Passive Investors, Active Traders and Strategic Delegation of Price Discovery

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December 2009

CWPE 0951
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Original version: December 2009

May 2010 update: Contact address for comments changed to m.jezek.04@cantab.net

Abstract

This paper points out that there is little theoretical or empirical support for the concept of rational performance-chasing equilibrium that appeared in the recent literature. A more accurate model of current active market dynamics involves investor confusion, which is partly driven by some managers’ performance manipulation. Unlike the former, the incentive structure in the latter model is fragile and not robust to social learning. More rationality means more passive informationless investing, which may ultimately lead to reduced price efficiency and greater misallocation of real resources. The recent growth of index products may continue unabated since there is no invisible hand that would limit it.

JEL Classification: G10, G11, G14

Keywords: Passive vs. active investing, performance manipulation, market efficiency

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1 Introduction

There has been a steady growth of index funds since the 1970s. Whether index mutual funds or the more recent exchange traded funds (ETFs), these investment vehicles allow individual and institutional investors alike to track a broadly defined index with minimum management fees. Prior to that, practically all investors engaged in active portfolio management, whether directly or through an intermediary, hoping to select securities that would deliver better than average returns. Crucially, such investing involves acquisition of information and its analysis. In contrast, indexing is an informationless strategy involving little more than mechanical buying and selling of securities proportional to their weight in the index. It passively relies on the ability of the rest of the market to determine prices correctly. For the real economy to operate efficiently, it is desirable that financial markets deliver price discovery without excessive volatility and provide liquidity at low cost. Therefore, the ratio of active and passive trading in the market is of great economic importance.

The objective of this paper is twofold. Firstly, we address the problem of incentives to go active. It is often glossed over in the literature but in our view, surprisingly many models treat the issue in an unsatisfactory manner. What might appear as a subtle point in academic discourse has in fact profound implications for practical portfolio management, consumer protection and regulation of institutions. Notably, we point out that the incentives are not necessarily related to market efficiency as is often claimed. Secondly, we present a model that integrates fragmented research on performance manipulation by fund managers and demonstrates how powerful such practices can be, given high attrition in the industry, in creating and sustaining confusion (‘alpha illusion’) on the part of unsophisticated investors. Specifically, we seek to improve on the rational performance-chasing story of Berk and Green (2004) and point out that such models tacitly overstate the amount of alpha in the system. Without alpha illusion, market activity and the price discovery machinery would grind to a halt. Up to a certain level, inefficiencies at the individual level may paradoxically be desirable in order to eliminate potentially much more serious inefficiencies at the aggregate level. Whether in academic research or policymaking, we emphasize that it is a sine qua non for a realistic paradigm of financial market activity to explicitly recognize such an incentive structure. For instance, the conclusion of Berk and Green (2004) or Berk (2005) that active mutual fund managers as a group are justly compensated for the value they add to their investors becomes rather tenuous under closer scrutiny and, implicitly, such models might give us a false sense of security regarding the stability of the price discovery process.

Compared to passive index tracking, active strategies are associated with less risk diversification, larger transaction costs and capital gains taxes due to higher portfolio turnover and, most importantly, with relatively high fees for delegated portfolio management. Still, the size of the active management industry does not seem to reflect that. In the US, for instance, the number of stock/hybrid mutual funds, most of them actively managed, is similar to the number of all publicly traded stocks. French (2008) estimates the annual aggregate cost of active investing in the US stock market between 1980 and 2006, over and above the cost of passive investing, at 0.67 % of the total market capitalization. High compensation in the

\(^1\) See www.ici.org.
\(^2\) See www.wilshire.com.
\(^3\) He also shows, for instance, that investors turned over the entire stock market portfolio more than twice in 2007. Bogle (2008) estimates the total cost of financial intermediation in the US and poses critical questions about the design and efficiency of the sector. Indeed, many of them were shortly answered as the financial
industry has attracted top talent and natural questions arise regarding social optimality of such allocation of scarce resources.\textsuperscript{4} The absorption of human capital by markets for financial capital has presumably improved the quality of price discovery. Public good aside, however, we do not fully understand the investors’ (private) incentives to pursue active rather than passive strategies.

There has been a long search for the appropriate financial markets paradigm in terms of price discovery and for many, the jury is still out. Fama (1970) defines an efficient market as one in which prices always fully reflect available information. When any investor comes to the market to trade on relevant information, others have almost surely traded on that already and so it is impossible to consistently achieve abnormal profits. The Efficient Market Hypothesis (EMH), however, is little more than a conjecture about the outcome of the complex price formation mechanism in financial markets and its proper empirical testing is difficult.\textsuperscript{5} A closer examination of the market structure suggests that in theory, markets might easily not be efficient. Grossman and Stiglitz (1980), for instance, argue that markets cannot be informationally efficient all the time since there would be no incentives for costly private acquisition of information if it were immediately and fully reflected in prices.\textsuperscript{6} Taking the latter paradigm to the real world, does it mean investors should go active?

Sharpe (1991) states succinctly the basic arithmetic behind active portfolio management. In summary, the average active investor cannot beat the market gross of costs and, in fact, will be beaten by it net of costs. While that does not preclude some investors from consistently outperforming the market, many academic studies have found little empirical evidence of their existence\textsuperscript{7} although some authors claim to have identified fund managers who have been able to deliver significant alpha.\textsuperscript{8} Indeed, identification of true skills is no less of a daunting task for experts than for unsophisticated investors. Nomen est omen, Weisman (2002) exposes investment strategies employed primarily but not exclusively by hedge funds that mask the true nature of the underlying return-generating process and tend to create alpha illusion for a sufficiently long time to allow for rent extraction from unsuspecting investors. He concludes that the resulting biases of traditional performance measures have “significant negative implications for both the asset allocation process and the validity of considerable academic research” (p. 91).

It seems rational for an investor who believes in the EMH, based on selected academic studies, to strategically delegate (costly) price discovery to the rest of the market and go passive. If all investors did so, however, would the EMH still hold? Would price discovery be of the same quality if no information was generated through costly analyses by professional investors and prices were determined, perhaps, by a handful of insiders? The textbook of Bodie et al. (2008, p. 11), which is consistent with the curriculum of the Chartered Financial Analyst (CFA) Program, states:

\textit{If the [EMH] were taken to the extreme, there would be no point in active security analysis.}\footnote{Samuelson (1974) provides an early such challenge to the universal efforts to ‘beat the market’.}  

\textsuperscript{4}Samuelson (1974) provides an early such challenge to the universal efforts to ‘beat the market’.  

\textsuperscript{5}This is because any test of the EMH jointly tests the equilibrium asset pricing model.  

\textsuperscript{6}Muendler (2007) demonstrates the possibility of a fully revealing equilibrium with costly information acquisition if there is a finite number of investors who can adjust the size of their portfolios through intertemporal consumption decisions.  

\textsuperscript{7}See e.g. Treynor and Mazuy (1966), Jensen (1968), Malkiel (1995), Carhart (1997) or Quigley and Sinquefield (2000).  

\textsuperscript{8}See e.g. Cohen et al. (2005), Kosowski et al. (2006), Chen and Liang (2007), Jiang et al. (2007), Cuthbertson et al. (2008) or Cremers and Petajisto (2009).
ysis; only fools would commit resources to actively analyze securities. Without ongoing security analysis, however, prices eventually would depart from ‘correct’ values, creating new incentives for experts to move in. Therefore, even in environments as competitive as the financial markets, we may observe only near-efficiency, and profit opportunities may exist for especially diligent and creative investors.

This conventional wisdom implies a natural limit to indexing.

The structure of our paper is as follows. Section 2 argues that although the corrective mechanism suggested above may be intuitively appealing, we should not rely on such rational equilibrating forces in the real world. Indeed, Woolley and Bird (2003) claim that a high level of indexing supplemented with quasi-indexing by active managers who strictly control their tracking error relative to a given benchmark has already contributed to excessive and wasteful investments, which ultimately results in lower economic growth and lower investor returns along with higher volatility associated with bubbles. We formalize and further develop the underlying story. This continues in Section 3 which formally states a fundamental alpha constraint independent of price efficiency and argues that the model of active portfolio management by Berk and Green (2004) is driven purely by violating that constraint with little empirical justification. The importance of clarifying the latter cannot be overstated since such models might be incorrectly interpreted as academic endorsement of ‘rational equilibrium performance chasing’.

In contrast, presenting an alternative paradigm, Section 4 models performance manipulation by active fund managers seeking to profit from client-related rather than securities-related informational asymmetries and it shows that the empirical attrition rate among active fund managers combined with such manipulative techniques is sufficient to generate the illusion that active managers tend to beat the market. Boundedly rational investors in a complex environment may not see through that easily. However, index products are a financial innovation which exploits transparency of modern capital markets and, rather than increasing complexity, combines simplicity with robust performance. Section 5 discusses whether such investors could learn their way out of their predicament, possibly ‘externally’ through enhanced protection of consumers as well as shareholders of institutional investors. Section 6 concludes, stressing that the recent steady growth of index products indicates that social learning or more broadly defined innovation diffusion is under way. This might ultimately affect the efficiency with which capital markets service the real economy and there is rather little policymakers could do to move the system out of such an equilibrium.

2 Capital Markets and the Passive Investment Strategy

Price discovery is an important externality from trading in financial markets. The efficient market mechanism is supposed to average out individual forecast errors and send out the best available signals about fundamentals. Therefore, it is of great import to understand the incentives for (costly) information acquisition, its analysis and synthesis and subsequent production of new information in the form of forecasts and informed opinions. We argue that some financial markets, in particular those allowing for relatively precise replication of the market portfolio, are characterized by incentives not to acquire and produce information about the securities traded, regardless of possible mispricing. This seems especially the case of the stock market and to a lesser extent the bond market.

The ‘wisdom of the crowd’ is often sought after in much wider areas of human activity. See e.g. the prediction markets at www.intrade.com.
Doran et al. (2010, p. 175) summarize the findings of their survey distributed to over 4,000 finance professors in the US as follows:

They show little agreement regarding the semi-strong form market efficiency. Despite their ostensible disagreement, their investing objectives suggest they generally believe that markets are semi-strong form efficient; twice as many of them passively invest as actively invest. We also come to the surprising conclusion that a finance professor’s opinion of market efficiency has little influence on his decision to actively or passively invest. ... This contradicts the fundamental notion that the active versus passive decision is driven by an assessment of the market’s efficiency.

While a believer in the semi-strong EMH should indeed go passive (absent inside information), we stress that even without such a belief and with ample time at hand for active analysis, the decisions are rather unsurprising. What is perhaps surprising is active investment strategies of many university endowment funds in markets where ultra-cheap passive alternatives are readily available. The following detailed analysis is an attempt to demystify the active/passive controversy.

2.1 Sitting on the Shoulders of Giants

First, we start with a simple example. There are two firms in the economy. In the current period, firm 1 and firm 2 issue shares and pay out terminating dividends in the next period. No alternative assets that enable transfer of consumption over time are available. There are two equally wealthy investors and the supply of securities is perfectly observable to both. Investor \( X \) is of poor intelligence and little education but his virtue is that he knows that he does not know. Investor \( A \), in contrast, has a skyrocketing IQ, excellent education and she is aware of both. \( X \) knows who \( A \) is and vice versa.

