Shifts in Portfolio Preferences of International Investors: an Application to Sovereign Wealth Funds*

Filipa Sá† Trinity College, University of Cambridge
Francesca Viani‡ European University Institute
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Abstract

Reversals in capital inflows can have severe economic consequences. This paper develops a dynamic general equilibrium model to analyse the effect on interest rates, asset prices, investment, consumption, output, the exchange rate and the current account of a shift in portfolio preferences of foreign investors. The model has two countries and two asset classes (equities and bonds). It is characterized by imperfect substitutability between assets and allows for endogenous adjustment in interest rates and asset prices. Therefore, it accounts for capital gains arising from equity price movements, in addition to valuation effects caused by changes in the exchange rate. To illustrate the mechanics of the model, we calibrate it to analyse the consequences of an increase in the importance of Sovereign Wealth Funds (SWFs). Specifically, we ask what would happen if ‘excess’ reserves held by Emerging Markets were transferred from central banks to SWFs. We look separately at two diversification paths: one in which SWFs keep the same allocation across bonds and equities as central banks, but move away from dollar assets (path 1); and another in which they choose the same currency composition as central banks, but shift from US bonds to US equities (path 2). In path 1, the dollar depreciates and US net debt falls on impact and increases in the long run. In path 2, the dollar depreciates and US net debt increases in the long run. In both cases, there is a reduction in the ‘exorbitant privilege’, i.e., the excess return the US receives on its assets over what it pays on its liabilities. The model is applicable to other episodes in which foreign investors change the composition of their portfolios.

Keywords: portfolio preferences, sudden stops, imperfect substitutability, global imbalances, sovereign wealth funds

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†fgs22@cam.ac.uk
‡Francesca.Viani@EUI.eu
1 Introduction

Reversals in capital inflows can have severe consequences for the real economy and the financial sector, as the empirical literature on sudden stops illustrates. This explains why global imbalances are seen as one of the greatest vulnerabilities in the international monetary system. The sustainability of global imbalances has been a major focus of the academic literature on international finance and is a contentious topic. Some studies (for example, Blanchard, Giavazzi and Sá (2005) and Obstfeld and Rogoff (2007)) find that global imbalances will not persist because the United States must stabilize its external debt level, which would require a large depreciation of the dollar. Other studies find that global imbalances can persist for a long period of time because of differences in financial market development that make US assets attractive to foreign investors (for example, Caballero et al (2008) and Forbes (2010)) or because of a persistent return differential between US and foreign assets - the so-called ‘exorbitant privilege’ (Gourinchas and Rey (2007)).

The recent financial crisis exposed severe weaknesses in the US financial and regulatory system. It would not have been surprising if investors had responded by reducing their holdings of US assets. However, while foreign investors did sell US equities and corporate debt during the crisis, their demand for US government debt increased sharply. This suggests that, even though the crisis started in the US, investors are still attracted by the safety and liquidity of US assets. Is this situation likely to continue in the future? Or is foreign investors’ appetite for US assets likely to diminish as US debt levels increase and tighter financial market regulations are adopted? What would be the implications for the dollar exchange rate, global imbalances and asset prices of a shift in the preferences of foreign investors away from US assets?

This paper develops a framework for understanding the implications for the dollar, interest rates, asset markets and external debt of a shift in the portfolio preferences of foreign investors. It develops a dynamic general equilibrium model with two regions (the US and the rest of the world - ROW), each specialized in the production of a different variety of goods. A distinctive feature of the model is the presence of two asset classes: equities and bonds. This allows to study the implications of two types of changes in the portfolio preferences of foreign investors: a reduction in their preference for US assets and a diversification away from US debt and into US equity assets. The real exchange rate determines the allocation of goods and assets across the two regions. Because each region issues both equities and bonds, there are four assets in total: US equities, US bonds, ROW equities, and ROW bonds. These assets are imperfect substitutes and their demand follows the specification in Blanchard, Giavazzi and Sá (2005), where the share of wealth invested in each asset has an exogenous and an endogenous component. The exogenous component represents shocks to portfolio preferences. The endogenous component captures the reaction of asset demands to changes in the relative expected returns of different assets\(^1\).

\(^1\)A growing body of literature derives portfolio holdings from optimization principles in stochastic general equilibrium settings (see, for instance, Devereux and Sutherland (2006) and Heathcote and Perri (2007)). For our purposes, it proves more convenient not to derive portfolio shares from microfoundations. We rather choose to embed in a general equilibrium framework the specification of asset demands that characterize standard portfolio-balance mod-
The general equilibrium nature of the model allows for endogenous adjustments in interest rates and asset prices. Therefore, we can see how they are affected by shifts in the preferences of foreign investors and what the implications are for the ‘exorbitant privilege’. Following any shock to portfolio preferences, both equity and bond prices adjust to clear asset markets. Over time, investors rebalance their portfolios in response to changes in expected returns. This endogeneity of interest rates and asset prices is the key difference between our model and the one in Blanchard, Giavazzi and Sá (2005), where interest rates are exogenous.

Our model is general enough to be usable for a variety of experiments. It could be calibrated to countries outside the US. For example, it could be used to study the implications of the sudden reversals in capital flows that occurred in Iceland and Greece during the global financial crisis and to analyse the consequences for other countries with high debt levels if foreign investors were to withdraw their investment. To illustrate how the model works, we use it to analyse the implications of an expansion in Sovereign Wealth Funds (SWFs).

SWFs are government-owned investment funds, set up for a variety of purposes, for example to transform the income from non-renewable natural resources into a diversified portfolio of assets or to increase the return on foreign exchange reserves. SWFs are becoming increasingly important in the international monetary system and are estimated to have between 2.1 and 3 trillion US dollars of assets under management. While this is relatively small in comparison with total global financial assets, estimated at 194 trillion dollars in 2006 (IMF (2008b)), it is a sizable amount and exceeds the size of hedge funds, estimated at 1.7 trillion dollars. Moreover, SWFs are projected to grow rapidly in the next decade and to have around 12 trillion dollars of assets under management in 2015 (Morgan Stanley (2008)).

Information about the portfolio structure of SWFs is relatively limited, since there is no uniform public disclosure of their assets and investment strategies. However, the information available suggests that the portfolios of SWFs are typically more diversified than traditional reserves held by central banks, with a larger share invested in equities and a wider geographical dispersion. Given these differences in investment strategies, a shift of reserve assets from central banks to SWFs could have implications for asset prices, the flow of funds between countries, exchange rates, and the evolution of global imbalances. In particular, SWFs may increasingly diversify away from dollar assets. This might lead to a reduction in capital inflows into the US, a depreciation of the dollar and an increase in returns on dollar assets. SWFs may also diversify their portfolios away from low-risk, short term debt instruments, and into longer term equity assets, which might lead to changes in asset prices and rates of return.

It has been argued that the changes in asset returns generated by the growth in SWFs might...
induce a reduction in the so-called ‘exorbitant privilege’ of the US. This term has been used by Gourinchas and Rey (2007) to denote the fact that the US receives higher returns on its foreign assets than it pays on its foreign liabilities. This excess return can be decomposed in two elements: a return effect - within each asset class, the return that the US pays to foreigners is smaller than the return that foreigners pay to the US; and a composition effect - the US tends to invest more in foreign equities, while foreigners tend to invest more in US bonds. The growth in SWFs may lead to a reduction in both components of the ‘exorbitant privilege’.

We calibrate the model to match asset holdings and returns observed in the data. We simulate a scenario where all ‘excess reserves’ currently held by central banks in Emerging Market Economies (EMEs) are transferred to SWFs, where ‘excess reserves’ are defined as being above the level that would be required for liquidity purposes. We consider two diversification paths: one in which SWFs keep the same asset allocation as central banks, i.e., the same investment shares in equities and bonds, but diversify away from dollar assets (path 1); and another in which they keep the same currency composition, but shift towards a riskier portfolio in the US market, with a larger share invested in US equities and a smaller share invested in US bonds (path 2). We focus on the implications for the dollar exchange rate, the US trade deficit and net debt, and the ‘exorbitant privilege’. The main purposes of our analysis are to provide a qualitative assessment of how changes in portfolio preferences of foreign investors affect asset returns, consumption, investment, the exchange rate, the trade deficit and net debt; to provide a rough quantification of the magnitude of the adjustment in these variables; and to understand the channels through which these effects occur.

