Abstract. Extending the basic model of two-stage cumulative innovation with asymmetric information to include ‘experimentation’ by second-stage firms, we find that the costs of a strong (versus weak) intellectual property (IP) regime may be substantially increased. In addition, these costs increase as experimentation becomes cheaper and as the differential between high and low value second-stage innovations grows, with the result that a weak IP regime is more likely to be optimal. Thus, technological change which reduces the cost of encountering and trialling new ‘ideas’ implies a reduction in the socially optimal level of IP rights such as patent and copyright.

Keywords: Cumulative Innovation, Hold-Up, Experimentation, Intellectual Property

JEL codes: K3, L5, O3
1. Introduction

[The] 90-minute documentary [Wanderlust] ... was also a window into the frustrations of making a clip-intensive film dependent on copyright clearance, which has become hugely expensive in the past decade. Initial quotations for the necessary sequences came to more than $450,000, which would have raised by half the cost of the IFC film. ... “Paramount wanted $20,000 for 119 seconds of Paper Moon”, Ms. Sams said. “The studios are so afraid of exploitation that they set boundaries no one will cross. Even after the prices were cut, we were $150,000 in the hole.”¹

Cumulative innovation and creativity, whereby new work build upon old, is a pervasive phenomenon. However, it was not until recently that it received significant attention in the literature. The early papers of Green and Scotchmer (1995); Chang (1995) introduced a two-stage innovation model in which the second innovation is enabled by, or builds upon, the first. The papers primarily concerns itself with how rents are divided between innovators at the two stages, in particular with the extent to which the first innovator is (under-)compensated for her contribution (the option value) to the second innovation. They investigate how different policy levers related to intellectual property rights, in particular breadth², could be used to affect the bargaining (or its absence) between different innovators and hence the resulting payoffs.

A central feature of these models, as well as subsequent work that extended it (such as Scotchmer (1996)), was an assumption that knowledge of costs and returns, whether deterministic or stochastic, was shared equally by innovators at different stages (i.e. was common knowledge). With common knowledge all mutually beneficial transactions are concluded, using ex ante licenses where necessary to avoid the possibility of hold-up of second-stage innovators.

This assumption, however, is problematic. If all innovators share the same information why have different innovators at different stages and why be concerned with licenses and

²A monopoly right (intellectual property right) such as a patent or a copyright confers the right to exclude not simply direct copies but also products that are sufficiently similar. The term lagging/leading breath are often used to denote the space of inferior/superior (respectively) products that are excluded by the patent/copyright (i.e. taken as infringing the monopoly).
bargaining if a single innovator could just as easily carry out all research? The simple answer is that this assumption of common knowledge does not reflect reality: many different firms engage in innovation precisely because they have specialized skills and knowledge that make it effective for them rather than another firm to engage in a given area. Thus, in this paper we investigate cumulative innovation under asymmetric information, for example, where a first-stage innovator only has a probabilistic prior over the second-stage innovator’s cost/values but the second-stage innovator knows them precisely.

Our paper takes as a starting point a ‘basic’ model very similar to that presented by Bessen (2004). Second-stage firms are of two types (high and low value) with the type unobserved by first-stage innovators. With (strong) IP first-stage firms may require second-stage innovators to pay a royalty while with (weak) IP second-stage firms may produce without having to license from first-stage firms. As first-stage firms do not know the type of given second-stage innovator with (strong) IP there may be ‘licensing failure’ (that is the royalty may be set above the level that a second-stage firm is willing to pay). Thus, there is a trade-off: with IP more first-stage innovation takes place due to the extra royalty income received by first-stage firms but some second-stage innovation may be lost as a result of ‘licensing failure’ due to high royalty rates.

Such a trade-off is already familiar in the literature and our main reason for presenting it is to provide a benchmark and basis for the more complex ‘experimentation’ model presented in the second section. ‘Experimentation’ is used here broadly to cover any kind of trialling and experimentation activity that is likely to take place before a license can be obtained.
agreed. The logic here is that there are transactional costs and complexities involved in negotiating and executing a license that mean that it only takes place once some degree of ‘experimentation’ has taken place. Furthermore, experimentation benefits an innovator by increasing the probability of having a high value innovation but it is costly.

For example, a first-stage innovation might be a ‘tool’ which the second-stage innovator wishes to use in some manner but is unsure what the most beneficial use for this tool is or how valuable usage of the tools will be (this would be particularly relevant to Biotechnology where the issue of research ‘tool’ licensing is particularly prominent). ‘Experimentation’ in this case would correspond to trialling and investigation by the second-stage innovator in order to determine the best way to use the tool and/or how valuable such usage is. The more ‘experimentation’ a second-stage innovator does, the more likely the resulting use is a high value one.

Alternatively, one could imagine a first-stage innovation is a basic product that the second-stage innovator wishes to extend. Here again, the second-stage innovator while knowing that she wishes to extend a particular first-stage innovation may not necessarily have a clear idea as to how best to do this (or whether the particular idea she has is actually feasible). ‘Experimentation’ would then indicate the trialling and investigation necessary to reduce these uncertainties and improve the likelihood the result is a good one.

Real-world examples of such situations abound. Consider, for example, software where it is common for developers to expend significant time trying out and experimenting with an existing product or ‘library’ in order to determine whether they can extend it or integrate in the way that would be useful or fits with their existing needs. Significant effort may have to be expended before a formal license is concluded (for example, the license may depend on the exact intended usage), and the more time a user or developer spends experimenting the more likely the resulting application is a good one. The same logic applies to other information products such as databases, as well as to research tools in areas like the life sciences. Here a (potential) user may need to spend significant time exploring the content and features of the product, as well as trialling different ways to use
and apply it prior to agreeing a formal license, with the likelihood of a good application increasing in the time spent in this way.\footnote{Consider here the case of geographic data in the UK, where the primary source for such data is the Ordnance Survey. The Ordnance Survey offer a range of possible reuse licenses, termed Specific Use Contracts, each targeted at a particular area. In all there are 14 options with, for example, one for Navigation Products, one for Location Based Services, one for User Derived Datasets etc. The terms and conditions vary between the contracts and it is therefore important for a reuser to know which contract to choose. However at the early stages of the development of a new product or application it may costly (or simply impossible) to know exactly which contract should be used and it will only be later when much of this uncertainty has been resolved that it will be worth incurring the cost and effort of negotiation to agree with the Ordnance Survey which SUC to use.}

On a different topic, consider the example of a documentary film-maker wishing to make a film on a particular topic and requiring clips from a particular source.\footnote{More generally all composers whether classical or modern use previous musical, ideas, motifs, and melodies as parts of new works. See e.g. Malcolm Gladwell, The New Yorker, 2004-11-22, Something Borrowed: Should a charge of plagiarism ruin your life?, also http://www.low-life.fsnet.co.uk/copyright/part3.htm#copyrightinfringement for information about sampling and experimentation in hip-hop music. The film-maker will need to have expended significant time experimenting with the source footage and weaving it into their work before arriving at the point of seeking a formal license and the more time spent the better the likely end result. Similarly in music, particularly modern music, re-use, and the associated experimentation and trialling, is ubiquitous. In particular, in hip-hop, the act of ‘sampling’, whereby a small section of a previous work is directly copied and then repeated or reworked in some manner, is the very basis of the genre and, once again, the more time spent experimenting with a particular source track the better the resulting work.}

Returning to the model, the crucial point is that experimentation takes place prior to any kind of royalty negotiation. Hence there can be hold-up: the hold-up of the experimentation effort. As a result the presence of IP rights that require second-stage innovators to license may now have another cost in addition to that from traditional ‘licensing failure’: fearing high royalty rates second-stage firms will reduce the level of experimentation they do and thereby reduce the average quality of second-stage innovations (and this reduction may be large due to the interaction of ‘licensing failure’ and experimentation). Because this effect operates across all second-stage innovators its consequences for welfare may be substantially greater than the traditional ‘licensing failure’ problem (which only affects low value second-stage innovators).
Turning to the comparative statics, we find that, in general, the lower the experimentation costs or the larger the differential between high and low value second-stage innovations, the more likely it is that a regime without intellectual property rights will be preferable. Thus, in the context of this model, technological change which reduces the cost of encountering and trialling new ‘ideas’ should imply a reduction in the socially optimal level of intellectual property rights such as patents and copyright.

