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Depreciation Bias, Financial-Sector Fragility and Currency Risk

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Abstract: Do expected future exchange rate fluctuations affect current social welfare? In the third-generation approach to currency crises, financial fragility can trigger devaluation and default. Expected future depreciation is costly if it raises \textit{ex ante} real interest rates. Given the strong violation of uncovered interest parity, expected future outcomes’ current cost/benefit depends on the currency risk premium. I extend the static one-period Barro-Gordon welfare loss function to include expected future depreciation and show that, when foreign investors are risk-averse, depreciation bias is higher than the static case if aggregate demand is a function of \textit{ex ante} real rates. If demand depends on the \textit{ex post} real interest rate, average depreciation can be zero if current welfare is sufficiently sensitive to the state of the financial sector. In this stylised framework, depreciation bias can be mitigated even in the presence of time-inconsistency, and expected welfare may be higher.

JEL classification: E52, E58, F33, F37

Keywords: Depreciation bias, real interest rate, currency risk premium, social welfare

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1 Introduction

In the aftermath of recent financial crises in developing countries—South East Asia, Russia, Brazil, Turkey and Argentina being headline cases—a third-generation methodology for modelling currency crises is under development, focussing on illiquidity in the banking system and adverse spillovers from the financial sector to currency markets.\footnote{Illiquidity may be traced to financial liberalisation, the shorter foreign debt structure and the currency mismatch of assets and liabilities. See Calvo (1998, 2000), Calvo and Reinhart (2000), Chang and Velasco (2000a,b), Frankel and Rose (1996) and Rogoff (1999).} Financial fragility is manifest in the high observed correlation between exchange rate collapses and banking crises. Liquidity problems are amplified by strong systemic bias towards debt finance, especially towards intermediation by banks, leading to massive short term reversals of capital flows to developing economies.

As a result, forward-looking indicators of the state of the financial sector have been the object of attention alongside traditional current and capital account measures. A prominent such indicator is the \textit{ex ante} real interest rate on deposits. Kaminsky and Reinhart (1998) classify a large sample of recent banking and currency crises and find that, in both categories, real rates were significantly higher in the 18 months leading to the crisis compared to their average in tranquil periods. Expectations of future exchange rate devaluation can, therefore, affect current social welfare to the extent that they can influence the \textit{ex ante} real rate. Expectations of future devaluation can also be costly if they affect current nominal wages, as in Obstfeld (1994,1996) or the \textit{ex post} real interest rate, as in Eichengreen and Jeanne (1998).

In this paper I model this channel by extending the stylised Barro and Gordon (1983) one-period welfare loss function to include the current \textit{ex ante} real rate. The latter is related to expected future outcomes by the Fisher equation. Thus the loss function effectively depends on one-period-ahead depreciation expectations formed in the previous \textit{and} current period. In the conventional Barro-Gordon framework with sticky prices, expected future outcomes are irrelevant. Actual depreciation is implemented following the
supply shock realisation, while expected depreciation is fixed beforehand. With rational expectations, this induces equilibrium depreciation bias if the policymaker has a short-term expansionary motive. Assuming risk neutrality and zero forward discount bias, positive nominal interest differentials induce equal expected future depreciation of the home currency. However useful risk neutrality may be in describing boom phases when risk factors seem to have little role in discouraging capital inflows, it is clearly unrealistic in the context of rapid capital outflows leading to financial crises and country insolvency/default. Indeed, there is strong empirical evidence that the uncovered interest parity (UIP) no-arbitrage condition is violated: the impact of current nominal interest rate differentials on expected future exchange rates is absolutely smaller—sometimes also in the opposite direction—from what UIP would predict, reflecting investor risk aversion. I therefore restrict attention to exogenous and positive currency risk premia, capturing a range of underlying sources of country risk.\footnote{On the forward premium puzzle and peso problems see Isard (1995) and Lewis (1995). Positive interest differentials may reflect banking sector fragility, excessive foreign debt, political uncertainty, credibility problems, lack of lender of last resort, illiquidity, etc. For related anecdotal and empirical evidence in the context of the Asian and Latin American financial crises through 1998 see Krugman (2000).}

Purchasing power parity (PPP) is assumed to hold in the short-term, and the equilibrium influence of actual on expected future depreciation is obtained from the intersection of short-run aggregate demand and supply. There is no uncertainty about the loss function: the extended one-period Barro-Gordon framework is the social welfare benchmark. Unlike second-generation crisis models, there is no fixed cost triggering the devaluation decision. Under perfect capital mobility, the present cost/benefit of expected future depreciation depends on the magnitude of the risk premium and its impact on the \textit{ex ante} real interest rate.