\( X \), acting blindly with no analysis whatsoever, buys half of the shares in both companies, whatever their prices. \( A \) does research into the fundamentals of the projects to be undertaken and forecasts the terminal cash flows, which she rationally prefers to a blind bet or, equivalently, reliance on \( X \). She incurs cost \( c \) of the information acquisition. Depending on her forecasts and perceived risks, she allocates part of her wealth to stocks and consumes the rest. Assume there are equilibrium prices \( P_1 \) and \( P_2 \) such that \( A \) is willing to purchase half of the shares of each company. Now for any \( P_1 \) and \( P_2 \), whatever the terminal cash flows turn out to be, the return on the investment of \( X \) will be the same as that of \( A \) gross of \( c \) and the former will exceed the latter net of \( c \). As long as \( A \) lets \( X \) buy a scaled-down version of the market, at whatever prices, the former has lost.

It is possible that \( A \) finds out that company 1 has superior technology and management and its return on every dollar invested always exceeds that of company 2. In such a case, \( A \) will never invest in company 2, neither will \( X \) (who would then have to hold all its shares rather than a half) and the inefficient company efficiently exits the market. \( X \) still beats \( A \), however, and there is no (credible) mechanism through which \( A \) can unload part of \( c \) on \( X \) or drive it out of the public market. Not all information has to be acquired in a costly fashion, of course. Some is possessed ‘naturally’ and is distributed asymmetrically across the investor population. But even if \( A \) were the ‘natural insider’ and \( c = 0 \), her informational (and intellectual) superiority would not translate into superior investment performance.
2.2 Charity Begins at Home

Now consider a capital market where \( N \) securities are dynamically traded and their supply \( Q = (Q_1, \ldots, Q_N)' \) is publicly known. Let \( X \) be an index fund buying and selling all securities in proportion to their market supply. At time \( t_1 \), it buys a fraction \( x \) of the market at the current prices \( P = (P_1, \ldots, P_N)' \). Trivially, between then and any time \( t_2 \) with prices \( \tilde{P} \),

\[
\tilde{r}_X = \frac{\tilde{P}' \cdot x \cdot Q}{\tilde{P}' \cdot x \cdot Q} - 1 = \frac{\tilde{P}' \cdot Q}{\tilde{P}' \cdot Q} - 1 = \tilde{r}_M
\]

and so the rate of return on the purchased securities, \( \tilde{r}_X \), is the same as the rate of return on the entire market, \( \tilde{r}_M \). Similarly, let \( A \) be an active fund conducting security analysis and setting prices which \( X \), as it buys and sells fractions of the market, passively accepts. Assume for a moment that investors cannot trade directly and can only invest through the two intermediaries. Therefore, in the absence of (active) competition, we prevent \( A \) from taking advantage of \( X \) by setting prices away from its valuations.

Both funds are marked to market and a marginal investor decides which one to buy. If \( X \) holds the fraction \( x \in (0, 1) \) of the market at time \( t_1 \), \( A \)'s holdings are \( (1 - x) \cdot Q \) and so by (1), the return on the latter between then and any future time \( t_2 \) is \( \tilde{r}_M \). For simplicity, the services of \( X \) are free while \( A \) charges its investors active management fees. Hence, the return to the latter investors must be \( \tilde{r}_A = \tilde{r}_M + \alpha_A \) where \( \alpha_A < 0 \) reflects the costs of active management over the period. Since \( \tilde{r}_X \) first-order stochastically dominates \( \tilde{r}_A \), no rational nonsatiated expected utility maximizer, even if risk-seeking, chooses \( A \) over \( X \).

Investors may well realize the collective consequences of their individual actions for the informativeness of security prices and allocation of resources in the real economy, which will eventually negatively feed into capital market performance, but that is unlikely to overcome the coordination failure if there is a large number of (anonymous) investors. A swap of \( X \)'s zero alpha for \( A \)'s negative alpha is hard to rationalize without heroic assumptions about preferences and outside investment options. Instead, let us split \( A \) into a large number of active funds. This opens up a Pandora’s box of alpha marketing by their managers as it is now possible for their subset to achieve positive alpha although it must remain negative in the aggregate.\(^{11}\) A single active superfund made reality inescapable, suppressing any alpha dreams. In contrast, a less transparent system of competing professional fund managers may well attract a substantial number of investors by appealing to their psychological biases and exploiting the less informed. Awareness raising and learning, however, might undo such effects and reestablish the dominance of indexing.

2.3 The Art of Piggybacking in Practice

While information acquisition and pricing in some asset markets is of little relevance for social welfare,\(^ {12}\) capital markets are the backbone of market economies and, therefore, dominance

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\(^{10}\) Throughout, tildes denote random variables. In this example, \( X \) perfectly tracks the capitalization-weighted index of the entire market. For simplicity, the securities and their quantities are the same throughout the evaluation period. In practice, at least over longer periods, some disappear (bankruptcy, default, maturity), new ones are issued (IPO, new debt) and the quantities of the existing ones change (seasoned issues, conversions, repurchases, free float adjustments). In such a case, intermediate rebalancing of the market-replicating portfolio is needed.

\(^{11}\) Rational risk seekers would be attracted by the extra active risk.

\(^{12}\) See e.g. markets for collectibles, including stamp indices, at www.stanleygibbons.com.
of informationless strategies might undermine the efficiency of both. How relevant are our examples in practice?

Due to dynamic changes in the supply of securities, including ‘births’ and ‘deaths’, and their cross-sectionally as well as temporally varying liquidity, indexers’ free ride is not smooth. Whenever quantities change, indexers are forced to buy/sell individual securities and sell/buy a slice of the ‘old’ market. Their price impact and other transaction costs then result in tracking error. Some markets are harder to track than others and indexers often follow only their sufficiently liquid segments. For instance, the total bond market with a large dynamically changing number of issues and diverse liquidity is harder to replicate than the total stock market. Overall, however, passive benchmark trackers operate mostly with tight tracking errors and, crucially, at a fraction of active managers’ costs.

Therefore, as expected, the majority of active managers lag behind their passive counterparts. For instance, Standard & Poor’s (2009, p. 1) reports the performance of active mutual funds as follows:

Over the five year market cycle from 2004 to 2008, S&P 500 outperformed 71.9 % of actively managed large cap funds, S&P MidCap 400 outperformed 79.1 % of mid cap funds and S&P SmallCap 600 outperformed 85.5 % of small cap funds. These results are similar to that of the previous five year cycle from 1999 to 2003. ... Benchmark indices outperformed a majority of actively managed fixed income funds in all categories over a five-year horizon. Five year benchmark shortfall ranges from 2-3 % per annum for municipal bond funds to 1-5 % per annum for investment grade bond funds. ... [I]ndices outperform a majority of actively managed non-U.S. equity funds over the past five years.

Indeed, as many active managers come to terms with their grim reality, they often turn into so-called ‘closet trackers’. While charging active management fees, they tend to passively follow the index, expecting nothing, hoping for the best and being prepared for the worst.

Yet, all that does not seem to disturb the great majority of investors on a wild alpha chase. According to the Investment Company Institute (2009), for instance, the assets in equity index funds relative to the assets in all equity mutual funds rose from 8.9 % in 1999 to 11.3 % in 2004 to 13 % in 2008. More generally, the data in the report allow us to construct a time series of the combined assets managed by index funds and ETFs relative to the assets managed by all mutual funds, closed-end funds (CEFs) and ETFs. This approximate measure of explicit

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13Unless the supply of all securities changes proportionately or some of them drop out of the market worthless.
14E.g. multiday executions may reduce price impact but at the same time open a back door for tracking error as market life moves on. To offset some of the transaction costs, indexers often run a securities lending program. Technically, tracking error is defined by some authors as the difference between portfolio returns and the corresponding benchmark (index) returns [Black and Litterman (1992), Roll (1992)] but its more usual definition, embraced by practitioners, is the standard deviation of that difference [Satchell and Hwang (2001)]. Tracking error is also known as ‘active risk’ (for active managers, the former is a misnomer).
15To optimize subject to transaction costs and liquidity constraints, some indexers replace the full replication approach with stratified sampling, i.e. random selection from predefined subsets (strata) of the market. This informationless strategy is in line with the EMH as described in Malkiel (2007, p. 24): “[A] blindfolded monkey throwing darts at a newspaper’s financial pages could select a portfolio that would do just as well as one carefully selected by the experts.” Sampling is not unbiased if liquidity considerations affect the process.
16If the tracking error is small to negligible, the difference between the index return and a fund return comes down roughly to the fund’s total expense ratio, i.e. total expenses as a fraction of the assets.
17Adjusted for survivorship bias, net of fees, excluding loads.
18The mutual fund data in the report exclude mutual funds that primarily invest in other mutual funds but include mutual fund holdings of ETFs and CEFs. Note that such double counting, like closet tracking, could only reinforce the point we are making.
Indexation by the above US investment companies increased from 6% in 1999 to 9.1% in 2004 to 11% in 2008. The recent growth of index funds and particularly ETFs coincides with a steady decline in direct holdings by individuals. For instance, French (2008) shows that approximately 60% of US stocks were held either directly by individuals or by mutual funds, CEFs and ETFs between 1999 and 2006. Direct holdings fell from more than a third of the market to less than a quarter in favour of the investment companies during that period. Still, such social learning is rather underwhelming and is most likely inhibited by exploitation of informational asymmetries by the active management industry, both directly and through the media.

Notably, passive strategies are embraced more by institutional investors. Calculations based on data provided in French (2008) reveal that about 25% of the stock market was held in public/endowment funds and defined benefit/contribution pension plans throughout the period considered above and about a tenth of the market was steadily under their passive management. The remaining roughly 15% of the stock market was in the ownership of employees, banks, insurance companies and hedge funds. In sum, although exact data are not available, a conservative estimate is that about a fifth or less of the US stock market has recently been under (openly) passive management.

2.4 Limits to Indexing

Historically, capital markets experienced the extreme of universal active management. Could the other extreme of universal passive management ever occur? Almost certainly not. While indexing might resemble buying and holding any broadly diversified portfolio, such an approach usually involves initial strategic security selection. In contrast, indexers give up on any security selection or market timing and strategically delegate price discovery to the rest of the market, passively rebalancing at the time of index reconstitution. They do not attempt to beat the market, they want to be the market, whatever the prices. Therefore, if all but one investors were passive, the remaining one would possess pricing power that would allow her to (actively) buy low and sell high over time while accommodating indexers’ asynchronous liquidity trading needs. If a new security were issued, *ceteris paribus*, all passive investors would sell a slice of the ‘old’ market to their active counterpart in order to buy the new security and the active trader would also buy the residual supply of the latter. At the end of the day, everybody would hold a slice of the ‘new’ market, after which index trading and pricing by the active trader would resume. Thus, at the minimum, there are incentives for the active segment to grow until it becomes competitive.

That, however, is unlikely to deliver fair pricing in the EMH sense. Trading would be infrequent, with entire indices typically changing hands, and arrival of new company-specific

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19Index funds and ETFs hold predominantly equity while the stock market is considerably smaller than the bond market in the US. However, paucity of reliable data on institutional investors’ strategies prevents us from numerically estimating the indexation of the latter.

20This is typical of unit investment trusts rather than index funds or ETFs.

21This does not apply to execution quality as indexers strive to minimize their price impact and time their trades around index changes to keep tracking error down. In fact, since market makers know that they do not trade on insider information, indexers are often able to secure preferential prices relative to market bid-ask spreads. Cf. Kyle (1985).

22By submitting limit orders around some (arbitrary) prices to keep an order-driven market liquid or, equivalently, by becoming a securities dealer who makes a quote-driven market by setting bid and ask prices.

23In this simplified exposition, as few as two competitors à la Bertrand might suffice.
information in the public domain would mostly be followed by no trades at all. More generally, competition with respect to exploitation of the ovine indexers does not necessarily imply socially optimal, or any, acquisition of information about fundamental values and its analysis. Naturally, the theoretical possibility of the blind leading the blind in an information vacuum is not pertinent to reality. This is because the competitive speculators realize that they hold an inventory of claims to cash flows, however uncertain and distant in time, and so some informed valuation of these will always be conducted. The quality of the process is nevertheless a potential cause of concern.

