Adiabatic Control of Atomic Dressed States for Transport and Sensing

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We describe forms of adiabatic transport that arise for dressed-state atoms in optical lattices. Focussing on the limit of weak tunnel-coupling between nearest-neighbour lattice sites, we explain how adiabatic variation of optical dressing allows control of atomic motion between lattice sites: allowing adiabatic particle transport in a direction that depends on the internal state, and force measurements via spectroscopic preparation and readout. For uniformly filled bands these systems display topologically quantised particle transport. An implementation of the dressing scheme using optical transitions in alkaline-earth atoms is discussed as well as its favorable features for precise force sensing.

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The topology of energy bands [1, 2] is a concept that has had profound influence in recent years, in the areas of both solid state systems and ultra-cold atomic gases. In ultra-cold gases, important experimental progress has been made in realising physics related to the integer quantum Hall effect, by contructing two-dimensional (2D) lattice models [3] whose energy bands have nontrivial topology [4, 5], as characterized by a nonzero Chern number [6]. Indeed, a nonzero Chern number has recently been measured in transport studies of bosons [5].

Closely related to quantised Hall transport of 2D systems is the quantised particle transport of (quasi)-one-dimensional (1D) systems under time-periodic adiabatic drives. In such "Thouless pumps"[7], the number of particles transported along the 1D system is also quantised according to a Chern number, defined over a periodic 2D parameter space spanned by the quasi-momentum across the 1D Brillouin zone and by a time-dependent periodic parameter varied over one cycle.

Ultra-cold gases provide an ideal setting in which to realise such adiabatic pumping. They afford very flexible control of the lattice potential, the possibility to vary parameters in time, and have access to very precise probing tools [8]. Although theoretical proposals have illustrated ways to achieve quantised adiabatic transport using optical superlattices [9–11] these have been limited to far-detuned implementations that couple to atoms in a spin-independent manner.

In this Letter, we describe the new features that arise in optical lattices involving optically dressed states of internal "spin" states of the atoms, within a model proposed in Ref. [12] and recently realised experimentally [13, 14]. Although motivated by Thouless pumping, and inheriting all features of this quantized pump, our results will not be restricted to filled bands. We shall emphasize a local description which shows how adiabatic control of dressed states can lead to novel and useful consequences. Notably, the direction of adiabatic transport depends on the spin-state of the atom. Moreover, the coupling of spin and orbital degrees of freedom facilitates force measurements using only spectroscopic control. The local description also allows one to understand in simple terms the role of inter-atomic interactions. We discuss an implementation using the long-lived clock states in alkaline earth atoms (AEA)[15–17].

We consider a model for a spin-orbit coupled atomic gas of the form proposed in Ref. [12], which uses M long-lived internal states to implement a synthetic dimension. The model is illustrated in Fig. 1(a). The atoms are prepared in the lowest band of a 1D optical lattice. The horizontal links represent tunnel coupling, $-t$, between neighbouring lattice sites at positions $x = \ldots, -1, 0, 1, 2, \ldots$, and are taken to be the same for all internal states as is appropriate for state-independent lattices. The vertical sites correspond to the $s = 1, 2, \ldots M$ internal states which form the synthetic dimension.

We consider the case of cyclic coupling where the state s is coupled to both $s-1$ and $s+1$ with s interpreted modulo M (i.e. $s = M+1$ is equivalent to $s = 1$). We choose the coupling from s to $s+1$ to be $\Omega_x^{s,s+1} = -\Omega e^{i\phi_{s,s+1}(x,\tau)}$ with uniform amplitude Ω. We shall require two features of the phases $\phi_{s,s+1}(x,\tau)$. First, the phases should be spatially dependent, leading to coupling of "spin" and spatial degrees of freedom. We take

$$\phi_{s,s+1}(x,\tau) = \phi_{s,s+1}(0,\tau) + x\Phi\,, \qquad (1)$$

for which the model maps to the Harper model in a square lattice at "flux" of Φ through each plaquette. Since we consider neutral atoms it is convenient to measure flux in dimensionless variables. Throughout we use the convention that the "flux" threading any loop is the phase picked up as a particle is transported around the loop. Hence one flux quantum is 2π.