A related expectational framework was studied by Krugman (1996), who argued that self-fulfilling crises may be an artifact of the unrealistic treatment of private agents’ expectations. If fundamentals deteriorate deterministically,
and expected future devaluation is costly because it raises the \textit{ex ante} real interest rate, then multiple equilibria cannot exist. However, as pointed out by Kehoe (1996), Krugman’s result relies on the loss function being sensitive to future expectations only. Jeanne and Masson (2000) argue that the policymaker’s decision is sensitive to the private sector’s depreciation expectations formed \textit{during}, as well as before the financial crisis. They show that, if the devaluation expectations affecting the present cost/benefit of a currency peg are formed in the last \textit{and} current periods, then devaluation expectations can become chaotic and a complete information model can admit an arbitrarily large number of equilibria.\footnote{Morris and Shin (1998) show that an alternative sufficient condition for uniqueness is lack of common knowledge about the fundamentals. Information asymmetries also produce real effects of balance sheet fluctuations, as in Bernanke and Gertler (1989).} Meyer et al. (2001) argue that incorporating future inflation expectations into forward-looking policy rules may also be warranted in closed-economy models because of lags in the monetary transmission mechanism.

The main results are as follows. If UIP holds, expected future outcomes cannot affect current welfare because \textit{ex ante} real rates are tied to the world level. I study two cases of UIP violation where the risk premium is positive. First, if aggregate demand is a function of \textit{ex ante} real rates then equilibrium depreciation bias is unambiguously higher than under the static one-period loss function, and expected social welfare is always lower. The intuition is that actual depreciation lowers real interest rates regardless of the risk premium. Second, if aggregate demand depends on the \textit{ex post} real rate, then average depreciation can be zero if current welfare is responsive to the \textit{ex ante} real rate and aggregate demand is more sensitive to the \textit{ex post} real rate than aggregate supply is to unanticipated depreciation. The appropriate sensitivity parameter \textit{increases} in the policymaker’s short-term expansionary motive and \textit{decreases} in the currency risk premium. Given sufficient risk aversion by foreign investors, this result suggests that expected inflation/depreciation bias may be eliminated \textit{even} in the presence of time-inconsistency if financial
sector fragility matters. In expected social welfare terms, the extended one-period loss function may dominate the static one provided the policymaker has a short-term expansionary motive.

In the remainder of the paper, Section 2 motivates the extended loss function and presents the model. Section 3 derives its equilibrium properties and evaluates expected social welfare. Section 4 concludes.

2 The model

2.1 The one-period static benchmark

In Barro and Gordon's (1983) one-period model applied to an open economy, $s_t$, $p_t$ and $p_t^*$ are the (log) nominal spot exchange rate and domestic and foreign price levels in period $t$. Assuming absolute PPP holds ($s_t = p_t - p_t^*$), normalising $p_t^* = 1$ and differencing implies $\Delta s_t = \pi_t$, so actual depreciation and inflation are equivalent. The static one-period welfare loss function is:

$$L_t = \frac{1}{2} (y_t - y^*)^2 + \frac{\chi}{2} \pi_t^2,$$

where $y_t$ and $\pi_t$ are the (log) period-$t$ output level and depreciation rate.

The depreciation target is set to zero, i.e. a fixed exchange rate, and relative depreciation aversion normalised to $\chi > 0$, wlog. The output target $y^* \geq 0$ exceeds the home potential output level $\overline{y}$ by $k = y^* - \overline{y} \geq 0$, capturing the policymaker's incentive for a surprise devaluation. Output is supply-determined by a short-run linear Phillips curve:

$$y_t = \overline{y} + \alpha(\pi_t - E_{t-1}\pi_t) + z_t$$

The slope $\alpha \geq 0$ captures the short-run output-depreciation trade-off, that is the open economy's sacrifice ratio. Supply shocks $z_t$ are distributed iid with conditional mean zero, standard error $\sigma_z$ and uncorrelated with $\pi_t$. Depreciation expectations $E_{t-1}\pi_t$, and hence nominal wages for period $t$, are
set in \( t - 1 \) before observing \( z_t \); but the policymaker sets \( \pi_t \) after the shock. Minimising (1) subject to (2) yields the one-period equilibrium outcomes:

\[
\pi_t = \frac{\alpha^2 E_{t-1}\pi_t + k\alpha - \alpha z_t}{\alpha^2 + \chi}, \quad y_t = \frac{\alpha \chi E_{t-1}\pi_t - k\alpha^2 - \chi z_t}{\alpha^2 + \chi}
\]

