td>B</td>
<td>0.04</td>
<td>0.28</td>
<td>4.78</td>
<td>0.006</td>
<td>0.011</td>
<td>–</td>
</tr>
<tr>
<td>C</td>
<td>0.45</td>
<td>0.88</td>
<td>0.05</td>
<td>0.02</td>
<td>0.008</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 4.3: Partition and diffusion coefficients calculated for the Darken profile A-C.

<table>
<thead>
<tr>
<th></th>
<th>Wagner</th>
<th>Uhrenius</th>
<th>Wada</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partition coefficient</td>
<td>0.968</td>
<td>1.01</td>
<td>1.42</td>
<td>1.86</td>
</tr>
<tr>
<td>$D_o$/mm$^2$s$^{-1}$</td>
<td>$5.33 \times 10^{-9}$</td>
<td>$4.76 \times 10^{-9}$</td>
<td>$4.84 \times 10^{-9}$</td>
<td>$4.84 \times 10^{-9}$</td>
</tr>
<tr>
<td>$D_e$/mm$^2$s$^{-1}$</td>
<td>$5.11 \times 10^{-9}$</td>
<td>$5.15 \times 10^{-9}$</td>
<td>$5.60 \times 10^{-9}$</td>
<td>$5.60 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Figure 4.3: Calculated Darken profiles using calculated values of the partition coefficient. a) All methods b) Close up showing profiles due to Wagner and Uhrenius. This joint was held at 1050°C for 14 days.

in concentrated solutions. This is proved by the fact that if an experimental value of the partition coefficient is taken from the Darken experimental profiles, then excellent agreement is obtained (Figure 4.5).

The Darken profile A–B can also be predicted using the model of Wada et al. These
Figure 4.4: Darken profile A–C produced using the interaction coefficients of Kucera and Stransky.

<table>
<thead>
<tr>
<th>Interaction coefficient</th>
<th>Baganis and Kirkaldy</th>
<th>Kucera and Stransky</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{Mn}$</td>
<td>-3.83</td>
<td>-3.41</td>
</tr>
<tr>
<td>$\epsilon_{Si}$</td>
<td>-0.73</td>
<td>7.85</td>
</tr>
<tr>
<td>$\epsilon_{Cr}$</td>
<td>-4.62</td>
<td>-9.51</td>
</tr>
<tr>
<td>$\epsilon_{Mo}$</td>
<td>-9.65</td>
<td>-9.56</td>
</tr>
</tbody>
</table>

Table 4.4: Comparison of the interaction coefficients calculated at 1323K.

results, together with those using an experimental value of the partition coefficient are shown in Figure 4.6.

The model will also indicate the effect of time and temperature on the diffusion. Figure 4.7(a) shows the effect of increasing time and Fig(4.7(b)) indicates the effect of increasing temperature. For these calculations the compositions of the steels used in the Darken experiment for weld A–C are used.

Having proved the reliability of the equations, the model can be extended with confidence to other more dilute systems, especially those whose thermodynamics are more widely researched. Of special interest are the chromium steels which are of greater commercial importance than those used in the Darken experiment. Wada et al. (1972) have also produced an equation for the system Fe–Cr–C for the composition range 0–12 wt% Cr and 0–1 wt% C and give the equation:-
Figure 4.5: Profiles produced using an experimental value of the partition coefficient.

![Graph showing experimental profiles](image)

Figure 4.6: Darken profile A-B a) using model of Wada et al. to calculate the partition coefficient, b) using a partition coefficient taken from the Darken data. This joint was held at 1050°C for 13 days.

\[
\ln \Gamma_c = \frac{2300}{T} - 2.24 + \left( \frac{179}{T} \right) \text{wt}\% C - \left( \frac{102}{T} - 0.033 \right) \text{wt}\% Cr
\]  

(4.21)

4.6 Extension of the model to the ferrite phase field

This model is only applicable for the joints held in the austenite phase field. Kimmins
(1989) has produced profiles for the $2\frac{1}{4}$CrMo/$\frac{3}{2}$CrMoV dissimilar metal weld which are illustrated in Figure 4.8(a) held at 700°C for 16 hours i.e. in the ferrite phase field. If these profiles are modelled using the current equation, it can be seen from Fig 4.8(b) that they are an underestimate. The reason for this discrepancy is that the austenite model deals only with the carbon in solution which is available for diffusion. In ferrite this amount is very small and hence the model cannot account for the steepness of the profiles. Therefore there must be some other process occurring which is accounting for the increase in the amount of carbon available for diffusion. On the low alloy side of the weld this process must be carbon release and on the high alloy side it must be carbon absorbing, such as the dissolution and precipitation of carbides. A detailed investigation into diffusion across dissimilar metal welds in the ferrite phase field is required in order to understand what is going on. Any successful model for predicting the extent of diffusion in ferrite must take these processes into consideration.
**Figure 4.8:** a) Experimental profile produced by Kimmins (1989). This joint was held at 700°C for 16 hours b) Prediction of this profile using current model.
5.1 Weld joint selection

The aim of the experimental work was to study the development of microstructure in dissimilar metal welds after tempering at a series of temperatures. The program of experimental work was intended to simulate the development of carburised and decarburised regions at the weld interface and to provide experimental data for the subsequent modelling of the carburisation and decarburisation effect. For this purpose four experimental joints were chosen; a mild steel/1CrMo, mild steel/2\(\frac{1}{4}\)CrMo, 2\(\frac{1}{4}\)CrMo/modified 9CrMo and a 2\(\frac{1}{4}\)CrMo/12CrMo joint. The first two joints were selected for the simplicity of the system, i.e. if the diffusion of carbon is driven by the difference in chromium concentration across the interface, then the modelling should be facilitated by the fact that one side of the weld, mild steel, contains no chromium. The second two joints were chosen because of their applicability to transition joints in actual power plant.

5.2 Weld preparation and manufacture

The mild steel/1CrMo and mild steel/2\(\frac{1}{4}\)CrMo joints were made to the weld design shown in Figure 5.1(a), whilst the 2\(\frac{1}{4}\)CrMo/9CrMo and 2\(\frac{1}{4}\)CrMo/12CrMo joints were welded to the design in Figure 5.1(b).

Details of the welding conditions for all of the specimens are given in Table 5.1.

<table>
<thead>
<tr>
<th>Weld joint</th>
<th>Preheat temperature</th>
<th>Interpass temperature</th>
<th>Number of passes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel/1CrMo</td>
<td>150-200°C</td>
<td>250°C max</td>
<td>5</td>
</tr>
<tr>
<td>Mild steel/2(\frac{1}{4})CrMo</td>
<td>200-300°C</td>
<td>350°C max</td>
<td>5</td>
</tr>
<tr>
<td>2(\frac{1}{4})CrMo/9CrMo</td>
<td>100°C</td>
<td>100°C</td>
<td>61</td>
</tr>
<tr>
<td>2(\frac{1}{4})CrMo/12CrMo</td>
<td>100°C</td>
<td>100°C</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 5.1: Welding conditions for all of the joints.

The resultant chemical compositions of the weld metal and parent plate for each joint are given in Table 5.2. Post weld heat treatment was not given to any of the joints prior to specimen sectioning.
Figure 5.1: a) Weld design for the mild steel/lCrMo and mild steel/2tCrMo joints b) weld design for 2\textfrac{1}{4}CrMo/9CrMo and 2\textfrac{1}{4}CrMo/12CrMo joints.

<table>
<thead>
<tr>
<th>Material</th>
<th>Composition /wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>Mild steel plate</td>
<td>0.20</td>
</tr>
<tr>
<td>1CrMo weld</td>
<td>0.056</td>
</tr>
<tr>
<td>Mild steel plate</td>
<td>0.23</td>
</tr>
<tr>
<td>2\textfrac{1}{4}CrMo weld</td>
<td>0.079</td>
</tr>
<tr>
<td>2\textfrac{1}{4}CrMo/9CrMo weld</td>
<td>0.108</td>
</tr>
<tr>
<td>2\textfrac{1}{4}CrMo/12CrMo weld</td>
<td>0.08</td>
</tr>
<tr>
<td>12CrMo pipe</td>
<td>0.20</td>
</tr>
<tr>
<td>2\textfrac{1}{4}CrMo weld</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 5.2: Chemical compositions of all of the weld joints.

5.3 Heat treatment program

In order to provide diffusion couples that could easily be studied, the welds were sectioned
parallel to the fusion line to the approximate dimensions shown in Figure 5.2.

In order to prevent decarburisation, scaling and contamination during heat treatment, the samples were sealed in quartz tubes under a partial pressure of argon and placed in ceramic boats in the furnace. Three temperatures were chosen at which to carry out the heat treatments. The highest heat treatment temperature was 700°C for mild steel/1CrMo and mild steel/\(2\frac{1}{4}\)CrMo, 730°C for \(2\frac{1}{4}\)CrMo/9CrMo and \(2\frac{1}{4}\)CrMo/12CrMo joints. The lowest corresponded to the working temperature of the power plant (565°C) and an intermediate heat treatment temperature of 620°C was also studied. Heat treatments were carried out at these temperatures for up to 10,000 hours for each weld combination. The specimens were quenched to room temperature in water at the completion of heat treatment.

5.4 Optical microscopy

After heat treatment the specimens were mounted in conductive Bakelite, ground to 120 grit and polished to a 1\(\mu\)m finish. An etch of 2% nital (concentrated nitric acid in methanol) was used for the mild steel/1CrMo and mild steel/\(2\frac{1}{4}\)CrMo joints. The \(2\frac{1}{4}\)CrMo/9CrMo and \(2\frac{1}{4}\)CrMo/12CrMo joints were etched in Villelas reagent (5ml HCl, 1g Picric acid, 100ml methanol). Optical micrographs were taken on Ilford PanF film using an Olympus BHM microscope.

5.5 Hardness measurements.

Vickers hardness measurements were taken at 0.25mm intervals across the weld interface.
at right angles to the fusion line from the parent plate to the weld metal. A Leitz Miniload microhardness machine was used with a load of 0.981 N and loading and dwell times of 15 seconds each. At every point three measurements were made and an average taken.

5.6 Measurement of decarburised zone width

There are two methods by which the decarburised zone width can be measured, either from optical micrographs or from hardness profiles taken across the weld interface.

5.6.1 Optical measurement of decarburised zones

Figure 5.3 illustrates how the decarburised zone width can be measured from optical micrographs taken of the weld interface area. The decarburised zone is characterised by the growth of large ferrite grains. The width of these grains was measured at several points on the micrograph and an average taken to give a mean decarburised zone width for that specimen.

![Optical measurement of the decarburised zone width.](image)

5.6.2 Hardness profile measurement of decarburised zone width

The principle behind this method is that the carburisation and decarburisation, which has taken place during the heat treatment of the joint, will produce hard and soft zones immediately adjacent to each other at the weld interface. Therefore, if a hardness profile is taken across the interface region at right angles to the fusion line, then a profile as illustrated in Figure 5.4 would be expected. It is therefore possible to measure a decarburised zone width from these plots. However, it was found that, in practice, there was so much scatter in the results that often it
was difficult to distinguish clearly the carburised and decarburised regions. For this reason the decarburised zone width data presented has all been measured optically.

5.7 Scanning electron microscopy (SEM)

Imaging in the SEM was carried out in a Camscan S4 microscope operated at 30kV using secondary electrons and 45° tilt. The specimens were polished and deeply etched to reveal the carbide structure which then protruded above the matrix. Chemical analyses were taken at 0.25mm intervals across the weld interface at right angles to the fusion line. A live time of 100 seconds was used with an average count rate of 1000 counts per second and a dead time of less than 20%. The operating voltage was 20kV. The working distance was kept at 30mm and the specimen tilted at an angle of 45° to the detector. The data were analysed using the LINK ZAF4 programmes for microanalysis, which correct the data for atomic number and account for overlapping peaks by fitting standard profiles.

5.8 Transmission electron microscopy (TEM)

Transmission electron microscopy (TEM) was used to identify individual carbides in the region of the weld close to the fusion line. The carbide phases can, of course, in most cases, be identified uniquely using electron diffraction, but this is in practice very difficult when examining large numbers of small particles. The carbides are often too thick or too thin to get reasonable diffraction data, and in some cases, the diffraction patterns can be ambiguous.

A common alternative method used routinely in this kind of research is based on energy dispersive X-ray (EDX) microanalysis. This technique for carbide identification is carried out
in the TEM and it enables chemical analyses to be taken from individual carbides. For these procedures, carbon extraction replicas were used instead of thin foils in order to avoid spurious X-rays from the matrix.

The carbon replica extraction procedure developed by Smith and Nutting (1957) was used to produce thin carbon films suitable for examination in the TEM. The metal surface was first polished and etched so that the structure was clearly visible under the optical microscope. A thin carbon film was then evaporated onto the metal surface. The thin film was removed by etching in a solution of 5% chloral (hydrochloric acid in methanol) at a potential of 1.5V until the film began to lift away from the surface. The specimen was then washed in alcohol before being transferred to a bath of distilled water where the replica was floated off and carefully collected on copper grids. The replicas were examined in the TEM using an accelerating voltage of 120kV. The diffraction patterns were taken using selected area apertures and chemical compositions were determined using EDX analysis with a specimen tilt of 35° towards the X-ray detector. A live time of 100 seconds was used during which the dead time was not allowed to exceed 25%. The data were analysed using the LINK RTS2 FLS programmes for microanalysis.

In this research, the majority of the carbide identification was carried out using EDX analysis with diffraction patterns providing valuable identity verification data.
CHAPTER 6

Experimental results

6.1 Introduction

The experimental work described here was all conducted on joints in the ferrite phase field to try and gain some understanding into the processes governing the diffusion of carbon. The work described in Chapter 3 highlighted the fact that the mechanism by which carbon is released for diffusion from the low alloy side and the mechanism by which it is taken up on the high alloy side needs to be understood thoroughly in order to develop a predictive model. The purpose of this work was to make an extensive characterisation of the microstructure and in particular the development of any decarburised zone in four different joints heat treated for varying times at three different temperatures. All welds were received in the as-welded condition with no post weld heat treatment having been given to any of the weld combinations. The micrographs in Figure 6.1 indicate the initial microstructure of all of the welds studied. The mild steel plate in both Figure 6.1(a) and 6.1(b) has a ferrite/pearlite structure in the unaffected parent and a martensitic reheated heat affected zone (HAZ) microstructure. The weld bead microstructure of both the 1CrMo and 2\(\frac{1}{4}\)CrMo is fully bainitic. The 9CrMo and 12CrMo parent plate illustrated in Figure 6.1(c) and 6.1(d) is martensitic. These micrographs also illustrate the bainitic nature of the 2\(\frac{1}{4}\)CrMo weld metal in both joints.

6.2 Measurement of the decarburised zone width

As described previously, there are two methods by which the decarburised zone width can be measured, optical measurement and hardness profile measurement.

6.2.1 Optical measurement of the decarburised zone width

Example micrographs for the four joints are presented in Figure 6.2. These joints were all heat treated at the post weld heat treatment temperature of the welds for 64 hours. The decarburised zone width is taken to be the distance from the fusion line to the end of the large ferrite grains.

6.2.1.1 Effect of time

The decarburised zone width plots against root time for all of the joints at their respective heat treatment temperatures are given in Figure 6.3. As would be expected all of these plots show that there is an increase in the amount of decarburisation with time. This point is also
Figure 6.1: As-received microstructures

a) mild steel/1CrMo

b) mild steel/2\frac{1}{4}CrMo

Figure 6.1: As-received microstructures
Figure 6.1: As-received microstructures

c) $2\frac{1}{4}$CrMo/9CrMo

d) $2\frac{1}{4}$CrMo/12CrMo
a) mild steel/1CrMo - 64 hours at 700°C

b) mild steel/2\frac{1}{4}CrMo - 64 hours at 700°C

**Figure 6.2:** Optical micrographs from specimens heat treated at the post weld heat treatment temperature for 64 hours
c) 2\frac{1}{4}CrMo/9CrMo - 64 hours at 730°C

Figure 6.2: Optical micrographs from specimens heat treated at the post weld heat treatment temperature for 64 hours

d) 2\frac{1}{4}CrMo/12CrMo - 64 hours at 730°C

Figure 6.2: Optical micrographs from specimens heat treated at the post weld heat treatment temperature for 64 hours
Figure 6.3: Decarburised zone width plots for the four joints heat treated at the post weld heat treatment temperature for the joints.

a) mild steel/1CrMo - 700°C
b) mild steel/2\(\frac{1}{4}\)CrMo - 700°C
c) 2\(\frac{1}{4}\)CrMo/9CrMo - 730°C
d) 2\(\frac{1}{4}\)CrMo/12CrMo - 730°C
illustrated in the micrographs of Figure 6.4. However, parabolic growth rate theory as described in Chapter 3 would predict that:

\[ \eta^0 \propto \sqrt{t} \]

where \( \eta^0 \) is the decarburised zone width. Therefore plots such as these should be linear. These plots therefore indicate that the classical parabolic equations do not apply in this case as there is clearly a levelling off of the profiles at longer times.

6.2.1.2 Effect of temperature

The effect of temperature is illustrated in the micrographs of Figure 6.5 and the plot of Figure 6.6. Figure 6.5a indicates that although changes in the microstructure are occurring in the joint at 565°C, no decarburised zone width could be measured optically at this temperature. These indicate that the extent of diffusion is less at lower temperatures. This is what would be expected as diffusion is a thermally activated process (section 7.3).

6.2.1.3 Effect of alloy content

A comparison of the relative extent of decarburisation between joints is quite difficult due to the error in the experimental measurements. Plots for all of the joints at the two temperatures where decarburised zone widths could be measured are given in Figure 6.7. At the post weld heat treatment temperature, the following sequence is suggested:

\[ 2\frac{1}{4}\text{CrMo/12CrMo} > (2\frac{1}{4}\text{CrMo/9CrMo} \& \text{mild steel}/2\frac{1}{4}\text{CrMo} ) > \text{mild steel}/1\text{CrMo} \]

It is not really possible to distinguish between the \( 2\frac{1}{4}\text{CrMo/9CrMo} \) joint and the mild steel/\( 2\frac{1}{4}\text{CrMo} \) joint as the curves overlap within the experimental error at several points. At 620°C:

\( \text{mild steel}/2\frac{1}{4}\text{CrMo} \) and \( \text{mild steel}/1\text{CrMo} \) > \( (2\frac{1}{4}\text{CrMo/9CrMo} \text{ and } 2\frac{1}{4}\text{CrMo/12CrMo}) \)

(for the 620°C comparison the two steels have been grouped together as it is not possible to say which have the greater decarburised zone widths within the experimental error.)

The first sequence is probably as would be expected given that the sequence is showing a decrease in the difference in alloy content between both sides of the weld and temperature from left to right. However, the sequence at 620°C shows that less decarburisation occurs in the two alloy systems that have the greater difference in chromium concentration across the interface, \( 2\frac{1}{4}\text{CrMo/9CrMo} \) and \( 2\frac{1}{4}\text{CrMo/12CrMo} \).

6.2.1.4 Problems in optical measurement of the decarburised zone width

The major problem with the optical measurement is determining where the decarburised zone width finishes. This is more difficult in the two cases where the decarburisation is in
the weld metal as the large ferrite grains tend to follow the weld bead shape as illustrated in Figure 6.8. In these cases an estimate had to be made.

6.3 Hardness measurements of the decarburised zone width

Hardness profiles for the mild steel/2\(\frac{1}{4}\)CrMo joint are presented in Figure 6.9. These illustrate the unaged profile together with three taken at increasing times at 700°C. The error bars on this plot indicate the standard deviation from the mean value of the three readings taken at each point. The decarburised zone widths measured using this technique are compared with those measured optically in Table 6.1. It can be seen that this method is highly inaccurate owing to the scatter produced in the microhardness data and the fact that this is only a scan across a small area and it is not representative of the whole sample. The variation of the decarburised zone width in the weld metal as described in the previous section is also a problem. For these reasons the decarburised zone width measurements were entirely made using the optical technique.