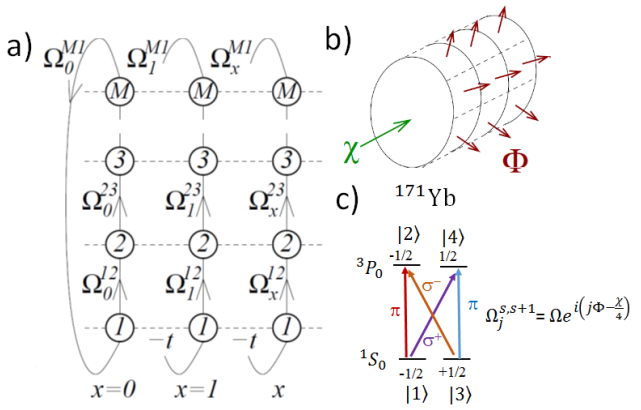

FIG. 1. Schematic illustration of the model. (a) The numbers denote the internal state $s = 1, \ldots M$, spanning the vertical synthetic dimension. Vertical links represent the cyclic Rabi coupling. Horizontal links represent the tunnel coupling of neighbouring lattice sites, $x = \ldots, -1, 0, 1, 2, \ldots$. (b) The coupling phases are such that the system can be viewed as a cylinder with a flux χ threading the periodic loop at $x = 0$, and a flux Φ threading each square plaquette on the surface. (c) Implementation of the four level cyclic scheme using the clock states in ^{171}Yb (nuclear spin $I = 1/2$) as bare states: $|1\rangle = |^1S_0, I_z = -1/2\rangle$, $|2\rangle = |^3P_0, I_z = -1/2\rangle$, $|3\rangle = |^1S_0, I_z = 1/2\rangle$, $|4\rangle = |^3P_0, I_z = 1/2\rangle$.

Second, it should be possible to vary the phase

$$\chi(\tau) \equiv -\sum_{s=1}^{M} \phi_{s,s+1}(0,\tau), \tag{2}$$

in real time τ. This phase has the simple interpretation as the flux through the periodic loop in the synthetic dimension at $x = 0$. [See Fig. 1(b).] For $M \geq 3$ internal states χ is a non-trivial, gauge-invariant phase that influences the spectrum.

As shown in Ref. [12] the vertical links and the "flux" Φ can be created via hyperfine states coupled by Raman transitions in a far-detuned optical lattice. This implementation was recently realized for $M = 3$ in Refs. [13, 14]. Alternative implementations, allowing larger M, can be realized using the long-lived clock states, $^1S_0 - {}^3P_0$, of AEA in a "magic" wavelength optical lattice [19]. A direct one-photon transition is enough to couple the levels while imparting enough momentum to generate a significant Φ. One ideal realization of the $M = 4$ case can be done with ^{171}Yb (with nuclear spin $I = 1/2$) as shown in Fig. 1c. Its simple level structure guarantees that the Rabi frequencies $|\Omega^{12}/\Omega^{34}| = |\Omega^{23}/\Omega^{41}| = 1$. $|\Omega^{23} \neq \Omega^{12}|$ but the difference can be easily compensated by adjusting the intensities of the circularly polarized laser beams. Physically, flux through the periodic loop is set by the relative phases of the M different Rabi couplings, so is readily controllable in experiments. The temperature needed for our proposal is just to avoid population of higher bands. This temperature is determined by the band gap which can be several kHz and currently easily achieved in most cold atom experiments. Note that we shall not require any type of quasi-momentum resolution, so thermal population of the lowest band is not a problem. The only requirement is to be able to reach laser frequency stability of a few Hz in order to vary χ at a rate slower than the tunneling. This type of laser frequency stability can be achieved with current laser technology as demonstrated in recent clock experiments. Those have achieved record levels of stability and residual laser drift less than mHz/s [15–17].

