Structural Interactions in Spatial Panels*

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Abstract

Traditionally, research has been devoted almost exclusively to estimation of underlying structural models without adequate attention to the drivers of diffusion and interaction across cross section and spatial units. We review some new methodologies in this emerging area and demonstrate their use in measurement and inferences on cross section and spatial interactions. Limitations and potential enhancements of the existing methods are discussed, and several directions for new research are highlighted.

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1 Introduction[1]

Spatial or cross-section dependence is a common feature of most economic applications involving either a cross-section of economic agents or a macro

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panel. Increasingly, availability of data on spatial panels provides the explicit opportunity to understand and model such cross-section and spatial dependence. Two distinct econometric approaches have been proposed in the literature to model spatial dependence. A popular characterisation, originally developed in the regional science and geography literatures, but with increasing economic applications, is based on spatial weights matrix, the elements of this matrix represent the direction and strength of spillovers between each pair of units. Alternatively, multifactor approaches which assume cross section dependence can be explained by a finite number of unobserved common factors that affect all units (regions, economic agents, etc.) are gaining increasing popularity. This paper informs the emerging debate as to which of these two approaches is more convenient and useful in applied studies.[2] In particular, we argue that spatial weights matrices with relatively unrestricted interactions are more appropriate in applications when spatial dependence is structural, in the sense that the observation units are not exchangeable. In other words, spatial dependence is driven, at least partially, by the location of the units in some observed (or even, notional and abstract) space. Further, we discuss several new econometric methods for inference on spatial dependence in the above setting, and illustrate their relative merits based on an application to cross-member interactions within a committee setting.

The idea behind spatial weights matrix is that there are spillover effects across the economic agents because of spatial or other forms of local cross section dependence. Such a matrix, $\boldsymbol{W}$, is square $(n \times n)$ with zero diagonal elements, and where the off-diagonal elements w_{ij} represent the spillover from unit j to unit i $(i, j = 1, \dots, n)$. Panel data regression models with such spatially correlated error structures have been estimated using maximum likelihood techniques (Anselin, 1988; Baltagi *et al.*, 2006; Kapoor *et al.*, 2007), or generalized method of moments (Kelejian and Prucha, 1999; Conley, 1999; Fingleton, 2007). Kelejian and Prucha (2007) also extend the GMM methodology to nonparametric estimation of a heteroscedasticity and autocorrelation consistent cross section covariance matrix, for applications

[2] In an invited session in honour of Cheng Hsiao at the recently concluded 15th International Panel Data Conference (Bonn, 2009), Badi H Baltagi provided an extensive review of the current literature on "Spatial Panels". Two important questions were actively debated in the general discussions following the talk. First, how useful is explicit modeling of spatial dependence using spatial weights matrices? Second, do multifactor aproaches provide more versatility in such modeling, and are there substantial advantages of interpretability attached to spatial weights matrices?

where an instrumental variable procedure has been used to estimate the regression coefficients.

At the same time as spatial weights characterise cross section dependence in useful ways, their measurement has a significant effect on the estimation of a spatial dependence model (Anselin, 2002; Fingleton, 2003). Measurement is typically based on underlying notions of distance between cross section units. These differ widely across applications, depending not only on the specific economic context but also on availability of data. Spatial contiguity (resting upon implicit assumptions about contagious processes) using a binary representation is a frequent choice. Further, in many applications, there are multiple possible choices and substantial uncertainty regarding the appropriate choice of distance measures. However, while the existing literature contains an implicit acknowledgment of these problems, most empirical studies treat spatial dependence in a superficial manner assuming inflexible diffusion processes in terms of known, fixed and arbitrary spatial weights matrices (Giacomini and Granger, 2004). The problem of choosing spatial weights becomes a key issue in many economic applications; apart from geographic distances, notions of economic distance (Conley, 1999; Pesaran *et al.*, 2004, Holly *et al.*, 2008), socio-cultural distance (Conley and Topa, 2002; Bhattacharjee and Jensen-Butler, 2005), and transportation costs and time (Gibbons and Machin, 2005; Bhattacharjee and Jensen-Butler, 2005) have been highlighted in the literature. The uncertainty regarding the choice of metric space and location, closely related to the measurement of spatial weights, have been addressed in the literature (Conley and Topa, 2002, 2003; Conley and Molinari, 2007). Related issues regarding endogeneity of locations have also been addressed (Pinkse *et al.*, 2002; Kelejian and Prucha, 2004; Pesaran and Tosetti, 2007).

On the other end, spatial panel regression models under multifactor error structure have been addressed by maximum likelihood (Bai, 2009), principal component analysis (Coakley *et al.*, 2002), or the common correlated effects approach (Pesaran, 2006). Factor models are potentially powerful in that they do not require strong and unverifiable *a priori* assumptions on the nature of spatial dependence. However, there are two potential limitations. First, a factor representation is equivalent to exchangebility of the observation units,[3] which is not a reasonable assumption in many applications. For example, in many spatial applications, the location of the units in space plays

[3] See, for example, de Finetti (1931) and Hewitt and Savage (1955).

a key role in modelling and interpretation, and these units cannot therefore be assumed to be exchangeable. In this paper, we use the term *structural spatial dependence* to describe situations where a factor representation does not provide an adequate description of spatial dependence, or in other words, the observation units are not exchangeable. Second, even when an approximate factor representation can be obtained, it is often the case that the identified factors cannot be related in any satistactory way to interpretable individual features or time effects. Thus, economic interpretation of factor models is often a considerable challenge.

The above two characterisations of cross section dependence, namely spatial weights and common factors, are not mutually exclusive. As discussed above, factor models typically only provide a partial expanation for cross section dependence, and therefore it is often observed that residuals from estimated factor models display substantial cross section correlation. Furthermore, Pesaran and Tosetti (2007) consider a panel data model where both sources of cross section dependence exist and show that, under certain restrictions on the nature of dependence, the common correlated effects approach (Pesaran, 2006) still works.

While the above literature addressed cross section dependence in various ways, it has focused mainly on estimation of the regression coefficients in the underlying model, treating the cross section dependence as a nuisance parameter. Estimation and inferences on the magnitude and strength of spillovers and interactions has been largely ignored. However, there are many instances in which inferences about the nature of the interaction is of independent interest. For example, understanding empirically the precise form of spillovers and diffusion between observational units is an important objective of the studies on economic growth and convergence in a cross-country panel setting. Likewise, studying cross-member interactions in a commitee or network setting is a crucial counterpart to the development of theories of economic networks; see, for example, Dutta and Jackson (2003) and Goyal (2007). The empirical contribution of this paper will be based on an application of the second kind.

By contrast to the above literature, we take a nonparametric view on the nature and strength of spatial diffusion and cross section interaction. We focus explicitly on several new methods for estimating spatial weights (or interactions) that are consistent with an observed pattern of spatial (or cross sectional) dependence. Once these interactions have been estimated they can be subjected to interpretation in order to identify the true nature of spatial dependence, representing a significant departure from the usual practice of assuming a priori the nature of spatial interactions. The methods are illustrated with an application to monetary policy making within the Bank of England's monetary policy committee (MPC).

The paper is organised as follows. In Section 2, we describe our model and the considered econometric methods. First, we follow Bhattacharjee and Jensen-Butler (2005) and describe estimation of the spatial weights matrix in a spatial error model. We emphasize that estimation of spatial weights consistent with an estimated pattern of spatial autocorrelations is a partially identified problem, and therefore structural constaints are required for precise estimation; symmetry of the spatial weights matrix constitutes such a valid set of identifying restrictions. Second, based on Bhattacharjee and Holly (2008a), we consider estimation and inference on interactions under moment restrictions which explicitly exploit the spatio-temporal nature of panel data on economic agents. Third, we extend the above methods to allow for spatial effects that may be partly driven by unobserved common factors. Following this (Section 4), we develop an application to decision making within the MPC, where members are allowed to have unrestricted interactions. Our empirical analysis illustrates each of the above methods and further, provides interesting inferences for spatial dynamics within a committee setting. Finally, Section 5 concludes, highlighting strengths and weaknesses of the discussed methods, as well as areas of new research.

2 Model and methodologies

The spatial weights matrix is one of the most convenient ways to summarise spatial relationships in the data. With conventional geographical data, the spatial weights matrix reflects the intensity of the geo-spatial relationship between observations in a neighborhood, for example, the distances between neighbors, the lengths of shared border, or whether they fall into a specified directional class such as north/ south. Standard spatial autocorrelation statistics compare the spatial weights to the covariance relationship at pairs of locations. Spatial autocorrelation that is more positive than expected from random assignment indicate the clustering of similar values across geo-space, while significant negative spatial autocorrelation indicates that neighboring values are more dissimilar than expected by chance.