Taking expectations at \( t - 1 \), average depreciation and output are just:

\[
E_{t-1}\pi_t = \frac{k\alpha}{\chi}, \quad E_{t-1}y_t = \overline{y}
\]

Expected depreciation bias increases in \( k \), the degree of the policymaker’s time-inconsistency, or short-term incentive to push output above potential. On average, a fixed exchange rate can only be maintained if the policymaker can credibly commit to make \textit{any} non-zero depreciation rate prohibitively expensive by setting \( \chi \to \infty \), that is to pay the price of a recession regardless of the magnitude of the supply shock, and/or if short-run aggregate supply is vertical \((\alpha = 0)\).\footnote{Calvo (2000) and Chang and Velasco (2002) argue that dollarization can also serve as a commitment device in an open economy. See also Gale and Vives (2002) for lender-of-last-resort issues.} The resulting expected social welfare losses in the static Barro-Gordon case are:

\[
E_{t-1}L_t = \frac{\alpha^2 + \chi}{2\chi} k^2 + \frac{\chi}{2(\alpha^2 + \chi)} \sigma_z^2
\]

Expected welfare losses are increasing at the square of the degree of time inconsistency. If \( k = 0 \), then \( E_{t-1}L_t \) only depends on depreciation and output fluctuations due to supply shock variability.

\section*{2.2 Extending the one-period loss function}

In the escape clause tradition of Flood and Isard (1989) and Obstfeld (1994, 1996, 1997) and Jeanne (1997), the impact of the short-term macroeconomic juncture on the state of fundamentals is such that, by introducing
non-linearity as an extra fixed cost of actual depreciation, there can be self-fulfilling speculative attacks and perhaps contagion from other countries suffering crises. Provided \( k > 0 \), these models generate multiple equilibria for expected depreciation which are self-fulfilling. Multiplicity is sensitive to the policymaker targeting short-term output above potential. In turn, the government’s incentive to overcome time-inconsistency justifies the extra cost of exchange rate realignment.\(^5\)

Second-generation currency crisis models introduce a fixed cost of actual exchange rate fluctuation to the one-period loss function:

\[
L_t = \frac{1}{2}(y_t - y^*)^2 + \frac{\chi}{2}\pi_t^2 + c_t(\pi_t)
\]

(5)

Any depreciation in period \( t \) results in \( c_t(\cdot) = \tau \), while any appreciation leads to \( c_t(\cdot) = \zeta \). Both \( \tau \) and \( \zeta \) are positive constants, not necessarily equal, and the actual marginal loss is zero. Importantly, the timing is effectively one-period because agents form their expectations in period \( t - 1 \), before the incidence of the crisis. If this feature is relaxed, then multiple equilibria can arise without the need for the policymaker to have a short-term surprise (time-inconsistency) motive.

In contrast, in this paper I follow Jeanne and Masson (2000) and assume that current welfare depends on the period-\( t \) expectational error \( (\pi_t - E_{t-1}\pi_t) \) and expectations for period \( t + 1 \) \( (E_t\pi_{t+1}) \). Whereas \( E_{t-1}\pi_t \) is set in \( t - 1 \) and built into period-\( t \) nominal wage contracts, \( E_t\pi_{t+1} \) is determined contemporaneously with \( \pi_t \). In particular, I incorporate expected future outcomes into the period-\( t \) loss function through the additional term \( c_t = c r_t^A \), where \( r_t^A \) is the ex ante short-term real interest rate. This is linked to expected future depreciation by the Fisher relation \( \dot{r}_t = r_t^A + E_t\pi_{t+1} \). The marginal cost/benefit of higher/lower real rates is taken to be a positive constant \( c \).

\[
\mathcal{L}_t = \frac{1}{2} (y_t - y^*)^2 + \frac{\chi}{2} \pi_t^2 + c r_t^4, \quad c > 0
\]  

(6)

