Towards a silent fan: an investigation of low-speed fan aeroacoustics

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This dissertation is submitted for the degree of

Doctor of Philosophy
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- does not exceed the prescribed word limit for the Degree Committee of the Department of Engineering.

All numerical calculations presented were computed by me using my own code except the LINSUB MATLAB routine and the 3D RANS CFD simulation, as specified in the text.

This dissertation contains 86 figures and approximately 39,000 words.

Timothy James Newman

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Abstract

The noise (unwanted sound) from fans of all sizes, operating in close proximity to people, can be a design constraint due to annoyance or, in the worse cases, health damage. Of the total noise, aeroacoustic noise — produced by unsteadiness in the air — often represents a significant source and is intrinsically linked to the aerodynamic features of the flow field. In this work, the aeroacoustics of low-speed fans are investigated using a compact mixed-flow fan as a test case. The low-speed regime is less developed compared to large-scale, high-speed machines and is increasingly relevant to applications such as micro air vehicles, small wind turbines, and other environmental comfort technologies found in buildings or vehicles.

The test case fan Reynolds number is of the order of $10^4$ which is a couple of orders lower than those generally found in gas turbines. Its main sources are therefore best identified experimentally in the absence of proven alternative methods. In order to do this, a way of quantifying fan noise is developed in tandem with control of the aerodynamic operating point. Following a study of sources of the significant broadband and tonal noise, a low-order noise prediction scheme is developed and applied to predict tonal noise with reference to Reynolds number effects.

The new, duct-based rig and method has several advantages over the existing sound power measurement rig built to the ISO 5136 standard at Dyson. The approach, which makes no assumptions about the relative power of different modes, has resulted in a rig that is much shorter. Unlike the ISO rig, it is capable of accurate narrow-band tone measurements with sources which excite strong non-plane-wave duct modes (as the modal structure of the sound is determined) for the frequencies of interest.

Tests have been carried out at different operating points with a range of geometry modifications produced with 3D printing techniques. In terms of tonal sources which particularly impact sound quality, the mixed-flow impeller alone produces tones due to very high sensitivity to inflow distortion of the mean flow (giving unsteady blade loading). This
means that the product inlet must be designed very carefully to optimally condition the flow. Periodicity in the impeller outlet flow produces rotor-stator interaction tones even with a number of guide vanes chosen to satisfy the Tyler-Sofrin theory cut-off criteria. This is thought to be due to abrupt radius change after the guide vanes in the rig (while the theory assumes constant radius). In the product, abrupt radius change also occurs.

The sensitivity of the broadband level to inflow turbulence was confirmed to be low in the rig, although the in-product inflow appears much less ideal. The main broadband noise source in rig tests is suggested to be impeller self-noise as only small reductions in rotor-stator interaction noise are achieved with far fewer vanes.

The low-order modelling scheme to understand the fundamental unsteady loading noise mechanism compares well to experiments for sample rotor-stator interaction tones. The velocity fluctuations which induce this noise, measured experimentally with a 2D hotwire, are shown to increase in intensity as Reynolds number is reduced towards $10^4$. This is due to a higher importance of viscosity which can give boundary layers that are thicker and liable to laminar separation. Surface treatments such as boundary layer trips could be used to prevent such separation and potentially reduce noise.

Based on the thesis findings, further tests, simulations and possible design modifications are suggested to understand and reduce the important noise sources.
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Nomenclature

Roman Symbols

\( A \) Mode amplitude
\( a_0 \) Speed of sound
\( B \) Number of rotor blades
\( C_{m,n} \) Normalisation factor
\( c_{1,2} \) Impeller absolute velocity at inlet, outlet
\( c \) Blade chord
\( D \) Diameter
\( E \) Hotwire voltage
\( F^\pm \) Cascade response function
\( f_i \) General function number \( i \)
\( \Delta f \) Frequency bin/resolution
\( \hat{H} \) Transfer function
\( I_x \) Time-averaged axial intensity
\( J_m \) Bessel functions of the first kind order \( m \)
\( K \) Kernel matrix
\( k \) Integer
\( k_{m,n} \) Axial wavenumber
\( k^2_{1,2} \) Squared yaw factor for HW 1 and 2
\( L \) Characteristic length
\( M \) Mach number

