ABSTRACT

In this paper, we propose a Generalized Kalman Filtered Compressive Sensing (Generalized-KFCS) framework to reconstruct a video sequence, which relaxes the assumption of a slowly changing sparsity pattern in Kalman Filtered Compressive Sensing [1, 2, 3, 4]. In the proposed framework, we employ motion estimation to achieve the estimation of the state transition matrix for the Kalman filter, and then reconstruct the video sequence via the Kalman filter in conjunction with compressive sensing. In addition, we propose a novel method to directly apply motion estimation to compressively sensed samples without reconstructing the video sequence. Simulation results demonstrate the superiority of our algorithm for practical video reconstruction.

1. INTRODUCTION

Most natural signals have sparse representations when expressed in some certain basis. For such a signal, Compressive Sensing (CS) [5, 6] enables a sub-Nyquist approach to recover the whole signal with a limited number of random linear projections. CS is thus attractive for applications with high data acquisition cost, e.g., imaging at non-visible wavelengths [7, 8] and sampling in wireless sensor networks [9, 10].

The problem of reconstructing time sequences of spatially sparse signals arises in many applications including taking video using a CS camera (e.g., a single pixel camera [11]), real-time Magnetic Resonance Imaging (MRI) [7], and channel equalization in communications [12]. For the video application, applying CS at each time separately is the most straightforward method. It can be performed online and has low-complexity, but the high temporal correlation present is not exploited by a conventional CS process. An alternative approach is to treat the whole video sequence as a single 3D signal and apply batch CS processing [7, 13]. However, it not only significantly increases the computational complexity, but also causes delays as the reconstruction is not performed until the projections for the sequence have been gathered.

In [1], Vaswani considers casual and recursive reconstruction of time-varying sequence and proposes a method known as Kalman Filtered Compressed Sensing (KFCS). She assumes that the sparsity pattern of the signal changes slowly over time, and the idea is to apply CS on the Kalman filtered residual. When this assumption holds, the signal residual is more sparse than the original signal, and thus better reconstruction performance is achieved. In [2] the work is extended to a practical MRI sequence, which is nearly sparse (compressible), rather than ideally so. The work in [3] is also related but focuses on exact reconstruction using fewer noiseless measurements, and the extension for the noisy case is demonstrated in [4]. Other work employing side information from key frames to improve the current reconstruction includes [14, 15] and other reweighting based work including [8, 16].

Vaswani’s approach has been shown to be successful in many applications where the support of the signal changes slowly in time, e.g., the MRI case [2]. However, in many practical video sequences, the support of the signal changes much more rapidly than for the MRI sequence so that the assumption underlying KFCS is not valid. In this paper, we propose a more general framework where the assumption of only a slow change of signal support is not necessary. The key to relaxing the assumption used in KFCS is the use of Motion Estimation (ME) which has been widely used for video compression [17]. In addition, we also propose a novel method to directly estimate motion from the subsamples of a video sequence without first recovering the video. Simulation results show that the proposed approach outperforms the state-of-art approaches.

2. PROBLEM FORMULATION

As in a conventional CS setting, we sample a time sequence of \( T \) sparse signals via:

\[
y_t = \Phi_t x_t + e_t = \Phi_t \Psi s_t + e_t = A_t s_t + e_t, \quad (1)
\]

where \( t = 1, \ldots, T \) is the time index of each frame, \( x_t \in \mathbb{R}^N \) is the \( t \)-th signal, \( y_t \in \mathbb{R}^M (M \ll N) \) is the \( t \)-th observation.
vector, \( e_t \in \mathbb{R}^M \) is a noise term, \( \Phi_t \in \mathbb{R}^{M \times N} \) represents the sensing matrix, \( \Psi \in \mathbb{R}^{N \times N} \) is a sparsifying basis, \( s_t \in \mathbb{R}^N \) is the sparse representation which has only \( K \) (\( K \ll N \)) non-zero coefficients and \( A_t = \Phi_t \Psi \) is the equivalent projection matrix.

To recover the sparse representation \( s_t \) from \( y_t \) is an ill-posed inverse problem. However, CS asserts that we can solve the problem with an overwhelming probability of success if \( A_t \) satisfies the Restricted Isometry Property (RIP) [18]. It has been shown that the RIP is satisfied for various matrices, such as an i.i.d. Gaussian matrix or a Bernoulli matrix [19]. However, in the sampling process when using such matrices with an image, one has to explicitly use all the pixels, although the number of measurements needed will appear to be reduced.

