

# On preparation of viscous pore fluids for dynamic centrifuge modelling

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## Abstract

*The scaling laws that arise from dynamic centrifuge modelling contain an inconsistency between the scaling of time for dynamic events and diffusion events. This problem can be resolved by reducing the permeability of the soil, with the help of high viscosity pore fluids. Hydroxypropyl methylcellulose is a water soluble cellulose ether that is widely used to create such fluids. In this paper, the effects that concentration, temperature, ageing, and shearing rate have on the viscosity of hydroxypropyl methylcellulose solutions are examined and equations that quantify them are presented. This information is meant to act as a guideline in preparing high viscosity pore fluids for dynamic centrifuge tests.*

## 1. INTRODUCTION

In geotechnical centrifuge modelling, experiments are performed using a scaled down model. Its behaviour corresponds to the response of a non scaled prototype that represents a simplified version of reality. The scaled down model is  $N$  times smaller than the prototype and is subjected to a centrifugal acceleration of  $N$  times the acceleration of gravity. In that way, stresses and strains in the model match those at homologous points of the prototype. The resulting accurate reproduction of soil behaviour ensures the validity of the results, as long as the boundary conditions are accounted for.

However, before data acquired from the model can be used to assess the behaviour of the prototype, suitable scaling laws must be applied. Schofield [1980, 1981] and more recently Madabhushi [2014] report the most important of these laws for both static and dynamic events.

An inconsistency exists in the scaling for time. For a scaling factor of  $n$ , if  $L$  signifies length,  $a$  acceleration,  $t$  time, and the subscripts  $m$  and  $p$  correspond to ‘model’ and ‘prototype’ respectively, then dynamic time is scaled as

follows:

$$t_m = \sqrt{\frac{L_m}{a_m}} = \sqrt{\frac{\frac{L_m}{n}}{a_p n}} = \frac{t_p}{n} \quad (1)$$

When considering diffusion phenomena, solving the corresponding one dimensional differential equation for consolidation, a dimensionless time factor  $T_v = C_v \frac{t}{d^2}$  can be introduced.  $C_v$  is the coefficient of consolidation,  $t$  is time, and  $d$  is the distance considered. If the same dimensionless time factor is to exist both in the model and the prototype, the scaling of time is derived as follows:

$$C_{vm} \frac{t_m}{d_m^2} = C_{vp} \frac{t_p}{d_p^2} \quad (2)$$

As long as the same soil is used in the model as in the prototype,  $C_{vm} = C_{vp}$ . Moreover,  $d_m = \frac{d_p}{n}$ . Therefore:

$$t_m = \frac{t_p}{n^2} \quad (3)$$

Consequently, if water is used as the pore fluid during dynamic centrifuge tests, the soil in the model will inevitably appear to be  $N$  times more permeable than its counterpart in prototype scale.

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This incompatibility between dynamic and diffusion time can be successfully resolved by scaling down the coefficient of consolidation in the model by  $n$ . The coefficient of consolidation can be expressed as:

$$C_v = \frac{k}{m_v \rho_f g} \quad (4)$$

with  $k$  being the permeability and  $m_v$  the compressibility of the soil,  $\rho_f$  the density of the pore fluid, and  $g$  the acceleration of gravity. If  $k$  is reduced by  $n$ ,  $C_v$  will be properly scaled. Permeability is defined as:

$$k = \frac{\kappa_s \rho_f g}{\mu} \quad (5)$$

with  $\kappa_s$  being the intrinsic permeability of the soil and  $\mu$  the dynamic viscosity of the fluid. Permeability can be scaled either by reducing the size of the soil grains (thus scaling down  $\kappa_s$ ) or through viscosity scaling (increasing  $\mu$ ) [Lambe and Whitman, 1982].

Zeng et al. [1998] note that smaller grain size could lead to a change in soil strength and stress - strain behaviour. On the contrary, triaxial tests performed by Dewoolkar et al. [1999] show that the use of hydroxyl methyl-cellulose (HPMC) solutions does not affect the soil's constitutive behaviour. As a result, viscosity scaling is the preferential means towards reducing permeability.

For the scaling laws of diffusion and dynamic time to match, a pore fluid  $N$  times more viscous than water must be used. Silicone oils or aquatic solutions of polymers have been used in the past as high viscosity pore fluids in centrifuge modelling. Zeng et al. [1998] showed that these fluids satisfy the scaling requirements for permeability. HPMC aquatic solutions are used in centrifuge centres worldwide as pore fluids, since they can provide the necessary viscosity scaling for a wide range of scaling factors, while being inexpensive and fairly easy to prepare, use, and dispose of. Moreover, they offer the significant advantage of having a density very close to that of water, for the usual viscosity range required in

centrifuge modelling [Stewart et al., 1998]. Finally, Dewoolkar et al. [1999] further bolster the use of HPMC solutions in centrifuge testing by successfully performing modelling of models using HPMC pore fluids.