Our results show that in path 1 (currency diversification) the dollar depreciates in the period immediately after the shock, leading to a reduction in the US trade deficit and net debt. In subsequent periods, the return on US assets must increase to clear asset markets. This generates a rebalancing of the portfolios of foreign investors towards holding more dollar assets, which leads to an appreciation of the dollar. The ‘exorbitant privilege’ of the US (the difference between the return it receives on its foreign assets and the return it pays on its foreign liabilities) decreases, and US net debt increases over time. In path 2 (asset diversification) the dollar depreciates and the US trade deficit decreases. However, US net debt increases over time due to a reduction in the ‘exorbitant privilege’.

Qualitatively, our results can be compared with the findings of an exercise conducted by the IMF (IMF (2008a)). It assumes that between 25 and 50 percent of new foreign currency inflows in countries that have recently established SWFs will be invested by those SWFs. The exercise is calibrated for two diversified portfolios: one which mimics the composition of Norway’s Government Pension Fund; and another which is based on information on asset allocation and currency composition provided in market analysis. These two stylized portfolios are compared with a scenario where assets are kept as central bank reserves. The results, derived using the IMF’s Global Integrated Monetary Fiscal Model, suggest that the US real interest rate would increase by 10 to 20 bp, the dollar would depreciate by 2 to 5 percent, and the US current account deficit would
improve by 0.25 to 0.5 pp of GDP. In the rest of the world, real interest rates would fall, currencies would appreciate, and domestic demand would increase. These results are qualitatively similar to ours.

This rest of the paper is organized as follows. The structure of the model is explained in section 2. Section 3 presents the baseline parameters used in the calibration and section 4 describes the scenarios used in the simulations. The results of the baseline simulations are discussed in section 5, while section 6 presents a number of robustness checks. Section 7 concludes.

2 The Model

This section describes the general equilibrium model. The core structure of the model is based on Gosh (2007) and Meredith (2007). The model consists of two regions: the US and the rest of the world (ROW), each fully specialized in the production of one homogeneous good. In each region, there are two types of assets: equities and bonds. Equities are modelled as claims on the capital stock. Bonds are issued by the government, who must balance its budget every period.

Each country is populated by a representative firm and two types of representative households: entrepreneurs and portfolio investors. Firms in both regions produce output using capital and labour, and adjust their productive capacity by increasing or decreasing their stock of capital. We abstract from any nominal rigidity and from real economic growth. Entrepreneurs manage the firms and have all their wealth invested in home equities. Portfolio investors invest in bonds and equities at home and abroad. They supply labour inelastically and decide on the proportion of their income to be allocated to consumption and portfolio investment. They receive income in the form of wages and returns on their portfolio and pay taxes or receive transfers from the government.

The general equilibrium nature of the model lets any adjustment in interest rates and asset prices be determined endogenously, as asset demands react to changes in the relative expected returns of different assets. The supply of equities is determined by firms’ investment in physical capital. Because equity prices are determined endogenously, the model is able to account for capital gains on equity holdings, in addition to valuation effects caused by changes in the exchange rate.

2 We write the model with two types of households to ensure internal consistency. For the model to be internally consistent, firms should discount future profits using the discount factor of the consumers who manage them. The steady-state discount factor of portfolio investors is a function of the returns on all assets in which they invest (home and foreign equities and bonds). However, the discount factor of US firms should equal, in steady-state, the user cost of capital, which coincides with the rate of return on US equities. If there were only portfolio investors in the economy, for the discount factor of US portfolio consumers to equal the discount factor of US firms, the steady-state returns on all four assets (home and foreign equities and bonds) would have to be the same. This would be an unattractive feature since we are interested in matching the return differential on assets observed in the data. We solve this problem by including two types of representative households: portfolio investors, who invest in all assets, and entrepreneurs, who only invest in home equities and manage the firms. The model is internally consistent since the discount factor of entrepreneurs equals the discount factor of the firms.
2.1 Consumers

The size of the world population is normalized to 1, with a fraction \( n \) in the US and \( (1 - n) \) in the rest of the world. There are two types of representative households in each economy: entrepreneurs and portfolio investors. Entrepreneurs manage the firms and invest all their wealth in home equities. Portfolio investors supply labour inelastically, pay lump-sum taxes (or receive transfers), and invest their wealth in both equities and bonds, at home and abroad. We denote the fraction of entrepreneurs in the US population by \( \alpha^E \) and the fraction of entrepreneurs in the ROW population by \( \alpha^{E*} \).

Both entrepreneurs and portfolio investors decide on how much to consume given their wealth. US households derive utility from consuming the following CES bundle of US and ROW-produced goods:

\[
C_t = \left\{ (\rho)^{1/\theta} \left( C_{US,t} \right)^{\frac{\theta-1}{\theta}} + (1 - \rho)^{1/\theta} \left( C_{ROW,t} \right)^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}
\]

where \( \rho > 0.5 \) is a parameter capturing the degree of home bias in consumption and \( \theta \) is the elasticity of substitution between goods produced in different regions. Consumption in the ROW is analogously defined, with starred variables denoting the corresponding quantities consumed by ROW households.

The consumer price indices can be derived from the households’ cost minimization problem. Taking the home good as the numeraire, the consumer price index for the US is given by:

\[
P_t = \left\{ \rho + (1 - \rho) e_t^{\theta-1} \right\}^{\frac{1}{1-\theta}}
\]

where \( e_t \) is the real exchange rate between the US and ROW, defined as the relative price of the goods produced in the two regions, so that an increase in the exchange rate represents an appreciation of the dollar.

The demands of US consumers for domestically and foreign produced goods are obtained from standard cost minimization subject to 1:

\[
C_{US,t} = \rho (P_t)^{\theta} C_t
\]

\[
C_{ROW,t} = (1 - \rho) (e_t P_t)^{\theta} C_t
\]

An appreciation of the dollar makes ROW goods less expensive to US consumers and US goods more expensive to ROW households, shifting world demand from US to ROW-produced goods. In this sense, the real exchange rate determines the allocation of goods across markets.

Consumers optimally decide to allocate their income between consumption and savings. The utility maximization problem for entrepreneurs is given by:
\[
\max_{\{C_E\}_{s=t}^\infty} E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \log \left( C_t^E \right) \right\}
\]

s.t. \( V_t^E = \tau_t^E V_{t-1}^E - P_t C_t^E \)

where \( V^E \) represents the financial wealth of US entrepreneurs, and \( \tau_t^E \) is the rate of return on their portfolio defined in local currency. Because entrepreneurs invest all their wealth in home equities, the rate of return on their portfolio equals the return on home equities.

The utility maximization problem for portfolio holders is given by:

\[
\max_{\{C_P\}_{s=t}^\infty} E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \log \left( C_t^P \right) \right\}
\]

s.t. \( V_t^P = R_t^P V_{t-1}^P - P_t C_t^P + w_t - \tau_t \)

The key difference relative to the problem of entrepreneurs is in the budget constraint, which now includes wage earning \( w_t \) net of lump-sum taxes \( \tau_t \). \( R_t^P \) is the rate of return on the portfolio, which includes home and foreign equities and bonds.

The first order conditions for utility maximization deliver the standard Euler equations:

\[ 1 = \beta E_t \left\{ \frac{R_{t+1}^i}{P_{t+1} C_{t+1}^i} \right\} \]

where \( i = E, P \) for entrepreneurs and portfolio investors, respectively.

The assumption of logarithmic utility implies that consumption expenditure for entrepreneurs is optimally determined as:

\[ P_t C_t^E = (1 - \beta) R_t^E V_{t-1}^E \]

Similarly, for portfolio investors:

\[ P_t C_t^P = (1 - \beta) \left\{ R_t^P V_{t-1}^P + H_t \right\} \]

where \( H_t \) is the present discounted value of lifetime human wealth, in the form of labour income net of taxes.