This approach therefore adds another dimension to the question of how profit is divided between innovators at different stages. Seen in this light, it also has direct analogies with existing results related to the question of whether second-stage innovations should be infringing (I) or non-infringing (NI). For example, Denicolo (2000), who extends Green and Scotchmer’s model with patent races at each stage, finds that in some circumstances it will be better to make second-stage innovations non-infringing (in this model one trades off faster second-stage innovation with non-infringement against faster first-stage innovation when there is infringement).

It also has a close connection to the recent work of Bessen and Maskin (2006). Similar to this paper they investigate the welfare impact of ‘licensing failure’ due to asymmetric information in a model of cumulative innovation. Similar to us they show that, with cumulative innovation, in contrast to what occurs in a ‘one-shot’ model, IP may, in some circumstances, reduce rather than increase innovation (and social welfare). However their focus is rather different from ours (complementarities in research rather than experimentation) and their results arise for different reasons. Specifically, in their model there are multiple stages with (the same) two firms at each stage. Each may choose to participate or not in researching the current innovation and the next innovation stage is reached if, and only if, research at the current stage is successful, with success an increasing function of the number of participating firms. As a result their is an ‘externality’ from participation in a given stage: though the value of success at the current stage accrues only to the winning firm by enabling subsequent stages (some of which may be won by the other firm) success also increases the other firms expected revenue. As a result, when one firm is excluded from subsequent stages due to ‘licensing failure’ under an IP regime the effect on welfare can be far more severe than in the one-stage case.
Likewise the present paper also has a connection to the recent paper of Polanski (2007). That presents a ‘centipede-type’ model of k-stage cumulative innovation and compares ‘Open-Source’ (OS) and ‘Proprietary’ (PR) production. The key assumptions there are that (a) producers derive some direct benefit from product improvements independent of any sales income (without this ‘Open Source’ would never work), and (b) there is only ex-post bargaining between stage producers in the ‘Proprietary’ mode which generates ‘hold-up’ problem effects – at each stage a given producer has sunk her costs before bargaining with the next stage producer begins and this problem can ‘cumulate’ over the innovation chain. Together these generate the main result that either mode of production, in the right circumstances, can be dominant – in the sense of permitting production when the other does not (though obviously for differing parameter values). Again, while some of the results in this paper have a similar flavour to those in Polanski (2007), our model differs substantially in way they arise. Specifically here there are only two stages and the main ‘hold-up’ issue arises in relation to the second-stage innovators ‘experimentation’ effort and its interaction with licensing failure in the presence of imperfect information about the types of second-stage innovators.

Finally, we should point out that our results are of relevance to a variety of recent policy debates. For example, in December 2006 the Gowers Review of Intellectual Property which had been setup by the UK government to examine the UK’s current IP regime, provided, as one its recommendations (no. 11), that “Directive 2001/29/EC [the EU Copyright ‘InfoSoc’ Directive] be amended to allow for an exception for creative, transformative or derivative works, within the parameters of the Berne Three Step Test.” Such a ‘transformative use exception’ would correspond very closely to the weak IP regime considered in the model presented here. Meanwhile in 2005 in the United States, the Supreme Court in *Merck KGaA v. Integra Life Sciences I, Ltd*10 created a very broad research exemption in relation to pre-clinical R&D. Such a change again corresponds closely in the model to a move towards a weak IP regime in which a second-stage product would not infringe on a first-stage firm’s patent.

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10The full opinion is available at http://www.supremecourtus.gov/opinions/04slipopinion.html.
2. A Basic Model of Two-Stage Cumulative Innovation

2.1. The Model. We adopt a simple model of two stage innovation in which the second innovation builds upon the first in some manner – either as an application or as an extension of it. All agents are risk-neutral and act to maximize profits.

Innovations are described by their net (stand-alone) value \( v \) (revenue minus costs). Because our interest lies in examining the trade-off between innovation at different stages we make no distinction between social and private value (i.e. there are no deadweight losses) and \( v \) may be taken to be both.

We assume the base (first) innovation has two possible (stand-alone) values: low \((v^L_1)\) and high \((v^H_1)\) with probability \(p\), \((1 - p)\) respectively. We assume that \(v^L_1 < 0\) so that without some additional source of revenue, for example from licensing (see below), the innovation will not be produced. High value innovations have positive stand-alone value, \(v^H_1 > 0\), and so do not require an outside source of revenue in order to be profitable.

Second-stage innovations also take two values: low \((v^L_2)\) and high \((v^H_2)\) with probability \(q\), \((1 - q)\) respectively and \(v^H_2 > v^L_2 > 0\). While the value of a second-stage innovation is known to the innovator who produces it, the value is not known to the owner of the first-stage innovation which it builds upon (this could occur because of imperfect information regarding revenue, costs or both). Without loss of generality we shall assume that the number (or measure) of second-stage innovations per first-stage innovation is one (having \(N\) second-stage innovations per first-stage innovation would just require replacing \(v^H_2\) with \(Nv^H_2\) and \(v^L_2\) with \(Nv^L_2\)). We also assume that \(v^L_1 + v^L_2 \geq 0\) – this ensures that whatever the value of \(q\) the overall value generated by a first-stage innovation is positive (the overall value is the stand-alone plus the value of dependent second-stage innovations).\(^{11}\)

2.1.1. Intellectual Property Rights and Licensing. We wish to consider two regimes: one with (strong) intellectual property rights (IP) and one with weak intellectual property rights (WIP). With intellectual property rights every second-stage innovator will require a license from the relevant first-stage innovator in order to market her product, while without intellectual property rights she may market freely without payment or licence.\(^{12}\)

\(^{11}\)Allowing values of \(v^L_1\) less than \(v^L_2\) does not alter the analysis in any significant way but brings extra complexity to the statement and proof of propositions.

\(^{12}\)Given that we are dealing with cumulative innovation some readers might prefer the infringing (I) vs. non-infringing (NI) dichotomy with its implication of a distinction between ‘horizontal’ imitation and ‘vertical’ improvement or application of a product.
We assume that the stand-alone value of the first and second-stage innovators \( (v_i) \) are unaffected by the intellectual property rights regime. Note here, that strictly, we only require that the sum of the first-stage and (dependent) second-stage innovation values is unaffected by the IP regime as we can incorporate transfers via changes in the probability \( p \) that a first-stage innovation is low value (under strong IP any gains by second-stage firms at the expense of first stage firms will be taken back through the royalty so \( p \) only plays a role in the weak IP regime). Thus, simple business stealing, in which the total combined rents of the two stages remain unchanged, but where the second-stage takes rents from the first-stage, is permitted in this model. However, we do not allow for the general erosion of overall rents under the weak IP regime due to a general increase in direct product market competition.\(^{13}\) It is therefore best to take the weak IP regime as corresponding to non-infringement where distinct improvements or applications do not need to license but where there is still of protection against direct copying.

Finally, we take the licence to define a lump-sum royalty payment \( r \). This assumption is without loss of generality since, in this model, an innovation is entirely defined by its net value \( v \) and there are no other attributes available to use in designing a mechanism to discriminate between types of second-stage innovator.\(^{14}\) The royalty is set ex-ante, that is prior to the second-stage innovator’s decision to invest, and is in the form of a take-it-or-leave it offer by the first-stage innovator.

2.1.2. Sequence of Actions. The sequence of actions in the model is:

1. Nature determines the value type of the first-stage innovator.
2. A first-stage innovator decide whether to invest. If the first-stage innovator does not invest the game ends and all payoffs are zero. Assuming the first-stage innovator invests the game continues.

\(^{13}\)This is similar to the approach taken explicitly or implicitly in much of the previous literature, for example Green and Scotchmer (1995); Chang (1995); Scotchmer (1996); Denicolo (2000); Bessen (2004). We would also note that with rent dissipation one must immediately provide for a distinction between private rent and social value further complicating the analysis, and distracting from the focus of this paper which is on the welfare effects of IP purely as they operate through the impact on innovation leaving aside the (classic and well-known) direct effect of IP on welfare due to the deadweight loss of reduced access.