Centrifuge tests that have been performed both with water and a viscous solution as the pore fluid, exemplify the need for viscosity scaling. Dewoolkar et al. [1999] examined saturated level sand beds and recorded completely different responses. For the same earthquake motion, the bed that was saturated with an HPMC solution liquefied fully, whereas the one that was saturated with water did not. Peiris et al. [1998] showed that during dynamic centrifuge tests, the use of water as pore fluid can limit the generation of excess pore pressures in sand formations below gravel embankments, lowering the recorded crest settlement significantly. Chian and Madabhushi [2010] examined the flotation of a tunnel in liquefiable soil using pore fluids of different viscosities. The use of lower than required viscosity was found to result in more significant pore fluid migration which lead to an increase in the uplift of the buried structure. In all cases, not correctly performing viscosity scaling was shown to lead to untrustworthy experimental results.

All in all, viscosity scaling is crucial to obtaining meaningful results from dynamic centrifuge tests. Extra care should be taken to make sure that the viscosity level is corresponding to the correct scaling factor. Stewart et al. [1998] present useful equations for the preparation of HPMC solutions that are to be used as viscous pore fluids. In this paper, the properties of these solutions are revisited, using METHOCEL F50 Food Gradient HPMC, available from the Dow Chemical Company. Deviations observed from what Stewart et al. [1998] report could be due to changes in the composition of the product or due to differences in the preparation of the solutions.

## 2. EXPERIMENTAL METHODS

HPMC is a type of water soluble cellulose ether that can be used to create high viscosity pore

fluids. The results presented here were obtained using HPMC in powder form, marketed as METHOCEL F50 Food Gradient and provided by the Dow Chemical Company. Salient properties are given by Dow [2002]. The characterisation F50 indicates a dynamic viscosity of  $50 \text{ mPa} \cdot \text{s}$  for an aqueous solution of 2% concentration, at  $20^\circ\text{C}$ .

Several methods can be employed to prepare the aqueous solutions. Provided there is sufficient time, HPMC can be gradually poured on the surface of deaired water. Using this method, the water can be kept at room temperature. However, only a small quantity of HPMC should be dispersed over the water surface each time, in order to avoid the creation of lumps, whose gelatinous surface obstructs full hydration. An alternative method that is suggested by Dow [2002] is the 'hot/cold' technique. This procedure is based on the fact that METHOCEL HPMC powder does not hydrate in high temperatures. The technique is briefly described below:

- Firstly,  $1/5$  to  $1/3$  of the required volume of water needs to be heated to  $90^\circ\text{C}$  (Fig.1a).
- The required HPMC powder is added to the heated water, which is continuously agitated (Fig.1b).
- Agitation continues until the HPMC particles are wetted and evenly distributed in the water.
- The remaining volume of water that is kept at room temperature is gradually added while agitation persists (Fig.1c). As a result, the temperature of the mix drops. Hydration and consequent development of viscosity occurs at temperatures of  $25^\circ\text{C}$  or lower.
- Agitation should continue for at least half an hour after temperatures below  $25^\circ\text{C}$  are reached (Fig.1d).

The above technique makes use of the entire volume of water for the hydration of HPMC, rather than just its surface, and is hence a much faster alternative, especially in the colder regions of the world. Nevertheless, it requires heating apparatus and provision for the vol-

ume of water that might get lost as water vapour. Moreover, the mix needs to be deaired again because of the prolonged agitation.

Calculating mass fractions involves using the ratio of the mass of HPMC over the mass of the mix. However, the concentrations of solutions described in this report are calculated by using the mass of water as the denominator. This choice is made so that concentrations can be more easily used to calculate the necessary amounts of water and HPMC when preparing viscous pore fluids. Concentrations are presented in percentages.

Temperature measurements were made using a Checktemp 1 thermometer by Hanna Instruments, with an accuracy of  $\pm 0.3^\circ\text{C}$ .

Viscosity was measured with an LV DV-I+ viscometer, manufactured by Brookfield. Three measurements were made, at a range of shearing rates and the average was recorded. In all cases, the change in viscosity was below  $4 \text{ mPa} \cdot \text{s}$ . The importance of the shearing rate is further discussed in 3.4.

### 3. RESULTS AND DISCUSSION

#### 3.1. Viscosity change with concentration

An increase in concentration of HPMC results in a more viscous fluid. From now on, the term viscosity will be used to describe dynamic viscosity. Measurements for aquatic solutions with different HPMC concentrations are presented in figure 2. All measurements were made at  $20^\circ\text{C}$ .