### 2.2 Firms

Firms in both countries are fully specialized in the production of the regional good, which is available for consumption and investment in both countries. They produce using a constant returns to scale technology combining capital and labour. The US production function is given by:
\[ Y_t = A_t K_t^\eta (L_t)^{1-\eta} \]

\( Y_t \) is the output of the US-produced good, \( A_t \) is an exogenous productivity term, \( K_t \) is the capital input and \( L_t \) is the labour input. Since we assume that only portfolio investors supply labour and that labour supply is inelastic, for the labour market to clear in equilibrium the labour input must equal the fraction of these investors in the economy, i.e. \( L_t = (1 - \alpha^E) \). A share \( \eta \) of output is paid to capital and the remaining is paid to labour.

Firms adjust their productive capacity by deciding the optimal amount of physical investment, \( I_t \), so as to maximize current and future cash flows. US firms solve the following problem:

\[
\max_{\{L_s, I_t, K_{t+1}\}} E_t \left\{ \sum_{s=t}^{\infty} \prod_{j=t+1}^{s} \Omega_{j,j+1} \left( A_s K_s^\eta (L_s)^{1-\eta} - w_s L_s - P_s I_s \left( 1 + \frac{I_s}{K_s} \right) - \delta P_s K_s \right) \right\} 
\]

\[ s.t.: K_{s+1} - K_s = I_s \]

where \( w_s \) denotes the wage, \( \left( \frac{I_s}{K_s} \right) \) is the linear homogeneous installation cost of capital, \( \delta \) is the depreciation rate, and \( \Omega_{j,j+1} \) the discount factor of US entrepreneurs, used by firms to discount future cash flows. \( P_s \) is the price index of the US investment bundle, which includes US and ROW-produced goods, built using the same CES aggregator used for the consumption good:

\[ I_t = \left\{ \rho^{1/\theta} (I_{US,t})^{\theta-1} + (1 - \rho)^{1/\theta} (I_{ROW,t})^{\theta-1} \right\}^{\theta/(\theta-1)} \]

Standard cost minimization by firms delivers their demands for US and ROW-produced goods:

\[ I_{US,t} = \rho P_t^\theta I_t \]

\[ I_{ROW,t} = (1 - \rho) (e_t P_t)^\theta I_t \]

The price of the US investment bundle coincides with the price of the US consumption bundle, \( P_t \), given by equation 2.

The firms’ problem in equations 5 and 6 can be stated recursively and its first order conditions written as:

\[ w_t = A_t (1 - \eta) K_t^\eta (1 - \alpha^E)^{-\eta} \]

\[ q_t = P_t \left( 1 + 2 \frac{I_t}{K_t} \right) \]
where $q_t$ is defined as the marginal value of capital and coincides with the price of US equities. Equation 9 determines the wage as the marginal product of labour and equation 10 determines the optimal amount of investment by US firms. Consistent with the standard q-theory, it implies that US firms increase their capital stock when the marginal value of capital $q_t$ exceeds the replacement cost of capital $P_t$. Equation 11 is an arbitrage condition stating that the marginal value of one unit of capital must be equal to the expected discounted value of returns one period ahead, which includes capital gains from equity ownership.

The realized gross return on US equities are given by:

$$r_t^E = \frac{q_t + \eta A_t K_t^{\eta-1}(1 - \alpha^E)^{1-\eta} - \delta P_t + \phi P_t \left( \frac{I_t}{K_t} \right)^2}{q_{t-1}}$$

Each period US equities pay out the realized marginal product of capital plus capital gains or losses due to adjustments in the price of equities.

### 2.3 Government

In each period governments in both regions finance public expenditure and pay out the interest on the outstanding stock of public debt either by selling new bonds at the current market price or by levying taxes on home portfolio investors. We follow Meredith (2007) in assuming that taxation is lump-sum in order to abstract from distortionary effects on capital accumulation. The US government budget constraint is given by:

$$G_t + B_{t-1} = P_t^B B_t + (1 - \alpha^E)\tau_t$$

Because taxes are paid only by portfolio investors, the amount of lump sum taxes raised by the government equals $(1 - \alpha^E)\tau_t$. For simplicity, we assume that $G_t$ and $B_t$ are exogenous and constant over time. Therefore, a reduction in the price of bonds, $P_t^B$, leads to an increase in lump sum taxes. In addition, we follow Meredith (2007) in assuming that the government consumes only the domestic good. This is a simplification, but is consistent with the evidence on home bias in government expenditure.

### 2.4 Portfolio Allocation

Entrepreneurs manage firms and allocate all their wealth to home equities. Portfolio investors make two types of decisions regarding their portfolios: they decide on the geographical composition (how much to invest in US and ROW assets) and on the asset composition (how much to invest
in equities and bonds). In what follows, all asset returns and prices are measured in units of the domestic good.

Bonds are issued by the governments in the US and ROW and pay out one unit of the good produced in the country in which they are issued. Therefore, the rates of return on US and ROW bonds are:

\[ r_t^B = \frac{1}{P_{t-1}^B} \quad \text{and} \quad r_t^{B*} = \frac{1}{P_{t-1}^{B*}} \]

where \( P^B \) and \( P^{B*} \) are the prices of the bonds expressed in US and ROW currencies.

Equities represent the ownership of one unit of capital of US or ROW firms. The rate of return on US equities is given by equation 12. A similar expression gives the rate of return on ROW equities, \( r_t^{E*} \).

We denote by \( \alpha_t \) the share of US financial wealth invested in US assets. From this, a fraction \( \beta_t \) is allocated to equities and a fraction \( (1 - \beta_t) \) is allocated to bonds.\(^3\) Similarly, from the share of US wealth invested in ROW assets, a fraction \( \gamma_t \) is allocated to equities and a fraction \( (1 - \gamma_t) \) is allocated to bonds. The shares for ROW portfolio investors, denoted with a star, are defined analogously.

Asset demands are characterized by imperfect substitutability between different assets and follow a similar specification to the one used in Blanchard, Giavazzi and Sá (2005). They have two components: an exogenous component, representing shocks to portfolio preferences, and an endogenous component, capturing the response of asset demands to changes in the relative returns of different assets. More specifically, \( \beta_t \), the fraction of wealth that US investors invest in equities in the US market, is given by:

\[ \beta_t = b^{\beta} \left\{ E_t \left( \frac{r_{t+1}^E}{r_{t+1}^B} \right) - \frac{r^E}{r^B} \right\} + s_t^{\beta} \]

where non time-indexed returns denote steady state values. The first term captures the reaction of \( \beta_t \) to changes in expected relative returns: if the return on US equities rises relative to the return on US bonds, investors allocate a relatively larger fraction of their wealth to equities. The parameter \( b^{\beta} \) captures the degree of substitutability between different assets (in this case between US equities and US bonds). A higher degree of substitutability makes portfolio shares more responsive to changes in expected relative returns. The second term, \( s_t^{\beta} \), is an exogenous shock to portfolio preferences.

Similarly, \( \gamma_t \), the fraction of wealth that US households invest in equities in the foreign market, is given by:

\[ \gamma_t = b^{\gamma} \left\{ E_t \left( \frac{r_{t+1}^{E*}}{r_{t+1}^{B*}} \right) - \frac{r^{E*}}{r^{B*}} \right\} + s_t^{\gamma} \]

\(^3\) According to these definitions, at time \( t \) US portfolio investors invest \( \alpha_t \beta_t V_t^P \) dollars in US equities and \( \alpha_t (1 - \beta_t) V_t^P \) dollars in US bonds.
An increase in the return on foreign equities relative to foreign bonds, increases the share of wealth that US investors invest in equities in the foreign market.\(^4\)

Turning to the currency composition of portfolios, the share of wealth that US portfolio investors invest in the domestic asset market, \(\alpha_t\), is given by:

\[
\alpha_t = b^{\alpha} \left\{ E_t \left( \frac{\beta_t r^E_{t+1} + (1 - \beta_t) r^B_{t+1}}{\gamma_t r^E_{t+1} + (1 - \gamma_t) r^B_{t+1}} \cdot \frac{e_{t+1}}{e_t} \right) - \frac{\beta r^E + (1 - \beta) r^B}{\gamma r^E + (1 - \gamma) r^B} \right\} + s^{\alpha}_t
\]  

(16)

The share of financial wealth invested by US portfolio holders in the domestic market is increasing in the relative expected return on domestic assets, given the proportion of wealth allocated to bonds and equities in each market. An expected appreciation of the dollar increases the relative return on dollar assets. Therefore, in our model, the real exchange rate determines not only the allocation of consumption across US and ROW goods, but also the allocation of wealth across US and ROW assets.\(^5\) \(^6\)

Given these definitions of portfolio shares, we can express the total rate of return of US portfolio investors as:

\[
R^P_t = \alpha_{t-1} \beta_{t-1} r^E_t + \frac{(1 - \alpha_{t-1}) \gamma_{t-1} r^E_{t+1}}{e_t} \cdot e_{t-1} + \alpha_{t-1}(1 - \beta_{t-1}) r^B_t + \frac{(1 - \alpha_{t-1}) (1 - \gamma_{t-1}) r^B_{t+1}}{e_t} \cdot e_{t-1}
\]  

(17)

Notice that valuation effects stemming from changes in the exchange rate affect the return US portfolio investors receive on their holdings of foreign assets.