<table>
<thead>
<tr>
<th>Time /hours</th>
<th>Hardness measurement</th>
<th>Optical measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>0.5</td>
<td>0.85</td>
</tr>
<tr>
<td>128</td>
<td>0.72</td>
<td>1.85</td>
</tr>
<tr>
<td>256</td>
<td>0.75</td>
<td>1.95</td>
</tr>
<tr>
<td>512</td>
<td>1.75</td>
<td>2.31</td>
</tr>
<tr>
<td>1024</td>
<td>1.75</td>
<td>2.30</td>
</tr>
</tbody>
</table>

Table 6.1: Comparison of decarburised zone widths measured using the hardness and optical techniques for the mild steel/2\(\frac{1}{4}\)CrMo joint at 700°C.

6.4 Measurement of the carburised zone width

The carburised zone width i.e. the region on the high alloy side of the weld interface where the carbon content is greater than average is rather more difficult to measure optically, as the micrographs illustrate. For this reason this measurement is made using the hardness profiles. Hardness profiles in the carburised region are given in Figure 6.10. Although this method suffers from the same problems of inaccuracy as described for the decarburised zone width, it does give an indication of the extent of carburisation compared with decarburisation for the same sample.
Figure 6.4: Optical micrographs for mild steel/2\textsuperscript{1/4}CrMo specimens heat treated at 700°C for various times.

a) Mild steel/2\textsuperscript{1/4}CrMo - 32 hours at 700°C

b) Mild steel/2\textsuperscript{1/4}CrMo - 512 hours at 700°C
c) mild steel/$2\frac{1}{4}$CrMo - 2000 hours at 700°C

**Figure 6.4:** Optical micrographs for mild steel/$2\frac{1}{4}$CrMo specimens heat treated at 700°C for various times

a) mild steel/$2\frac{1}{4}$CrMo - 512 hours at 565°C

**Figure 6.5:** Optical micrographs for mild steel/$2\frac{1}{4}$CrMo specimens heat treated at various temperatures for 512 hours
b) mild steel/$2\frac{1}{4}$CrMo - 512 hours at 620°C

Figure 6.5: Optical micrographs for mild steel/$2\frac{1}{4}$CrMo specimens heat treated at various temperatures for 512 hours

$2\frac{1}{4}$CrMo

Decarburised zone

Mild steel

500 μm

C) mild steel/$2\frac{1}{4}$CrMo - 512 hours at 700°C

Figure 6.5: Optical micrographs for mild steel/$2\frac{1}{4}$CrMo specimens heat treated at various temperatures for 512 hours
Figure 6.6: Effect of temperature on the decarburised zone width for the mild steel/$2\frac{1}{4}$CrMo joint
Figure 6.7: Effect of alloy content on the decarburised zone width for all of the joints at a) post weld heat treatment temperature and b) 620°C
Figure 6.8: 9CrMo/2 1/4CrMo joint heat treated at 730°C for 64 hours indicating the difficulties in measuring the decarburised zone width.
Figure 6.9: Hardness profiles for the mild steel/$2\frac{1}{4}$CrMo joint heat treated at 700°C
This illustrates that the zones formed on either side of the interface are unequal and carburisation is less extensive than decarburisation.

6.5 Characterisation of microstructure

From these initial results a clearer understanding of the microstructural changes occurring in the joints is required to try and explain the apparently anomalous observations concerning the shape of the decarburised zone width curve and the effect of alloy content. To this end electron microscopy techniques were employed. A scanning electron micrograph across a mild steel/2\(\frac{1}{4}\)CrMo joint at 700°C (Figure 6.12) illustrates the change in microstructure and specifically carbide size, shape and volume fraction. The parent plate, which was initially a ferrite/pearlite structure still shows this structure although the pearlite is now spheroidised. The decarburised zone is characterised by the large ferrite grains which are virtually precipitate free. At the interface there is a sharp transition between the precipitate free (low alloy) side and the precipitate rich (high alloy) side. Figure 6.12(c) illustrates the morphology and volume fraction of carbides in this region. When compared with the micrograph of the weld metal in Figure 6.12(d) it can be seen that the precipitation is more extensive at the weld interface. Figure 6.13 compares the interface region for the other three joints at this temperature. The main point to notice from these micrographs is that the size and volume fraction of carbides is lower in the 2\(\frac{1}{4}\)CrMo/9CrMo and 2\(\frac{1}{4}\)CrMo/12CrMo joints than it is in the mild steel/1CrMo and mild steel/2\(\frac{1}{4}\)CrMo joints.

A more extensive study of the carbides at the interface was made using transmission electron microscopy. The carbides could be identified either by electron diffraction or the EDX technique as described in Chapter 5. Typical extraction replicas from the 2\(\frac{1}{4}\)CrMo side of the mild steel/2\(\frac{1}{4}\)CrMo steel weld are presented in Figure 6.14 showing the morphology of the carbides present and typical diffraction patterns from the major types. Representative chemical analyses are given in Table 6.2. Carbide identification was carried out initially for the mild steel/2\(\frac{1}{4}\)CrMo joint at 700°C. Figure 6.15 illustrates the fraction of carbide types of the total identified at each heat treatment. It is interesting to note in this figure that the less stable cementite is reappearing in the specimen after its initial dissolution earlier in the precipitation sequence. The literature reviewed earlier appeared to predict that the structure was tending towards one of M\(_{23}\)C\(_6\) and M\(_6\)C (Figure 6.16). Results from the mild steel/2\(\frac{1}{4}\)CrMo joint at 620°C are presented in Figure 6.17 which indicates a similar reversal in the precipitation sequence at longer times. An investigation was then carried out on the other joints to ascertain whether the same phenomenon could be observed. The 1CrMo maintained a structure of...
Figure 6.10: Hardness profiles for the mild steel/2\frac{1}{4}CrMo joint heat treated at 700°C showing the carburised region.
Figure 6.11: Comparison of carburised and decarburised zone widths for the mild steel/2\(\frac{1}{4}\)CrMo joint heat treated at 700°C.
Figure 6.12: SEM micrographs of mild steel/$2\frac{1}{4}$CrMo joint heat treated at 700°C for 1024 hours

a) Parent plate

b) Interface region

Figure 6.12: SEM micrographs of mild steel/$2\frac{1}{4}$CrMo joint heat treated at 700°C for 1024 hours

80
c) Detail of carbides in the interface region

d) Weld metal

**Figure 6.12:** SEM micrographs of mild steel/$\frac{5}{4}$CrMo joint heat treated at 700°C for 1024 hours
Figure 6.13: SEM micrographs of all joints heat treated at the post weld heat treatment temperature for 1024 hours

a) Mild steel/1CrMo – 700°C

b) Mild steel/2\(\frac{1}{4}\)CrMo – 700°C

---

Figure 6.13: SEM micrographs of all joints heat treated at the post weld heat treatment temperature for 1024 hours
c) $2\frac{1}{4}$CrMo/9CrMo - 730°C

d) $2\frac{1}{4}$CrMo/12CrMo - 730°C

Figure 6.13: SEM micrographs of all joints heat treated at the post weld heat treatment temperature for 1024 hours
a) Morphology of Fe₃C particle in 2 ⁴/₄CrMo

b) Morphology of Mo₂C particle in 2 ⁴/₄CrMo

Figure 6.14: TEM micrographs of 2 ⁴/₄CrMo joint heat treated at 700°C
c) Morphology of $M_{23}C_6$ particle in $2\frac{1}{4}$CrMo

d) Morphology of $M_7C_3$ particle in $2\frac{1}{4}$CrMo

Figure 6.14: TEM micrographs of $2\frac{1}{4}$CrMo joint heat treated at 700°C
e) Morphology of $\text{M}_6\text{C}$ in $2\frac{1}{4}\text{CrMo}$

**Figure 6.14:** TEM micrographs of $2\frac{1}{4}\text{CrMo}$ joint heat treated at $700^\circ\text{C}$
Figure 6.15: Fraction of each carbide type found at the various heat treatment times in 2\textsuperscript{1/4}CrMo heat treated at 700°C.

Figure 6.16: Predicted carbides in 2\textsuperscript{1/4}CrMo heat treated at 700°C from the work of Baker and Nutting (1959).
cementite in a ferrite matrix up until the longest times. The carbide present in the 12CrMo was $M_{23}C_6$ for all of the specimens up until 10,000 hours. The 9CrMo side of the $2\frac{1}{4}$CrMo/9CrMo joint showed varying proportions of $M_{23}C_6$ and $M_7C_3$ as illustrated in Figure 6.18. The presence of $M_7C_3$ is consistent with MTDATA calculations for this joint (Section 2.8.3) but no other authors have reported the presence of $M_{23}C_6$ and $M_7C_3$ concurrently in the microstructure even at the longest tempering times.

<table>
<thead>
<tr>
<th>Carbide</th>
<th>Carbide composition /wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fe</td>
</tr>
<tr>
<td>$Fe_3C$</td>
<td>65.1</td>
</tr>
<tr>
<td>$Mo_2C$</td>
<td>18.0</td>
</tr>
<tr>
<td>$M_{23}C_6$</td>
<td>40.6</td>
</tr>
<tr>
<td>$M_7C_3$</td>
<td>54.2</td>
</tr>
<tr>
<td>$M_6C$</td>
<td>39.0</td>
</tr>
</tbody>
</table>

Table 6.2: Typical EDX compositions of the carbides found in $2\frac{1}{4}$CrMo heat treated at 700°C. It should be noted that this table neglects carbon which cannot be detected on the equipment used which has a Be window on the EDX detector.

The size and distribution of the carbides in the joints varied with time and temperature as illustrated in Figure 6.19 and Figure 6.20. These micrographs are of the 12CrMo joint in which the carbide type remained as $M_{23}C_6$. It can be seen that with increasing time at the same temperature there is an increase in the particle size and a decrease in the particle volume fraction. With increasing temperature the particle size increases and the volume fraction decreases.

6.6 Summary of results presented

The salient results from this experimental work are:-

1. The decarburised zone width levels off on a plot against root time where parabolic growth theory would predict linear behaviour.
2. The precipitation sequences on the high alloy side of the weld are changed, reverting back to carbide phases which had previously dissolved.
These results require discussion and explanation in order that they may be incorporated into any successful model.

Consider what is occurring during the diffusion of carbon across the interface. Effectively, the average carbon concentration in the interface region is decreasing on the low alloy side and increasing on the high alloy side. This results in local changes in the bulk carbon concentration. If MTDATA calculations are made for the $2\frac{1}{4}$CrMo steel by increasing the carbon concentration of the alloy then the volume fractions and types of carbide depicted in Figure 6.21 are predicted. Thus it can be seen that the lower order carbides become stable again as the carbon concentration is increased. This is what is effectively occurring in the welded joint together with the normal precipitation sequence of Figure 6.16 superimposed. The reason that the 12CrMo side remains as $M_{23}C_6$ is that MTDATA calculations predict that the average carbon concentration in the joint would have to rise to 3wt% in order to reprecipitate cementite (Figure 6.22). The conclusion is that the change in precipitation sequences on the high alloy side of dissimilar metal welds is a result of the local increase in carbon concentration. This, in turn, is going to affect the driving force for diffusion across the joint as will be described in Section 7.7.4 and will influence the decarburised zone width plot.

Other results that need to be able to be predicted are:-

1. The decarburised zone width increases with both time and temperature.
2. The effect of alloy content on the extent of decarburisation.
3. The extent of carburisation.
Figure 6.17: Fraction of each carbide type found at the various heat treatment times in $2\frac{1}{4}$CrMo heat treated at 620°C

Figure 6.18: Fraction of each carbide type found at the various heat treatment times in 9CrMo heat treated at 730°C
Figure 6.19: Carbon extraction replicas from 12CrMo heat treated at 730°C for various times

a) 16 hours

b) 256 hours

Figure 6.19: Carbon extraction replicas from 12CrMo heat treated at 730°C for various times
c) 2048 hours

**Figure 6.19:** Carbon extraction replicas from 12CrMo heat treated at 730°C for various times

a) 565°C

**Figure 6.20:** Carbon extraction replicas from 12CrMo heat treated for 2048 hours at various temperatures
Figure 6.20: Carbon extraction replicas from 12CrMo heat treated for 2048 hours at various temperatures

b) 620°C

c) 730°C

**Figure 6.20:** Carbon extraction replicas from 12CrMo heat treated for 2048 hours at various temperatures
Figure 6.21: MTDATA predictions of carbon types and volume fraction in 2½CrMo with increasing carbon concentration

Figure 6.22: MTDATA predictions of carbon types and volume fraction in 12CrMo with increasing carbon concentration
7.1 Model for the determination of the decarburised zone width

A search of the literature has revealed no model which can successfully predict decarburised zone widths in the ferrite phase field because none of the models takes into account the dissolution and precipitation of carbides which occurs on either side of the interface. In this work, a theory has been developed which attempts to account for these processes.

The carbides on either side of the weld interface are represented as particles of carbide in a ferrite matrix as illustrated in Figure 7.1. The substitutional alloy content is such that diffusion will occur from the $\alpha$ to the $\beta$ side of the weld interface.

![Figure 7.1: Illustration of the particulate nature of the model.](image)

The decarburised zone width is defined as the distance from the weld interface to the carbide particle that is dissolving at any time $t$ after diffusion has been allowed to occur. The initial distance from the interface is $z_0^A$ (as illustrated in Figure 7.2). With increasing time, the particle, A1, will progressively dissolve so that the distance $z_0^A$ increases to $z_i^A$. The decarburised zone width, $\eta^\alpha$, is therefore given by:

$$\eta^\alpha = z_i^A$$ (7.1)

There are two models that have been developed to determine a value of $z_i^A$: one which approximates constant concentration gradients in the ferrite and another which is based on
exponential profile. Both of these profiles are illustrated in Figure 7.3 together with the variety of carbon concentration terms. Thus \( x^{\alpha A} \) is the carbon concentration of the carbide which is in equilibrium with the ferrite on the low alloy side of the weld interface and \( x^{\beta A} \) is the carbon concentration of the ferrite which is in equilibrium with the carbide on the low alloy side of the weld interface. Similarly on the high alloy side of the weld \( x^{B\beta} \) is the carbon concentration in the carbide which is in equilibrium with ferrite and \( x^{B\alpha} \) is the carbon concentration in the ferrite which is in equilibrium with the carbide. These carbon concentrations can be calculated using MTDATA (1989). The interface carbon concentrations, \( x^{\alpha} \) and \( x^{\beta} \) are also illustrated in Figure 7.3. \( x^{\alpha} \) is therefore the concentration of carbon in \( \alpha \) at the \( \alpha/\beta \) interface and similarly \( x^{\beta} \) is the carbon concentration in \( \beta \) at the \( \alpha/\beta \) interface.

### 7.1.1 Constant concentration gradient model

If a linear concentration profile is assumed as illustrated in Figure 7.3(a) then mass conservation requires that for the \( i \)th particle \( A \),

\[
V_i^A(x^{A\alpha} - x^{A\beta}) = \frac{(x^{\alpha A} - x^{\alpha})D_\alpha}{z_i^A}
\]  
(7.2)

where \( V_i = \frac{\partial z_i^A}{\partial t} \), so that

\[
\frac{\partial z_i^A}{\partial t}(x^{A\alpha} - x^{A\beta}) = \frac{(x^{\alpha A} - x^{\alpha})D_\alpha}{z_i^A}
\]  
(7.3)

Integrating this expression:

\[
\int_{z_i^A}^{z_i^A} z_i^A \partial z_i^A = \int_0^t D_\alpha \frac{(x^{\alpha A} - x^{\alpha})}{(x^{A\alpha} - x^{A\beta})} \partial t
\]  
(7.4)
Thus

$$\frac{1}{2}\{(z^A_t)^2 - (z^A_0)^2\} = D_\alpha \frac{(x^{\alpha A} - x^{\alpha B})}{(x^{A \alpha} - x^{\alpha A})} t$$  \hspace{1cm} (7.5)

For the other side of the weld, the expression is similarly:

$$\frac{1}{2}\{(z^B_j)^2 - (z^B_0)^2\} = D_\beta \frac{(x^{\beta A} - x^{\beta B})}{(x^{B \beta} - x^{\beta B})} t$$  \hspace{1cm} (7.6)

where \(z^B_0\) is the initial distance from the interface to particle B and \(z^B_j\) is the distance to the growing particle at some time \(t\). Rearrangement of these equations yields the following expressions for \(z^A_t\) and \(z^B_j\),

$$z^A_t = \sqrt{\frac{2D_\alpha (x^{\alpha A} - x^{\alpha B})t}{(x^{A \alpha} - x^{\alpha A})} + z^A_0^2}$$  \hspace{1cm} (7.7)

$$z^B_j = \sqrt{\frac{2D_\beta (x^{\beta A} - x^{\beta B})t}{(x^{B \beta} - x^{\beta B})} + z^B_0^2}$$  \hspace{1cm} (7.8)

Now, for mass balance to be satisfied, the area of the two shaded triangles in Figure 7.3(a) must be equal. Therefore,

$$\frac{1}{2}z^A_t(x^{\alpha A} - x^{\alpha B}) = \frac{1}{2}z^B_j(x^{\beta A} - x^{\beta B})$$  \hspace{1cm} (7.9)

$$\frac{(x^{\alpha A} - x^{\alpha B})}{(x^{\beta A} - x^{\beta B})} = \frac{z^B_j}{z^A_t}$$  \hspace{1cm} (7.10)
A partition coefficient, $k$ can be defined such that

$$ k = \frac{x^\alpha}{x^\beta} $$

(7.11)

which rearranges to

$$ x^\beta = k x^\alpha $$

(7.12)

Therefore, equations 7.12 and 7.10 can be solved simultaneously to calculate the interface compositions $x^\alpha$ viz.,

$$ x^\alpha = \frac{(z_i A + z_j B)}{k z_i A + z_j B} $$

(7.13)

7.1.2 Exponential concentration profile

It was shown from the previous section that the flux of carbon from the $i$th particle is given by:

$$ \frac{\partial x^A_i}{\partial t} \left(x^A_i - x^A_A\right) = D \left[\frac{\partial x}{\partial z}\right]_{z=0} $$

(7.14)

and for the other side of the interface:

$$ \frac{\partial x^B_j}{\partial t} \left(x^B_j - x^B_B\right) = -D \left[\frac{\partial x}{\partial z}\right]_{z=0} $$

(7.15)

If an exponential concentration profile is assumed as shown in Figure 7.3(b) then an expression for $\frac{\partial x}{\partial z}$ can be obtained from the Laplace solution for Fick's second law.

$$ x^\alpha = A_1 + B_1 \text{erf} \left(\frac{z}{2\sqrt{D_{\alpha} t}}\right) $$

(7.16)

$$ x^\beta = A_2 + B_2 \text{erf} \left(\frac{|z|}{2\sqrt{D_{\beta} t}}\right) $$

(7.17)

where $A_1, B_1, A_2$ and $B_2$ are constants determined by the boundary conditions. Details of the calculation of the constants are given in Chapter 3.

Therefore, from equation (7.14):

$$ D_{\alpha} \left[\frac{\partial x^\alpha}{\partial z}\right]_{z=0} = D_{\alpha} \left(\frac{\partial \left(\text{erf} \left(\frac{z}{2\sqrt{D_{\alpha} t}}\right)\right)}{\partial z}\right) $$

(7.18)

so,

$$ D_{\alpha} \left[\frac{\partial x^\alpha}{\partial z}\right]_{z=0} = \frac{2}{\sqrt{\pi}} D_{\alpha} B_1 \exp \left(\frac{z^2}{4D_{\alpha} t}\right) \frac{1}{2\sqrt{D_{\alpha} t}} $$

(7.19)

when $z = 0$

$$ D_{\alpha} \left[\frac{\partial x^\alpha}{\partial z}\right]_{z=0} = \frac{2}{\sqrt{\pi}} D_{\alpha} B_1 \frac{1}{2\sqrt{D_{\alpha} t}} $$

(7.20)
therefore, substituting into (7.14)
\[
(x^{A_\alpha} - x^{A_A}) \frac{\partial x^{A_A}}{\partial t} = \frac{D_\alpha B_1}{\sqrt{\pi D_\alpha t}}
\]  
(7.22)
Rearrangement yields:
\[
(x^{A_\alpha} - x^{A_A}) \partial z = \frac{D_\alpha B_1}{\sqrt{\pi D_\alpha t}} \partial t
\]
(7.23)
and on integration
\[
(x^{A_\alpha} - x^{A_A}) \int_{z_0}^{z_1} \partial z = \int_0^t \frac{D_\alpha B_1}{\sqrt{\pi D_\alpha t}} \partial t
\]
(7.24)
\[
(x^{A_\alpha} - x^{A_A})(z_1^A - z_0^A) = \frac{2\sqrt{D_\alpha t}B_1}{\sqrt{\pi}}
\]
(7.25)
therefore,
\[
z_1^A = 2\sqrt{\frac{D_\alpha}{\pi}} \frac{B_1}{(x^{A_\alpha} - x^{A_A})} + z_0^A
\]
(7.26)
where from Chapter 3,
\[
A_1 = \frac{\beta B(x^{A_A})}{k+1} = x^{A_\beta} + \ldots
\]
and
\[
B_1 = x^{A_A} - A_1
\]
Similarly, for the other side of the weld:
\[
z_2^B = 2\sqrt{\frac{D_\beta t}{\pi}} \frac{-B_2}{(x^{B_\beta} - x^{B_B})} + z_0^B
\]
(7.27)
where
\[
B_2 = -B_1
\]
therefore
\[
z_2^B = 2\sqrt{\frac{D_\beta t}{\pi}} \frac{(x^{A_\alpha} - x^{A_\beta})}{(x^{B_\beta} - x^{B_B})} + z_0^B
\]
(7.28)

7.2 Summary of calculations.

The following equations are a summary of the relevant equations for each model as derived in the previous sections. These are the equations that are utilised in the computer model.