\(^{14}\)For example, there are no quantities on which to base a non-linear pricing scheme (fixed fee plus per unit fee royalty). For the same reason there is no opportunity to use type-contingent menus, or any other form of more complex licensing agreement, to increase total royalty income by discriminating between high and low value innovators.
Player & Second-Stage Innovator  
<table>
<thead>
<tr>
<th>First Stage Innovator</th>
<th>Value Type</th>
<th>Action</th>
<th>Low (q)</th>
<th>High (1-q)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low (p)</td>
<td>NI</td>
<td>$v_H^L$, 0</td>
<td>$v_H^L + r$, $v_L^H - r$</td>
</tr>
<tr>
<td></td>
<td>High (1-p)</td>
<td>NI</td>
<td>$v_H^L$, 0</td>
<td>$v_H^L + r$, $v_L^H - r$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>$v_L^L$, 0</td>
<td>$v_L^L + r$, $v_H^L - r$</td>
</tr>
</tbody>
</table>

Table 1. Action and Payoff Matrix Assuming First-Stage Innovator Invests. (I/NI = Invest/Do Not Invest, r = Royalty Rate)

(3) The first-stage innovator sets the royalty rate $r$ (under the no/weak IP regime second-stage innovations do not infringe and so the de facto royalty rate is 0).

(4) Nature determines the value type of a second-stage innovator.

(5) Given this royalty rate second-stage firms decide whether to invest.

(6) Payoffs are realized.

The action/payoff matrix is summarized in Table 1.

2.2. Solving the Model. Define a constant, $\alpha$, as follows:

$$\alpha \equiv \frac{v_H^H - v_L^L}{v_H^L}$$

Proposition 2.1. With intellectual property rights, the game defined above has the following Subgame Perfect Nash equilibria. A second-stage innovator invests if and only if its realized value is greater than or equal to the royalty rate (i.e. net profits are non-negative).

A first-stage innovator invests and sets a low royalty rate (RL), $r_L = v_L^L$ if the probability of a low value innovation ($q$) is greater than $\alpha$ and a high royalty rate (RH) $r_H = v_H^H$ if $q \leq \alpha$. When $q = \alpha$ the first-stage innovator may set any royalty of the form $r_L$ with probability $x$ and $r_H$ with probability $1 - x$, $x \in [0,1]$. Thus, there always exist a pure strategy equilibrium and except when $q = \alpha$, this equilibrium is unique.

Proof. See appendix. □

Proposition 2.2. Without intellectual property rights the game above has the following solution: both types of second-stage innovators invest but, of first-stage innovators, only those that have ‘high-value’ innovations invest (there are $1 - p$ of these type).

Proof. Trivial. □

2.3. Welfare. To determine welfare we need to know the ‘trade-off’ between first and second-stage innovations that occurs when revenue is allocated from one to the other
by licensing. As stated above, without royalty income from second-stage innovations a proportion \( p \) of first-stage innovations are not produced with average (stand-alone) value \( v^L_1 \). The remaining innovations \((1-p)\) are produced irrespective of whether royalty revenue is received and have average value \( v^H_1 \).

Let us now consider social welfare in the four possible situations given by \((IP, RL)\), \((IP, RH)\), \((WIP, RL)\), \((WIP,RH)\) as well as the difference in welfare between an intellectual property regime and a weak intellectual property regime \((IP-WIP)\). Due to our earlier assumption welfare is determined by calculating total net value. Define for convenience \( v_1 = pv^L_1 + (1-p)v^H_1 \), the average first-stage innovator value (if all innovate), and \( v_2 = qv^L_2 + (1-q)v^H_2 \), the average second-stage innovator value (if all innovate). We summarize the welfare situation in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>RL</th>
<th>RH</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP</td>
<td>( v_1 + v_2 )</td>
<td>( v_1 + (1-q)(v^H_2) )</td>
</tr>
<tr>
<td>WIP</td>
<td>( (1-p)(v^H_1 + v_2) )</td>
<td>( (1-p)(v^H_1 + v_2) )</td>
</tr>
<tr>
<td>IP - WIP</td>
<td>( p(v^L_1 + v^L_2) + p(v_2 - r_L) \geq 0 )</td>
<td>( p(v^L_1 + (1-q)v^H_2) - (1-p)qv^L_2 )</td>
</tr>
</tbody>
</table>

Table 2. Welfare in the Basic Model

2.4. Policy Implications.

**Proposition 2.3.** When a low royalty will be set \((q \geq \alpha)\) an IP regime is optimal.

**Proof.** In the low royalty \((RL)\) situation all second-stage innovations will be produced whether there is IP or not. In that case one wishes to maximize returns to the first innovator and patents do this by transferring rents via licensing. Formally in the low royalty case the welfare difference between patents and no patents \((IP-WIP)\) is:

\[
p(v^L_1 + r_L) + p(v_2 - r_L)
\]

Both of the terms in brackets are positive implying that the intellectual property regime delivers higher welfare than the weak intellectual property \((WIP)\) regime. \(\square\)

The situation when the high royalty will be set is less clear. First, define \( \beta \) as the proportion of the royalty payment to a low-value first-stage innovator that would be ‘used up’ in paying their extra costs:

\[
\beta \equiv \frac{-v^L_1}{(1-q)r_H}
\]
Note that $v^L_1$ is negative and must be less in absolute terms than the royalty received $(1 - q)r_H$ as we are assuming that the royalty enables low value first-stage innovators to produce. Under this definition $\beta = 1$ corresponds to the case where all of the royalty paid to a low-cost first-stage innovator being used to pay their ‘extra’ costs while $\beta \approx 0$ means all of the royalty payment is being retained as extra profits (and welfare).

**Proposition 2.4.** When a high royalty will be set ($q < \alpha$) an intellectual property regime will be preferable to a weak intellectual property (WIP) regime if and only if (NB: in fact with equality one would be indifferent):

$$ p \geq \frac{qv^L_2 + qv^H_2}{(1 - \beta)(1 - q)v^H_2} \geq (1 - p)qv^L_2 $$

(2.1)

$$ p(v^L_1 + (1 - q)v^H_2) \geq (1 - p)qv^L_2 $$

Making $p$ the subject of this inequality and using $\beta$ we obtain the stated result. \hfill \square

Proof. From Table 2 an IP regime yields higher welfare than an WIP regime if and only if:

We represent the import of these propositions graphically in Figure 1, a diagram which shows optimal policy regions as a function of the exogenous probabilities of low value first-stage ($p$) and second-stage ($q$) innovations.

**Remarks:** in the high royalty case (RH) $q$ is the proportion of second-stage innovations that do not occur with intellectual property rights (due to high royalties and the resulting licensing failure) while $p$ is the proportion of first-stage innovations that do not occur without intellectual property rights. As first-stage innovations enable second-stage ones when we lose a first-stage innovation we lose all dependent second-stage ones as well. Due to this, when $\beta$ is low for weak intellectual property rights to be preferable $q$ must be substantially higher than $p$. It is only then that the cost of intellectual property rights, in terms of lost second-stage innovations, will outweigh the gains in terms of more first-stage (and dependent second-stage) innovations.

As $\beta$ increases the area in which weak intellectual property rights are preferable will increase, with the line separating the two regions moving upwards. In the limit as $\beta$ tends to $1$ – which corresponds to all royalty income being used by a low value first-stage
innovator to pay costs – the marginal $p$ tends to 1, that is, it is optimal to have intellectual property rights only if all first-stage innovations are of a low value type.

3. A Model of Cumulative Innovation with Experimentation

3.1. The Model. The ‘experimentation’ model differs from the ‘basic’ model presented in the previous section only in the addition of a single extra period in which experimentation by second-stage firms takes place prior to any royalty setting.\(^{15}\) Formally, there is the
following sequence of actions (modifications compared to the basic model are bolded for clarity):

(1) Nature determines the value type of the first-stage innovator.

(2) A first-stage innovator decide whether to invest. If the first-stage innovator does not invest the game ends and all payoffs are zero. Assuming the first-stage innovator invests the game continues.

(3) **Second-stage innovators chooses their level of experimentation** $k$. (One could think of this, for example, as the number of first-stage products a second-stage firms chooses to investigate via purchase, observation etc).

- Experimentation has constant marginal cost $\tau$.
- Knowledge of the experimentation level chosen by a second-stage firm. There are two possibilities regarding the knowledge of the experimentation level available to first-stage innovators. In the first case the first-stage innovator does observe the experimentation level. In the second case the first-stage innovator does not observe the experimentation level. In what follows we focus on the case where the experimentation level is unobserved as we feel this is more realistic though the results are unchanged (and simpler to derive) when it is observed.

(4) The first-stage innovator sets the royalty rate $r$ (under the no/weak IP regime second-stage innovations do not infringe and so the de facto royalty rate is 0).