Two expressions are produced that fit the data in a satisfactory way. A power law is first used as a potential curve to fit the data. The equation used is described below:

$$V(\text{mPa} \cdot \text{s}) = 8.017 \cdot c(\%)^{2.715} \quad (6)$$

Viscosity  $V$  is calculated in  $\text{mPa} \cdot \text{s}$  and concentration  $c$  is given in percentage.

As seen in figure 2, the power law estimation of viscosity fits the data in a satisfactory way ( $R^2 = 0.9917$ ). However, its predictions tend to be low for concentrations above 2.8%

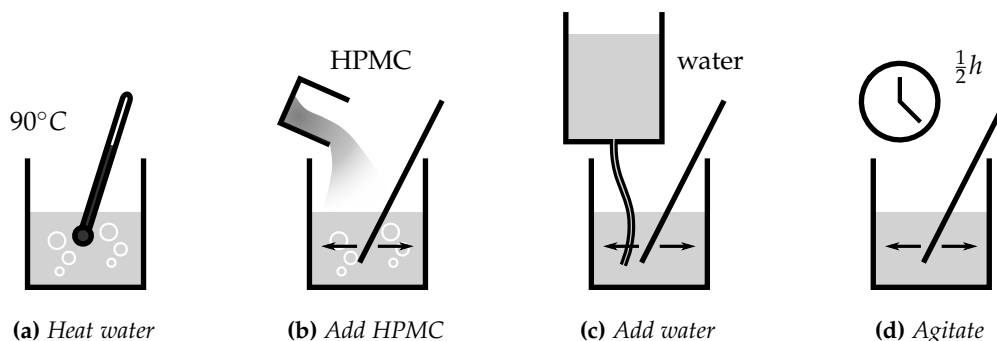


Figure 1: 'hot/cold' method

and slightly high for concentrations around 2.2%. Whereas higher concentrations are not of much interest for dynamic centrifuge modelling, concentrations below 2.4% are important. Dow [2002] suggests that the 8<sup>th</sup> root of viscosity should be a linear function of concentration. Following this suggestion, an 8<sup>th</sup> order fit is tried as an alternative. The equation for this approximation is as follows:

$$V(mPa \cdot s) = (0.268 \cdot c(\%) + 1.094)^8 \quad (7)$$

Viscosity is calculated in  $mPa \cdot s$  and concentration is given in percentage.

The 8<sup>th</sup> order approximation ( $R^2 = 0.9945$ ) indeed seems to be a better fit than the power law. The 8<sup>th</sup> root of viscosity is plotted versus concentration in figure 3. In this domain it is more obvious that the linear approximation corresponding to equation 7 better fits the data.

Stewart et al. [1998] also produced a similar expression for METHOCEL grade F. However, their equation systematically underpredicts viscosity (fig.2). Differences in calculating concentrations have been taken into account. It is possible that the formulation of METHOCEL grade F is now different from sixteen years ago, when the equation of Stewart et al. was derived. Dow [2002] suggests that for mass fraction of 2%, viscosity of  $50 mPa \cdot s$  should be obtained at  $20^\circ C$ . Whereas this is achieved by the measurements depicted in figures 2 and 3, the equation given by Stewart et al. predicts a lower value. Another possible reason for this inconsistency might be the fact that

Stewart et al. used in their solutions benzoic acid, equal to 1% of HPMC powder mass, to prevent microbial growth. Although HPMC solutions are considered stable in a range of pH values, they can reduce in viscosity in an acidic environment.

On the whole, equation 7 is suggested as the expression to correlate viscosity and concentration.

### 3.2. Viscosity change with temperature

HPMC aquatic solutions tend to become thinner with increasing temperature. In this section, the change of viscosity with temperature will be quantified.

The centrifuge of the Schofield Centre at the University of Cambridge does not have a temperature control system for the model. Hence, an estimation of the temperature on the day of the test needs to be made. The expected range of temperatures in the centrifuge pit throughout the year is between  $10^\circ C$  and  $25^\circ C$ .

It should be noted that if heated sufficiently, aqueous HPMC solutions can gel. The gelation process is completely reversible upon cooling. Gelation temperatures for METHOCEL F50 are well above the expected range of temperatures in dynamic centrifuge modelling [Dow, 2002]. As a result, the gelation process is not considered in this study.

