

Title: Spatial Metrics to Study Urban Patterns in Growing and Shrinking Cities

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Spatial Metrics to Study Urban Patterns in Growing and Shrinking Cities

Abstract

This paper reviews existing literature on spatial metrics, presenting a portfolio of metrics addressing the spatial patterns of growing and shrinking cities and discussing their potential and limitations. A wide and diverse set of spatial metrics was found. While these metrics address most of the identified spatial patterns of urban growth, spatial metrics used in urban shrinkage studies are much scarcer and not nearly sufficient to provide a comprehensive assessment of its spatial patterns. The paper concludes that there is a great potential for the development of new spatial metrics or mixed indicators, particularly in shrinkage contexts. The paper builds on recent literature that has been prolific in reviewing and developing metrics for particular spatial patterns (notably patterns of urban sprawl), but considers a very broad and multidisciplinary set of metrics, and focuses not only on the outcomes of urban growth but also on those of the increasingly common shrinking phenomenon.

Keywords: spatial metrics, spatial patterns, urban growth, urban shrinkage

1. Introduction

The study of cities' form and structure is recently re-gaining a central attention in the urban planning debate (Pinho & Oliveira, 2011). The increasing focus on sustainable development has strengthened the importance of the physical dimension of urban areas; and both European and North American literature are becoming increasingly concerned with the analysis of urban patterns, with particular emphasis on quantitative methods (Clifton et al., 2008; Dieleman & Wegener, 2004; Huang et al., 2007).

Along with these sustainability concerns, two other factors are also considered to have been essential to the most recent advances in quantitative analyses of urban patterns. These are the developments in geographical information systems and in information

technology; and the increasing quality and availability of spatially referenced data, notably with the development of remote sensing techniques (Clifton et al., 2008; Herold et al., 2003; Huang et al., 2007; Larkham, 2006).

This debate is gaining further interest and importance as urban spatial patterns of development of European and North American cities appear to have been changing considerably during the last decades (Beauregard, 2009; Kabisch & Haase, 2011; Turok & Mykhnenko, 2007). After a 20th century marked by intense and widespread growth of urban areas, mainly led by industrialization and technological development, and the consequent urban migration and extensive suburbanization; recent urban development trends anticipate more diversified trajectories for Western cities in the 21st century, with increasing inter-city competition in which some cities will tend to grow at the cost of others, depending on a wide and complex set of factors that are still not totally clear.

Processes of growth and shrinkage may occur simultaneously in the same urban system, leading to an increasing geographic polarization among and within cities (Banzhaf et al., 2006; Oswalt & Rieniets, 2006; Pallagst, 2005), and urban planning theories and tools should be prepared to deal with both these processes. This calls for a new perspective on the development of cities: one that places growth and shrinkage side by side, as equally valid and natural trajectories of urban development.

Using such an approach, and focusing on quantitative methods of urban form and structure, the authors carried out a literature review on the use of spatial metrics to study the patterns of urban growth and urban shrinkage (Reis et al., 2014). A wide range of spatial metrics was found in the international literature, from simple geometric measures to more complex indicators, developed and applied in several research fields – such as Geography, Planning or Ecology – and addressing different features and dimensions of the urban space (Huang et al., 2007; Reis & Silva, 2015; Reis et al., 2014; Schneider

&Woodcock, 2008; Schwarz, 2010). This paper develops and expands this literature review, putting together a broad portfolio of spatial metrics that is not restricted to a particular method or disciplinary background. Moreover, it seeks to perform a preliminary analysis of these metrics regarding their potential to address the particular spatial patterns of growing and shrinking cities.

Spatial metrics have been used for different purposes, such as characterizing urban patterns in order to support planning policy; comparing physical patterns of different cities or regions; or understanding the spatial-temporal patterns of urban development. They have also been increasingly used together with other methods, such as remote sensing techniques or imbedded in urban growth models (Banzhaf et al., 2009; Herold et al., 2005; Silva et al., 2008; Van de Voorde et al., 2009).

The main purpose of this paper is therefore to analyse and discuss the state of the art regarding spatial metrics used to quantify the spatial patterns of both urban growth and urban shrinkage, building a broad portfolio of metrics which can be useful for urban researchers and practitioners. The methodology will consist of: (1) an overview of the literature on the spatial patterns of growth and shrinkage; (2) a systematic review of the most important spatial metrics referred to in studies of urban form; and (3) bring together the previous findings in order to discuss the current state of research on spatial metrics to analyse the patterns and processes of growth and shrinkage, and find the main gaps that can be further explored in future research.

The paper is divided in five parts. After this general introduction, Section 2 addresses the changing patterns of urban development, presenting an overview of the literature on urban growth and urban shrinkage. A particular emphasis is set on the spatial patterns of these two phenomena, and the main spatial features that characterize growing and shrinking cities are presented. In Section 3 a thorough literature review on spatial metrics

for the analysis of urban form is presented, building on a previous review of spatial metrics carried out by the authors (Reis et al., 2014). Section 4 consists of a discussion of the main findings of the joint analysis of these two reviews, particularly regarding the adequacy of spatial metrics to evaluate the patterns of growing and shrinking cities, as mentioned above. In Section 6 the main conclusions of this study are summarized, as well as the most important topics for future research.

2. The study of urban change: growth and shrinkage

2.1. Urban growth and urban shrinkage: main concepts

As referred above, the spatial patterns of development of cities in Western countries have been changing during the last decades (Kabisch & Haase, 2011; Turok & Mykhnenko, 2007). Recent literature emphasizes the differences in urbanization trajectories of cities in different regions, different countries and even within countries, with higher inter-city competition in which some cities will tend to grow at the cost of others. These more diversified urban development trends of cities in developed countries have also brought about quite diversified planning discourses, from the reurbanization and revival of smart cities, mainly seen on Western Europe and on the largest US urban areas; to the urban decline of former US and Western European mining and industrial cities and to the severe decline of important Eastern European agglomerations.

After many decades of planning policy and practice assuming and promoting continuous urban growth (Bontje, 2004; Popper & Popper, 2002; Rieniets, 2005; Sousa, 2010), this growth paradigm appears to be changing as urban shrinkage is becoming an emergent subject in urban and regional planning (Pallagst, 2010). Processes of growth and shrinkage tend to occur simultaneously, leading to an increasing geographic polarization

among and within cities (Banzhaf et al., 2006; Oswalt & Rieniets, 2006; Pallagst, 2005), and urban planning theories and tools should be prepared to deal with both these processes.

Urban growth has always been one of the most prominent topics of planning. Numerous authors even argue that modern urban and regional planning has arisen in response to the social and economic problems caused by intense population growth in 19th century cities during the Industrial Revolution (Hall & Tewdwr-Jones, 2011; Wegener et al., 2007). A few decades later, during the first half of the 20th century, planners and rural conservationists became increasingly concerned with the uncontrolled spread out of cities across rural hinterlands, fuelling the first discourses towards controlling and preventing urban sprawl (Hall & Tewdwr-Jones, 2011). Concerns about the consequences of sprawl continued to be a central area of study by urban planners, geographers and landscape ecologists until the present day, triggered by extensive low density suburbanization and excessive land consumption in some European and North American cities.

Urban growth is, however, far from being a clear concept. There are at least three different and widely studied concepts of urban growth in the urban and regional planning literature, related respectively to population change, economic performance, and the spatial expansion of urban areas. The social-demographic dimension of urban growth focuses on demographic trends and migration. The first includes the natural population growth rates, depending mainly on fertility and mortality rates (affected by natural or socio-economic factors), while the second depends on the capacity of a city to attract residents from other cities or rural settlements (Rieniets, 2009; Turok & Mykhnenko, 2007).

Urban economic growth normally regards a city's economic performance, taking into consideration a set of economic variables such as growth in employment, income levels, GDP, or housing prices (Cheshire & Magrini, 2009; Glaeser & Gottlieb, 2006); while spatial growth concerns changes on the geographic space occupied by built structures and human activities and is often associated to terms such as "urbanisation" and "urban expansion"(Clifton et al., 2008). The spatial dimension of urban growth is perhaps the most widely studied in urban and regional planning, especially in disciplines like Spatial Planning, Urban Design, Physical Geography or Urban Morphology (Clifton et al., 2008; Cutsinger et al., 2005; Marshall & Gong, 2009; Reilly et al., 2009; Schneider & Woodcock, 2008 among others).

In this paper the focus will be set on both the spatial and the socio-demographic dimensions of urban growth (the economic concept of growth will not be directly addressed). For the purposes of this research,urban growth will thus be defined as *the process through which a city changes its spatial structure as a result of an increase in population, normally but not necessarily accompanied by the expansion of its urbanised area.*

Despite population decline has always been present throughout urban history, the study of urban shrinkage as a natural and accepted pattern of urban development has been somewhat neglected on the urban and regional planning debate. Planning theories, practices and methodologies have mainly been developed on the assumption of continuous and enduring urban growth, and policy actions have been – and to a certain extent still are – mostly designed upon the desirability of growth (Bontje, 2004; Hollander et al., 2009; Rieniets, 2005; Rink & Kabisch, 2009; Sousa, 2010; Wiechmann & Pallagst, 2012). It was not until quite recently that this "growth paradigm" of urban planning has started to change, as cities with declining populations are becoming more and more common in Western developed countries. Indeed, during the last decade, the

study of urban shrinkage has become an important research topic of urban and regional planning both in Europe and in North America, and a focus of attention of both researchers and practitioners.

The concept of urban shrinkage is not consensual, despite virtually all definitions of shrinkage found in the literature are closely related to population decline. The inclusion of minimum thresholds for a city's population change (Hollander et al., 2009; Pallagst, 2010) or the spatial scale of shrinkage (namely whether urban shrinkage applies only to cities whose total population has been decreasing, or also to parts of the city that are declining even when the overall population of the urban area is still growing) (Rieniets, 2009) are two of the most cited issues in defining urban shrinkage.

Other authors have attempted to develop more comprehensive definitions of shrinkage (to a certain extent following the approaches used for urban growth), including other factors such as “economic performance” or “physical shrinkage” of the built-up area. These definitions are however not easy to operationalize, since the economic implications of shrinkage are still not very clear and nor are its physical patterns (Hollander et al., 2009; Wolff, 2010). Therefore, the following definition for shrinking cities will be adopted for the purposes of this research: *territories experiencing population decrease, due to various reasons, and that may or may not have started to spatially shrink*(suggested by Sousa, 2010, p. 54).