Constant concentration gradient
\[
z_1^A = \sqrt{\frac{2D_\alpha(x^{A_\alpha} - x^{A_\beta})t}{(x^{A_\alpha} - x^{A_A})} + z_0^{A^2}}
\]
(7.29)
In order to solve these equations knowledge is required of the diffusion coefficient of carbon in ferrite, \(D_a\) and \(D_B\) and the partition coefficient, \(k\).

### 7.3 Calculation of the carbon diffusion coefficient in ferrite

The model used in this analysis is due to McLellan et al. (1965). The carbon is assumed to occupy both the tetrahedral and octahedral sites in the ferrite lattice. The observed diffusivity is therefore a weighted sum of the three possible diffusion paths:

i) from an octahedral (\(O\)) site to another \(O\) site will be via a tetrahedral (\(T\)) hole, giving \(O-T-O\);

ii) an atom on a position \(T\) has two alternative paths \(T-O-T\) and \(T-T\).

The diffusion coefficient is given by :

\[
D = \phi D_a^{O-T-O} + (1 - \phi) f D_b^{T-T} + (1 - \phi)(1 - f) D_b^{T-O-T}
\]  

(7.37)

where \(\phi\) is the fraction of the total interstitial atoms in the octahedral sites and \(f\) is the fraction of the remaining interstitial atoms which jump via the \(T-T\) route. The superscripts in the diffusion coefficients indicate the particular jump paths for the carbon atoms. McLellan et al. (1965) give the following expressions for \(\phi\) and \(f\):

\[
\phi = 1 - \left( \frac{1}{2} e^{\frac{E}{RT}} e^{-\frac{\Delta S}{RT}} + 1 \right)^{-1}
\]

100
and

\[ f = 0.86 \]

In this expression, \( \Delta E = E_u^T - E_u^O \) where \( E_u^T \) is the change in Gibbs free energy when a solute atom is placed in a tetrahedral site and \( E_u^T \) is the change in Gibbs free energy when a solute atom is placed in an octahedral site. Similarly \( \Delta S = S_u^T - S_u^O \) where \( S_u^T \) is the change in vibrational entropy when a solvent atom is inserted in a tetrahedral site and \( S_u^O \) is the change in vibrational entropy on insertion of a solute atom into an octahedral site.

7.4 Calculation of the partition coefficient

For the linear model, the following equation was derived for calculating the interfacial carbon concentration, \( x^{\alpha\beta} \):

\[
x^{\alpha\beta} = \frac{(z_i^A x^{\alpha A} + z_j^B x^{\beta B})}{k z_j^B + z_i^A}
\]

In order for there to be any diffusion at all, \( x^{\alpha A} \) must be greater than \( x^{\alpha\beta} \), as Figure 7.3 illustrates. Therefore, from the above equation, for diffusion to start:

\[
x^{\alpha A} > \frac{(z_i^A x^{\alpha A} + z_j^B x^{\beta B})}{k z_j^B + z_i^A}
\]

Rearrangement of this equation yields the following condition for diffusion using the linear gradient model:

\[
x^{\alpha A} k > x^{\beta B} \tag{7.38}
\]

This provides a criterion for predicting a value of \( k \) that:

\[
k > \frac{x^{\beta B}}{x^{\alpha A}}
\]

which enables the best estimate to be made of the methods available to be made. In general it has been found that the models of Wagner and Wada et al. give very similar values for the lower chromium concentration systems (mild steel/1CrMo and mild steel/2CrMo) but that the Wagner model gives a better estimate for the higher chromium concentrations. These models are exactly as have been described previously in Chapter 4.

7.5 Carburised zone width.

On the high alloy side of the weld, the situation is as illustrated in Figure 7.4. Carbon is diffusing away from the low alloy side of the weld causing the carbide B1 to grow. The value of \( z_j^B \) (i.e. the distance to which the particle B1 has grown in time \( t \)) is increasing with time as...
the carbide grows and therefore a carburised zone is forming on this side of the interface. In a similar manner to the decarburised zone, a carburised zone width can also be defined as:

\[ \eta^\beta = (z_j^B - z_o^B) + d_{20} \]  

(7.39)

i.e. the gain in size of the particle as a result of diffusion from the low alloy side \((z_j^B - z_o^B)\) plus the original size of the particle, \(d_{20}\).

![Figure 7.4: Schematic representation of the high alloy side of the weld.](image)

7.6 Calculation of diffusion away from the interface.

Consider what is happening on the high alloy side of the interface. A carbon concentration gradient builds up in this region due to the diffusion of carbon from the low alloy side. Thus, it would be expected that carbon would diffuse away from the interface on the high alloy side of the weld under the influence of this carbon gradient. Figure 7.5 outlines the equilibrium carbon concentrations present between the matrix and alloy carbides on the high alloy side of the interface. Now mass balance requires that:

\[-(x_{B_2B}^{B_2\beta} - x_{B_2B}^{B_2\beta}) \frac{\partial z}{\partial t} = \frac{D(x_{B_2B}^{B_2\beta} - x_{B_2B}^{B_2\beta})}{z} \]  

(7.40)

i.e. the mass absorbed by the carbide must be equal to the mass arriving by diffusion. In this expression the subscript 1 refers to the first slice and the subscript 2 to the second slice, therefore \(x_{B_2B}^{B_2\beta}\) is the equilibrium carbon concentration in the carbide B, in equilibrium with ferrite and \(x_{B_2B}^{B_2\beta}\) is the equilibrium concentration in the ferrite in equilibrium with the carbide.
Similarly, $B_1$ refers to the carbide in the first slice from the interface.

Integrating the above expression:

$$
\int_{z=z_0}^{z} z \, dz = \int_{0}^{t} \frac{d\beta}{Dt} \left( \frac{x^{\beta}B_1 - x^{\beta}B_2}{x^{\beta}B_2 - x^{\beta}B_2} \right)
$$

$z_0$ is the initial distance between the carbides and $z$ is the distance between carbides at some time $t$ after the particle $B_2$ has been allowed to grow (i.e. $z$ decreases with time).

Therefore,

$$
\frac{1}{2} z^2 - \frac{1}{2} z_0^2 = \left( \frac{x^{\beta}B_1 - x^{\beta}B_2}{x^{\beta}B_2 - x^{\beta}B_2} \right) Dt
$$

$$
z^2 = \left( \frac{x^{\beta}B_1 - x^{\beta}B_2}{x^{\beta}B_2 - x^{\beta}B_2} \right) 2Dt + z_0^2
$$

therefore

$$
z = \left( \frac{x^{\beta}B_1 - x^{\beta}B_2}{x^{\beta}B_2 - x^{\beta}B_2} \right)^{\frac{1}{2}} 2Dt + z_0^2
$$

The amount of carbon transferred, $\Delta c$, in time $t$ is therefore given by:

$$
\Delta c = (z_0 - z)(x^{B_2\beta} - x^{\beta}B_2)
$$
This amount is then subtracted from $\bar{\tau}_\beta$ to give a new value for the average carbon concentration on the high alloy side.

7.7 Application of the theory to a computer model

The basic set-up for the computer model is illustrated in Figure 7.6.

![Figure 7.6: Set-up dimensions for the computer model.](image)

It is assumed that the diffusion is only occurring in one dimension perpendicular to the weld interface, so the carbide particles are considered to dissolve or grow in this direction. The dimensions of the model are also illustrated in Figure 7.6. $D_1$ and $D_5$ are the widths of the ferrite slabs on the $\alpha$ and $\beta$ sides of the interface respectively, $D_3$ is the inter-particle spacing on the $\alpha$ side and $D_6$ is the inter-particle spacing in $\beta$. $D_2$ is the particle size on the $\alpha$ side and $D_4$ the particle size on the $\beta$ side of the weld interface. As diffusion occurs from $\alpha$ to $\beta$, particle A1 will dissolve and particle B1 will grow. With reference to Figure 7.2, particle A1 will decrease by an amount $(z_i^A - z_j^A)$ in any time interval, whilst particle B1 will grow by an amount $(z_j^B - z_o^B)$. As the particles dissolve and D2, the particle size, decreases, there comes a point when $D2=0$ and the particle has dissolved. When this occurs the program will step to the next particle, A2, on the low alloy side as illustrated in Figure 7.7. The decarburised zone width will therefore increase in a step-like fashion by an amount $D1 + \frac{D3}{2}$ as far as the calculations are concerned.

7.7.1 Calculation of the modelled dimensions

For this model, it is initially assumed that each ferrite slab contains the equilibrium volume fraction of carbide. Therefore, on the $\alpha$ side, there is $v_\alpha$ volume fraction of carbide and on the $\beta$
side, \( v_\beta \) volume fraction of carbide. These equilibrium volume fractions can be calculated using MTDATA. The value of \( D_2 \) and \( D_4 \), the particle sizes, are initially set at an arbitrary value of 5\( \mu m \) and thus all of the other distances can be determined from the input volume fractions, \( v_\alpha \) and \( v_\beta \), viz.,

\[
D_1 = \frac{D_2}{v_\alpha} \quad \text{and} \quad D_5 = \frac{D_4}{v_\beta} \tag{7.46}
\]
\[
D_3 = D_1 - D_2 \quad \text{and} \quad D_6 = D_5 - D_4 \tag{7.47}
\]

The particle size of 5\( \mu m \) was chosen as it was fairly representative of actual particle sizes taking into account the clustering of the particles found experimentally. As the particles on the low alloy side of the weld start dissolving, then obviously the particle is getting smaller and \( D_2 \) is decreasing by an amount \((z_f^A - z_1^A)\) for every time increment. Conversely on the high alloy side of the weld, the particle size, \( D_4 \), increases with time. Now, the volume fractions of carbide on either side of the interface can be calculated using a rearrangement of equations (7.46). Thus,

\[
v_\alpha = \frac{D_2}{D_1} \quad \text{and} \quad v_\beta = \frac{D_4}{D_5}
\]

and therefore, as \( D_1 \) and \( D_5 \) are constant until a particle has dissolved completely (see section 7.10), the value of \( v_\alpha \) will decrease and the value of \( v_\beta \) will correspondingly increase.

7.7.2 Calculation of the average carbon concentrations

A consequence of the change in volume fractions of carbide on either side of the interface is that the average carbon concentrations in each of the slabs are also going to change; decreasing
on the low alloy side of the interface and increasing on the high alloy side. These changes can be calculated using the formulae:

\[
\bar{x}_\alpha = x^A_\alpha v_\alpha + x^\alpha (1 - v_\alpha) \tag{7.48}
\]

\[
\bar{x}_\beta = x^B_\beta v_\beta + x^\beta (1 - v_\beta) \tag{7.49}
\]

That is, the average carbon concentration in \( \alpha \), \( \bar{x}_\alpha \), is equal to the equilibrium carbon concentration in the carbide, \( x^A_\alpha \), multiplied by the volume fraction of carbide present \( (v_\alpha) \), plus the equilibrium carbon concentration in the ferrite, \( x^\alpha \), multiplied by the volume fraction of ferrite \( (1-v_\alpha) \), and similarly for the \( \beta \) side of the interface.

**7.7.3 Calculation of the partition coefficient.**

If the average carbon concentration changes then the change must affect the calculation of the partition coefficient. Consequently, a new value has to be calculated at the end of every time step using one of the methods previously outlined.

**7.7.4 Changes in equilibrium carbon concentrations.**

As the diffusion of carbon proceeds there is a change in the bulk carbon concentration on either side of the interface. This, in turn, means that the equilibrium carbon concentrations between the carbides and the matrix are constantly changing. However, if MTDATA calculations are run allowing for changes in bulk carbon concentration, then the changes in \( x^A_\alpha \), \( x^\beta \), \( x^A \) and \( x^\beta \) can be plotted as functions of the carbon concentration. An example plot for \( x^A_\alpha \) for mild steel is shown in Figure 7.8 for three different temperatures. Equations can be produced, by this method, for every alloy at every temperature so that the equilibrium carbon concentrations can be recalculated as the carbon concentration changes. When calculating curves such as these, it is important to ensure that the carbon concentration calculated in the program does not exceed the limit of the carbon concentrations for which data points have been calculated. For this reason the plots cover a wide range of carbon concentrations to ensure that accurate values of the equilibrium carbon concentrations are calculated. The equations for all of the alloys are given with the program listing in Appendix III.

Another consequence of the change in bulk carbon concentration is that the carbide that is precipitating also changes as has been observed experimentally. This change in carbide precipitation sequence can also be predicted by MTDATA. Table 7.1 shows that as the carbon concentration is increased there is a reversion in the sequence to carbides that have previously dissolved. This will cause another change in the equilibrium carbon concentrations which can be accounted for in the program by allowing the equations for calculation of \( x^\beta \) and \( x^\beta \) to...
Figure 7.8: Plots of $x_{Ar}$ as a function of the bulk carbon concentration for mild steel at 700°C, 620°C and 565°C.

Table 7.1: MTDATA predictions showing the change in type of carbide with increasing carbon content. The numbers in the boxes indicate the equilibrium volume fractions.

change once the average carbon concentration (predicted by MTDATA) has exceeded that required for lower order carbide precipitation.

The effect of this can be seen in Figure 7.9. This shows that the profiles level off after cementite precipitation and therefore it would be expected that the rate of diffusion or the width of the decarburised zone would also tend to increase at a slower rate, as observed experimentally.
7.8 Testing the model

The model was tested for four experimental joints, mild steel/1CrMo, mild steel/21/4CrMo, 21/4CrMo/9CrMo and 21/4CrMo/12CrMo. The exact steel compositions used in the calculations are given in Table 7.2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Composition /wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>Mild steel/1CrMo weld</td>
<td></td>
</tr>
<tr>
<td>Mild steel plate</td>
<td>0.20</td>
</tr>
<tr>
<td>1CrMo weld</td>
<td>0.056</td>
</tr>
<tr>
<td>Mild steel/21/4CrMo weld</td>
<td></td>
</tr>
<tr>
<td>Mild steel plate</td>
<td>0.23</td>
</tr>
<tr>
<td>21/4CrMo weld</td>
<td>0.079</td>
</tr>
<tr>
<td>21/4CrMo/9CrMo weld</td>
<td></td>
</tr>
<tr>
<td>9CrMo pipe</td>
<td>0.108</td>
</tr>
<tr>
<td>21/4CrMo weld</td>
<td>0.08</td>
</tr>
<tr>
<td>21/4CrMo/12CrMo weld</td>
<td></td>
</tr>
<tr>
<td>12CrMo pipe</td>
<td>0.20</td>
</tr>
<tr>
<td>21/4CrMo weld</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 7.2: Chemical compositions of all of the weld joints.
The convention adopted for all of these joints is that the low alloy (α) side of the weld combination will be written first followed by the high alloy (β) side i.e. mild steel/2\(\frac{1}{4}\)CrMo. Table 7.3 illustrates this further as well as giving the carbides expected in each combination and the change of carbide, if any, that would be expected as the carbon concentration is increased.

<table>
<thead>
<tr>
<th>Alloy combination</th>
<th>α side</th>
<th>β side</th>
<th>carbide A</th>
<th>carbide B</th>
<th>Change in carbide B to</th>
</tr>
</thead>
<tbody>
<tr>
<td>mild steel/1CrMo</td>
<td>mild steel</td>
<td>1CrMo</td>
<td>Fe(_3)C</td>
<td>M(_{23})C(_6)</td>
<td>Fe(_3)C</td>
</tr>
<tr>
<td>mild steel/2(\frac{1}{4})CrMo</td>
<td>mild steel</td>
<td>2(\frac{1}{4})CrMo</td>
<td>Fe(_3)C</td>
<td>M(_{23})C(_6)</td>
<td>Fe(_3)C</td>
</tr>
<tr>
<td>2(\frac{1}{4})CrMo/9CrMo</td>
<td>2(\frac{1}{4})CrMo</td>
<td>9CrMo</td>
<td>M(_{23})C(_6)</td>
<td>M(_7)C(_3)</td>
<td>no change</td>
</tr>
<tr>
<td>2(\frac{1}{4})CrMo/12CrMo</td>
<td>2(\frac{1}{4})CrMo</td>
<td>12CrMo</td>
<td>M(_{23})C(_6)</td>
<td>M(_{23})C(_6)</td>
<td>no change</td>
</tr>
</tbody>
</table>

Table 7.3: Classification of alloy combinations.

The volume fractions of carbide phases present are calculated from the weight fraction of phase output by MTDATA and using the density of that phase to calculate the resultant volume fractions. The weight fractions and volume fractions of the equilibrium carbide present are tabulated in Table 7.4 for each alloy combination. It is assumed in this model that only one carbide is present at any time. In the case of 2\(\frac{1}{4}\)CrMo, more than one carbide is expected at equilibrium. The carbide with the largest volume fraction, M\(_{23}\)C\(_6\), is therefore chosen. Similarly, as the increase in carbon content causes a change in the carbide precipitating, it is assumed that Fe\(_3\)C is the sole carbide present.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Alloy combination</th>
<th>carbide A</th>
<th>carbide B</th>
<th>wt. frac A</th>
<th>wt. frac B</th>
<th>(v_\alpha)</th>
<th>(v_\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>700°C</td>
<td>mild steel/2(\frac{1}{4})CrMo</td>
<td>Fe(_3)C</td>
<td>M(_{23})C(_6)</td>
<td>0.0326</td>
<td>0.0155</td>
<td>0.0337</td>
<td>0.0178</td>
</tr>
<tr>
<td>700°C</td>
<td>mild steel/1CrMo</td>
<td>Fe(_3)C</td>
<td>M(_{23})C(_6)</td>
<td>0.0326</td>
<td>0.0109</td>
<td>0.0337</td>
<td>0.0111</td>
</tr>
<tr>
<td>730°C</td>
<td>2(\frac{1}{4})CrMo/9CrMo</td>
<td>M(_{23})C(_6)</td>
<td>M(_7)C(_3)</td>
<td>0.0153</td>
<td>0.0209</td>
<td>0.0175</td>
<td>0.0240</td>
</tr>
<tr>
<td>730°C</td>
<td>2(\frac{1}{4})CrMo/12CrMo</td>
<td>M(_{23})C(_6)</td>
<td>M(_{23})C(_6)</td>
<td>0.0153</td>
<td>0.0385</td>
<td>0.0175</td>
<td>0.0450</td>
</tr>
</tbody>
</table>

Table 7.4: Input data for volume fractions for all the weld combinations. The density of M\(_{23}\)C\(_6\) is taken to be 6.996 g/cm\(^3\), the density of Fe\(_3\)C as 7.704 g/cm\(^3\) and the density of M\(_7\)C\(_3\) as 6.965 g/cm\(^3\), Andrews et al. (1967)

7.9 One particle system

In order to illustrate clearly what the program is doing, the first results presented are for a one particle system (i.e. there is one particle dissolving and one growing on either side of
the interface). All of these calculations were carried out using the exponential model and the experimental joint mild steel/2\(\frac{1}{4}\)CrMo at a temperature of 700°C. As a recap, mild steel is the \(\alpha\) side of the interface with the carbide Fe\(_3\)C dissolving and 2\(\frac{1}{4}\)CrMo is the \(\beta\) side of the interface with the carbide M\(_{23}\)C\(_6\) precipitating. For a one particle system the carbon concentration on the \(\beta\) side does not increase sufficiently to cause precipitation of Fe\(_3\)C. Figure 7.10 illustrates the variations in the particle sizes.