To make the ideas concrete we focus on $M = 4$ internal states and $\Phi = \pi/2$, but the key features appear in more general cases. Without loss of generality, we can choose a gauge in which the phases are uniform, with

$$\phi_{s,s+1}(x,\tau) = \phi(x,\tau) \equiv -\chi(\tau)/4 + x\pi/2. \tag{3}$$

We consider first the limit of vanishing tunnel-coupling $t = 0$, for which the sites x can be treated independently. The Hamiltonian describing the local Rabi couplings in the rotating wave approximation is

$$\hat{H}_\Omega = \sum_x \sum_{s=1}^{4} \left[\Omega_x |s+1\rangle_x\langle s| + \Omega_x^* |s\rangle_x\langle s+1|\right], \tag{4}$$

with $\Omega_x \equiv -\Omega e^{i\phi(x,\tau)}$. The eigenstates are the dressed states

$$|k_s\rangle_x = \frac{1}{2}\sum_{s=1}^{4} e^{ik_s s}|s\rangle_x \tag{5}$$

labelled by the allowed wavevectors along the synthetic direction, $k_s \in \{0, \pi/2, \pi, 3\pi/2\}$. The wave functions take the same form for all x, but their energies vary with position according to

$$\epsilon_{x,k_s} = -2\Omega\cos(k_s - x\pi/2 + \chi/4)\,. \tag{6}$$

Note that the change $\chi \to \chi' = \chi + 2m\pi$ and $k_s \to k_s' = k_s - m\pi/2$, with m an integer, leaves the spectrum unchanged and reflects its gauge invariance.

For isolated lattice sites, $t = 0$, one can readily envisage ways to prepare the atoms in a given dressed state. For example, this can be accomplished by slowly ramping up the Rabi coupling Ω from zero while keeping the lasers slightly detuned from resonance to introduce energy offsets that are proportional to s. This generates the net rotating-frame Hamiltonian $\hat{H}_\delta + \hat{H}_\Omega$, with

$$\hat{H}_\delta = \delta \sum_x \sum_s s|s\rangle_x\langle s|\,. \tag{7}$$

For an atom at site x initially in internal state s, turning on Ω slowly compared to δ/h will adiabatically transfer it

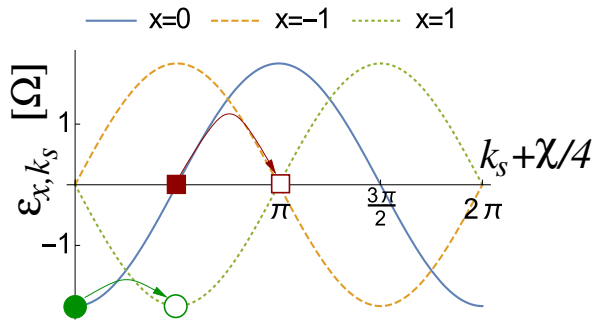

FIG. 2. Dressed-state energies (6) for vanishing tunneling $t = 0$, on sites $x = 0, -1, 1$ as a function of $k_s + \chi(\tau)/4$. At the allowed $k_s \in \{0, \pi/2, \pi, 3\pi/2\}$ degeneracies between states on neighbouring sites, $|\Delta x| = 1$, appear only for $\chi(\tau) = \pi$ (modulo 2π). These degeneracies are split by $t \neq 0$: a particle initially in state $|k_s = 0\rangle_{x=0}$ at $\chi(0) = 0$ (filled circle) is then transferred adiabatically to the state $|k_s = 0\rangle_{x=1}$ as $\chi(\tau)$ increases to 2π (open circle); a particle initially in $|k_s = \pi/2\rangle_{x=0}$ at $\chi(0) = 0$ (filled square) is transferred adiabatically to $|k_s = \pi/2\rangle_{x=-1}$ at $\chi(\tau) = 2\pi$ (open square).