2.2. Urban growth patterns

The literature on the patterns of urban growth is very extensive including a wide range of studies in many different disciplines from Geography and Urban Planning to Landscape Ecology or Urban Modelling. Subjects like Urban Morphology have also been interested in studying the spatial structure of cities and its changes over time, regarding

not only their physical shape (natural or built), but also the interaction between humans and physical features, including land uses, functions, and behaviours (Kropf, 2009). In this review we found a large set of spatial features that characterize urban growth, shown in Table 1. In order to facilitate the analysis we subdivided these patterns of growth in four main groups: Expansion, Sprawl, Polycentrism and Densification/Coalescence. It's important to note that these groups do not intend to constitute any formal categorization of urban growth, they simply correspond to the main processes of urban change in which the respective literature was focusing on. Moreover the different patterns of growth presented in the table below were defined by the authors based on an extensive review of the literature. These should not be interpreted as the only possible division of growth patterns nor as mutually exclusive characteristics. Different approaches can be found in the literature, notably studies focusing on different typologies of urban sprawl (see for instance Cutsinger et al., 2005; Ewing et al., 2002; Sarzynski et al., 2013; Torrens, 2008).

[insertTable 1 - Spatial patterns of urban growth]

As it was referred above, urban expansion is a very common definition of urban growth, and the increase in the urbanized areas is one of the most straightforward – and also one of the most cited – urban growth patterns. Urban expansion can be easily operationalized by identifying the conversion of non-urban into urban land use types over time, or by measuring the decrease in green areas or pervious surfaces. This pattern of growth is particularly popular in Ecology – notably in studies of the impacts of urbanization in natural landscapes and ecosystems – and in Urban Modelling (Herold et al., 2005; Silva et al., 2008; Wilson et al., 2003).

Urban sprawl is also closely connected to urban expansion, yet these are different concepts: while urban sprawl implies expansion, its spatial features go beyond it. Although it is probably the most studied urban growth pattern in Planning and

Geography, the exact definition of sprawl is rather ambiguous (Frenkel & Ashkenazi, 2008). Several authors have pointed out the different dimensions of sprawl and often suggested a set of quantitative methods and metrics to measure them (Ewing et al., 2002; Galster et al., 2001; Sarzynski et al., 2014; Torrens, 2008). Although the characteristics of sprawl are still not totally clear or consensual, some agreement exists on recognizing extensive urbanization, low density, single use, fragmentation/scatter or poor accessibility as some of its main spatial features (see Table1).

A different pattern of urban growth that is often seen as an alternative to urban sprawl is the polycentric model of urban development. Although some authors do not make a clear distinction between a multi-nodal structure of sub-centres and a dispersed and apparently unorganized sprawl pattern (Gordon & Richardson, 1996, 1997), others see in the polycentric pattern a potential for compact development (Anas et al., 1998; Ewing, 1997; Martens, 2006). Polycentric urban growth is characterized by the growth of “outlying” settlements, resulting in sub-centres formation although the characteristics and thresholds – such as their size, specialization, spatial location or the distance and degree of interdependence between sub-centres – used to define a “sub-centre” – may not always be clear and has been subject to discussion in the literature (Champion, 2001; Martens, 2006; Parr, 2004; Yang et al., 2012).

Other authors refer another type of urban growth based on densification, i.e. through “infill development” and increasing density. In this case, urban growth can be accomplished without significant spatial expansion, for instance through increasing population density or by redevelopment of existing areas using higher built-up densities¹.

A considerably different approach to urban growth patterns is concerned with the physical shape of the built structures and with the way they fill the urban space. This

¹It is important to note that these are two different patterns: an increase in built-up density does not necessarily correspond to higher population densities and vice versa.

eminently morphological approach relies on the idea that although built-up structures grow into very complex and irregular patterns, these patterns tend to repeat themselves at different levels of hierarchy and at different spatial scales, resembling fractals. Fractal structures, although apparently chaotic, follow a well-defined spatial organization principle that can be quantified (Batty & Longley, 1994; Frankhauser, 1998, 2004). In this context, notions and tools from fractal geometry have also been used to characterize and forecast urban growth patterns (Batty, 2008; De Keersmaecker et al., 2003).

2.3. Urban Shrinkage patterns

The literature on patterns of shrinkage is far less extensive than the literature on patterns of growth. This is in part because the systematic study of shrinkage is quite recent, but also because spatial patterns of shrinkage tend to be less clear. Normally urban areas do not shrink spatially when they lose population, and even when a decrease in urbanized areas is observed, it occurs with considerable time lags as built structures only disappear as a result of demolition policies or long term degradation (Oswalt, 2005; Pallagst, 2005; Rieniets, 2005, 2009; Siedentop & Fina, 2008). There are, however, some common spatial features that our literature review found to be characteristic of shrinking cities, the most commonly referred of these are shown in Table 2).

[insert Table 2 - Spatial patterns of urban shrinkage]

One of the most common spatial features of shrinking cities is the presence of high rates of housing vacancy, as a result of population out-migration to other cities or to other areas of the city. Vacant housing space over a long period of time is either demolished or decays, creating a fragmented housing geography with a small-scale perforation of the housing fabric often accompanied by degradation of the built structures (Reilly et al., 2009; Schwarz et al., 2010). In a similar way, mining closures, deindustrialization and the

close down of manufacture industries often leave shrinking cities with large scale brownfield sites, both in the central city and in the suburbs (Schwarz et al., 2010; Siedentop & Fina, 2008). In some severe cases of urban shrinkage large-scale demolition of the housing stock as a result of planning policies may take place as well (Haase et al., 2007; Schwarz et al., 2010).

On a larger spatial scale, shrinking cities are often characterized by a perforated and fragmented urban landscape, with abandoned lands and low-density settlements (Schwarz et al., 2010). According to Hollander et al. (2009) the patterns of shrinkage at the city level are varied: the hollowing-out of the inner city compared to its suburbs is one of the most commonly referred patterns of shrinkage in US and European cities, but very different patterns can be found as well. Indeed, urban shrinkage often represents a heterogeneous spatial phenomenon throughout the city, where some of its parts can even grow slightly, while others stagnate or shrink, although the latter situations have to prevail in order to result in an overall shrinking process (Hollander et al., 2009; Sousa, 2010).

Moreover, urban shrinkage is often accompanied by sprawl in the urban peripheries, resulting in urban areas where less people and fewer activities are spread out across a more extensive territory (Couch et al., 2005; Rieniets, 2005; Siedentop & Fina, 2008). The physical patterns of shrinkage sprawl are quite similar to those of urban sprawl in a growing context, resulting in a fragmented and perforated territory with low-density development and increasing vacancy and deteriorating urban fabric in inner city locations, although this last effect is usually more severe in shrinking cities. This process presents some similarities to the “desurbanisation” stage of the widely referred cyclic model of urban development introduced by van den Berg and colleagues (Frenkel, 2007; Kabisch & Haase, 2011; Van Den Berg, 1982), but it may correspond to longer term or even permanent processes in shrinking cities.

3. Review of spatial metrics

3.1. Methodology

This section presents a literature review on spatial metrics. This analysis builds on a previous review of metrics carried out by the authors (Reis et al., 2014), presenting an extended, updated and more thorough portfolio of spatial metrics used to measure the urban growth and urban shrinkage patterns identified on Section 2. In order to cover as many metrics as possible, independently of the subject area or methodology used, a broad definition of spatial metrics has been adopted. Spatial metrics are defined in this paper as *the quantitative measures used to assess the spatial characteristics of urban settlements and structures*” (Reis et al., 2014).

Similarly to the work carried out by Reis et al. (2014), the methodology for this review consisted of a first research in multidisciplinary databases by keywords (mainly two databases were used: “Scopus” and “Web of Knowledge”), followed by a second research considering the references and citations of selected papers. Some of the most used keywords were “metrics”, “urban form”, “urban growth” and “urban shrinkage”. Only metrics used in studies with empirical applications published over the last 15 years were surveyed. Moreover, the results have been restricted with regard to the scale of analysis. Although we considered metrics that use a broad range of scales (regional, urban, neighbourhood), metrics using the scale of the building, common in urban design and typological approaches, were left out of this study.

Given the large number and the wide diversity of the metrics found, they were assembled into three groups, based on the area of knowledge and methodological

approach to urban form in which the metrics were developed. These groups (or “types of metrics”) are:

1. landscape metrics;
2. geo-spatial metrics;
3. spatial statistics.

It is important to notice that these groups do not intend to constitute a universal classification or a typology of metrics, but rather to group metrics considering their disciplinary background and the broad methodological approach they use, in order to facilitate the analysis. Moreover, some of the metrics from different groups are based on similar principles, their objects of study sometimes overlap and some metrics were even developed based on (or influenced by) metrics from a different group².

This literature review found a total of 160 metrics (41 landscape metrics, 108 geo-spatial metrics and 11 spatial statistics), the great majority of which were used in studies of urban growth. The following sections will present these metrics in more detail. A full list of all the metrics reviewed, featuring their description, calculation method and the context (urban growth, shrinkage or other) in which they were applied is provided in an online appendix.

Finally, although this study intends to cover a diverse and a comprehensive set of metrics, it is important to mention that there are other metrics that were not reviewed in detail here, including different metrics and methods of spatial data analysis in the spatial statistics and econometrics literature (for more complete reviews see Getis et al., 2004; O'Sullivan & Unwin, 2010) and complex methods of land classification used in remote sensing (Yang, 2011) or metrics focusing on other subjects such as transport and accessibility (Cerda, 2009; Curtis & Scheurer, 2010).

² A previous review (Reis et al., 2014) included a 4th group of ‘accessibility metrics’, which was not considered here for its limited relevance for the study of urban growth and shrinkage.

3.2. Landscape metrics

Landscape metrics have been used since the 1980s in landscape ecology to quantify the shape and pattern of vegetation (Clifton et al., 2008; Herold et al., 2005; McGarigal & Marks, 1995). Landscape ecologists are primarily concerned with environmental protection and resource conservation (Clifton et al., 2008; Turner, 2005), and thus landscape metrics have been traditionally used to quantify several aspects of landscape configuration and composition, focusing primarily on types of land cover rather than land use.

Landscape metrics have been, however, increasingly used to study urban patterns. Indeed, several authors highlight the usefulness of spatial metrics adapted from landscape ecology to represent spatial urban characteristics (Aguilera et al., 2011; Herold et al., 2005; Herold et al., 2003; Herold et al., 2002; Schneider & Woodcock, 2008; Schwarz, 2010), to link economic processes to land use patterns (Parker et al., 2001, referred to by Herold et al., 2005) and also in combination with urban growth models. According to Clifton et al. (2008), spatial metrics adapted from landscape ecology differ from other urban form indicators in two main aspects: they often rely on data derived from aerial photography and satellite remote sensing, and they use “patches” (i.e. polygons with homogeneous characteristics for a specific landscape property) as the basic unit of analysis.