Measurements of viscosity with changing

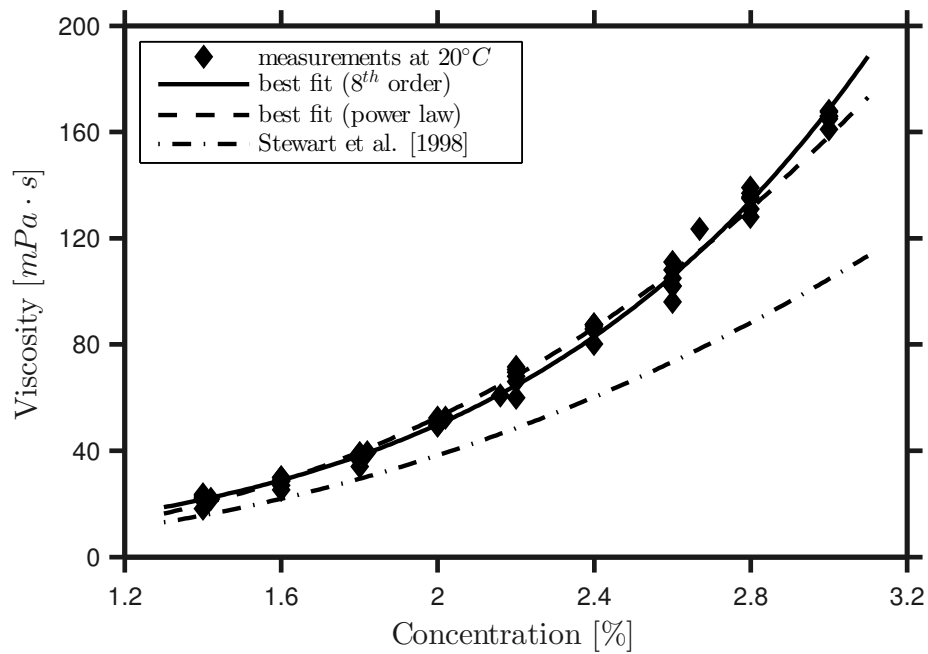


Figure 2: Viscosity change with concentration

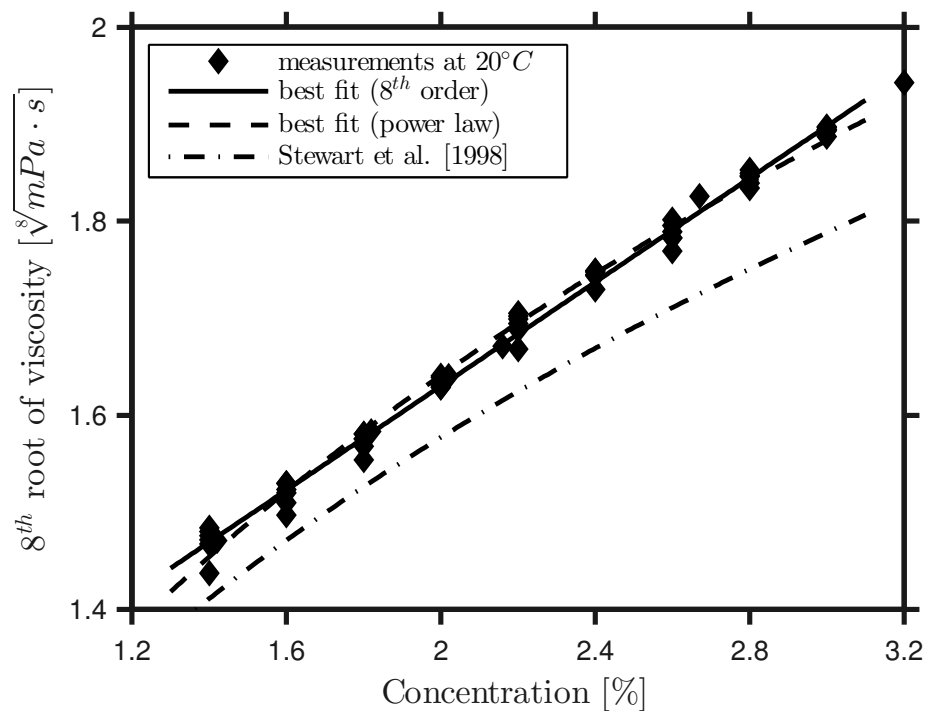


Figure 3: 8<sup>th</sup> root of viscosity change with concentration

temperature for samples of three different concentrations are presented in figure 4. The behaviour is similar for all three samples, with viscosity decreasing as temperatures get higher. Indeed, if all measurements for each sample are normalised with their corresponding viscosity at 20°C, they fall on a single curve (Fig.5). 20°C are chosen for the normalisation as that is approximately the average temperature on the Schofield Centre’s lab floor, where solutions are prepared. The following expression is computed as the best fit:

$$\frac{V_{T^{\circ}C}}{V_{20^{\circ}C}} = 2.44e^{-0.045 \cdot T(^{\circ}C)} \quad (8)$$

where  $V_{T^{\circ}C}$  is the viscosity at the desired temperature of  $T(^{\circ}C)$  and  $V_{20^{\circ}C}$  is the viscosity at 20°C.

Stewart et al. [1998] presented an equivalent expression to describe the change of viscosity with temperature. When normalised in the same way (Fig.5), it is shown to be similar to the expression described above.