The relevance of the extra term can be justified as follows. First, in the real economy higher \textit{ex ante} real rates adversely affect current investment and future output, \textit{ceteris paribus}. Second, the private sector's debt burden increases, damaging corporate balance sheets and raising the prospect of a financial crisis.\(^6\) Moreover, to the extent that higher real rates are due to expected future depreciation, rather than an increase in foreign interest rates, they may trigger abrupt reversals in short-term international capital flows—Calvo's (1998, 2000) \textit{sudden stops}—which can push the country to insolvency and default. Refinancing foreign-denominated liabilities then becomes more expensive, aggravating any existing liquidity problems, which in turn intensifies speculation against the currency. The negative balance-sheet effects of the currency plunge may lead to a collapse in domestic investment. Conversely, lower \textit{ex ante} real rates have a beneficial actual effect. The impact of expected future depreciation on the \textit{ex ante} real rate is determined by the degree of foreign investors' risk aversion, to which I now turn.

2.3 Foreign investor risk aversion and UIP violation

Expected future depreciation is related to the nominal interest differential by the uncovered interest rate parity (UIP) no-arbitrage condition. If investors are risk-neutral, domestic and foreign assets are perfect substitutes and forward exchange rates are unbiased predictors of future spot exchange rates, then under perfect capital mobility expected returns in the two currencies are equalised. Denote the domestic nominal interest rate for period-\(t\) deposits maturing at \(t + 1\) by \(i_t\) and the (constant) foreign nominal rate by \(i^*\):

\[
E_t \Delta s_{t+1} = E_t \pi_{t+1} \approx i_t - i^*
\]

(7)

\(^6\) Other indicators of financial-sector (overlending) problems include the ratio of domestic credit to nominal GDP, deposits at commercial banks, the ratios of lending to deposit rates and M2 to foreign exchange reserves, the M2 multiplier, etc.
Positive nominal interest rate differentials \((i_t - i^* > 0)\) imply that the home currency is expected to depreciate by the same amount. However, the ensemble of UIP conditions is very unrealistic, not least because of the failure of risk neutrality to account for the capital inflow and outflow patterns in developing countries. If investors are risk-averse, (7) generalises to:

\[
E_t \pi_{t+1} = i_t - i^* - \xi_t, \quad \xi_t \geq 0
\]

where \(\xi_t \equiv f_t - E_t s_{t+1}\) is the home currency’s risk premium measuring deviations from UIP, in other words the forward prediction bias component of market expectations. For analytical purposes, I assume interest differentials are strictly positive to exclude the case \(i_t = i^*\), and adopt the equivalent multiplicative formulation:

\[
E_t \pi_{t+1} = \delta_t (i_t - i^*), \quad \delta_t \leq 1
\]

UIP holds iff \(\delta_t = 1\ (\xi_t = 0)\); \(\delta_t\) and \(\xi_t\) are inversely related. So as to simplify the exposition in a one-period setup, in the remainder of this paper I drop the time subscripts and focus on an exogenously given range of positive risk premia, reflecting the UIP violation relevant for developing countries.\(^7\) Assuming \(i_t > i^*\), this is the range \(\xi > 0\) corresponding to \(\delta < 1\). Compared to UIP, positive risk premia induce smaller average depreciation of the home currency if \(\delta \in (0, 1)\), and average appreciation if \(\delta < 0\). The case \(\delta = 0\) is ruled out as it implies \(E_t \pi_{t+1} = 0\) independently of the interest rate differential. Applying (9) to \(r^A_t = i_t - E_t \pi_{t+1}\) and solving for \(r^A_t\) yields:

\[
r^A_t = \delta i^* + (1 - \delta) i_t = i^* + \frac{1 - \delta}{\delta} E_t \pi_{t+1}
\]

\[
\Rightarrow \frac{dr^A_t}{dE_t \pi_{t+1}} = \frac{1 - \delta}{\delta}
\]

\(^7\)For a dynamic model with endogenous risk premia see Céspedes et al. (2000), who also distinguish between financial fragility and mibustness.
Substituting (10) in extended welfare loss function (6) expresses period-$t$ losses as a function of actual and expected future depreciation:

$$
\mathcal{L}_t = \frac{1}{2}(y_t - y^*)^2 + \frac{\chi}{2} \sigma_t^2 + c t^* + \frac{c(1 - \delta)}{\delta} E_t \bar{\pi}_{t+1}
$$

(12)