\( M \) Mode decomposition matrix

\( m \) Azimuthal mode number

\( N \) Number of propagating modes, harmonic number of BPF

\( n \) Radial mode number

\( P \) Acoustic power

\( p \) Pressure, acoustic pressure

\( R \) Reference radius

\( r \) Radial distance

\( r_k \) Integer

\( \hat{r} \) Reference signal

\( Re \) Reynolds number

\( \hat{S}_{i,j} \) Spectrum (for \( i = j \)) or cross-spectrum (for \( i \neq j \))

\( S \) Cross-sectional area, number of stator blades

\( S \) Scattering matrix

\( s \) Blade spacing

\( T \) Torque

\( t \) Time

\( U \) Mean velocity in the \( x \)-direction

\( u_{ref} \) Impeller blade speed at reference radius

\( u_1, u_2 \) Impeller blade speed at inlet, outlet

\( u \) Acoustic velocity in the \( x \)-direction

\( V \) Mean velocity in the \( y \)-direction

\( v \) Acoustic velocity in the \( y \)-direction

\( W \) Resultant velocity of \( U \) and \( V \)

\( w_1, w_2 \) Impeller relative velocity at inlet, outlet
\( w \)  Input upwash (perpendicular to chord-line) perturbation
\( W_c \)  Compressor work
\( W_e \)  Effective HW cooling velocity
\( W_{cal} \)  Calibration jet velocity
\( z_{m,n} \)  Bessel function order \( m \) derivative zeros

**Greek Symbols**

\( \alpha \)  Wavenumber in the \( x \)-direction, flow rate coefficient
\( \beta \)  Wavenumber in the \( y \)-direction
\( \Gamma \)  Vorticity fluctuation amplitude
\( \Delta \)  Finite change
\( \epsilon \)  Expansibility factor
\( \eta \)  Efficiency
\( \theta \)  Angle
\( \kappa \)  Condition number
\( \lambda \)  Reduced frequency
\( \mu \)  Dynamic viscosity
\( \nu \)  Kinematic viscosity
\( \xi \)  Stagger angle
\( \rho \)  Density, reflection coefficient
\( \sigma \)  Inter-blade phase angle
\( \tau \)  Transmission coefficient
\( \tau_A \)  Moment of external forces
\( \varphi \)  Vane angular location
\( \phi \)  Flow coefficient
\( \psi \)  Pressure rise coefficient
\( \Omega \)  Impeller angular velocity in radians per second
\( \omega \) Angular frequency

**Subscripts**

- \( 0 \) Mean flow conditions
- \( a \) Atmospheric conditions
- \( bm \) Rig bellmouth-related value
- \( d \) Rig duct-related value, design value
- \( g \) Global parameter definition
- \( h \) Hub
- \( m, n \) Mode numbers
- \( op \) Orifice plate value
- \( ref \) Reference value
- \( rms \) Root mean square value
- \( s \) Shroud

**Superscripts**

- \( + \) Travelling away from source or downstream
- \( - \) Travelling towards source or upstream
- \( s \) Source

**Abbreviations**

- **BPF** Blade passing frequency
- **CAE** Computer-aided engineering
- **HVAC** Heating, Ventilation and Air Conditioning
- **LE** Leading edge
- **MF** Mixed-flow
- **OGV** Outlet guide vane
- **PWL** Sound power level
- **RPM** Revolutions per minute
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<tr>
<td>SLA</td>
<td>Stereolithography</td>
</tr>
<tr>
<td>SLS</td>
<td>Selective laser sintering</td>
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<tr>
<td>SPL</td>
<td>Sound pressure level</td>
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<tr>
<td>TE</td>
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Chapter 1  Introduction