Since spatial weights are usually defined by conventional measures of geographic (or economic) distances between observation units, or various measures of contiguity, a spatial weights matrix is typically a exogenous (and known) square matrix with zero diagonal elements and positive symmetric elements away from the diagonal. Given a spatial weights matrix $\boldsymbol{W}$, dynamic spatial regression models are constructed in ways analogous to standard time series analysis, with the spatial lag of a vector $\underline{y}$ of observations for all units defined as the vector $\boldsymbol{W}\underline{y}$. Once such a regression model has been set up, inferences can be drawn using a variety of methods, including maximum likelihood and GMM; see, for example, Anselin (1999) and Anselin *et al.* (2003) for further discussion.

However, as discussed earlier, there is in most real applications considerable uncertainty regarding measurement of these distances. Worse still, inferences drawn using mis-measured spatial weights are typically biased (Anselin, 2002; Fingleton, 2003) and further, inferences from mis-specified spatial models are also inadequate (Baltagi *et al.*, 2009). Thus, while convential spatial econometric methods have treated the spatial weights as known, estimation of the spatial weights matrix is emerging as an important and active area of research (Dubin, 2009).

2.1 Spatial error model with unrestricted structural dependence

Studies in spatial econometrics typically distinguish between two different kinds of spatial effects in regression models for cross-sectional and panel data – spatial interaction (spatial autocorrelation) and spatial structure (spatial heterogeneity). While the study of spatial structure is similar to the traditional treatment of coefficient heterogeneity in econometrics, spatial interaction is usually modeled through a spatial weights matrix. Given a particular choice of the spatial weights matrix, there are two important and distinct ways in which spatial interaction is modelled in spatial regression analysis – the spatial error model and the spatial lag model.

We consider the spatial error model with spatial autoregressive errors, where the response variable (y) is explained by the effects of explanatory variables ($\boldsymbol{X}$) and spatial spillover of errors from other units.[4] The model

[4]By contrast, the spatial lag model includes the spatial lag of the response variable, $\boldsymbol{W}\underline{y}$, as an additional regressor.

and its reduced form are described as follows:

$$\begin{aligned} \underline{y}_t &= \boldsymbol{X}_t\underline{\beta} + \underline{u}_t, \quad t = 1, \ldots, T, & (1)\\ \underline{u}_t &= \boldsymbol{W}\underline{u}_t + \underline{\varepsilon}_t, & (2)\\ &\Longrightarrow \underline{y}_t = \boldsymbol{X}_t\underline{\beta} + (\boldsymbol{I} - \boldsymbol{W})^{-1}\underline{\varepsilon}_t, \end{aligned}$$

where there are T time periods ($t = 1, \ldots, T$) and n units ($i = 1, \ldots, n$),
$\underline{y}_t$ is the $n \times 1$ vector of the response variable in period t,
$\boldsymbol{W}$ is an unknown spatial weights matrix of dimension $n \times n$, and
$\underline{\varepsilon}_t$ is the $n \times 1$ vector of independent but possibly heteroscedastic spatial errors.

We first make the following four assumptions.

Assumption 1: *We assume that the spatial errors, $\underline{\varepsilon}_t$, are iid (independent and identically distributed) across time. However, we allow for heteroscedasticity across regions, so that $\mathbb{E}(\underline{\varepsilon}_t\underline{\varepsilon}_t') = \boldsymbol{\Sigma} = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2)$, and $\sigma_i^2 > 0$ for all $i = 1, \ldots, n$.*

The uncorrelatedness of the spatial errors across the units is crucial. **Assumption 1** ensures that all spatial autocorrelation in the model is solely due to spatial diffusion described by the spatial weights matrix.

Assumption 2: *The spatial weights matrix $\boldsymbol{W}$ is unknown and possibly asymmetric.* $\boldsymbol{W}$ has *zero diagonal elements and there are no restrictions on the off-diagonal elements (i.e., they could be either positive or negative).*

At the moment, we retain the flexibility of a possibly asymmetric spatial weights matrix. Our most significant point of departure from the literature is in the assumption of an unknown spatial weights matrix, which we intend to conduct inferences on. We do not impose a non-negativity constraint on the off-diagonal elements of $\boldsymbol{W}$.

Assumption 3: $(\boldsymbol{I} - \boldsymbol{W})$ *is non-singular, where $\boldsymbol{I}$ is the identity matrix. This is a standard assumption in the literature, and required for identification in the reduced form.*

Under **Assumption 3**, we have:

$$\mathbb{E}(\underline{u}.\underline{u}') = (\boldsymbol{I} - \boldsymbol{W})^{-1}.\boldsymbol{\Sigma}.(\boldsymbol{I} - \boldsymbol{W})^{-1'}.$$

2.2 Estimation under structural constraints

Bhattacharjee and Jensen-Butler (2005) consider estimation of a spatial weights matrix in the above spatial error model with spatial autoregressive errors.

They consider a setting where a given set of cross section units have fixed but unrestricted interactions; these interactions are inherently structural in that the units are not exchangeable, and the interactions are therefore related to an underlying structural economic model. In addition to Assumptions 1-3 above, they make the following assumption.

Assumption 4: *For the spatial error model (1, 2), the population spatial autocovariance matrix, $\mathbb{E}\left(\underline{u}.\underline{u}'\right)$, is unknown and positive definite with probability one, but otherwise has a completely unrestricted structure. Further, there exists a consistent estimator, $\widehat{\boldsymbol{\Gamma}}$, of the population spatial autocovariance matrix $\mathbb{E}\left(\underline{u}.\underline{u}'\right)$.*

Thus, Bhattacharjee and Jensen-Butler (2005) take an estimation method for the underlying regression model as given. Based on residuals from these estimates, a consistent estimator is first obtained. This estimator $\widehat{\boldsymbol{\Gamma}}$ is then used to estimate the unknown spatial weights matrix. They show that without any structural constraints on the spatial weights matrix,[5] the estimation problem is only partially identified, up to an orthogonal transformation of interactions. Specifically, under Assumptions 1-4, the matrix

$$\boldsymbol{V} = (\boldsymbol{I} - \boldsymbol{W})' .diag\left(\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \ldots, \frac{1}{\sigma_K}\right) \tag{3}$$

is consistently estimated, upto an orthogonal transformation, by $\widehat{\Gamma}^{-1/2} = \widehat{\boldsymbol{E}}.\widehat{\boldsymbol{\Lambda}}^{-1/2}.\widehat{\boldsymbol{E}}^{T}$, where $\widehat{\boldsymbol{E}}$ and $\widehat{\boldsymbol{\Lambda}}$ contain the eigenvectors and eigenvalues respectively of the estimated spatial autocovariance matrix $\widehat{\Gamma}$.[6] In other words, $\widehat{\Gamma}^{-1/2}$ is a consistent estimator of $\boldsymbol{VT}$ for some unknown square orthogonal matrix $\boldsymbol{T}$.

Since $\boldsymbol{T}$ is an arbitrary orthogonal matrix, it has $n\left(n-1\right)/2$ free elements. Hence, the spatial weights matrix $\boldsymbol{W}$ can be preceisely estimated under additional structural constraints. Symmetry of the spatial weights matrix constitutes one set of valid identifying restrictions. Note that, the assumption of a symmetric spatial weights matrix is natural in many applications, and spatial econometric studies routinely assume symmtric spatial

[5] Other than the condition that $(\boldsymbol{I} - \boldsymbol{W})$ is nonsingular which is required for identification in the reduced form.

[6] Here, $\mathbf{A}^{1/2}$ denotes the symmetric square root of a positive definite matrix $\mathbf{A}$, and $\mathbf{A}^{-1/2}$ denotes its inverse. In other words, $\mathbf{A}^{-1/2}$ has the same eigenvectors as $\mathbf{A}$, but with the eigenvalues replaced by reciprocal of the square root of the corresponding eigenvalues of $\mathbf{A}$.

weights based on geographical or economic distances. Of course, depending of the application, one can postulate other sets of structural constraints. We will develop such an alternative set of constraints in our application later in the paper.

Under such structural assumptions, Bhattacharjee and Jensen-Butler (2005) describe inference methods and an algorithm for estimation of the unknown spatial weights matrix. Estimation requires application of the "gradient projection" algorithm (Jennrich, 2001) which optimises any objective function over the group of orthogonal transformations of a given matrix. Convergence is fast and the algorithm is easily programmable.[7]

However, the above method has two important limitations. First, the identifying restrictions of symmetry (or other alternate structural assumptions) may be too strong in some applications. Second, and more importantly, these structural constraints are not generally testable. Further, standard errors in the above method have to be estimated by a bootstrap procedure which can be cumbersome.