### 3.3. Viscosity change with time

High viscosity HPMC solutions are typically prepared in advance of a centrifuge test. Saturating the soil with the pore fluid, loading the model on the centrifuge and connecting all the instrumentation can take a few days. Hence, it is useful to know whether viscosity can change with time.

Dow stresses that METHOCEL cellulose ether products are very resistant to microorganisms and enzymes. Indeed, HPMC products are often used as coating for pills that have to pass through the intestinal track, proving their stability in hostile biochemical environments. Consequently, the HPMC solutions prepared for dynamic centrifuge modelling are expected to maintain their viscosity at a constant value for a prolonged period of time.

In order to assess the sensitivity of HPMC solutions to ageing, the viscosity of three samples of different concentrations was measured at 20°C several times within a month. As can be seen in figure 6, there was no significant

change in viscosity. As a result, it is safe to assume that within the time scale of a dynamic centrifuge test the viscosity of the prepared HPMC pore fluid will not be affected by ageing.

### 3.4. Viscosity change with shearing rate

HPMC aqueous solutions are non - Newtonian fluids, exhibiting a decrease in apparent viscosity with increasing shear rate (pseudoplastic behaviour). In figure 7, the change in viscosity of HPMC solutions with increasing shear rate is presented. The numbers on the curves signify different viscosity types. For low shearing rates the fluids appear to be Newtonian, not changing significantly in viscosity, especially within the viscosity range normally used in centrifuge modelling (below 100  $mPa \cdot s$ ).

In order to assess whether the decrease in viscosity with increasing shearing rate is important for dynamic centrifuge modelling, estimations of the maximum shearing rate reached during a dynamic centrifuge test are made below.

#### 3.4.1 During the earthquake

During the earthquake, shearing rate will be estimated using one dimensional wave propagation. Assume a uniform, damped soil layer resting on a rigid plate. The only axis,  $z$ , starts from the surface of the soil and points downwards. The depth of the deposit is  $H$  and its damping ratio is  $\zeta$ . Horizontal displacement is termed  $u$  and time  $t$ . At the surface, the shear stress must be zero:

$$G \frac{\partial u}{\partial z} \Big|_{z=0} = 0 \quad (9)$$

At the bedrock, a known harmonic acceleration time history is applied:

$$\frac{\partial^2 u}{\partial t^2} \Big|_{z=H} = a \cdot e^{i\omega t} \quad (10)$$

where  $a$  is the amplitude of the imposed acceleration and  $\omega = 2\pi f$ , with  $f$  being the frequency of the imposed motion.

