Electoral Uncertainty and Public Goods*

Toke S. Aidt†    Jayasri Dutta‡

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Abstract

This paper argues that uncertain or random voter turnout plays a key role in mediating conflicts of interest between voters and politicians on the one hand and heterogenous groups of voters on the other. Random voter turnout creates an incentive for politicians to seek consensus because it is unclear ex ante who will hold the majority among those who turn out to vote. We argue that this leads to efficient provision of public goods and that it protects minority groups against the tyranny of the majority. We also argue that compulsory voting may not be desirable because it reduces randomness in turnouts.

Keywords: Political Agency, Performance Voting, Turnout Uncertainty, Public Finance.

JEL Classifications: D72; D78; H41.

1 Introduction

Political processes are designed to resolve conflicts among groups of citizens with different and often conflicting objectives and goals. The manner in

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†Faculty of Economics, University of Cambridge, Austin Robinson Building, Sidgwick Avenue, Cambridge CB3 9DD, Tel.: +44 1223 335231. E-mail: Toke.Aidt@econ.cam.ac.uk.

‡Department of Economics, University of Birmingham, Tel.: +44 0121 4146640. E-mail: J.Dutta@bham.ac.uk.
which such conflicts are resolved is of interest not only because it informs normative discussions of fairness and legitimacy but also because it helps predict outcomes under different institutional arrangements. Voting is the cornerstone of most political processes and the properties of different types of voting systems have attracted a lot of attention from public choice scholars.\footnote{See, e.g., Mueller (2003).} This includes the fundamental question of why voters vote.\footnote{See Dhillon and Peralta (2002) or Aidt (2000) for a discussion of this literature.}

While there exists a substantial body of research on this question, the issue of voter turnout is largely ignored in applied work on the political economy of fiscal policy. The canonical models – the median voter model, the probabilistic voting model as well as the various agency models of elections – all assume that voters turn out to vote in each election.\footnote{For a good introduction to these models, see, e.g., Persson and Tabellini (2000) or Hettich and Winer (1999, chapter 2).}

Yet, the evidence suggests otherwise. Not only is average voter turnout low in many countries, it also fluctuates substantially over time and space. To illustrate this point, Table 1 records average turnout rates in national elections along with the coefficient of variation for 25 countries for the period 1970 to 2000.\footnote{The coefficient of variation is defined as the standard deviation divided by the mean multiplied by 100.} Of particular importance to the argument of this paper is the fact that the coefficient of variation – a direct measure of turnout uncertainty – is substantial in many countries and often exceeds 5% of the average.

Another important fact about electoral turnout is the large variation in the average turnout of voters with different demographic and socioeconomic characteristics. This is clearly illustrated by Table 2 which documents large and variable gaps in inter-group electoral participation in western democracies during the period 1996 to 1999.

In this paper, we argue that uncertain voter turnout has important but largely overlooked implications for how particular political processes mediate conflicts of interest between voters and politicians on the one hand and heterogeneous groups of voters on the other. A simple example can illustrate our reasoning. Consider a country in which decisions are made by simple majority rule. The country is divided into two regions, called North and South, and is populated by 100 voters distributed with 60 voters in the South and 40 voters in the North. So, if turnout were certain (and all voters showed up to vote), then politicians could win elections simply by pandering to the
<table>
<thead>
<tr>
<th>Country</th>
<th>mean</th>
<th>coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>83.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Austria</td>
<td>83.3</td>
<td>7.2</td>
</tr>
<tr>
<td>Belgium</td>
<td>87.6</td>
<td>3.8</td>
</tr>
<tr>
<td>Canada</td>
<td>65.7</td>
<td>6.6</td>
</tr>
<tr>
<td>Denmark</td>
<td>84.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Finland</td>
<td>77.3</td>
<td>8.5</td>
</tr>
<tr>
<td>France</td>
<td>63.9</td>
<td>7.5</td>
</tr>
<tr>
<td>Germany</td>
<td>78.9</td>
<td>7.3</td>
</tr>
<tr>
<td>Greece</td>
<td>84.6</td>
<td>2.1</td>
</tr>
<tr>
<td>Iceland</td>
<td>88.9</td>
<td>2.0</td>
</tr>
<tr>
<td>Italy</td>
<td>92.4</td>
<td>2.8</td>
</tr>
<tr>
<td>Japan</td>
<td>67.5</td>
<td>13.8</td>
</tr>
<tr>
<td>South Korea</td>
<td>74.3</td>
<td>7.9</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>65.1</td>
<td>9.2</td>
</tr>
<tr>
<td>Mexico</td>
<td>54.8</td>
<td>18.2</td>
</tr>
<tr>
<td>Netherlands</td>
<td>80.8</td>
<td>7.7</td>
</tr>
<tr>
<td>Norway</td>
<td>79.7</td>
<td>4.0</td>
</tr>
<tr>
<td>Portugal</td>
<td>81.1</td>
<td>7.8</td>
</tr>
<tr>
<td>Spain</td>
<td>76.8</td>
<td>5.8</td>
</tr>
<tr>
<td>Sweden</td>
<td>85.1</td>
<td>4.0</td>
</tr>
<tr>
<td>Switzerland</td>
<td>40.3</td>
<td>9.9</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>72.9</td>
<td>3.2</td>
</tr>
<tr>
<td>United States</td>
<td>45.1</td>
<td>17.4</td>
</tr>
<tr>
<td>New Zealand</td>
<td>82.4</td>
<td>4.8</td>
</tr>
<tr>
<td>All countries</td>
<td>74.6</td>
<td>19.5</td>
</tr>
</tbody>
</table>