2.5 Equilibrium and Balance of Payments Dynamics

Equilibrium requires goods and asset markets to clear. Combining equations 3, 4, 7, 8, and their foreign analogues, we can write market clearing conditions for the goods produced in the US and

\(^4\)The ROW analogues of equations 14 and 15 are:

\[
\beta^*_t = b^{\beta*} \left\{ E_t \left( \frac{r^E_{t+1}}{r^B_{t+1}} \right) - \frac{r^E_{t+1}}{r^B_{t+1}} \right\} + s^{\beta*}_t
\]

\[
\gamma^*_t = b^{\gamma*} \left\{ E_t \left( \frac{r^E_{t+1}}{r^B_{t+1}} \right) - \frac{r^E_{t+1}}{r^B_{t+1}} \right\} + s^{\gamma*}_t
\]

Expressing relative returns in deviations from steady state values simplifies the calibration of the model without affecting its dynamics.

\(^5\)The analogue of equation 16 for the share of ROW wealth invested in ROW assets, \(\alpha^*_t\), is given by:

\[
\alpha^*_t = b^{\alpha*} \left\{ E_t \left( \frac{\beta^*_t r^E_{t+1} + (1 - \beta^*_t) r^B_{t+1}}{\gamma^*_t r^E_{t+1} + (1 - \gamma^*_t) r^B_{t+1}} \cdot \frac{e_t}{e_{t+1}} \right) - \frac{\beta^* r^E + (1 - \beta^*) r^B}{\gamma^* r^E + (1 - \gamma^*) r^B} \right\} + s^{\alpha*}_t
\]

\(^6\)An alternative way to model the portfolio allocation problem would be to choose one of the four assets (for example, US bonds) to be the reference asset and have investors decide how much to allocate to each asset depending on its return relative to the reference asset. However, this specification does not allow to differentiate between a shock to the currency composition (how much to invest in the US versus ROW) and a shock to the asset composition (how much to invest in equities versus bonds). The specification we adopt for portfolio shares allows us to differentiate between these two types of shocks: a shock to \(\beta_t\) changes the asset composition without affecting the currency composition, while a shock to \(\alpha_t\) changes the currency composition without affecting the asset composition.
in ROW as:

\[
nA_tK_t^n(1 - \alpha^E)^{1-\eta} = \rho n (P_t) \theta \left[ C_t + \delta K_t + I_t \left( 1 + \phi \frac{I_t}{K_t} \right) \right] + \\
+ (1 - \rho^*) (1 - n) \left( \frac{P_i^*}{e_t} \right)^{\theta} \left[ C_t^* + \delta K_t^* + I_t^* \left( 1 + \phi \frac{I_t^*}{K_t^*} \right) \right] + nG_t
\]

\[
(1 - n)A_t^*K_t^{*n}(1 - \alpha^{*E})^{1-\eta} = (1 - \rho) n (P_t e_t)^{\theta} \left[ C_t + \delta K_t + I_t \left( 1 + \phi \frac{I_t}{K_t} \right) \right] + \\
+ \rho^* (1 - n) \left( \frac{P_i^*}{e_t} \right)^{\theta} \left[ C_t^* + \delta K_t^* + I_t^* \left( 1 + \phi \frac{I_t^*}{K_t^*} \right) \right] + (1 - n)G_t^*
\]

Since equity is a claim on the stock of capital, the supply of equities in each region equals the value of the capital stock in that region. Hence, the market clearing condition for the US equity market is given by:

\[
(1 - \alpha^E)n\alpha_t\beta_tV_t^P + \alpha^E nV_t^{E*} + \frac{(1 - \alpha^{*E})(1 - n) (1 - \alpha^*_t\beta_t^*V_t^{P*})}{e_t} = nq_tK_t
\]

This condition states that demand for US equities must equal supply. The first term on the left hand side gives the demand for US equities by US portfolio investors. There is a fraction \((1 - \alpha^E)n\) of these investors in the world population who invest a fraction \(\alpha\beta\) of their wealth in US equities.

The second term gives the demand by US entrepreneurs, which equal a fraction \(\alpha^E n\) of the world population and invest all their wealth in home equities. Finally, the third term gives the demand by foreign portfolio investors, which equal a fraction \((1 - \alpha^E)(1 - n)\) of the world population and invest a share \((1 - \alpha)\gamma\) of their wealth in US equities.

The market clearing condition for the ROW equity market is similarly given by:

\[
(1 - \alpha^{*E})n(1 - \alpha_t)\gamma_tV_t^{P*} + \alpha^{*E}(1 - n)V_t^{*E} + (1 - \alpha^{*E})(1 - n)\alpha_t^*\beta_t^*V_t^{*P*} = (1 - n)q_t^*K_t^*
\]

The market clearing conditions for the US and ROW bond markets are given by:

\[
(1 - \alpha^E)n\alpha_t(1 - \beta_t) V_t^P + \frac{(1 - \alpha^{*E})(1 - n) (1 - \alpha^*_t) (1 - \gamma^*_t) V_t^{P*}}{e_t} = nB_t P_t^B
\]

\[
(1 - \alpha^E)n (1 - \alpha_t) (1 - \gamma_t) V_t^P e_t + (1 - \alpha^{*E})(1 - n)\alpha_t^* (1 - \beta_t^*) V_t^{*P*} = (1 - n)B_t^* P_t^{*B*}
\]

The net debt position of the US is equal to the value of the stock of US assets, including equities and bonds, minus the value of financial wealth of US households:
\[ F_t = q_t K_t + P_t^B B_t - (1 - \alpha^E) V_t^P - \alpha^E V_t^E \]  \hfill (18)

The US trade deficit equals the difference between total expenditure and total output:

\[ TD_t = P_t \left[ G_t + \delta K_t + I_t \left( 1 + \frac{I_t}{K_t} \right) + G_t \right] - A (K_t)^\eta (1 - \alpha^E)^{1-\eta} \]  \hfill (19)

Using the market clearing conditions for the equity and bond markets, we can rewrite equation 18 as:

\[
F_t = \left[ r_t^E \gamma_{t-1}^* + r_t^B \left( 1 - \gamma_{t-1}^* \right) \right] F_{t-1} + TD_t - (1 - \alpha^E) n (1 - \alpha_{t-1}) V_{t-1}^P : \\
\left\{ \frac{r_t^E e_{t-1}}{e_t} \gamma_{t-1} + \frac{r_t^B e_{t-1}}{e_t} \left( 1 - \gamma_{t-1} \right) - r_t^E \gamma_{t-1}^* - r_t^B \left( 1 - \gamma_{t-1}^* \right) \right\}
\]

This equation describes the dynamics of US net debt. The two terms on the right hand side are standard: net debt next period equals the return the US pays on its existing stock of external net debt plus the trade deficit. The last term captures the effect on US net debt of changes in returns on US assets and liabilities, and embeds valuation effects stemming from exchange rate adjustments. A higher positive spread between the return on US assets and liabilities implies a lower accumulation of net external debt. An appreciation of the dollar reduces the dollar value of the returns the US receives on its foreign assets, contributing to a rise in net debt.