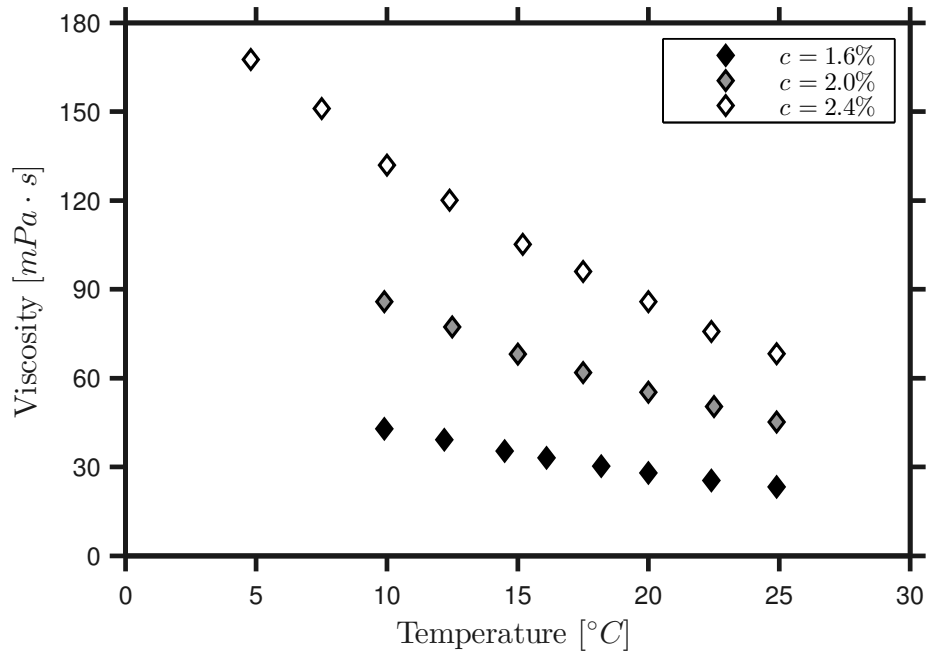


Figure 4: Viscosity change with temperature

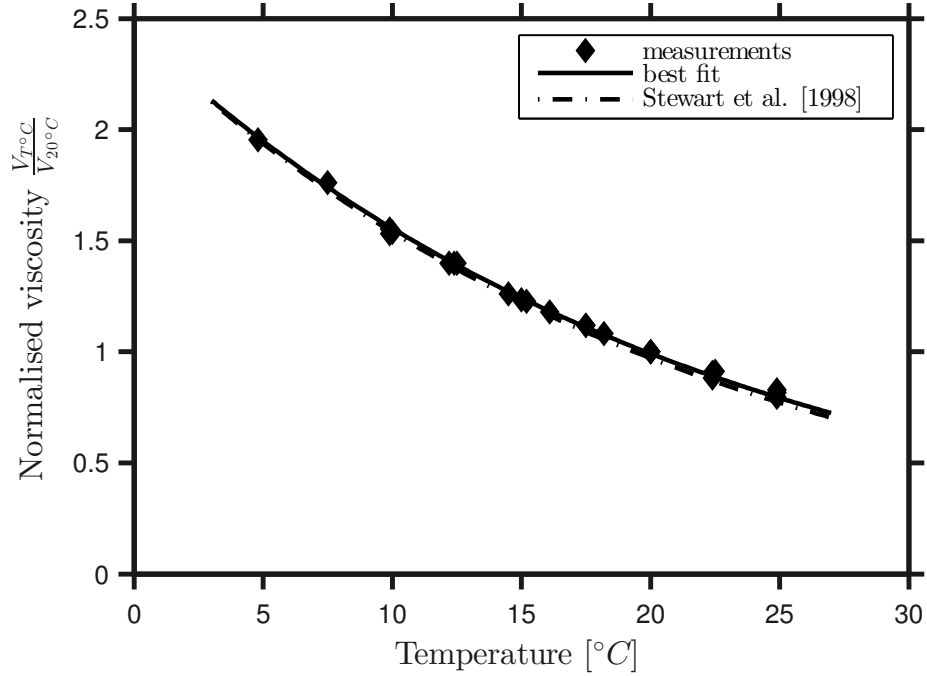


Figure 5: Normalised viscosity change with temperature

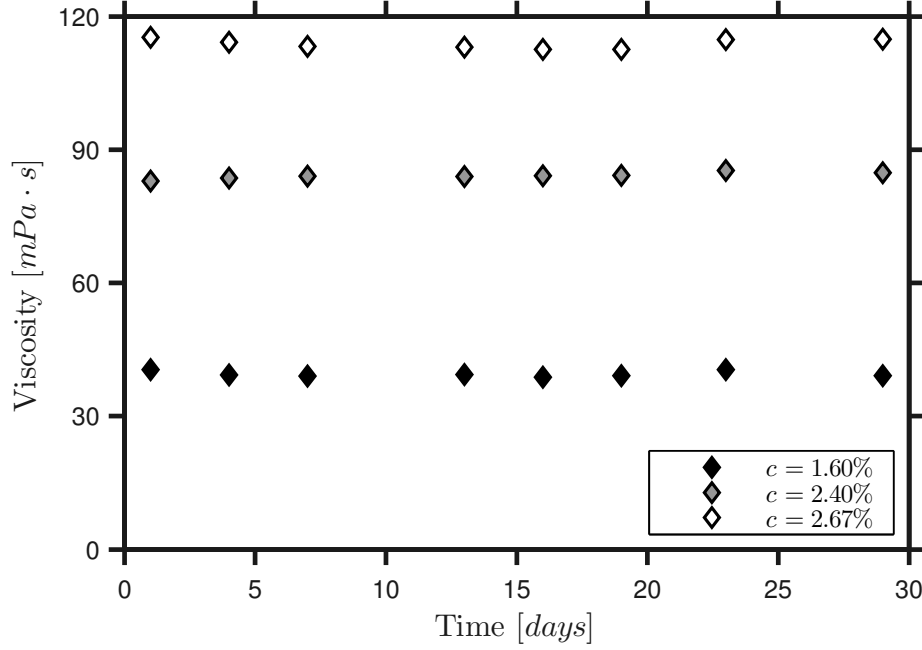


Figure 6: Viscosity change with time

The solution of the wave equation is of the following form [Kramer, 1996]:

$$u(z, t) = A \cdot e^{i(\omega t + k^* z)} + B \cdot e^{i(\omega t - k^* z)} \quad (11)$$

with  $k^* = \frac{\omega}{V_s \sqrt{1 + 2i\xi}}$ , a complex wave number,  $V_s$  being the shear wave velocity. For usual values of  $\xi$ , it is safe to use  $k^* \approx \frac{\omega}{V_s} (1 - i\xi) = k(1 - i\xi)$ .