2.3 GMM based inferences on endogenous interactions

In the same setting as above, Bhattacharjee and Holly (2008a) developed an alternative GMM based methodology for estimating spatial or interaction weights matrices which are unrestricted except for the validity of the included instruments and other moment conditions. This method assumes a nonempty set of other cross section units, correlated with the units under consideration, but which may change over time, expand or even vanish. Specifically, motivated by the system GMM approach (Arellano and Bond, 1991; Blundell and Bond (1998), they use these additional cross section (or spatial) units to constitute instruments, in addition to temporal lags normally available as instruments in a panel data setting. Bhattacharjee and Holly (2008a) also extend their methodology to a model with interval censored responses.

Like Bhattacharjee and Jensen-Butler (2005), the methodology in Bhattacharjee and Holly (2008a) relies on an estimator for the underlying regression model

$$\underline{y}_t = \boldsymbol{X}_t \underline{\beta} + \underline{u}_t.$$

The residuals, $\widetilde{u}_{it}$, from this estimated model are obtained. Then, for a given

[7] An implementation of the algorithm in Matlab is available from the authors on request.

cross section unit i, the following regression model in latent errors follows

$$\widetilde{u}_{it} = \widetilde{u}\,'_{(-i)t} w_{(i)} + \varepsilon_{it},$$

where $\widetilde{u}_{(-i)}$ denotes the vector of residuals for the cross section units other than i, $w_{(i)}$ is the i-th row of $\boldsymbol{W}$ transposed (ignoring the diagonal element, which is zero by construction), and observations run over $t = 1, \ldots, T$. At each subsequent estimation step, they estimate $w_{(i)}$, an index row of $\boldsymbol{W}$. The same procedure is repeated for each cross section unit in turn, and the whole $\boldsymbol{W}$ matrix is therefore estimated.

The main issue with the estimation of the above model is the endogeneity of the regressors, $\widetilde{u}_{(-i)t}$. In order to address this issue, Bhattacharjee and Holly (2008a) draw on the connection between the setting here and the standard dynamic panel data model. Here, we have (spatially) lagged endogenous variables as regressors, but the observations are not sampled at equi-spaced points on the time axis. Rather, the locations of our units lie in a multi-dimensional, and possibly abstract, space without any clear notion of ordering or spacing between observations. At the same time, one can often imagine that potential nonzero interaction weights imply that $u_{1t}, u_{2t}, \ldots, u_{nt}$ are regression errors from (1), at time t, on a collection of observation units who are not located very far away in space. In many applications, there may also be, potentially specific to the time period, additional units who are located further away (like those at higher lags in the dynamic panel data model), who are correlated with the above set of endogenous variables, but not with the idiosyncratic errors $\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{nt}$ from the interaction error equation (2).

In social networks agents who have weak ties with other agents may act as instruments for groups of agents that share strong ties (Granovetter, 1973, Goyal, 2005). In panel data on cross-sections of countries or regions, such a set may include other countries not included in the analysis either because of irregular availability of data or because they are outside the purview of the analysis. Similarly, in geography and regional studies, observations at a finer spatial scale may constitute such instruments.

Assumption 5: *There is, specific to a particular time point t, a collection of instruments $\overline{u}_i^{(t)} = \left(u_{it}^{(1)}, u_{it}^{(2)}, \ldots, u_{it}^{(k_t)}\right)$, with corresponding $\sum_{t=1}^{T} k_t$ moment conditions*

$$E\left(\overline{u}_i^{(t)}.\varepsilon_{it}\right) = 0 \quad i = 1, 2, \ldots, n; t = 1, 2, \ldots, T. \tag{4}$$

The validity of these potentially large number of instruments can be checked using, for example, the Sargan-Hansen J-test (Hansen, 1982). However, weak instruments may also potentially provide a problem here. Similar to Arellano and Bond (1991), and assuming a first order autoregressive structure in the errors of the interactions model, a further set of moment conditions are obtained.

Assumption 6: *Assume a first order autoregressive model*

$$\varepsilon_{it} = \alpha\varepsilon_{i,t-1} + e_{it} \quad i = 1, 2, \ldots, n; t = 2, 3, \ldots, T,$$
$$E\left(e_{it}e_{is}\right) = 0 \quad t \neq s,$$

so that an additional $n\left(T-1\right)\left(T-2\right)/2$ *linear moment conditions follow*

$$E\left(u^{(t-2)}.\varepsilon_{it}\right) = 0 \quad t = 3, \ldots, T, \tag{5}$$

where $u^{(t-2)} = \left(u_1^{(t-2)}, u_2^{(t-2)}, \ldots, u_n^{(t-2)}\right)$ *and* $u_i^{(t-2)} = \left(u_{i1}, u_{i2}, \ldots, u_{i,t-2}\right)$ *for* $i = 1, 2, \ldots, n$.

Estimation can now follow along stnadard lines. First, we estimate the underlying regression model (1) using an optimisation based method such as maximum likelihood, least squares or GMM, and collect residuals. Next, we estimate the interactions error model (2) using a two-step GMM estimator. The weights matrix is estimated using the outer product from moment conditions evaluated at an initial consistent estimator, which is the GMM estimator using the identity weighting matrix. The validity of this multi-step procedure follow from Newey (1984).

Bhattacharjee and Holly (2008a) also extend their methodology to the censored regression model. This is achieved by making a control function assumption.

Assumption 7: *Assume the interval censored observation scheme*

$$\begin{aligned} \text{Observations} &: \left(\left[u_{0it}, u_{1it}\right], \widetilde{u}_{(-i)t}, z_{it}\right) \\ P\left(\widetilde{u}_{it} \in \left[u_{0it}, u_{1it}\right]\right) &= 1, \end{aligned} \tag{6}$$

where $z_{it} = \left(\overline{u}_i^{(t)}.u^{(t-2)}\right)$ *is a set of instruments. Further, the following conditions hold:*

$$\begin{aligned} \widetilde{u}_{(-i)t} &= z'b + \delta; E\left(z\delta\right) = 0; \\ \varepsilon &= \delta'\gamma + \upsilon; \upsilon \perp \varepsilon, \delta; \upsilon \sim N\left(0, \sigma_\upsilon^2\right). \end{aligned} \tag{7}$$

The interval inequalities $P\left(\widetilde{u}_{it} \in [u_{0it}, u_{1it}]\right) = 1$ can be expressed as

$$
\begin{aligned}
D_0 &= 1\left(-u_{0it} + \widetilde{u}\,'_{(-i)t} w_{(i)} + \delta'\gamma + \upsilon \geq 0\right) \quad \text{and} && (8)\\
D_1 &= 1\left(u_{1it} - \widetilde{u}\,'_{(-i)t} w_{(i)} - \delta'\gamma - \upsilon \geq 0\right) && (9)
\end{aligned}
$$

Assuming exogenous censoring intervals, and using the control function approach (Blundell and Smith, 1986), the following moment conditions are obtained:

$$
\begin{aligned}
E\left[z\left(\widetilde{u}_{(-i)t} - z'b\right)\right] &= 0\\
\\
E\left[R_0\left(D_0, u_{0it}, \widetilde{u}_{(-i)t}, \left(\widetilde{u}_{(-i)t} - z'b\right), w, \gamma, \sigma_\upsilon\right)\widetilde{u}_{(-i)t}\right] &= 0\\
E\left[R_0\left(D_0, u_{0it}, \widetilde{u}_{(-i)t}, \left(\widetilde{u}_{(-i)t} - z'b\right), w, \gamma, \sigma_\upsilon\right)\left(\widetilde{u}_{(-i)t} - z'b\right)\right] &= 0 \quad (10)\\
E\left[R_0\left(D_0, u_{0it}, \widetilde{u}_{(-i)t}, \left(\widetilde{u}_{(-i)t} - z'b\right), w, \gamma, \sigma_\upsilon\right)u_{0it}\right] &= 0\\
\\
E\left[R_1\left(D_1, u_{1it}, \widetilde{u}_{(-i)t}, \left(\widetilde{u}_{(-i)t} - z'b\right), w, \gamma, \sigma_\upsilon\right)\widetilde{u}_{(-i)t}\right] &= 0\\
E\left[R_1\left(D_1, u_{1it}, \widetilde{u}_{(-i)t}, \left(\widetilde{u}_{(-i)t} - z'b\right), w, \gamma, \sigma_\upsilon\right)\left(\widetilde{u}_{(-i)t} - z'b\right)\right] &= 0\\
E\left[R_1\left(D_1, u_{1it}, \widetilde{u}_{(-i)t}, \left(\widetilde{u}_{(-i)t} - z'b\right), w, \gamma, \sigma_\upsilon\right)u_{1it}\right] &= 0,
\end{aligned}
$$