The steady state of the model is characterized by zero physical capital investment \((I = I^* = 0)\) and constant portfolio shares. In steady state the current account of the two regions must be balanced, and equation 20 reads:

\[
[r^E \gamma^* + r^B (1 - \gamma^*) - 1] F - (1 - \alpha^E) n (1 - \alpha) V^P [r^E \gamma + r^B (1 - \gamma) - r^E \gamma^* - r^B (1 - \gamma^*)] + TD = 0
\]

We linearize the model to the first order around the steady state, and solve it using a numerical linear solver. The next section describes the calibration of the parameter values in steady state.

### 3 An application: Growth of Sovereign Wealth Funds

#### 3.1 Calibration

We calibrate the model in the steady state to match asset returns and portfolio shares computed from the data, the ratio of US net external debt to GDP, and the ratio of US and ROW private consumption to GDP. Table 1 lists all parameter values used in the calibration.

The calculation of the portfolio shares is explained in detail in Appendix A. Consistent with the evidence in Gourinichas and Rey (2007), the share that US investors allocate to foreign equities
(56%) is substantially larger than the share that foreign investors allocate to US equities (31%). In this sense, the US can be characterized as a ‘venture capitalist’. The steady state annual gross rates of returns on different assets are obtained from Forbes (2010), who presents rates of return disaggregated by three assets classes: FDI, portfolio equities and bonds. We treat FDI and portfolio equities as a single asset class and aggregate the returns on FDI and portfolio equities in Forbes (2010) by weighting them by the proportion of these types of assets on US external assets and liabilities using the data in Lane and Milesi-Ferretti (2007).

With this parameterization for asset shares and rates of returns, the model generates an exorbitant privilege equal to 3.85% in steady state. This is close to the value 3.32% computed by Gourinchas and Rey (2007) for the period 1973-2004. Gourinchas an Rey decompose the privilege into two components: a return effect, due to the fact that, within each asset class, the US receives higher returns on its foreign assets than it pays on its liabilities; and a composition effect, due to the fact that the composition of US portfolio is skewed towards high-yielding equity assets, while its liabilities are composed mostly of low-yielding debt. Table 2 presents the values for this decomposition generated by our model in steady state and compares them with the values in Gourinchas and Rey. We obtain that most of the exorbitant privilege (2.65%) is due to the return effect, consistent with the findings in Gourinchas and Rey.

We normalize the steady state exchange rate and US total factor productivity to 1 for simplicity. In our benchmark calibration, we set the elasticity of substitution between US and ROW-produced goods, $\theta$, to 0.97, which is the median value of long-run price elasticities of aggregate trade flows for the US and other G7 countries estimated by several studies and reported in Hooper and Marquez (1995). The parameters capturing the degree of substitutability between assets, $b^3, b^5, b^3, b^7, b^7$, and $b^7$, are set to 1 following the central scenario in Blanchard, Giavazzi, and Sá (2005). As part of the sensitivity analysis, we check the robustness of our results to changes in $\theta$ and the $b^3$.

We set the shares of entrepreneurs in the US and ROW economies, $\alpha^E$ and $\alpha^{E,*}$, to equal 20% and calibrate the relative wealth of entrepreneurs and portfolio holders to equal 20% as well. We also impose that the steady state values of the ratio of US net debt to GDP, US consumption to GDP and ROW consumption to US GDP match the values obtained from the data. Finally, we set $n$ to equal the ratio of US population to world population, obtained from the US Census Bureau.

---

7 The exorbitant privilege is given by the difference between the return the US receives on its assets and the return it pays on its liabilities, namely $[\frac{[\bar{r}^{E,*} - \bar{r}^{B,*}]}{2} + \frac{(1 - \bar{r}^{E,*})}{2}] - [\frac{[\bar{r}^{E} - \bar{r}^{B}]}{2} + \frac{(1 - \bar{r}^{E})}{2}]$. The return effect arises from the difference between the return rates on assets and liabilities, evaluated at the average portfolio weights, i.e., $[\frac{[\bar{r}^{E,*} - \bar{r}^{E}]}{2} + \frac{(1 - \bar{r}^{E,*})}{2}] + \frac{(1 - \bar{r}^{E})}{2}$. The composition effect arises from the difference between the composition of US portfolio and liabilities, evaluated at the average return, i.e., $[\bar{r}^{E,*} - \bar{r}^{E}] - (1 - \bar{r}^{E,*})$.

8 The share of entrepreneurs is in the range of values commonly used by the literature on financial frictions and is close to values observed in the data (0.1-0.2). We tried alternative values for these shares and obtained very similar results.
3.2 Shocks to Portfolio Preferences

To study the impact that growth in SWFs is likely to have on asset prices and returns, the level of US net debt and the dollar exchange rate, we need to make an assumption about the potential size of SWFs. A natural assumption is that the amount of ‘excess reserves’ now held by central banks will be managed by SWFs in the future. ‘Excess reserves’ are defined as being in excess of what would be justified for liquidity purposes. A rule of thumb frequently used to estimate the size of ‘excess reserves’ is the Greenspan-Guidotti rule, according to which reserves should cover short term external debt. Using this rule, we estimate that the amount of ‘excess reserves’ held by central banks in emerging markets is around 3 trillion dollars, about the same as the current size of SWFs. We use our model to study what will happen if these 3 trillion dollars of ‘excess reserves’ are managed by SWFs rather than central banks.

There are two margins along which SWFs may diversify their portfolios relative to central banks: currency diversification (away from dollars towards other currencies) and cross-asset diversification (away from bonds towards equities). We consider two paths: one in which the currency composition changes and the asset composition remains constant (path 1), and one in which the asset composition changes and the currency composition remains constant (path 2).

To compute by how much the portfolio shares would change under each of these paths, we need information on the currency and asset composition of the portfolios of central banks and SWFs. For the currency composition, we use data from the IMF COFER dataset. For the asset composition, we use the data reported in IMF (2008a). This information is presented in Table 3. SWFs allocate a much smaller percentage of their wealth to dollar assets than central banks (38% compared to 60%) and allocate most of their wealth to equities.

3.2.1 Path 1. Shock to Currency Composition

Given the currency composition of the portfolios of SWFs and central banks reported in Table 3, if 3 trillion dollars of ‘excess reserves’ held by central banks in emerging markets start being managed by SWFs, the amount of wealth that foreign investors invest in dollars will be reduced by $(0.6 - 0.38) \cdot 3 = 0.66$ trillion dollars. This corresponds to $\frac{0.66}{14.2} \cdot 100 = 4.65\%$ of US GDP.

In terms of the parameters of our model, this shock can be seen as a reduction in the share of wealth that ROW portfolio investors invest in the US market, i.e., a reduction in $(1 - \alpha^*)$. The change in $s^{\alpha^*}$ that generates a reallocation of wealth from dollars to other currencies equal to 4.65% of US GDP is given by:

$$\Delta s^{\alpha^*} = \frac{4.65}{100} \cdot \frac{US_{GDP}}{(1 - n) \cdot (1 - \alpha^{E*}) \cdot VP^* \cdot E} = \frac{0.66}{(1 - n) \cdot (1 - \alpha^{E*}) \cdot VP^* \cdot E} \quad (21)$$

The denominator in this expression is total wealth of ROW portfolio investors: there is a proportion $(1 - n) \cdot (1 - \alpha^{E*})$ of these investors in the world economy, each with wealth equal to $VP^*$. This expression gives us the size of the shock for path 1.
3.2.2 Path 2. Shock to Asset Composition

A shift of 3 trillion dollars of ‘excess reserves’ from central banks to SWFs reduces the amount that foreign investors invest in US bonds and increases the amount that they invest in US equities by $(1 - 0.29) \times 3 \times 0.6 = 1.278$ trillion dollars. This corresponds to $\frac{1.278}{14.2} \times 100 = 9\%$ of US GDP.

In terms of the parameters of our model, this corresponds to an increase in $\gamma^*$, the share of wealth that ROW portfolio investors invest in equities in the US market. The change in $s^*$ that delivers an increase in investment in equities equal to 9\% of US GDP is given by:

$$\Delta s^* = \frac{9}{100} \cdot \frac{US\_GDP}{(1 - n) \cdot (1 - \alpha^{E^*}) \cdot VP^E \cdot E} = \frac{1.278}{(1 - n) \cdot (1 - \alpha^{E^*}) \cdot VP^E \cdot E} \quad (22)$$

This gives us the size of the shock for path 2.