Note: coefficient of variation = (std. deviation/mean)*100

Table 1: The average turnout rates and the coefficient of variation (Parliamentarian elections) in selected OECD countries 1970-99
<table>
<thead>
<tr>
<th>Social Group</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>age-groups</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>younger (18-30)</td>
<td>72.5</td>
<td>72.9</td>
</tr>
<tr>
<td>middle</td>
<td>85.1</td>
<td>83.9</td>
</tr>
<tr>
<td>older (65+)</td>
<td>86.9</td>
<td>83.2</td>
</tr>
<tr>
<td>Income-groups</td>
<td></td>
<td></td>
</tr>
<tr>
<td>low</td>
<td>82.1</td>
<td>76.9</td>
</tr>
<tr>
<td>middle</td>
<td>82.3</td>
<td>81.3</td>
</tr>
<tr>
<td>high</td>
<td>83.9</td>
<td>80.7</td>
</tr>
<tr>
<td>Urbanization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rural</td>
<td>79.9</td>
<td>76.6</td>
</tr>
<tr>
<td>City</td>
<td>82.3</td>
<td>80.7</td>
</tr>
<tr>
<td>Suburbs</td>
<td>86.5</td>
<td>86.3</td>
</tr>
<tr>
<td>Education-groups</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incomplete primary</td>
<td>77.1</td>
<td>70.9</td>
</tr>
<tr>
<td>Primary</td>
<td>77.3</td>
<td>74.7</td>
</tr>
<tr>
<td>Secondary</td>
<td>79.4</td>
<td>79.4</td>
</tr>
<tr>
<td>Graduate</td>
<td>85.9</td>
<td>84.2</td>
</tr>
<tr>
<td>Work status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full time employed</td>
<td>84.4</td>
<td>78.1</td>
</tr>
<tr>
<td>Part time employed</td>
<td>83.8</td>
<td>86.1</td>
</tr>
<tr>
<td>Unemployed</td>
<td>67.5</td>
<td>64.5</td>
</tr>
<tr>
<td>Students</td>
<td>71.7</td>
<td>74.7</td>
</tr>
<tr>
<td>Retired</td>
<td>84.9</td>
<td>81.1</td>
</tr>
</tbody>
</table>


Table 2: Turnout rates in general elections by gender for different social groups in 19 countries 1996-99
Table 3: The number of voters who turn out to vote in the two regions as a function of the weather.

<table>
<thead>
<tr>
<th></th>
<th>South</th>
<th>North</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good</td>
<td>Foul</td>
</tr>
<tr>
<td>Good</td>
<td>60 40</td>
<td>30 40</td>
</tr>
<tr>
<td>Foul</td>
<td>60 20</td>
<td>30 20</td>
</tr>
</tbody>
</table>

Note: The first entry in each cell is for the South.

South. Suppose, however, that turnout depends on weather conditions and that turnout is only half if the weather is foul. Moreover, suppose that the probability of "good weather" is $\frac{1}{2}$, that the probability of "foul weather" is $\frac{1}{2}$ and that weather conditions are independent across regions. Table 3 shows the number of voters who turn out to vote in each region as a function of the weather. We observe that region South holds the majority in three out of four cases (i.e., with probability $\frac{3}{4}$) and that region North – the minority region – holds the majority among those who show up to vote with probability $\frac{1}{4}$. So, ex ante, politicians might be wary pandering only to the majority of the South: if the weather turns out to be good in the North and foul in the South, the voters of region North will be casting the decisive vote.

Building on this logic, we explore the consequences of uncertain voter turnout for electoral accountability and competition between heterogeneous groups of voters. We do so within one of the canonical models of electoral politics, namely the so-called retrospective voting model. This model was suggested by Barro (1973) and further developed by Ferejohn (1986), and has been extensively used by Persson and Tabellini (2000, 2003) and many others in recent work on comparative public finance.\(^5\) The model portrays elections as a vehicle through which voters (the principal) can dismiss or replace under-performing politicians (the agent) at election day. The main purpose of elections within this conception is to hold politicians accountable ex post for the choices they made while in office, i.e., the model highlights the accountability role of elections. In situations with heterogeneous groups of voters, elections serve the additional purpose of aggregating conflicting preferences.\(^6\) In this case, the model typically predicts that policy outcomes are

\(^5\)See e.g., Persson et al. (1997) and Coate and Morris (1999).

\(^6\)Generally, elections serve a number of different roles. They aggregate preferences and information, they select politicians and they allow voters to hold the selected politicians.
biased in favor of the majority group at the expense of the minority. Moreover, competition between groups of voters allows politicians far too much leeway and renders electoral accountability ineffective (Ferejohn, 1986; Aidt and Magris, 2006). Turnout uncertainty changes all of this in fundamental ways.

Turnout uncertainty has the two surprising implications. Firstly, politicians always implement policies that satisfy the demands of all voters including minority groups. Secondly, voters, in turn, make demands that politicians want to satisfy. We call this strategic consensus.\(^7\) Strategic consensus insures politicians against turnout risk and voters against partisan choices that ignore the interests of minority groups. In contrast to an economy with certain electoral turnout, the interest of the minority is always included in the political calculus. Using this logic, we study the classical public finance choice between targeted transfers and universal public goods.\(^8\) In an economy characterized by turnout uncertainty, we show that i) universal public goods are only provided if the Lindahl-Samuelson condition is satisfied, ii) the minority is protected from exploitation by the majority and iii) in economies with sufficiently high public sector productivity, the minority group is at least as well off as the majority group and often strictly better off. None of this is true if voter turnout is non-random.