Even if prices reflected ‘all available information’ according to the EMH, some socially useful information might never see daylight. It seems quite likely that a rich price formation process involving multiple informed opinions, especially those of potential long-term holders of securities, averages out idiosyncratic forecast errors and sends higher-quality information to the real economy. It is then desirable that there be a relatively large number of sophisticated investors who actively collect ‘all available information’ and produce new information in the form of insights about securities for their prices to be fair. Thus, reaching the limits of indexing seems neither socially optimal nor beneficial for the indexer population. However, maximally rational capital markets provide incentives for investors to index, counterfactually, until that limit is reached.

3 Investment Skills: Models and Reality

Consider a market in which $I$ fund managers follow buy-and-hold investment strategies over the time interval $[t_1, t_2]$. At time $t_1$, manager $i$ constructs a portfolio of securities worth the fraction $w_i$ of the total market capitalization. Passive managers replicate the market and incur no costs while active managers conduct security analysis to identify mispricing and charge active management fees. Then, the following intuitive result holds.

**Proposition 1:** Let there be a risk-averse investor with preferences over final wealth who selects a fund manager. Given $\alpha_i = 0$ for indexers,

$$\sum_{i=1}^{I} w_i \cdot \alpha_i \leq 0 \quad (2)$$

where $\alpha_i$ is defined as the value added by manager $i$ over and above the index as measured by the

i) expected excess return;

ii) absolute or relative gain in the investor’s expected wealth;

iii) absolute or relative gain in the investor’s expected utility.

Except for the extreme case of universal passive investing, the inequality is strict.

**Proof:** In the Appendix.

This confirms active portfolio management as a negative-sum alpha contest. It does not necessarily refer to the market for all existing securities, it can be any segment of it which constitutes an index. Without further information, whatever the investor’s prior belief about

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\[24\] I.e. with all participants being rational [Rubinstein (2001)].
the index performance, there are no rational incentives to pick an active manager who in turn picks mispriced securities to beat the index. The Appendix explains this in more detail.

Let us consider full trading dynamics. Firstly, (2) holds between any two adjacent trading points, with the weights and alphas generally varying over time. Even under continuous trading, as \( t_2 \to t_1 \), the constraint must hold over each infinitesimal buy-and-hold period. Thus, it applies to investors rebalancing quarterly or annually as well as to algorithmic traders, currently major providers of market liquidity,\(^{25}\) for whom the relevant time period is measured in microseconds. Secondly, if there are intermediate cash flows in and out of the market, e.g. due to dividends, repurchases or new issues, the proper benchmark return reflecting what the average investor earns in the market over any period of time is the dollar-weighted (internal) rate of return rather than the geometric average of market rates of return over given subperiods.\(^{26}\) Analogously, at a lower level of aggregation, if a fund experiences intermediate inflows and outflows, the average return its investors earn over a given period is the dollar-weighted return. The alternative buy-and-hold return, or the geometric average of fund returns over given subperiods, only reflects how the initially invested dollar has performed, irrespective of the timing and magnitude of further flows.

Fund returns often serve as the basis for skills measurement because inflows and outflows are largely outside of the control of the manager. However, the evaluation of investment skills is a daunting task whether the manager’s track record is summarized by fund returns or the dollar-weighted return. If the manager’s mandate restricts her investments to the market(s) under consideration, she must pick securities based on perceived risk-return characteristics with any given assets under management and against any active-passive decisions of the other managers, while those clearly affect her investment opportunity set through (2). For instance, a skilled manager with superior past returns who is open to new investors may attract strong flows and experience alpha decay as a consequence of increased \( w_i \). Thus, if investors chase past winners, an upward bias of (positive) alpha estimates needs to be taken into account. In sum, even if the intellectual abilities of all \( I \) managers were constant over time, their investment skills would most likely be time-varying.

In general, the market is a complex dynamic system riddled with higher-order beliefs and herding tendencies where profitable strategies lose their glamour sooner rather than later. Any exploitable profit opportunity is only temporary as proprietary knowledge eventually leaks. Even the so-called ‘behavioural’ strategies taking advantage of anomalies caused by investor irrationality eventually become unprofitable as too many investors pursue them in search for profit, including those irrational investors who have previously created them, thus chasing their own past errors. While their irrationality will stay, its market manifestation will continually change. Therefore, investment skills are not fortuitous adherence to a momentarily successful investment style but the ability to keep ahead of the curve and continually identify new successful strategies as the old ones die out.

In our framework, the EMH implies that alphas of indexers are zero and alphas of active traders are negative. However, the reverse implication does not hold although the absence of (positive) alpha is popularly interpreted as evidence of market efficiency. With equally (un)skilled active managers, i.e. each of them outsmarting the rest equally often and by equal amounts, the market would be alpha-free regardless of the amplitude and frequency of

\(^{25}\)For instance, Financial Times (2009) reported that high-frequency trading accounted for as much as 73% of US daily equity volume.

\(^{26}\)Dichev (2007) with Keswani and Stolin (2008) provide empirical evidence on the difference between investor returns and market returns.
oscillation of (disequilibrium) prices around fundamentals. The latter are determined by the true rational equilibrium asset pricing model that market participants could collectively fail to implement most of the time, contrary to the EMH, thus creating excessive volatility. The roller-coaster ride in Figure 1 might well provide an example of that.

Grossman and Stiglitz (1980), using an ad hoc partial equilibrium model with rational expectations, make the point that some ‘equilibrium degree of disequilibrium’ is needed so that those who expend resources to obtain information are compensated. However, (2) implies that such compensation can only come from the presence of exploitable investors, regardless of how far away from fundamentals prices drift. In a maximally rational market, such investors would index and no compensation for security analysis and selection would be possible.

The trillion-dollar question is who the exploitable investors are. If there were a large

\[\text{Figure 1: The S&P 500 stock index since 1975.}\]

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27Essentially, the EMH proposes that the true asset pricing model in the economy is fed with all available information efficiently. Thus, the aforementioned joint hypothesis problem implies that model specification is part and parcel of any empirical test of the EMH, making a stand-alone rejection of the latter practically impossible. Fama (1970) acknowledges this on p. 384.

28In a statistical/econometric analogy, the market might not act as an efficient estimator of the ‘true’ value and yet provide little incentive for fundamental trading that would improve the properties of the estimator.

29Inter alia, their uninformed investors do not observe the risky asset supply. We address modern transparent securities markets which do not have such a feature. Thus, indexers can focus on free ‘quantity discovery’ rather than costly price discovery.

30E.g. French (2008) estimates that the present value of the costs of active investing in the US stock market is over 10% of its current capitalization, which alone roughly corresponds to the stated amount.
number of unskilled active traders constructing their active portfolios randomly and independently both in the cross section and over time, they would tend to hold an expensive index fund in the aggregate, from which sophisticated investors could not draw their alpha. The latter group can only thrive in the presence of systematic underperformers, i.e. traders who believe they act independently (and smartly) but do not realize that others know something about them that they do not know about themselves. For instance, their actions may be correlated through fads and sentiment, thus possibly allowing sophisticated investors to make superior picks at their expense. The underperformers may be reluctant to learn and admit the truth about themselves and/or there may, in theory rather than practice, be a steady arrival of new unsuccessful alpha contestants replacing the old ones who rationally update their overoptimistic priors and promptly quit.

For instance, individual investors are often presumed to be less skilled than professional managers. They usually lack expertise, chase styles successful in the past and trade on infotainment. If it is the case, then the aforementioned recent fall in direct investing in the US stock market in parallel with the growth of index funds and ETFs is likely to have depressed alphas in the residual active segment of the market. If such a trend continues, a rational response of many sophisticated and previously active investors might be to go passive, either openly or in a ‘closet’ fashion, as the ‘source’ of their own alpha disappears. Such indexing acceleration might bring about price discovery deterioration. The change in the structure of market participants yet again illustrates how external effects can cause time variation of alphas of managers with time-invariant intellectual abilities.

Investors can switch between funds and, if we introduce an outside investment opportunity, they can time the market by moving in and out, hoping to buy low and sell high. Each investor’s (dollar-weighted) return in the market will typically differ from fund returns, if only because of idiosyncratic cash flows on account of labour income or liquidity shocks. Alternatively, fund picking and switching might be delegated to fund of funds managers and, similarly, market timing could be conducted by managers with a mandate to invest outside of the market. In the latter case, however, the principle remains the same. Namely, to maximize alpha while in the market, which would increase the ‘broader alpha’ defined over the market and the outside option.

The alpha we have used so far is related to security selection within the market(s) under consideration. In contrast, pure market timing or tactical asset allocation is equivalent to

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31 Pure liquidity traders would trade ETFs rather than attempt to pick the most mispriced securities.
32 Formally, skilled investors must construct zero-cost portfolios \( q^* \), as in (15) in the Appendix, that are profitable in (true) expectation.
33 For example, they often incorrectly interpret ‘hot tips’ from analysts and the media as strategic information rather than entertainment or, in some cases, manipulation.
34 Barras et al. (2010) find that a quarter of US domestic equity mutual funds have statistically significant negative alpha and the proportion of skilled funds decreased from 14.4 % in 1990 to statistically insignificant 0.6 % in 2006 (ignoring the additional negative effect of loads and taxes).
35 Cremers and Petajisto (2009) find a rapid increase in the proportion of closet indexers among US all-equity mutual funds from the late 1980s to 2003, suggesting that they have only been a relatively recent phenomenon.
36 Gruber (1996) suggests that apart from being misled by advertising/broker advice, some investors may be trapped in underperforming funds for tax reasons or due to pension plan restrictions. However, that does not explain why future investors should get into such a position in the first place. Indeed, the following one and a half decades showed a massive surge in index products. Also, informationally disadvantaged retail investors have been empowered by the internet.
37 Throughout, direct investors are simply assumed to be managers of their own single-investor funds.
dynamic trading of index funds. In the latter, the (imperfect) analogy to the passive informationless approach towards security selection is gradual investing over time to average out market-timing errors. However, in light of the previous discussion regarding the acquisition and analysis of information about individual issuers, even ‘active’ market timing is likely to contribute rather superficially to the quality of price discovery since timers of a broad market focus on macro risks and are most unlikely to conduct continual detailed fundamental analysis and due diligence of every single issuer. At the extreme, investors may seek exposure to the entire market in a classical CAPM style and base their periodic consumption/savings decisions on its long-term historical performance.

3.1 Rational Alpha-Chasing Equilibrium of Berk and Green (2004)

For our purposes, it is necessary to evaluate skills net of expenses. However, some authors define alpha based on returns gross of expenses, in which case the manager (direct investor) is considered skilled even if her fees (transaction and opportunity costs) exceed the value added.

In a prominent paper, Berk and Green (2004) present a fully rational model with no information asymmetry in which both mutual fund managers and their investors learn about the abilities of the former from past observed returns and flows of investor funds compete away any returns going forward that are expected to exceed the passive benchmark. Before costs, alpha of each manager is constant but since the costs are an increasing strictly convex function of the assets under management, which is to reflect the increasing difficulty in finding and, due to price impact, buying undervalued securities, flows of funds towards managers who turn out to be more skilled reduce the expected investor return to the index return. Similarly, if investors learn that their fund is underperforming the index, outflows will continue until the expected investor return matches the index.

The authors thus illustrate that managerial skills are compatible with no persistence in fund returns and argue that in a rational competitive market, active fund managers are naturally rewarded for their talents through fees capturing any value added on top of the index. The motto/subtitle of Berk (2005), promoting Berk and Green (2004), is: “Most active managers are skilled.” Concretely, their model tells them that with minimum assets under management, 80 % of managers entering the market are capable of adding value in excess of the fees charged.