(5) Nature determines the value type of a second-stage innovator. As before there are two types of stage 2 firms, high and low value: $v^H_2, v^L_2$. However, here:

- **The probability, $q$, that a second-stage firm is low value is a function of the experimentation level**: $q \equiv q(k)$.
- Properties of $q(k)$: $q' \leq 0$ (otherwise there is no benefit from experimentation). There are diminishing returns to experimentation: $q'' \geq 0$. If no
experimenter entrusts all firms are of low value type \( q(0) = 1 \). The functional form \( q(k) \) is assumed to be common knowledge.

(6) Given this royalty rate second-stage firms decide whether to invest.

(7) Payoffs are realized.

The new action/payoff matrix is shown in Table 3.

### 3.2. Solving the Model.

Define, as in the basic model, a high royalty to be equal to the value of a high-value second-stage innovation: \( r_H = v_H^2 \), and a low royalty to be equal to the value of a low-value second-stage innovation: \( r_L = v_L^2 \).

We begin with a set of preliminary propositions which detail the players’ best responses before moving on to characterise the equilibrium under both (strong) IP and weak IP (WIP).

**Proposition 3.1** (Second-stage innovator’s investment strategies). A second-stage innovator with value \( v_X \) facing a royalty of \( r \) will invest if and only if \( v_X \geq r \).

**Proof.** Just as in the original model second-stage innovator’s move with full knowledge of all variables. In this case an innovator of type X invests if and only if net profits from investing, \( v_X - r - k\tau \) are greater than \( -k\tau \) the payoff from not investing (experimentation costs are sunk). Hence the investment strategies are the same as in the basic model: a second-stage innovator invests if and only if \( v_X \geq r \). \( \square \)

**Proposition 3.2** (First-stage Best-Response Royalty). Under the IP regime, a first-stage innovator, whose belief about the experimentation level is given by the cdf \( F(k) \) and where

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Player} & \text{Second-Stage Innovator} & \text{Sample (k)} \\
\hline
\text{First-Stage Innovator} & & \\
\hline
\text{Value Type} & \text{Action} & \text{Low (q(k))} & \text{High (1-q(k))} \\
\hline
\text{Low (p)} & \text{NI} & v_L^1 - k\tau & v_L^1 + r, v_L^2 - r - k\tau \\
\text{High (1-p)} & \text{I} & v_L^1 + r, v_L^2 - r - k\tau & v_L^1 - k\tau, v_L^2 + r, v_L^2 - r - k\tau \\
\hline
\end{array}
\]

Table 3. Action and Payoff Matrix Assuming First-Stage Innovator Invests (I/NI = Invest/Do Not Invest, \( r \) = Royalty Rate)

\[\text{It should be noted that this assumption is not important for the results and has simply been made for simplicity. It is easy to adapt all of the results to the case where even under zero experimentation there is a non-zero probability of having a high-value innovation.}\]
$\bar{q} = \mathbb{E}_F(q(k))$, will set a royalty of the form:

$$r(k) = \begin{cases} 
    r_L = v^L_2, & \bar{q} > \alpha \\
    r_H = v^H_2, & \bar{q} < \alpha \\
\end{cases}$$

mixed strategy $(r_H, r_L)$ with prob $(x, 1-x), x \in [0,1], \bar{q} = \alpha$

where $\alpha$ is as in the basic model, that is the probability such that a first-stage firm is indifferent between setting a high and a low royalty rate:

$$\alpha \equiv \frac{v^H_2 - v^L_2}{v^H_2}$$

Proof. See appendix. \hfill \Box

Remark 3.3 (Definition of $k_\alpha$). If a first-stage innovator believes second-stage innovators all play the same pure strategy, $k$, then we can replace the conditions of the form $\bar{q} <, =, > \alpha$ with the condition that $k >, =, < k_\alpha$ (note the inversion of ordering), where the constant $k_\alpha$, is the experimentation level such that $q(k_\alpha) = \alpha$.

Proposition 3.4 (Second-Stage Experimentation Level). Under an IP regime the second-stage innovators best response to a royalty of $r$, including ‘composite’ royalties of the form $r = xv^H_2 + (1-x)v^L_2, x \in [0,1]$ (that is mixed royalty with $r_H$ played with probability $x$), is as follows:

$$k = \begin{cases} 
    k_2, & r \leq r_L = v^L_2 \\
    k_r, & r_L < r < r_H \\
    0, & r \geq r_H = v^H_2 \\
\end{cases}$$

where $k_r$ is defined implicitly by: \footnote{If $q'(0) > -\infty$ then for values of $r$ sufficiently close to $r_H = v^H_2$ this equation will have no solution. In such cases define $k_r = 0$.}

$$q'(k_r) = \frac{-\tau}{v^H_2 - r}$$

And $k_2$ is given as follows: \footnote{We use the subscript 2 because this is the level of experimentation undertaken in the case where both types of second-stage innovators find it worthwhile to invest.}

$$k_2 = k_{rL} = k_{v^L_2} \Rightarrow q'(k_2) = \frac{-\tau}{v^H_2 - v^L_2}$$

Proof. See appendix. \hfill \Box
Theorem 3.5. With intellectual property rights (IP) the perfect Bayesian equilibrium of the game defined above falls into one of two cases:

(i) **Low royalty case** \( (k_2 \leq k_\alpha) \)

1. First-stage innovators: both high and low value types invest, believe that second-stage innovators sample at level \( k_2 \) and set a low royalty rate.
2. Second-stage innovators: sample at level \( k_2 \) and both high and low value types invest.

(ii) **Mixed royalty case** \( (k_2 > k_\alpha) \)

1. First-stage innovators: both high and low value types invest, believe that second-stage innovators sample at level \( k_\alpha \) and set a mixed royalty rate consisting of a high royalty \( (r_H) \) with probability \( x_\alpha \) and a low royalty \( (r_L) \) with probability \( (1 - x_\alpha) \) where:\(^{19}\)

\[
x_\alpha = 1 - \frac{\tau}{-q'(k_\alpha)(v_H^2 - v_L^2)}
\]

2. Second-stage innovators: sample at level \( k_\alpha \) and invest if and only if the realized value of their innovation is greater than the royalty rate (though the first-stage innovator is playing a mixed strategy the second-stage innovator knows the royalty rate with certainty at the point of investment).

Proof. See appendix. \( \Box \)

Proposition 3.6 (Equilibrium under weak IP). Under weak IP the ‘experimentation’ model has the following solution: second-stage innovators sample at level \( k_2 \) and both types of second-stage innovators invest. Of first-stage innovators, those that have ‘high-value’ innovations invest (there are \( 1 - p \) of these type) and those with ‘low-value’ innovations do not.

Proof. Trivial. (Second-stage experimentation best-response correspondences have already been derived in Proposition 3.4). \( \Box \)

Remark 3.7. Recall that \( k_2 \) is the experimentation level undertaken by a second-stage firm in the case when both high and low value second-stage innovators invest (so it occurs

\(^{19}\)Note examining the definition of \( k_2 \) shows that \( k_\alpha < k_2 \) guarantees that \( x_\alpha \) is non-negative.
either in the case where there is weak IP or when the royalty is sufficiently low. It is also, therefore, the experimentation level which maximizes expected second-stage innovation value and, for that reason, the socially optimal experimentation level.

3.3. Welfare. For the welfare calculations we proceed as in the original model. A proportion $p$ of first-stage innovations are low value ($v^L_1 < 0$) and only occur when there is royalty income. Analogously to the basic model define $v_1 = pv^L_1 + (1 - p)v^H_1$ and $v_2(k) = -k\tau + (1 - q(k))v^H_2 + q(k)v^L_2$ (the expected value generated by a second-stage innovator experimenting at level $k$).

Proposition 3.8. [The Optimal Regime in the Low Royalty Case] In the low royalty case ($k_2 < k_\alpha$) it is optimal to have an IP regime (compared to weak IP one). Specifically if the proportion ($p$) of first-stage innovation that is lost without IP is positive then welfare is higher with IP (otherwise $p = 0$ and both regimes generate the same level of welfare).