The rest of the paper is organized as follows. In section 2, we present the model and discuss the main assumptions. In section 3, we present the main analysis. In section 4, we present the results. In section 5, we discuss the broader implications of the analysis. The appendix contains some derivations and proofs.

2 The Model

Society consists of two groups of voters, \(i = 1, 2\); politicians are indicated by index 0. Voters and politicians have an infinite time horizon. Time is indexed by \(t\). A group is defined as a subset of voters who are affected in the same way by public policy. Group affiliation may be determined by observable accountable. We focus on the first and last of these roles, but acknowledge that the other roles are also important in practice.

\(^7\)See Aidt and Dutta (2004).

\(^8\)This classical question has received substantial attention in the recent literature on positive public finance (e.g., Persson and Tabellini, 1999; Lizzeri and Persico, 1998).
characteristics such as geographical location, age, gender, or religion. Per-period utility, $u_{it}$, is discounted with the common discount factor $\beta \in (0, 1)$. There are $n_1$ voters in group 1 and $n_2$ voters in group 2. The size of the total (voter) population is $n = n_1 + n_2$. Assuming that $n_1 > n_2$, we refer to group 1 as the majority and group 2 as the minority, although either group may in actual fact hold the majority among those who turn out to vote (see below).

Each period, the politician collects taxes up to a maximum of $T$, spends some of this on universal public goods and/or group-specific transfer payments, and keeps the rest for himself as a political rent.\(^9\) Denoting the cost of providing utilities to the two groups of voters $c_t$, we can write the politician’s per-period political rent as

$$u_{0t} = T - c_t$$

if in office, and $u_{0t} = 0$ otherwise. Politicians apply the same discount factor as voters.

The politician, elected at $t$, cannot make binding promises on the level and pattern of public spending before he enters office. Since his own payoff decreases with $c_t$, he would, in the absence of further incentives, choose $c_t = 0$ and provide no amenities to the electorate. Voters know this, and threaten to vote retrospectively against a politician who does not provide them with a minimum level of utility. At the beginning of each period, voters in each group announce a performance standard, denoted $x_{1t}$ and $x_{2t}$. At the election at the end of the period, they then vote in favor of reelection of the incumbent politician if, and only if the policy implementation observed generates at least that level of utility, i.e., if, and only if $u_{it} \geq x_{it}$.

Importantly, neither group can guarantee to turn out in full force at elections. Suppose a politician delivers on the performance standard set by group 1, who holds the majority ex ante but fails to deliver on the standard set by group 2 ($u_{2t} < x_{2t}$). On the day of the election, $\tilde{n}_{it}$ voters from group $i$ actually show up to vote, and the politician can lose his bid for reelection if $\tilde{n}_{2t} > \tilde{n}_{1t}$. The central assumption of our analysis is that electoral turnout is uncertain. Voters can commit to vote according to the announced

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\(^9\)This formulation of the conflict of interest between voters and politicians is due to Persson et al. (1997) and used extensively in Persson and Tabellini (2000). It should be understood as a metaphor for the more general phenomenon that politicians can divert their efforts towards activities that are not in the interests of their electorate.
performance standards if they show up to vote, but cannot commit to a particular turnout rate. This is captured by the following assumption.

**Assumption 1** Electoral turnout, $\tilde{n}_t = (\tilde{n}_{1t}, \tilde{n}_{2t})$, is random. The ex ante probability that the turnout of group 1 is greater than that of group 2, $P(\tilde{n}_{1t} \geq \tilde{n}_{2t})$, is equal to $p$ and constant over time. Moreover, $p \in (0, 1)$.

Here, we specify the parameter $p$ directly. It can be derived from more basic considerations, however. In this analysis, it is important that $0 < p < 1$, so that neither group can guarantee reelection. This is more likely to be the case when turnout shocks are correlated within groups. This is, for example, the case when differences in weather conditions affect the turnout of voters in different geographical locations or foul weather keeps certain types of voters, such as the poor or old, at home (Roemer, 1998), and when differences in group sizes are not too large (e.g., because election districts are designed to be of equal size).

The game between the incumbent politician and the two groups of voters unfolds over time as illustrated in Figure 1. At the beginning of each
period, voters in each group announce the (utility) standard that the politician needs to satisfy to get their votes in the next election. The standards are chosen by the two groups non-cooperatively and at the same time. The politician observes the standards and determines whether to comply, and if so, how many standards to meet. We denote the set of actions available to the politician by \( A = \{(00), (10), (01), (11)\} \) with elements \( a_t = (00) \) (meet neither standard); \( a_t = (10) \) (meet group 1’s standard only); \( a_t = (01) \) (meet group 2’s standard only); and \( a_t = (11) \) (meet both standards). At the end of the period, a new election is held and voters randomly turn up to vote. Those who turn up vote according to the announced performance standard. The politician either wins or loses. In the latter case, he is replaced by an identical challenger; in the former case, he gets (at least) another term in office. After the election, the game continues to the next period where a similar sequence of events takes place.