into the s^{th} lowest energy dressed state of $\hat{H}_\Omega$ once $\Omega \gg \delta$. (It may be advantageous to simultaneously vary the detunings $\delta \to 0$ over this ramp.) For $\chi = 0$, and starting from $s = 1$ on site x this is the state with $k_s = x\pi/2$. Reversing this protocol will allow measurements of the dressed state occupations, since each dressed state will be adiabatically mapped to a different internal state s.

Now, imagine that the atom located on site x in a deep lattice, $t = 0$, has been prepared in a dressed state $|k_s\rangle_x$. Consider reducing the lattice depth to introduce weak tunnel coupling $t \ll \Omega$. The tunnel coupling conserves the synthetic momentum k_s, so, for typical values of χ, the state $|k_s\rangle_x$ is out of resonance from the neighbouring states, $\epsilon_{x,k_s} \neq \epsilon_{x\pm1,k_s}$. Since the energy offset is of order Ω, for $t \ll \Omega$ the energy eigenstates are well described by the *localised* states $|k_s\rangle_x$. We note that the periodicity of ϵ_{x,k_s} under $x \to x + 4$ requires the energy eigenstates to be extended Bloch waves. However, the bandwidth of these states is of order t^4/Ω^3 which for now we assume to be small. (For $t/h \sim 100$Hz and $\Omega/h = 10$kHz, this bandwidth is of order 10^{-4}Hz $\times\, h$.)

The key feature that allows adiabatic transfer is that, by varying the phase $\chi(\tau)$, neighbouring states can be brought into resonance and the tunnel coupling restored. This is illustrated in Fig. 2, which shows the variation of the energy levels (Eq. 6) at sites $x = 0, 1$ and -1 as a function of $k_s + \chi(\tau)/4$. Consider a particle that is prepared in the state $|k_s = 0\rangle_{x=0}$ for $\chi(0) = 0$, denoted by the filled circle in Fig. 2. As $\chi(\tau)$ is increased from 0 the energy of this state increases smoothly until it encounters a crossing with the state $|k_s = 0\rangle_{x=1}$ at $\chi(\tau) = \pi$. For non-zero tunnel coupling, $-t$, these two states anti-cross with gap $2t$. So if $\chi(\tau)$ is varied slowly compared to $2t/h$ the particle will follow the ground state, ending at $\chi(\tau) = 2\pi$ in the state $|k_s = 0\rangle_{x=1}$ (open circle in Fig. 2). Thus the particle is adiabatically transported in the lattice, in a direction determined by the sign of $d\chi/d\tau$. This encapsulates the local picture of the adiabatic pumping protocol. It is a robust process, with each particle transferred by one lattice constant as $\chi(\tau) = \chi(0) + 2\pi$, within the assumption of adiabatic evolution.

Moreover, this adiabatic transfer has the feature that the direction of motion depends on which dressed state the particle occupies, k_s. For example, a particle starting in the state $|k_s = \pi/2\rangle_{x=0}$ at $\chi = 0$ (filled square in Fig. 2) will be transferred to the state $|k_s = \pi/2\rangle_{x=-1}$ (open square in Fig. 2) if $\chi(\tau)$ is adiabatically increased to $\chi(\tau) = 2\pi$. This internal-state dependence contrasts with prior pumping protocols based on scalar optical lattices. It can be used as a way to separate spin states in an adiabatic manner: while the states $k_s = 0, \pi$ move to the right, the states $k_s = \pi/2, 3\pi/2$ move to the left when $\chi(\tau) = \chi(0) + 2\pi$.