In the review of empirical studies, 41 different landscape metrics were found (see Table A1 in appendix online). These include quite different types of metrics; from simple geometrical measures (e.g. *patch area*) to more complex indicators based on perimeter-

area ratios (e.g. *fractal dimension*, *shape index*) or on statistical measures (e.g. *Shannon's diversity and evenness indexes*).

These metrics also aim at analysing very different morphological characteristics of the urban landscape. Taking this into account and building upon previous classifications by several authors (Aguilera et al., 2011; Frenkel & Ashkenazi, 2008; Huang et al., 2007; McGarigal & Marks, 1995; Schneider & Woodcock, 2008; Seto & Fragkias, 2005; Weng, 2007), these metrics can be divided into the following four categories: shape irregularity, fragmentation, diversity and other (Table 3).

[insert Table 3 - Landscape metrics organized by categories]

Shape irregularity includes the metrics that assess whether an urban settlement has a regular or even shape or if, on the contrary, it has a complex shape with a ragged edge. They can be used to characterize a single patch (e.g. *fractal dimension*³ or *shape index*) or at the landscape level (e.g. *landscape shape index*, *edge density* or *area weighted mean patch fractal dimension*). The metrics most often used to analyse shape irregularity are *area weighted mean patch fractal dimension*, *edge density*, *area weighted mean shape index* and *landscape shape index*.

Fragmentation metrics measure the extent to which urban settlements – or patches – are close together (aggregated) or dispersed (fragmented). These metrics are used at the landscape level. A fragmented landscape is normally characterized by a higher number of patches, with a smaller average size and located further away from each other. The metrics mostly used to measure fragmentation are *mean patch size*, *number of patches*, *patch density* and *contagion index*.

³ The fractal dimension used in landscape ecology is not the same metric used in applications of fractal geometry. Although these fractal dimensions are based on principles from fractal geometry, the calculation method is quite different (see table A1, in appendix online).

Diversity metrics focus more on the composition of the urban landscape rather than on its shape. The most used metrics are *Shannon's diversity* and *evenness indexes*, which measure the distribution of different patch types (for instance land use types) throughout the urban area. Other metrics include the *largest patch index*, measuring the relative importance of the largest patch (which may be useful to study for instance the importance of the urban centre), and the *compactness index*, which uses a concept of compactness based on both fragmentation and shape irregularity.

3.3. Geo-spatial metrics

Geo-spatial metrics include metrics mostly used by urban planners and geographers and normally developed specifically to measure urban spatial patterns. These metrics are very diverse regarding both their complexity (from basic statistical measurements to more complex indicators) and the specific feature of the urban built environment they aim to measure.

An important difference between these metrics and the metrics from landscape ecology is that while the latter include a set of metrics that evolved in a “top-down” type of approach, being developed by a set of researchers in one particular subject and subsequently transferred to multiple case studies and software (in most of the cases using the same mathematical formulations); the former tend to be developed for particular case studies. They do not have, therefore, such a transferable potential due to the customization to each particular case study and, accordingly, to each geo-spatial subject (i.e. geospatial metrics in geography, architecture or planning can have very different assumptions, methods of collecting/processing data, scales of analysis and variables used, even if they all measure the same specific spatial feature).

A set of 108 different metrics were found in this review (see table A2 in appendix online), aiming to measure many different features of urban areas, such as land use diversity or fragmentation. Table 4 shows nine categories of geo-spatial metrics considering the urban morphological features they intend to measure.

[insert Table 4 – Geo-spatial metrics organized by categories]

Fragmentation metrics assess the extent to which urban settlements are more continuous and compact or more scattered across the territory. They take into account different characteristics of the urban areas, such as the ratio between built-up and vacant areas (e.g. *ratio of open space*, *gross leapfrog index*), or the geographic position of new built-up areas in relation to existing ones (e.g. *leapfrog*, *continuity*, *clustering*).

This category also includes the fractal dimension. Fractal dimension measures the extent to which built areas fill the two-dimensional space, varying between 1 – the Euclidean dimension of a line, with length but no width – and 2 – the dimension of a plane, with length and width. In other words, it represents the extent to which geographical objects fill more space than a line but less than a plane (Frankhauser, 2004; Longley & Mesev, 2000), using estimation methods that verify the extent to which an observed pattern follows fractal logic⁴.

Density metrics measure the density of built up development or the intensity of particular land uses in an urban area or in different sub-areas, normally using ratios of population, number of activities or residential units per sub-area of development.

Land use diversity metrics measure whether an urban settlement is more mixed or mono-functional, normally counting the number of different land uses present (e.g.

⁴There are several ways to calculate the fractal dimension of an urban area, using different measures (fractal relations) and through different algorithms, therefore there can be more than one fractal dimension for the same urban area, depending on the method used. Some of the most used methods are the box counting method (Batty and Longley, 1994; Shen, 2002; Terzi and Kaya, 2011), the dilation method (Frankhauser, 1998; De Keersmaecker et al. (2003); Terzi and Kaya, 2011), or the correlation analysis (Frankhauser, 1998; De Keersmaecker et al. (2003)).

segregated land use, land use diversity). There are however metrics using different and more complex methods, notably the *land use diversity index*, which evaluates the evenness of the distribution of land uses based on the concept of entropy (Knaap et al., 2007).

Metrics of Centrality measure the degree to which urban development occurs close to the central business district, assuming implicitly a monocentric urban structure (e.g. *distance to CBD, centrality, centrality index*); or the extent of this monocentric structure (e.g. *core-dominated nuclearity*). Metrics of Accessibility normally focus on the proximity between different activities or land uses in an urban area. As referred before, it's important to note that there are other more complex measures of accessibility that were not included in this review.

Connectivity metrics were designed based on the notion that sprawled patterns contain winding streets, *cul-de-sacs* and excessively large blocks, which reduce the connectivity between different places in an urban community (Song & Knaap, 2004). Inequality measures assess whether certain attributes (for instance houses, jobs or other activities) are evenly distributed across the urban space or if they are disproportionately located in some areas.

The category Spatial network analysis includes three different sub-categories corresponding to different methods: Space syntax (Hillier et al., 1976), Multiple centrality assessment (Porta et al., 2006c) and other dual graph approaches. Network analysis has been used in geography for a long time (Volchenkov & Blanchard, 2008) with a wide range of research in urban studies since the sixties (Porta et al., 2006a). It consists in representing cities as networks, in which identifiable urban elements (e.g. settlements, locations, intersections) are regarded as nodes in a planar graph and the connections between pairs of nodes (e.g. roads, transport lines) are represented as edges.

After the construction of a graph, it can then be studied using several tools and measures of graph analysis.

A set of metrics – mainly topological centrality measures – can then be extracted from the graph in order to quantify the relative accessibility of each space in the system. In this review,²¹ spatial network analysis metrics were found in empirical studies. The most commonly used metrics are *connectivity*, *integration*, *intelligibility* and *synergy*. Further details on Spatial Network Analysis methods can be found in Volchenkov and Blanchard (2008), Porta et al. (2006a, 2006c) or Hillier (1996), among others.

The category “Other metrics” includes metrics that, for quantifying particular features of urban areas, do not belong to any of the four categories above. These comprise very diverse metrics, from measure of the proportion of urban development along major roads (*highway strip*), to metrics of vacancy or assessing the degree of monocentricity/polycentricity on an urban area (*median and mean contour polycentricity*).

3.4. Spatial statistics

The field of spatial statistics is concerned with the mathematical and statistical descriptors of spatial structure, focusing on the nature of spatial data (Getis et al., 2004). In other words, spatial statistics are metrics based on statistical tools, used to assess the distribution of events across space. These metrics are often used in combination with regression and spatial econometric models, but are also used to characterize particular spatial patterns of urban settlements, such as diversity or fragmentation.

This literature review found 11 spatial statistics, which we divided in four categories (Table 5). Regression metrics normally correspond to *density gradients* used to determine the spatial profile of land use change through time, and are often calculated by regressing

density against distance from the city centre, using the Ordinary Least-Squares (OLS) method (Torrens, 2008).

[insert Table 5 – Spatial statistics metrics organized by categories]

The concept of spatial autocorrelation (or spatial dependence) relates to the idea that data from near locations are more likely to be similar than data from more distant locations (Haining et al., 2010; O'Sullivan & Unwin, 2010). Spatial autocorrelation metrics are useful to measure, for instance, urban decentralization patterns – whether certain types of areas (e.g. density, land use types, activities) are evenly (or randomly) distributed across the urban area or clustered – and have been used to study urban sprawl (Torrens, 2008; Tsai, 2005) or patterns of re-urbanisation (Porat et al., 2012). The *Moran's coefficient (I)* and *Local Moran coefficient (I_i)* are the two most used autocorrelation measures according to this review.

Evenness of distribution metrics measure the inequality of an attribute distribution (e.g. population or employment) by spatial units in a metropolitan area. For instance, high values of the *Gini coefficient* (i.e. close to 1) mean that population or employment density is extremely high in fewer sub-areas whereas values close to 0 indicate that these attributes are evenly distributed across the urban area. This metric, however, does not take into account the spatial location of these attributes, contrarily to metrics based on spatial autocorrelation.

Two other metrics were found – *number of fragments* and *spatial index* – that measure the extent to which an attribute (e.g. activity type) is fragmented across different locations. It is important to notice that other metrics and methods of spatial data analysis can be found in the literature on spatial statistics and econometrics (for such review see, for instance, Getis et al., 2004; Haining et al., 2010; O'Sullivan & Unwin, 2010). However, these metrics are, clearly, beyond the nature and remit of this review.

4. Metrics for growth and shrinkage

The literature review showed that the state of international research on urban growth and shrinkage is still not at the same level. Urban growth has always been at the centre of urban and regional planning debate, and holds a very wide and multidisciplinary set of studies, theories and methods, that have been developed since the beginning of last century. Literature on shrinkage, on the contrary, is generally quite recent and although it has been rapidly gaining importance in recent years, it is still much less developed than the literature on growth.

This is particularly clear regarding the spatial patterns of these two phenomena, and it is even more significant if we consider quantitative approaches, as the literature review on spatial metrics clearly showed: 125 spatial metrics were found in studies of urban growth, a much larger number than the 15 metrics used to quantify spatial patterns of urban shrinkage. The reasons for this discrepancy will be discussed further in this section.