3 Analysis

Ultimately, our goal is to understand the role of turnout uncertainty in shaping competition between heterogeneous groups of voters. The first step towards this goal is to characterize the so-called political cost function. The second step is to characterize equilibrium outcomes of the game described above. We restrict attention to history-independent subgame perfect Nash equilibria.\(^{10}\)

The Political Cost Function The political cost function defines the minimum expenditure the politician must incur to provide voters with a given utility level. Specifically, we define \( C(x_{1t}, x_{2t}) \) as the minimum cost of simultaneously providing utility levels \( u_{1t} \geq x_{1t} \) and \( u_{2t} \geq x_{2t} \) to voters in the two groups at time \( t \). Likewise, we define \( C_i(x_{it}) \) as the minimum cost of separately providing the utility level \( u_{it} \geq x_{it} \) to group \( i \), \( i = 1, 2 \).

\(^{10}\)Formally, the model describes a dynamic common agency game with absorbing states and perfect information. The two groups of voters are principals, and the elected politician their common agent. Uncertainty in rewards arises from uncertainty about which of the two principals will have the “casting vote”, or final say, in the only reward available: re-election. There is no aggregate uncertainty, as one of the principals will have the casting vote for sure.
The politician can please voters by providing a universal public good, $g_t$, or targeted, lump sum transfers, $\tau_{it} \geq 0$, $i = 1, 2$, or a combination of the two.\footnote{We do not allow transfers to be targeted specifically to individual voters, but only to groups of voters in different geographical locations, age-groups, professions, etc.} The public good is produced by a linear technology

$$g_t = Ak_t,$$  \hspace{1cm} (1)

where $k_t$ denotes the tax revenues devoted to the production of the public good. $A > 0$ is a productivity parameter that captures the efficiency of the public sector. The public budget constraint requires that

$$n_1\tau_{1t} + n_2\tau_{2t} + k_t \leq T$$  \hspace{1cm} (2)

for each $t$. We assume that all voters value the universal public good in the same way, and that utility is linear in public and private goods:

$$u_{it} = g_t + \tau_{it}; \hspace{0.5cm} i = 1, 2.$$  \hspace{1cm} (3)

We note that all voters like the public goods and dislike political rents, but disagree on who should have the transfers.

A utilitarian social planner would provide the public good if, and only if the Lindahl-Samuelson condition, saying the sum of the marginal benefit of the public good ($An$) exceeds the marginal cost of producing the good (1), holds:

$$An - 1 \geq 0.$$  \hspace{1cm} (4)

Moreover, given that the marginal utility of transfer payments is constant and invariant across groups, a social planner has not any particular reason to redistribute income.

The self-interested politician can choose the level and composition of public spending as he likes, but needs to take into account that if more is spent on targeted transfers, less is available for public goods and political rents. The politician also keeps in mind that transfers must be provided to all voters in the relevant group, implying that the cost of targeted redistribution is sensitive to group sizes. In contrast, public goods allow the politician to satisfy all voter demands simultaneously irrespective of group sizes. This distinction turns out to be important.

We derive the political cost function in the Appendix and summarize the key features below. The cost-efficient method of meeting the performance
standards depends on $A$, the productivity level in the public sector, relative to the size of the two groups, as follows:

1. Let $A < \frac{1}{n}$. The political cost function is:

\begin{align*}
C(x_{1t}, x_{2t}) &= n_1 x_{1t} + n_2 x_{2t}; \\
C_1(x_{1t}) &= n_1 x_{1t}; \\
C_2(x_{2t}) &= n_2 x_{2t}.
\end{align*}

Here, $A$ is sufficiently low that the Lindahl-Samuelson condition is violated. For the politician, this implies that transfers are the cheapest way to buy voter approval. Consequently, no public goods are provided if voters in group $i$ ask for the utility level $x_{it}$, the cost to the politician of providing the utility level is $n_i x_{it}$.

2. Let $\frac{1}{n} \leq A < \frac{1}{m}$. The political cost function is

\begin{align*}
C(x_{1t}, x_{2t}) &= \min\left(\frac{x_{1t} + n_1 (x_{1t} - \min[x_{1t}, x_{2t}]) + n_2 (x_{2t} - \min[x_{1t}, x_{2t}])}{A}\right) + n_2 x_{2t}; \\
C_1(x_{1t}) &= n_1 x_{1t}; \\
C_2(x_{2t}) &= n_2 x_{2t}.
\end{align*}

Here, the politician provides public goods only if he wishes to satisfy both standards. In particular, he satisfy the demands of the least demanding group with public goods and top up with a targeted transfer to the more demanding group. On the other hand, if he only wants to satisfy the standard of one group, the cheapest way to do so is to provide targeted transfers to that group only.

3. Let $\frac{1}{m_1} \leq A < \frac{1}{n_2}$. The political cost function is

\begin{align*}
C(x_{1t}, x_{2t}) &= \frac{x_{1t}}{A} + n_2 \max(x_{2t} - x_{1t}, 0); \\
C_1(x_{1t}) &= \frac{x_{1t}}{A}; \\
C_2(x_{2t}) &= n_2 x_{2t}.
\end{align*}

Here, the political costs are minimized by satisfying group 1 – the majority – with public goods, and meeting further demands from the minority with transfers.
4. Let $A \geq \frac{1}{m^2}$. The political cost function is

$$C(x_{1t}, x_{2t}) = \max \frac{x_{1t} \cup x_{2t}}{A};$$

(14)

$$C_1(x_{1t}) = \frac{x_{1t}}{A};$$

(15)

$$C_2(x_{2t}) = \frac{x_{2t}}{A}. $$

(16)

Here, the productivity of the public sector is so high that all demands are met by public goods rather than transfers. A politician who wants to meet the standard of one group will automatically provide (some) utility to the other.