Another technique called random sampling [9, 20], in which only a small, uniformly distributed, randomly chosen fraction of the coefficients is captured, can help to achieve simpler implementation in hardware. The entries of such a sensing matrix \( \Phi_t \in \mathbb{R}^{M \times N} \) are all zeros except for \( M \) entries with the value of one in \( M \) different columns and rows. Applying such a sensing matrix for images is effectively taking \( M \) pixels at random.

The CS reconstruction for the noisy case as in (1) can be formulated as a Basis Pursuit De-Noising (BPDN) problem:

\[
\min_{s_t} \frac{1}{2} \| y_t - A_t s_t \|_2^2 + \gamma \| s_t \|_1,
\]

where \( \gamma > 0 \) is a regularization parameter which determines the tradeoff between sparsity and data consistency. A conventional CS approach is to apply (2) for each frame in the time sequence. But this is inefficient because it completely ignores the temporal correlation between frames. In this paper, we propose to combine the use of ME and a Kalman Filter (KF) to effectively exploit the dependencies between frames for CS captured video sequences.

\[
\text{MAD}(d_m) = \sum_{p \in B_m} D(p, d_m),
\]

where \( p \) is the paired pixel index in \( B_m \), \( d_m \) is a MV for the block, and \( D \) is the function to calculate the pixel difference value for each index \( p \). The \( d_m \) that minimizes \( \text{MAD}(d_m) \) is taken as the MV for the current block. The function \( D \) is defined as:

\[
D(p, d_m) = \begin{cases} 
|x_{ref}(p + d_m) - x_{cur}(p)|, & \text{if } x_{cur}(p) \geq 0 \\
0, & \text{elsewhere,}
\end{cases}
\]

\[
|x_{ref}(p + d_m)| + |x_{cur}(p)| - \max(|x_{cur}(p)|, |x_{ref}(p + d_m)|).
\]

3. PROPOSED METHOD

Our proposed method uses the framework shown in Figure 1. The CS measurements \( y_t \) are first pre-processed to determine an initial current frame \( x_{t,ini} \). Then ME is carried out between \( x_{t,ini} \) and the reconstructed previous frame \( x_{t-1} \), which yields a transition matrix \( F_t \) for implementing a KF. When \( t = 1 \), \( x_0 \) comes from the result of performing conventional CS. The output of a KF, \( s_{t,KF} \), together with residual-based BPDN or Modified BPDN [4], is used to derive the reconstructed images. This section will discuss in detail each of the main modules.

3.1. Motion Estimation for CS Captured Video

ME is a core step in traditional video coding. For an overview of ME techniques we refer to [17], and we observe that the most popular ME method is the Block Matching (BM) algo-

\[
\text{Fig. 1. Framework for Generalized-KFCS}
\]

\[
\text{rithm. The idea is to divide the current frame into small macro blocks and then each block is compared with the corresponding blocks in a searching window in the previous frame. The best match is determined based on cost function criteria. A Motion Vector (MV) is then created to describe the movement from the position of the current block to that of the best matching block.}

There is one barrier to utilising ME in our work. In contrast to the traditional video coding problem, full resolution frames are not available for CS captured video. To deal with this problem, [8] proposes to do a coarse reconstruction first and then apply ME to this intermediate resolution frame. To derive the transition matrix \( F_t \), ME based on low resolution frames is carried out in [21]. To the best of our knowledge, there is no existing ME algorithm that can be applied directly to the subsamples obtained by CS. Here, for the random sampling strategy, we propose a method of direct ME from subsamples without performing any reconstruction beforehand.

To do ME on CS captured video, we need to pre-process the subsamples as follows: 1) allocate each sample to the original index (indices are available from the measurement matrix \( \Phi \), 2) pad the image frame with a negative constant \( \omega \) at each position with a missing pixel. Then if we represent the padded current frame \( x_{cur} \) and the previously reconstructed reference frame \( x_{ref} \) as matrices, the BM algorithm can be applied to them using a modified method for computing the cost function. Specifically, we divide \( x_{cur} \) into \( m \) blocks and denote the set of pixel indices in each block as \( B_m \). Then we calculate the Mean Absolute Difference (MAD) [8] over each block \( B_m \) as follows:

\[
\text{MAD}(d_m) = \sum_{p \in B_m} D(p, d_m),
\]

where \( p \) is the paired pixel index in \( B_m \), \( d_m \) is a MV for the block, and \( D \) is the function to calculate the pixel difference value for each index \( p \). The \( d_m \) that minimizes \( \text{MAD}(d_m) \) is taken as the MV for the current block. The function \( D \) is defined as:

\[
D(p, d_m) = \begin{cases} 
|x_{ref}(p + d_m) - x_{cur}(p)|, & \text{if } x_{cur}(p) \geq 0 \\
0, & \text{elsewhere,}
\end{cases}
\]

\[
|x_{ref}(p + d_m)| + |x_{cur}(p)| - \max(|x_{cur}(p)|, |x_{ref}(p + d_m)|).
\]

\[
\text{Fig. 1. Framework for Generalized-KFCS}
\]
3.2. Motion Estimation Enhanced Kalman Filter