The effect of $E_t \bar{\pi}_{t+1}$ on welfare in period-$t$ depends on its impact on $r_t^A$. First, if UIP holds ($\delta = 1$) then $r_t^A = i^*$: the foreign interest rate is exogenous, so expected future depreciation has no effect on the *ex ante* real rate. Second, if the risk premium is positive and $\delta \in (0, 1)$ then expected future depreciation ($E_t \bar{\pi}_{t+1} > 0$) implies higher $r_t^A$. As argued above, this is costly in period $t$ because it induces financial-sector fragility, domestic currency substitution and foreign capital flight. The magnitude of constant $c$ then reflects expected future depreciation aversion. Conversely, expected future appreciation ($E_t \bar{\pi}_{t+1} < 0$) may strengthen bank balance sheets, stimulate real and portfolio investment and encourage capital inflows. Third, if UIP is violated and $\delta < 0$ then positive interest rate differentials imply the home currency is expected to appreciate. This raises the *ex ante* real rate and lowers period-$t$ welfare, all else equal. Finally, if the currency risk premium is negative ($\xi < 0, \delta > 1$) then (11) implies that expected future depreciation lowers real rates. This may reflect the notion that, in contrast to emerging markets, in industrial countries devaluations are frequently viewed as expansionary. In the last two cases, the magnitude of $c$ captures a relative social preference for expected future depreciation via the financial-sector channel.

### 3 Equilibrium depreciation bias

#### 3.1 First-order conditions

The necessary optimality condition for minimising the extended one-period loss function (12) is:
\[
(y_t - y^*) \frac{\partial y_t}{\partial \pi_t} + \chi \pi_t + c \frac{\partial r^A_t}{\partial \pi_t} =
(y_t - y^*) \frac{\partial y_t}{\partial \pi_t} + \chi \pi_t + c \frac{dr^A_t}{dE_t \pi_{t+1}} \frac{\partial E_t \pi_{t+1}}{\partial \pi_t} = 0 \tag{13}
\]

\[\frac{\partial y_t}{\partial \pi_t} = \alpha\] from short-run aggregate supply function (2). From (11), if \(\delta \in (0,1)\) then \(dr^A_t/dE_t \pi_{t+1} > 0\) and decreasing in \(\delta\). Equivalently, the sensitivity of the \textit{ex ante} real rate to expected future depreciation increases in the risk premium \(\xi\). It is only zero if foreign investors are risk-neutral and UIP holds.

### 3.1.1 Aggregate demand a function of \textit{ex ante} real interest rates

In order to compute \(\partial E_t \pi_{t+1}/\partial \pi_t\) in (13) and avoid an infinite forward-induction problem, I first consider a linear aggregate demand function of the \textit{ex ante} short-term real interest rate:

\[y_t = \beta - \gamma r^A_t + u_t \quad , \gamma > 0 \tag{14}\]

where \(u_t \sim (0,\sigma_u^2)\) is a mean-zero, serially uncorrelated aggregate demand shock.\(^8\) In equilibrium, domestic aggregate demand must equal aggregate output. Substituting \(r^A_t\) from (10) in (14) and equating with (2):

\[
\beta - \gamma y^* + \frac{\gamma (\delta - 1)}{\delta} E_t \pi_{t+1} + u_t = \gamma + \alpha(\pi_t - E_{t-1} \pi_t) + z_t
\]

Therefore, the marginal impact of actual on expected future depreciation under the extended one-period loss function is given by:

\[
\frac{\partial E_t \pi_{t+1}}{\partial \pi_t} = \frac{\alpha \delta}{\gamma (\delta - 1)} \tag{15}
\]

\(^8\)This is general aggregate demand function \(y_t = \beta + \lambda(s_t + p^*_t - p_t) - \gamma r^A_t + u_t\) with short-run PPP imposed.
\[ \partial E_t \pi_{t+1} / \partial \pi_t \] is undefined if UIP holds (\( \delta = 1 \)). However, as argued above, then \( r_t^A = i^* \) and the \textit{ex ante} real rate does not affect welfare in period \( t \). Otherwise, \((15)\) is negative for all \( \delta \in (0, 1) \) and positive for \( \delta < 0 \). The equilibrium effect of actual on expected future depreciation is smaller the more sensitive aggregate demand is to the \textit{ex ante} real interest rate, and bigger the more sensitive aggregate supply is to unanticipated exchange rate fluctuations.