Air-moving fans, such as those pictured in Figure 1.1, can increasingly be found in use all around us. A flow of air is generated by a rotating arrangement of blades, often referred to as a rotor or impeller, which acts on the air and leads to an increase in its pressure. Fans impart a relatively small increase in pressure to a flow with high volume compared to compressors, which give a substantial pressure rise to a flow of relatively low volume. The rotor is usually contained within a housing or casing which channels the air to be harnessed for a range of purposes; from propulsion to improving environmental comfort. Depending on the application requirements, fan size can vary greatly. For example, at the large end of the scale the turbofan engine pictured in Figure 1.1c contains a 3m high-speed fan to help propel an aircraft by accelerating vast quantities of air rearwards. At the other end of the scale, the Dyson desk fan pictured in Figure 1.1b contains a 10cm impeller to generate a cooling air flow. Huge investment and development by the aerospace industry over many decades has meant that fan aerodynamics for large-scale, high-speed applications are very well understood. The same cannot be said about small-scale, low-speed fans which are much less developed. Therefore, a key focus of the present thesis is to understand what changes and what stays the same when one ‘miniaturises’ a fan.
The Dyson desk fan, serving as an example of a small air-moving appliance, is designed for use in a home or office environment in close proximity to people. The noise from such devices is a key design constraint as unacceptable levels of noise can render it unusable. An ideal, ‘silent’ fan - producing sound only at a level below the lower limit of human hearing – is considered the ultimate ‘standard of excellence’ for the fan technology considered in this work. Of the total noise, a certain proportion is produced purely by virtue of the air flowing unsteadily and interacting with stationary or rotating surfaces. This type of aerodynamically-generated sound is referred to as aeroacoustic, and is the focus of the noise investigations in the present thesis.

1.1 Motivation

Aeroacoustic noise is a design constraint for all of the applications shown in Figure 1.1 due to its effect on people. The severity of the impact of noise in general can range from annoyance up to health damage [1]. In order to achieve an acceptable noise level, compromises are often

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†Image by Julian Herzog via Wikimedia Commons
made in the overall design to contain or absorb noise once it has been produced. The most elegant and efficient design for low noise would aim to stop it being produced in the first place. Therefore, reduction at the source is the ideal solution and is taken as a basis for this work.

Much of the knowledge for high-speed applications is useful to understand low-speed applications but its limitations are not well known. A better appreciation of fundamental differences in flow physics between the two regimes has many implications beyond the scope of small appliances. The regime is relevant at the low speeds found in HVAC systems, and is expected to become increasingly important as the scale of machines, from unmanned aerial vehicles to wind turbines [2], reduces in the future. Micro air vehicles (MAVs) are currently being developed for a range of commercial and military applications. Companies such as Google and Amazon [3] are already testing drones which can deliver small parcels to customers’ homes more quickly than conventional delivery networks. Minimising the noise produced by these technologies in particular will be of great importance to achieve public acceptance. Oil and gas company ExxonMobil uses UAVs to survey and inspect pipelines. Military research programmes to develop small drones which can monitor hazardous environments have been carried out worldwide, for example by DARPA in the US [4]. Technological advances in these areas, in addition to improving our fundamental understanding of the regime in question, can feed back into high-speed technology.

1.2 Aeroacoustics of low-speed fans

Dyson specialises in the design and manufacture of small air-moving appliances for a variety of applications. The Dyson desk fan is taken as a test case low-speed, air-moving appliance which is compact and for which noise is a key design constraint.

1.2.1 Description of the test case

The device’s overall function, illustrated in Figure 1.2b, can be broken down into the internal flow which enters through vents in the base of the device and leaves through a narrow annular gap in the hoop, and the external cooling flow which is produced as a result of this
annular jet. The external flow is at a low enough speed that the sound produced by the product is expected to be dominated by internal sources. Variants of the product are manufactured by Dyson which have the option to heat the air before it exits the hoop.