The KF [22] is one of the most popular data fusion algorithms. It offers a way to exploit the temporal correlation and recursively estimate a sequence of signals. Here, we employ it in the sparse domain as follows:

\[
\hat{s}_t |_{t-1} = F_{t,s} \hat{s}_{t-1} |_{t-1} \quad (5)
\]

\[
P_t |_{t-1} = F_{t,s} P_{t-1} |_{t-1} F_{t,s}^T + Q \quad (6)
\]

\[
K_t = P_t |_{t-1} (\Phi_t \Psi_t)^T (\Phi_t \Psi_t P_{t-1} |_{t-1} (\Phi_t \Psi_t)^T + R_t)^{-1} \quad (7)
\]

\[
\hat{s}_t |_t = \hat{s}_t |_{t-1} + K_t (y_t - \Phi_t \Psi_t \hat{s}_t |_{t-1}) \quad (8)
\]

\[
P_t |_t = P_t |_{t-1} - K_t (\Phi_t \Psi_t) P_t |_{t-1}, \quad (9)
\]

where the subscript \( t \) denotes the time index, \( t | t-1 \) represents the prediction for time \( t \) from time \( t-1 \), and \( t | t \) is the final update value for time \( t \). The meaning for each term in the equations is as follows: \( \hat{s} \) denotes the estimated sparse signal vector, \( F_{t,s} \) is the state transition matrix from time \( t-1 \) to \( t \) in the sparse domain, \( P \) is the error covariance matrix, \( Q \) is the process noise covariance matrix, \( K \) is the Kalman gain, \( \Psi_t \) is the measurement matrix, \( \Psi_t \) is the sparsifying basis, \( R \) is the measurement noise covariance matrix, \( y \) is the measurement vector, and the superscript \( T \) is the transposition operator.

The equations have two stages, i.e., the prediction stage (5)-(6) and the measurement update stage (7)-(9). For a CS captured video sequence, given the estimated previous frame, we can get the prediction and update of the current frame using the KF equations, provided that the transition matrix \( F_{t,s} \) is known. The KFCS framework proposed in [1] runs an initial KF with \( F_{t,s} = I \), where \( I \) is an identity matrix, since the sparsity pattern is assumed to be changing sufficiently slowly. In order to derive a generalized KFCS framework without enforcing an unchanging support, we consider the use of an actual transition matrix to capture the change of support.

Recall that in the previous step, ME provides us the MVs that indicate the movements of blocks. With the MV values, the time-domain transition matrix \( F_t \) can be derived as a permutation matrix, whose entries are all 0 except that in row \( j \) the entry \( i \) equals 1 (where \( i \) is the pixel index in \( x_{t-1} \) and \( j \) is its corresponding index after moving according to the MV). Then the sparse domain transition matrix is \( F_{t,s} = \Psi^{-1} F_t \Psi \). If we set \( \hat{s}_{t-1} |_{t-1} \) as the reconstructed previous frame, then \( \hat{s}_t |_t \) is the estimated sparse signal after the KF, i.e., \( s_{t,KF} \).

3.3. Compressive Sensing on the Residual

To further improve the performance, we apply CS on the residual from the KF, which is more sparse than the original signal. With the result after the KF, i.e., \( s_{t,KF} \), the residual is calculated as:

\[
y_{t,res} = y_t - \Phi_t \Psi s_{t,KF}. \quad (10)
\]

Then we propose two alternative methods to recover the residual signal using \( y_{t,res} \). The first way is to utilize BPDN as in equation (2) using \( y_{t,res} \) as the measurements. The second one is to employ a Modified BPDN [4] as follows:

\[
\min_{s_t} ||(s_t)_\Lambda||_1, \text{ s.t. } ||y_{t,res} - \Phi_t s_{t,KF}||_2 \leq \varepsilon. \quad (11)
\]

where \( \varepsilon \) is an upper bound on the size of the noisy contribution, \( (s_t)_\Lambda \) refers to the sub-vector of \( s_t \) that contains the elements with index in \( \Lambda \) and \( \Lambda^c \) denotes the complement of the set \( \Lambda \). Here, we derive \( \Lambda \) as the 99%-energy support [3] of \( s_{t,KF} \), rather than the support of \( s_{t-1} \). Either approach yields a residual signal, denoted by \( s_{t,res} \) and \( s_{t,res,mod} \), respectively. Then the reconstructed frame is the sum of the KF result and the reconstructed residual signal.