What is important for what follows is that the political cost function is sub-additive, i.e., $C(x_{1t}, x_{2t}) \leq C(x_{1t}) + C(x_{2t})$. In plain words this means that it is at least as cheap for the politician to satisfy the performance standards of the two groups jointly as it is doing it separately. Sub-additivity arises from the fundamental role of public goods. Imagine that a politician wants to provide utility to one group of voters only. He can do this by making transfers to this group. If he wants to provide utility to both groups, it may be cheaper to provide universal public goods. The fact that public goods can be used to provide utility to everybody at the same time allows the cost function to be sub-additive. When, as in case 1, it is inefficient to provide public goods and the politician uses transfers to satisfy the demands of voters, the political cost function becomes additive, i.e., $C(x_{1t}, x_{2t}) = C(x_{1t}) + C(x_{2t})$.

**Equilibrium**  Our model of the political economy of fiscal choices under turnout uncertainty is a special case of a more general model studied in Aidt and Dutta (2004). In that paper, we show that all equilibrium paths of the game described above have a property called strategic consensus: the politician prefers to meet all performance standards at all times, all groups of voters vote for the incumbent, and the incumbent is reelected with certainty, irrespective of turnout shocks. This is a surprising result as intuition would lead one to think that it must at least sometimes – e.g., when it is very unlikely that a group is ever going to hold the majority among those who turn out to vote – be optimal for a politician to be partisan and focus on one group only. This intuition is, however, wrong. Whenever the politician
is willing to implement a “partisan” outcome, the disfavored group has an incentive to lower its standard to induce the politician to make a “partisan” choice in its favor. This logic continues until the standards are such that the politician is just willing to implement a policy that satisfies both groups. The result is strategic consensus.

Although all equilibrium paths display strategic consensus, the distribution of payoffs depends critically on the properties of the political cost function. In an economy with sub-additive political costs, the following characterization result holds.\textsuperscript{12}

**Proposition 1 (Sub-additive Costs)** If the political cost function is sub-additive, then the distribution of payoffs is determined by the following conditions:

$$
\text{(SC}_1^+\text{)} \quad C(x_{1t}, x_{2t}) = \beta T;
$$

$$
\text{(SC}_2^+\text{)} \quad C_1(x_{1t}) \geq \beta p T;
$$

$$
\text{(SC}_3^+\text{)} \quad C_2(x_{2t}) \geq \beta (1 - p) T.
$$

Moreover, (SC\textsuperscript{+}_2\text{)} and (SC\textsuperscript{+}_3\text{)} hold with equality for an additive political cost function. Along all equilibrium paths, the politician receives payoffs $(1 - \beta)T$ per period.

The proposition tells us how the payoff is divided between voters and the politician (i.e., how large the political rent is) and how the remainder is divided between the two groups of voters. The politician always gets the political rent $(1 - \beta)T$ per period. The remaining share of tax revenues, $\beta T$, is devoted to the task of generating utilities to the voters. Importantly, this distribution of payoffs is unaffected by turnout uncertainty. Thus, strategic consensus provides the politician with “full insurance” against random voter turnout and voters with insurance against “partisan” choices by the politician. When the political cost function is additive, the allocation of utility between the two groups of voters is uniquely determined by $p$. In contrast, economies with strictly sub-additive costs exhibit multiple equilibria in performance standards at each $t$.

\textsuperscript{12}We state proposition 1 without proof. For a proof, see Aidt and Dutta (2004; theorem 1 and proposition 1).
4 Results

A number of interesting results about the composition of public spending and the surprising role of turnout uncertainty in mediating inter-group conflict flow directly from this analysis. The results are valid for any $p \in (0, 1)$ and so do not depend on the precise distribution of the turnout shocks. To prove the results, we combine proposition 1 with the specific cost function we derived above. This is done in the appendix. Here, we focus on the general insights and the intuition behind them.

**Proposition 2 (Public goods)** Along any equilibrium path, public goods are provided ($g_{i} > 0$) if, and only if

\[ nA > 1. \]  \hspace{1cm} (17)

**Proof.** The first statement follows directly from the political cost function $C(x_{1t}, x_{2t})$ and the fact that all equilibrium paths exhibits strategic consensus $\blacksquare$

**Proposition 3 (Transfers)** Along any equilibrium path, transfers are used only if

\[ 0 < A < \frac{1}{n_{2}}. \]  \hspace{1cm} (18)

Further, only the minority gets transfers if $A \in (\frac{1}{n_{2}}, \frac{1}{n_{1}})$.

**Proof.** The first statement follows from the political cost function $C(x_{1t}, x_{2t})$. The second statement follows from the fact that $A \in (\frac{1}{n_{1}}, \frac{1}{n_{2}})$ implies that $\tau_{1} = 0$ minimizes costs for any attainable $x_{1t}$ and $x_{2t} \blacksquare$

Proposition 2 demonstrates that the politician only provides public goods if the Lindahl-Samuelson condition is satisfied. In this sense, strategic consensus implies efficient provision of public goods. It is clear, however, that the politician supplies less public goods than the social planner, who spends all tax revenues on the purpose ($g^{*}_{i} = AT$ for $A \geq \frac{1}{n}$). Under-provision arises, as in Persson et al. (1997), because voters must allow their politicians to divert some funds, which could otherwise have been spent on public goods (or transfers), in order to discipline them not to expropriate everything. More surprisingly, for $A \in [\frac{1}{n}, \frac{1}{n}]$, all attainable equilibrium paths over-provide public goods relative to the wishes of the majority who for this ranges of productivity levels prefers transfers to public goods. This happens because
the politician finds it cheaper to satisfy the demands of one of the groups with public goods.