These adiabatically prepared dressed states are highly sensitive to external forces along the 1D lattice and offer the interesting potential to detect them using *spectroscopy*. Forces could arise from external influences (e.g. gravity, or magnetic fields) or from inter-atomic interactions. We shall first illustrate the ideas for an external force, F_x, such as gravity, that provides an internal-state-independent energy difference $\Delta V = F_x a$ between neighbouring lattice sites (a is the lattice spacing).

Note that in the above pumping protocol if the phase χ is varied from $\chi = 0$ to $\chi = \pi$ (not as far as 2π), then an atom initially in state $|k_s = 0\rangle_{x=0}$ will evolve into the state $(1/\sqrt{2})\,[|k_s = 0\rangle_{x=0} + |k_s = 0\rangle_{x=1}]$ (this in-phase combination is selected by the tunnel coupling, $-t$). In the presence of an additional energy offset $\Delta V = F_x a$ between neighbouring lattice sites, adiabatic evolution to $\chi = \pi$ loads the atom in the ground state $|\psi\rangle_+ = \sin(\theta/2)|k_s = 0\rangle_{x=0} + \cos(\theta/2)|k_s = 0\rangle_{x=1}$ where $\theta = \sin^{-1}(t/\sqrt{(\Delta V/2)^2 + t^2})$. One can envisage various ways to extract ΔV from subsequent measurements. One way is to measure the mean occupations $\sin^2(\theta/2)$ and $\cos^2(\theta/2)$ of the two states $|k_s = 0\rangle_{x=0}$ and $|k_s = 0\rangle_{x=1}$, which depend linearly on $\Delta V/t$ for small ΔV: $\sin^2(\theta/2) = 1 - \cos^2(\theta/2) \simeq \frac{1}{2}\,[1 - \Delta V/(2t) + \ldots]$. Rapidly ramping up the 1D optical lattice to $t = 0$ freezes the particles in given lattice sites: $|k_s = 0\rangle_{x=0}$ is the local groundstate but $|k_s = 0\rangle_{x=1}$ is an excited state, so on reverting from $\chi = \pi$ to $\chi = 0$ and then removing the coupling $\Omega \to 0$ adiabatically in the presence of the detunings (Eq. 7) the dressed states evolve into different internal states s which are readily detected spectroscopically. Another possibility is to start from the state $|\psi_+\rangle$ and ramp up the lattice to suppress tunneling $t = 0$ for a time τ_R, during which the system performs Ramsey oscillations between $|\psi_\pm\rangle$ at frequency $\Delta V/h$. These can be measured once t is restored by reversing the preparation sequence.

Currently, precise local force sensing protocols with alkali atoms rely on measurements of Bloch-oscillations. The experiments use spin-polarized Fermi gases to suppress s-wave collisions but require reaching ultralow temperatures $T < T_F \ll t$ (T_F the Fermi temperature) in order to resolve the oscillations [20, 21]. Our dressed state approach allows measurements of $\Delta V/h$ using the same spectroscopic methods as those developed in atomic clocks and it is not limited to quantum degenerate conditions. In fact it can be implemented in current AEA optical lattice clocks which can operate at conditions of density and temperature where interaction effects can be suppressed. Thus taking advantage of the high insensitivity to magnetic field fluctuations and limited spontaneous emission offered by AEA clocks [18] our method has the potential of reaching at least one order of magnitude larger sensitivity than time-of-flight based protocols.