3.3 Baseline Results

For the baseline results, we calibrate the steady state using the numbers in Table 1. In this section, we show impulse responses for all the key variables in the model: US and ROW asset prices and returns, investment, capital stock, GDP, wages, consumption, the exchange rate, US trade deficit and net debt, and the exorbitant privilege. Looking at the full set of impulse responses allows us to understand the mechanisms through which the shocks operate. For the robustness checks we focus only on the responses of the exchange rate, US trade deficit and net debt and the exorbitant privilege.

3.3.1 Path 1. Shock to Currency Composition

Given the steady state parameters in Table 1 and our assumption about the size of the shock to portfolio preferences, equation 21 implies an increase in the share that foreign investors invest in ROW assets ($\alpha^*$) from 74\% to 74.56\%. This is a small increase and we should not expect it to have a large impact. We could assume a larger shock, for example, if we believe that currency diversification by SWFs will lead to herding behaviour, inducing other investors to also move away from dollar assets.\(^9\)

We expect that, as ROW investors shift demand from dollar assets to ROW assets, the price of US assets should fall and the price of ROW assets should rise. Chart 1 plots the evolution of the prices of US and ROW assets. There is a reduction in the price of US equities and bonds and an increase in the price of ROW equities and bonds. Since foreign investors are less willing to invest in dollar assets, we would expect the return on these assets to rise so that the US can continue attracting foreign investment and be able to maintain its current account balance. Chart 2 (a) and (b) shows the response of the return on US equities and bonds. The return on equities includes capital gains or losses arising from movements in equity prices. Because the price of US equities falls in the first period after the shock, the return on US equities also falls, but it rises after

\(^9\)See Corsetti et al (2004) for a model in which a large trader may influence the actions of small traders.
that. The return on US bonds rises in response to the shock. The returns on ROW assets decrease (except in the first period after the shock), as illustrated in Chart 2 (c) and (d). Because portfolio shares respond to movements in expected returns, changes in asset returns generate further changes in the portfolio shares over time. For example, as the expected return on US assets increases and the expected return on ROW assets falls, US investors invest a larger share of their wealth in US assets, i.e., $\alpha$ rises.

Chart 3 (a) to (c) illustrates the response of US investment, capital stock and GDP. Investment is driven by the marginal value of capital, $q_t$, which coincides with the price of US equities. Therefore, the evolution of investment in Chart 9 mirrors the evolution of the price of US equities in Chart 1. Because the price of US equities fall, investment also falls, leading to a reduction in the US capital stock and GDP. The opposite effects happen in ROW, as shown in Chart 3 (d) to (f).

Chart 4 shows the evolution of wages and consumption in the US and ROW. The reduction in the capital stock in the US reduces the marginal product of labour, which is equal to the wage. The wealth of US portfolio investors falls, both because of the fall in the wage and because of the fall in the return on their portfolio, $R^P_t$, given by equation 17. Since there is home bias in portfolio investment ($\alpha > 0.5$) and the return on US assets falls, $R^P_t$ falls. The decrease in the wealth of portfolio investors leads to a reduction in their consumption. Turning to US entrepreneurs, they invest all their wealth in home equities. Therefore, the dynamics of their wealth is entirely determined by the return on US equities, given in Chart 2 (a). This return falls in the period after the shock due to the capital losses generated by the fall in the price of US equities, but rises afterwards in order to attract foreign investment and maintain the current account balance. Thus, consumption of US entrepreneurs falls in the period following the shock and rises afterwards. Chart 4 (d) shows the evolution of aggregate consumption by US households. This is dominated by consumption of portfolio holders, since in our calibration they represent 80% of the US population. For ROW, we obtain the opposite effects on wages and consumption.

The exchange rate is defined as the relative price of US and foreign-produced goods. Its evolution, shown in Chart 5, is determined by the relative demand for US and foreign-produced goods, including both consumption and investment demands. Charts 3 and 4 show that investment and consumption fall in the US and increase in ROW as a result of the shock. Because there is home bias in consumption and investment ($\rho > 0.5$), this implies a reduction in the world demand for US-produced goods and an increase in the world demand for foreign-produced goods. Therefore, the exchange rate depreciates following the shock. Given our calibration, we obtain an immediate depreciation of 0.58%. This is a small effect but it is not surprising given the small size of the shock that we are assuming. Following this initial depreciation, the exchange rate appreciates again, following the increase in investment in the US and the decrease in investment in ROW. These changes in investment are driven by the changes in asset prices. As the return in US equities increases to attract foreign investment back into the US and maintain the current account balance, the demand for US equities increases, leading to an increase in their price, a recovery in investment and an appreciation of the dollar.
Chart 6 shows the evolution of the trade deficit, which is obtained as the difference between total expenditure and total output (equation 19). The dynamics of the trade deficit is dominated by the evolution of consumption in the US and ROW (Chart 4 (d) and (h)). The trade deficit falls significantly following the shock (by 0.24 percentage points of GDP) and continues falling afterwards, as US consumers reduce their consumption of both home and foreign-produced goods and ROW consumers increase their consumption of both varieties of goods.

The evolution of the US ‘exorbitant privilege’ is shown in Chart 7. The ‘exorbitant privilege’ rises in the period following the shock because capital gains increase the return the US receives on its investment in ROW equities and capital losses reduce the return the US pays on US equities. After the first period the privilege is reduced, as the return the US receives on its foreign assets falls and the return it pays on its foreign liabilities increases. Table 4 shows the long run decomposition of the privilege. The spread between the return on US external assets and liabilities falls from 3.85% to 3.7%. This is fully explained by a reduction in the return effect.

The evolution of US net debt is given in Chart 8 and can be explained by changes in the different terms in equation 20. The initial depreciation and the reduction in the trade deficit leads to a fall in net debt equal to 1.9 percentage points of GDP. The rapid increase in US net debt in subsequent periods can be explained by different factors. First, there is an increase in the return the US pays on its existing stock of debt, because the returns on US equities and bonds increase. Second, the reduction in the spread between the return on US assets and liabilities (the ‘exorbitant privilege’) implies a higher accumulation of US net debt over time. Finally, the expected appreciation of the dollar following the initial depreciation reduces the dollar value of the returns the US receives on its foreign assets, contributing to a rise in net debt.

3.3.2 Path 2. Shock to Asset Composition

For path 2, we introduce a shock to the asset composition of the portfolios of foreign investors, assuming that they keep the same share of investment in dollar assets, but diversify away from bonds into equities. Given the parameter values we chose, equation 22 gives an increase in the share foreign investors allocate to equities in the US market, $\gamma^*$, from 31% to 32.11%.

Charts 9 and 10 plot the evolution of the prices and returns of US and ROW assets. Substitution away from US bonds into US equities by ROW investors leads to an increase in the price of US equities and a reduction in the price of US bonds. The return on US equities increases in the first period after the shock reflecting capital gains caused by the increase in the price of equities and falls in subsequent periods. The return on US bonds rises in order to attract investors and clear the market for US bonds. The changes in relative asset returns lead to changes in portfolio shares, generating small movements in the prices and returns of ROW assets. In particular, the decrease on the expected relative return on US equities versus US bonds reduces the return that US investors receive when they invest in the US (given that their portfolios at home consist mostly of equities), leading to a fall in the share of wealth that they invest in the US, $\alpha$. At the same time, foreign
investors now receive a higher return on their investment in the US, because they moved away from bonds into higher yielding equities. This induces them to invest a higher fraction of their wealth in US assets, i.e., $\alpha^*$ decreases. In our calibration, the first effect dominates and demand for ROW assets increases, leading to an increase in their prices.

Chart 11 plots the response of investment, the capital stock and GDP. As before, the evolution of investment mirrors the evolution of the price of US equities, given in Chart 9 (a). Investment rises in the period after the shock, since the increase in demand for US equities drives up their price. This leads to an increase in the capital stock and GDP. In subsequent periods, investors rebalance their portfolios again towards bonds, as a response to the increase in the relative return on US bonds versus US equities. For this reason, the price of US equities falls and investment falls, leading to a reduction in the capital stock and GDP. The increase in investment in ROW can be attributed to the increase in the price of ROW equities documented in Chart 9 (c).