where

$$
\begin{aligned}
& R_0\left(D_0, u_{0it}, \widetilde{u}_{(-i)t}, \delta, w, \gamma, \sigma_\upsilon\right)\\
={}& D_0 \frac{\phi\left[\left(-u_{0it} + \widetilde{u}\,'_{(-i)t} w_{(i)} + \delta'\gamma\right)/\sigma_\upsilon\right]}{\Phi\left[\left(-u_{0it} + \widetilde{u}\,'_{(-i)t} w_{(i)} + \delta'\gamma\right)/\sigma_\upsilon\right]}\\
& + (1 - D_0)\frac{-\phi\left[\left(-u_{0it} + \widetilde{u}\,'_{(-i)t} w_{(i)} + \delta'\gamma\right)/\sigma_\upsilon\right]}{1 - \Phi\left[\left(-u_{0it} + \widetilde{u}\,'_{(-i)t} w_{(i)} + \delta'\gamma\right)/\sigma_\upsilon\right]}, \text{ and}\\
& R_1\left(D_1, u_{1it}, \widetilde{u}_{(-i)t}, \delta, w, \gamma, \sigma_\upsilon\right)\\
={}& D_1 \frac{\phi\left[\left(u_{1it} - \widetilde{u}\,'_{(-i)t} w_{(i)} - \delta'\gamma\right)/\sigma_\upsilon\right]}{\Phi\left[\left(u_{1it} - \widetilde{u}\,'_{(-i)t} w_{(i)} - \delta'\gamma\right)/\sigma_\upsilon\right]}\\
& + (1 - D_1)\frac{-\phi\left[\left(u_{1it} - \widetilde{u}\,'_{(-i)t} w_{(i)} - \delta'\gamma\right)/\sigma_\upsilon\right]}{1 - \Phi\left[\left(u_{1it} - \widetilde{u}\,'_{(-i)t} w_{(i)} - \delta'\gamma\right)/\sigma_\upsilon\right]},
\end{aligned}
$$

and ϕ and Φ are the pdf and cdf of the standard normal distribution respectively. As before, GMM estimation can now along standard lines.

In a simple way, the above methodology exploits the panel nature of the data as well as spatial interactions to obtain robust inferences in the presence of potential endogeneity. This is particularly important in microeconomic and spatial contexts where the positions of economic agents or regions in geographical and quality space are determined strategically, and therefore endogenously, as a result of repeated cross section interactions; see also Pinkse *et al.* (2002) and Conley and Topa (2003).

The GMM based methods discussed above are potentially quite powerful. They also have the advantage that unverifiable assumptions on the structure of $\boldsymbol{W}$ are not required. At the same time, there is the potential of a weak instruments problem. It is an empirical question as to which sets of identifying assumptions, structural restrictions as in Bhattacharjee and Jensen-Butler (2005) or moment restrictions as in Bhattacharjee and Holly (2008a), is more appropriate in the context of any specific application.

2.4 Presence of unobserved common factors

The above two methodologies are based on the *structural spatial dependence* assumption that the spatial units under observation are not exchangeable. However, just as a pure factor model usually explains only a part of the spatial dependence, a pure structural dependence assumption can also be problematic. Such an assumption would imply that whatever structural drivers lead to spatial autocorrelation, whether geographic distance or something more abstract, it is uncorrelated with the regressors included in the model. These drivers shape a particular pattern of spatial interaction, which then affects the spatial diffusion of shocks, and which in turn we can identify and infer upon using the above methods. Clearly, this is a strong assumption.

Further, some spatial interactions can also potentially be driven by common factors, and there can a combination of both structural drivers and unobserved factors.[8] Pesaran and Tosetti (2007) consider a model where, in addition to spatial or network interactions described by a weights matrix, there are unobserved common factors; see also Holly *et al.* (2008). Their

[8] While the literature on spatial econometrics has been silent on this crucial question, there is some discussion of related issues in the literature on regional growth and convergence; see, for example, Evans and Karras (1996).

estimation is based on the common correlated effects approach (Pesaran, 2006) where, in addition to the usual regressors, linear combinations of unobserved factors are approximated by cross section averages of the dependent and explanatory variables. Defining notions on weak and strong cross section dependence, Pesaran and Tosetti (2007) show that the common correlated effects method provides consistent estimates of the slope coefficient under both forms of dependence. Here, we are interested in inferences on spatial interactions under the model:

$$\begin{aligned} y_{it} &= \eta_i + \beta_i' x_{it} + \alpha_i^{(y)} \overline{y}_t + \alpha_i^{(x)\prime} \overline{x}_t + u_{it}, \\ u_t &= \boldsymbol{W} u_t + \varepsilon_t, \quad t = 1, 2, \ldots, T, \end{aligned} \tag{11}$$

where our original model (1) is simply augmented with cross section averages of y_{it} and x_{it} ($\overline{y}_t$ and $\overline{x}_t$ respectively).

While inference in Pesaran and Tosetti (2007) assumes certain structural constraints on the weights matrix, our network interactions are unrestricted except for the identifying assumption that the matrix $(\boldsymbol{I} - \boldsymbol{W})$ is nonsingular. Instead, we would achieve identification through the moment conditions given in (4) and (5). Under these moment conditions, GMM estimation is straightforward.

Also, as in the previous subsection, we can accommodate interval censored responses, by assuming either that censoring intervals are exogenous and then conducting GMM estimation under the moment conditions (10).

This combination of spatial weights estimation with the multifactor model is very powerful. It will allow us to infer on the relative importance of factor based and structural explantions of spatial dependence. Further, it is now possible to additionally infer on the drivers of the structural spatial effects as well as on endogenous network architectures.

3 Application: Interactions within the Bank of England's MPC

We develop the methodological approach described above in the context of a particular form of interaction. In this case it is the decisions that a Committee makes on interest rates for the conduct of monetary policy.

3.1 A simple model of committee decision making within the MPC

Our model for the inflation process is structured as follows:

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + \epsilon_t \quad (12)$$

$$y_t = \beta_1 y_{t-1} - \beta_2 (r_{t-1} - \pi_{t-1}) + \eta_t. \quad (13)$$

Here, π_t is the inflation rate in period t, y_t is the output gap (the difference between the log of output and the log of potential output), and r_t the nominal interest rate. η_t, a supply shock and ε_t, a demand shock, are iid shocks in period t not observable in period $t-1$. The coefficients α and β_2 are positive; β_1 $(0 < \beta_1 < 1)$ measures the degree of persistence in the output gap. The output gap depends negatively on the real lagged interest rate. The change in inflation depends on the lagged output gap. The output gap is normalised to zero in the long run.

If the policymaker only targets inflation, the central bank can (in expectation) use the current interest rate to hit the target for inflation two periods hence. So the intertemporal problem can be written as a sequence of single period problems. In this case (Svensson, 1997):

$$L_t = \frac{1}{2} \left[\pi_{t+2|t} - \pi^* \right]^2, \quad (14)$$

where $\pi_{t+2|t}$ is the forecast of inflation at time period $t+2$ based on information available in period t.

Then the rule for setting the interest rate by the monetary authority is:

$$r_t = \left(\pi_{t|t} - \pi^* \right) + \frac{1}{\alpha \beta_2} \pi_{t+1|t} + \frac{\beta_1}{\beta_2} y_{t|t}. \quad (15)$$

Next, we model the decision making process within the MPC. A standard way of understanding how a committee comes to a decision is that each member reacts independently to a ‘signal’ coming from the economy and makes an appropriate decision in the light of this signal and the particular preferences/expertise of the member. A voting method then generates a decision that is implemented. In practice there is also cross committee dependence. Before a decision is made there is a shared discussion of the state of the world as seen by each of the members. In this section we model the possible interactions between members of a committee as one in which interaction occurs in the form of deliberation. Views are exchanged about the interpretation of signals and an individual member may decide to revise his view depending upon how much weight he places on his own and the views of others.

This process can be cast as a simple signal extraction problem within a highly stylised framework. Let the j-th MPC member formulate an (unbiased) estimate of, say, the output gap, y_t^j. We adopt this notation here since we wish to consider situations where $j = 1, \ldots, m$ members could be a subset of a Committee of N members. Then the underlying model for the j-th member is:

$$\begin{aligned} y_t^j &= \beta_j . x_t^j + \omega_t^j \text{ with } \omega_t^j \backsim N(0, \sigma_{\omega^j}^2) \text{ and} \\ E\left(y_t^j\right) &= \beta_j . x_t^j = y_t, \text{ for } j = 1, \ldots, m. \end{aligned} \tag{16}$$

The initial estimates of output gap for each individual member are unbiased. Further, since ω_t^j reflects private views not shared by other committee members, we would normally expect that $E(\omega_t^j . \omega_t^k) = 0$, for $j \neq k$. However, in case there is strategic interaction between committee members j and k, ω_t^j and ω_t^k can be correlated.