Their story suffers from the following weaknesses. Firstly, their equilibrating mechanism stands and falls with the presumption that most (and all surviving) managers tend to outperform the index before costs/fees are taken off. The more skilled the manager is, the more assets she attracts and hence the larger the equilibrium amount of profit she generates over and above the passive alternative. More precisely, the authors work with the parametrization of 1.5 % for fees and 6.5 % for the average alpha. Managers with alpha below 3 % fail to attract

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38 This is essentially selection of asset classes in the ‘broader market’ in each subperiod and so Proposition 1 applies to that market. Note, however, that it might not be practically ‘indexable’, e.g. if the outside option is cash.

39 This is obviously less effective in a realistic time frame with exogenous cash flow shocks. Constantinides (1979) shows that the special case of dollar-cost averaging is suboptimal.

40 This paper is singled out because it is one of the best known and most widely cited equilibrium models of rational performance chasing. As always, only interesting papers are worth a critique and the objective of this paper is to improve upon the Berk-Green story while acknowledging their contribution.

41 They assume that the manager’s portfolio is always of the same risk as the index. (At the end, they briefly address fund volatility but risk plays no relevant role in the model.)
enough assets to cover their fixed costs and go out of business. However, by Proposition 1, seeking alpha is a predatory strategy and their model is silent about who the prey in the capital market ecosystem is. If all mutual funds are above market average before costs/fees, then some systematic losers must operate in the background, refusing to take the hint. If that is the case, then the advertised full rationality of the model is necessarily selective. Secondly, they ignore the negative effects of active risk on rational risk-averse investors.

Lastly and most importantly, they conclude that “the rational model [is] consistent with much of the empirical evidence.” (p. 1293) Similarly, Berk (2005, p. 31) argues: “[M]uch of what we observe about the behavior of actively managed mutual funds is consistent with a world populated by rational value-maximizing investors who compete with each other. . . . [R]esearchers . . . have drawn the erroneous conclusion that active managers add little value. Given the overall levels of manager compensation, one would expect that managers in aggregate should have significant levels of skill and thus add considerable value. . . . [D]ata are indeed consistent with many skilled managers who add considerable value but capture this themselves in the fees they charge.” However, the empirical evidence on the (under)performance of active mutual funds presented in Subsection 2.3 indicates quite the opposite. Similarly, Fama and French (2010) show that the aggregate portfolio of actively managed US equity mutual funds is close to the market portfolio and so the costs of active management simply reduce the aggregate returns of their investors relative to the market.

The Berk-Green paradigm is best captured by the corporate finance analogy they use to motivate their model. In particular, they compare mutual fund managers with corporate managers searching for positive net present value opportunities in real investments. In both groups, it is the skilled managers and not the competing suppliers of funds who should capture the rent on account of scarce skills. However, the active market dynamics is very different from this story.

Firstly, a good corporate manager will run the business efficiently and may spot market opportunities that would remain unexploited if she were incompetent, ceteris paribus. Her superior capital budgeting translates into higher business value without necessarily reducing, one for one, the value of other businesses. As a group, corporate managers create value and play a competitive but positive-sum game. In contrast, active fund managers do not create wealth in the sense that value added to one investor is value taken from another one, irrespective of whether pricing is fair or not. Price discovery and liquidity, which require a certain amount of active trading, are externalities that in the medium and long run improve returns on the index, not on the aggregated zero-cost active portfolios. Even if active fund managers represented the highest intellectual calibre the world has to offer, they could easily end up in a ‘stalemate of the masters’ with all investors paying something for nothing. Secondly, competition for status is a zero-sum game in any area of life but while it may be rational to stay a below-average corporate manager (or driver, cook, artist, student, researcher, etc.), below-average fund managers are not compatible with full rationality because of the freely available option for investors to become (weakly) above average.

The authors simply assume that fund managers ‘generate’ alpha, thus inviting rational investors to compete away any free lunch until an equilibrium is established in which managers are generously rewarded for their productive efforts and talents. This assumption is crucial in their model in the sense that if it is relaxed, active trading stops. The next section argues for an alternative paradigm of active trading which involves investor confusion, partly driven by

\footnote{Cf. (15) in the Appendix.}
performance manipulation. We believe that it not only better describes the current market
dynamics but also reveals the potential instability of such an incentive structure in practice.

4 The Active-Passive Portfolio Choice in Practice

The central problem any investor faces is how to tell true alpha from false one. The cum-fees return going forward that manager $i$ delivers is

$$\tilde{r}_i = \tilde{r}_X + \tilde{\epsilon}_i$$

and the proportional fee over the same period is $f_i$. As before, we assume for simplicity that indexers charge $f_i = 0$. Thus, the investor effectively pays the fraction $f_i$ of her wealth for the proportional gamble $\tilde{\epsilon}_i$. Inherently, this is certain money for a highly uncertain product.

In practice, it may translate into the following problem. The investment committee of a university endowment fund makes a strategic asset allocation decision based on the university objectives and the characteristics of major asset classes inferred from historical data, which results in a billion dollars to be allocated to ‘indexable’ stocks and fixed income instruments. If, for example, active management costs 30-150 bp more than passive management, then the decision being made is effectively whether to pay 3-15 million dollars per year to active managers who promise to trade around the relevant indices in such a way that the private benefits exceed the costs.

As Section 3 revealed, the rational uninformed prior for $\tilde{\epsilon}_i$ is a fair gamble loaded with pure active risk. Therefore, the committee members must possess credible information $\Psi$ that makes $\tilde{\epsilon}_i|\Psi$ more favourable and justifies their decision to pay $f_i$. Typically, $\Psi$ contains the track record of the manager(s) as summarized by past returns and assets under management, possibly including detailed portfolio allocations, and stated investment strategies going forward. The committee members then have the unenviable task to model, whether formally or simply in their minds, the nonstationary social processes in the active segment of the market and try to forecast managerial alpha over the coming investment period.

This is where the issue of fiduciary responsibility arises. Star managers who tend to come out as winners from the negative-sum alpha contests can be viewed as a desirable addition to the universe of all available assets in the sense that their human capital enables profitable exploitation of disequilibria in the market for the other assets. It is, therefore, legitimate to seek them. At the same time, however, verification and monitoring of the quality of their investment process are challenging at best. Importantly, rational active investing is conditional on bounded rationality of some other market participants and so $\Psi$ must necessarily involve information on that.

4.1 Complexity and Alpha Illusion

The purpose of this section is to demonstrate how seemingly minor errors made by investors in a complex environment can have a profound impact on the market landscape and, para-

\footnote{Note that $\tilde{\epsilon}_i$ corresponds to the last term in (17) in the Appendix.}

\footnote{All possible fees are expressed as a fraction of the initial wealth paid at the end of the period.}

\footnote{In practice, the latter can be expected below 10 bp for large institutional investors.}

\footnote{While price discovery and liquidity that active management produces may be public goods, the university, alumni donors, benefactors and other stakeholders certainly do not wish to pay for it, if only because education, research and leadership that the university produces serve the public good no less. Generally, the structure of investor incentives is summarized by the title of Subsection 2.2.}
doxically, keep the price discovery process alive. Managerial ability is no less difficult to establish than misvaluation of securities in the market and before costs, delegation of active portfolio management is merely a switch from one zero-sum game to another. However, while the former is supposed to be played by professionals, the latter is played largely by amateurs. Curiously, the latter investors implicitly assume the role of experts on selecting experts whose expertise they usually do not have and can therefore verify with only limited confidence.

Thus, complexity is on the side of the managers, advisers and marketeers. We shall now explore some basic techniques and strategies of sophisticated investment companies and their managers that can, intentionally or inadvertently, impede alpha learning by unsophisticated investors.

### 4.1.1 Incubation and Strategic Closures

Consider an active fund and let $A_t$ be its net asset value (NAV) per share at time $t$, i.e. the ex-fees value of a dollar invested in the fund at its inception. Let $X_t$ be the corresponding value of a dollar invested in the market index at the same time. The total performance index of the active manager is then

$$a_t \equiv \frac{A_t}{X_t}.$$  \hspace{1cm} (4)

The fund management company institutes a shutdown rule for its funds to prevent excessive losses and shed the worst underperformers. Moreover, investors react to certain patterns in performance and once its medium-term deterioration hits their tolerance threshold, the fund again folds and the manager is fired or, if it can pacify a sufficient proportion of investors, she may be replaced by a new manager with a stellar track record. In the case of discontinuation, the fund is liquidated and its shareholders receive cash redemptions or, subject to their approval, it may be merged with a more successful fund from the same fund family.

While the shutdown rule reflects a natural effort of the fund management company to identify and select the best talent, it often involves deliberate incubation of artificial star performers capable of attracting strong flows and thus generating profits through fees. The company sets up a large number of small funds, provides them with seed capital and lets them operate for some time, possibly a few years, out of public sight. After that, it starts to advertise the successful ones aggressively and, being the principal shareholder, shuts down the rest. Before the culling, the company may boost the performance of incubator funds through cross-subsidization within the fund family by means of preferential distribution of fees and allocation of underpriced IPOs, often from investment banks having soft dollar arrangements with large funds from the same family. While funds continue to drop out in the post-incubation period and the resulting survivorship bias keeps increasing with managerial tenure, incubation can render it especially severe as the fund management company can

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47 In practice, there is an additional (expensive) layer of managers who specialize in picking star funds for their funds of funds. Then, the investor only needs to become an expert on selecting experts on selecting experts. Unsurprisingly, there exist e.g. funds of funds of hedge funds, the so-called F3s, with triple-layer fees.

48 The ethical aspects of this practice in the mutual fund industry are addressed by Ackermann and Loughran (2007) or Palmiter and Taha (2009). The latter provide technical details of the incubation process and its regulatory limits. Evans (2010) estimates that almost a quarter of US domestic equity mutual funds between 1996 and 2005 were incubated, their incubation performance exceeded that of young non-incubated funds by 3.5 % p.a. on a risk-adjusted basis and the outperformance vanished after they hatched.

49 This involves excess commissions to broker-dealers in exchange for additional services and favours, such as research or ‘hot’ IPO allocations.
manufacture ‘alpha’ in a Monte Carlo fashion with little risk to the reputation of the fund family.50

Let us consider managers who have no investment skills and, their efforts notwithstanding, end up constructing dadaistic portfolios that are equivalent to random collections of securities.51 Then, we can model their investment process as

$$\frac{dA}{A} = \frac{dX}{X} - f \cdot dt + \sigma \cdot dW$$

where

$$\frac{dX}{X} = \mu_X \cdot dt + \sigma_X \cdot dW_X,$$

dW · dWX = 0, i.e. W and WX are uncorrelated standard Wiener processes, and f denotes fees as a fraction of net assets under management. By Itô’s Lemma, we obtain the process for the performance index

$$\frac{da}{a} = -f \cdot dt + \sigma \cdot dW.$$ (7)

If there is no incubation, a0 = 1. If, on the other hand, the fund incubated for period κ, then the truly initial condition is a−κ = 1 and assuming the investment process under incubation follows (5) with the post-incubation parameters (−f, σ) replaced by (µκ, σκ), we obtain52

$$a_0 \sim \logN \left[ \left( \mu_\kappa - \frac{\sigma_\kappa^2}{2} \right) \cdot \kappa, \sigma_\kappa^2 \cdot \kappa \right].$$

Cross-subsidization leads to µκ > −f.