The increase in the capital stock in the US raises the marginal product of labour, which is equal to the wage. In spite of the increase in wages, consumption of US portfolio holders falls because the reduction in the price of US bonds requires an increase in lump sum taxes in order for the government budget constraint (equation 13) to be satisfied. This reduces the wealth of US portfolio investors. Consumption of US entrepreneurs is driven by the return on US equities, plotted in Chart 10 (a). Consumption rises in the first period after the shock and falls in subsequent periods. Aggregate consumption in the US mirrors the evolution of consumption of US portfolio holders, since these represent 80% of the US population in our calibration.

The evolution of the dollar exchange rate, given in Chart 13, is determined by the relative demands of US and ROW-produced goods. We have seen that the shock increases investment both in the US and ROW in the period after the shock and reduces it in subsequent periods. For aggregate consumption, we have seen that it decreases in both regions immediately after the shock. In subsequent periods, aggregate consumption falls in the US and rises in ROW. Because there is home bias in consumption and investment, this implies a depreciation of the dollar and a reduction in the US trade deficit (Chart 14).

Chart 15 shows the evolution of the US ‘exorbitant privilege’. Because foreign investors moved away from US bonds into higher yielding US equities, the US must pay a higher return on its liabilities and its ‘exorbitant privilege’ is reduced. This effect diminishes over time as investors rebalance their portfolios in response to endogenous changes in asset returns. Table 5 shows the quantification of the short run and long run effects on the US ‘exorbitant privilege’.

The evolution of US net debt, given in Chart 16, can be interpreted by changes in the different elements of equation 20. The depreciation of the dollar reduces net debt through two channels: first, it reduces the trade deficit; second, it increases the dollar value of the return the US receives on its foreign assets. But there is a counterbalancing effect coming from the reduction in the ‘exorbitant privilege’. The reduction in the spread between the return the US receives on its foreign assets and the return it pays on its foreign liabilities leads to a higher accumulation of net external debt. In our calibration, this effect dominates and US net debt increases over time.
3.4 Robustness checks

3.4.1 Degree of substitutability between assets

To test the robustness of our results to different assumptions about the degree of substitutability between assets, we simulate path 1 under different values of the parameter $b$. In particular, we follow Blanchard, Giavazzi, and Sá (2005) and set $b = 1$ and $b = 0.1$. To show the effect of a very limited degree of substitutability we also use $b = 0.0001$.

With a lower degree of substitutability between assets, asset demands are less responsive to changes in relative returns. Therefore, asset prices and returns have to move more in order for asset markets to clear. The price of US equities falls by more and the price of ROW equities rises by more when there is a lower degree of substitutability between assets. Because investment is driven by the price of equities, a low degree of substitutability increases the divergence in the response of US and ROW investment. Bond price movements and the consequent fiscal effects are amplified as well, which rises the gap between US and ROW consumption. Therefore, the lower the degree of substitutability between assets, the bigger the drop in the relative demand for the US and the ROW-produced goods, the bigger the fall in their relative price, and the larger the depreciation of the exchange rate, as Chart 17 illustrates.

A larger depreciation of the dollar makes the US trade deficit fall by more, while more volatile asset prices amplify movements in the exorbitant privilege, as depicted in Charts 18 and 19. The higher dollar depreciation, pronounced reduction in the US trade deficit, and the higher initial increase in the privilege associated with low asset substitutability, amplify the initial reduction in US net debt (Chart 20). Over time, however, US net debt rises by more when the degree of substitutability between assets is low because the larger reduction in the ‘exorbitant privilege’ facilitates a progressive transfer of financial wealth from the US to ROW.

3.4.2 Elasticity of substitution between goods

We have also looked at the sensitivity of our results to different values of the elasticity of substitution between US and ROW-produced goods ($\theta$). We compare the results obtained for path 1 with $\theta = 0.97$ (the benchmark) and $\theta = 0.6$ (following Kollmann (2006)).

The lower the elasticity of substitution between US and ROW-produced goods, the larger the exchange rate depreciation required to absorb the excess demand for ROW-produced goods that opens up following a fall in ROW demand for US assets (Chart 21). Since with a low elasticity of substitution the relative demand for US and ROW-produced goods is less reactive to changes in their relative price, the depreciation of the dollar generates a smaller reduction in the US trade deficit when $\theta = 0.6$ (Chart 22). For this reason, we would expect a smaller reduction in US net debt when the elasticity of substitutions between goods is low. However, there is an additional effect which operates through exchange rate valuation effects. The depreciation of the exchange rate increases the dollar value of the return the US receives on its external assets. This effect is stronger when $\theta$ is low because in that case the depreciation is larger. This effect works towards
reducing US net debt (Chart 23).

4 Conclusions

Our analysis highlights the channels through which changes in the portfolio allocation of foreign investors may impact on asset prices and returns, consumption, investment, the exchange rate and net debt. The framework we use allows for endogenous determination of asset prices and returns and portfolio rebalancing in response to changes in asset returns. In addition, the dynamics of net external debt incorporates valuation effects arising from movements in the exchange rate.

To illustrate the mechanics of the model, we look at the impact of an expansion in Sovereign Wealth Funds under two different scenarios: in one scenario, foreign investors move away from US assets but keep the same share of investment in equities and bonds; in another scenario, they do not change the currency composition of their portfolios, but move away from US bonds into US equities. In the first scenario, the dollar depreciates in the period immediately after the shock, leading to a reduction in the US trade deficit and net debt. In subsequent periods, the return on US assets must increase to clear asset markets. This generates a rebalancing of the portfolios of foreign investors towards holding more dollar assets, which leads to an appreciation of the dollar. The ‘exorbitant privilege’ in the US, i.e., the difference between the return it receives on its foreign assets and the return it pays on its foreign liabilities, decreases, and US net debt increases over time. In the second scenario, the dollar depreciates and the US trade deficit decreases. However, US net debt increases over time due to a reduction in the ‘exorbitant privilege’.

The model is general enough to be applicable to other situations. For example, it could be used to study the impact of the sudden stop episodes experienced in Iceland and Greece during the recent global financial crisis.
References


Appendix. Construction of Portfolio Shares

Share of US wealth invested in the US market ($\alpha$)

In the first quarter of 2008, the value of US financial wealth was 44.1 US trillion, from Table L100 in Federal Reserve (2008). The value of US-owned assets abroad was 17.6 US trillion, according to BEA (2008). Combining these two numbers, the share of US wealth invested in the US market, $\alpha$, is given by:

$$\alpha = 1 - \frac{17.6}{44.1} = 0.60$$

Share of ROW wealth invested in the ROW market ($\alpha^*$)

From IMF (2008b), total world financial wealth in 2006 (equal to the sum of stock market capitalization and the value of debt securities) was equal to 120 US trillion. Subtracting the value of US financial wealth, we obtain a value of ROW financial wealth equal to 75.9 US trillion. According to BEA (2008), the value of foreign holdings of US assets in 2007 was equal to 20.1 US trillion. Therefore, the share of ROW wealth invested in ROW assets ($\alpha^*$) is equal to:

$$\alpha^* = 1 - \frac{20.1}{75.9} = 0.74$$

Share of US wealth in ROW market allocated to equities ($\gamma$)

Using the data from Lane and Milesi-Ferretti (2007), we compute the share of US foreign assets allocated to equities (the remainder is allocated to bonds). The data distinguishes between FDI and portfolio equities. Because the only difference between the two is the degree of ownership, we consider them as a single asset class. This gives a value $\gamma = 0.56$.

Share of ROW wealth in the US market allocated to equities ($\gamma^*$)

In a similar way, we can use the data from Lane and Milesi-Ferretti to compute the share of US liabilities allocated to equities, which corresponds to the share of ROW assets in the US allocated to equities. With this calculation we obtain $\gamma^* = 0.31$.