Applying the conditions given in equations 9 and 10 to equation 11, the following expression can be obtained:

$$u(z, t) = -\frac{a}{\omega^2} \frac{\cos(k^* z)}{\cos(k^* H)} e^{i\omega t} \quad (12)$$

The shearing rate can be calculated as follows if the displacements are known:

$$\dot{\gamma}(z, t) = \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial t} \right) = ik^* \frac{a}{\omega} \frac{\sin(k^* z)}{\cos(k^* H)} \cdot e^{i\omega t} \quad (13)$$

The maximum value that shearing rate can take is of interest in this analysis. As a result, only the modulus of shearing rate is important:

$$|\dot{\gamma}(z, t)| = \left| ik^* \frac{a}{\omega} \right| \cdot \left| \frac{\sin(k^* z)}{\cos(k^* H)} \right| \cdot |e^{i\omega t}| \quad (14)$$

Since  $k^* \approx k(1 - i\xi)$  and knowing that

$$|\sin(x + iy)| = \sqrt{\sin^2 x + \sinh^2 y} \text{ and}$$

$|\cos(x + iy)| = \sqrt{\cos^2 x + \sinh^2 y}$ , equation 14 can be rewritten as:

$$|\dot{\gamma}(z, t)| = \frac{ka}{\omega} \frac{\sqrt{\sin^2(kz) + \sinh^2(\xi kz)}}{\sqrt{\cos^2(kH) + \sinh^2(\xi kH)}} |\xi + i| \quad (15)$$

For small values of  $\xi$  the above expression can be simplified as:

$$|\dot{\gamma}| = \frac{ka}{\omega} \sqrt{\xi^2 + 1} \sqrt{\frac{\sin^2(kz) + (\xi kz)^2}{\cos^2(kH) + (\xi kH)^2}} \quad (16)$$

The above equation can be used to make an estimation of the upper limit of strain rate during a dynamic centrifuge test. In the worst case, the height of the soil layer is such that the first natural frequency is excited:

$$H \approx \frac{V_s}{4f} \quad (17)$$

Depth  $z$  is expressed as a fraction of  $H$ :

$$z = \beta \cdot H = \beta \cdot \frac{V_s}{4f} \quad (18)$$



with  $\beta \in [0, 1]$ . Using this expression, equation 16 can be rewritten as:

$$|\dot{\gamma}| = \frac{2a\sqrt{\xi^2+1}}{\xi\pi V_s} \sqrt{\sin^2\left(\beta\frac{\pi}{2}\right) + \left(\xi\beta\frac{\pi}{2}\right)^2} \quad (19)$$

which takes its maximum value for  $\beta = 1$ :

$$|\dot{\gamma}|_{max} = \frac{2a\sqrt{\xi^2+1}}{\xi\pi V_s} \sqrt{1 + \left(\xi\frac{\pi}{2}\right)^2} \quad (20)$$

It is assumed that even while exciting the first natural frequency, damping ratio remains low,  $\xi \approx 3\%$ . Furthermore, a high value for base acceleration is used. If at prototype scale the excitation at the bedrock was  $0.4 g = 3.924 \frac{m}{s^2}$ , then at model scale, for a centrifugal acceleration of  $50 g : a \approx 200 \frac{m}{s^2}$ . Shear wave velocity is assigned a low value:  $V_s \approx 100 \frac{m}{s}$ . Following this conservative scenario, the maximum value of shearing rate is:

$$|\dot{\gamma}|_{max} \approx 40 s^{-1} \quad (21)$$

Consequently, usual high viscosity fluids used in centrifuge modelling can be assumed to act as Newtonian fluids during an earthquake (Fig. 7).

### 3.4.2 Post earthquake

In the case of modelling liquefiable soil, the excess pore pressures produced during shaking will dissipate after the earthquake and flow will occur. In this section a conservative estimation for the shearing rate of pore fluid during post earthquake reconsolidation will be made. It is assumed that fluid is only moving upwards. If the vertical axis is termed  $z$  and the horizontal  $x$ , then the shearing rate  $\dot{\gamma}$  can be defined as:

$$\dot{\gamma} = \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial t} \right) \quad (22)$$

The actual fluid velocity  $\frac{\partial z}{\partial t}$  can be associated to the artificial velocity  $v$ , that is used for Darcy's law, as follows [Atkinson and Bransby, 1978]:

$$v = \frac{e}{1+e} \cdot \frac{\partial z}{\partial t} \quad (23)$$

where  $e$  is the voids ratio.

Assuming laminar flow, Darcy's law can be used to provide an estimate for the shearing rate:

$$v = ki \quad \therefore \quad \frac{\partial z}{\partial t} = \frac{1+e}{e} \cdot ki \quad (24)$$

where  $k$  is the coefficient of permeability and  $i$  is the hydraulic gradient.