3.4. Motion Estimation Enhanced KFCS (Generalized-KFCS)

Based on the previous discussions, the proposed generalized-KFCS algorithm is shown in Algorithm 1. The input covariance matrix \( Q \) is estimated by using a training sequence. Specifically, for a sequence \( s_t (t = 1, ..., T) \), we first zero out the “compressible” coefficients by setting all coefficients below a threshold to zero. The method of selecting the threshold is described in [2]. Then we calculate the transition matrices \( F_t \) and \( F_{t,s} (t = 1, ..., T) \) using ME. The estimation is then computed as:

\[
Q = \sigma^2_{sys} \mathbf{I},
\]

where

\[
\sigma^2_{sys} = \frac{1}{\sum ||s_t||_1} \sum_t \sum_i (s_{t,i} - F_{t,s} \hat{s}_{t-1,i})^2, \quad \delta_i := \{ t : s_{t,i} - F_{t,s} \hat{s}_{t-1,i} \neq 0 \}, i \text{ is the coefficient index. Note that } \sigma_{sys} \text{ is set to zero if } \sum_i ||\delta_i|| \text{ is zero. This process is a simple extension of [2].}
\]

In contrast to the KFCS algorithm [1, 2] and the modified CS algorithm [3, 4], we take account of the motion between
frames, therefore the requirement for a slowly changing support set is relaxed. The proposed algorithm can run accurately as long as ME can be performed properly. Even so, it still requires some correlation between frames (this is also why we do not use the conventional CS for each frame), but we will show that our algorithm outperforms KFCS and modified BPDN when a relatively fast changing support set exists.

4. SIMULATION RESULTS

The proposed algorithm is evaluated on the foreman video sequence [23]. The frames in the sequence are resized to $64 \times 64$ to ease implementation issues. Normalized Square Error (NSE) is used for evaluation, which is defined as: $NSE := \frac{||x_t - \hat{x}_t||^2}{||x_t||^2}$. We compare the performance of our algorithm with the conventional CS method, the KFCS algorithm [2] and the modified BPDN algorithm [4]. We label modified BPDN as modCS, our algorithm with BPDN in step 5 as Generalized-KFCS and our algorithm with modified BPDN in step 5 as Generalized-KF-modCS. We consider two different CS implementations: 1) i.i.d. Gaussian sensing matrices for different time frames; 2) i.i.d. random sampling matrices for different time frames. We employ a 4-level 2D wavelet transform for sparsifying. The parameters are set as: $\gamma = 10$ [24], $\sigma_{obs}^2 = 25$, $\epsilon = 5$, $\omega = -10$. The results are obtained by averaging over 50 trials.

Figure 2 illustrates the performance of various reconstruction methods for a fixed sampling rate ($M/N = 0.4$) (a) using Gaussian projection matrices; (b) using random sampling matrices.

Fig. 2. Comparison of various reconstruction methods for a fixed sampling rate ($M/N = 0.4$) (a) using Gaussian projection matrices; (b) using random sampling matrices.

Fig. 3. Comparison of various reconstruction methods for different sampling rates (a) using Gaussian projection matrices; (b) using random sampling matrices.

employ direct ME, still maintains its good performance. Note that the different NSE values for KFCS on the first frame occurs in both figures because the algorithm starts its iterations from the first frame, but for other algorithms we just initialize using the conventional CS for the first frame. Also note that to aid clarity, different scales for the y axes are used in the figures. When the random sampling is employed, our algorithm has worse performance but less complexity than with Gaussian sampling.

In Figure 3, we evaluate our algorithm for different sampling rates. The results shown are for frame 10. From Figure 3(a), we can observe that both versions of Generalized-KFCS are superior to the other algorithms, except that modified BPDN performs slightly better when the sampling rate approaches 0.2. However, when random sampling is employed as in Figure 3(b), both versions of our method always have better performance than the others. For example, to achieve the NSE of 0.02, the two alternatives of the generalized-KFCS need only a sampling rate of less than 0.25, while conventional CS requires a sampling rate of 0.3 and the others require a sampling rate of about 0.35. It is noticeable that the fewer measurements we take, the more gain can be obtained by using our proposed method.

5. CONCLUSIONS

In this paper, we propose a generalized KFCS framework for causal reconstruction of a CS captured video sequence. We relax the assumption of a slowly changing signal support in KFCS by including an actual estimated transition matrix for the KF. A framework for direct ME from CS subsamples is also developed when the random sampling method is utilized. Experiments demonstrate the advantage of the proposed method in the reconstruction of an actual video sequence.

6. REFERENCES