Share of US wealth in US market allocated to equities ($\beta$)

Using data from Table L100 in Federal Reserve (2008), we can construct the overall shares of wealth that US investors allocate to equities and bonds, considering both the domestic and the foreign markets. This gives us:

\begin{align*}
\text{share}^{US.E} &= 0.73 \\
\text{share}^{US.B} &= 0.27
\end{align*}

Combining these shares with $\alpha$ and $\gamma$, it is possible to compute the share of US wealth allocated to equities in the US market, $\beta$:

\begin{align*}
\text{share}^{US.E} &= \beta \cdot \alpha + \gamma \cdot (1 - \alpha) \\
0.73 &= \beta \cdot 0.60 + 0.56 \cdot 0.40 \\
\beta &= 0.84
\end{align*}

Share of ROW wealth in ROW market allocated to equities ($\beta^*$)

Data for $\beta^*$ is calculated in a similar way to $\beta$. First, we need to obtain the overall shares of
wealth that foreign investors allocate to equities and bonds, considering both the US and ROW markets. For the Euro Area, we can obtain these shares from Table 3.1 in ECB (2008):

\[ \text{share}^{EA,E} = 0.72 \]
\[ \text{share}^{EA,B} = 0.28 \]

For Japan, we can use data from Bank of Japan (2008):

\[ \text{share}^{Japan,E} = 0.71 \]
\[ \text{share}^{Japan,B} = 0.29 \]

The shares for the Euro Area and Japan are very similar. We take the Euro Area shares as representative of the ROW:

\[ \alpha^{ROW,E} = 0.72 \]
\[ \alpha^{ROW,B} = 0.28 \]

Combining these shares with \( \alpha^* \) and \( \gamma^* \), we compute the share of ROW wealth allocated to equities in the ROW market, \( \beta^* \):

\[
\text{share}^{ROW,E} = \gamma^* (1 - \alpha^*) + \beta^* \alpha^* = 0.72 = 0.31 \times 0.26 + \beta^* \times 0.74
\]
\[ \beta^* = 0.86 \]
Path 1. Shock to Currency Composition

Chart 1 (a) Price of US Equities

Chart 1 (b) Price of US Bonds

Chart 1 (c) Price of ROW Equities

Chart 1 (d) Price of ROW Bonds

Chart 2 (a) Return on US Equities

Chart 2 (b) Return on US Bonds

Chart 2 (c) Return on ROW Equities

Chart 2 (d) Return on ROW Bonds
Path 2. Shock to Asset Composition
Chart 12 (a) US Wages
Chart 12 (b) Consumption US Portfolio Holders
Chart 12 (c) Consumption US Entrepreneurs
Chart 12 (d) US Aggregate Consumption
Chart 12 (e) ROW Wages
Chart 12 (f) Consumption ROW Portfolio Holders
Chart 12 (g) Consumption ROW Entrepreneurs
Chart 12 (h) ROW Aggregate Consumption

Chart 13 Dollar Exchange Rate
Robustness checks: Degree of substitutability between assets ($b$)
Robustness checks: Elasticity of substitution between goods ($\theta$)
Table 1. Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portfolio shares</strong></td>
<td></td>
<td>Authors’ calculations in Appendix A</td>
</tr>
<tr>
<td>Share of US wealth invested in the US market</td>
<td>$\alpha$</td>
<td>0.6</td>
</tr>
<tr>
<td>Share of ROW wealth invested in the ROW market</td>
<td>$\alpha^*$</td>
<td>0.74</td>
</tr>
<tr>
<td>Share of US wealth in the US market allocated to equities</td>
<td>$\beta$</td>
<td>0.84</td>
</tr>
<tr>
<td>Share of ROW wealth in ROW market allocated to equities</td>
<td>$\beta^*$</td>
<td>0.86</td>
</tr>
<tr>
<td>Share of US wealth in ROW market allocated to equities</td>
<td>$\gamma$</td>
<td>0.56</td>
</tr>
<tr>
<td>Share of ROW wealth in the US market allocated to equities</td>
<td>$\gamma^*$</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>Rates of return</strong></td>
<td></td>
<td>Forbes (2008)</td>
</tr>
<tr>
<td>Rate of return on US equities</td>
<td>$r^E$</td>
<td>1.067</td>
</tr>
<tr>
<td>Rate of return on ROW equities</td>
<td>$r^{E*}$</td>
<td>1.124</td>
</tr>
<tr>
<td>Rate of return on US bonds</td>
<td>$r^B$</td>
<td>1.046</td>
</tr>
<tr>
<td>Rate of return on ROW bonds</td>
<td>$r^{B*}$</td>
<td>1.049</td>
</tr>
<tr>
<td><strong>Other parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US home bias in goods</td>
<td>$\rho$</td>
<td>0.7</td>
</tr>
<tr>
<td>US capital share</td>
<td>$\eta$</td>
<td>0.39</td>
</tr>
<tr>
<td>ROW capital share</td>
<td>$\eta^*$</td>
<td>0.39</td>
</tr>
<tr>
<td>Elasticity of substitution between goods</td>
<td>$\theta$</td>
<td>0.97</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.05</td>
</tr>
<tr>
<td>Installation cost of capital</td>
<td>$\phi$</td>
<td>6</td>
</tr>
<tr>
<td>Degree of substitutability between assets</td>
<td>$b$</td>
<td>1</td>
</tr>
<tr>
<td>US productivity</td>
<td>$A$</td>
<td>1</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>$E$</td>
<td>1</td>
</tr>
<tr>
<td>Parameter</td>
<td>Symbol</td>
<td>Value</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>US net debt to GDP ratio</td>
<td>F/Y</td>
<td>0.17</td>
</tr>
<tr>
<td>US consumption to GDP ratio</td>
<td>C/Y</td>
<td>0.9</td>
</tr>
<tr>
<td>ROW consumption to US GDP ratio</td>
<td>C^*/Y</td>
<td>2.08</td>
</tr>
<tr>
<td>Share of entrepreneurs in US economy</td>
<td>α^E</td>
<td>0.2</td>
</tr>
<tr>
<td>Share of entrepreneurs in ROW economy</td>
<td>α^E*</td>
<td>0.2</td>
</tr>
<tr>
<td>Relative wealth of entrepreneurs in US economy</td>
<td>V^E/V^P</td>
<td>0.2</td>
</tr>
<tr>
<td>Relative wealth of entrepreneurs in ROW economy</td>
<td>V^E*/V^P*</td>
<td>0.2</td>
</tr>
<tr>
<td>Relative size of US population</td>
<td>n</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Table 2.** Decomposition of the US exorbitant privilege

<table>
<thead>
<tr>
<th></th>
<th>Our calibration</th>
<th>Gourinchas and Rey (2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3.85%</td>
<td>3.32%</td>
</tr>
<tr>
<td>Return effect</td>
<td>2.65%</td>
<td>2.45%</td>
</tr>
<tr>
<td>Composition effect</td>
<td>1.2%</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.

**Table 3.** Currency and asset composition of the portfolios of central banks and SWFs

<table>
<thead>
<tr>
<th></th>
<th>Central Banks</th>
<th>SWFs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Currency composition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USD</td>
<td>60%</td>
<td>38%</td>
</tr>
<tr>
<td>Other</td>
<td>40%</td>
<td>62%</td>
</tr>
<tr>
<td><strong>Asset composition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>0%</td>
<td>71%</td>
</tr>
<tr>
<td>Bonds</td>
<td>100%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Sources: Data on the currency composition of central banks’ reserves is from the IMF COFER dataset (numbers for developing countries). Data on the currency and asset composition of SWFs’ portfolios is from IMF (2008a).
### Table 4. Path 1 (Currency diversification) – Effect on the US exorbitant privilege

<table>
<thead>
<tr>
<th>Exorbitant privilege</th>
<th>Initial calibration</th>
<th>Initial impact (at T=1)</th>
<th>Long run impact (at T=100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3.85%</td>
<td>4.05%</td>
<td>3.7%</td>
</tr>
<tr>
<td>Return effect</td>
<td>2.65%</td>
<td>2.82%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Composition effect</td>
<td>1.2%</td>
<td>1.23%</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.

### Table 5. Path 2 (Asset diversification) – Effect on the US exorbitant privilege

<table>
<thead>
<tr>
<th>Exorbitant privilege</th>
<th>Initial calibration</th>
<th>Initial impact (at T=1)</th>
<th>Long run impact (at T=100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3.85%</td>
<td>3.72%</td>
<td>3.8%</td>
</tr>
<tr>
<td>Return effect</td>
<td>2.65%</td>
<td>2.5%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Composition effect</td>
<td>1.2%</td>
<td>1.22%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.