A comparison of propositions 2 and 3 shows how the politician makes use of the two policy instruments in economies with different productivity levels and group sizes. Begin by considering an economy with an inefficient public sector \( (A < \frac{1}{n}) \). In this economy, the politician prefers to use targeted transfers to get reelected. This makes the political cost function additive, and voters in each group receive actuarially fair insurance against partisan choices. In particular, the voters’ share of total revenue \( (\beta T) \) is divided between the two groups according to their probability of being pivotal, i.e., each member of group 1 gets \( \frac{p \beta T}{n_1} \), while each member of groups 2 gets \( \frac{(1-p)\beta T}{n_2} \).

An implication, then, is that the majority is unable to (fully) expropriate (with the help of the politician) the wealth of the minority. Turnout uncertainty plays a critical role in generating this outcome. To see this, suppose, as in Ferejohn (1986), that voters *always* turn out to vote. Since reelection requires the support of the majority only, the wealth of the minority is expropriated completely by the politician who redistributes some to the majority and keeps the rest for himself.\(^{13}\) Turnout uncertainty protects the minority against this because there is a chance that it is, in fact, the minority that holds the majority among those who show up to vote.

Contrast this with an economy with high public sector productivity \( (A \geq \frac{1}{n_2}) \). In this economy, the politician prefers to satisfy all demands from voters by public goods and the political cost function becomes (strictly) subadditive. An immediate implication is that all voters are treated equally and turnout uncertainty is no longer necessary to protect the minority from expropriation. To see why, return to the situation where turnout is certain and the politician can win the election by pleasing only the majority group. Since the cheapest way to do so is to provide public goods, everybody— including voters in the minority group— get the same benefits, even if the politician were to attempt to implement a “partisan” outcome.\(^{14}\)

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\(^{13}\)If there are more than two groups, then outcomes are even worse. The politician will be looking for a minimum winning coalition. Competition to get included in this coalition provides a strong incentive for groups to offer their votes at a discount and sparks a process of underbidding leading to the result that none of the groups get any transfers at equilibrium.

\(^{14}\)Persson and Tabellini (2000, chapter 9) and Aidt and Magris (2006) make a similar point.
Only in an economy with an intermediate productivity level \((A \in (\frac{1}{n_1}, \frac{1}{n_2}))\), the politician prefers to use a combination of public goods and transfers (to at most one group) to please voters. While for \(A \in (\frac{1}{n_1}, \frac{1}{n_1})\), the direction of the transfer depends on the particular equilibrium path attained, only the minority receives transfers when \(A \in [\frac{1}{n_1}, \frac{1}{n_2})\). This observation has a somewhat surprising implication.

**Proposition 4 (Minority welfare)** Along any equilibrium path, the minority is at least as well off as the majority if

\[
A \geq \frac{1}{n_1}. \tag{19}
\]

**Proof.** To establish this, we note that \(\frac{1}{n_2} > A \geq \frac{1}{n_1}\) implies

\[
C(x_{1t}, x_{2t}) = \frac{x_{1t}}{A} + n_2 \max(x_{2t} - x_{1t}, 0) = \beta T. \tag{20}
\]

Hence, \(g_t = x_{1t}, \tau_{1t} = 0\) and \(\tau_{2t} \geq 0\). This implies that

\[
u_{2t} \geq g_t = u_{1t} = x_{1t}. \tag{21}
\]

If \(A \geq \frac{1}{n_2}\), we have \(g_t = \max(x_{1t}, x_{2t})\), and \(\tau_{1t} = \tau_{2t} = 0\) all \(t\) implying that \(u_{1t} = u_{2t}\).

The result derives from the fact that it is often too expensive for politicians to satisfy the demands of the majority with transfers: the group is simply too large. Conversely, it is too expensive to satisfy additional demands by the minority with (more) public goods when \(A < \frac{1}{n_2}\). Hence, for \(\frac{1}{n_2} > A \geq \frac{1}{n_1}\), the politician provides public goods to please the majority. The minority, of course, also benefits from this, and, in addition, in some, but not all equilibria, gets a transfer.\(^{15}\)

\(^{15}\)In the analysis, we assumed that public goods can be produced with constant returns to scale. This assumption allows us to illustrate the main implications of turnout uncertainty for the composition of public spending in a simple and transparent way. The case with decreasing returns is more complex but yields very similar results. In particular, in a “large” economy, where large refers to the tax raising capacity of the economy \((T)\), the socially optimal level of the public good is provided along all equilibria paths with strategic consensus, yet redistribution via targeted transfers (in many cases going to both groups) takes place because political costs are additive on the margin.
5 Discussion

The analysis shows that turnout uncertainty has important implications for the fiscal choices made by self-interested politicians. In the absence of such uncertainty, politicians are tempted to be "partisan" in the sense of pleasing only those groups which are strictly required to secure reelection. Turnout uncertainty, on the other hand, induces politicians to seek consensus outcomes. The reason is simply that politicians cannot be sure ex ante who will hold the majority among those who turn out to vote. This significantly changes the dynamics of inter-group competition for political favors and implies that the political process aggregates the preferences of heterogeneous groups of voters in such a way that all interests are given some weight.