Investors do not observe funds under incubation but the track records of those hatched become observable in retrospect and are backfilled into relevant databases. What investors see is

$$a^*_T \equiv a_t|\Theta_t$$

where T = t + κ ≥ κ is the manager’s tenure and Θt conditions on the development of the process a on [0, t]. Defining the running minimum of the process as

$$\bar{a}_t = \min_{s \in [0,t]} a_s,$$

we can state the following result.

50 In the lightly regulated alternative investment industry, hedge fund incubators are frequently set up to help managers establish an attractive track record using their personal capital. Those who succeed have their performance records audited and can begin to legally market their newly created funds to potential outside investors. If a hedge fund seeding company/incubator (effectively a multi-manager hedge fund or a fund of hedge funds) distributes seed capital across a number of managers in return for a share of their future fee income (on top of the seed investment income), its losses on unsuccessful managers are often covered multiple times by the profits resulting from flows to successful ones. Regarding the demand for seed capital, e.g. Financial Times (2008) quoted the COO of the incubation arm of a major fund of funds operator as saying: “We’re seeing a lot more investment opportunities than we’d expected. When we started out, we expected to maybe see 200 managers a year. We now expect to see between 500 and 1,000 this year.”

51 Obtained through effectively random draws of q̂ in (15) in the Appendix.

52 There is a simpler and less risky way to boost the initial track record. For instance, two managers may hold the index and, on top of that, enter opposite index futures positions. At the end of the incubation period, the unlucky fund is discontinued and the managers jointly launch the successful one. In such a case, a0 > 1 almost surely. However, incubation does not necessarily involve such scheming and may be genuinely perceived as a competitive selection process by many participants. We model the latter.
Proposition 2: Let the shutdown rule imply
\[ \Theta_t = \{ a_0 \geq c_0, \ a_t > c \} \] (11)
where \( c_0 \) and \( c \) are constants such that \( c_0 > c > 0 \). Then, the density of \( a_t^* \) is given by
\[ f_{a_t^*} (\hat{a}|c_0, c) = \frac{\Phi [\xi_1 (\hat{a}, c_0)] - \xi_2 (\hat{a}, c) \cdot \Phi [\xi_3 (\hat{a}, c_0, c)]}{\left(1 - \mathbb{P}_t^1\right) \cdot \xi_4 (\hat{a}) \cdot \Phi [\xi_5 (c_0)]} \] (12)
for \( \hat{a} > c \) and zero otherwise, where \( \Phi \) is the standard normal cumulative distribution function and the auxiliary functions \( \xi_1, \ldots, \xi_5 \) as well as \( \mathbb{P}_t^1 \), the probability of a post-incubation closure by time \( t \), are given in the Appendix.

Proof: In the Appendix.

Thus, if only those managers whose performance index equals at least \( c_0 \) at the end of the incubation period go public and if they go out of business once their performance index falls to the critical level \( c \) afterwards, we can easily contrast the performance density of survivors of a given tenure with the corresponding density implied by the true performance process. Also, we can obtain the proportion of the survivors outperforming the index.\(^{53}\)

Figure 2 shows the results obtained under the following assumptions. Under incubation, \( \mu_{\kappa} = 0 \) through cross-subsidization and relatively risky incubation strategies lead to \( \sigma_{\kappa} = 15 \% \). Incubator funds take off only if they outperform the index at the end of their one-year testing period.\(^{54}\) Fees are 1 % of the net assets under management and \( \sigma = 8 \% \).\(^{55}\) Once the total performance of a manager falls short of the index by 22.5 %, she goes out of business.

As a result, incubator funds face a 53% termination probability. The bottom right graph plots \( \mathbb{P}_t^1 \), the probability that a manager is forced out of business by a given post-incubation date, and contrasts it with the corresponding probability in the no-incubation scenario in which \( a_0 = 1 \). According to Malkiel and Saha (2005), the annual rate of attrition among US mutual funds in 1994-2003 was mostly 5-8 %. Similarly, Standard & Poor’s (2009) reports that only 73.37 % of US domestic equity mutual funds survived their performance evaluation period 2004-2008. The attrition in our no-incubation scenario is slightly less than the above, with e.g. the 5-year survival probability being 75 %. Moreover, mortality falls substantially in the incubation scenario (assuming the same \( c \)) and so in a population of incubated and nonincubated funds, the survival rate would be between the two.

While not all closures are necessarily performance-related, few in the majority that are take place because of success, especially in the first several years of the fund’s existence. Therefore, our simplified shutdown rule may well provide a realistic, albeit crude, approximation of the selection process in the mutual fund industry.\(^{56}\) Most importantly, the purpose of this exercise is to illustrate that if investors and unsophisticated experts make the relatively innocent error of relying on the visible and ignoring the largely invisible, it is sufficient to make them believe in the superiority of active portfolio management in general and/or in their own ability to find a skilled manager.

\(^{53}\)Using quadrature.

\(^{54}\)In practice, longer incubation periods enable marketing selected funds based on their string of annual successes. On the other hand, it involves greater time costs and career risks for the managers.

\(^{55}\)By (5) and (6), the net asset value \( A \) per share follows a geometric Brownian motion with the volatility parameter \( \sigma_A = (\sigma_X^2 + \sigma_t^2)^{1/2} \). For instance, \( \sigma_A = 17 \% \) under the realistic assumption of \( \sigma_X = 15 \% \).

\(^{56}\)The concept of survival in general is analyzed by Brown et al. (1995).
Figure 2: Survivors’ vs. true performance process. The top graph’s horizontal and vertical axes are $100 \cdot (\hat{a} - 1)$ and $f(\hat{a})$, respectively, where $f$ denotes the densities of $a_6^*(\kappa = 1)$, $a_5^*(\kappa \downarrow 0)$ and $a_5^*$; the legend also applies to the bottom left graph. Parameters: $f = 0.01$, $\sigma = 0.08$, $c_0 = 1$, $c = 0.775$ and $\mu_\kappa = 0$. If incubated, $\sigma_\kappa = 0.15$. If not, $\sigma_\kappa \downarrow 0$.

The top graph juxtaposes the performance density of survivors after 5 years of public existence with the density of the underlying performance process (7) started at $t = 0$ with $a_0 = 1$. It also demonstrates how aggressive incubation can make a lasting impact on the total performance of a manager/fund and perpetuate the aforementioned belief that overall, active managers tend to beat indexers. The bottom left graph indeed shows that in a population consisting of incubated and nonincubated funds, the majority of surviving managers appear to be beating the index over their career. Thus, it is possible for a fund management company to operate a large number of funds with mortality not exceeding that of the overall market such that to the naive eye, most of its managers appear to deliver positive alpha even if it is negative in reality.

In addition, let us note that the distribution of survivors’ performance exhibits high positive skewness, which may be an attractive feature per se, although the skewness of the true performance process is substantially lower. For instance, if investors have Cumulative Prospect

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57 The more recent performance of such funds will tend to be worse than the total performance. However, the latter may well bias the overall expectations of naive investors upwards, especially if advertising of such funds stresses the geometric average of fund returns rather than the dollar-weighted return as assets grow due to new inflows. Since incubator funds start small and attract public flows only if successful, the incubation effect may be naively misinterpreted as the size effect reducing the investment opportunities of a talented manager.

58 If $\sigma$ goes up, i.e. the tracking error rises, and/or $c$ is increased, mortality grows while the performance of survivors improves, and vice versa.
Theory preferences as in Barberis and Huang (2008) or form optimal beliefs/expectations as in Brunnermeier et al. (2007), the extra skewness may further raise their perceived value of active portfolio management. There may be a variety of psychological biases in the investor population that are hard to model and test empirically. Animal spirits may well be the major driver of market activity. Overconfidence may bring many investors to active trading and biased self-attribution, coupled with wishful thinking, may keep them there much longer than would be expected from a rational learner who has the option to index.

We stress a single prominent aspect of delegated portfolio management that may serve as a basis for such biases, of which the supply side takes strategic advantage and which may in itself be sufficient to explain why so many investors seem to miss the fine point made by Proposition 1. Considering that survivorship bias was for a long time ignored in empirical studies by scientists, it is perhaps not surprising that this might still be a major source of confusion among less sophisticated investors.

So far, we have considered the performance process of managers who effectively trade on noise. If there exist systematically skilled managers, they would tend to fail less frequently and, conversely, their systematically unskilled counterparts would tend to disappear more frequently. The troubling aspect of searching for alpha is that even if investors managed to perfectly correct for the incubation and survivorship bias, which is not always possible in reality, a quarter of noise-trading managers are still beating the index after 15 years, which exceeds the lifetime of a majority of funds. Adjustment for higher volatility would reduce the proportion of alpha candidates but in general, allowing for time variation of alpha as discussed in Section 3, there is little hope for meaningful statistical analysis of performance using time series of typical length or, alternatively, the estimation risks would mostly run too high for rational decision makers unless they face a truly exceptional track record.

4.1.2 Peso Problem Engineering

While some managers maximize returns relative to a benchmark index, others follow absolute return strategies which aim to deliver returns in excess of the riskfree rate in bull and bear markets alike. This is particularly the case of many hedge funds. Such zero-beta strategies are supposed to deliver ‘portable alpha’, i.e. a welcome addition to one’s overall portfolio that brings superior returns without increasing exposure to the market or other major risk factors. Thus, instead of a traditional active mutual fund, the investor might buy an index fund and add portable alpha on top of that.

Conceptually, however, there is little difference from nonzero-beta strategies that claim to deliver alpha, especially if portable alpha comes from positions in the same market. For instance, the so-called 130/30 strategy now employed by some mutual funds combines the index with a long/short portfolio promising to deliver portable alpha. The total portfolio satisfies (15) in the Appendix and so Proposition 1 places restrictions on the sign and magnitude of such alphas across the market.

59For details, see Elton et al. (1996).
60After 5 years, 64.4 % of the managers underperform the index and it becomes 69.5 % if \( f = 1.5 \% \). Note that this is still less than the realized underperformance of active mutual funds reported in Subsection 2.3.
61The term is largely a misnomer, however, since a lot of hedge funds today pursue directional strategies.
63This combines exposure to the index with short and long positions each worth 30 % of the portfolio. Such strategies are generalized as 1X0/X0 with X being a single-digit number.
64Thus, the above example with a university endowment fund carries over if it e.g. combines allocations to
Moreover, the typically opaque nature of hedge funds exacerbates the informational asymmetry between managers and investors, of which the former may take advantage to boost their performance record and attract/retain capital through the Peso Problem$^{65}$ engineering, i.e. entering into positions with a high probability of (smaller) success and a low probability of (large) failure. Since occurrence of the rare event usually ends the manager’s career (with the current fund), the surviving population consists of managers with improved track record on account of this practice even though they add no value if the securities are fairly priced. The option-like structure of the manager’s compensation (limited liability, absence of clawback provisions) may often make such a strategy tempting since it translates into a high probability of superior income and a low probability of positive income from the outside employment option.

Brown et al. (1999) and Malkiel and Saha (2005) raise the point that the annual attrition rate among hedge funds seems to be about 15–20%. Figure 3 shows the typical profile of survivors engineering the Peso Problem in a high-mortality environment. The results are obtained by writing index option strangles priced by the Black-Scholes-Merton formula but should be generally viewed as a result of a wide range of intrinsically similar and largely independent strategies such as statistical ‘arbitrage’ (pair trading), merger ‘arbitrage’, fixed income ‘arbitrage’, etc.$^{66}$

Margin requirements impose a maximum on the number of options the manager can write and in the simulations, the manager exploits only about a third of the maximum during incubation and half of it post-incubation. Therefore, the highest ex-ante performance in the figure should not be viewed as the limit such strategies can deliver. Quite on the contrary, the manager could collect much more premium (or convert cash into higher-yielding assets) and substantially improve the overall performance in the luckiest states. On the other hand, she would then increase the frequency of forced intermediate liquidations when unable to meet margin calls, which would reshape the distributions.