Proof. See appendix. □

This result has a simple intuition behind it. The low royalty case encompasses the situation where the experimentation level is fairly low even when the royalty rate faced by second-stage firms is small ($k_2 \leq k_\alpha$) – this may occur because experimentation is costly ($\tau$ is high) or generates little benefit ($v^H_2$ and $v^L_2$ are close). As a result most second-stage innovations are low value and so a first-stage innovator sets a low royalty rate ($r^L_2$). Hence (a) there is no ‘licensing failure’ and (b) all second-stage firms sample at the optimum rate ($k_2$). Taken together these mean that, just as with the low royalty case of the simpler model, there are no costs to having strong IP. Since, thanks to the licensing income, there is more (by an amount $p$) first stage innovation under strong IP than under weak IP the strong IP regime is clearly better.

Proposition 3.9. [The Optimal Regime in the Mixed Royalty Case] In the mixed royalty case ($k_2 \geq k_\alpha$) it is optimal to have an IP regime rather than a weak/no (WIP) regime if the proportion ($p$) of first-stage innovation that does not occur under no/weak IP is
sufficiently high, specifically:

\[ p \geq p^m = \frac{(v_2(k_2) - v_2(k_\alpha)) + x_\alpha q(k_\alpha)v_L^2}{(v_2(k_2) - v_2(k_\alpha)) + x_\alpha q(k_\alpha)v_L^2 + (v_2(k_\alpha) - x_\alpha q(k_\alpha)v_L^2 - (v_L^2))} \]

\[ = \frac{\text{Reduced Exper. Cost} + \text{Licensing Failure Cost}}{\text{Reduced Exper. Cost} + \text{Licensing Failure Cost} + \text{Surplus from Extra 1st Stage}} \]

(3.2)

**Proof.** See appendix.

**Remark 3.10.** Reduced Experimentation Cost: \( v_2(k_2) \) is the average value of second-stage innovations when second-stage firms sample at the unrestricted (and optimal) level \( k_2 \). Under the IP regime second-stage firms only sample at level \( k_\alpha \) because of the higher (average) royalty. Thus, the average value of a second-stage innovation is less under the IP regime compared to the weak IP regime due to this reduced experimentation precisely by the amount: \( v_2(k_2) - v_2(k_\alpha) \) (NB: obviously this only applies to those second-stage innovations associated with the \((1 - p)\) first-stage innovations which are produced under both the IP and the weak IP regime.)

Licensing Failure Cost: licensing failure occurs when a second-stage firm with a low-value innovation is faced with a high royalty rate. Under the IP regime \( x_\alpha \) is the probability that a high royalty is set by a first-stage innovator \( q(k_\alpha) \) is the probability a second-stage firm has a low-value innovation. Thus \( x_\alpha q(k_\alpha) \) is the probability that licensing failure occurs and when it does the loss equals the potential value of the second-stage innovation: \( v_L^2 \).

Surplus from Extra First-Stage Innovation: the plus side of the IP regime is the extra first (and dependent) second-stage innovation that happens because first-stage innovators receive higher incomes. There are a proportion \( p \) of low (standalone) value first-stage innovators, who will only invest under the (strong) IP regime. For each such innovation the net surplus generated equals the surplus generated by the second-stage firms plus the net (stand-alone) surplus of a first-stage firm. The expected second-stage surplus equals the average value if all second-stage firms produced (when experimenting at \( k_\alpha \): \( v_2(k_\alpha) \), minus the surplus of those second-stage firms who are held-up: \( x_\alpha q(k_\alpha)v_L^2 \). Finally the net standalone surplus of a first-stage firm is \( v_L^2 < 0 \).
Finally, compare equation (3.1) with equation (2.1) from the basic model. The main, and most obvious, difference is that, as well as the standard ‘licensing failure cost’ of (strong) IP, there is another, additional, cost in the form of ‘reduced experimentation’ (and reduced average value of second-stage innovations).

**Corollary 3.11.** Extending $p^m = 0$ to the low royalty case ($k_2 \leq k_\alpha$) by defining $p^m = 0$ if $k_2 \leq k_\alpha$, we have that an IP regime is optimal if $p > p^m$ and a weak IP is optimal if $p < p^m$.

**3.4. Policy Implications.** Since we do not have any precise estimates for the exogenous parameters such as the experimentation cost ($\tau$) or the values of second-stage innovations ($v^H_2$, etc) we cannot make direct statements about which regime would yield higher welfare for a given industry. Instead our approach will be to pick a ‘dependent’ variable to focus on (in our case $p$, the proportion of first-stage innovation ‘lost’ under weak IP) and then derive the ‘break-even’ or marginal $p^m$ such that if $p = p^m$ society is indifferent in welfare terms between the two regimes.

Our next step is to investigate the comparative statics of the marginal $p$ ($p^m$) with respect to exogenous variables, in particular the cost of experimentation ($\tau$) and the relative value of high ($v^H_2$) and low type ($v^L_2$) second-stage innovations.

Our general results are summarized in Figure 2 and Figure 3 (the formal details are in Propositions 3.12 and 3.14 below). As we note in the captions one can only indicate the general form as any specific form for $p^m$ will depend on the functional form for $q$ and of course the values of the other exogenous parameters. Nevertheless, both diagrams have clear implications.

As detailed in Corollary 3.15, the first diagram shows that, as experimentation costs become low, it becomes more likely that a weak IP regime is preferable. Specifically, it would only be in industries in which the proportion of first-stage innovation dependent on income from second-stage developments was very high that one would want strong IP. This case corresponds to a situation where the initial innovations have little standalone use and provide value almost entirely via reuse as tools or components in other products and applications. Conversely, in industries where a reasonable number of first-stage product
have substantial standalone value\textsuperscript{20} it will be optimal to have a weak IP regime in which second-stage firms are free to reuse first-stage innovations without needing to license.

Coming to the second figure, as detailed in Corollary 3.16 this shows that, as high-value second-stage innovations become more important relative to low value ones (the distribution is more skewed of innovation values are more skewed), it is more likely that a weak IP regime is socially optimal. Specifically, unless the proportion of first-stage innovators dependent on income from second-stage firms is very high (approximately 100% for very skewed value distributions) it will be optimal to have a weak IP regime.

\textsuperscript{20}For example, the music industry.
Figure 3. Marginal $p^m$ as a function of $v_2^H$ (or equivalently, for fixed $v_2^L$: $v_2^L - v_2^H$). For the same reasons given in relation to Figure 2 ticks on the $v_2^H$ axis have been omitted. However to give the reader some sense of proportion we note that $\tau = 0.5, v_2^L = 1.0, q(k) = e^{-k}$ and $p^m = 0$ below approximately 3.0.

Proposition 3.12. The experimentation levels $k_\alpha$ and $k_2$ have the following comparative statics:

\[
\frac{dk_\alpha}{d\tau} = 0 \quad (3.3)
\]
\[
\frac{dk_\alpha}{dv_2^H} < 0 \quad (3.4)
\]
\[
\frac{dk_2}{d\tau} < 0 \quad (3.5)
\]
\[
\frac{dk_2}{dv_2^H} > 0 \quad (3.6)
\]

And taking limits:

\[
\lim_{\tau \to \infty} k_2 = 0, \quad \lim_{v_2^H \to v_2^L} k_2 = 0, \quad \lim_{\tau \to 0} k_2 = \infty, \quad \lim_{v_2^H \to \infty} k_2 = \infty \quad (3.7)
\]
\[
\lim_{v_2^H \to \infty} k_\alpha = 0 \quad (3.8)
\]
Proof. Recall that we have:

\[ q(k_\alpha) = \frac{v_H^2 - v_L^2}{v_H^2} \]

\[ q'(k_2) = \frac{-\tau}{v_H^2 - v_L^2} \]

Given that \( q' < 0 \) and \( q'' > 0 \) the results following trivially by simple differentiation. \( \square \)

Remark 3.13. The intuition behind these results is straightforward. \( k_\alpha \) is the level of experimentation that leaves a first-stage innovator indifferent between charging a high and a low royalty rate. As such it is a function only of the relative values of the two types of innovation (and of \( q \)) and does not depend on the cost of experimentation at all.

The intuition in the second case is a little more complicated. If we increase \( v_H^2 \) keeping \( v_L^2 \) constant we increase the differential between high and low value second-stage innovations. Then the net change in revenue for a first-stage innovator’s from switching to a high royalty rate must increase (loss of royalty revenue from low-value second-stage innovations is lower relative to royalty from high-value second-stage innovations). Hence, the proportion of high value second-stage innovations \((1 - q(k))\) at which the switch to a high royalty rate is made is smaller and the corresponding level of experimentation \((k_\alpha)\) is smaller.