![Figure 7.10](image)

**Figure 7.10:** Change in D2 and D4 with root time for a one particle system using the exponential model: mild steel/2\(\frac{1}{4}\)CrMo, carbide A=Fe\(_3\)C, carbide B=M\(_{23}\)C\(_6\), 700°C

Both D2 and D4 start at the initial value set (i.e. 5\(\mu\)m), D2 decreases until the particle size reaches zero when it has dissolved and D4 increases in a linear fashion. As a result of the changes in particle size, the volume fractions of carbide (Figure 7.11(a)) and the average carbon content (Figure 7.11b) in the slice also behave in a similar manner.

The reason that the average carbon content in the low alloy side does not reach zero when the carbide has dissolved is due to the carbon concentration in the ferrite, from equation 7.48, contributing to the average carbon concentration. For the one particle system, the value of \(z^A_i\) and \(z^B_i\) are also increasing giving rise to the corresponding graphs for carburised and decarburised zone widths (Figure 7.12 and Figure 7.13).

### 7.10 Multi–particle system

During the dissolution of one particle all of the other variables discussed earlier remain constant. However, once a particle has dissolved the value of D1 is stepped as shown in Figure 7.14 and the value of D2 is reset to the original value. It is at this point that all of the
Figure 7.11: a) Change in $v_\alpha$ and $v_\beta$ with root time for a one particle system. b) Change in $x_\alpha$ and $x_\beta$ with root time for a one particle system using the exponential model: mild steel/2%CrMo, carbide A=Fe$_3$C, carbide B=M$_{23}$C$_6$, 700°C.

Figure 7.12: Change in $z_i^A$ and $z_i^B$ with root time for a one particle system using the exponential model: mild steel/2%CrMo, carbide A=Fe$_3$C, carbide B=M$_{23}$C$_6$, 700°C.

equilibrium carbide concentrations, volume fractions of carbides and average carbon concentrations are recalculated.

In the following calculations the same conditions have been used as for the one particle system calculations (i.e. mild steel/2%CrMo, carbide A=Fe$_3$C, carbide B=M$_{23}$C$_6$, 700°C). However, once the average carbon concentration $x_\beta$ has exceeded 0.4 wt%, the carbide B becomes
Figure 7.13: Change in carburised and decarburised zone widths between initial and final conditions for a one particle system using the exponential model: mild steel/\(2\frac{1}{4}\)CrMo, carbide A=Fe\(_3\)C, carbide B=M\(_{23}\)C\(_6\), 700°C.

Fe\(_3\)C. The carbon concentration at which the cementite appears was calculated using MTDATA. Figure 7.15 illustrates the changes that occur in the interfacial concentrations \(x^A\) and \(x^B\).

Figure 7.15: Change in interfacial carbon concentrations with root time: mild steel/\(2\frac{1}{4}\)CrMo, carbide A=Fe\(_3\)C, carbide B=M\(_{23}\)C\(_6\)→Fe\(_3\)C when \(x^B \geq 0.4\)wt%, 700°C.

It can be seen from these diagrams that the values of these variables are converging as successive particles are dissolved. This translates to a levelling off of the respective concentration gradients as illustrated in Figure 7.16.

The 'flattening' of the carbon concentration gradients accounts for the levelling off of the decarburised zone width against root time plots observed experimentally, because the driving
force for diffusion is levelling off. The carbon concentrations in the carbide $x^{B\beta}$ and $x^{A\alpha}$ also vary with carbon concentration as illustrated in Figure 7.17 and predicted by MTDATA plots such as Figure 7.8.

Figure 7.18 illustrates how the decarburised zone width increases with time in a step-like fashion. Although all of the results presented here are from the exponential model, the linear model predicts the same trends.

7.11 Conclusions

Two models were presented in this chapter for the prediction of decarburised zone widths. From the experimental work of Chapter 6, two major requirements of these models were that they account for the precipitation sequences occurring in dissimilar metal welds during tempering and that they predict non-parabolic growth of the decarburised zone. Both of the models
Figure 7.16: Change in carbon concentration gradients: mild steel/\(2\frac{1}{4} \text{CrMo}\), carbide A=Fe₃C, carbide B=M₂₃C₆→Fe₃C when \(\bar{x}_p \geq 0.4\text{wt\%}, 700^\circ\text{C}\).

Figure 7.17: Change in \(x^{B_\beta}\) and \(x^{A_\alpha}\) with root time: mild steel/\(2\frac{1}{4} \text{CrMo}\), carbide A=Fe₃C, carbide B=M₂₃C₆→Fe₃C when \(\bar{x}_p \geq 0.4\text{wt\%}, 700^\circ\text{C}\).

successfully meet this criteria.
Figure 7.18: Change in decarburised zone width with root time: mild steel/2\frac{1}{4}CrMo, carbide A=Fe_3C, carbide B=M_{23}C_6\rightarrow Fe_3C when $\bar{\delta} \geq 0.4\text{wt\%}$, 700°C.
CHAPTER 8

Applications of the model

8.1 Introduction

As a result of the experimental work into the diffusion of carbon across dissimilar ferritic/ferritic welds, there were four points that arose that required addressing and modelling:

i) The decarburised zone width should show non-parabolic kinetics i.e. it should level off on a plot against root time.

ii) The decarburised zone width should increase with both time and temperature.

iii) The model should be able to predict the extent of decarburisation with respect to the difference in substitutional alloy content across the welded joint.

iv) The model should be able to predict the extent of carburisation in relation to the extent of decarburisation.

The work presented here is intended to illustrate how the models cope with these points and to go on to explain how it has found industrial application in the selection of material for dissimilar metal joints.

8.2 Levelling of the decarburised zone

This is probably the most significant result regarding these dissimilar metal joints from the point of view of the manufacturer as it suggests that a maximum decarburised zone width is reached rather than a continual growth over the service life of the weld. The reason for the levelling off is thought to be a result of the change in bulk carbon concentrations of the alloys at the interface. This causes both a change in the carbide that is precipitating and a change in the interfacial carbon concentrations resulting in a levelling of the respective carbon concentration gradients as was illustrated in Figure 7.9. It is therefore necessary to account for these changes in the model, which is achieved via the MTDATA calculations described in the previous chapter. As an illustration of this, plots are presented in Figure 8.1 where the interfacial carbon concentrations $x^A_\alpha$, $x^A_\beta$, $x^B_\beta$ and $x^B_\beta$ are kept constant (Figure 8.1a) and are allowed to change according to the equations calculated using MTDATA. These plots show how the decarburised zone widths level off as a result of changes in the equilibrium carbon concentrations owing to a change in the bulk carbon concentration. It should be noted from Figure 8.1 that the decarburised zone width does not start at zero time but at a value
corresponding to $z_o^A$. The smooth profile is obtained by joining the tops of the steps of a plot such as Figure 7.18.

8.3 Decarburised zone increase with time and temperature

8.3.1 Linear model

Predicted and experimental values of the decarburised zone width are compared for the linear model in Figure 8.2 for the mild steel/1CrMo, mild steel/2\(\frac{1}{4}\)CrMo joint and the 2\(\frac{1}{4}\)CrMo/9CrMo joint. These plots illustrate that the linear model successfully predicts an increase in the decarburised zone width with both time and temperature for both these joints. The prediction of the actual decarburised zone width values, although rather high, is fairly accurate, especially at the shorter times.

8.3.2 Problems with the linear model

8.3.2.1 Mass balance breakdown

The reason for this is illustrated in Figure 8.3. As each successive particle is dissolved the gradient is instantaneously switched from situation 1 to situation 2 and a certain amount of carbon is apparently created from nowhere. At the shorter times or lower temperatures where few particles have been dissolved, this discrepancy is negligible but is exacerbated as more particles dissolve at longer times or higher temperatures.

Figure 8.1: Illustration of the effect of a) keeping the equilibrium carbon concentrations constant and b) allowing them to change as a function of the bulk carbon concentration. This calculation was carried out for the mild steel/2\(\frac{1}{4}\)CrMo joint at 700°C using the linear model, $d_{20} = 5\mu m$, and using the Wagner model to calculate the partition coefficient.
Figure 8.2: Predicted plots for the linear model: Mild steel/1CrMo a) at 620°C b) at 700°C, mild steel/2\%CrMo c) at 620°C d) at 700°C, 2\%CrMo/9CrMo e) at 620°C f) at 730°C. For all of these calculations the Wagner model was used to calculate the partition coefficient and \( d_{20} = 5\mu m \)
8.3.2.2 Stopping the model

The plots of Figure 8.2 showed that the prediction of the decarburised zone width stops before the time allowed for diffusion has lapsed. (In all these simulations the program was run for 10,000 hours.) If reference is made back to the equations for the calculation of the diffusion
distances:—

\[ z_i^A = \sqrt{\frac{2D_\alpha (x_\alpha^A - x_\alpha^B)t}{(x_\alpha^A - x_\alpha^B)}} + z_0^A \quad (8.1) \]

\[ z_j^B = \sqrt{\frac{2D_\beta (x_\beta^A - x_\beta^B)t}{(x_\beta^A - x_\beta^B)}} + z_0^B \quad (8.2) \]

it can be seen that the model will breakdown if

\[ x_\alpha^A < x_\alpha^B \]

or

\[ x_\beta^A < x_\beta^B \]

In relation to the relevant carbon concentration gradients, this effectively means that diffusion is no longer going to take place from \( \alpha \) to \( \beta \) (Figure 8.4). This results in the model stopping ‘prematurely’. The effect of this is most dramatic for the 2 1/4CrMo/9CrMo joint at 620°C which has stopped after only one particle has been dissolved.

![Figure 8.4: Concentration gradients that cause the model to stop prematurely.](image)

8.3.2.3 Problem with the 2 1/4CrMo/12CrMo joint

The problem with this joint in the linear model is that the model stops before the first particle has dissolved for the reasons mentioned in the previous section. The calculations of \( x_\beta^B \) and \( x_\beta^B \) are proceeding within the limits of the MTDATA calculations but are causing \( x_\beta^B \) to be greater than \( x_\beta^A \) quite quickly. If the values of \( x_\beta^B \) and \( x_\beta^B \) are allowed to remain constant then the plot in Figure 8.5 is obtained which proves that the problem simply lies in
Figure 8.5: $\frac{1}{2}$CrMo/12CrMo joint at 730°C illustrating the effect of keeping the equilibrium carbon concentrations constant; $d_{20} = 5 \mu m$, Wagner model used for calculation of $k$.

The conclusion is therefore that, although the linear model gives accurate results for some joints it cannot be relied upon for joints with higher solute concentrations and calculations at longer times and higher temperatures.

8.3.3 Exponential model

The exponential model also predicts an increase in decarburised zone width with both time and temperature as illustrated in Figure 8.6 for the mild steel/$\frac{1}{2}$CrMo joint at 700°C and 620°C.

In comparison with the experimental data (Figure 8.7) it can be seen that these predictions are slightly low. However, the exponential model does have a major application in that it allows comparisons to be made between different joints.

8.4 Effect of the difference in substitutional alloy concentration

In order to be able to compare alloy systems, the calculations have to be made using the same method for the calculation of the partition coefficient and the same value of the particle size. The exponential model is more sensitive to particle size than the linear model in that the slower kinetics at lower temperatures require a smaller particle size to be set. This is more in keeping with what is observed experimentally. The actual values of the decarburised zone width are then not really relevant as long as a comparison can be made. The exponential model
allows this to be done because it does not suffer from the same problems of mass balance that the linear model does so it can run for all of the systems. The experimental data presented in Chapter 6 revealed the following sequences for the degree of carburisation:

At the post weld heat treatment temperature:-

\[ 2\frac{1}{4}\text{CrMo/12CrMo} > (2\frac{1}{4}\text{CrMo/9CrMo} \& \text{mild steel/2\frac{1}{4}CrMo}) > \text{mild steel/1CrMo} \]

At 620°C:-

(mild steel/2\frac{1}{4}CrMo and mild steel/1CrMo) > (2\frac{1}{4}CrMo/9CrMo and 2\frac{1}{4}CrMo/12CrMo)

The graphs presented in Figure 8.8 show the calculations made from the exponential model revealing the same types of sequences in the extent of decarburisation.

There are two interesting things to note from these plots. Firstly, the model successfully predicts the change in sequence of the alloys between the two temperatures. It might be expected that the 2\frac{1}{4}CrMo/12CrMo and 2\frac{1}{4}CrMo/9CrMo would still exhibit higher degrees of decarburisation than either of the mild steel joints at both temperatures. Christoffel and Curran also noted this phenomenon, but were unable to explain it other than to indicate that it was not the absolute difference in alloy content that was important, as might be expected, but the particular alloy combinations involved. In an attempt to try and account for these changes, the initial carbon concentrations at the interface have been plotted for the mild steel/2\frac{1}{4}CrMo and 2\frac{1}{4}CrMo/12CrMo at both temperatures, Figure 8.9. Both joints show steeper gradients at the higher temperature which would be expected as the degree of decarburisation is greater at higher temperatures. However, the mild steel/2\frac{1}{4}CrMo joint shows steeper

![Figure 8.6: Mild steel/2\frac{1}{4}CrMo joint decarburised zone width predictions using the exponential model; \( d_{20} = 5\mu m \). The Wagner model was used to calculate the partition coefficient.](image)
Figure 8.1: Predicted plots for the exponential model at the post weld heat treatment temperature a) Mild steel/1CrMo b) Mild steel/2\frac{1}{4}CrMo c) 2\frac{1}{4}CrMo/9CrMo d) 2\frac{1}{4}CrMo/12CrMo. For all of these calculations the Wagner model was used to calculate the partition coefficient and $d_{20} = 5\mu m$.
gradients than the $2\frac{1}{4}$CrMo/12CrMo at both temperatures. Also, in all cases $z^{\alpha A} > z^{\beta B}$ except for the $2\frac{1}{4}$CrMo/12CrMo joint at 620°C, where $z^{\alpha A} < z^{\beta B}$. The factors governing the extent of decarburisation would therefore seem to be a complex balance between the type of carbide on either side of the weld interface and the values of $z^{\alpha A}$ and $z^{\beta B}$.

Figure 8.9: Carbon concentration gradients for a) mild steel/$2\frac{1}{4}$CrMo at 700°C and 620°C and b)$2\frac{1}{4}$CrMo/12CrMo at 730°C and 620°C
Secondly, the model predicts the crossing over of the plots for the $2\frac{1}{4}$CrMo/9CrMo and mild steel/$2\frac{1}{4}$CrMo joints. It was mentioned in Chapter 6 that it was impossible to determine which of these joints had the greater degree of decarburisation because the plots overlap, Figure 8.10.

![Graph showing decarburised zone widths](image)

**Figure 8.10:** Experimental decarburised zone widths for the mild steel/$2\frac{1}{4}$CrMo and $2\frac{1}{4}$CrMo/9CrMo joint at the post weld heat treatment temperature.

If the exponential model is run to longer times the plot of Figure 8.11 is obtained which shows a similar overlapping. This phenomenon was also observed on running the linear model to longer times. The mild steel/$2\frac{1}{4}$CrMo joint is therefore seen to start off more quickly than the $2\frac{1}{4}$CrMo/9CrMo joint but will level out more quickly. This has to be an important consideration in dissimilar metal weld design and the ability of this model to able to predict factors such as this is extremely useful.

### 8.5 The effect of carburisation

The predicted carburised zone width is shown in Figure 8.12 for the mild steel/$2\frac{1}{4}$CrMo joint at 700°C.

It can be seen that the carburised zone width levels off in a similar manner to the experimental results. There is, however, a slight discontinuity in the graph corresponding to the onset of cementite precipitation where the values of $x^\beta B$ and $z^B B$ are changed. The plot of the predicted decarburised and carburised zone width of Figure 8.13 for this joint indicates that
Mild steel/2.25CrMo

![Graph](image)

**Figure 8.11:** Exponential model predictions of the decarburised zone width for the 2\(\frac{1}{4}\)CrMo/9CrMo and mild steel/2\(\frac{1}{4}\)CrMo joint showing the overlapping of the plots.

![Graph](image)

**Figure 8.12:** Carburisation effect for the mild steel/2\(\frac{1}{4}\)CrMo joint at 700°C.

the carburisation effect is less than the decarburisation effect as was noted experimentally in Figure 6.11.

8.6 Conclusions and further applications of the model

The conclusions from the results of the modelling work presented here are:-

1. The model will predict the non-parabolic kinetics of the decarburisation process. This is a result of allowing the equilibrium carbon concentrations in the ferrite and carbides to change as a function of the bulk carbon concentration.
Figure 8.13: Comparison of predicted carburised and decarburised zone widths for the mild steel/2\(\frac{1}{4}\)CrMo joint at 700°C using the linear model.

2. The linear model will predict an 'upper bound' for the extent of decarburisation in a joint. However, owing to assumptions made about the mass balance on changing a particle and the difficulty of obtaining accurate thermodynamic data, it is recommended that this model is only used for simple systems and not any system containing more than 9wt% chromium.

3. The exponential model predicts much slower kinetics, but, as a result, predictions can be made for more alloy systems. This means that this model can be used to compare the extent of decarburisation in different alloy systems and those containing higher chromium concentrations. A general rule seems to be that decarburisation is minimised if there is the same type of carbide on either side of the interface e.g. \(\text{M}_{23}\text{C}_6\) in the \(2\frac{1}{4}\)CrMo/12CrMo joint at 620°C gives less decarburisation than \(\text{Fe}_3\text{C}\) and \(\text{M}_{23}\text{C}_6\) for a mild steel/2\(\frac{1}{4}\)CrMo joint. This also accounts for the reason why the mild steel/2\(\frac{1}{4}\)CrMo joint levels off more quickly than the 2\(\frac{1}{4}\)CrMo/9CrMo joint at the post weld heat treatment temperature because the carbide type changes to being the same on either side.

Now that the models have been established, further applications for them can be envisaged. The ability to compare the extent of decarburisation in different joints makes the exponential model a useful tool in alloy design for dissimilar metal welds. In fact, this model has already found industrial application in this area. This property of the model can also be extended to actually designing alloys that will produce the smallest decarburised zone widths.

As decarburisation can occur during post weld heat treatment and during subsequent service it should be possible to change the temperature during the calculation and monitor the effect of an initial decarburised zone width on subsequent diffusion.
CHAPTER 9
Conclusions and Further Work

9.1 Conclusions

The aim of this work was to study the diffusion of carbon across dissimilar steel welds and ultimately to be able to model this process in both the austenite and ferrite phase fields. The major conclusions can be summarised as follows:-

1. Modelling of carbon diffusion in austenite can be achieved by the combination of equations to describe diffusion in semi-infinite media. The only limitation to this is the prediction of the partition coefficient, especially in concentrated solutions.

2. Modelling of carbon diffusion in ferrite is more complicated as the carbon available for diffusion is tied up in the form of carbides. Any model for carbon diffusion in ferrite must take this into account.

3. In ferritic/ferritic joints, the decarburised zone widths do not exhibit the expected parabolic kinetics but level off on a plot against root time.

4. The transport of carbon across the weld interface causes a reversion in the normal precipitation sequence.

5. The effect of the change in the bulk carbon concentration on either side of the interface causes the decarburised zone width to level off.

6. Two models have been developed for the prediction of decarburised zone widths, one which assumes a linear carbon concentration gradient and one which assumes an exponential profile.

7. The linear model for the prediction of decarburised zone widths will predict an 'upper bound' for the lower alloy combinations.

8. The exponential model will successfully predict trends in the extent of decarburisation although the actual values tend to be low. This has enabled this model to find industrial application as a tool in materials selection for dissimilar metal welds.

9. A general principle is that the carbon migration at dissimilar ferritic steel joints can be minimised by ensuring that the same kind of carbide is stable on either side of the joint. This phenomenon also explains the deviation from the parabolic rule at long times.