First it is important to mention that the multidisciplinary review of the spatial metrics used in studies of urban growth and urban shrinkage carried out in this paper found a large set of metrics with clearly different purposes, calculation methods and disciplinary backgrounds. Building on the work of Reis et al. (2014), these metrics were divided into three different types: landscape metrics, initially developed by landscape ecologists but increasingly used in urban analysis; geo-spatial metrics, a very diverse set of metrics that have normally been developed specifically for urban studies; and spatial statistics, which were mostly developed for spatial data analysis by statisticians and geographers but are also used in studies of urban growth.

Landscape metrics are quite developed in the literature, with a strong and well-documented body of research using these metrics to quantify urban patterns, especially

patterns of growth or of urbanization(Reis et al., 2014). These metrics have been widely applied and tested on different situations and used in a large set of cities around the world (Aguilera et al., 2011; Herold et al., 2005; Huang et al., 2007; Schneider & Woodcock, 2008; Schwarz, 2010; Wu et al., 2011). Their results have been broadly discussed and their methods are quite standardized, facilitating empirical applications and the comparison of different case studies.

Landscape metrics are, however, sometimes criticized for relying too much on ecology principles not being the most adequate to study some specific urban processes. This is particularly evident for smaller scale (or intra-urban) phenomena – landscape metrics usually use regional scales – and for processes involving population movements, socioeconomic variables or governance structures (Herold et al., 2005; Schneider & Woodcock, 2008; Schwarz, 2010).

Geo-spatial metrics is by far the most wide and diverse of the three types of spatial metrics considered here. Although each individual metric has not been applied in a large number of case studies (some metrics were specifically designed for a particular case study, and are therefore less transferable), the total number and diversity makes these metrics extremely relevant. Some of these metrics present quite interesting measures of urban patterns, and have the advantage of having been mostly developed specifically for urban studies. It is important to note that some metrics from syntax of space have, according to this review, a higher number of empirical applications than the other geo-spatial metrics, which can be explained by the fact that they tend to be linked with commercial software developed during the past 20 years and therefore being used as a toolbox for different case studies.

Spatial statistics is a field of study developed mainly by geographers and spatialeconometricians that encompasses a number of methods of spatial data

analysis(Getis et al., 2004), and whose thorough literature review was not carried out in this research. Instead, we presented a few metrics that have been used in the study of urban growth patterns. These metrics were mostly used to characterize the evenness/inequality of the distribution of an attribute, and to find patterns of spatial clustering or fragmentation of these attributes across an urban area(Reis et al., 2014).

One of the main objectives of this literature review is to compile and present available spatial metrics and to discuss their potential to characterize the identified spatial patterns of urban growth and shrinkage. Table 6 presents a summary of the metrics found, indicating whether they were used in studies of urban growth, in studies of urban shrinkage or in studies of urban patterns not explicitly related to any of these two phenomena.

[insertTable 6 – Summary of metrics]

Regarding urban growth, the review carried out in this paper shows that there is a wide and diverse set of spatial metrics in the literature addressing most of its identified spatial patterns. Measuring the spatial configuration and composition of urban expansion patterns is the goal of most studies using landscape metrics.

The study of spatial patterns of urban sprawl has also been the subject of extensive research, including several studies developing and applying all types of spatial metrics. Many landscape metrics were used to study patterns of sprawl, particularly those measuring shape irregularity and fragmentation. The importance of the urban sprawl literature is perhaps even more evident considering geo-spatial metrics.Indeed a significant part of the studies using these metrics focus on sprawl (Crawford, 2007; Frenkel & Ashkenazi, 2008; Galster et al., 2001; Hasse & Lathrop, 2003; Knaap et al., 2007; Sarzynski et al., 2014; Song & Knaap, 2004; Torrens, 2008).Therefore there is a great number of geo-spatial metrics measuring the most well-known physical patterns of

the sprawl phenomenon – particularly those of the categories fragmentation, density, land use diversity, centrality and poor accessibility) – some of these were developed specifically to measure this phenomenon. Sprawl patterns were also studied with spatial statistics, notably the *Moran's I* (Torrens, 2008).

Other metrics were found characterizing other particular patterns of growth, but patterns related to urban expansion and to urban sprawl (as defined before) are by far the most cited. It is therefore fair to say that there is a wide body of literature using spatial metrics to characterize the main physical patterns of urban growth. The main challenge appears to be the selection, adaptation and/or combination of some of these metrics, in order to create one or more spatial metrics or mixed indicators able to provide a more holistic and accurate assessment of specific growth patterns at different spatial scales.

Contrarily to growth, the study of physical patterns of shrinkage is still quite recent, lacking quantitative approaches and systematic methods. In a literature review that covered more than 50 studies about urban shrinkage, only 4 empirical studies were found using spatial metrics (Bontje, 2004; Couch et al., 2005; Kabisch et al., 2006; Schetke & Haase, 2008). This is in accordance with the findings of the review presented above, which clearly suggested a lack of research on spatial patterns of shrinkage. Most of the few empirical studies of urban shrinkage found using quantitative approaches focus on modelling the shrinkage process based on household residential preferences, considering both demographic trends and location attractiveness (Banzhaf et al., 2006; Haase et al., 2012; Haase et al., 2010; Kabisch et al., 2006; Lauf et al., 2012; Schwarz & Haase, 2010; Schwarz et al., 2010). The four empirical studies using spatial metrics encompass a total of only 15 metrics: 2 landscape metrics and 13 geo-spatial metrics (Table 6), which makes the development of quantitative methods (including spatial metrics/mixed indicators) to assess shrinkage patterns one of the main challenges for future research.

The reasons for the few metrics developed so far or for the absence of a mixed indicator that can tackle different characteristics of the spatial patterns of shrinkage, as far as this review is concerned, may be related to three factors. Firstly, spatial patterns of urban shrinkage are indeed quite complex. Shrinkage in spatial terms is not the opposite of growth: contemporary urban areas do not tend to physically shrink when they lose population but, on the contrary, they often continue to grow (or sprawl) at a lower density (Oswalt, 2005; Pallagst, 2005; Rieniets, 2005, 2009; Siedentop & Fina, 2008). Moreover, even when built-up areas decrease as a result of abandonment, long-term degradation and the consequent demolition, this occurs with considerable time lags from the moment population started to decline, increasing the difficulty of the analysis.

The second reason relates to the fact that the systematic study of urban shrinkage in urban planning is relatively recent and has been to a large extent fuelled by planning practice (see, for instance, Oswalt, 2006) and therefore has been more concerned with the causes of shrinkage and with policy actions to deal with this process, than studying or measuring its patterns. Finally, it is not easy to find good quality quantitative data to support the development of urban shrinkage metrics. This is because shrinkage patterns require a wide range of variables (built-up/physical structures but also demographic, land use, and socio-economic data), and a high level of disaggregation since some of these patterns occur at different scales, including very local (block or neighbourhood) levels.

Despite these gaps, a few metrics were found capable of evaluating some of the spatial patterns of urban shrinkage identified before. Three geo-spatial metrics appear to be particularly relevant, as they address residential vacancies, demolition and urban renewal.

It is important to point out that this review looked into the state of the art on the use of spatial metrics, in particular discussing the number and diversity of spatial metrics used in empirical studies and to what extent the spatial metrics literature is taking into account

the spatial patterns of urban change, from a “growth and shrinkage” perspective. Another not less important issue is the quality of these metrics or, in other words, whether they are actually useful or efficient in measuring particular spatial patterns.

This is an important question because it is clear on the tables presented above that there are several metrics addressing similar spatial patterns which might, on the one hand, present levels of correlation amongst each other; and, on the other, perform differently in terms of the usefulness of their results (Orenstein et al., 2014; Schwarz, 2010). Moreover, the applicability and performance of indicators depends highly on other factors such as the type and quality of data they require or the scale of analysis (both in terms of extension and grain) (Šímová & Gdulová, 2012). Therefore the quality and usefulness of metrics might depend on the data available, its aggregation level and the scale of analysis of particular contexts or case studies. The review and discussion carried out in this paper provides a good starting point for future research focusing on these questions, testing some of the metrics presented here in different empirical case studies.

5. Concluding Remarks and further research

This paper presented two literature reviews. The first review focused on the phenomena of urban growth and shrinkage and identified the main spatial patterns of growing and shrinking cities. Building on previous work by Reis et al. (2014), the second review presented an extensive and multidisciplinary portfolio of spatial metrics used to quantify these patterns. The paper then discussed the potential and limitations of spatial metrics to assess spatial patterns of urban growth and shrinkage. Despite the prolific literature on metrics to study urban growth and shrinkage from very distinct perspectives and backgrounds, this study identified important gaps.

Regarding urban growth, we found a range of spatial metrics addressing some of the most important spatial patterns identified, although one could ask if there are no other spatial patterns occurring less frequently and therefore less prone to research. Nevertheless, these metrics may need to be adapted to the particular conditions of growth in different case studies and to different spatial scales. The combination of different metrics on a single multidimensional indicator, perhaps with the inclusion of socio-economic and demographic variables as well, is certainly an interesting topic for future research.

The gaps in urban shrinkage literature are very clear, with only a few studies found using quantitative approaches to measure its spatial patterns, and even fewer using spatial metrics. In this literature we found, however, metrics focusing on the most important patterns of shrinkage (notably vacancy, demolition and fragmentation), providing positive prospects for the development of new metrics.

Several authors argue that much still needs to be done in the development of new spatial metrics, either improving the existing ones or creating indices that aggregate information of different metrics, to achieve robust measures to assess urban growth patterns (Aguilera et al., 2011; Herold et al., 2002; Huang et al., 2007; Huang et al., 2009). This is even more important for the study of shrinking cities, in which the small number of studies using quantitative methods to measure their spatial patterns makes the development of such methods one of the most important challenges ahead on this topic. As referred above, a further assessment of the usefulness of indicators for particular research contexts and goals, perhaps through a more detailed classification of metrics, would also be important and useful for researchers studying urban form (see, for instance, Frenkel & Orenstein, 2011). In this sense, the review carried out in this paper may be a good starting point for a better understanding of the quantitative indices of urban form

and for the development of new metrics or mixed indicators that can reveal more thoroughly the spatial patterns of growing and shrinking cities.

Moreover, it may be useful to extend the literature to other types of metrics including demographic and socio-economic indicators (this review considered spatially-explicit metrics only) that could be combined with some of the spatial metrics presented here in the form of a mixed indicator. The latter could be particularly useful for shrinkage metrics, since urban shrinkage, driven in the majority of cases by profound demographic changes, would benefit from the emergence of new studies combining the physical dimensions of shrinkage with socio-economic and demographic variables (for instance, Banzhaf et al., 2006; Buzar et al., 2007; Kabisch et al., 2006; Schetke & Haase, 2008; Sousa, 2010).