The internal process of deliberation between the members of the Committee reveals to everyone individual views of the output gap brought to the meeting.[9] At the end of discussion and deliberation, an agreed estimate, y_t^b, of the output gap is agreed upon. This common estimate (y_t^b) is a weighted average of the initial estimates for the m committee members, the weights reflecting their relative importance or seniority within the committee. Therefore

$$y_t^b = y_t + \omega_t^b \text{ with } \omega_t^b \backsim N(0, \sigma_{\omega^b}^2) \tag{17}$$

is also unbiased for the unknown true output gap.

For the j-th member, the final estimate of y_t that minimises the forecast error variance and combines optimally the central bank estimate (y_t^b) and the private estimate (y_t^j) is given by:

$$y_t^{dj} = y_t^b + \kappa^j (y_t^j - y_t^b), \tag{18}$$

[9] Austin-Smith and Banks (1996) point out that we need each committee member to be open in revealing his estimate of the output gap and sincere in casting a vote for an interest rate decision that corresponds to the infomation available. Although we consider only the one period problem here, in a multi-period context we assume that reputational considerations are sufficiently powerful to ensure fair play.

where κ^j is:

$$\kappa^j = \frac{\sigma^2_{\omega^b}}{\sigma^2_{\omega^b} + \sigma^2_{\omega^j}}. \tag{19}$$

Clearly the more confident the committee member is in her own judgement the smaller $\sigma^2_{\omega^j}$, and the less weight is attached to the collective forecast.

This final estimate shows how members may differ about the size of the output gap. Committee members may also differ in their views on the effect of interest rates on inflation and output gap. This implies member-specific effects $\alpha_j\beta_{2j}$ and β_{2j} respectively.

Then, the decision rule for the j-th member can be written as:

$$i_{jt} = (\pi_{t|t} - \pi^*) + \frac{1}{\alpha_j\beta_{2j}}\pi_{t+1|t} + \frac{\beta_1}{\beta_{2j}}y_{t|t} + \varsigma_{jt}, \tag{20}$$

where $y_{t|t} = y_t^b$ is the average (forecast) of current output gap, and

$$\varsigma_{jt} = \frac{\beta_1}{\beta_{2j}}.\kappa^j.(y_t^j - y_{t|t}) \tag{21}$$

represents the effect of the deviation of the j-th member's initial estimate of output gap from the common estimate.

There are two important features of the ς_{jt}'s, which are crucial for our empirical analysis. First, ς_{jt} need not be a zero mean process and in general captures the extent to which the j-th member deviates from the central interest rate projection. Hence, ς_{jt} can be expressed in fixed effects form as

$$\varsigma_{jt} = \phi_j + \gamma_{jt}.$$

Second, the ς_{jt}'s are uncorrelated across different meetings for the same policy maker, but are correlated across members of the committee. This is in turn because of two reasons: (a) they are related to each other through the common estimate ($y_{t|t}$), which is a linear combination of individual estimates, and (b) there may be strategic interactions among committee members, in which case $E(\omega_t^j.\omega_t^k) \neq 0$ for some $j \neq k$.

Therefore, our model implies the following decision rule for the j-th member:

$$i_{jt} = (\pi_{t|t} - \pi^*) + \phi_j + \frac{1}{\alpha_j\beta_{2j}}\pi_{t+1|t} + \frac{\beta_1}{\beta_{2j}}y_{t|t} + \underline{\beta}_x^j.\underline{x}_t^j + \beta_\sigma^j.\sigma_{\gamma_t} + u_{jt}, \tag{22}$$

where ϕ_j (the fixed effect) indicates whether the j-th member is a hawk (high values) or a dove, $\underline{x}_t^j$ denotes indicators included in the j-th member's estimate of output gap (with heterogeneity both in the choice of the variables and in their effects), σ_{γ_t} is a measure of the uncertainty associated with future macroeconomic climate and β_σ^j denotes the j-th member's response to such uncertainty, and u_{jt}'s are zero mean errors with heteroscedastic variances across members; the magnitude of the variance reflects how activist a particular member is. For member j, u_{jt}'s are uncorrelated across meetings. However, u_{jt}'s are correlated across members, because of (a) deliberation within the committee, and (b) strategic interaction between members.

3.2 Data and sample period

The primary objective of the empirical study is to understand cross member interaction in decision making at the Bank of England's MPC, within the context of the model of committee decision making presented in the previous section. Importantly, our framework allows for heterogeneity among the MPC members and the limited dependent nature of preferred interest rate decisions. Our dependent variables are the decisions of the individual members of the MPC. The source for these data are the minutes of the MPC meetings.

TABLE 1: Voting records of selected MPC members

Member	Meetings	Votes			Dissent		
		Lower	No change	Raise	Total	High	Low
Buiter	36	10	10	16	17	9	8
Clementi	63	14	39	10	4	3	1
George	74	15	51	6	0	0	0
Julius	45	18	25	2	14	0	14
King	85	14	50	21	12	12	0

Since mid-1997, when data on the votes of individual members started being made publicly available, the MPC has met once a month to decide on the base rate for the next month.[10] Over most of this period, the MPC has had 9 members at any time: the Governor (of the Bank of England), 4 internal members (senior staff at the Bank of England) and 4 external members.

[10] The MPC met twice in September 2001. The special meeting was called after the events of 09/11.

External members were usually appointed for a period ranging from 3 to 4 years. Because of changes in the external members, the composition of the MPC has changed reasonably frequently. To facilitate study of heterogeneity and interaction within the MPC, we focus on 5 selected members, including the Governor, 2 internal and 2 external members. The longest such period when the same 5 members have concurrently served in the MPC is the 33 month period from September 1997 to May 2000. The 5 MPC members who served during this period are: George (the Governor), Clementi and King (the 2 internal members) and Buiter and Julius (the 2 external members). The voting pattern of these selected MPC members suggest substantial variation (Table 1).[11]

In order to explain the observed votes of the 5 selected members, we collected information on the kinds of data that the MPC looked at for each monthly meeting. The important issue was to ensure that we conditioned only on what information was actually available at the time of each meeting. Assessing monetary policy decisions in the presence of uncertainty about forecast levels of inflation and the output gap (including uncertainty both in forecast output levels and perception about potential output) requires collection of real-time data available to the policymakers when interest rate decisions are made as well as measures of forecast uncertainty. This contrasts with many studies of monetary policy which are based on realised (and subsequently revised) measures of economic activity (see Orphanides, 2003).

We collected information on unemployment (where this typically refers to unemployment three months prior to the MPC meeting), as well data on the underlying state of asset markets (housing prices, share prices and exchange rates). We measure unemployment by the year-on-year change in International Labor Organization (ILO) rate of unemployment, lagged 3 months. The ILO rate of unemployment is computed using 3 months rolling average estimates of the number of ILO-unemployed persons and size of labour force (ILO definition), both collected from the Office of National Statistics (ONS) Labour Force Survey. Housing prices are measured by the

[11] For example, of the 45 meetings which Julius attended, the votes for 14 were against the consensus decision, and all of these were for a lower interest rate. On the other hand, King disagreed with the consensus decision in 12 of the 82 meetings he attended, voting for a higher interest rate each time. Buiter dissented in 17 meetings out of 36, voting on 8 occasions for a lower interest rate and 9 times in favour of a higher one. See also King (2002) and Gerlach-Kristen (2004).

year-on-year growth rates of the Nationwide housing prices index (seasonally adjusted) for the previous month (Source: Nationwide). Share prices and exchange rates are measured by the year-on-year growth rate of the FTSE 100 share index and the effective exchange rate respectively at the end of the previous month (Source: Bank of England). The other current information included in the model is the current level of inflation – measured by the year-on-year growth rate of RPIX inflation lagged 2 months (Source: ONS).

Our empirical model also includes expected rates of future inflation and forecasts of current and future output. One difficulty with using the Bank's forecasts of inflation is that they are not sufficiently informative. By definition, the Bank targets inflation over a two year horizon, so it always publishes a forecast in which (in expectation) inflation hits the target in two years time. To do anything else would be internally inconsistent. Instead, as a measure of future inflation, we use the 4 year ahead inflation expectations implicit in bond markets at the time of the MPC meeting, data on which can be inferred from the Bank of England's forward yield curve estimates obtained from index linked bonds.[12] For current output, we use annual growth of 2-month-lagged monthly GDP published by the National Institute of Economic and Social Research (NIESR) and for one-year-ahead forecast GDP growth, we use the Bank of England's model based mean quarterly forecasts.