Our model has some interesting similarities with the probabilistic voting model, in particular as applied to redistributive politics by Lindbeck and Weibull (1987), Dixit and Londregan (1996), and Hettich and Winer (1988, 1999). This body of research studies the incentives of competing political parties to target monetary transfers (or tax concessions) at specific groups of voters in order to "buy" votes. A key result is that transfers are targeted at swing voters, i.e., groups of voters whose voting probabilities are particularly sensitive to additional benefits because they are not ideologically committed. In our model, competition is between groups of voters, rather than between political parties. The equilibrium payoff of a group is increasing in its likelihood of casting the decisive vote in the election. This is much in the spirit of probabilistic voting where groups of voters are rewarded according to how sensitive their vote decisions are at the margin. From a theoretical point of view, it is also interesting to notice that under turnout uncertainty, an equilibrium exists in our model under mild conditions on the political cost function. Existence of equilibria in the probabilistic voting model requires that voters' utility functions are sufficiently concave (see, e.g., Lindbeck and Weibull (1987, Theorem 2) or Lin et al. (1999)). While the utility functions in our application satisfy this condition, existence of equilibrium does not require this in our model.

An important insight of the model is that turnout uncertainty limits redistribution and protects the minority. Standard political economy explanations for why the majority (typically the poor with income below the average) does not expropriate the wealth of the minority (typically the rich with income above the average) are based either on the notion that taxation is distor-
tionary making it too costly for the majority to demand complete equalization of after-tax income (Richard and Meltzer, 1981; Winer and Rutherford, 1993) or on the notion that the rich can organize pressure groups and protect themselves that way against high taxes (Becker, 1983; Aidt, 2003). As an alternative to this, Roemer (1995) demonstrates that two-party competition can limit redistribution if the policy space has two dimensions and voters care sufficiently about a non-economic issue such as religion or race. The idea is intuitive: the party representing the poor, which in the absence of the non-economic issue would propose a tax rate of one, can enhance the welfare of its constituency by attracting votes from among those rich who care sufficiently about its position on the non-economic issue by proposing a more lenient tax policy. Finally, Corneo and Gruner (2000) provide a sociological explanation. They argue that social status is positively correlated with income, and appeal to the idea that fear of losing social status as a result of less income inequality might induce middle class voters to block redistributive policies that expropriate the wealth of the rich. Our model demonstrates that turnout uncertainty can provide an alternative answer to the puzzle of why redistribution is limited in a democracy.

Randomness in turnouts plays a positive role in our model and might be socially desirable. This has an interesting implication for the design of voting systems. In some countries, including Australia, voting is compulsory. The aim of this policy is to increase average turnout and to ensure that all citizens are participating in the political process. However, it has the downside, which is very clear from Table 1, of reducing turnout uncertainty. This makes "partisan outcomes" rather than "consensus outcomes" more likely. Thus, our analysis suggest a new trade off: compulsory voting guarantees high average turnout, but eliminates turnout uncertainty.

References


Appendix

The political cost functions  To simplify notation, we omit all time subscripts. Write the cost to the politician if $k$ is invested in the public good, $\tau_1$ is transferred to Group 1 and $\tau_2$ is transferred to Group 2 as

$$c(k, \tau_1, \tau_2) = k + n_1\tau_1 + n_2\tau_2.$$  

Let $x = \{x_1, x_2\}$ be the utility standards announced by voters. The least cost of satisfying both standards is the solution to the following problem:

$$C(x_1, x_2) = \min_{k \geq 0, \tau_1 \geq 0, \tau_2 \geq 0} c(k, \tau_1, \tau_2)$$
subject to $Ak + \tau_1 \geq x_1$, $Ak + \tau_1 \geq x_2$ and the public budget constraint. Similarly, we can define the least cost of providing utility levels satisfying one of the standards only as

$$C_i(x_i) = \min_{k \geq 0, \tau_i \geq 0} c(k, \tau_i, 0)$$

subject to $Ak + \tau_i \geq x_i$ and the public budget constraint. It is clear that feasibility requires that $C(x_1, x_2) \leq T$ and $C_i(x_i) \leq T$. The solutions to these problems depend on the size of $A$ relative to $n_1$, $n_2$ and $n$. Consider first the derivation of $C(x_1, x_2)$. Logically there are five ways in which the politician can provide utility to the two groups. The case with $g > 0$, $t_1 > 0$ and $t_2 > 0$ can, however, be ruled out immediately because of the linear production technology. If the politician wants to target transfers to both groups, it must be cheaper to do so than providing any public goods at all. If, on the other hand, the politician wants to provide public goods, it must be cheaper to satisfy the demands of at least one group completely with public goods. This leaves us with four cases to consider:

1. $g = 0$, $\tau_1 > 0$ and $\tau_2 > 0$ with costs $C(1) = n_1x_1 + n_2x_2$.
2. $g > 0$, $\tau_1 = 0$ and $\tau_2 \geq 0$ with costs $C(2) = \frac{x_1}{A} + n_2(x_2 - x_1)$ for $x_2 \geq x_1$.
3. $g > 0$, $\tau_1 \geq 0$ and $\tau_2 = 0$ with costs $C(3) = \frac{x_2}{A} + n_1(x_1 - x_2)$ for $x_1 \geq x_2$.
4. $g > 0$, $\tau_1 = 0$ and $\tau_2 = 0$ with costs $C(4) = \frac{\max(x_1, x_2)}{A}$.