Formally, the total performance index of the manager is

$$a_t \equiv A_t \cdot e^{-(r+(\kappa)}$$

and the now extended shutdown rule implies

$$\Theta_t = \{a_0 \geq c_0, \hspace{0.5cm} a_t > c, \hspace{0.5cm} a_{Ht} > c_H\}$$

where $a_{Ht} = A_t/H_t$ in which $H_t$ is the year-end level of the NAV that needs to be attained for at least some performance fees to be paid out and $c_H > 0$ is the level of $a_{Ht}$ at which the fund is closed down.$^{67}$ More formal details are in the Appendix and what follows is a summary of the problem.

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$^{65}$See e.g. Krasker (1980) or Veronesi (2004). The now generic term refers to what seemed for a long time like market inefficiency inviting for carry trade between the Mexican peso and the US dollar, until one day in 1976 the out-of-sample tail risk of a sizable peso devaluation finally entered in the sample, wiping out the previous steady gains.

$^{66}$In theory, arbitrage involves zero risk of loss and positive probability of profit. In practice, these characteristics are usually not satisfied but the term is still used if it generates abnormal risk-adjusted returns. Such ‘arbitrage’ is in the eye of the beholder. What one interprets as underrated high-yield bonds may be seen as the Peso Problem by others.

$^{67}$This reflects the practical observation that hedge fund managers ‘deep under water’ have incentives to close down and open a new fund.
Figure 3: Survivors' vs. true performance process. The distributions of $100 \cdot (\tilde{a} - 1)$ with $\tilde{a} = a_6^*$ (top left: $\kappa = 1$), $\tilde{a} = a_5^*$ (top right: $\kappa = 0$) and $\tilde{a} = a_5$ (bottom left) are plotted as histograms with bins 1 percentage point wide. Parameters: $d = 1/252$, $r = f_M = 1\%$, $f_P = 10\%$, $k = 2.85$, $\nu = 1/2$, $k_\kappa = 1.35$, $\nu_\kappa = 0.36$, $c = 3/4$ and $c_H = 2/3$. 1,000,000 index trajectories were randomly generated with $2 \cdot \mu_X = \sigma_X = 15\%$. Details in the Appendix.

The manager has an underlying position in cash or cash equivalents such as T-bills. First, she structures an incubation bet on the index which has a 50% probability of being correct and survives her one-year trial period only if all the shorted options in the fund's portfolio expire worthless. In such a case, she emerges publicly with a 16% return for the previous year, advertising a 15% ‘alpha’ given the riskfree rate of 1 %. Then, at the start of each year, she writes options that expire worthless at the end of the year with the probability of almost 85 %. The management fee is 1 % of the NAV and in addition, the manager charges a 10% performance fee on the fund's return over and above the riskfree rate, taking the high-water mark of year-end NAVs as the basis for its calculation. The fund folds if the NAV falls short of a cash position by a quarter or if it falls short of the hurdle for performance fees by a third.

The figure illustrates with striking clarity how survival of the fittest can be observationally equivalent to survival of the luckiest.\textsuperscript{68} Note that survivors will generally exhibit little

\textsuperscript{68}Just like before, fund closures effectively serve as a ‘stop loss’ on the true process. Success is visible, failure less so, especially if there is limited communication among investors of folded funds. In addition to the incubation bias at the start of their life cycle, hedge funds do not have to report the liquidation value at the end of the life cycle and so our knowledge of the severity of ‘blow-ups’ in the industry is limited. To keep the door open for a new hedge fund, managers attempt to frame liquidations positively as the ‘best wealth-preserving action in an unprecedented situation’. There is no comprehensive hedge fund database that would allow unbiased evaluation of the industry’s past performance and so, unfortunately, any empirical study is only well-intentioned guesswork.
correlation with the index and the track record of the large proportion of best performers in particular will show practically no volatility. This all may create the impression of portable alpha. However, what naive investors using static risk measurement techniques often perceive as near-zero beta is in fact ‘asymmetric beta’ which is, in absolute value, low in most states but very high in others due to nonlinear option-like payoffs of this type of strategies. Or it may generally be ‘alternative beta’ of an omitted risk factor. In a high-attrition world with the average managerial tenure close to the average time to extreme events, investors are largely in the dark.

The manager can adopt highly complex strategies and as long as marketing manages to be economical with the truth about the underlying investment process, unsuspecting investors may be attracted by in-sample performance, thus buying an out-of-sample disaster since such funds often operate as catastrophic risk insurers.69 The fact that some sophisticated investors and academics70 are well aware of performance manipulation techniques does not preclude the majority of less sophisticated investors from failing to detect them. In contrast, rational investors who wish to add such a payoff structure to their portfolio will obtain it through passive exposure and increase it by the amount of saved hedge fund fees. Indeed, recently there has been a movement towards passive replication of hedge fund strategies.71

Protection of proprietary trading strategies characteristic of the hedge fund industry reduces transparency and introduces risks of performance manipulation on top of the active trading risks discussed previously.72 Consequently, rational investors will demand extra opacity premium, which further reduces ‘aggregate alpha’, thus constituting an additional incentive for them not to go active within ‘indexable’ markets and, instead, obtain (leveraged) index exposure involving no active management and performance fees. The recent rise of innovative investment vehicles specially designed for the latter therefore greatly facilitates rationalization of portfolio choice. Managers who are (momentarily) truly skilled may find it difficult to credibly differentiate themselves from the high-turnover mass of their lucky naive and/or opportunistic counterparts.

Note that the Peso Problem has been used in the literature to argue that some rational models fail to fit the observed reality because rational individuals are not fooled by the sample and expect catastrophic events that have not materialized yet,73 while here it supports the argument that reality fails to fit the rational model because many boundedly rational investors are fooled by the sample.

In general, we present a behavioural alternative to rational alpha-chasing models, arguing that investors tend to suffer from ‘aggregate alpha illusion’. Minor errors on the part of nonexperts and less sophisticated professionals making nontrivial investment decisions can be a major driver of active trading on information. At the same time, however, such activity is not robust to learning, enhanced consumer protection and improved risk management rules for institutional investors. The simplicity of the argument for indexing may well accelerate the recent trend towards passive informationless investing, thus undermining socially beneficial

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69 Similarly to ‘closet trackers’, they hope to go unnoticed for as long as possible and charge fees.
70 See Lo (2001), Weisman (2002), Goetzmann et al. (2007) or Foster and Young (2008, 2009). The latter show that returns-based contracts separating truly skilled managers from their mimics are impossible to design under a ‘black-box’ investment process.
72 Even with full portfolio disclosure, performance-manipulating managers could employ dynamic trading strategies that replicate option payoffs but elude investors without deep expertise in the area [Lo (2001)]. Or, in line with footnote 66, managers can always claim that they short options because they are overpriced.
73 See e.g. Rietz (1988).
5 Discussion and Policy Recommendations

Even if financial markets did operate efficiently in the past, a (desirable) universal increase in financial literacy and (correct) appeals of index fund groups to investor rationality might bring about a gradual paradigm shift resulting in invalidation of previous empirical findings, poorer price discovery and insufficient liquidity. No less importantly, ever-increasing passive ownership of the economy’s productive capacities could put a strain on the efficiency of corporate governance. Overall, the relative merits of market-based and bank-based financial systems would be rewritten.74

Should open and ‘closet’ indexers become the dominant market force and marginalize fundamental investors, relative pricing of securities would tend to deteriorate in quality.75 Also, informationless flows of funds in and out of the market would have the tendency to inflate and deflate the overall price level with relatively little regard for fundamentals. The remaining active segment might not be able or willing to correct that. For instance, given practical limits of arbitrage, sophisticated investors who become aware of a forming bubble may find it rational to withdraw from the market altogether rather than resort to uncertain shorting strategies or, alternatively, they may strategically ride the bubble with the hope of a timely exit.76 While it is possible to attract liquidity to the market,77 it is not so with price discovery for obvious verification reasons.

Therefore, despite large uncertainty about relevant social costs and benefits, it seems prudent that public policies implementing the social therapy to moderate widespread alpha chasing should be designed in a two-way fashion. On the one hand, targeted financial education should provide low/middle-income individuals, university endowment funds and charitable foundations with unbiased information about the true nature and historical record of active management strategies and, for instance, retirement plans should be legally obliged to provide low-cost index funds as a default option.78 Since stakes are high, more transparency and accountability is needed in the process of delegated portfolio management. While individual investors are free to take the personal risks associated with alpha bets, agents in charge of institutional investors should have clearly defined fiduciary duties protecting the principals and the reasons behind the choice of an active manager operating in an ‘indexable’ market should be subject to scrutiny.

In general, gullibility of vulnerable groups in the population should be reduced by raising awareness of the role of chance, including statistical (in)significance of alpha estimates based on time series of typical length, and the presence of a strong survivorship bias. In particular, it is important to reveal and intensively popularize the techniques used by the active management industry to market its alpha through the introduction of favourable biases.

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74For a review, see Levine (2002).
75Woolley and Bird (2003) offer a concrete case study of such mispricing due to captive demand coming from indexers and ‘tight trackers’.
76By a dictum attributed to Keynes, “the market can stay irrational longer than you can stay solvent.” See e.g. Shleifer and Vishny (1997) or Abreu and Brunnermeier (2003).
77For instance, liquidity rebates have recently been paid by exchanges to active traders supplying liquidity, effectively charging those who demand it. See Financial Times (2009).
78The latter is in line with the broader public policy approach ‘Nudge’ as proposed by Thaler and Sunstein (2008).
into traditional/intuitive performance measures. The bulk of the requisite enlightenment is likely to come directly from the index fund industry, independent financial advisors and independent media. However, there is room for greater involvement of securities and exchange commissions, central banks and other government agencies as well as the broader educational system in the process. Most likely, the effect of such a public contribution to the exposition of alpha marketing tricks would be magnified through further dissemination of any relevant material by the aforementioned private channels.

On the other hand, provided that adequate regulatory and supervisory standards exist and financial stability concerns are addressed, policymakers should embrace a *laissez-jouer* approach towards high net worth individuals and other entities willing and able to bear, on a strictly private basis, the inherent costs and risks of the negative-sum alpha contests. As long as the latter parties do not seek to externalize such internalities through moral hazard *vis-à-vis* shareholders or taxpayers, such a solution seems efficient in the broader social sense and incentive-compatible from the perspective of political economy. Although the active management industry may resemble a casino internally, there is necessarily a moment at which it serves as a utility externally. Besides its crucial allocative role, the capital market may have a major impact on macroeconomic volatility and so the quality of the signals it transmits to the wider economy is of primary importance to policymakers.

Should passive investing come to dominate the market overwhelmingly at some point, the resulting poor acquisition and analysis of information coupled with low liquidity would most likely pose a greater threat to social welfare than excessive acquisition and analysis of information financed by those who stand ready, by revealed preference, to absorb potential losses from such activity. For instance, there are likely to be nonnegligible social benefits in having an industry of systemically unimportant benchmark-unrestricted hedge funds that readily assume informed leveraged and short positions in individual securities as well as entire asset classes and strategic policymaking should, at the minimum, erect no barrier to entry of qualified investors.

The economic literature labels several empirical phenomena that do not square with the *‘homo oeconomicus’* model as ‘puzzles’. If that is the criterion, then the dominance of active trading in capital markets deserves its place on the list. Regarding its complete disappearance, however, we should be careful what we wish for, lest it come true.