Processes of urban shrinkage are becoming more and more frequent in cities in developed countries, particularly in Europe and North America, often occurring together with urban growth within the same city or urban system. This calls for a renewed perspective of planning and urban analysis: one that defines theories, methods and policies that are adaptive to both these phenomena. If it is true that some of the methods (including spatial metrics) developed focusing mostly on growth can be equally used in a shrinking context, some particular patterns of urban shrinkage are still clearly lacking proper understanding and appropriate assessment methods. We believe the review and discussion carried out in this paper might constitute a starting point for the development of more robust methods of urban analysis that are appropriate both for growing and shrinking areas.

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Table 1 - Spatial patterns of urban growth

Spatial patterns of urban growth		Sources	
Expansion	Increase in urbanized area (also measured as greenfield/pervious area consumption)	Frenkel and Askenazi (2008)	Pham et al. (2011)
		Hahs and McDonnell (2006)	Schneider and Woodcock (2008)
		Herold et al. (2002)	Seto and Fragkias (2005)
		Herold et al. (2003)	Sexton et al. (2012)
		Herold et al. (2005)	Silva et al. (2008)
			Torrrens (2008)
	New development adjacent to urbanised areas	Sun et al. (2013)	Pham et al. (2011)
		Shi et al. (2011)	Wilson et al. (2003)
	Size of urban area	Schneider and Woodcock (2008)	
		Tsai (2005)	
	Low density (built up; housing units; lot size)	Crawford (2007)	Lowry and Lowry (2014)
		Ewing et al. (2002)	Sarzynsky et al. (2014)
		Ewing (1997)	Schneider and Woodcock (2008)
		Knaap et al. (2007)	Song and Knaap (2004)
	Low density (population/households; jobs)	Ewing et al. (2002)	Sarzynsky et al. (2014)
		Frenkel and Askenazi (2008)	Schneider and Woodcock (2008)
		Huang et al. (2007)	Torrrens (2008)
		Lowry and Lowry (2014)	Tsai (2005)
	Single use development (land use segregation/mix)	Crawford (2007)	Knaap et al. (2007)
		Ewing (1997)	Lowry and Lowry (2014)
		Ewing et al. (2002)	Sarzynsky et al. (2014)
		Frenkel and Askenazi (2008)	Song and Knaap (2004)
		Galster et al. (2001)	Torrrens (2008)
		Hahs and McDonnell (2006)	
	Fragmentation (leapfrog/discontinuous development; low compaction)	Aguilera et al. (2011)	Pham et al. (2011)
		Crawford (2007)	Sarzynsky et al. (2014)
		Frenkel and Askenazi (2008)	Schneider and Woodcock (2008)
		Galster et al. (2001)	Seto and Fragkias (2005)
		Hahs and McDonnell (2006)	Shi et al. (2011)
		Herold et al. (2002)	Sun et al. (2013)
		Herold et al. (2003)	Torrrens (2008)
		Herold et al. (2005)	
		Huang et al. (2007)	Wilson et al. (2003)
	Shape irregularity/complexity	Aguilera et al. (2011)	Herold et al. (2005)
		Ewing (1997)	Huang et al. (2007)
		Frenkel and Askenazi (2008)	Pham et al. (2011)
		Hahs and McDonnell (2006)	Seto and Fragkias (2005)
		Herold et al. (2002)	Torrrens (2008)
		Herold et al. (2003)	
	Poor accessibility (also measured as low proximity or high average distance between activities)	Frenkel and Askenazi (2008)	Sarzynsky et al. (2014) ⁶
		Galster et al. (2001) ²	Song and Knaap (2004)
		Knaap et al. (2007)	Torrrens (2008)
		Lowry and Lowry (2014) ⁵	
	Inequality/ low concentration	Galster et al. (2001)	Tsai (2005)
		Sarzynsky et al. (2014)	
	Low centrality (development outside main centre; population decentralisation)	Ewing et al. (2002)	Hahs and McDonnell (2006)
		Frenkel and Askenazi (2008)	Huang et al. (2007)
		Galster et al. (2001)	Sarzynsky et al. (2014)
	Absence of centralities (low clustering; decentralisation)	Ewing et al. (2002)	Torrrens (2008)
		Galster et al. (2001)	Tsai (2005)
		Martellozo and Clarke (2011)	
	Low connectivity (street connectivity; block size)	Ewing et al. (2002)	Lowry and Lowry (2014) ⁷
		Knaap et al. (2007)	Song and Knaap (2004)
	Linear development (or along main roads)	Aguilera et al. (2011)	Wilson et al. (2003)
		Crawford (2007)	
	Outlying/secondary centre formation	Portnov and Schwartz (2009)	Yang et al. (2012)
		Wilson et al. (2003)	
	Nuclearity (low levels also used to characterize sprawl)	Sarzynsky et al. (2014)	
		Galster et al. (2001)	
	Infill (built up area; increase in residential units or road network density)	Couch et al. (2005)	Sun et al. (2013)
		Hahs and McDonnell (2006)	Shi et al. (2011)
		Pham et al. (2011)	Wilson et al. (2003)
	Infill (population, jobs, activities)	Herold et al. (2003)	
		Hahs and McDonnell (2006)	
	Increase in non-residential land uses	Aguilera et al. (2011)	

⁵Defined as 'centrality' by the authors.

⁶Defined as 'proximity' by the authors.

⁷Defined as 'accessibility' by the authors.

Table 2- Spatial patterns of urban shrinkage

Spatial patterns of urban shrinkage	Sources	
Residential vacancy	Ahrens (2005) Haase et al. (2007) Nevin (2004) Kabish et al. (2006) Bontjie (2004) Haase et al. (2013) Pinho et al. (2010)	Rieniets (2009) Schetke and Haase (2008) Schwarz et al. (2010) Siedentop and Fina (2008) Sousa (2010) Wiechmann and Pallagst (2012)
Urban decay (proportion of decaying buildings)	Nevin (2004) Rieniets (2009) Haase et al. (2012)	Schetke and Haase (2008) Schwarz et al. (2010)
Vacant industrial land (increasing urban brownfield area)	Schwarz et al. (2010) Haase et al. (2012)	Siedentop and Fina (2008) Sousa (2010)
Urban perforation (spatial heterogeneity, small scale fragmentation)	Cunningham-Sabot and Fol (2007) Haase et al. (2007) Hollander et al. (2009) Haase et al. (2012)	Schetke and Haase (2008) Schwarz et al. (2010) Siedentop and Fina (2008) Sousa (2010)
Large-scale demolition	Haase et al. (2007) Schetke and Haase (2008) Kabish et al. (2006)	Schwartz et al. (2010) Wiechmann and Pallagst (2012)
Commercial vacancy	Schetke and Haase (2008) Haase et al. (2012)	
Increasing open spaces	Schetke and Haase (2008)	

Table 3 - Landscape metrics organized by categories (the values in brackets correspond to the number of empirical papers using that metric). Adapted from Reis et al. (2014).

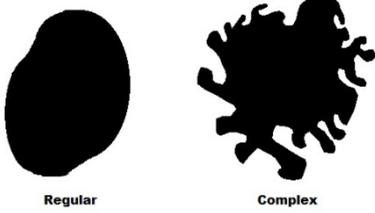
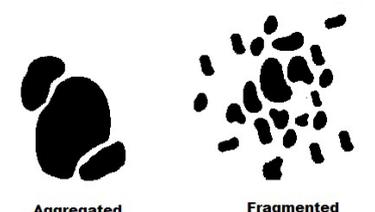
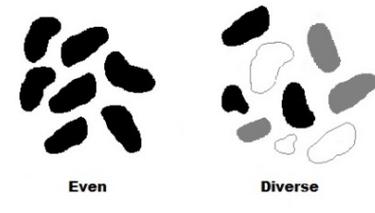
Category	Meaning	Metrics
<p>Shape irregularity</p>  <p>Regular Complex</p>	<p>Measures whether an urban settlement has a regular shape or a complex shape with a ragged edge</p>	<p>AWMP Fractal dimension (10) Edge density (8) AWM Shape index (4) Landscape shape index (6) Fractal dimension (4) Comp. index of the largest patch (3) Shape index (1) Mean shape index (1) Square pixel (1) Mean perimeter-area ratio (1) Mean radius of gyration (1) Edge to interior ratio (1)</p>
<p>Fragmentation</p>  <p>Aggregated Fragmented</p>	<p>Measures the extent to which urban settlements (or patches) are close together (aggregated) or dispersed (fragmented).</p> <p>These metrics are used at the landscape level.</p>	<p>Mean patch size (14) Number of patches (12) Patch density (9) Contagion index (7) Mean nearest neighbour distance (6) Landscape expansion index (2) Intersp. andjustap. index (2) Mean landscape expansion index (1) AWM Landscape expansion index (1) Mean nearest neighbour distance standard deviation (1) Change in density of urban land (1) Percent. Like of adjacency (1) Length of common edge (1)</p>
<p>Diversity</p>  <p>Even Diverse</p>	<p>Measures the relative distribution of different urban characteristics (e.g. land uses). More focused on the composition of the urban landscape.</p>	<p>Shannon's diversity index (5) Shannon's evenness index (2) Patch size standard deviation (3) Patch size coefficient variation (2) Patch richness (2) Contrasting edge ratio (1) Contrasting edge proportion (1) Mean dispersion (1) Diversity index (1) Simpson's diversity index (2) Simpson's evenness index (1)</p>
	<p>Measures both complexity and fragmentation</p>	<p>Compactness index (3)</p>
<p>Other metrics</p>	<p>Area metrics</p>	<p>Change in size of urban area (1) Urban area (12)</p>
	<p>Other</p>	<p>Largest patch index (7) Patch cohesion index (1)</p>

Table 4 - Geo-spatial metrics, organized by categories (the values in brackets correspond to the number of empirical papers using that metric). Adapted from Reis et al. (2014).