Note that, if actual depreciation—hence also inflation—was known to follow a pure random walk process \((\pi_{t+1} = \pi_t + \epsilon_{t+1}, \text{where } \epsilon \text{ is white noise})\) then \( \partial E_t \pi_{t+1} / \partial \pi_t = 1 \), implying \( \delta^* = \frac{\gamma}{\gamma - \alpha} \). Thus, in principle a random walk is consistent with both positive and negative currency risk premia, depending on the relative slopes of the short-run aggregate demand and supply curves.\(^9\)

Alternatively, equating aggregate demand and supply and solving for \( r_t^A \):

\[
r_t^A = \frac{\beta - \gamma}{\gamma} - \frac{\alpha}{\gamma} \left( \pi_t - E_{t-1} \pi_t \right) + \frac{u_t - z_t}{\gamma}
\]

\( \text{Ceteris paribus,} \) favorable demand shocks \((u_t > 0)\) and adverse supply shocks \((z_t < 0)\) raise \( r_t^A \). Surprise depreciation lowers it; in equilibrium, \( \partial r_t^A / \partial \pi_t = -\alpha / \gamma < 0 \) regardless of the currency risk premium.\(^10\) Intuitively, specifying aggregate demand as a function of the \textit{ex ante} real rate means that actual depreciation can only lower \( r_t^A \), thereby improving welfare through the financial-sector channel. The converse is true for actual appreciation.

To obtain the extended one-period optimum, substitute aggregate supply \((2)\) and equilibrium expression \((15)\) into first-order condition \((13)\):

\[
\alpha^2 \left( \pi_t - E_{t-1} \pi_t \right) - \alpha k + \alpha z_t + \chi \pi_t - \frac{\epsilon \alpha}{\gamma} = 0
\]

\(^9\)An extension along these lines is beyond the scope of this paper. The necessary sustainability condition \( E_t \pi_{t+1} = \pi_t \) is a logistic mapping from actual to expected future depreciation. Solving establishes \( \pi_t^* = 0 \) as one of its two fixed points. The second is generically non-zero, and the resulting exchange rate dynamics can be chaotic for realistic parameter values.

\(^{10}\)Equivalently, multiplying \((11)\) by \((15)\) the effect of \( \delta \) cancels out.
Taking expectations at $t-1$ and solving for $E_{t-1}\pi_t$ yields:

$$E_{t-1}\pi_t = \frac{\alpha k}{\chi} + \frac{c\alpha}{\chi \gamma}$$

Equilibrium $E_{t-1}\pi_t$ is positive and exceeds its value under the standard Barro-Gordon loss function by $c\alpha/\chi \gamma$. Average depreciation is higher because a weaker home currency unambiguously improves period-$t$ welfare via the ex ante real interest rate channel. Excess depreciation bias is decreasing in $\gamma$ and increasing in $\alpha$, all else equal. Meanwhile, substituting (18) into aggregate supply function (2) and taking expectations yields $E_{t-1}y_t = \bar{y}$: average output is unchanged at its potential level.

3.1.2 Aggregate demand a function of ex post real interest rates

Alternatively, I now evaluate $\partial E_t\pi_{t+1}/\partial \pi_t$ assuming that aggregate demand is strictly decreasing in the ex post short-term real interest rate:

$$y_t = \beta - \gamma r_t^P + u_t, \quad \gamma > 0$$

where $r_t^P = \bar{r}_t - \pi_t$ and aggregate demand shock $u_t$ is defined as in (14). Once more, domestic aggregate demand must clear aggregate supply in each period:

$$\beta - \gamma (\bar{r}_t - \pi_t) + u_t = \bar{y} + \alpha (\pi_t - E_{t-1}\pi_t) + z_t$$

Substitute out $i_t$ using (9) and differentiate $E_t\pi_{t+1}$ with respect to $\pi_t$ to obtain the equilibrium sensitivity of $E_t\pi_{t+1}$ to $\pi_t$, analogous to (15) above:

$$\frac{\partial E_t\pi_{t+1}}{\partial \pi_t} = \frac{(\gamma - \alpha)\delta}{\gamma}$$

(20)

Applying (20) and $dr_t^p/dE_t\pi_{t+1} = (1 - \delta)/\delta$ into first-order condition (13) yields the equilibrium expression:

$$\alpha^2(\pi_t - E_{t-1}\pi_t) - \alpha k + \alpha z_t + \chi \pi_t + \frac{c(1 - \delta)(\gamma - \alpha)}{\gamma} = 0$$

(21)
Unlike corresponding expression (17), the last term is not unambiguously signed. Now $\partial r^d_t / \partial \pi_t = (1 - \delta)(\gamma - \alpha) / \gamma$, which depends on the relative slopes of the short-run aggregate demand and supply curves as well as on the risk premium. Taking expectations at $t - 1$ and solving for $E_{t-1}\pi_t$:

$$E_{t-1}\pi_t = \frac{\alpha k}{\chi} + \frac{c(\delta - 1)(\gamma - \alpha)}{\gamma \chi}$$

(22)

There are three cases: (i) If UIP holds ($\delta = 1$) there is zero average depreciation associated with the real interest rate term in the loss function.