### 6 Conclusion

Alas, there seems to be no invisible hand leading capital market participants towards socially optimal acquisition of raw information and production of new information about security issuers, considerable uncertainty regarding the actual optimum notwithstanding. In a classical coordination problem, the passive/active ratio may drift up indefinitely with little regard for social consequences. The notion that at some point active management suddenly becomes more attractive due to market inefficiency is a fallacy.

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79 I.e. a ‘let-them-play’ policy.
80 Some may enjoy investing for its own sake, perhaps being rationally risk-seeking above a certain wealth level or perceiving hedge fund investing as a status symbol.
81 Apart from precluding moral hazard, this increases competition in information acquisition and its analysis. Efficiency should not be compromised as economies of scale are likely to be exhausted long before a fund becomes systemically important.
Whatever the prior beliefs of investors about the performance of a particular market index, they unconditionally view active management as a costly fair gamble on top of that, i.e. favouring alpha products over beta products is a strategy minimizing risk-adjusted returns irrespective of price inefficiencies. Such an undesirable incentive structure is the downside of modern transparent organized markets with relatively slow variation in the securities supply.

We argue that models à la Berk and Green (2004) incorrectly rationalize the active management sector and do not promote our understanding of the true nature of market activity. This has important implications not only for practical portfolio allocation but also for any prospect of informed policymaking. In search for a more realistic paradigm of active trading, we provide a model illustrating that if investors overweigh the characteristics of survivors and underestimate the effect of natural and strategic closures, such a relatively minor error may be sufficient to make them sway towards active management, which they would probably not do if properly instructed. This insight implies a very fragile incentive structure as it is now easier than ever before for both individual and institutional investors to optimize their investment decisions.

Despite the complexity of capital markets, the argument for indexing is simple and powerful. While policymakers could push the passive/active ratio further up with the stroke of a pen, they cannot stop or reverse its recent upward trend. Economists may agree that active investors tend to overtrade and more long-term fundamental investing based on security analysis, without excessive trading, would be preferable. However, passive informationless investing is very different from that.

We do not observe the quality of price discovery and cannot easily measure it, since we do not know what the underlying fundamental values are. If we did, we might as well be living in a world with no history of a spectacular failure of central planning, which goes back to Hayek (1945). However, even in its abstract form, the argument that dwindling participation in the search for fundamentals in capital markets will ultimately produce less precise information about security issuers seems compelling. The ‘wisdom of the crowd’ is likely to be superior to the ‘wisdom of a few’ and when fundamental investors become indexers, price discovery will suffer.

Following the burst of the technological bubble of the 1990s and the financial crisis of the late 2000s, there have been concerns about the ability of capital markets to conduct efficient price level discovery. Figure 1 illustrates why. If indexing becomes prevalent, further concerns regarding the efficiency of relative price discovery are likely to arise. Defenders of efficient markets often invoke rationality behind price formation. Yet, paradoxically, increased investor rationality might well render markets less efficient.

Indeed, this may be viewed as a broader issue touching the heart of economics. For instance, rational schools of thought argue quite convincingly about the ineffectiveness of macroeconomic policies which are implicitly based on the presumption that active macroeco-
nomic management can fool people over and over again. Yet, the very same economists are often staunch defenders of the EMH, implying that active asset management has been doing precisely that for decades, adding no value but subtracting fees on the way. While the relative significance of such errors may be subject to debate, the inherent inconsistencies carried by such joint propositions are far from negligible and deserve ‘joint attention’.

Whether indexing manages to gradually transform capital markets in the coming years is yet to be seen but it is obvious that the current market dynamics need not be robust against social learning or more broadly defined innovation diffusion, and the innovative index products might eventually become standard. Young (2009) shows possible adoption dynamics resulting from such social processes and we know from history that even simple ideas may take surprisingly long to be embraced, rational incentives notwithstanding. This is an important issue for policymakers as well as an interesting avenue for academic research, since price discovery in capital markets is central to the efficiency of the real economy. Should the proverbial pendulum, as a result of increased rationality of individual decisions, swing from current hyperactivity to hyperpassivity, capital markets would not be what they used to be, for better and worse.

**A Appendix**

**A.1 Proof and Explanation of Proposition 1**

At time $t_1$, the total market supply of securities is $Q = \sum_{i=1}^{I} q_i$, their prices are $P > 0$ and the portfolio of each investor can be expressed as

$$q_i = w_i \cdot Q + q_i^a$$

where

$$w_i = \frac{P' \cdot q_i}{P' \cdot Q} \in (0, 1)$$

is the value of the portfolio as a fraction of total market capitalization, with $\sum_{i=1}^{I} w_i = 1$. Thus, the investor’s portfolio can be viewed as a combination of a passive portfolio and a zero-cost active portfolio, the first and the second term in (15), respectively. $P' \cdot q_i^a = 0$ follows immediately from (15) and (16).

The rate of return on portfolio $i$ over the time interval $[t_1, t_2]$ is then

$$\hat{r}_i = \frac{P' \cdot q_i}{P' \cdot q_i} - 1 = \left[ \frac{P' \cdot Q}{P' \cdot Q} - 1 \right] + \frac{P' \cdot q_i^a}{P' \cdot q_i},$$

with $P$ denoting prices at $t_2$ and the bracketed expression being the market index return $\hat{r}_X$. It follows from (15) that $\sum_{i=1}^{I} q_i^a = 0$ and so (17) gives

$$\hat{r}_X = \sum_{i=1}^{I} w_i \cdot \hat{r}_i.$$
i) Taking the expectation of (18) and defining $\alpha_i \leq E(\tilde{r}_i - \tilde{r}_X)$ gives (2).

ii) This follows immediately if $u$ is defined as linear in iii).

iii) At $t_1$, the investor has wealth $W \leq \min_i \{w_i\} \cdot P' \cdot Q$ which she invests in the market and may have additional nonmarket wealth $\tilde{N}$. At $t_2$, those become $\tilde{W}_i = W \cdot (1 + \tilde{r}_i)$ and $\tilde{N}$. She has preferences over $\tilde{N} + \tilde{W}_i$ represented by the von Neumann-Morgenstern utility function $u$ such that $u' > 0$ and $u'' < 0$. (18) implies $\tilde{W}_X = \sum_{i=1}^I w_i \cdot \tilde{W}_i$ and so by Jensen’s inequality,

$$Eu\left(\tilde{N} + \tilde{W}_X\right) = Eu\left[\sum_{i=1}^I w_i \cdot \left(\tilde{N} + \tilde{W}_i\right)\right] \geq \sum_{i=1}^I w_i \cdot Eu\left(\tilde{N} + \tilde{W}_i\right).$$

Rearranging,

$$\sum_{i=1}^I w_i \cdot \left[Eu\left(\tilde{N} + \tilde{W}_i\right) - Eu\left(\tilde{N} + \tilde{W}_X\right)\right] \leq 0.$$ 

Then, defining $\alpha_i$ as weakly smaller than the corresponding bracketed expression gives (2).

Index funds deliver $\tilde{r}_i = \tilde{r}_X$ to investors and so their $\alpha_i = 0$. Because of fees, active managers pass strictly less than $\tilde{r}_i$ to their investors and so in such cases, the definitional inequalities for $\alpha_i$ are strict. Consequently, the inequality in (2) is strict unless all managers index.

For economy of notation and space, the symbol ‘alpha’ is used very loosely for risk-unadjusted performance improvement in i) and ii). In iii), instead of ‘traditional alpha’ defined as extra expected return in the CAPM framework [Jensen (1968)], we seek a more flexible concept of value added by the manager so that individual implicit risk adjustment of excess returns takes place in the expected utility framework. Thus, positive (negative) alpha means that the manager delivers an increase (decrease) of the certainty equivalent rate of return to the investor. The value and even the sign of alpha depends on individual preferences but knowing all the alphas, each investor can construct her unique ranking of the funds. In all cases, alphas are defined ex-fees, which is the relevant measure of performance enhancement for investors.

Note that $\alpha_i$ may be positive in i) and ii) while being negative in iii). This would be the case when the extra risk taken by the manager is not justified by the excess returns delivered, i.e. the expected return increases but its certainty equivalent decreases. The reverse is equally possible. In general, it is possible for some active managers to identify disequilibrium prices allowing them to construct portfolios with higher expected returns and lower risk than the market portfolio. However, like the former, the latter cannot be achieved by all managers because concentration of portfolios cannot reduce their aggregate riskiness. Quite on the contrary, diversification is the only free lunch in the market and active managers tend to miss on that. In sum, active management cannot deliver excess returns in the aggregate and in addition to being costly, it generates extra active risk that must be distributed somehow among the investors.

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87 This is without loss of generality since $u$ is unique only up to a positive affine transformation.
88 Or, more generally, in multifactor models [Fama and French (1993), Carhart (1997)].
89 We do not operate with the weighted sum/average of the excess certainty equivalent rates of return, for it has little economic meaning to the individual.
90 While risk averters suffer a welfare loss, risk seekers would prefer active risk per se. Throughout, we restrict
Now we consider the time interval \([t, T]\) with intermediate trading. This can be broken into buy-and-hold subintervals, possibly of different length. Let us temporarily impose the restriction that at any intermediate point \(\tau\), \(w_{\tau} = (w_{1\tau}, \ldots, w_{I\tau})'\) is unaffected by flows.\(^{91}\)

We denote the return on fund \(i\) over the time interval as \(\tilde{r}_{iT}\) and the return on the market index as \(\tilde{r}_{XT}\). Since the latter is necessarily a weighted average of the former returns, the constraint

\[
\sum_{i=1}^{I} w_{it} \cdot (\tilde{r}_{iT} - \tilde{r}_{XT}) = 0
\]

must hold for any \(w_t\). The (gross) fund and index returns over the entire period are simply products of (gross) subperiod fund and index returns, respectively. Before the subtraction of management fees, manager \(i\) adds \(\tilde{r}_{iT} - \tilde{r}_{XT}\) on top of \(\tilde{r}_{XT}\) with the goal of achieving the highest return and the lowest risk for the fund. However, (20) compared with (18) implies that Proposition 1 holds over \([t, T]\) and so active managers necessarily participate in a negative-sum alpha contest.

Such a contest is held over each subperiod, suggesting that the extra returns managers deliver per unit of time generally do not have stationary statistical properties. Firstly, realizations of returns lead to intermediate shifts in \(w\) and hence in subperiod alphas. Secondly, if we relax the restriction on intermediate flows, those will generally affect \(w\), possibly in a major way. Thirdly, (‘closet’) indexing of a manager with any skills leads to \(\tilde{r}_i = \tilde{r}_X\) and through the constraint, it affects the investment opportunities of the residual active managers. And last but not least, relative intellectual abilities of managers may change over time as well, thus affecting their alphas. All this introduces \textit{extra risks} to the process of forecasting managerial skills.

\section*{A.2 Proof of Proposition 2}

By Itô’s Lemma, (7) implies

\[
dY = -\left(f + \frac{\sigma^2}{2}\right) \cdot dt + \sigma \cdot dW
\]

where \(Y = \log \frac{a}{c}\). Let

\[
Z_t = \begin{cases} 
Y_t & \text{if } t < t_0 \\
0 & \text{if } t \geq t_0 
\end{cases}
\]

where \(t_0 = \min\{t : Y_t = 0\}\) is the first time at which \(Y_t\) hits the absorbing barrier of zero.\(^{92}\)

We first treat the initial condition \(Z_0 = z_0 > 0\) as fixed. The density \(f_Z(z, t)\) of the process \(Z\) satisfies the Fokker-Planck partial differential equation (Kolmogorov forward equation)

\[
\frac{\partial f_Z}{\partial t} = \left(f + \frac{\sigma^2}{2}\right) \cdot \frac{\partial f_Z}{\partial z} + \frac{\sigma^2}{2} \cdot \frac{\partial^2 f_Z}{\partial z^2}
\]

our analysis to risk-averse investors, assuming rational risk seekers could always flip a coin over index fund returns as a cheaper substitute for pure active risk. In fact, the European roulette wheel has 37 pockets, which gives the probability of zero about 2.7\% and the house edge drops to 1.35\% if the \textit{en prison} rule applies. Casinos can apparently offer more thrill for less money than many a fund manager.