Category	Meaning	Metrics		
Fragmentation	Considers the relation between built up settlements or blocks and open areas. Measures the extent to which urban settlements are more continuous and concentrated or more scattered (fragmented).	Fractal dimension (6)	Continuity (1)	
		Index of clustering (3)	Clustering (1)	
		Ratio of open space (2)	Dispersion index (1)	
		Leapfrog index (2)	H indicator (1)	
		Degree of sealing (1)	Hrel indicator (1)	
		Gross leapfrog index (1)	Area index (1)	
		Net leapfrog index (1)	Cluster index (1)	
		Land consumption index (I) (1)	Shape index (R) (1)	
		Fraction of imperv. surface (1)	Compactness (1)	
		Peripheral density (1)	Coefficient of variation (1)	
Density	Measures the density of built up development, infrastructure, people or activities in an area, or the intensity of particular land uses.	Population density (7)	Clark's dens. gradient (1)	
		Residential density (5)	Road network density (1)	
		Lot size (4)	Urban density index (1)	
		Floor space (2)	Ratio density of people (1)	
		Job density (2)	Av. household size (1)	
		Single family dwellings dens. (1)	Res. dev. existing UA (1)	
Land use diversity	Measures the relative distribution of different land uses.	Segregated land use (2)	Mix actual (1)	
		Land use diversity (1)	Mix zoned (1)	
		Land consumption index (II) (1)	Mixed uses (1)	
		Land use diversity index (1)	Mix (1)	
		Total greenery (1)	Urb. LU change (1)	
		Neighb. rec. area (1)	Area neighb. green (1)	
Centrality	Measures the relative position of settlements in relation to the whole urban area.	Centrality index (2)	Distance to CBD (I) (1)	
		Index of remoteness (1)	Distance to CBD (II) (1)	
		Spatial isolation index (1)	Centrality (1)	
		Centralization index (1)	Core-dominated nucl. (1)	
		Nuclearity (1)	H indicator (1)	
			Hrel indicator (1)	
Accessibility	Measure the spatial distribution of activities focusing on the proximity between land uses in an urban area	Commercial distance (3)	Med. dist. to schools (1)	
		Commercial ped. access. (2)	Transit ped. access (2)	
		Bus distance (2)	Weighted av. proximity (1)	
		Park distance (2)	Dist. to roads (1)	
		Proximity (same LU) (1)	Dist. to pr. school (1)	
		Proximity (dif. LU) (1)	Dist. to shopping (1)	
		Community node inacces. (1)	Degree of isolation (1)	
Connectivity	Measures the connectivity between different places in an urban community	Internal (street) connectivity (3)	Blocks (1)	
		External connectivity (2)	Length cul-de-sacs (2)	
		Blocks perimeter (2)	Dendritic street pattern (1)	
Inequality	Measures inequality in the distribution of attributes.	Concentration (1)	Relative entropy (1)	
		Delta index (1)	Batty's entropy (1)	
Spatial network analysis	Syntax of space	Measures developed in Space Syntax or in related methods.	Integration (10)	Number of axial lines (4)
			Connectivity (9)	Control (3)
			Mean depth (6)	Grid axiality (2)
			Synergy (5)	Axial ringiness (2)
			Intelligibility (5)	Real rel. asymmetry (1)
			Mean axial lines length (4)	Choice (1)
	Different dual graph approach	Also uses dual graph, but with a different method for the construction of the axial map.	Number of nodes (1)	Clustering coefficient (1)
			Average degree (1)	Efficiency (1)
			Characteristic path length (1)	
Multiple centrality assessment	Uses a primal graph, more common in other spatial network analysis approaches.	Closeness centrality (4)	Straightness centrality (4)	
		Betweenness centrality (4)	Information centrality (3)	
Other metrics	Metrics that quantify particular features of urban areas, not included in the other categories.	Highway strip index (2)	Res. vacancy (2)	
		Median contour poliocentricity (1)	Orientation index (1)	
		Mean contour poliocentricity (1)	B-ratio (1)	
		Peak ratio (1)	A-ratio (1)	
		Share of renovated houses (1)	Share of demolition (1)	

Table 5 - Spatial statistics metrics organized by categories (the values in brackets correspond to the number of empirical papers using that metric).Adapted from Reis et al. (2014).

Category	Meaning	Metrics
Regression metrics	Based on regression methods.	Density gradient by OLS regression (1)
Spatial autocorrelation	Measure whether certain attributes are evenly (or randomly) distributed across the urban area or clustered.	Moran's I (5) Local Moran (Ii) (2) Geary coefficient (1) Getis-OrdGi (1) Getis-OrdGi* (1)
Evenness of distribution	Measure the inequality of an attribute distribution.	Gini coefficient (1) Locational Gini coefficient (1) Location quotient (1)
Spatial fragmentation/clustering	Fragmentation of an attribute across different locations	Number of fragments (1) Spatial index (1)

Table 6 - Summary of metrics

	Total number of metrics: 41
Landscape Metrics	In studies of urban growth: 41
	In studies of urban shrinkage: 2
	In other studies: 0
	Total number of metrics: 108
Geo-spatial Metrics	In studies of urban growth: 76
	In studies of urban shrinkage: 13
	In other studies: 31
	Total number of metrics: 11
Spatial Statistics	In studies of urban growth: 6
	In studies of urban shrinkage: 0
	In other studies: 7
	Total number of metrics: 160

APPENDIX – Full lists of spatial metrics

Table A1 – Landscape Metrics

1. Landscape Metrics					
Group / Metric		Measurement	Empirical applications	Interpretation	Study focus⁸
Area	Change in size of urban area	Difference in urban area between time periods	Schneider and Woodcock (2008)	Increase of urban area indicates growth.	PUG
	Change in density of urban land	Difference in ratio of urban expansion to all land	Schneider and Woodcock (2008)	Larger increase in built-up land density indicates infilling, smaller increase indicates more dispersion.	PUG
	Urban area (also percentage of landscape; class area)	Share in area of urban land or of a particular land use type	Herold et al. (2002; 2003; 2005); Aguilera et al. (2011); Schneider and Woodcock (2008); Guerois and Pulmain (2008); Kassanko et al. (2006); Schneider et al. (2005); Wu et al. (2011); Weng (2007); Hahs et al. (2006); Pham et al. (2011)	The proportion of the total area that has a particular characteristic (e.g. land use type)	PUG
Number of patches		$NP=n_i$ $n_i=\text{number of patches of the same type}$	Schwartz (2010) Aguilera et al. (2011) Seto and Fragkias (2005) Herold et al. (2003) Herold et al. (2005) Huang S et al (2009) Yu and Ng (2007) Deng et al. (2009) Hahs et al. (2006) Torrens (2008) Pham et al. (2011)	Increases with the creation of different nuclei (fragmentation) and decreases if urban areas merge into continuous urban fabric (aggregation/ compactness)	PUG
Mean patch size		$MPS = \frac{\sum_{j=1}^n a_{ij}}{n_i}$ $[0; \infty]$ <p>a_{ij}=patch area of correspondent patch type; n_i=number of patches of the same type</p>	Schwartz (2010) Aguilera et al. (2011) Herold et al. (2005) Seto and Fragkias (2005) Schneider et al. (2005) Frenkel and Ashkenazi (2008) Silva et al. (2008) Wu et al. (2011) Huang S et al (2009) Weng (2007) Yu and Ng (2007) Deng et al. (2009) Irwin and Bockstael (2007) Sun et al. (2013)	Inverse trend to the NP. High MPS: aggregation; patches are clustering to form large patches. Low MPS: fragmented; if decreases with time, it means that urban growth results in the creation of new nuclei rather than by envelopment or annexation to old settlements	PUG
Patch size standard deviation		$PSSD = \sqrt{\frac{\sum_{j=1}^n \left[a_{ij} - \frac{\sum_{j=1}^n a_{ij}}{n_i} \right]^2}{n_i}}$ $[0; \infty]$	Schwartz (2010) Herold et al. (2002) Herold et al. (2005)	High PSSD: larger differences in patch size between patches of the same type PSSD=0: all patches have the same size	PUG
Patch size coefficient variation		$PSCV = \frac{PSSD}{MPS}$ $[0; \infty]$	Schwartz (2010) Seto and Fragkias (2005)	Similar to PSSD, but normalised considering MPS	PUG
Patch density		$PD = \frac{n_i}{A}$ $n_i=\text{number of patches of the same type};$ $A=\text{total landscape area}$	Herold et al. (2002; 2005); Schneider and Woodcock (2008) Wu et al. (2011) Weng (2007) Ji et al. (2006) Deng et al. (2009) Sun et al. (2013) Irwin and Bockstael	Similar to NP, but considering the total area High PD: fragmentation, scatter Low PD: infilling, aggregation	PUG

⁸PUG: metrics used to study patterns of urban growth; PUS: metrics used to study patterns of urban shrinkage; UP: metrics used to study urban spatial patterns but not necessarily related to growth or shrinkage.