Coming to \( k_2 \), which is the optimal level of experimentation (and that performed under a low or zero royalty), we have unsurprisingly that as the cost of experimentation goes down the amount of experimentation goes up. Similarly, an increase in the relative size of a high value innovation compared to a low value one, increases the benefit of experimentation and therefore increases the amount of experimentation performed.

Combining the differentials with the limits we have that (a) keeping other variables fixed there exists a unique finite \( \tau^* \) such that for \( \tau < \tau^*, k_2 > k_\alpha \) and a mixed royalty is set (conversely for \( \tau > \tau^* \) a low royalty is set and \( p^m = 0 \)); (b) similarly there exists a unique \( v^* \) such that for \( v_H^2 > v^*, k_2 > k_\alpha \) and a mixed royalty is set (conversely for \( v_H^2 < v_H^2* \) a low royalty is set and \( p^m = 0 \)). This then demonstrates the validity of the right-hand part of Figure 2 and the left-hand part of Figure 3 where we have \( p^m = 0 \).

What occurs then if \( k_2 > k_\alpha \) and we are in the mixed royalty case?
Proposition 3.14. Assuming $k_2 > k_\alpha$ (i.e. $\tau$ sufficiently small or $v_2^H$ sufficiently large) then:

$$p \geq p^m \equiv \frac{(v_2(k_2) - v_2(k_\alpha)) + x_\alpha q(k_\alpha) v_2^I}{(v_2(k_2) - v_2(k_\alpha)) + x_\alpha q(k_\alpha) v_2^I + ((v_2(k_\alpha) - x_\alpha q(k_\alpha) v_2^I) - (-v_1^I))}$$

And we have that:

$$\frac{dp^m}{d\tau} < 0$$
$$\frac{dp^m}{dv_2^H} > 0$$

That is, the marginal level of first-stage innovation lost under weak IP (that is the level such that above this an IP regime is optimal) is (a) decreasing in experimentation costs (b) increasing in the relative size of high value to low value second-stage innovations.

Proof. See appendix. □

Informally this result can be explained as follows. Reductions in experimentation costs will increase the ‘optimal’ level of experimentation ($k_2$) relative to the restricted level of experimentation ($k_\alpha$). This in turn increases the cost of intellectual property rights arising from (a) loss of second-stage innovations due to licensing failure ($x_\alpha \cdot q(k_\alpha)$); (b) lower average value of second-stage innovations ($v_2(k_2) - v_2(k_\alpha)$); while having no effect on the surplus from extra first-stage innovations under IP. As a result the welfare under weak IP rises relative to the welfare under IP and the marginal $p$ must rise.

Similarly if the relative size of high value second-stage innovation compared to a low value one rises this (a) increases the ‘optimal’ level of experimentation ($k_2$) relative to the restricted level of experimentation ($k_\alpha$) (b) directly increases the benefit of experimentation. This again increases the reduced experimentation cost and the licensing failure cost but reduces the surplus from second-stage innovations under IP. As a result welfare under weak IP rises relative to that under IP and the marginal $p$ must rise.

This result then establishes the validity of the rest of Figures 2 and 3 and implies the following corollaries regarding how the optimal policy regime in relation to intellectual property rights varies in response to changes in the exogenous environment:
Corollary 3.15. Reducing experimentation costs make it more likely that a freer (weak intellectual property rights) regime will be optimal.

Proof. Follows from previous propositions as summarised in Figure 2. □

Corollary 3.16. Increasing the differential between high and low value second-stage innovations (which could be interpreted as experimentation becoming more important for product quality) makes it more likely that a freer (weak intellectual property rights) regime will be optimal.

Proof. Follows from previous propositions as summarised in Figure 3. □

Remark 3.17. Most studies of the value of intellectual property rights (copyrights or patents) indicate that their distribution is highly skewed with a few very high value works and many low value works. This suggests that $v_{2}^{H} \gg v_{2}^{L}$.

4. Conclusion

In this paper we have shown how asymmetric information about the value of follow-on innovations, combined with intellectual property rights such as patents, can result in licensing failure and hold-up. Presenting the policy decision as a choice between having or not having intellectual property rights, we have shown that, in contrast to parts of the previous literature, in some circumstances it may be optimal not to have intellectual property rights. For whilst intellectual property rights help transfer income from second-stage to first-stage innovators they can also lead to licensing failure and hold-up with a resulting reduction in second-stage innovation.

The first part of the paper presented a basic model, which illustrated the already familiar trade-off when strengthening IP between reduced second-stage innovation as a result of licensing failure and increased first-stage innovation. This formed the basis for the model of cumulative innovation and experimentation which is the focus of this paper. Here, the existence of a perfect Bayesian equilibrium was shown along with the fact that, stronger IP, may have additional costs beyond those from licensing failure in the form of reducing experimentation below be socially optimal level.

Therefore, in addition to the basic trade-off mentioned above between more first-stage innovations and fewer second-stage ones, there is the additional factor: the average value of
second-stage innovations have a lower average value due to a lower level of experimentation. Furthermore, this ‘reduced experimentation cost’ may be substantially larger than the costs of licensing failure. Examining the comparative statics, it was shown that, the lower the cost of experimentation and the greater the differential between the low and high values of second-stage innovations, the more likely it is that a regime with weak intellectual property rights, in which second-stage firms did not have to license, was socially optimal.

Thus, technological change which reduces the cost of encountering and trialling new ‘ideas’ should imply a reduction in the socially optimal level of intellectual property rights such as patents and copyright. A perfect case of such technological change in recent years can be found in the rapid advances in computers and communications. These advances have, for example, dramatically reduced the cost of accessing and re-using cultural material, such as music and film, as well as greatly increasing the number of ‘ideas’ that a software developer can encounter and trial. Concrete policy actions that could be taken in line with these conclusions include extending ‘fair-use’ (fair-dealing) provisions in copyright law to increase the degree of reuse that would be permitted without the need to seek permission and excluding software and business methods from patentability.

Finally, we should emphasize that there remains plentiful scope to improve and extend the present paper. For instance, it was assumed that the non-royalty income for the first-stage and second-stage innovator was unaffected by the intellectual property rights regime. However this is unlikely to be the case and the model could be improved by the inclusion of the direct effect of no (or weaker) intellectual property rights on the revenue of the first-stage (and second-stage) innovator.

It would also be useful to extend the analysis to the case of a continuous distribution of innovation values, as well as to investigate the consequences of making experimentation costs a function of the intellectual property rights regime. It would also be valuable to examine what occurs when the structure of innovation is more complex, for example by having second-stage inventions incorporate many first-stage innovations (a componentized model) or having heterogeneity across innovations with some developments used more than others. Finally, one of the most important extensions would be to properly integrate transaction costs into the analysis. Transaction costs relating to both the acquisition of

---

21 As discussed in detail above, while we do allow for business stealing between the first and second-stage innovators we do not allow for general rent dissipation from wider product market competition.
information and the execution of contracts are significant and without them we lack a key
element for the furtherance of our understanding of the process of innovation both in this
model and in general.

A. Proofs

A.1. Proof of Proposition 2.1.

Proof. We are considering only subgame perfect nash equilibria so we may begin at the
final stage of the game and work backwards. Given a royalty level of \( r \), at the final stage,
a second-stage innovator of type X faces a payoff of \( v_X^2 - r \) if she invests and 0 if she does
not. Thus, a second-stage innovator, seeking to maximize profits will invest if and only
if \( v_X^2 \geq r \) (formally, they are indifferent if \( r = v_X^2 \). However if they do not invest when
\( v_X^2 = 0 \) there will be no equilibrium of the overall game).

Given this, by simple dominance and focusing on pure strategies, a first-stage innovator
must EITHER (a) set a low royalty rate \( r_L = v_L^2 \) which will lead to investment by all
second-stage innovations; OR (b) set a high royalty rate \( r_H = v_H^2 \) which will result in
investment only by high value second-stage innovations. In the first case the payoff is \( r_L \)
while in the second it is \( (1 - q)r_H \). Thus, a low royalty rate should be chosen if and only
if (assuming that if payoffs are equal a low royalty is chosen):

\[
 r_L \geq (1 - q)r_H \iff q \geq \frac{r_H - r_L}{r_H} = \alpha
\]

Since any mixed royalty strategy must consist of some combination of \( r_L \) and \( r_H \) we
have immediately that a proper mixed strategy is only possible when \( r_L = (1 - q)r_H \), that
is if \( q = \alpha \).