\(^{91}\)This only allows for market inflows and outflows distributed proportionately across all funds.

\(^{92}\)An introduction to conditional diffusions is e.g. in Grimmett and Stirzaker (2001).
for \( z > 0 \) subject to the boundary conditions
\[
  \begin{align*}
  &f_Z(0, t) = 0 \quad \text{for } t \geq 0 \\
  &f_Z(z, 0) = \delta_{z_0}(z) \quad \text{for } z \geq 0
  \end{align*}
\]
(24)
(25)

where \( \delta_{z_0} \) is the Dirac delta function centered at \( z_0 \). The unique solution turns out to be
\[
f_Z(z, t | z_0) = \frac{\phi \left[ d_-(z, t, z_0) \right]}{\sigma \cdot \sqrt{t}} - \frac{\phi \left[ d_+(z, t, z_0) \right]}{\sigma \cdot \sqrt{t}} \cdot e^{z_0 \cdot (1 + 2 \cdot f \cdot \sigma^{-2})} \tag{26}
\]

where
\[
d_{\pm}(z, t, z_0) = \frac{z \pm z_0 + \left( f + \frac{\sigma^2}{2} \right) \cdot t}{\sigma \cdot \sqrt{t}}
\]

and \( \phi(x) = \left( 2 \cdot \pi \cdot e^{x^2} \right)^{-\frac{1}{2}} \) is the standard normal probability density function. Hence
\[
\text{Prob}(Z_t = 0 | Z_0 = z_0) = 1 - \int_0^\infty f_Z(z, t | z_0) \, dz = \\
= \Phi \left[ \frac{\left( f + \frac{\sigma^2}{2} \right) \cdot t - z_0}{\sigma \cdot \sqrt{t}} \right] + \Phi \left[ -\frac{\left( f + \frac{\sigma^2}{2} \right) \cdot t + z_0}{\sigma \cdot \sqrt{t}} \right] \cdot e^{z_0 \cdot (1 + 2 \cdot f \cdot \sigma^{-2})} \tag{27}
\]

where \( \Phi \) is the standard normal cumulative distribution function.

By (8), the density of \( Z_0 = \log \frac{a_0}{c} | a_0 \geq c_0 \) is
\[
f_{Z_0}(z_0) = \frac{(2 \cdot \pi \cdot \kappa \cdot \sigma^2)^{-\frac{1}{2}}}{\sqrt{\left( \mu_\kappa - \frac{\sigma^2}{2} / \kappa - \log c_0 \right) \sigma^2 \kappa}} \cdot e^{-\frac{1}{2} \left[ z_0 - \left( \mu_\kappa - \frac{\sigma^2}{2} / \kappa - \log c_0 \right) \sigma^2 \kappa \right]^2} \tag{28}
\]

for \( z_0 \geq \log \frac{c_0}{c} \) and zero otherwise. Using (28), we integrate out \( z_0 \) from (26) and (27) to obtain the distribution of \( Z_t \) as characterized by the probability mass
\[
\mathbb{P}_t^\dagger = \text{Prob}(Z_t = 0) = \mathbb{E}_{Z_0} \left[ \text{Prob}(Z_t = 0 | Z_0) \right] \tag{29}
\]

at \( z = 0 \), the density \( \mathbb{E}_{Z_0} \left[ f_Z(z, t | Z_0) \right] \) for \( z > 0 \) and zero otherwise. For \( T > \kappa \), the distribution of \( a_T^\dagger = c \cdot e^{Z_T - \kappa} | Z_T - \kappa > 0 \), ignoring the mass of failed funds/managers, is then characterized by the density
\[
f_{a_T^\dagger}(\hat{a}) = \frac{\mathbb{E}_{Z_0} \left[ f_Z \left( \log \frac{\hat{a}}{c}, t | Z_0 \right) \right]}{\hat{a} \cdot \left( 1 - \mathbb{P}_t^\dagger \right)} \tag{30}
\]
for $\hat{a} > c$ and zero otherwise, which can be expressed as (12) with\textsuperscript{93}

$$\xi_1(\hat{a}, c_0) = \frac{D}{\sqrt{2\pi}} \left( \log \hat{a} + \frac{f}{\sigma^2} + \frac{\mu_\kappa}{\sigma^2} \right) - \frac{D}{\sqrt{2\pi}} \cdot \log c_0$$

(31)

$$\xi_2(\hat{a}, c) = \exp \left\{ \frac{2}{\sigma^2} \cdot \log \frac{\hat{a}}{c} \cdot \left( \log c - \frac{f}{\sigma^2} - \frac{\mu_\kappa}{\sigma^2} \right) \right\}$$

(32)

$$\xi_3(\hat{a}, c_0, c) = \xi_1(\hat{a}, c_0) - 2 \cdot \frac{\log \hat{a}}{\sigma^2}.$$  

(33)

$$\xi_4(\hat{a}) = \hat{a} \cdot \left[ 2 \cdot \pi \cdot \left( \sigma^2 \cdot t + \sigma^2 \cdot \kappa \right) \right]^{\frac{1}{2}} \cdot \frac{[\log \hat{a} + (\sigma^2 / 2) \cdot t - (\mu_\kappa - \sigma^2 / 2) \cdot t]^2}{2 (\sigma^2 + \sigma^2 \cdot \kappa)}$$

(34)

$$\xi_5(c_0) = \frac{(\mu_\kappa - \sigma^2 / 2) \cdot \kappa - \log c_0}{\sigma^2 \cdot \kappa}.$$ 

(35)

where $D = \sigma^{-2} \cdot t^{-1} + \sigma^{-2} \cdot \kappa^{-1}$.

### A.3 Peso Problem Engineering

Assume that trading is continuous and derivatives positions are marked to market on a daily basis. The index value $X_t$ follows (6). Let

$$[t] = \max \{ s : s \leq t \& s \in \mathbb{N}_0 \}$$

(36)

where $t \in \{0, d, 2d, \ldots\}$ is time measured in years and $d^{-1} \in \mathbb{N}$ is the number of trading days per year. At each $t = [t]$, the manager enters a short strangle position by writing out-of-the-money European call and put index options that are equally likely to expire in the money at probability $2 \cdot \Phi(-k/2)$. This is achieved with the strike price $X_{[t]} \cdot K_-$ for calls and $X_{[t]} \cdot K_+$ for puts where

$$K_\pm = e^{\mu_- - (\sigma X / 2 - \sigma X \pm k)}.$$ 

(37)

At time $t$, the value of a call and put, respectively, with expiration at $[t] + 1$ is

$$C_t^{[t]+1} = X_t \cdot \Phi(\delta_-) - X_{[t]} \cdot K_- \cdot \Phi(\delta_-) \cdot e^{-r \cdot ([t] + 1 - t)}$$

(38)

$$P_t^{[t]+1} = X_{[t]} \cdot K_+ \cdot \Phi(-\delta_+) \cdot e^{-r \cdot ([t] + 1 - t)} - X_t \cdot \Phi(-\delta_+)$$

(39)

by the Black-Scholes-Merton formula where

$$\delta_{\tau \pm} = \frac{\log(X_t / X_{[t]}) - \mu_\tau + (\sigma X / 2) \cdot (\sigma X \pm k)}{\sigma X \cdot \sqrt{[t] + 1 - \tau}} + \frac{r}{\sigma X} \pm \frac{\sigma X}{2}$$

and $r$ is the riskfree rate of return on cash or its equivalents. At expiration time $t = [t]$, the value of a single strangle is

$$C_t^* + P_t^* = \max \{ X_t - X_{t-1} \cdot K_-, X_{t-1} \cdot K_+ - X_t, 0 \}.$$ 

\textsuperscript{93}To obtain the scaling factor for (30) and (12), we evaluate $P_t^*$ in (29) by quadrature.
The margin requirements follow Chicago Board Options Exchange (2000). At time $t$, the balance on the margin account per strangle must be

$$M_t = \begin{cases} 
M_t^C + P_t^{[t]+1} & \text{if } M_t^C > M_t^P \\
M_t^P + C_t^{[t]+1} & \text{if } M_t^C \leq M_t^P 
\end{cases}$$

(41)

where

$$M_t^C = C_t^{[t]+1} + \max \{ 0.1 \cdot X_t, 0.15 \cdot X_t - \max \{ X_{[t]} \cdot K_+ - X_t, 0 \} \}$$

(42)

$$M_t^P = P_t^{[t]+1} + \max \{ 0.1 \cdot X_{[t]} \cdot K_+, 0.15 \cdot X_t - \max \{ X_t - X_{[t]} \cdot K_+, 0 \} \} .$$

(43)

At $t = [t]$, the manager writes $n_t = \nu \cdot m_t$ strangles where $m_t$ is their maximum number determined by margin requirements and $\nu \in [0, 1]$. At $t > [t]$, the number of shorted strangles is

$$n_t = \min \{ \nu \cdot m_t, m_{[t]+d}, \ldots, m_t \} ,$$

(44)

i.e. margin calls may force the fund to realize losses through liquidations prior to expiration.

At $t \geq 0$, the long cash position per share of the fund is

$$B_t = A_t + n_t \cdot (C_t^{[t]+1} + P_t^{[t]+1}) .$$

(45)

At $t > 0$, marking to market gives the NAV plus fees currently due

$$A_t^+ = B_{t-d} \cdot e^r \cdot d - n_{t-d} \cdot \left( C_{t-\delta}^{[t-\delta]+1} + P_{t-\delta}^{[t-\delta]+1} \right) .$$

(46)

Deducting the management fees ($f_M$) and potentially performance fees ($f_P$) then gives

$$A_t = (1 - f_M \cdot d) \cdot A_t^+ - f_P \cdot \max \{ A_t^+ - H_t, 0 \} \cdot I(t = [t])$$

(47)

where

$$H_t = \max \{ A_0, A_1, \ldots, A_{[t-d]} \} \cdot e^r$$

(48)

combines a high-water mark with a hurdle rate and the indicator function $I$ returns unity if true and zero otherwise. Initially, $A_0^+ = A_0$. From $m_t = B_t(m_t)/M_t$, having replaced $n_t$ with $m_t$ in (45), we obtain

$$m_t = \frac{A_t}{M_t - C_t^{[t]+1} - P_t^{[t]+1}} .$$

(49)

The fund goes through one-year incubation ($\kappa = 1$) with parameters $k_\kappa$ and $\nu_\kappa$ rather than $k$ and $\nu$. Given $A_{-1} = 1$, $A_0$ conditional on no intermediate liquidations and all options expiring worthless at $t = 0$ (both conditions shortened as ‘ace’) is

$$A_0 | \text{ace} = \left[ 1 + n_{-1} \cdot (C_{-1}^d + P_{-1}^d) \right] \cdot e^{-f_M}$$

(50)

where management fees, for simplicity, are charged continuously and there are no performance fees over the incubation period. We then set $c_0 = (A_0 | \text{ace}) \cdot e^{-r}$ in (14). Alternatively, no incubation takes place ($\kappa = 0$), in which case $A_0 = c_0 = 1$. In the latter, we also simulate the true process by suspending the shutdown rule altogether. Parameter values are under Figure 3.
References