Since an upper estimate for shearing rate is sought after, the critical hydraulic gradient, for which the seepage forces become equal to the submerged weight of the grains, will be used:

$$i_c = \frac{\gamma}{\gamma_w} - 1 = \frac{G_s - 1}{1 + e} \quad (25)$$

with  $\gamma$  being the unit weight of the soil,  $\gamma_w$  the unit weight of water, and  $G_s$  the specific gravity of the soil's solids.

Combining equations 22, 24, and 25, the following can be deduced for the shearing rate at the critical hydraulic gradient:

$$\dot{\gamma}_c = \frac{G_s - 1}{e} \frac{k}{\partial x} \quad (26)$$

The pore fluid is flowing through the soil particles. One can imagine equivalent tiny pipes through which the fluid moves. The diameter of these fictional pipes can be used as  $\partial x$  in equation 26. For this estimation,  $\partial x$  is assumed to be roughly equal to  $d_{10}$  of the soil (90% by weight of the soil is composed of grains of larger diameter than  $d_{10}$ ).

In the case of Hostun sand,  $G_s = 2.65$ ,  $e_{min} = 0.555$ ,  $k = 1 \frac{mm}{s}$ , and  $d_{10} = 0.3 mm$ . The maximum shearing rate is calculated using equation 26:

$$\dot{\gamma}_{max} \approx 10 s^{-1} \quad (27)$$

Consequently, it is safe to assume that during post earthquake reconsolidation, high viscosity fluids used in dynamic centrifuge modelling behave as Newtonian fluids.

### 3.5. Effect of increased viscosity on damping ratio

Bolton and Wilson [1990] showed that increasing pore fluid viscosity can lead to an increase in damping ratio by a factor of 2 for silicon oil

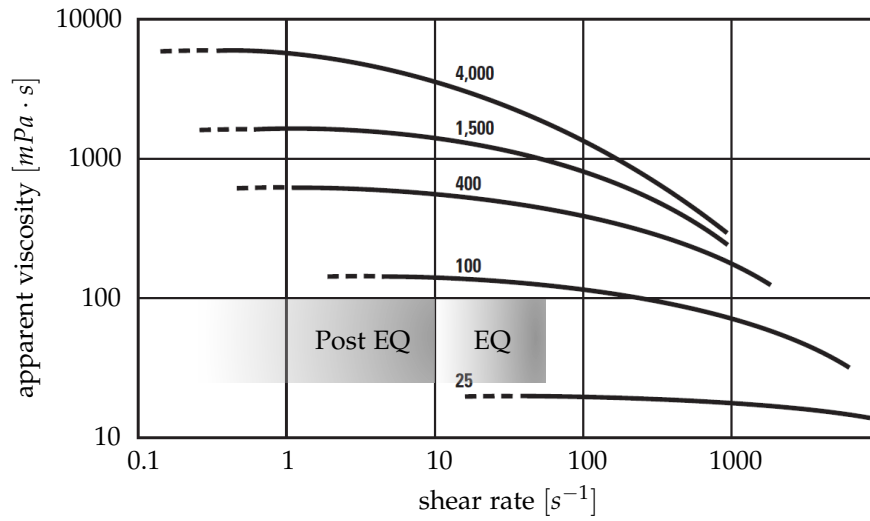


Figure 7: Apparent viscosity change with shear rate, at 20°C. Following Dow [2002]

of 100 cS kinematic viscosity at 75 Hz. However, Ellis et al. [2000] stress that viscous damping is only important when strain amplitudes are very small and loading frequency is high. For large strain events like those stemming from earthquake loading, they suggest that the contribution of increased fluid viscosity to the total damping can be neglected. In fact, for shear strains larger than  $2 \cdot 10^{-4}$ , they report that the increased skeleton damping is enough to mask any effect of pore fluid viscous damping.

#### 4. CONCLUSIONS

Careful pore fluid viscosity scaling is crucial to obtaining meaningful experimental results through dynamic centrifuge modelling.

In this paper, guidelines for the preparation of high viscosity HPMC pore fluids intended to be used for dynamic centrifuge modelling are given. The effects of concentration, temperature, ageing, and shearing rate on the viscosity of aqueous HPMC solutions are examined.

Equations that describe the variation of viscosity with concentration and temperature are presented. It is suggested that estimations for both the required viscosity and the expected

temperature during the test are made. Thereafter, the viscosity of the required solution at 20°C can be found using equation 8. Given the viscosity at 20°C, the necessary concentration can be calculated using equation 7. As the volume of pore fluid required to saturate the soil is known, the computed concentration can be used to estimate the weight of HPMC powder needed.

Ageing is shown not to affect HPMC solutions, at least within a time frame of one month. It is also demonstrated that it is safe to assume that HPMC solutions used in dynamic centrifuge modelling behave as Newtonian fluids, at least in the case of tests performed using Hostun sand. Quick estimations for the upper limits of shearing rate during and after a seismic event can be made using equations 20 and 26 respectively.

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