(ii) With a positive risk premium ($\delta < 1$), if short-run aggregate supply is more sensitive to $\pi_t - E_{t-1}\pi_t$ than aggregate demand is to the ex post real interest rate ($\gamma < \alpha$), then the contribution to bias of the additional term in the loss function is positive: average depreciation is greater than under the static Barro-Gordon loss function by the second term in (22).

(iii) In contrast, if $\delta < 1$ and $\gamma > \alpha$ then the extra term’s contribution is negative: average depreciation is smaller than under the static loss function. When $\gamma > \alpha$, a positive risk premium mitigates depreciation bias because actual depreciation implies higher ex ante real rates for all $\delta < 1$ ($\xi > 0$), thus lowering current social welfare. Indeed, if short-run aggregate supply is very steep (small $\alpha$) and aggregate demand is very responsive to $r^d_t$ (large $\gamma$), then (22) implies the home currency may appreciate on average.\(^{11}\) It follows that equilibrium depreciation bias can be zero even if the policymaker has a short-term expansionary motive ($k > 0$), provided period-$t$ welfare is sufficiently sensitive to the ex ante real rate. The appropriate level of sensitivity for $E_{t-1}\pi_t = 0$ is given by:

$$c^* = \frac{k\alpha \gamma}{(\delta - 1)(\alpha - \gamma)}$$

(23)

The knife-edge level $c^*$ is increasing in $k$ and independent of the actual depreciation aversion coefficient $\chi$. Recall from (12) that the interpretation

\(^{11}\)Positive interest differentials may also induce expected future appreciation if $\delta < 0$. 

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of \( c \) as a coefficient of expected future depreciation aversion is conditional on the magnitude of the risk premium. The impact of the latter on \( c^* \) is 
\[ \frac{\partial c^*}{\partial \delta} = k\alpha\gamma/[(\delta - 1)^2(\gamma - \alpha)] > 0 \] 
for all \( \gamma > \alpha \). Thus \( c^* \) is increasing in \( \delta \); the nearer foreign exchange markets are to the UIP no-arbitrage limit \((\delta \to 1, \xi \to 0)\), the greater the sensitivity of period-\( t \) welfare to expected future exchange rate fluctuation required for zero average depreciation, all else equal. The intuition for why \( c^* \) becomes infinite as \( \delta \to 1 \) is that the extra average depreciation term in (22) then tends to zero.

### 3.2 Comparative expected social welfare evaluation

The equilibrium expected welfare losses under static loss function (1) are in equation (4). These are now compared to the outcomes under extended one-period loss function (6):

\[ E_{t-1}L_t = \frac{1}{2} E_{t-1}(y_t - y^*)^2 + \frac{\chi}{2} E_{t-1}^2 \pi_t^2 + c E_{t-1} r_t^A \]  

(24)

There are two cases, corresponding to the aggregate demand function specification (14) or (19). Inspecting their first-order conditions, respectively (17) and (21), makes clear that the extra term \( E_{t-1} r_t^A \) contributes a constant to each optimisation. Consequently, because the variance of equilibrium inflation and output are unchanged from the static loss function, any change in expected welfare is due only to the contribution of the \( E_{t-1} \pi_t^2 \) term, hence to the equilibrium expected depreciation rates (18) and (22).