		(2007)		
Largest patch index	$LPI = \frac{\max(a_{ij})}{A} 100$ <p>Max(aij)=area of largest patch; A=total urban area [0;100%]</p>	Herold et al. (2003) Huang S et al (2009) Yu and Ng (2007) Deng et al. (2009) Hahs et al. (2006) Schetke and Haase (2008) Pham et al. (2011)	LPI approaches 0 when largest patch becomes smaller; LPI=100 when total landscape consists of a single patch of corresponding type	PUG PUS
Fractal dimension	Fractal Dimension $FD = \frac{2 \ln p_i}{\ln s_i}$ <p>s_i=area patch i; p_i= perimeter patch i; N=total nr of patches [1;2]</p>	Schwartz (2010) Frenkel and Ashkenazi (2008) Herold et al. (2002) Hahs et al. (2006)	FD describes the complexity of a patch by perimeter-area proportion.	PUG
	Area weighted mean patch fractal dimension $AWMPFD = \frac{\sum_{i=1}^{i=N} 2 \ln 0.25 p_i / \ln s_i}{N} \times \frac{s_i}{\sum_{i=1}^{i=N} s_i}$ <p>s_i=area patch i; p_i= perimeter patch i; N=total nr of patches [1;2]</p>	Schwartz (2010) Huang et al. (2007) Seto and Fragkias (2005) Herold et al. (2002) Herold et al. (2003) Herold et al. (2005) Wu et al. (2011) Huang S et al (2009) Yu and Ng (2007) Pham et al. (2011)	AWMPFD describes the shape complexity or the raggedness of the urban boundary, weighting larger patches higher. Ranges between 1 and 2: 1:simple shapes 2: complex shapes	PUG
Shape Index	Shape Index <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> Vector $SHAPE = \frac{p_{ij}}{2\sqrt{\pi} \cdot a_{ij}}$ </div> <div style="border: 1px solid black; padding: 5px;"> Raster $SHAPE = \frac{0.25 p_{ij}}{\sqrt{a_{ij}}}$ </div> </div> <p>p=perimeter; a=area [1;∞]</p>	Frenkel and Ashkenazi (2008)	Defines shape irregularity. SHAPE=1 when patch is circular (vector) or square (raster) and increases with irregularity	PUG
	Area weighted mean shape index $AWMSI = \frac{\sum_{i=1}^{i=N} p_i / 4\sqrt{s_i}}{N} \times \frac{s_i}{\sum_{i=1}^{i=N} s_i}$ <p>s_i=area patch i; p_i= perimeter patch i; N=total nr of patches (raster) [1;∞]</p>	Schwartz (2010) Huang et al. (2007) Huang S et al (2009) Yu and Ng (2007)	AWMSI is the average shape index of patches of the correspondent type, wighted by patch area, so that larger patches weight more than small ones.	PUG
	Landscape shape index $LSI = \frac{0.25E}{\sqrt{A}}$ <p>E=sum of landsc boundary and edge segments; A=total landsc area (raster) [1;∞]</p>	Schneider et al. (2005) Silva et al. (2008) Wu et al. (2011) Deng et al. (2009) Hahs et al. (2006) Sun et al. (2013)	LSI describes the irregularity of the complete landscape. SHAPE_MN measures the ratio between perimeter of a patch and the perimeter of the simplest patch in the same area	PUG
	Mean Shape index $SHAPE_MN = \frac{\sum_{j=1}^n p_{ij} / \min p_{ij}}{n_i}$ <p>p=patch perimter; minp=minimum perimeter of patch</p>	Aguilera et al. (2011)		PUG
Square pixel	$SqP = 1 - \frac{1}{LSI}$	Wu et al. (2011)	Normalised perimeter-area ratio that measures shape complexity of whole landscape or specific patch type.	PUG
Edge density	<p>Sum of edge density of all patches (including landscape boarder) divided by the total landscape area [0;∞]</p> $ED = \frac{\sum p_i}{A}$	Schwartz (2010) Herold et al. (2002) Herold et al. (2003) Seto and Fragkias (2005) Wu et al. (2011) Huang S et al (2009) Deng et al. (2009) Pham et al. (2011)	Measures the complexity or raggedness of the landscape High ED: ragged ED decreases when urban areas fuse together and boundaries dissolve and increases with new nuclei	PUG
Contagion Index	$CONTAG = 1 - \frac{\left[\sum_{i=1}^m \sum_{k=1}^m (P_i) \left(\frac{g_{ik}}{\sum_{k=1}^m g_{ik}} \right) \right] \left[\ln(P_i) \left(\frac{g_{ik}}{\sum_{k=1}^m g_{ik}} \right) \right]}{2 \ln(m)} \right] (100)$ <p>P_i=proportion of landsc occupied with patch type i; g_{ik}=nr of adgencies between pixels of classes i and k; i,k are different patch types; m is the nr of patch types [0;100%]</p>	Herold et al. (2002) Herold et al. (2003) Herold et al. (2005) Wu et al. (2011) Huang S et al (2009) Yu and Ng (2007) Torrens (2008)	Describes fragmentation of a landscape by the probability of a patch type being adjacent to another patch type. Low CONTAG: landscape consisting of large and less fragmented patches High CONTAG: Landscape with a great number of small or highly fragmented patches	PUG

Compactness index	$CI = \frac{\sum_i P_i / p_i}{N^2} = \frac{\sum_i 2\pi\sqrt{s_i/\pi} / p_i}{N^2}$ <p>s_i and p_i are area and perimeter of patch i; P_i=perimeter of a circle with area of s_i; N=nr of patches</p>	Schwartz (2010) Huang et al. (2007) Li and Yeh (2004)	Measures individual patch shape and fragmentation of total area. High CI: regular patch shape and compact urban area (lower patch number)	PUG	
Compactness index of the largest patch	$CILP = \frac{2\pi\sqrt{s/p}}{p}$ <p>s and p are area and perimeter of largest patch</p>	Schwartz (2010) Huang et al. (2007) Li and Yeh (2004)	Similar to CI, but applied only to the largest patch. Represents the overall shape of the urban centre.	PUG	
Shannon's evenness index	$SHEI = \frac{m - \sum_{i=1}^m (P_i \circ \ln P_i)}{\ln m}$ <p>m= different patch types; P_i= proportion of landscape area occupied by patches of type i.</p>	Weng (2007) Deng et al. (2009)	Patch diversity determined by the distribution of the proportion of different LU types in a landscape. Low: uneven distribution of area among patch types (0 when there's only 1 patch) SHDI=1: distribution of area among patch types is perfectly even.	PUG	
Shannon's diversity index	$SHDI = - \sum_{i=1}^m (P_i \circ \ln P_i)$ <p>[0;∞] m= different patch types; P_i= proportion of landscape area occupied by patches of type i.</p>	Wu et al. (2011) Yu and Ng (2007) Deng et al. (2009) Torrens (2008) Schetke and Haase (2008)	Measures diversity of patch types in a landscape determined by the nr of different patch types and the proportional distribution of area among patch types SHDI increases if the number of different patches increases or the area distribution among patches is more even. SHDI=0: only one patch (no diversity). Higher SHDI – more diversity.	PUG PUS	
Simpson's evenness index	$SIEI = \frac{1 - \sum_{i=1}^m P_i^2}{1 - (\frac{1}{m})}$ <p>[0;1] m= different patch types; P_i= proportion of landscape area occupied by patches of type i.</p>	Torrens (2008)	SIEI nears zero when distribution of area among different activities is uneven (SIEI=0: landscape has only a single patch of activity). SIEI=1: the distribution of area is even.	PUG	
Simpson's diversity index	$SIDI = 1 - \sum_{i=1}^m P_i^2$ <p>m= different patch types; P_i= proportion of landscape area occupied by patches of type i.</p>	Hahs and McDonnell (2006) Lowry and Lowry (2014)	Represents the probability that any two patches selected at random will be different types. Higher SIDI: greater diversity	PUG	
Patch richness	$PR=m$ <p>m=nr of different patch types in the study area</p>	Hahs et al. (2006) Lowry and Lowry (2014)	Is a measure of diversity, in terms of the number of different patch types.	PUG	
Patch cohesion index	$COHESION = \left[1 - \frac{\sum_{j=1}^m P_{ij}}{\sum_{j=1}^m P_{ij} \sqrt{a_{ij}}} \right] \left[1 - \frac{1}{\sqrt{A}} \right]^{-1} (100)$ <p>p_{ij} =perimeter of patch ij in terms of number of cell surfaces; a_{ij}=area of patch ij in terms of nr of cells; A=total nr of cells in the landsc. [0;100]</p>	Yu and Ng (2007)	Measures the physical connectedness of the corresponding patch type. Increases as the patch type becomes more clumped or aggregated in its distribution (more physically connected).	PUG	
Nearest neighbour distance	Mean nearest neighbour distance	$MNN = \frac{\sum_{i=1}^m \sum_{j=1}^{n'} h_{ij}}{N'}$ <p>h=distance to nearest patch (edge-to-edge); N'=total nr of patches that have neighbour; m=nr of patch types [0;∞]</p>	Herold et al. (2005) Herold et al. (2003) Silva et al. (2008) Aguilera et al. (2011) Pham et al. (2011) Sun et al. (2013)	Measures the average distance between patches of the same patch type. Higher MNN = higher dispersion	PUG
	Mean nearest neighbour distance std deviation	$NNSD = \sqrt{\frac{\sum_{i=1}^m \sum_{j=1}^{n'} h_{ij}^2 - \left(\frac{\sum_{i=1}^m \sum_{j=1}^{n'} h_{ij}}{N'} \right)^2}{N'}}$ <p>[0;∞]</p>	Herold et al. (2005)	Measures variability in MNN. NNSD=0: all patches have the same MNN. NNSD increases with irregularity of MNN	PUG

Mean radius of gyration		$MRG = \sum_{r=1}^z \frac{h_{ijr}}{z}$ <p>h_{ijr}=distance between cell ijr located in patch ij and the centroid of the patch; z=nr of cells in the patch ij</p>	Aguilera et al. (2011)	Measures the mean shape of patches in terms of roundness Low: roundness (compaction) High: elongation	PUG
Landscape expansion index	Landscape expansion index	$LEI = 100 \times \frac{A_o}{A_o + A_v}$ <p>A_o=intersection of the buffer zone of the new patch with occupied land; A_v= intersection of the buffer zone of the new patch with vacant land [0;100]</p>	Liu et al. (2010) Shi et al. (2012)	Measures the proportion of a buffer around the edge of a new patch that intersects old built-up area. LEI defines three types of urban growth: infilling [50,100]; edge-expansion [0-50]; outlying [0]	PUG
	Mean landscape expansion index	$MEI = \sum_{i=1}^N \frac{LEI_i}{N}$ <p>N=total number of newly grown patches</p>	Liu et al. (2010)	MEI is the simple average of all newly grown patches. AWMEI is similar to MEI but weighting the area of each patch.	PUG
	Area Weighted mean landscape expansion index	$AWMEI = \sum_{i=1}^N LEI_i \times \left(\frac{a_i}{A}\right)$ <p>a_i=area of new patch i; A_i=total area of all new patches</p>	Liu et al. (2010)	Higher MEI or AWMEI: lower expansion, the landscape tends to be more compact.	PUG
Mean perimeter to area ratio		$\frac{\sum_i \frac{l_{ik}}{a_{ik}}}{n_k}$ <p>l_{ik}=perimeter of patch i of LU k; a_{ik}=area of patch i of LU k; n_k=total nr of patches in land use k</p>	Irwin and Bockstael (2007)	Captures the mean size and shape of patches, holding constant the total nr of patches. Increase of PAR of a focal land use: increasing complexity or the addition of smaller patches.	PUG
Contrasting edge ratio		$\frac{e_{kj}}{e_{kk}}$ <p>e_{kj}=total edge length shared between cells with focal LU k and contrasting LU j; e_{kk}= total edge length shared between cells with same LU k</p>	Irwin and Bockstael (2007)	Measure the interspersion of a focal and contrasting LU by measuring the total length of their shared border.	PUG
Contrasting edge proportion		$\frac{e_{kj}}{e_{kj} + e_{kk}}$ <p>Same as above; varies between 0 and 1 [0;1]</p>	Irwin and Bockstael (2007)	Measure the interspersion of a focal and contrasting LU by measuring the total length of their shared border. Normalized by total length of like and contrasting edges.	PUG
Mean dispersion		$\frac{\sum_i p_{jik}}{n_k}$ <p>p_{jik}=proportion of cells of contrasting LU j that are within a specified distance of cell i with focal LU k [0;1]</p>	Irwin and Bockstael (2007)	Measures the mean proportion of a contrasting LU within a given neighbourhood of a focal LU cell.	PUG
Diversity index		$H = - \sum_k (P_k) \ln P_k$ <p>P_k=proportion of landscape in cover type k</p>	Geoghegan et al. (1997)	Measures the extent to which landscape is dominated by a few or many land uses	PUG
Edge to interior ratio (fragmentation index)		$R = \sum \frac{P_i}{A_i}$ <p>P_i and A_i are perimeter and area of patches of land cover type i</p>	Geoghegan et al. (1997)	Reflects patch size and shape	PUG
Interspersion and juxtaposition index		$IJI = \frac{- \sum_{i=1}^{m'} \sum_{k=i+1}^{m'} \left[\left(\frac{e_{ik}}{E}\right) \ln \left(\frac{e_{ik}}{E}\right) \right]}{\ln \left[\frac{1}{2} m'(m' - 1) \right]} \cdot 100$ <p>m' :nr LU types, including a landscape border. e_{ik}: length of edge between i and k; E:total edge length in the study area.</p>	Torrens (2008) Lowry and Lowry (2014)	Similar to contagion, but for patches instead of pixels.	PUG
Percentage like of adjacency		$PLADJ = \frac{\sum_{i=1}^m g_{ii}}{\sum_{i=1}^m \sum_{k=1}^m g_{ik}}$ <p>G_{ii}: nr.of like adjacencies between pixels of patch type i; g_{ik}: nr.of adjacencies between pixels of patchtypes i and k; m: nr. of pixels</p>	Pham et al. (2011)	Measures the degree of aggregation of patch types. PLADJ=0: maximum disaggregated pattern in the class or no like adjacencies; PLADJ=100: computed areas cover a single class or all adjacencies are in the same class (maximally contagious). Low PLADJ: high fragmentation or	PUG