Note that $C(2) = C(3) = C(4)$ if $x_1 = x_2$. Suppose that $x_1 \geq x_2$. Then, we get

$$(A1) \quad C(1) \leq (\leq) C(3) \iff A \leq (\leq) \frac{1}{n},$$

$$(A2) \quad C(3) \leq (\leq) C(4) \iff A \leq (\leq) \frac{1}{n_1}.$$  

Suppose that $x_2 \geq x_1$. Then, we get

$$(A3) \quad C(1) \leq (\leq) C(2) \iff A \leq (\leq) \frac{1}{n},$$

$$(A4) \quad C(2) \leq (\leq) C(4) \iff A \leq (\leq) \frac{1}{n_2}.$$  

Note that $\frac{1}{n} \leq \frac{1}{n_1} < \frac{1}{n_2}$. We can now derive the cost function for the 4 cases stated in the text.
1. Let $A < \frac{1}{n}$. It follows from (A1) – (A4) that

\[ C(1) < C(3) < C(4) \quad \text{for} \quad x_1 \geq x_2, \]
\[ C(1) < C(2) < C(4) \quad \text{for} \quad x_2 > x_1. \]

Hence,
\[ C(x_1, x_2) = n_1x_1 + n_2x_2. \]

2. Let $A \in [\frac{1}{n_1}, \frac{1}{n_2})$. It follows from (A1) – (A4) that

\[ C(3) \leq C(1) \quad \text{and} \quad C(3) < C(4) \quad \text{for} \quad x_1 \geq x_2, \]
\[ C(2) \leq C(1) \quad \text{and} \quad C(2) < C(4) \quad \text{for} \quad x_2 > x_1. \]

Hence, defining $x^{\min} = \min\{x_1, x_2\}$, we can write
\[ C(x_1, x_2) = \frac{x^{\min}}{A} + n_2(x_2 - x^{\min}) + n_1(x_1 - x^{\min}). \]

3. Let $A \in [\frac{1}{n_1}, \frac{1}{n_2})$. It follows from (A1) – (A4) that

\[ C(4) \leq C(3) < C(1) \quad \text{for} \quad x_1 \geq x_2, \]
\[ C(2) < C(1) \quad \text{and} \quad C(2) < C(4) \quad \text{for} \quad x_2 > x_1. \]

Hence, we get
\[ C(x_1, x_2) = \frac{x_1}{A} + n_2(\max\{x_2 - x_1, 0\}). \]

4. Let $A \geq \frac{1}{n_2}$. It follows from (A1) – (A4) that

\[ C(4) < C(3) < C(1) \quad \text{for} \quad x_1 \geq x_2, \]
\[ C(4) \leq C(2) < C(1) \quad \text{for} \quad x_2 > x_1. \]

Hence, we get
\[ C(x_1, x_2) = \frac{\max\{x_1, x_2\}}{A}. \]

To derive $C(x_i)$, we note that the relevant cases are

1. $g = 0$ and $\tau_i > 0$ with costs $C(1) = n_ix_i$.
2. $g > 0$ and $\tau_i = 0$ with costs $C(2) = \frac{x_i}{A}$.

It follows that $C(x_i) = n_ix_i$ for $A < \frac{1}{n_i}$ and that $C(x_i) = \frac{x_i}{A}$ for $A \geq \frac{1}{n_i}$.
Equilibria Combining proposition 1 with the political cost function derived above, we can show the following.

1. Let $A < \frac{1}{n}$. The political cost function is additive. It follows from proposition 1 that the equilibrium allocation is unique and given by
   
   \[ u_{1t} = \tau_{1t} = \frac{\nu_1 T}{n_1}, \quad u_{2t} = \tau_{2t} = \frac{(1-p)\beta T}{n_2} \quad \text{and} \quad u_{0t} = (1 - \beta) T \]
   
   for all $t$.

2. Let $\frac{1}{n} \leq A < \frac{1}{n_1}$. The political cost function is strictly sub-additive. Proposition 1 then implies that there exist many possible equilibrium paths, but along all of these the politician wants to satisfy the standard of the least demanding group by public goods and then “top-up” the utility of the other with transfers. This implies that some public goods are always provided and that at most one group receives transfers. The politician always gets $u_{0t} = (1 - \beta) T$ for all $t$.

3. Let $\frac{1}{n_1} \leq A < \frac{1}{n_2}$. The political cost function is strictly sub-additive and provision levels and the size of the transfer vary across equilibria. The direction of the transfer, if any, is, however, uniquely determined: it goes towards the minority. The politician always gets $u_{0t} = (1 - \beta) T$ for all $t$.

4. Let $A \geq \frac{1}{n_2}$. Since the productivity of the public sector is high enough all demands are met by public goods rather than by transfers. Although the cost function is (strictly) sub-additive, all equilibrium paths generate the same utility allocation, namely $g_t = u_{1t} = u_{2t} = A \beta T$ and $u_{0t} = (1 - \beta) T$ for all $t$.\footnote{This outcome can be supported by many different performance standards.} A politician who wants to meet the standard of one group will automatically provide (some) utility to the other. An implication, then, is that the utility allocation is independent of $p$.