Finally, total royalty income to a first-stage innovator is at least \( r_L = v_L^2 \). Thus, total
net income for a low-value first-stage innovator is at least \( v_L^1 + r_L = v_L^L + v_L^L > 0 \) (by
assumption) – and net income for a high-value first-stage innovator is obviously greater.
Hence both types of first-stage innovator will invest. \( \square \)

A.2. Proof of Proposition 3.2.

Proof. Given a first-stage innovator believes \( F(k) \), the expected probability that a second-
stage firm is low value is \( EF(q(k)) = \bar{q} \). By subgame perfection a first-stage innovator
knows that, once a second-stage firm discovers its type, its best response to a given royalty will be as stated in Proposition 3.1. In particular, if the royalty rate is set to be less than or equal to the second-stage low value ($v_L$) all second-stage innovators will license, if a royalty is above this but less than or equal to the second-stage high value ($v_H$) then only high value firms will license ($1-\bar{q}$ of them) and if the royalty is higher than this no second-stage firms will license. Then, letting $G(r)$ be the cumulative distribution function over royalties representing the first-stage innovator’s mixed strategy, the expected payoff to a first-stage innovator is:

$$\Pi_1(G(r)) = \int_{0}^{v_L} r \cdot dG(r) + (1-\bar{q}) \int_{v_L}^{v_H} r \cdot dG(r) + 0 \cdot \int_{v_H}^{\infty} r \cdot dG(r)$$

Maximizing with respect to $G(r)$ immediately gives that, just as for the basic model, an optimal mixed strategy can only consist of some combination of the pure strategy $r_L = v_L$ and the pure strategy $r_H = v_H$. Let us suppose that these two pure strategies, $r_H, r_L$, are played with probability $x, 1-x$ respectively. Revenue from royalties is then:

$$r_L(1-x) + (1-\bar{q})r_H x = r_L + x \cdot ((1-\bar{q})r_H - r_L)$$

Maximizing revenue requires $x = 0$ if the term in brackets is less than zero, $x = 1$ if the term in brackets is greater than 0, and allows any value of $x$ if the term in brackets is zero. By the definition of $\alpha$ (see above) these conditions correspond precisely to $\bar{q}$ (the expected probability of a low value innovation) being less than, greater than or equal to $\alpha$. Hence, the first-stage innovator’s royalty response as a function of their belief about the level of experimentation is of the form stated.

**A.3. Proof of Proposition 3.4.**

**Proof.** Using the optimal investment stage determined in Proposition 3.1, for a given experimentation level $k$, payoffs as a function of the royalty levels are as in Table 4.

Suppose second-stage innovator plays a strategy given by the cdf $F(k)$ and a first-stage innovator sets a royalty defined by a cumulative distribution function $G(r)$. Then the

<table>
<thead>
<tr>
<th>$\Pi(k)$</th>
<th>$r \leq r_L$</th>
<th>$r_L &lt; r &lt; r_H$</th>
<th>$r \geq r_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-k\tau - r + q(k)v_L + (1-q(k))v_H$</td>
<td>$-k\tau + (1-q(k))(v_H - r)$</td>
<td>$-k\tau$</td>
</tr>
</tbody>
</table>

Table 4. Payoff for Second Stage Innovator
payoff to a second-stage innovator is as follows (where expectations are taken with respect to $F$ and $q$ is short for $q(k)$):

$$\Pi_2(F(k)) = \mathbb{E} \left( -\tau k + \int_0^{r_L} qv_2^L \, dq + (1 - q)v_2^H - rdG(r) + \int_{r_L}^{r_H} (1 - q)(v_2^H - r)dG(r) + \int_{r_H}^{\infty} 0dG(r) \right)$$

$$= \mathbb{E} \left( -\tau k - q(G(r_H)v_2^H - G(r_L)v_2^L - \int_{r_L}^{r_H} rdG(r)) + G(r_H)v_2^H - \int_{r_H}^{\infty} rdG(r) \right)$$

Claim: Second-stage innovators play pure strategies.

Proof: $q$ is convex so $-q$ is concave. Suppose we have a mixed strategy $F(k)$ with $\mathbb{E}(k) = \bar{k}$ then $-\bar{q} = \mathbb{E}(-q(k)) \leq -q(\bar{k})$ with equality if and only if $F(k)$ is a point distribution (i.e. corresponds to a pure strategy). Substituting:

$$\Pi_2(F(k)) = \mathbb{E}_F( -\tau k + G(r_H)v_2^H - q(G(r_H)v_2^H - G(r_L)v_2^L) - \int_{r_L}^{r_H} rdG(r)$$

$$\leq -\tau \bar{k} - \bar{q} \cdot (+ve) + \text{const}$$

(With equality iff and only if $F(k)$ is a point distribution with $k = \bar{k}$ with probability 1). Thus for any properly mixed strategy $F(k)$ we can always achieve a higher payoff by playing the pure strategy $\bar{k} = \mathbb{E}(k)$.

Thus, in what follows we may confine our attention to pure strategies $k$. Returning to the payoff function we first note that if royalty (or royalties in a mixed strategy) are all greater than $r_H$ (formally the support of $G(r)$ lies entirely above $r_H$) then the optimal experimentation level is zero ($\Pi_2(k) = -k\tau$).

When this is not the case we have the first order condition is:22

$$q'(k) = \frac{-\tau}{G(r_H)v_2^H - G(r_L)v_2^L - \int_{r_L}^{r_H} rdG(r)}$$

For ease of reference define $S$ as the denominator in the previous equation. We shall look at several special cases as follows:

---

22The second order condition, $\Pi'' \leq 0$, is easily checked: $\Pi'' = -q''(k) \cdot (+ve) < 0$ since, by assumption, $q''(k) > 0$. 
(i) \( r \leq r_L \). Then \( G(r_H) = G(r_L) = 1 \) and we have \( S = v_H^2 - v_L^2 \). The profit-maximizing \( k \) therefore equals \( k_2 \) where (as defined above):

\[
q'(k_2) = \frac{-\tau}{v_H^2 - v_L^2}
\]

The intuition here is simple: both firms always invest and pay the royalty. Thus, in terms of the payoff, experimentation will only affect the value type and the experimentation level will be chosen so that the marginal gain in terms of lower costs, \( q'(k)(v_H^2 - v_L^2) \), equals the marginal experimentation costs, \( \tau \).

(ii) \( r_L < r < r_H \). Here \( G(r_L) = 0, G(r_H) = 1 \) and we have \( S = v_H^2 - r \) and the optimal \( k \equiv k_r \) solves:

\[
q'(k_r) = \frac{-\tau}{v_H^2 - r}
\]

(iii) \( r_H \) played with probability \( x \) and \( r_L \) with probability \( (1 - x) \). Then \( G(r_L) = (1 - x), G(r_H) = 1 \). Define the ‘composite’ royalty \( r = xr_H + (1-x)r_L = xv_H^2 + (1-x)v_L^2 \) then we have \( S = v_H^2 - (1-x)v_L^2 - xr_H = (1-x)(v_H^2 - v_L^2) = v_H^2 - r \). So the optimal experimentation level is \( k \equiv k_r \) where \( r \) is the composite royalty.

\( \square \)

A.4. Proof of Theorem 3.5.

Proof. We will solve for a subgame perfect Bayesian nash equilibrium by recursing backwards through the game.

In previous propositions we have already derived the best-response correspondences (where the royalty best-response is defined in terms of beliefs about experimentation rather than the actual experimentation level). We have also shown second-stage firms will always play a pure strategy (i.e. choose a single experimentation level). Furthermore, at the experimentation stage all second-stage firms are the same, hence all second-stage firms will choose the same pure experimentation strategy. Thus, a first-stage innovator’s beliefs (to be consistent) must be single-valued and we may rewrite the royalty best-response
correspondence in terms of their belief as to the experimentation level \( k \):\(^{23}\)

\[
r(k) = \begin{cases} 
  r_L = v_L^H, & k < k_\alpha \\
  r_H = v_H^2, & k > k_\alpha \\
  \text{mixed strategy } (r_H, r_L) \text{ with prob } (x, 1-x), x \in [0,1], & k = k_\alpha 
\end{cases}
\]

**Case 1:** \( k_2 \leq k_\alpha \). There are three possibilities for the beliefs of a first stage innovator regarding the experimentation level of second-stage firms:

(i) \( k > k_\alpha \). Hence the first-stage innovator would set a high royalty rate. Then second-stage innovator’s best response is \( k = 0 \) and beliefs will be inconsistent. Thus, there cannot be an equilibrium with such beliefs.