Finally, uncertainty in future macroeconomic environment and private perceptions about the importance of such uncertainty plays an important role in the model developed in this paper. The extent to which there is uncertainty about the forecast of the Bank of England can be inferred from the fan charts published in the Inflation Report. As a measure of uncertainty in the future macroeconomic environment, we use the standard deviation of the one-year-ahead forecast. These measures are obtained from the Bank of England's fan charts of output; details regarding these measures are discussed elsewhere (Britton *et al.*, 1998).

3.3 The empirical model

We start with the model of individual voting behaviour within the MPC developed in the previous section (**??**). The model includes individual specific heterogeneity in the fixed effects, in the coefficients of inflation and output

[12] We use the four year expected inflation figure because the two year figure is not available for the entire sample period. In practice the inflation yield curve tends to be very flat after two years.

gap, and in the effect of uncertainty. We aim to estimate this model in a form where the dependent variable is the j-th member's preferred change in the (base) interest rate. In other words, our dependent variable, v_{jt}, represents the deviation of the preferred interest rate for the j-th member (at the meeting in month t) from the current (base) rate of interest r_{t-1}:

$$v_{jt} = i_{jt} - r_{t-1}.$$

Therefore, we estimate the following empirical model of individual decision making within the MPC:

$$\begin{aligned} v_{jt} &= \phi_j + \beta_j^{(r)}.\triangle r_{t-1} + \beta_j^{(\pi 0)}.\pi_t + \beta_j^{(\pi 4)}.\pi_{t+4|t} + \beta_j^{(y0)}.y_{t|t} \\ &\quad + \beta_j^{(y1)}.y_{t+1|t} + \beta_j^{(\sigma)}.\sigma\left(y_{t+1|t}\right) + \underline{\lambda}_j^{/}.\underline{Z}_t + u_{jt}, \end{aligned} \tag{23}$$

where $\underline{Z}_t$ represents current observations on unemployment ($\triangle u_t$) and the underlying state of asset markets: housing, equity and the foreign exchange market ($P_{hsg,t}$, $P_{FTSE,t}$ and $P_{exch,t}$ respectively). Standard deviation of the one-year ahead forecast of output growth is denoted by $\sigma\left(y_{t+1|t}\right)$; this term is included to incorporate the notion that the stance of monetary policy may depend on the uncertainty relating to forecast future levels of output and inflation. As discussed in the previous section, increased uncertainty about the current state of the economy will tend to bias policy towards caution in changing interest rates. In particular, this strand of the literature suggests that optimal monetary policy may be more cautious (rather than activist) under greater uncertainty in the forecast or real-time estimates of output gap and inflation (see Issing, 2002; Aoki, 2003; and Orphanides, 2003). Since, as previously discussed, the published inflation forecast is not sufficiently informative, we confined ourselves to uncertainty relating to forecasts of future output growth.

However, there are two important additional features of our data generating process that render the estimation exercise nonstandard. First, the dependent variable is observed in the form of votes, which are highly clustered interval censored outcomes based on the underlying decision rules. Second, and importantly in our context, the regression errors are potentially interrelated across the members. Therefore, we augment the empirical model (23) with a model for interaction between the error terms for different members

$$u_t = \boldsymbol{W} u_t + \varepsilon_t, \tag{24}$$

where $\boldsymbol{W}$ is a $(m \times m)$ matrix of interaction weights with zero diagonal elements and unrestricted entries on the off-diagonals, subject to the constraint that $(\boldsymbol{I} - \boldsymbol{W})$ is nonsingular, and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{mt})'$ is a vector of uncorrelated errors that are possibly heteroscedastic with

$$E\left(\varepsilon_t \varepsilon_t'\right) = \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_m^2 \end{bmatrix}.$$

Votes of MPC members are highly clustered, with a majority of the votes proposing no change in the base rate. The final decisions on interest rate changes are all similarly clustered. For the Bank of England's MPC as a whole over the period June 1997 to March 2005, 69 per cent of the meetings decided to keep the base rate at its current level, 14 per cent recommended a rise of 25 basis points, 13 per cent recommended a reduction of 25 basis points, and the remaining 4 per cent a reduction of 50 basis points.

This clustering has to be taken into account when studying individual votes and committee decisions of the MPC. We do not observe changes in interest rates on a continuous or unrestricted scale, we have a non-continuous or limited dependent variable. Moreover, changes in interest rates are in multiples of 25 basis points. Therefore, we use an interval regression framework for analysis; other authors have used other limited dependent variable frameworks, like the logit/ probit or multinomial logit/ probit framework to analyse monetary policy decisions. Our choice of model is based on the need to use all the information that is available when monetary policy decisions are made, as well as problems relating to model specification and interpretation of multinomial logit models (Greene, 1993). We also explored an ordered and multinomial logit formulations, and found the broad empirical conclusions to be similar.

Therefore, the observed dependent variable in our case, $v_{jt,obs}$, is the truncated version of the latent policy response variable of the j-th member, v_{jt}, which we model as

$$\begin{aligned} v_{jt,obs} &= -0.25 \quad \text{if} \quad v_{jt} \in [-0.375, -0.20) \\ &= 0 \qquad\;\; \text{if} \quad v_{jt} \in [-0.20, 0.20] \\ &= 0.25 \quad\;\; \text{if} \quad v_{jt} \in (0.20, 0.375]\,, \text{ and} \\ v_{jt} &\in (v_{jt,obs} - 0.125, v_{jt,obs} + 0.125] \text{ whenever } |v_{jt,obs}| > 0.325 \end{aligned} \tag{25}$$

The wider truncation interval when there is a vote for no change in interest rates (*ie.*, for $v_{jt,obs} = 0$) may be interpreted as reflecting the conservative stance of monetary policy under uncertainty with a bias in favour of leaving interest rates unchanged.

3.4 Results

Under the maintained assumptions that (a) regression errors are uncorrelated across meetings, and (b) the response variable is interval censored, estimation of the policy reaction function for each member (23) is an application of interval regression (Amemiya, 1973). In our case, however, we have an additional feature that the errors are potentially correlated across members. If we can estimate the covariance matrix of these residuals, then we can use a standard GLS procedure by transforming both the dependent variable and the regressors by premultiplying with the symmetric square root of this covariance matrix. However, the dependent variable is interval censored and has to be placed at its conditional expectation given current parameter estimates and its censoring interval. This sets the stage for the next round of iteration. Now, the dependent variable is no longer censored; hence, a standard SURE methodology can be applied.

Estimating the covariance matrix at the outset is also nonstandard. Because the response variable is interval censored the residuals also exhibit similar limited dependence.[13] We use the Expectation-Maximisation algorithm (Dempster *et al.*, 1977; McLachlan and Krishnan, 1997) for estimation. At the outset, we estimate the model using standard interval estimation separately for each member and collect residuals. We invoke the Expectation step of the EM algorithm and obtain expected values of the residuals given that they lie in the respective intervals. Since we focus on five MPC members, for each monthly meeting, we have to obtain conditional expectations by integrating the pdf of the 5-variate normal distribution with the given estimated covariance matrix.

[13] For example, suppose the observed response for the j-th member in a given month t is 0.25. By our assumed censoring mechanism (25), this response is assigned to the interval $(0.20, 0.375]$. Suppose also that the linear prediction of the policy response, based on estimates of the interval regression model is $\widehat{v}_{jt} = 0.22$. Then the residual $v_{jt} - \widehat{v}_{jt}$ cannot be assigned a single numerical value, but can be assigned to the interval $(0.20 - 0.22, 0.375 - 0.22]$. In other words, the residual is interval censored: $v_{jt} - \widehat{v}_{jt} \in (-0.02, 0.155]$.

Iterating the above method till convergence provides us maximum likelihood estimates of the policy reaction function for each of the five members, under standard assumptions, specifically multivariate normality of the cross member errors. The covariance matrix of the errors is unrestricted.