9.2 Further work

Possible further work on dissimilar metal welds could lie in the following areas:-
1. Finding ways of healing the microstructure.

9.2.1 Healing the decarburised microstructure

A small study into the feasibility of healing the decarburised microstructures was started during the course of this work. The ‘healing’ process involves reheating the microstructure into the austenite phase field to dissolve the carbides and then cool to room temperature. The initial trials were carried out on two mild steel/$2\frac{1}{4}$CrMo specimens which had received a heat treatment of 4096 hours at 700°C. MTDATA calculations revealed that full dissolution of the carbides should occur at 900°C, so a heat treatment of 1 hour at 950°C was applied to ensure complete dissolution and equilibration of the microstructure. One of the specimens was then quenched into water and the other air cooled to room temperature. The starting microstructure and resultant microstructures are illustrated in Figure 9.1. Hardness profiles across all three joints are provided in Figure 9.2.

It can be seen that the quenched sample, whilst regaining some of its original hardness has cracked at the position corresponding to the peak in the hardness profile. However, the air cooled sample, although still being relatively soft, has a more level profile, indicating that the discontinuity at the interface has decreased. However, more work is required to discover whether other properties such as toughness have recovered.

9.2.2 Prediction of decarburisation across ferritic/austenitic joints

The ferritic/austenitic type of dissimilar metal welds, although not studied at all in this work, make up a large proportion of the welds found in power station boilers. Diffusion in this case occurs from the ferrite to the austenitic side of the weld. Using the program for calculating diffusion profiles across austenitic/austenitic joints and combining this with the program for calculating the extent of diffusion in ferritic/ferritic joints, it should be possible to make predictions for the ferritic/austenitic system.
Figure 9.1: Interface microstructure for the mild steel/2\frac{1}{4}CrMo joint a) initial microstructure, b) air cooled microstructure c) quenched microstructure.
c) Quenched microstructure

Figure 9.1: Interface microstructure for the mild steel/2$\frac{1}{4}$CrMo joint a) initial microstructure, b) air cooled microstructure c) quenched microstructure.

Figure 9.2: Hardness profile across all three joints
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Appendix I

The differential of the error function

This differential is required when solving the boundary condition of continuous flux across the interface and the derivation of the solution is as follows:

\[ \text{erf}(bz) = \frac{2}{\sqrt{\pi}} \int_0^{bz} \exp\{-x^2\} dx \]

where in this case,

\[ b = \frac{1}{2\sqrt{Dt}} \]
\[ x = \frac{z}{2\sqrt{Dt}} \]

If \( \text{erf}(bz) \) is defined as \( f(z) \) then,

\[ f(z) = \frac{2}{\sqrt{\pi}} \int_0^{bz} \exp\{-x^2\} dx \]

Also if \( y = bz \) then,

\[ g(y) = \frac{2}{\sqrt{\pi}} \int_0^{y} \exp\{-x^2\} dx \]

Therefore, \( f(z) = g(y) \), so

\[ \frac{\partial f}{\partial z} = \frac{\partial g}{\partial z} = \frac{\partial g}{\partial y} \frac{\partial y}{\partial z} \]

Therefore

\[ \frac{\partial f}{\partial z} = \frac{2}{\sqrt{\pi}} \exp\{-x^2\} b \]
\[ \frac{\partial \{\text{erf}(bz)\}}{\partial z} = \frac{2}{\sqrt{\pi}} \exp\{-\frac{z^2}{4Dt}\} \frac{1}{2\sqrt{Dt}} \]
\[ \frac{\partial \{ \text{erf}\left(\frac{z}{2\sqrt{Dt}}\right) \}}{\partial z} = \frac{2}{\sqrt{\pi}} \exp\{-\frac{z^2}{4Dt}\} \frac{1}{2\sqrt{Dt}} \]
Appendix II
Program listing for calculating diffusion profiles with austenite on both sides of the interface
This program is for considering the diffusion couple
with austenite on both sides of the interface

The alpha side of the interface is the high carbon activity side
The theta side of the interface is the low carbon activity side
A1 is the constant calculated from the boundary conditions in Chapter 3
B1 is the constant calculated from the boundary conditions in Chapter 3
CA is the carbon concentration in alpha at some position X after some time T
CG is the carbon concentration in theta at some position X after some time T
CAO is the carbon concentration in the high carbon activity side of the weld distant from the interface
CGO is the carbon concentration in the low carbon activity side of the weld distant from the interface
CAG is the carbon concentration at the weld junction on the alpha side
CGA is the carbon concentration at the weld junction on the theta side
DA is the diffusion coefficient in the alpha component
DG is the diffusion coefficient in the theta component
K TEMP is the absolute temperature
THR is the time in hours
T is the time in seconds
W is the carbon interaction energy in austenite
WX1 is the weight percent of element X on the high carbon activity side.
WX2 is the weight percent of element X on the low carbon activity side.
XX1 is the mole fraction of element X on the low carbon activity side.
XX2 is the mole fraction of element X on the high carbon activity side.
X is the distance on either side of the interface in metres
XMM is the distance on either side of the interface in millimetres

IMPLICIT DOUBLE PRECISION(A-H,K,N-Z), INTEGER(I,J,L,M)
DIMENSION D1(30), D2(30), Y1(30), Y2(30), YC1(30), YC2(30)
AVER=2.0

Subroutines LOGO, DATA, DATA1 and DATA2 and DDTA all collect and display the data with options to make any changes
Subroutine OMEGA calculates the carbon-carbon interaction energy for each side of the weld

CALL LOGO(AVER)
OPEN(2, FILE='AUS.RES')
CALL DATA(K TEMP, THR)
CALL DATA1(WC1, WMN1, WSI1, WNI1, WCR1, WMO1, WW1, WNB1, WCO1)
CALL DATA2(WC2, WMN2, WSI2, WNI2, WCR2, WMO2, WW2, WNB2, WCO2)
CALL OMEGA(WC1, WSI1, WMN1, WNI1, WCR1, WMO1, WW1)
CALL OMEGA(WC2, WSI2, WMN2, WNI2, WCR2, WMO2, WW2)
WRITE(*,**)' Is the diffusion driven by: '
WRITE(*,**)' 1. Silicon'
WRITE(*,**)' 2. Chromium'
CALL REEDI(JY)

500 CALL DDTA(WC1, WMN1, WSI1, WNI1, WCR1, WMO1, WW1, WNB1, WCO1,
& WC2, WMN2, WSI2, WNI2, WCR2, WMO2, WW2, WNB2, WCO2, K TEMP, THR, W1, W2, JY)

Subroutine CON converts the data from weight per cent to mole fraction
CALL CON(WC1, WMN1, WSI1, WNI1, WCR1, WMO1, WW1, WNB1, WCO1,
& XC1, XMN1, XSI1, XNI1, XCR1, XM01, XW1, XV1, XNB1, XCO1)
These equations calculate the initial guesses of CAG and CAG in order that a value of the partition coefficient can be calculated (Section 4.4)

The subroutine DIFFF calculates the diffusion coefficient of carbon in austenite

\begin{align*}
CAG &= XCl \times 0.66D0 \\
CGA &= XC2 \times 1.33D0
\end{align*}

\begin{verbatim}
IF (CAG .LT. CAO) THEN
   CALL DIFFF(W1,CAG,CAO,KTEMP,DA)
ELSE
   CALL DIFFF(W1,CAO,CAG,KTEMP,DA)
ENDIF
IF (CGO .LT. CGA) THEN
   CALL DIFFF(W2,CGO,CGA,KTEMP,DG)
ELSE
   CALL DIFFF(W2,CGA,CGO,KTEMP,DG)
ENDIF
\end{verbatim}

The KFUN subroutines calculate the partition coefficient by one of the methods below

\begin{verbatim}
WRITE(*,*)' For the calculation of partition coefficient,'
WRITE(*,*)' Option 1 = Wagner'
WRITE(*,*)' Option 2 = Uhrenius'
WRITE(*,*)' Option 3 = Wada et al.'
WRITE(*,*)' Option 4 = Experimental value'
WRITE(*,*)' Select the method number to continue.'
CALL REEDI(J)
IF (J .EQ. 1) THEN
   CALL KFUN(CAO,XMN1,XSI1,XN11,XCR1,XM01,XCU1,XW1,XV1,XNB1,XCO1,CGO,XMN2,XSI2,XN12,XCR2,XMO2,XCU2,XW2,XV2,XNB2,XCO2,KTEMP,K)
ELSEIF (J .EQ. 2 .AND. JY .EQ. 1) THEN
   CALL KFUN2(XSI1,CAO,XSI2,CGO,KTEMP,K)
ELSEIF (J .EQ. 2 .AND. JY .EQ. 2) THEN
   CALL KFUN6(XCR1,CAO,XCR2,CGO,KTEMP,K)
ELSEIF (J .EQ. 3 .AND. JY .EQ. 1) THEN
   CALL KFUN4(WSI1,WCl,KTEMP,WC2,WSI2,K)
ELSEIF (J .EQ. 3 .AND. JY .EQ. 2) THEN
   CALL KFUN7(WCR1,WC1,KTEMP,WC2,WCR2,K)
ELSEIF (J .EQ. 5) THEN
   WRITE(*,*)' Input value of k.'
   CALL REED(K)
   IF (K .GT. 1.0D+02 .OR. K .LT. 0.0D0) THEN
      CALL BOUND(K,1.0D+02,0.0D0)
   ENDIF
ENDIF
\end{verbatim}

\begin{verbatim}
A1 = (CGO + (CAO*DSQRT(DA/DG)))/(K + DSQRT(DA/DG))
CAG = A1
CGA = K*A1
IF (CAG .LT. CAO) THEN
   CALL DIFFF(W1,CAG,CAO,KTEMP,DA)
ELSE
   CALL DIFFF(W1,CAO,CAG,KTEMP,DA)
ENDIF
\end{verbatim}

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IF (CGO .LT. CGA) THEN
   CALL DIFFF(W2,CGO,CGA,KTEMP,DGN)
ELSE
   CALL DIFFF(W2,CGA,CGO,KTEMP,DGN)
ENDIF

DAN and DGN are the values of the diffusion coefficient produced in the iteration process

\[ N = \frac{DA}{DAN} \]

This IF-ELSE loop determines whether the iteration continues

IF ( N .LT. 1.05 .AND. N .GT. 0.95) THEN
   GOTO 30
ELSE
   DA=DAN
   DG=DGN
   GOTO 100
ENDIF

30 T=THR*3600

This DO loop produces the values of CA and CG at various values of XMM

\[ M = 0 \]

DO 40 I = 0,25
   X=I*1.0D-03
   XMM=X*1.0D+03
   A1=(CGO+(CAO*DSQRT(DA/DG)))/(K+DSQRT(DA/DG))
   B1=CAO-A1
   CA=A1+(B1*DERF((X/(2.0*DSQRT(DA*T)))))
   CG=(K*A1)-(DSQRT(DA/DG)*B1*DERF((X/(2.0*DSQRT(DG*T)))))
   CAG=A1
   CGA=K*A1
   NEG=-XMM
   M = M + 1
   D1(M)=XMM
   Y1(M)=CA
   D2(M)=NEG
   Y2(M)=CG
40 CONTINUE

All of the calculations are made in mole fraction and subroutine CONVE converts the concentrations back to wt%:

\[ \text{CALL CONVE(M,Y1,XMN1,XSI1,XNI1,XCR1,XMO1,XW1,XV1,XNB1,XCO1,} \]
\[ \text{& YC1)} \]
\[ \text{CALL CONVE(M,Y2,XMN2,XSI2,XNI2,XCR2,XMO2,XW2,XV2,XNB2,XCO2,} \]
\[ \text{& YC2)} \]

The results are printed to the file AUS.RES in subroutine RESULT, but there is also an option to print to the screen, subroutine RESP or for a plot, subroutine PLOT

\[ \text{CALL RESULT(K,DA,DG,T,THR,D1,YC1,YC2,M)} \]
\[ \text{WRITE(*, *)' Results are in the file AUS.RES'} \]
\[ \text{WRITE(*,*)' Do you want a screen printout?'} \]
\[ \text{WRITE(*,*)' 1 _______ YES '} \]
\[ \text{WRITE(*,*)' 2 ______ NO '} \]
\[ \text{CALL REEDI(LYES)} \]
IF (LYES .EQ. 1) THEN
   CALL RESP(K,CAG,CGA,DA,DG,T,THR,D1,YC1,YC2,M)
ELSE

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The data routines input the data to the program. Every value is checked to ensure that it lies within the preset bounds for that variable. If it does not the option is given to enter another value or continue with the value inputed, although no guarantee is given for the success of the run!

SUBROUTINE DATA(KTEMP,THR)
IMPLICIT DOUBLE PRECISION (A-H,K,N-Z), INTEGER(I,J,L,M)
WRITE(*,8)
8 FORMAT(10X,' Please enter the following information:-')
WRITE(*,1)
1 FORMAT(10X,' Temperature in Kelvin')
CALL REED(KTEMP)
IF (KTEMP .GT. 1700 .OR. KTEMP .LT. 900) THEN
CALL BOUND(KTEMP,1700.0D0,900.0D0)
ENDIF
WRITE(*,4)
4 FORMAT(10X ' Time in hours')
CALL REED(THR)
IF (THR .GT. 1.0D99 .OR. THR .LT. 1.0D-99) THEN
CALL BOUND(THR,1.0D99,1.0D-99)
ENDIF
RETURN
END

SUBROUTINE DATA1(WC1,WMN1,WSI1,WN1,WCR1,WMO1,WW1,WV1,WNB1,WCO1)
IMPLICIT DOUBLE PRECISION (A-H,K,N-Z), INTEGER(I,J,L,M)
WRITE(*,1)
1 FORMAT(10X,' Please enter the following information:-')
WRITE(*,2)
2 FORMAT(10X,' Carbon conc. in low C activity steel in wt %')
CALL REED(WC1)
IF (WC1 .GT. 5 .OR. WC1 .LT. 0.02) THEN
    CALL BOUND(WC1,5.0D0,2.0D-2)
ENDIF
WRITE(*,3)
3 FORMAT(1OX,' Manganese cone. in low C activity steel in wt %')
CALL REED(WMN1)
IF (WMN1 .GT. 15 .OR. WMN1 .LT. 0.02) THEN
    CALL BOUND(WMN1,15.0D0,2.0D-2)
ENDIF
WRITE(*,4)
4 FORMAT(1OX,' Silicon cone. in low C activity steel in wt %')
CALL REED(WS1l)
IF (WS11 .GT. 15 .OR. WS11 .LT. 0.02) THEN
    CALL BOUND(WS1l,15.0D0,2.0D-3)
ENDIF
WRITE(*,5)
5 FORMAT(1OX,' Nickel cone. in low C activity steel in wt %')
CALL REED(WN1l)
IF (WN1l .GT. 15 .OR. WN1l .LT. 0.0) THEN
    CALL BOUND(WN1l,15.0D0,0.0D0)
ENDIF
WRITE(*,6)
6 FORMAT(1OX,' Chromium cone. in low C activity steel in wt %')
CALL REED(WCR1)
IF (WCR1 .GT. 15 .OR. WCR1 .LT. 0.0) THEN
    CALL BOUND(WCR1,15.0D0,0.0D0)
ENDIF
WRITE(*,7)
7 FORMAT(1OX,' Molybdenum cone. in low C activity steel in wt %')
CALL REED(WMO1)
IF (WMO1 .GT. 15 .OR. WMO1 .LT. 0.0) THEN
    CALL BOUND(WMO1,15.0D0,0.0D0)
ENDIF
WRITE(*,8)
8 FORMAT(1OX,' Tungsten cone. in low C activity steel in wt %')
CALL REED(WW1)
IF (WW1 .GT. 15 .OR. WW1 .LT. 0.0) THEN
    CALL BOUND(WW1,15.0D0,0.0D0)
ENDIF
WRITE(*,9)
9 FORMAT(1OX,' Vanadium cone. in low C activity steel in wt %')
CALL REED(WV1)
IF (WV1 .GT. 15 .OR. WV1 .LT. 0.0) THEN
    CALL BOUND(WV1,15.0D0,0.0D0)
ENDIF
WRITE(*,10)
10 FORMAT(1OX,' Niobium cone. in low C activity steel in wt %')
CALL REED(WNB1)
IF (WNB1 .GT. 15 .OR. WNB1 .LT. 0.0) THEN
    CALL BOUND(WNB1,15.0D0,0.0D0)
ENDIF
WRITE(*,11)
11 FORMAT(1OX,' Cobalt cone. in low C activity steel in wt %')
CALL REED(WCO1)
IF (WCO1 .GT. 15 .OR. WCO1 .LT. 0.0) THEN

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CALL BOUND(WC01,15.0D0,0.0D0)
ENDIF
RETURN
END

SUBROUTINE DATA2(WC2,WMN2,WSI2,WNI2,WCR2,WM02,WW2,WV2,WNB2,WC02)
IMPLICIT DOUBLE PRECISION (A-H,K,N-Z), INTEGER(I,J,L,M)
WRITE(*,1)
1 FORMAT(10X,' Please enter the following information:-')
WRITE(*,2)
2 FORMAT(10X,' Carbon cone. in high C activity steel in wt %')
CALL REED(WC2)
IF (WC2 .GT. 5 .OR. WC2 .LT. 0.02) THEN
   CALL BOUND(WC2,5.0D0,2.0D-2)
ENDIF
WRITE(*,3)
3 FORMAT(10X,' Manganese cone. in high C activity steel in wt %')
CALL REED(WMN2)
IF (WMN2 .GT. 15 .OR. WMN2 .LT. 0.02) THEN
   CALL BOUND(WMN2,15.0D0,2.0D-2)
ENDIF
WRITE(*,4)
4 FORMAT(10X,' Silicon cone. in high C activity steel in wt %')
CALL REED(WSI2)
IF (WSI2 .GT. 15 .OR. WSI2 .LT. 0.02) THEN
   CALL BOUND(WSI2,15.0D0,2.0D-2)
ENDIF
WRITE(*,5)
5 FORMAT(10X,' Nickel cone. in high C activity steel in wt %')
CALL REED(WNI2)
IF (WNI2 .GT. 15 .OR. WNI2 .LT. 0.0) THEN
   CALL BOUND(WNI2,15.0D0,0.0D0)
ENDIF
WRITE(*,6)
6 FORMAT(10X,' Chromium cone. in high C activity steel in wt %')
CALL REED(WCR2)
IF (WCR2 .GT. 20 .OR. WCR2 .LT. 0.0) THEN
   CALL BOUND(WCR2,20.0D0,0.0D0)
ENDIF
WRITE(*,7)
7 FORMAT(10X,' Molybdenum cone. in high C activity steel in wt %')
CALL REED(WM02)
IF (WM02 .GT. 15 .OR. WM02 .LT. 0.0) THEN
   CALL BOUND(WM02,15.0D0,0.0D0)
ENDIF
WRITE(*,9)
9 FORMAT(10X,' Tungsten cone. in high C activity steel in wt %')
CALL REED(WW2)
IF (WW2 .GT. 15 .OR. WW2 .LT. 0.0) THEN
   CALL BOUND(WW2,15.0D0,0.0D0)
ENDIF
WRITE(*,10)
10 FORMAT(10X,' Vanadium cone. in high C activity steel in wt %')

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CALL REED(WV2)
IF (WV2 .GT. 15 .OR. WV2 .LT. 0.0) THEN
    CALL BOUND(WV2,15.0D0,0.0D0)
ENDIF
WRITE(*,11)
11 FORMAT(10X,' Niobium conc. in high C activity steel in wt %')
CALL REED(WNB2)
IF (WNB2 .GT. 15 .OR. WNB2 .LT. 0.0) THEN
    CALL BOUND(WNB2,15.0D0,0.0D0)
ENDIF
WRITE(* ,12)
12 FORMAT(10X,' Cobalt conc. in high C activity steel in wt %')
CALL REED(WCO2)
IF (WCO2 .GT. 15 .OR. WCO2 .LT. 0.0) THEN
    CALL BOUND(WCO2,15.0D0,0.0D0)
ENDIF
RETURN
END

This subroutine converts weight percent to mole fraction

SUBROUTINE CON(WC,WMN,WSI,WNI,WCR,WMO,WW,WV,WNB,WCO,
&
    XC,XMN,XSI,XNI,XCR,XMO,XW,XV,XNB,XCO)
IMPLICIT DOUBLE PRECISION (A-Z)
AC=12.01
AMN=54.94
ASI=28.09
ANI=58.71
ACR=52.00
AMO=95.94
AW=183.85
AV=50.94
ANB=92.91
ACO=58.93
AFE=55.85
ADD=WC+WMN+WSI+WNI+WCR+WMO,WW,WV,WNB+WCO
WFE=100.0D0-ADD
MC=WC/AC
MMN=WMN/AMN
MSI=WSI/ASI
MNI=WNI/ANI
MCR=WCR/ACR
MMO=WMO/AMO
MW=WW/AW
MV=WV/AV
MNB=WNB/ANB
MCO=WCO/ACO
MFE=WFE/AFE
SUM=MC+MMN+MSI+MNI+MCR+MMO+MW+MV+MNB+MCO+MFE
XC=MC/SUM
XMN=MMN/SUM
XSI=MSI/SUM
XNI=MNI/SUM
XCR=MCR/SUM

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This is a subroutine to calculate the carbon-carbon interaction energy in austenite as a function of alloy composition. The theory for this program is in Bhadeshia (1981 c).