The high sensitivity of the adiabatic protocol can additionally be used to measure inter-atomic interactions if they become relevant. Consider two atoms that start in the same internal state (e.g. $s = 1$) at two adjacent lattice sites (e.g. $x = 0, 1$). For weak onsite interaction, $|U| \ll t, \Omega$, the above preparation sequence and ramp to $\chi = \pi$ would place these atoms approximately in an equal superposition of the states $|k_s = 0\rangle_{x=0}|k_s = \pi/2\rangle_{x=1}$, $|k_s = 0, \pi/2\rangle_{x=1}$, $|k_s = 0\rangle_{x=0}|k_s = \pi/2\rangle_{x=2}$, and $|k_s = 0\rangle_{x=1}|k_s = \pi/2\rangle_{x=2}$. Since there is non-zero amplitude for both atoms to occupy $x = 1$, if tunneling is suddenly suppressed and the system is let to evolve for some time, the onsite interactions will generate Ramsey fringes with frequency U/h. The connection to force measurement with a single atom, described above, can be made precise by filling a superlattice of double-wells, such that only one atom is displaced at $\chi = \pi$. Note that only SU(M) symmetric interactions preserve k_s as a good quantum number. SU(M)-breaking interactions will further lead to detectable couplings to states with $k_s \neq 0, \pi/2$.

We have focussed on motion and force detection in the weak tunneling regime, $t \ll \Omega$. For $t \sim \Omega$ the eigenstates must be considered to be extended Bloch waves of the Harper model. They are characterized by the 2D wavevectors $(k_x, k_s + \chi/M)$ with continuous k_x and discrete $k_s \in \{2\pi/M \times \text{integer}\}$. At flux $\Phi = (2\pi)(p/q)$, with p and q relatively prime integers, the Harper model has a set of energy bands with topological character, as described by non-zero Chern number, $\mathcal{C}$ [6].

For the 1D model considered here, $\mathcal{C}$ sets the number of particles that move along the length of the system under the adiabatic evolution of $\chi = 0 \to 2\pi$ [7]. The resulting quantised transport for an insulating state with an integer number, α, of bands filled (1D filling $n_{1\mathrm{D}} = M\alpha/q$) is described by the application of the iconic results of Refs. [6, 7]. For $M = 4$, $\Phi = \pi/2$ (i.e. $p/q = 1/4$), the case $\alpha = 1$ corresponds to one particle per lattice site ($n_{1\mathrm{D}} = 1$). The lowest energy band of the Harper model at $t = \Omega$ has Chern number 1. Thus, precisely one particle transported along the 1D lattice for each cycle $\chi(\tau) = \chi(0) + 2\pi$.

This topological phase of the dressed atoms can be adiabatically prepared starting from vanishing Rabi coupling, $\Omega = 0$, and a band insulator of $n_{1\mathrm{D}} = 1$ fermion per lattice site in a single internal state, say $s = 1$. To do so, one simply ramps up the coupling Ω of Eq.4 in the presence of the detuning (Eq. 7) for $\chi \neq \pi$. It may seem surprising that one can adiabatically connect the trivial band insulator (at $\Omega = 0$) to an insulating state at $\Omega = t$ which is characterised by a non-zero Chern number. However, in this 1D setting k_s is discrete, so by ramping at fixed χ the system only explores certain lines through the 2D Brillouin zone. For $M = 4$, $\Phi = \pi/2$, for which $k_s \in \{0, \pm\pi/2, \pi\}$ the lowest band only has gap closings at $k_s + \chi/4 = \pm\pi/4, \pm 3\pi/4$. For $\chi \neq \pi$ the spectrum remains gapped and the system evolves adiabatically.

For a filled band the adiabatic transport is topologically protected, so is insensitive to weak perturbations, such as interparticle interactions with strength $|U| \lesssim t, \Omega$ and variations of the Rabi frequencies $\delta\Omega$ that are small compared to the the average $\bar{\Omega}$ and the tunnelling t (the energy scales that determine the band structure and band gaps). For $t \ll \bar{\Omega}$, where the "local picture" is valid these conditions can become restrictive since one needs $\delta\Omega \ll t \ll \bar{\Omega}$. As discussed above in this limit the use of ^{171}Yb could facilitate reaching homogeneous couplings.