TABLE 2: Interval Regression Estimates of policy reaction functions for the 5 MPC members

Variables	GOVERNOR	INTERNAL		EXTERNAL	
	George	**Clementi**	**King**	**Buiter**	**Julius**
$\triangle r_{t-1}$	-0.119	-0.107	-0.101	-0.164	-0.226^{*}
π_t	0.040	0.024	-0.050	0.051	0.110
$\pi_{t+4\|t}$	0.103^{**}	0.110^{**}	0.111^{**}	0.258^{**}	0.248^{**}
y_t	-0.016	-0.024	-0.052	-0.114	0.046^{+}
$y_{t+1\|t}$	0.211^{**}	0.223^{**}	0.186^{**}	0.216^{*}	0.047
$\triangle u_t$	-0.175	-0.244^{+}	-0.216^{+}	-0.645^{**}	-0.251^{**}
$P_{hsg,t}$	1.760^{**}	1.696^{*}	2.355^{**}	6.154^{**}	1.323
$P_{FTSE,t}$	0.620^{**}	0.412^{+}	0.614^{**}	1.229^{**}	-0.222
$P_{exch,t}$	0.003	0.009	0.007^{+}	-0.006	0.004
$\sigma\left(y_{t+1\|t}\right)$	-1.152^{*}	-0.420	0.378	-0.555	-0.981^{*}
constant	-0.054	-0.795	-1.313^{**}	-1.803	-0.716
Number of meetings	73	62	94	35	45
Good. of fit Wald χ^2	141.4	148.4	174.6	458.5	911.7
$Prob. > \chi^2(10)$	0.000	0.000	0.000	0.000	0.000
Log pseudo-likelihood	-37.98	-38.57	-66.17	-27.89	-11.70

** , *and $^{+}$– Significant at 1%, 5% and 10% level respectively.

These estimates are presented in Table 2. The estimates show substantial heterogeneity across the members of the MPC, which is discussed elsewhere (Bhattacharjee and Holly, 2008b).

Our focus here is on the cross-member interactions. Based on the above estimates, we obtain interval censored residuals using the initial censoring scheme. These are also placed at their expected values, conditional on estimates of the model parameters and their own respective censoring intervals. Similarly, policy reaction functions are estimated for other $(N-m)$ members who were in the committee in each month under study, for use as instruments later on. These are also placed at their conditional expected values.

Further, as discussed above, we use the residuals for the 5 selected members and invoke the EM algorithm to obtain the MLE of their spatial autocovariance matrix. The iterative estimation procedure converges quite fast (in

4 iterations). The estimated covariance matrix and the implied correlation matrix for the regression errors across the 5 selected members are reported in Table 3.

The estimated correlation matrix in Table 3 indicate very high correlation coefficients between regression errors corresponding to several pairs of MPC members. The CD (cross-section dependence) test (Pesaran, 2004) strongly rejects the null hypothesis of no cross-section dependence. Estimation of the spatial weights matrix would facilitate understanding of these interactions.

Table 3: Estimated MLEs for Mean Vector, Covariance Matrix and Correlation Matrix of Regression Errors (n = 33 months)[14]

A. REGRESSION ERRORS: MEAN VECTOR (MLE)

	George	**Clementi**	**King**	**Buiter**	**Julius**
	0.0041	−0.0174	−0.0070	0.0016	−0.0054

B. REGRESSION ERRORS: CORRELATION (COVARIANCE) MATRIX

	George	**Clementi**	**King**	**Buiter**	**Julius**
George	**1.00** (0.00829)				
Clementi	0.9989	**1.00** (0.01031)			
King	0.9923	0.9896	**1.00** (0.00871)		
Buiter	0.9573	0.9679	0.9558	**1.00** (0.00778)	
Julius	0.5184	0.4934	0.5182	0.2965	**1.00** (0.00050)

The stage is now set for estimating the matrix of cross member network interactions. This is done using the three methodologies described in the previous section.

First, we estimate the interaction (spatial) weights under suitable structural constraints, using the methodology in Bhattacharjee and Jensen-Butler (2005). Typically, one would then require either $m(m-1)/2$ constraints, or equivalently an appropriate objective function to fix the orthogonal transformation. Bhattacharjee and Jensen-Butler (2005) show that an useful set of constraints is symmetry of the spatial weights matrix $\boldsymbol{W}$. This constraint is, however, not useful in our case since we expect the strength of interaction

[14]Panel B reports the cross-member correlation matrix, with figures in parentheses on the diagonals representing the corresponding variances.

between MPC members often to be asymmetric. For example, it is plausible that an external member of the MPC arrives at her estimate of the output gap quite independently of what an internal member does, while the internal member may position himself strategically after assessing how the external member is likely to vote.

We, therefore, build up an alternative set of $m(m-1)/2 = 10$ constraints. Based on some ideas about the institutional setting of monetary policy decision making in the Bank of England, we choose the following sets of restrictions:

1. Row-standardisation: It is quite common in the spatial econometrics literature to work with a row standardised spatial weights matrix, where the rows sum to unity. This, however, is not strictly relevant in our context because some of the elements in the spatial weights matrix could be negative. Instead, we standardise rows so that the squares of the elements in each row sum to unity. This assumption gives us 5 constraints.

2. Homoscedasticity: Idiosyncratic error variances (σ_j^2's) are the same for George, Clementi and Buiter, and different and unequal error variances for King and Julius – $\sigma^2_{\text{George}} = \sigma^2_{\text{Clementi}} = \sigma^2_{\text{Buiter}}$ (2 restrictions)

3. Symmetry: Symmetric weights between the internal members and the Governor – $w_{\text{Clementi,King}} = w_{\text{King,Clementi}}$, $w_{\text{George,Clementi}} = w_{\text{Clementi,George}}$ and $w_{\text{George,King}} = w_{\text{King,George}}$ (3 restrictions).

The estimates of the spatial weights and idiosyncratic error variances are presented in Table 4. As one can see, the restrictions are approximatey satisfied. More importantly, confidence intervals based on the bootstrap indicate that quite a few of the spatial weights are significant. As discussed earlier, non-zero spatial weights in our model are indicative of (a) interaction due to deliberation and combined decision making within the MPC, and/ or (b) strategic interaction. Of particular significance are the negative spatial weights. While the process of discussion and agreement to a common estimate of the output gap would contribute to positive spatial weights, negative weights are almost certainly the outcome of strategic interaction. In this context, the negative spatial weights between the Governor and the external members (Buiter and Julius) are of particular importance. It would appear that the evidence from these estimates point towards strategic alignment of votes within the MPC.

Table 4: Estimated Weights Matrix under Structural Constraints

	George	Clementi	King	Buiter	Julius	Row SS	$\widehat{\sigma}_j$
George	**0**	0.642^{**} (0.043)	0.602^{**} (0.053)	-0.284^{**} (0.110)	-0.343^{**} (0.132)	**0.973**	2.74*e*-4
Clementi	0.638^{**} (0.042)	**0**	-0.600^{**} (0.223)	0.261^{*} (0.106)	0.277^{**} (0.085)	**0.911**	2.97*e*-4
King	0.618^{**} (0.097)	-0.608^{**} (0.235)	**0**	0.265^{*} (0.114)	0.322^{**} (0.092)	**0.926**	1.32*e*-3
Buiter	-0.562^{**} (0.172)	0.562^{**} (0.212)	0.542^{**} (0.124)	**0**	-0.297 (0.187)	**1.014**	3.18*e*-4
Julius	-0.564^{*} (0.228)	0.564^{*} (0.249)	0.555^{*} (0.227)	-0.249 (0.158)	**0**	**1.007**	1.63*e*-3

** , *and $^{+}$– Significant at 1%, 5% and 10% level respectively.
Bootstrap standard errors in parentheses.

Second, we estimate the spatial weights matrix under moment restrictions, using the methodology developed in Bhattacharjee and Holly (2008a). This is achieved by GMM, assuming that the censoring intervals are exogenous in the interaction model for the errors, and using moment conditions given in (10). In other words, instruments are derived from residuals for other members in the committee and lagged residuals of members included in the analysis (from lag 2 backwards). In the spirit of dynamic panel GMM estimators (Arellano and Bond, 1991; Blundell and Bond, 1998), the instrument set is therefore different for each month under analysis. The endogenous error models for each member are estimated separately, though the entire estimation exercise can be combined together within an unified GMM setup.