The answer is in Joules/mol.

SUBROUTINE OMEGA(WC,WSI,WMN,WNI,WMO,WCR,WV,W)
DOUBLE PRECISION C(8),P(8),B1,B2,Y(I),T10,T20,B3,XBAR
DOUBLE PRECISION WC,WSI,WMN,WNI,WMO,WCR,WV,W
INTEGER B5,I,U,B4
B3=0.0D+00
C(1)=WC/12.0115D+00
C(2)=WSI/28.09D+00
C(3)=WMN/54.94D+00
C(4)=WNl/58.71D+00
C(5)=WMO/95.94D+00
C(6)=WCR/52.0D+00
C(7)=WV/50.94D+00
C(8)=C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)
C(8)=100.0D+00-C(8)
C(8)=C(8)/55.84D+00
B 1=C(l )+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)
DO 107 U=2,7
Y(U)=C(U)C(8)
107 CONTINUE
DO 106 U=1,8
C(U)=C(U)/B 1
106 CONTINUE
XBAR=C(l)
XBAR=DINT(10000.0D+00*XBAR)
XBAR=XBAR/10000
B2=0.0D+00
T10=Y(2)*(-3)-Y(3)*2+Y(4)*12+Y(5)*(-9)+Y(6)*(-1)+Y(7)*(-12)
T20=3-Y(2)-37.5*Y(3)-6*Y(4)-26*Y(5)-19*Y(6)-44*Y(7)
P(2)=2013.0341+763.8167*C(2)+45802.87*C(2)**2-280061.63*C(2)**3
+3.864D+06*C(2)**4-2.4233D+07*C(2)**5+6.9547D+07*C(2)**6
P(3)=2012.067-1764.095*C(3)+6287.52*C(3)**2-21647.96*C(3)**3
+3.864D+06*C(2)**4-2.4233D+07*C(2)**5+6.9547D+07*C(2)**6
P(4)=2006.8017+2330.2424*C(4)-54915.32*C(4)**2+1.6216D+06*C(4)**3
+2.0119D+06*C(3)**4+3.1716D+07*C(3)**5-1.3885D+08*C(3)**6
P(5)=2006.834-2997.314*C(5)-37906.61*C(5)**2+1.0328D+06*C(5)**3
+1.3306D+07*C(5)**4+8.411D+07*C(5)**5-2.0826D+08*C(5)**6
P(6)=2012.367-9224.2655*C(6)+33657.8*C(6)**2-566827.83*C(6)**3
+8.5676D+06*C(6)**4-6.7482D+07*C(6)**5+2.0837D+08*C(6)**6
P(7)=2011.996-6247.9118*C(7)+5411.7566*C(7)**2+250118.1085*C(7)**3
+250118.1085*C(7)**4
DO 108 U=2,7
B3=B3+P(U)*Y(U)
B2 = B2 + Y(U)

CONTINUE
IF (B2 .EQ. 0.0D+00) GOTO 455
W = (B3/B2)*4.187
GOTO 456

W = 8054.0

CONTINUE
RETURN
END

This subroutine converts from mole fraction to weight percent.

SUBROUTINE CONVE(M,Y,XMN,XSI,XNI,XCR,XMO,XW,XV,XNB,
  XCO,YC)
IMPLICIT DOUBLE PRECISION(A-H,K,N-Z), INTEGER(I,J,L,M)
DIMENSION Y(30),YC(30)
AC = 12.01
AMN = 54.94
ASI = 28.09
ANI = 58.71
ACR = 52.00
AMO = 95.94
AW = 183.85
AV = 50.94
ANB = 92.91
ACO = 58.93
AFE = 55.85
ADD = Y(M) + XMN + XSI + XNI + XCR + XMO + XW + XV + XNB + XCO
XFE = 1.0D0 - ADD
NC = Y(M)*AC
MMN = XMN*AMN
NSI = XSI*ASI
NNI = XNI*ANI
NCR = XCR*ACR
NMO = XMO*AMO
NW = XW*AW
NV = XV*AV
NNB = XNB*ANB
NCO = XCO*ACO
NFE = XFE*AFE
SUM = NC + MMN + NSI + NNI + NCR + NMO + NW + NV + NNB + NCO + NFE
YC(M) = (NC/SUM)*100
RETURN
END

This program calculates the Wagner activity coefficients for the calculation of the value of k.

EX is the Wagner interaction parameter for element X.

AC is the activity coefficient of carbon.

LAC is the log of the activity coefficient.

SUBROUTINE KFUN(XC1,XMN1,XSI1,XNI1,XCR1,XMO1,XCU1,
  & XW1,XV1,XNB1,XCO1,XC2,XMN2,XSI2,XNI2,XCR2,XMO2,XCU2
  & ,XW2,XV2,XNB2,XCO2,KTEMP,K)
This program calculates the Uhrenius activity coefficients for the determination of the partition coefficient when Si is driving the diffusion. AY is the activity of carbon LAY is the log of the carbon activity YCR,YC,K1 AND K2 are defined in Uhrenius, B., 1977, Scandinavian Journal of Metallurgy, 6, 86-89.
R is the gas constant

SUBROUTINE KFUN2(XSI,WC,KTEMP,W2,WSI,K)
IMPLICIT DOUBLE PRECISION (A-Z)
EC=8910/KTEMP
EMN=-5070 /KTEMP
ESI=4.84-(7370/KTEMP)
ENI=-2.2+(7600/KTEMP)
ECR=24.4-(38400/KTEMP)
EMO=3.855-(17870/KTEMP)
ECU=4200 /KTEMP
EW=23.4-(36214/KTEMP)
EV=-24660/KTEMP
ENB=-28770/KTEMP
ECO=2800 /KTEMP

LAC1=(XSI*EC)+(XMN1*EMN)+(XS1*ESI)+(XN1*ENI)+(XCR1*ECR)+&
(XMO1*EMO1)+(XC1*ECU)+(XW1*EW)+(XV1*EV)+(XNB1*ENB)+(XCO1*ECO)

LAC2=(XSI*EC)+(XMN2*EMN)+(XS2*ESI)+(XN2*ENI)+(XCR2*ECR)+&
(XMO2*EMO2)+(XC2*ECU)+(XW2*EW)+(XV2*EV)+(XNB2*ENB)+(XCO2*ECO)

AC1=DEXP(LAC1)
AC2=DEXP(LAC2)
K=AC1/AC2
RETURN
END

This program calculates the partition coefficient as given by the data of Wada et al. These equations are for Si driving the diffusion.

SUBROUTINE KFUN3(WSI,WC,KTEMP,W2,WSI,K)
This program calculates the Uhrenius activity coefficients for the determination of the partition coefficient when Cr is driving the diffusion:

```
SUBROUTINE KFUN6(XCR1,XC1,XCR2,XC2,KTEMP,K)
IMPLICIT DOUBLE PRECISION (A-Z)
R=8.314
K1=42140+23.17*KTEMP
K2=-339+0.187*KTEMP
YCR1=XCR1/((1.0-XC1)
YC1=XC1/((1.0-XC1)
LAY1=(R*KTEMP*DLOG(YC1/((1.0-YC1)+(K1*YC1)+(K2*YCR1)))
& / (R*KTEMP)
AY1=DEXP(LAY1)
YCR2=XCR2/((1.0-XC2)
YC2=XC2/((1.0-XC2)
LAY2=(R*KTEMP*DLOG(YC2/((1.0-YC2)+(K1*YC2)+(K2*YCR2)))
& / (R*KTEMP)
AY2=DEXP(LAY2)
K=(AY1*XC2)/(AY2*XC1)
RETURN
END
```

This subroutine calulates the partition coefficient according to the data of Wada et al. when Cr is driving the diffusion:

```
SUBROUTINE KFUN7(WCR1,WC1,KTEMP,WC2,WCR2,K)
IMPLICIT DOUBLE PRECISION(A-H,K,N-Z), INTEGER(I,J,L,M)
AFC1=2300/KTEMP-2.24+((179/KTEMP)*WC1)-((102/KTEMP)-0.033)*WCR1
FC1=DEXP(AFC1)
AFC2=2300/KTEMP-2.24+((179/KTEMP)*WC2)-((102/KTEMP)-0.033)*WCR2
FC2=DEXP(AFC2)
K=FC1/FC2
RETURN
END
```

This program calculates the effective diffusivity of carbon in austenite, taking account
of the fact that the diffusivity is concentration dependent.

HH is Planck’s constant (Joules/sec)

KK is Boltzmann’s constant (Joules/Kelvin)

Z is the co-ordination of an interstitial site

PSI is the concentration dependence of the diffusion coefficient

THETA is the number of C atoms/number of Fe atoms

ACTIV is the activity of carbon in austenite

R is the gas constant

X is the mole fraction of carbon

SIGMA is the site exclusion probability

---

**SUBROUTINE DIFFF(W,XBAR,XGAG,T,ANS)**

**IMPLICIT DOUBLE PRECISION (A-H,K-Y), INTEGER(I,J,Z)**

**DOUBLE PRECISION DIFF(500), CARB(500)**

HH = 6.6262D-34

KK = 1.38062D-23

Z = 12

A5 = 1.0D+00

R = 8.31432D+00

II2 = 0

DASH = (KK * T / HH) * DEXP(-(21230.0D+00 / T)) * DEXP(-31.84D+00)

This DO loop specifies the variables for the subroutine TRAPE to evaluate the integral

**DO 9 II = 1,1000**

CARB(1) = XBAR

IF (II .GT. I) GOTO 1

GOTO 8

1

XINCR = (XGAG - XBAR) / 10

CARB(II) = CARB(II-1) + XINCR

(CARB(II) . GT. XGAG) GOTO 5

8

X = CARB(II)

II2 = II2 + 1

THETA = X / (A5 - X)

CALL LOGS(X, T, W, R, ACTIV)

ACTIV = DEXP(ACTIV)

CALL DIV(X, T, W, R, DACTIV)

DACTIV = DACTIV * ACTIV

DACTIV = DACTIV * A5 / ((A5 + THETA)**2)

SIGMA = A5 - DEXP(-((W) / (R * T)))

PSI = ACTIV * (A5 + Z * ((A5 + THETA) / (A5 - (A5 + Z / 2) * THETA) + (Z/2) * &

(A5 + Z/2) * (A5 - SIGMA) * THETA * THETA)) + (A5 + THETA) * DACTIV

DIFF(II) = DASH * PSI * 1.0D-04

9 CONTINUE

5

II3 = 0

**TRAPE is a subroutine for integration of a function**

CALL TRAPE(CARB, DIFF, ANS, II2)

ANS = ANS / (XGAG - XBAR)

RETURN

END

---

This function gives LN(ACTIVITY) of carbon in austenite

**SUBROUTINE LOGS(X,T,W,R,CT)**

**DOUBLE PRECISION J,DG,DUM,T,R,W,X,CT**

J = 1 - DEXP(-W / (R * T))

DG = DSQRT(1 - 2 * (1 + 2 * J) * X + (1 + 8 * J) * X * X)
This function gives the differential of \( \ln(\text{ACTIVITY}) \) of carbon in austenite with respect to \( X \).

\[
\text{SUBROUTINE DIV}(X,T,W,R,\text{DCG})
\]

\[
\begin{align*}
\text{DOUBLE PRECISION } & J, \text{DG, DDG, X, T, W, R, DCG} \\
J = & 1 - \exp\left(-\frac{W}{R \times T}\right) \\
\text{DG} = & \sqrt{1 - 2 \times (1 + 2 \times J) \times X + (1 + 8 \times J) \times X^2} \\
\text{DDG} = & \left(\frac{0.5}{\text{DG}}\right) \times (-2 - 4 \times J + 2 \times X + 16 \times J \times X) \\
\text{DCG} = & \left(\frac{(10/(1-2*X))+(5/X)}{(\text{DG} \times (1+3*X))}-\frac{(\text{DDG}+3)}{(\text{DG} \times (1+3*X))}-\frac{(\text{DDG}-3)}{(\text{DG} \times (1-3*X))}\right) \\
\end{align*}
\]

RETURN
END

The purpose of this subroutine is to compute the vector of integral values for a given general table of argument and function values.

\[ \text{USAGE - CALL TRAPE}(X,Y,Z,\text{NDIM}) \]

\[ \text{DESCRIPTION OF PARAMETERS} \]

\[ X \] - Double precision input vector of argument values

\[ Y \] - Double precision input vector of function values

\[ Z \] - The resulting DP vector of integral values

\[ \text{NDIM} \] - The dimension of vectors \( X, Y, Z \). NDIM MAX. 1000

\[ \text{METHOD} \]

Beginning with \( Z(1)=0 \), evaluation of vector \( z \) is done by means of the trapezoidal rule.

\[
\begin{align*}
\text{SUBROUTINE TRAPE}(X,Y,AN,\text{NDIM}) \\
\text{DOUBLE PRECISION } X(1000), Y(1000), Z(1000) \\
\text{DOUBLE PRECISION SUM1, SUM2, ANS} \\
\text{SUM2} = & 0.0 \\
\text{IF} (\text{NDIM}) & = 1, 3, 4 \\
\text{INTEGRATION LOOP} \\
1 & \text{DO } 2 \; I = 2, \text{NDIM} \\
& \text{SUM1} = \text{SUM2} \\
& \text{SUM2} = \text{SUM2} + \frac{5 \times 10^{-9} (X(I) - X(I-1)) (Y(I) + Y(I-1))}{2} \\
2 & \text{AZ}(I-1) = \text{SUM1} \\
3 & \text{AZ} = \text{NDIM} = \text{SUM2} \\
& \text{ANS} = \text{SUM2} \\
4 & \text{RETURN} \\
\end{align*}
\]

RETURN
END

This is a routine for calculating the error function.

\[
\text{DOUBLE PRECISION FUNCTION DERF}(Y) \\
\text{IMPLICIT REAL*8(A-H,O-Z)} \\
\text{INTEGER ISW, I} \\
\text{DOUBLE PRECISION P(5), Q(3), P1(8), Q1(7), P2(5), Q2(4)} \\
\text{COEFFICIENTS FOR } 0.0 \leq Y \leq 0.477 \\
\text{DATA P(1) = -0.44422647396874,} \\
& \text{P(2) = 1.0731707253648,} \\
\]

150
& P(3)/15.915606197771/,
& P(4)/374.81624081284/,
& P(5)/2.5612422994823D-02/
DATA Q(1)/17.903143558843/,
& Q(2)/124.82892031581/,
& Q(3)/332.17224470532/
COEFFICIENTS FOR .477 .LE. Y .LE. 4.0
DATA P1(1)/T.2117582508831/,
& P1(2)/43.162227222057/,
& P1(3)/152.98928504694/,
& P1(4)/339.32081673434/,
& P1(5)/451.91895371187/,
& P1(6)/300.45926102016/,
& P1(7)/-1.3686485738272D-07/,
& P1(8)/5.6419551747897/
DATA Q1(1)/77.000152935229/,
& Q1(2)/277.58544473499/,
& Q1(3)/638.98026446563/,
& Q1(4)/931.3540985061/,
& Q1(5)/790.95092532790/,
& Q1(6)/300.45926095698/,
& Q1(7)/12.782727319629/
COEFFICIENTS FOR 4.0 .LT. Y
DATA P2(1)/-.22695659353969/,
& P2(2)/-4.9473091062325D-02/,
& P2(3)/-2.9961070770354D-03/,
& P2(4)/-2.2319245973418D-03/,
& P2(5)/-2.7866130860965D-01/
DATA Q2(1)/1.0516751070679/,
& Q2(2)/1.9130892610783/,
& Q2(3)/1.0620923052847D-02/,
& Q2(4)/1.9873320181714/
CONSTANTS
DATA XMIN/1.0D-8/,XLARGE/5.6875DO/
DATA SSQPI/.56418958354776/
FIRST EXECUTABLE STATEMENT
X = Y
ISW = 1
IF (X.GE.0.0D0) GO TO 5
   ISW = -1
   X = -X
5
IF (X.LT. .477D0) GO TO 10
IF (X.LE.4.0D0) GO TO 25
   IF (X.LT.XLARGE) GO TO 35
      RES = 1.D0
      GO TO 50
ABS(Y) .LT. .477, EVALUATE APPROXIMATION FOR ERF
10
   IF (X.LT.XMIN) GO TO 20
   XSQ = X*X
   XNUM = P(5)
   DO 15 I=1,4
      XNUM = XNUM*XSQ+P(I)
   CONTINUE
15
   XDEN = ((Q(1)+XSQ)*XSQ+Q(2))*XSQ+Q(3)
   RES = X*XNUM/XDEN
```
20      GO TO 50
RES = X*P(4)/Q(3)
GO TO 50
.
.
25 .477 .LE. ABS(Y) .LE. 4.0 EVALUATE APPROXIMATION FOR ERF
    XSQ = X*X
    XNUM = P1(7)*X+P1(8)
    XDEN = X+Q1(7)
    DO 30 I=1,6
         XNUM = XNUM*X+P1(I)
         XDEN = XDEN*X+Q1(I)
    CONTINUE
    RES = XNUM/XDEN
GO TO 45
4.0 .LT. ABS(Y), EVALUATE APPROXIMATION FOR ERF
35    XSQ = X*X
    XI = 1.0D0/XSQ
    XNUM = P2(4)*XI+P2(5)
    XDEN = XI+Q2(4)
    DO 40 I=1,3
         XNUM = XNUM*XI+P2(I)
         XDEN = XDEN*XI+Q2(I)
    CONTINUE
    RES = (SSQPI+XI*XNUM/XDEN)/X
45    RES = RES*DEXP(-XSQ)
    RES = 1.0D0-RES
50    IF (ISW.EQ.-1) RES = -RES
    DERF = RES
    RETURN
END

DOUBLE PRECISION FUNCTION DERFC(A)
DOUBLE PRECISION A,DERF
    DERFC=1.0D+00-DERF(A)
    RETURN
END

This subroutine reads in a double precision number and checks that a character string has
not been input
SUBROUTINE REED(A)
IMPLICIT DOUBLE PRECISION(A-H,K,N-Z), INTEGER(I,J,L,M)
996 READ(*,*,ERR=999)A
GOTO 998
999 WRITE(*,997)
997 FORMAT(10X,' Your input is incorrect, please try again.'/)
GOTO 996
998 RETURN
END

This routine checks that the input value is an integer
SUBROUTINE REEDI(I)
```

152
INTEGER I
996 READ(*,*,ERR=999)I
GOTO 998
999 WRITE(*,997)
997 FORMAT(10X,' Your input is incorrect, please try again.'/)
GOTO 996
998 RETURN
END

This routine checks that the input data is within the bounds for that variable

SUBROUTINE BOUND(A,B,C)
IMPLICIT DOUBLE PRECISION(A-H,K,N-Z), INTEGER(I,J,L,M)
2 WRITE(*,1)B,C
1 FORMAT(10X,' Your input is out of bounds'/
& 12X,' The limits are 'D12.4,' to 'D12.4//'
& 12X,' Please input a new value or type 999 to continue with'/
& 12X,' your dubious choice of input! ')
CALL REED(D)
IF (D .EQ. 999)GOTO 3
A=D
IF (A .LT. C .OR. A .GT. B)GOTO 2
3 RETURN
END