			many individual urban units.	
Length of common edge	$R = \frac{l_c}{l}$ <p>l_c: length of common edge between a newly developed urban patch and an existing urban patch; l: perimeter of the newly developed urban patch. [0,1]</p>	Sun et al. (2013)	<p>Distinguishes different types of urban growth.</p> <p>Infilling: R > 0.5, development of new urban patch surrounded by at least 50% old urban area.</p> <p>Edge-expansion: 0 < R < 0.5, new urban area spreading out from the edge of an urban area and surrounded by less than 50%.</p> <p>Outlying growth: R = 0, new urban area without spatial connection to existing urban area</p>	PUG

Table A2 – Geo-spatial metrics

2. Geo-spatial metrics				
Group / Metric	Measurement	Empirical applications	Interpretation	Study focus
Fractal dimension	$D = \frac{1}{\ln R'} \ln \left[\frac{N(R')}{4} \right] \quad (9)$ <p>N'= number of occupied points on a rectangular grid, at radial distance R' from CBD</p>	Longley and Mesev (2000) Shen (2002) Frankhauser (2004) Frankhauser (1998) De Keersmaecker et al. (2003) Terzi and Kaya (2011)	Measures the extent to which built areas fill the space. Values between 1 (built areas fill the space of a line) and 2 (built areas completely fill the two-dimensional space).	PUG UP
Centrality index	$\text{Centrality} = \frac{\sum_{i=1}^{N-1} D_{i/N-1}}{R} = \frac{\sum_{i=1}^{n-1} D_{i/N-1}}{\sqrt{S/\pi}}$ <p>D_i=distance of centroid of patch i to centroid of the largest patch; R=radius of a circle with area S S=sum of area of all patches; N=nr of patches</p>	Schwarz (2010) Huang et al. (2007)	Measures the degree to which the urban development is close to CBD (defined as the centroid of the largest patch). It minimizes the effect of scale by dividing by R. Higher Cent. Index: more elongated the overall city is.	PUG
Ratio of open space	$ROS = \frac{S'}{S} 100$ <p>S'=sum of all holes within urban area S=sum of area of all patches</p>	Schwarz (2010) Huang et al. (2007)	Measure of porosity: measures the total ratio of open space (unclassified areas) compared to the urban area	PUG
Gross leapfrog index	$I_{gj} = \frac{A_i^{out}}{A_i^u}$ <p>A_i^{out}=leapfrog areas in settlement i; A_i^u=urban built-up area of settlement i</p>	Frenkel and Ashkenazi (2008)	Measures fragmentation or scatter. Similar o ROS	PUG
Net leapfrog index	$I_{ni} = \frac{R_i^{out}}{R_i}$ <p>R_i^{out}=resid. areas outside central built-up areas of settlement i; R_i=resid. area of settlement i</p>	Frenkel and Ashkenazi (2008)	Fragmentation of residential use. Similar to I _{gj} , but considering residential land use.	PUG
Land consumption index (I)	$LC_{town} = \frac{\sum A_{unit}}{N_{town}}$ <p>A_{unit}=parcel area of new residential unit; N_{town}=nr. of new residential units per region</p>	Crawford (2007)	Indicator of the average amount of land area occupied by each new residential unit. Higher values: sprawl	PUG
Index of remoteness	IR is defined as the aerial distance from a settlement to the closest major urban centre	Portnov and Erell(1998)	Measures whether a settlement is located close or isolated from a main regional centre.	PUG
Spatial isolation index	IS of an urban settlement is the number of settlements located within a commuting distance from it.	Portnov and Erell(1998)	Measures the potential for intra-regional economic and social interaction.	PUG
Index of clustering	$IC = \frac{SI}{IR}$ <p>IR: index of remoteness; SI: spatial isolation</p>	Portnov and Erell(1998)	Measures the combined effect of remoteness and spatial isolation. Tends to be higher value in central, densely populated areas; and lower in remote peripheral areas in which urban settlements are more scattered.	PUG
	$IC_1 = \frac{N}{IR}$ <p>N: number of other towns located in a commuting distance</p>	Portnov et al. (2000) Portnov and Schwartz (2009)		PUG
	$IC_2 = \frac{\sum P_i}{IR}$ <p>∑P_i: the total population of all towns located within a commuting distance</p>	Portnov et al. (2000) Portnov and Schwartz (2009)		PUG

⁹This expression, suggested by Longley and Mesev (2000), is only an example of the many methods used to measure the fractal dimension of an urban area. Different methods are described, for instance, in Batty and Longley (1994), Frankhauser (1998) and De Keersmaecker et al. (2003).

Leapfrog index	$LF_{\text{town}} = \frac{\sum Df_{\text{unit}}}{N_{\text{town}}}$ <p>Df_{unit}=distance from centre of each new resid. parcel to the centre of the existing resid. parcel</p>	Crawford (2007) Haase and Lathorp (2003)	Measures the extent to which growth occurs at a significant distance from existing residential units. Higher leapfrog: sprawl	PUG
Segregated land use index	$SL = \frac{\sum(X - NLU)}{N_{\text{town}}}$ <p>X=maximum nr of different land uses possible NLU=nr of different developed urban uses within 1500ft of each new resid. parcel centroid</p>	Crawford (2007) Haase and Lathorp (2003)	Measures the mix of land uses within reasonable walking distance of residential settings. Higher values: sprawl	PUG
Land use diversity index	$DIV_{\text{town}} = 1 - \frac{-\sum(p_{i,\text{unit}} \log(p_{i,\text{unit}}))/\log(X)}{N_{\text{town}}}$ <p>[0,1] p_i=percentage of land area in a particular land use class i; X= maximum nr of different land uses possible</p>	Crawford (2007)	Similar in principle to segregated land use: measures the degree of l.u. heterogeneity. 0: completely uneven l.u. distribution (ex. all residential) 1: perfectly even distribution	PUG
Highway strip index	$HS_{\text{town}} = \frac{\sum HB_{\text{unit}}}{N_{\text{town}}}$ <p>HB_{unit}=0: newly developed units outside a 300ft buffer around major roads; HB_{unit}=1: newly developed units within a 300ft buffer around major roads;</p>	Crawford (2007) Haase and Lathorp (2003)	Measures the proportion of development occurring along highway strips. Higher values: sprawl	PUG
Land consumption index (II)	$LCI = \frac{\% \Delta B_{(t1-t0)}}{\% \Delta [LU]_{(t1-t0)}}$ <p>%ΔB: percent change in built-up land; %Δ[LU]:percentage change in land use (housing units or business establishments); t0, t1: beginning time, ending time</p>	Ji et al. (2006)	Relates built-up area change to change in housing and commercial construction as major driving factors in urban land conversion.	PUG
Road network density	Length of existing public roads, ranging from local roads to motorways	Hahs and McDonnell (2006)	Assesses the amount of road infrastructure in an area.	PUG
Fraction imperv. surface	Average amount of impervious surface calculated at the sub-pixel level from the impervious surface fraction image	Hahs and McDonnell (2006)	Measures the proportion of impervious surface.	PUG
Distance to CBD (I)	Linear distance from the central business district	Hahs and McDonnell (2006)	Measures centrality of a settlement based on its linear distance to the main centre.	PUG
Ratio of the density of people	$\frac{PEOP}{\%URB} = \frac{PEOP}{(\%URB + 0.5)}$ <p>PEOP: pop. density;%URB: proportion of urban land cover</p>	Hahs and McDonnell (2006)	Describes the variation of population density in different areas with the same amount of urban land cover	PUG
Internal (or street) connectivity	$\text{Int_connectivity} = \frac{\text{nr. street intersections}}{\text{nr. intersections} + \text{nr. cul de sacs}}$	Song and Knaap (2004) Knaap et al. (2007) Lowry and Lowry (2014)	Compares the nr of intersections with the total number of streets of a neighbourhood. The higher the ratio the greater int. connectivity	PUG
Blocks perimeter	Median perimeter of blocks	Song and Knaap (2004) Lowry and Lowry (2014)	A neighbourhood with smaller block perimeter has a greater internal connectivity	PUG
Blocks	$\text{Blocks} = \frac{\text{nr. of blocks}}{\text{nr. of housing units}}$	Song and Knaap (2004)	Higher value: lower internal connectivity	PUG
Lenght cul-de-sacs	Median length of cul-de-sacs	Song and Knaap (2004) Lowry and Lowry (2014)	Shorter cul-de-sacs in a neighbourhood correspond to higher internal connectivity	PUG
External connectivity	Median distance between Ingress/Egress (access) points	Song and Knaap (2004) Knaap et al. (2007)	The shorter the distance, the greater the external connectivity	PUG
Dendritic street pattern	Ratio of cul-de-sacs to streets	Lowry and Lowry (2014)	Lower ratio in a neighbourhood correspond to higher connectivity	PUG
Lot size	Median lot size of single family dwelling (SFD) in the neighbourhood	Song and Knaap (2004) Knaap et al. (2007) Lowry and Lowry (2014) Ewing et al. (2002)	The smaller the lot size, the higher the density	PUG