It is easy to check the expected ex ante and ex post real rates are equal. Taking expectations and substituting aggregate demand functions (14) or (19) and expected depreciation from (18) or (22) into extended first-order condition (13) implies:

\[ E_{t-1} r_t^A = E_{t-1} r_t^P = \frac{\beta - \gamma}{\gamma} \]  

(25)

Requiring positive average real interest rates imposes the weak parameter restriction \( \beta > \gamma \). Substituting (18) and (25) into (24) and manipulating
yields expected welfare losses when aggregate demand depends on the \textit{ex ante} real interest rate:

\[
E_{t-1}L_t = \frac{\alpha^2 + \chi}{2\chi} k^2 + \frac{\chi}{2(\alpha^2 + \chi)} \sigma_z^2 + \frac{c}{\gamma} \left[ \beta - \bar{y} + \frac{\alpha^2}{2\chi} \left( \frac{c}{\gamma} + 2k \right) \right] \quad (26)
\]

This clearly exceeds \(E_{t-1}L_t\) in (4) for all \(c > 0\) provided average real interest rates are positive (\(\beta > \bar{y}\)). Extending the loss function by \(cr_t^A\) is thus always detrimental to welfare. Excess expected losses are increasing in \(c, k\) and \(\alpha\), decreasing in \(\chi\) and \(\gamma\), and independent of the risk premium.

In the second case, when aggregate demand depends on the \textit{ex post} real rate \(r_t^P\), equilibrium expected welfare losses are: \(^{12}\)

\[
E_{t-1}L_t = \frac{\alpha^2 + \chi}{2\chi} k^2 + \frac{\chi}{2(\alpha^2 + \chi)} \sigma_z^2 + \frac{c}{\gamma} \left[ \beta - \bar{y} + \frac{c(\delta - 1)^2(\gamma - \alpha)^2}{2\gamma\chi} + k \frac{\alpha(\delta - 1)(\gamma - \alpha)}{\chi} \right] \quad (27)
\]

The last term in square brackets is now of ambiguous sign, suggesting the possibility that expected welfare may be higher than under the static one-period loss function. In order for \(E_{t-1}L_t^P < E_{t-1}L_t\), the whole term in brackets must be negative. The implied restriction on \(c\) turns out to be:

\[
c < \frac{2\gamma\chi}{(\delta - 1)^2(\gamma - \alpha)^2} \left( \bar{y} - \beta + \alpha(1 - \delta)(\gamma - \alpha) \frac{k}{\chi} \right) \quad (28)
\]

Note that, because \(\beta > \bar{y}\), (28) is violated for all positive \(c\) if \(k = 0\). However, it may be satisfied if the short-term output target exceeds potential. \(^{12}\) Given \(k > 0\), inequality (28) is a sufficient condition for \(E_{t-1}L_t^P < E_{t-1}L_t\). A weaker necessary condition for this involves the expression in parentheses:

\[
E_{t-1}L_t < E_{t-1}L_t \quad \Rightarrow \quad k\alpha(1 - \delta)(\gamma - \alpha) > (\beta - \bar{y})\chi \quad (29)
\]

\(^{12}\)The algebraic derivation is available from the author upon request.
Therefore, in addition to \( k > 0 \), conducive parameter values for expected social welfare losses to be lower under the extended one-period loss function include small \( \delta \)—i.e. a large currency risk premium \( \xi \)—and \( \gamma > \alpha \). These \textit{ceteris paribus} parameter restrictions involve a high degree of foreign investor risk aversion and a sensitivity of aggregate demand to the \textit{ex post} real rate (\( \gamma \)) exceeding that of aggregate supply to unanticipated depreciation (\( \alpha \)). However, from (27) note also that the expected loss increment is multiplied by \( c/\gamma \), so any expected welfare gain due to large \( \gamma \) is bounded above.

4 Conclusion

This paper extended the static Barro-Gordon one-period welfare loss function to include expected future depreciation when foreign investors are risk-averse and financial-sector fragility is costly. Expected future outcomes then affect social welfare through the \textit{ex ante} real interest rate. Aggregate demand was considered to be a function of either \textit{ex ante} or \textit{ex post} real rates. If foreign investors are risk-averse, average depreciation can be zero if expected future depreciation is sufficiently costly, and the level of this cost was shown to decrease in the currency risk premium. Parameter restrictions for expected social welfare to be higher were also derived. Clearly, social welfare would be higher in the absence of time-inconsistency under both the static and the extended loss functions. However, when expected future outcomes matter and aggregate demand is a function of the \textit{ex post} real rate, a certain degree of short-term time-inconsistent behaviour by the home policymaker may result in higher expected social welfare. Taken together with second-generation models’ requirement that \( k > 0 \) for multiple equilibria to exist, this result suggests that explicit consideration of policymakers’ short-term incentives remains a key research priority for understanding financial crises. A final caveat is that any stylised model based on UIP violations may be misleading if \( \delta \) estimates are insignificant, and/or if regressing future exchange rate movements on current interest differentials has little explanatory power.
References


## Working Papers

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