The coupling between positional motion and the dressed states allows force detection with spectroscopic read-out also in this regime where the energy eigenstates must be viewed within band theory [22]. Consider a system of non-interacting atoms that fill a set of the Harper bands at a fixed χ (e.g. a fermionic band insulator), or that are uniformly distributed in k_x. Since the band is uniformly occupied, a force F_x does not lead to Bloch oscillations along the x-direction. However, it does lead to a current along the synthetic dimension, corresponding to a nonzero expectation value of $\hat{I}_s \equiv \frac{1}{\hbar}\frac{\partial \hat{H}_\Omega}{\partial \chi}$. This arises from the existence of an anomalous velocity associated with the Berry curvature[24, 25] of the occupied states. The mean synthetic current is $I_s \equiv \langle \hat{I}_s \rangle = -(Na/\hbar)\Sigma F_x$ with N the total number of atoms, a the lattice constant, and the dimensionless conductivity Σ determined by the average Berry curvature along the lines $(k_x, k_s + \chi/M)$. The dependence of Σ on χ is shown in Fig. 3 for $M = 4$, $\Phi = \pi/2$ and $n_{1\mathrm{D}} = 1$. For $t/\Omega \ll 1$ the Berry curvature is maximum close to $\chi = \pi$, which is where bandgaps close at $t \to 0$. While at any given χ this conductivity is not quantized, its integral $\int_0^{2\pi} \Sigma \, d\chi$ is the (integer) Chern number. Note that the eigenstates of $\hat{I}_s$ are the same as those of $\hat{H}_\Omega$ (Eq. 4), given by Eq. 5. Thus, their occupations — and therefore the mean synthetic current when weighted by the eigenval-

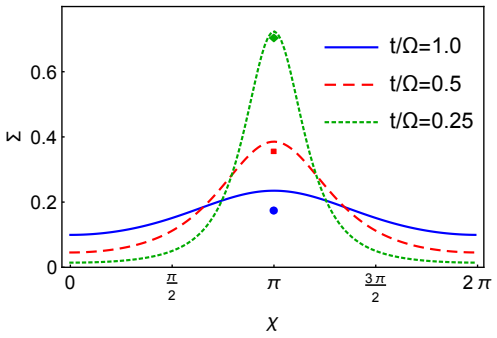

FIG. 3. Dimensionless conductivity Σ, describing the mean synthetic current I_s in response to a force F_x, as a function of the phase χ. ($M = 4$, $\Phi = \pi/2$ and $n_{1D} = 1$ particle per lattice site.) For weak tunneling, $t \ll \Omega$, the response at $\chi = \pi$ is well-described within the local picture (points).

ues $I_{x,k_s} = (\Omega/2\hbar)\sin(k_s - x\pi/2 + \chi/4)$ — can be measured by the adiabatic ramps described before, in which dressed states adiabatically return to different internal states s. For weak tunneling $t \ll \Omega$ this reduces to the two-state problem described above in the local description. The linear dependence of $\sin^2(\theta/2)$ on $\Delta V = F_x a$ for $\chi = \pi$ corresponds to a Berry-curvature induced synthetic current, $I_s = \frac{\Omega}{2\hbar}\frac{1}{\sqrt{2}}\sum_x \left[\sin^2(\theta/2) - \cos^2(\theta/2)\right] = -\frac{\Omega}{2\sqrt{2}\hbar}\frac{\Delta V}{2t} \times N = -\frac{Na}{\hbar}\frac{\Omega}{4\sqrt{2}t}F_x$ with N the number of atoms. This limiting result, $\Sigma = \frac{\Omega}{4\sqrt{2}t}$, is shown as points in Fig. 3, accurately describing Σ for $t/\Omega \ll 1$ [26].

In summary we have shown how an interesting and subtle concept from condensed matter physics (adiabatic pumps), implemented by the coherent optical dressing of internal atomic states, can lead to new forms of transport and of force-sensing. Those can arise even on a *local* level, not requiring the standard band-theoretical formulation and can be observed in non-degenerate gases, facilitated by the spectroscopic preparation and read-out.

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