This subroutine prints out all of the input data and gives the option to change any of the variables

SUBROUTINE DDTA(WCl,WMNl ,WSIl ,WNIl ,WCRl ,WMOl ,WWl ,WVl ,WNBl,WCOl,
& WC2,WMN2,WSI2,WNI2,WCRI,WMO2,WW2,WV2,WNB2,WCO2,KTEMP,THR,Wl ,W2,JY)
IMPLICIT DOUBLE PRECISION(A-H,K,N-Z), INTEGER(I,J,L,M)
WRITE(* ,300)WCl,WMNl,WSIl,WNIl,WCRl,WMOl,WWl,WVl,WNBl,WCOl,
& WC2,WMN2,WSI2,WNI2,WCRI,WMO2,WW2,WV2,WNB2,WCO2
WRITE(* ,400)KTEMP,THR,Wl,W2
300 FORMAT(
& 10X' The current values that have been inputed are:-'/
& 10X' Low C activity side of the weld'/
& 10X' 1. C ',F8.3,10X, ' wt %'/
& 10X' 10. Co ',F8.3/
& 10X' High C activity side of the weld'/
& 10X' 11. C ',F8.3,10X, ' wt %'/
& 10X' 20. Co ',F8.3)
400 FORMAT(10X,' 21. Temperature in Kelvin ',F8.3/
& 10X' 22. Time in hours ',F8.3/
& 10X' Carbon-carbon interaction energy in alpha ',D12.4/
& 10X' Carbon-carbon interaction energy in theta ',D12.4/)
WRITE(*,*)' Is the diffusion driven by: '
WRITE(*,*)' 1. Silicon'
WRITE(*,*)' 2. Chromium'
CALL REEDI(JY)
IF ( WSIl .LT. WSI2 .AND. JY .EQ. 1) THEN
  WRITE(* ,462)
  462 FORMAT(IOX,'*******************WARNING*******************',/
& 10X,' The Si values that you have input are incorrect'/
& 10X,' Please change them.')
ENDIF
IF ( WCR2 .LT. WCRI .AND. JY .EQ. 2) THEN
  WRITE(* ,463)
  463 FORMAT(IOX,'*******************WARNING*******************',/
& 10X,' The Cr values that you have input are incorrect'/
& 10X,' Please change them.')
ENDIF
WRITE(*,124)
124 FORMAT(IOX,' Do you wish to change any of this input?'20X,/ & 10X,' (Choose the item number, or 0 to continue) ')
CALL REEDI(JYES)
IF (JYES .EQ. 0) THEN
  RETURN
ENDIF
IF (JYES .EQ. 1) GOTO 1
IF (JYES .EQ. 2) GOTO 2
IF (JYES .EQ. 3) GOTO 3
IF (JYES .EQ. 4) GOTO 4
IF (JYES .EQ. 5) GOTO 5
IF (JYES .EQ. 6) GOTO 6
IF (JYES .EQ. 7) GOTO 7
IF (JYES .EQ. 8) GOTO 8
IF (JYES .EQ. 9) GOTO 9
IF (JYES .EQ. 10) GOTO 10
IF (JYES .EQ. 11) GOTO 11
IF (JYES .EQ. 12) GOTO 12
IF (JYES .EQ. 13) GOTO 13
IF (JYES .EQ. 14) GOTO 14
IF (JYES .EQ. 15) GOTO 15
IF (JYES .EQ. 16) GOTO 16
IF (JYES .EQ. 17) GOTO 17
IF (JYES .EQ. 18) GOTO 18
IF (JYES .EQ. 19) GOTO 19
IF (JYES .EQ. 20) GOTO 20
IF (JYES .EQ. 21) GOTO 21
IF (JYES .EQ. 22) GOTO 22
1 WRITE(*,100)
100 FORMAT(IOX,’ Carbon conc. in low C activity steel in wt %’) CALL REED(WCI)
IF (WCI .GT. 5 .OR. WCI .LT. 0.02) THEN
  CALL BOUND(WCI,5.0D0,2.0D-2)
ENDIF
GOTO 200
2 WRITE(*,102)
102 FORMAT(IOX,’ Manganese conc. in low C activity steel in wt %’)

154
CALL REED(WMN1)
IF (WMN1 .GT. 15 .OR. WMN1 .LT. 0.02) THEN
   CALL BOUND(WMN1,15.0D0,2.0D-2)
ENDIF
GOTO 200
3 WRITE(*,103)
103 FORMAT(10X,' Silicon conc. in low C activity steel in wt %'
CALL REED(WSI1)
IF (WSI1 .GT. 15 .OR. WSI1 .LT. 0.02) THEN
   CALL BOUND(WSI1,15.0D0,2.0D2)
ENDIF
GOTO 200
4 WRITE(*,104)
104 FORMAT(10X,' Nickel conc. in low C activity steel in wt %'
CALL REED(WNI1)
IF (WNI1 .GT. 15 .OR. WNI1 .LT. 0.0) THEN
   CALL BOUND(WNI1,15.0D0,0.0D0)
ENDIF
GOTO 200
5 WRITE(*,105)
105 FORMAT(10X,' Chromium conc. in low C activity steel in wt %'
CALL REED(WCR1)
IF (WCR1 .GT. 20 .OR. WCR1 .LT. 0.0) THEN
   CALL BOUND(WCR1,15.0D0,0.0D0)
ENDIF
GOTO 200
6 WRITE(*,106)
106 FORMAT(10X,' Molybdenum conc. in low C activity steel in wt %'
CALL REED(WMO1)
IF (WMO1 .GT. 15 .OR. WMO1 .LT. 0.0) THEN
   CALL BOUND(WMO1,15.0D0,0.0D0)
ENDIF
GOTO 200
7 WRITE(*,107)
107 FORMAT(10X,' Tungsten conc. in low C activity steel in wt %'
CALL REED(WW1)
IF (WW1 .GT. 15 .OR. WW1 .LT. 0.0) THEN
   CALL BOUND(WW1,15.0D0,0.0D0)
ENDIF
GOTO 200
8 WRITE(*,108)
108 FORMAT(10X,' Vanadium conc. in low C activity steel in wt %'
CALL REED(WV1)
IF (WV1 .GT. 15 .OR. WV1 .LT. 0.0) THEN
   CALL BOUND(WV1,15.0D0,0.0D0)
ENDIF
GOTO 200
9 WRITE(*,109)
109 FORMAT(10X,' Niobium conc. in low C activity steel in wt %'
CALL REED(WNB1)
IF (WNB1 .GT. 15 .OR. WNB1 .LT. 0.0) THEN
   CALL BOUND(WNB1,15.0D0,0.0D0)
ENDIF
GOTO 200
10 WRITE(*,110)
FORMAT(10X,' Cobalt conc. in low C activity steel in wt %')
CALL REED(WC01)
IF (WC01 .GT. 15 .OR. WC01 .LT. 0.0) THEN
  CALL BOUND(WC01,15.0D0,0.0D0)
ENDIF
WRITE(*,111)

FORMAT(10X,' Carbon conc. in high C activity steel in wt %')
CALL REED(WC2)
IF (WC2 .GT. 5 .OR. WC2 .LT. 0.02) THEN
  CALL BOUND(WC2,5.0D0,2.0D-2)
ENDIF
GOTO 200

WRITE(*,112)

FORMAT(10X,' Manganese conc. in high C activity steel in wt %')
CALL REED(WMN2)
IF (WMN2 .GT. 15 .OR. WMN2 .LT. 0.02) THEN
  CALL BOUND(WMN2,15.0D0,2.0D-2)
ENDIF
GOTO 200

WRITE(*,113)

FORMAT(10X,' Silicon conc. in high C activity steel in wt %')
CALL REED(WSI2)
IF (WSI2 .GT. 15 .OR. WSI2 .LT. 0.02) THEN
  CALL BOUND(WSI2,15.0D0,2.0D-2)
ENDIF
GOTO 200

WRITE(*,114)

FORMAT(10X,' Nickel conc. in high C activity steel in wt %')
CALL REED(WNI2)
IF (WNI2 .GT. 15 .OR. WNI2 .LT. 0.0) THEN
  CALL BOUND(WNI2,15.0D0,0.0D0)
ENDIF
GOTO 200

WRITE(*,115)

FORMAT(10X,' Chromium conc. in high C activity steel in wt %')
CALL REED(WCR2)
IF (WCR2 .GT. 20 .OR. WCR2 .LT. 0.0) THEN
  CALL BOUND(WCR2,20.0D0,0.0D0)
ENDIF
GOTO 200

WRITE(*,116)

FORMAT(10X,' Molybdenum conc. in high C activity steel in wt %')
CALL REED(WM02)
IF (WM02 .GT. 15 .OR. WM02 .LT. 0.0) THEN
  CALL BOUND(WM02,15.0D0,0.0D0)
ENDIF
GOTO 200

WRITE(*,117)

FORMAT(10X,' Tungsten conc. in high C activity steel in wt %')
CALL REED(WW2)
IF (WW2 .GT. 15 .OR. WW2 .LT. 0.0) THEN
  CALL BOUND(WW2,15.0D0,0.0D0)
ENDIF
GOTO 200

WRITE(*,118)
FORMAT(10X,'Vanadium conc. in high C activity steel in wt %')
CALL REED(WV2)
IF (WV2 .GT. 15 .OR. WV2 .LT. 0.0) THEN
   CALL BOUND(WV2,15.0D0,0.0D0)
ENDIF
GOTO 200
WRITE(*,119)
FORMAT(10X,' Niobium conc. in high C activity steel in wt %')
CALL REED(WNB2)
IF (WNB2 .GT. 15 .OR. WNB2 .LT. 0.0) THEN
   CALL BOUND(WNB2,15.0D0,0.0D0)
ENDIF
GOTO 200
WRITE(*,120)
FORMAT(10X,' Cobalt conc. in high C activity steel in wt %')
CALL REED(WCO2)
IF (WCO2 .GT. 15 .OR. WCO2 .LT. 0.0) THEN
   CALL BOUND(WCO2,15.0D0,0.0D0)
ENDIF
GOTO 200
WRITE(*,121)
FORMAT(10X,' Temperature in Kelvin')
CALL REED(KTEMP)
IF (KTEMP .GT. 1700 .OR. KTEMP .LT. 900) THEN
   CALL BOUND(KTEMP,1700.0D0,900.0D0)
ENDIF
GOTO 200
WRITE(*,122)
FORMAT(10X ' Time in hours')
CALL REED(THR)
IF (THR .GT. 1.0D99 .OR. THR .LT. 1.0D-99) THEN
   CALL BOUND(THR,1.0D99,1.0D-99)
ENDIF
GOTO 200
RETURN
END

This subroutine prints out the logo at the head of the program

SUBROUTINE LOGO(AVER)
IMPLICIT DOUBLE PRECISION (A-Z)
WRITE(*,1)
FORMAT(/)
WRITE(*,2)
FORMAT(10X,'************************** ********** ******** *****')
WRITE(*,3)
FORMAT(1OX,' **',44X,'**')
WRITE(*,3)
WRITE(*,4)AVER
FORMAT(10X,' **', CARBON DIFFUSION IN AUSTENITIC JOINTS ** ',/
8X' ** (Version ',F3.1,') ** ')
This subroutine prints the results to the file AUS.RES.

SUBROUTINE RESULT(K, DA, DG, T, THR, D1, YC1, D2, YC2, M)
IMPLICIT DOUBLE PRECISION (A-H,K,N-Z), INTEGER(I,J,L,M)
DIMENSION D1(30), YC1(30), D2(30), YC2(30)
WRITE(2,6) T, THR
6 FORMAT(' TIME, SECONDS =',D8.2, 'HOURS =',F5.0/
& ' MM CA - XMM CG ')
DO 10 I=1,M
   WRITE(2,22) D1(I), YC1(I), D2(I), YC2(I)
10 CONTINUE
22 FORMAT(4D12.4,I4)
WRITE(2,60) DA, DG, K
60 FORMAT(' DIFFUSION COEFFICIENT DA =',D12.4/
& ' DIFFUSION COEFFICIENT DG =',D12.4/
& ' PARTITION COEFFICIENT =',F12.4/
RETURN
END

This subroutine prints the results to the screen.

SUBROUTINE RESP(K, CAG, CGA, DA, DG, T, THR, D1, YC1, D2, YC2, M)
IMPLICIT DOUBLE PRECISION (A-H,K,N-Z), INTEGER(I,J,L,M)
DIMENSION D1(30), YC1(30), D2(30), YC2(30)
WRITE(* ,6) T, THR
6 FORMAT(' TIME, SECONDS =',D8.2, 'HOURS =',F5.0/
& ' MM CA - XMM CG ')
DO 10 I=1,M
   WRITE(*,22) D1(I), YC1(I), D2(I), YC2(I)
10 CONTINUE
22 FORMAT(4D12.4,I4)

This subroutine produces a plot using NAG graphics routines

SUBROUTINE PLOT(D1,YC1,D2,YC2)
IMPLICIT DOUBLE PRECISION (A-H,K,N-Z), INTEGER(I,J,L,M)
DIMENSION D1(30),YC1(30),D2(30),YC2(30)
MAX = 26
CALL XXXXXX
CALL J06VAF(1,6)
CALL J06WAF
OPEN (6,FILE='AUS2.ERR')
CALL J06XDF(1,2,1,1)
CALL J06XDF(2,3,1,1)
CALL J06XDF(3,4,1,1)
CALL J06YMF(1)
IF (YC2(26) .LT. YC1(1)) THEN
  YMIN= YC2(26)*0.8
ELSEIF (YC1(1) .LT. YC1(26)) THEN
  YMIN= YC1(1)*0.8
ELSE
  YMIN= YC1(26)*0.8
ENDIF
IF (YC1(26) .GT. YC2(1)) THEN
  YMAX= YC1(26)*1.2
ELSEIF (YC2(1) .GT. YC2(26)) THEN
  YMAX= YC2(1)*1.2
ELSE
  YMAX= YC2(26)*1.2
ENDIF
CALL J06WBF(-25.0D0,25.0D0,YMIN,YMAX,1)
CALL J06ACF
CALL J06YMF(2)
CALL J06CCF(D1,YC1,MAX,2,10)
234
CALL J06CCF(D2,YC2,MAX,2,10)
CALL J06YAF(0,YC1(1))
CALL J06YCF(0,YC2(1))
CALL J06YMF(1)
CALL J06AEF
CALL J06YMF(3)
CALL J06AJF(2,'Carbon concentration /wt%')
CALL J06AHF('CARBON CONCENTRATION PROFILE ')
CALL J06AJF(1,'Distance from interface /mm')
CALL J06WZF
RETURN
END
Appendix III

Program listing for calculating decarburised zone widths
with ferrite on both sides of the interface
The Ferrite Model – Flow diagram.

INPUT.
i) Steel chemical compositions.
ii) Equilibrium carbide compositions.
iii) Equilibrium volume fractions.
iv) Temperature.

PRELIMINARY CALCULATION STAGE.
i) Calculates carbon diffusion coefficient in ferrite.
ii) Calculates the partition coefficient.
iii) Calculates the geometry of the particles in the model.
iv) Calculates the interface compositions.

DO loop stepped with time.

CALCULATION.
i) Decarburised zone width.
ii) Carburised zone width.

Have any particles been dissolved?

YES

STEP THE VALUE OF D2 AND D1.

FEEDBACK CALCULATIONS.
i) Calculation of the volume fractions.
ii) Calculation of the average carbon concentrations.
iii) Calculation of the equilibrium carbon concentrations.
iii) Recalculation of the partition coefficient.
iv) Calculation of diffusion away from the interface on the high alloy side.

NO
This program is a particulate model for the diffusion of carbon across dissimilar metal joints in the ferrite phase field.

The nomenclature is described as follows:

\[ \text{ATHR} \] is the root of the time in hours
\[ \text{AVER} \] is the version number.
\[ \text{BA} \] is the number of the dissolving particle on the high carbon activity side
\[ \text{BB} \] is the number of the dissolving particle on the low carbon activity side.
\[ \text{CBARA} \] is the average carbon concentration in the high carbon activity slice \( (x_a) \)
\[ \text{CBARB} \] is the average carbon concentration in the low carbon activity slice \( (x_p) \)
\[ \text{CZW} \] is the carburised zone width \( (\eta^c) \)
\[ \text{DA} \] is the diffusion coefficient in the high carbon activity side.
\[ \text{DB} \] is the diffusion coefficient in the low carbon activity side.
\[ \text{DZW} \] is the decarburised zone width \( (\eta^a) \)
\[ \text{D1} \] is the slab size on the high carbon activity side of the interface
\[ \text{D10} \] is the initial slab size on the high carbon activity side of the interface
\[ \text{D2} \] is the particle size on the high carbon activity side
\[ \text{D20} \] is the initial particle size on the high carbon activity side
\[ \text{D3} \] is the interparticle spacing on the high carbon activity side
\[ \text{D4} \] is the particle size on the low carbon activity side of the interface
\[ \text{D40} \] is the initial particle size on the low carbon activity side
\[ \text{D5} \] is the slab size on the low carbon activity side
\[ \text{D6} \] is the interparticle spacing on the low carbon activity side
\[ \text{D60} \] is the interparticle spacing on the low carbon activity side
\[ \text{K} \] is the partition coefficient.
\[ \text{KTEMP} \] is the absolute temperature
\[ \text{T} \] is the time in hours
\[ \text{TSEC} \] is the time in seconds
\[ \text{VFEA} \] is the equilibrium volume fraction on the high carbon activity side (calculated from MTDATA)
\[ \text{VFA} \] is the volume fraction of carbide at time \( T \) on the high carbon activity side \( (v_a) \)
\[ \text{VFB} \] is the volume fraction of carbide at time \( T \) on the low carbon activity side \( (v_p) \)
\[ \text{WAAL} \] is the concentration of carbon in the carbide in equilibrium with the matrix on the high carbon activity side /wt% \( (x^A) \)
\[ \text{WAALE} \] is the initial value of \( \text{WAAL} \) when \( T=0 \)
\[ \text{WALA} \] is the concentration of carbon in the matrix in equilibrium with carbide on the high carbon activity side /wt% \( (x^oA) \)
\[ \text{WALAE} \] is the initial value of \( \text{WALA} \) when \( T=0 \)
\[ \text{WALBE} \] is the carbon concentration at the weld junction in the high carbon activity side /wt% \( (x^oB) \)
\[ \text{WBBE} \] is the concentration of carbon in the carbide in equilibrium with the matrix on the low carbon activity side /wt% \( (x^B) \)
\[ \text{WBBEE} \] is the initial value of \( \text{WBBE} \) when \( T=0 \)
\[ \text{WBEAL} \] is the carbon concentration at the weld junction in the low carbon activity side /wt% \( (x^B) \)
\[ \text{WBEB} \] is the concentration of carbon in the matrix in equilibrium with carbide on the low carbon activity side /wt% \( (x^oB) \)
\[ \text{WBEBE} \] is the initial value of \( \text{WBEB} \) i.e. when \( T=0 \)
\[ \text{WX1} \] is the weight percent of element \( X \) on the high carbon activity side.
\[ \text{WX2} \] is the weight percent of element \( X \) on the low carbon activity side.
\[ \text{XX1} \] is the mole fraction of element \( X \) on the high carbon activity side.
\[ \text{XX2} \] is the mole fraction of element \( X \) on the low carbon activity side.
\[ \text{ZIA} \] is the distance of the particle from the interface at the end of the time step on the high carbon activity side \( (z^c_i) \)
\[ \text{ZIAS} \] is the square root of \( \text{ZIA} \).
\[ \text{ZJB} \] is the distance of the particle from the interface at the end of the time step on the low
carbon activity side (z^B)
ZOA is the distance of the particle from the interface at the start of the time step on the high carbon activity side (z^A)
ZOB is the distance of the particle from the interface at the start of the time step on the low carbon activity side (z^B)