(ii) \( k < k_\alpha \). In this case the best response of a first-stage innovator is to set a low royalty \( r_L \) in which case second-stage firm must choose a experimentation level \( k = k_2 \). Thus, for beliefs to be consistent, a first-stage innovator must believe \( k = k_2 \) and the equilibrium is as claimed.

(iii) \( k = k_\alpha \). In this case a first-stage innovator’s best response correspondence consists of all mixed strategies: \( r_H \) with probability \( x \), \( r_L \) with probability \( 1-x \) for \( x \in [0,1] \). Now a second-stage innovator (if behaving optimally) never samples above the level \( k_2 \) and will sample strictly below \( k_2 \) if the first-stage innovator plays any strategy in which \( r_H \) is played with positive probability. Hence if beliefs are to be consistent we must have (a) \( k_2 = k_\alpha \) and (b) \( x = 0 \) (i.e. a low royalty is always set). In such a case the equilibrium is again as claimed.

**Case 2:** \( k_2 > k_\alpha \). There are three possibilities for the beliefs of a first stage innovator regarding the experimentation level of second-stage firms:

(i) \( k > k_\alpha \). Just as in the first case this leads to inconsistent beliefs and so cannot be an equilibrium.

(ii) \( k < k_\alpha \). In this case the best response of a first-stage innovator is to set a low royalty \( r_L \) in which case second-stage firm must choose a experimentation level \( k = k_2 \). But \( k_2 > k_\alpha \). Thus, beliefs will be inconsistent and this cannot be an equilibrium.

\(^{23}\) At the experimentation stage all second-stage firms are the same and their best-response correspondence is single-valued. Hence all second-stage firms must have the same experimentation strategy and a first-stage innovator’s belief
(iii) \( k = k_\alpha \). In this case a first-stage innovator best response correspondence consists of all mixed strategies: \( r_H \) with probability \( x \), \( r_L \) with probability \( 1 - x \) for \( x \in [0,1] \). Denote the corresponding composite royalty by \( r(x) = xr_H + (1-x)r_L \). Then for an equilibrium (with consistent beliefs) we must find an \( x \) such that the best-response experimentation level equals \( k_\alpha \). Formally, using the notation of Proposition 3.4 we must find an \( x \) such \( k_r(x) = k_\alpha \). The best response experimentation level is defined implicitly by:

\[
q'(k) = \frac{-\tau}{(1 - x)(v_H^2 - v_L^2)}
\]

Since \( q' < 0 \) we have, denoting \( k(x) \) as the implicit solution as a function of \( x \), that \( k'(x) < 0 \) (intuitively a higher average royalty lowers experimentation). Since \( k(0) = k_2 > k_\alpha \) and that \( k(1) = 0 \) (as \( x \to 1 \) the RHS of the above takes arbitrarily large negative values), by the intermediate value theorem and the monotonicity of \( k(x) \), there must exist a unique \( x_\alpha \in (0,1) \) such that \( k(x_\alpha) = k_\alpha \). Replacing \( q'(k) \) by \( q'(k_\alpha) \) and rearranging we have as claimed that:

\[
x_\alpha = 1 - \frac{\tau}{-q'(k_\alpha)(v_H^2 - v_L^2)}
\]

**First-stage innovators investment strategy:** finally as with our basic model first-stage innovators of both types invest because with royalty income net profits will be non-negative.

A.5. **Proof of Proposition 3.8.**

*Proof.* Analogously to the low royalty case in the basic model, in this situation all second-stage innovators invest so (a) there is no licensing failure (b) second-stage firms sample at the optimal level \( (k_2) \). At the same time, intellectual property allows some first-stage innovators to engage in production who wouldn’t be able to do so otherwise. Hence an IP regime will deliver higher welfare.

Formally, the welfare difference between the IP and WIP regime is net surplus associated with the \( p \) extra first-stage innovations that occur under IP:

\[
p((v_1^L + r_L) + (v_2(k_2) - r_L))
\]
Both the first term (by the assumption that the royalty is sufficient to allow production) and the second (since second-stage innovators are making non-negative profits) are positive. Hence, if $p > 0$ the sum is positive and welfare is higher with intellectual property.

\[ \square \]


**Proof.** In this case comparing the IP to the no/weak IP regime we have the following differences:

**(+) Under IP there are** $(p)$ **extra first-stage (and dependent second-stage) innovation because the royalty income allows some first-stage innovators to produce who would not otherwise:**

\[
p \left( v_1^L + v_2(k_\alpha) - x_\alpha q(k_\alpha)v_2^L \right)
\]

**surplus per extra first stage innovation**

**(−) For the** $(1 - p)$ **first-stage innovations that occur under both IP and no/weak IP there are fewer associated second-stage innovations due to licensing failure (licensing failure cost) and the innovations are of lower average value due to reduced experimentation (reduced experimentation cost):**

\[
-(1 - p) \left( \underbrace{v_2(k_2) - (v_2(k_\alpha))} + x_\alpha q(k_\alpha)v_2^L \right)
\]

Reduced Experimentation Cost Licensing Failure Cost

An IP regime is optimal compared to a weak IP (WIP) if the first effect is larger than the second (and vice versa):

\[
p(v_1^L + v_2(k_\alpha) - x_\alpha q(k_\alpha)v_2^L) - (1 - p)(v_2(k_2) - v_2(k_\alpha) + x_\alpha q(k_\alpha)v_2^L) \geq 0
\]

\[
\iff p \geq p^m \equiv \frac{(v_2(k_2) - v_2(k_\alpha)) + x_\alpha q(k_\alpha)v_2^L}{(v_2(k_2) - v_2(k_\alpha)) + x_\alpha q(k_\alpha)v_2^L + ((v_2(k_\alpha) - x_\alpha q(k_\alpha)v_2^L) - (v_1^L))}
\]

Where $p^m$ has been defined as the probability of a low value first-stage innovation which leaves one indifferent between having and not having intellectual property rights. \[ \square \]

Proof. Define:

\[ S = \text{Higher Experimentation Cost} = (v_2(k_2) - v_2(k_\alpha)) \]

\[ H = \text{Licensing Failure Cost} = x_\alpha q(k_\alpha) v_2^L \]

\[ E = \text{Surplus per Extra Stage 1} = v_2(k_\alpha) - x_\alpha q(k_\alpha) v_2^L - (-v_1^L) \]

Then,

\[ p^m = \frac{S + H}{S + H + E} \]

Examining the differentials of \( S, H, E \) we have:

\[
\frac{dS}{d\tau} = \frac{\partial}{\partial \tau} (v_2(k_2) - v_2(k_\alpha)) + \frac{dv_2(k_2)}{dk_2} \frac{dk_2}{d\tau} - \frac{dv_2(k_\alpha)}{dk_\alpha} \frac{dk_\alpha}{d\tau} = (-) + (+ \cdot -) + (+ \cdot 0) = -
\]

\[
\frac{dH}{d\tau} = \frac{dx_\alpha}{d\tau} (\cdots) + (\cdots) \frac{dk_\alpha}{d\tau} = (- \cdot +) + 0 = -
\]

\[
\frac{dE}{d\tau} = \frac{dE}{dk_\alpha} \frac{dk_\alpha}{d\tau} = (\cdots) \cdot 0 = 0
\]

Similarly,

\[
\frac{dS}{dv_2^H} = +
\]

\[
\frac{dH}{dv_2^H} = +
\]

\[
\frac{dE}{dv_2^H} = -
\]

For the last equation note, that by definition of \( k_\alpha, v_2(k_\alpha) = (1 + q(k_\alpha)) v_2^L - k_\alpha \tau \) and that for \( k < k_2, v'(k) > 0 \) so that:

\[
\frac{dv_2(k_\alpha)}{dv_2^H} = \frac{\partial v_2(k_\alpha)}{\partial v_2^H} + v'(k_\alpha) \frac{dk_\alpha}{dv_2^H} = 0 + (+ \cdot -) = -
\]

Putting these derivatives together with the derivative of \( p^m \) with respect to \( S, H, E \) we have the required result. \( \Box \)
References