Table 5: Estimated Spatial Weights Matrix, GMM

	George	Clementi	King	Buiter	Julius	Row SS	J-stat.
George	**0**	0.813^{**} (0.031)	0.184^{**} (0.057)	-0.008 (0.061)	-0.149^{*} (0.061)	**0.717**	9.09 (p=0.77)
Clementi	0.911^{**} (0.061)	**0**	-0.081 (0.073)	0.173^{**} (0.055)	0.159^{**} (0.057)	**0.892**	10.12 (p=0.68)
King	0.532^{**} (0.172)	-0.283 (0.216)	**0**	0.610^{**} (0.127)	0.540^{**} (0.120)	**1.027**	7.67 (p=0.86)
Buiter	-0.153 (0.277)	0.352 (0.233)	0.680^{**} (0.132)	**0**	-0.454^{**} (0.067)	**0.816**	11.36 (p=0.58)
Julius	-0.427^{**} (0.086)	0.481^{**} (0.121)	0.501^{**} (0.072)	-0.340^{**} (0.085)	**0**	**0.780**	10.92 (p=0.62)

** , *and $^{+}$– Significant at 1%, 5% and 10% level respectively.
HAC standard errors in parentheses.

The validity of the assumed moment conditions is checked using the Sargan-Hansen J-test for overidentifying restrictions (Hansen, 1982). The estimated interaction matrix is presented in Table 5. The reported estimates are numerically, and definitely in sign, similar to estimates of spatial weights under structural constraints (Table 4). At the same time, significance of some of the weights are different. Admittedly, the assumed structural restrictions on the weights matrix are not verifiable and can be violated in empirical applications. This observation further underscores an advantage of the GMM based methodology, subject to the validity of the assumed moment conditions. Further, the moment restrictions can be tested, as we have done here.

Third, we estimate structural spatial weights after allowing for common correlated factors. In the context of the current application, we suspect *a priori* that the MPC members may not be completely exchangeable. In other words, some structural connections may be important. At the same time, unobserved factors may drive some of the cross-section dependence. In order to explore the relative importance of these two channels, we estimate the model using the common correlated effects (CCE) methodology (Pesaran, 2006). In the current application, none of the variables in the RHS of the empirical policy rule (23) has cross-section variation, but we allow for complete slope heterogeneity. Therefore, we estimate a modified policy rule with cross-section averages of the dependent variable as an additional regressor. Very similar results are obtained when we use the median response as a proxy for this mean response. This also has the advantage of helping place an explicit interpretation to the additional regressor; hence, we report results for this modified decision rule:

$$\begin{aligned} v_{jt} &= \phi_j + \beta_j^{(r)}.\triangle r_{t-1} + \beta_j^{(\pi 0)}.\pi_t + \beta_j^{(\pi 4)}.\pi_{t+4|t} + \beta_j^{(y0)}.y_{t|t} \\ &\quad + \beta_j^{(y1)}.y_{t+1|t} + \beta_j^{(\sigma)}.\sigma\left(y_{t+1|t}\right) + \underline{\lambda}_j^{/}.\underline{Z}_t + \beta_j^{(CCE)}.\triangle r_t + u_{jt}, \end{aligned} \tag{26}$$

where as before

$$\begin{aligned} u_t &= \boldsymbol{W} u_t + \varepsilon_t; \\ E\left(\varepsilon_t \varepsilon_t'\right) &= \boldsymbol{\Sigma} = diag\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2\right). \end{aligned}$$

The covariance and correlation matrix of the residuals from this model is estimated in the same way as before, and reported in Table 6. It is interesting to note that, after allowing for common factors, the degree of structural cross-section dependence has significantly reduced, to the extent that the CD test (Pesaran, 2004) now fails to reject the null hypothesis of no spatial dependence.

Table 6: Estimated MLEs for Mean Vector, Covariance Matrix and Correlation Matrix of Regression Errors in CCE model (26)[15]

A. Regression Errors: Mean Vector (MLE)

	George	**Clementi**	**King**	**Buiter**	**Julius**
	$7.57e$-10	-0.0141	-0.0216	-0.0114	0.0121

B. Regression Errors: Correlation (Covariance) Matrix

	George	**Clementi**	**King**	**Buiter**	**Julius**
George	**1.00** (0.00019)				
Clementi	0.1144	**1.00** (0.00719)			
King	-0.0895	0.1089	**1.00** (0.00989)		
Buiter	-0.2021	0.0917	0.3145	**1.00** (0.00974)	
Julius	-0.1117	-0.0140	0.0440	-0.0435	**1.00** (0.01020)

However, some spatial correlations are quite large, and it is possible that a degree of structural spatial interactions may be present. In order to explore this, we estimate the structural spatial weights matrix by GMM, using the same moment conditions as before (10). The estimates and corresponding tests for overidentifying restrictions are reported in Table 7.

The estimates point to important structural interconnections between members of the MPC. Though the statistical significance of spatial interactions is weaker than those reported before (Tables 4 and 5), the direction and magnitude of the important network effects are broadly preserved. Further analysis of network connections within the Bank of England's MPC, and related inferences on network architecture and strategic behaviour, is beyond the scope of this paper.

Overall, we can conclude that the objective of studying spatial interactions is best served by allowing for both structural and factor based cross-section dependence. Further, we describe and illustrate several methods

[15] Panel B reports the cross-member correlation matrix, with figures in parentheses on the diagonals representing the corresponding variances.

based on spatial panel data that can be used to draw inferences on these spatial and interaction weights.

Table 7: Estimated Spatial Weights Matrix by GMM
(allowing for common unobserved factors – CCE)

	George	**Clementi**	**King**	**Buiter**	**Julius**	J-stat.
George	**0**	0.0015 (0.0087)	−0.0096 (0.0099)	−0.0060 (0.0121)	−0.0279* (0.0134)	8.77 (p=0.79)
Clementi	−0.4976 (0.5561)	**0**	0.1516 (0.1055)	0.2511+ (0.1365)	−0.1458 (0.1389)	8.46 (p=0.81)
King	0.2936 (1.4663)	0.2393+ (0.1431)	**0**	0.3319* (0.1467)	0.4021* (0.1769)	8.75 (p=0.79)
Buiter	2.2354+ (1.3537)	0.0326 (0.0994)	0.6435** (0.1198)	**0**	−0.0234 (0.1251)	10.78 (p=0.63)
Julius	−2.6098** (0.9904)	0.0105 (0.1136)	−0.0231 (0.1078)	0.1583 (0.1198)	**0**	8.73 (p=0.79)

** , *and +– Significant at 1%, 5% and 10% level respectively.
HAC standard errors in parentheses.

4 Conclusions

In this paper, we argue that the distinction between structural and factor based connections is very important in the study of spatial and cross-section interactions. Both of these channels offer alternative explantions for spatial correlation and can coexist in some models and applications. Further, while the assumption of exchangebility inherent in the factor model can be unreasonable in many spatial applications, assuming the absence of common unobserved factors also appears to be too strong.

We describe three methods to draw inferences on spatial (interaction) weights that explicitly address the above distinction. The first two methods are designed to draw inferences under the structural dependence assumption, one under structural constraints on the spatial weights matrix (Bhattacharjee and Jen-Butler, 2005) and the other under moment restrictions inspired by the system GMM literature (Bhattacharjee and Holly, 2008a). The third method allows for both structural and factor based dependence, where the unobserved factors are modelled using the common correlated effects methodology (Pesaran, 2006) and the structural spatial weights estimated using GMM methods propoesed in Bhattacharjee and Holly (2008a). The methods are illustrated using an application to committee decision making within the Bank of England's MPC. The application highlights the relative advantages and shortcomings of each of the methods, and helps draw useful inferences on the nature and strength of interactions.

Research in the above area, both empirical and econometric, is ongoing. More work needs to be done in combining the common factor approach with structural spatial interactions. Further research on the nature of spatial interactions would inform both the very active theoretical literature on economic networks, and provide additional empirical insights into the stability of different network architectures under assumptions on information sharing and bargaining. Also, the networks emerging from our work can be viewed as a sequence of weighted directed graphs, with connections condituioned on the assumed significance level. How research on such random graphs can aid inferences on connections in space and economic networks is a matter of further study.

Apart from the application developed here, Bhattacharjee and Jensen-Butler (2005) and Holly et al. (2008) have used related frameworks for empirical studies of housing markets. However, the applicability of the framework and methods would surely go beyond these couple of applications. For example, an important application area would be economic convergence of countries and regions. Previous research suggests that there are stable differences in productivity across regions in the EU. Potentially, such spatial inequality can be explained by technology transfer, where some regions are more efficient in generating new technology or technology absorption. In turn, technology transfer is often related to trade (imports and/ or exports), FDI etc., while technology absorption depends on human capital, R&D and similar features. However, theory provides no clear guidance as to which of these channels are more important, or indeed if relative importance varies across regions. Further, while some of the diffusion can be explained by the pull of common factors, there may be institutional features that enhance or depress technology transfer between specific pairs of regions. In empirical studies, it is useful to allow for technology transfer in relatively unrestricted manner and further, to infer on the strength and direction of inter-region diffusion while being agnostic about the specific drivers of such interaction; see Bhattacharjee and Jensen-Butler (2005) for a simulation study based on the US. Further research along similar lines will be important for our understanding of economic growth and convergence.

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