The subroutines for reading and displaying the data are the same as for the previous model and will not be repeated in this listing.

```fortran
IMPLICIT DOUBLE PRECISION (A,C-I,K,M-S,U-Z)
DOUBLE PRECISION TSEC
INTEGER J,L,BA,T,BB,JM
AVER=1.1

Subroutines LOGO, DATA1-3 and DDTA all collect and display the data with options to make any changes.
CALL LOGO(AVER)
CALL DATA1(WCl ,WMN1,WSI1,WN11,WCRI,WMO1,WW1,WVB1,WCO1)
CALL DATA2(WC2,WMN2,WSI2,WN12,WCRI2,WMO2,WW2,WVB2,WCO2)
CALL DATA3(KTEMP,WALAE,WBEBE,WAALE,WBBEE,VEAE,VEEB)

CALL DDTA(WCl,WMN1,WSI1,WN11,WCRI,WMO1,WW1,WVB1,WCO1,
& WC2,WMN2,WSI2,WN12,WCRI2,WMO2,WW2,WVB2,WCO2,KTEMP,WALAE,
& WBEBE,WAALE,WBBEE,VEAE,VEEB)

Subroutine CON converts the data from weight per cent to mole fraction.
CALL CON(WCl,WMN1,WSI1,WN11,WCRI,WMO1,WW1,WVB1,WCO1,
& XC1,XMN1,XSI1,XN11,XCR1,XMO1,XW1,XVB1,XCO1)
CALL CON(WC2,WMN2,WSI2,WN12,WCRI2,WMO2,WW2,WVB2,WCO2,
& XC2,XMN2,XSI2,XN12,XCR2,XMO2,XW2,XVB2,XCO2)

Subroutine DIFF calculates the diffusion coefficient of carbon in ferrite.
CALL DIFF(KTEMP,DA)
CALL DIFF(KTEMP,DB)

The KFUN subroutines calculate the partition coefficients by one of the methods below.
WRITE(*,*)' For the calculation of partition coefficient,'
WRITE(*,*)' Method 1 = Wagner'
WRITE(*,*)' Method 2 = Wada et al.'
CALL REEDI(J)
IF (J .EQ. 1) THEN
  CALL KFUN1(XC1,XMN1,XSI1,XN11,XCR1,XMO1,XW1,XVB1,XCO1,
  & XC2,XMN2,XSI2,XN12,XCR2,XMO2,XW2,XVB2,XCO2,KTEMP,K)
ELSEIF (J .EQ. 2) THEN
  CALL KFUN2(WCRI,WCl,KTEMP,WC2,WCRI2,K)
ENDIF

The distances D1-D6 are calculated from the inputed value of the particle size and the volume fraction.

High carbon activity side - setting up the dimensions of the model:

XY=5.0D-06
D2=XY
D20=D2
D10=D20/VFA
D1=D10
D3=(D1-D2)
ZOA=D3/2
ZIA=ZOA
WALA=WALAE
WAAL=WAALE

Low carbon activity side - setting up the dimensions of the model

D4=XY
D40=D4
D5=D4/VFB
D6=(D5-D4)
D60=D6
ZOB=D6/2+D4
ZJB=ZOB
WBEB=WBEBE
WBBE=WBBEE

The calculations are made of the interface concentrations using equations (7.12) and (7.13) for the linear model

WALBE=((ZOA*WALA)+(ZOB*WBEB))/((K*ZOB)+ZOA)
WBEAL=K*WALBE

T=0
BA=0
BB=0
JT=0
JA=0
JM=0
WRITE(*,32)
32 FORMAT(2X'ATHR','DMW',BX,'BA')

This DO loop calculates the distance of the carbide from the interface ZIA at various times T and then recalculates the values of the distances and the average carbon concentration and volume fractions.

DO 30 T=0,10000
   JT=JT+1
   TSEC=T**3600

The values of ZIA and ZJB are calculated using the equations (7.7) and (7.8)

ZIAS=((2.0D0*DA*(WALA-WALBE)*TSEC)/(WAAL-WALA))+ZOA**2
ZIA=DSQRT(ZIAS)
ZJBS=(DB*((WBEAL-WBEB)/(WBBE-WBEB))*TSEC*2.0D0)+ZOB**2
ZJB=DSQRT(ZJBS)
DZW=(ZIA-ZOA)+(ZOA)+(BA*D10)
CZW=(ZJB-ZOB)+((BB+1)*D40)

The calculated values of DZW and CZW are converted to millimetres

DZW=DZW*1.0D+03
CZW=CZW*1.0D+03
D6=D60-(ZJB-ZOB)
ATHR=(TSEC/3600)**0.5
IF (TSEC.EQ.0.0) THEN
   WRITE(*,31)ATHR,DZW,BA
ENDIF

This IF-ELSE loop checks to see if any particles have been dissolved in which case it goes
onto the next particle

\[
\text{IF } ((ZIA-ZOA) \geq (D20*(BA+1))) \text{THEN}
\]
\[
\begin{align*}
BA &= BA+1 \\
D1 &= D10*(BA+1) \\
D2 &= D20*(BA+1) \\
JA &= 1 \\
VFA &= (D2/D1) \\
VFB &= (D4/D5) \\
CBARA &= (WAAL*VFA + (WALA*(1.0-VFA))) \\
CBARB &= (WBBE*VFB + (WBEB*(1.0-VFB)))
\end{align*}
\]

\[
\text{ELSE}
\]
\[
\begin{align*}
D2 &= (D20*(BA+1)) - (ZIA-ZOA) \\
D4 &= (D40*(BB+1)) + (ZJB-ZOB) \\
VFA &= (D2/D1) \\
VFB &= (D4/D5)
\end{align*}
\]

The average carbon concentrations on either side of the weld are calculated using equations (7.48) and (7.49)

\[
\begin{align*}
CBARA &= (WAAL*VFA + (WALA*(1.0-VFA))) \\
CBARB &= (WBBE*VFB + (WBEB*(1.0-VFB)))
\end{align*}
\]

If a particle has just been dissolved then the interface concentrations are recalculated using the SUBROUTINE INTER

\[
\text{IF } (JA = 1) \text{ THEN}
\]
\[
\begin{align*}
\text{CALL INTER(WALA,WAAL,CBARA,CBEB,WBBE,CBEB,CBARB,ZJB,ZIA,} \\
&\quad \text{K,WALBE,WBEAL,JM)} \\
\text{IF } (JM = 1) \text{ GOTO 501} \\
\quad JA &= 0 \\
\text{WRITE(*,31)ATHR,DZW,BA}
\end{align*}
\]

This IF statement checks to see whether the space between particles on the low carbon activity side has been filled with carbide

\[
\text{IF } ((ZJB-ZOB) \gt (D60*(BB+1))) \text{ THEN}
\]
\[
\begin{align*}
BB &= BB+1 \\
D5 &= D5*(BB+1)
\end{align*}
\]

\text{ENDIF}

This IF routine checks that the space between the carbides has not been filled as a result of diffusion away from the interface

\[
\text{IF } ((D60-ZH) \lt D6) \text{ THEN}
\]
\[
\begin{align*}
\text{CALL BACK(DB,WEBB,WEBBE,SEC,WEBB,D6,CBEB,DELC)} \\
\end{align*}
\]

\text{ENDIF}

B=WALA-WALBE

B2=-B

\[
\begin{align*}
\text{CALL CON(CBARA,WMN1,WSI1,WN11,WC11,WM01,WW1,WN1,WB11,WC11,} \\
&\quad \text{X11,XML1,XML1,XSC1,XCM1,XML1,XDM1,XML1,XML1,XCO1)} \\
\text{CALL CON(CBEB,WMN2,WSI2,WN22,WC22,WM02,WW2,WN2,WB22,WC22,} \\
&\quad \text{X22,XML2,XML2,XSC2,XCM2,XML2,XDM2,XML2,XML2,XCO2)}
\end{align*}
\]

A new value of the partition coefficient is calculated as a result of the changing carbon concentrations

\[
\text{IF } (J = 1) \text{ THEN}
\]
\[
\text{CALL KFUN1(XC1,XML1,XML1,XSC1,XCM1,XML1,XDM1,XML1,XML1,} \\
&\quad \text{XCO1,} \\
&\quad \text{XC2,XML2,XML2,XSC2,XCM2,XML2,XDM2,XML2,XML2,XCO2)}
\]

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ELSEIF (J .EQ. 2) THEN
   CALL KFUN2(WCR1,CBARA,KTEMP,CBARB,WCR2,K)
ENDIF

ATHR= (TSEC/3600)**0.5

CONTINUE
WRITE(*,50)
FORMAT(/)
WRITE(*,*)'Type 0 to amend data or 1 to quit'
CALL REEDI(L)
IF (L .EQ. 0) THEN
   GOTO 500
ELSEIF (L .EQ. 1) THEN
   GOTO 501
ENDIF
STOP
501 END

This subroutine calculates the changes in the equilibrium carbon concentrations as a result of the change in carbon concentrations. The equations are calculated by fitting curves to data calculated using MTDATA. These equations are for the 9Cr/2\textfrac{1}{4}CrMo joint at 620°C. The rest of the equations are given at the end of this appendix.

SUBROUTINE INTER(WA, LA, WAAL, CBARA, WBEB, WBBE, CBARB, ZJB, ZIA, &
   K, WALBE, WBEAL, JM)
IMPLICIT DOUBLE PRECISION(A-I,K-Z)
INTEGER JM
WALA=3.5189D-05* (10**(5.138*CBARA))
WAAL=5.0834-(0.49266*CBARA)+(3.1529*CBARA **2)-24.958 &
*(CBARA **3)+62.901*(CBARA **4)
WBEB=2.2060D-04-(2.731D-04*CBARB)+ &
(2.2857D-04*(CBARB**2))
WBBE=8.6412-(0.22853*DLOG10(CBARB))
WALBE=((ZIA*WALA)+(ZJB*WBEB))/((K*ZJB)+ZIA)
WBEAL=K*WALBE
IF (WALA .LT. WALBE .OR. WBEAL .LT. WBEB) THEN
   JM=1
   GOTO 44
ENDIF
44 RETURN
END

This subroutine calculates the amount of diffusion that is occurring away from the interface on the high alloy side.

SUBROUTINE BACK(DB, WBEB, WBBEE, TSEC, WBBEE, D6, CBARB, DELC, ZH)
IMPLICIT DOUBLE PRECISION (A-Z)
ZH=DSQRT(((2*DB*(WBEB-WBEBE)*TSEC)/(WBBEE-WBEE))+D6**2)
DEFC=(D6-ZH)**(WBBEE-WBEBE)
CBARB=CBARB-DEFC
RETURN
END
This is a subroutine to calculate the Wagner activity coefficient. \( \epsilon_{ij} \) is the Wagner interaction parameter between element X and carbon.

```
SUBROUTINE KFUN1(XC1,XM1,XN1,XC11,XM11,XN11,XC12,XM12,XN12,
               & XCO1,XC2,XM2,XN2,XC21,XM21,XN21,XC22,XM22,XN22,XCO2,KTEMP,K)
IMPLICIT DOUBLE PRECISION (A-Z)
EC=8910/KTEMP
EMN=-5070/KTEMP
ESI=4.84-(7370/KTEMP)
ENI=-2.2+(7600/KTEMP)
ECR=24.4-(38400/KTEMP)
EMO=3.855-(17870/KTEMP)
EW=23.4-(36114/KTEMP)
EV=-24660jKTEMP
ENB=-28770jKTEMP
ECO=2800jKTEMP
LAC1=(XC1*EC)+(XM1*EMN)+(XS1*ESI)+(XN1*ENI)+(XCR1*ECR)+
& (XMO1*EMO)+(XW1*EW)+(XV1*EV)+(XN1B1*ENB)+(XCO1*ECO)
AC1=DEXP(LAC1)

LA

LAC2=(XC2*EC)+(XM2*EMN)+(XS2*ESI)+(XN2*ENI)+(XCR2*ECR)+
& (XMO2*EMO)+(XW2*EW)+(XV2*EV)+(XN2B1*ENB)+(XCO2*ECO)
AC2=DEXP(LAC2)
K=AC1/AC2
RETURN
END
```

This subroutine calculates the partition coefficient from the empirical equations of Wada et al. (1972)

AFC is the natural logarithm of the activity coefficient

```
AFC1=2300jKTEMP-2.24+ ((179jKTEMP)*WC1)-((102jKTEMP)-0.033)*WCR1
FC1=DEXP(AFC1)
```

```
AFC2=2300jKTEMP-2.24+ ((179jKTEMP)*WC2)-((102jKTEMP)-0.033)*WCR2
FC2=DEXP(AFC2)
K=FC1/FC2
RETURN
END
```

This subroutine calculates the diffusion coefficient of carbon in ferrite as described in Section (7.3) using the model due to McLellan et al. (1965)

DOTO is the diffusion coefficient for carbon atoms jumping from an octahedral to an octahedral site via a tetrahedral site

```
DOTO is the diffusion coefficient for carbon atoms jumping from an octahedral to an octahedral site via a tetrahedral site
```

D is the diffusion coefficient of carbon in ferrite
PHI and F are as defined in McLellan et al. (1965)

SUBROUTINE DIFF(KTEMP,D)
DOUBLE PRECISION R,KTEMP,PHI,DOTO,DTT,F,D
R=8.314
PHI=1.0D0-1.0/(0.5D0*DEXP(7.2D03*4.184/(R*KTEMP))
& *DEXP(4.4D0)+1.0D0)
DOTO=3.3D-07*DEXP(-19.3D03*4.184D0/(R*KTEMP))
DTT=3.0D-4*DEXP(-14.7D03*4.184D0/(R*KTEMP))
F=0.86D0
D= PHI*DOTO+(1.0D0-PHI)*F*DTT+(1.0D0-PHI)
& *(1.0D0-F)*DOTO
RETURN
END
Subroutine INTER for the other joints studied

Mild steel/$\frac{1}{2}$CrMo at 730°C

```
SUBROUTINE INTER(WALA, WAAL, CBARA, WBEB, WBBE, CBARB, ZJB, ZIA, K, WALBE, WBEAL, JM)
IMPLICIT DOUBLE PRECISION(A-H,K-Z)
INTEGER JM
WALA=(1.1004D-02)+(4.5057D-03*CBARA)-
   (4.2857D-03*CBARA**2)
WAAL=6.7006-(3.3143D-02*CBARA)+(5.714D-02*CBARA**2)
IF (CBARB .LT. 0.4) THEN
   WBEB=5.6098D-04 *(10** (2.9965*CBARB))
   WBBE=5.1090-(1.1479*CBARB)+(3.3126*CBARB**2)
ELSE
   WBEB=(4.6043D-03)+(6.7886D-03*CBARB)- (1.8571 D-03*CBARB **2)
   WBBE=6.8311-(0.45905*CBARB) + (0.29452*CBARB**2)
ENDIF
WALBE=((ZIA*WALA)+(ZJB*WBEB))/(ZIA+(K*ZJB))
WBEAL=K*WALBE
IF (WALA .LT. WALBE .OR. WBEAL .LT. WBEB) THEN
   JM=1
   GOTO 44
ENDIF
44 RETURN
END
```
Mild steel/1CrMo at 700°C

SUBROUTINE INTER(WALA, WAAL, CBARA, WBEB, WBBE, CBARB, ZJB, ZIA, & K, WALBE, WBEAL, JM)
IMPLICIT DOUBLE PRECISION(A-I, K-Z)
INTEGER JM
WALA=(1.1004D-02)+(4.5057D-03*CBARA)-(4.2857D-03*CBARA**2)
WAAL=6.7006-(3.3143D-02*CBARA)+(5.714D-02*CBARA**2)
IF (CBARB .LT. 0.3) THEN
  WBEB=-5.5371 D-04+(3.6744D-02*CBARB)-(1.013D-02*CBARB**2)
  WBBE=4.9424+(0.45291 *CBARB)
ELSE
  WBEB=(6.1877D-03)+(1.539D-02*CBARB)-(9.1372D-03*CBARB**2)
  WBBE=6.7857-(0.46469*CBARB)+(0.52338 *CBARB **2)
ENDIF
WALBE=((ZIA*WALA)+(ZJB*WBEB))/((K*ZJB)+ZIA)
WBEAL=K*WALBE
IF (WALA .LT. WALBE .OR. WBEAL .LT. WBEB) THEN
  JM=1
  GOTO 44
ENDIF
44 RETURN
END

Mild steel/1CrMo at 620°C

SUBROUTINE INTER(WALA, WAAL, CBARA, WBEB, WBBE, CBARB, ZJB, ZIA, & K, WALBE, WBEAL, JM)
IMPLICIT DOUBLE PRECISION(A-I, K-Z)
INTEGER JM
WALA=(3.6078D-03)+(4.3603D-03*CBARA)-(4.714D-03*CBARA**2)
WAAL=6.7006-(5.5714D-02*CBARA)+(8.5714D-02*CBARA**2)
IF (CBARB .LT. 0.1) THEN
  WBEB=2.5017D-04+(1.2517D-04*CBARB)+(1.5444D-03*CBARB**2)
  WBBE=4.9588-(0.13215*DLOG10(CBARB))
ELSE
  WBEB=(1.4114D-03)+(5.7409D-03*CBARB)-(2.8689D-03*CBARB**2)
  WBBE=6.6328-(0.42104*CBARB)
ENDIF
WALBE=((ZIA*WALA)+(ZJB*WBEB))/((K*ZJB)+ZIA)
WBEAL=K*WALBE
IF (WALA .LT. WALBE .OR. WBEAL .LT. WBEB) THEN
  JM=1
  GOTO 44
ENDIF
44 RETURN
END

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SUBROUTINE INTER(WALA, WAAL, CBARA, WBEB, WBBE, CBARB, ZJB, ZIA, K, WALBE, WBEAL, JM)
IMPLICIT DOUBLE PRECISION(A-I, K-Z)
INTEGER JM
WALA=1.4854D-03*(10**(2.10*CBARA))
WAAL=5.0441-(0.24299*CBARA)+(0.49609*(CBARA **2))
WBEB=2.2632D-03-(1.3115D-03*CBARB)+(4.5924D-03*(CBARB **2))
WBBE=8.6538-0.24565*DLOG10(CBARB)
WALBE=((ZIA*WALA)+(ZJB*WBEB))/((K*ZJB)+ZIA)
WBEAL=K*WALBE
IF (WALA.LT.WALBE .OR. WBEAL.LT.WBEB) THEN
JM=1
GOTO 44
ENDIF
44 RETURN
END

2\frac{1}{4}CrMo/12CrMo at 730°C

SUBROUTINE INTER(WALA, WAAL, CBARA, WBEB, WBBE, CBARB, ZJB, ZIA, K, WALBE, WBEAL, JM)
IMPLICIT DOUBLE PRECISION(A-I, K-Z)
INTEGER JM
WALA=1.4854D-03*(10**(2.10*CBARA))
WAAL=5.0441-(0.24299*CBARA)+(0.49609*(CBARA **2))
WBEB=1.0847D-03-(7.0130D-05*CBARB)+(2.7296D-03*(CBARB**2))
WBBE=5.4254+(0.42155*DLOG10(CBARB))
WALBE=((ZIA*WALA)+(ZJB*WBEB))/(ZIA+(K*ZJB))
WBEAL=K*WALBE
IF (WALA.LT.WALBE .OR. WBEAL.LT.WBEB) THEN
JM=1
GOTO 44
ENDIF
44 RETURN
END

2\frac{1}{4}CrMo/12CrMo at 620°C

SUBROUTINE INTER(WALA, WAAL, CBARA, WBEB, WBBE, CBARB, ZJB, ZIA, K, WALBE, WBEAL, JM)
IMPLICIT DOUBLE PRECISION(A-I, K-Z)
INTEGER JM
WALA=3.5189D-05*(10**(5.138*CBARA))
WAAL=5.0834-(0.49266*CBARA)+(3.1529*CBARA **2)-24.958*(CBARA **3)+
62.901*(CBARA **4)
WBEB=2.9018D-04-(1.1251D-03*CBARB)+(1.4576D-03*(CBARB**2))
WBBE=5.4562+(0.42155*DLOG10(CBARB))
WALBE=((ZIA*WALA)+(ZJB*WBEB))/((K*ZJB)+ZIA)
WBEAL=K*WALBE
IF (WALA .LT. WALBE .OR. WBEAL .LT. WBEB) THEN
   JM=1
   GOTO 44
ENDIF
44 RETURN
END