Figure 1.2 – Dyson desk fan product (model “AM01”) external flow illustration

As shown in the drawing in Figure 1.3, the main component in the base is the mixed-flow fan consisting of the impeller – driven directly by an electric motor - and outlet guide vanes. Desvard et al. [5] state that the mixed flow fan “is the predominant source of noise from the product and the most significant component of this noise is aerodynamic related”. Measurements show significant tones and broadband noise emitted by the mixed-flow fan when tested in isolation and when housed in the product. The mixed-flow configuration is typified by a significant increase in the mean radius of the flow within the impeller such that the flow leaves at an angle to the axis of rotation ~45°. The flow is then turned back towards the axial direction by the curvature of the flow passages before passing through the guide vanes. This achieves a high pressure ratio with a diffuser that is more compact than if the flow left the impeller at 90° to the axis.

Mixed-flow turbomachinery is suited to other applications which require a high mass flow at a high pressure ratio such as gas cooled nuclear reactors and hovercraft fans [6, p. 58]. They
are increasingly of interest to the aerospace industry as design methods evolve, and can be found in small P&WC turbofan engines manufactured for very light jets [7].

When using the product, the speed of the impeller is varied to control the flow rate through the machine. The design performance is optimised at a speed corresponding to the highest speed which is selectable in the product where the noise is highest - Table 1.1 shows key parameters at this operating point.

Table 1.1 – Dyson desk fan operating at the design point which corresponds to the upper limit of product rotational speeds

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate through machine</td>
<td>27 litres/s</td>
</tr>
<tr>
<td>Impeller rotational speed</td>
<td>8200 rpm</td>
</tr>
<tr>
<td>Fan stagnation pressure rise</td>
<td>450 Pa</td>
</tr>
<tr>
<td>Impeller shroud diameter at inlet, $D_{s1}$</td>
<td>53 mm</td>
</tr>
<tr>
<td>Impeller shroud diameter at outlet, $D_{s2}$</td>
<td>98 mm</td>
</tr>
<tr>
<td>Number of impeller blades, $B$</td>
<td>9</td>
</tr>
<tr>
<td>Number of guide vanes, $S$</td>
<td>22</td>
</tr>
</tbody>
</table>

To achieve low noise design, the operation of both the fan and the system as a whole must be
understood and optimised. Poor aerodynamic efficiency from non-ideal flow behaviour is intrinsically linked to aeroacoustic noise generation. The mechanisms behind this are discussed in the next section with reference to the differences in flow physics for the regime of interest.

1.2.2 Fans and compressors from high to low speed

In this section, ways of quantifying and understanding the mixed-flow fan’s aerodynamics and the closely-linked aeroacoustic noise are discussed. A parameter of particular importance is the Reynolds number which quantifies the relative importance of inertial and viscous forces in the fluid, and is defined as the ratio of the two:

\[
Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho UL}{\mu}
\]  

(1.1)

where \(U\) is a characteristic velocity, \(L\) is a characteristic length, and the fluid properties \(\rho\) and \(\mu\) have their usual definition. The highest Reynolds number, based on a mid-span impeller relative inlet velocity and streamwise blade chord (as indicated in Figure 1.4), is \(1 \times 10^5\) and reduces to around 30% of this value at the lowest speeds encountered. This means the test case is always operating in a low Reynolds number regime (i.e. in the range \(10^4 - 10^5\)) where the potential for significant transitional flow and/or separation is high.
Viscous boundary layers on stationary and rotating surfaces within the machine become more prominent at low Re for a number of reasons. Predictably, they become thicker by virtue of the fact that viscous forces are more influential. Less predictable is the effect of low Re on flow separation, transition and laminar flow. A boundary layer which starts out as laminar can do one of several things:

- Remain laminar \textit{ad finem}
- Become unstable and transition to turbulence without separation
- Become unstable, transition to turbulence and later separate (‘turbulent separation’)
- Separate without reattachment (‘laminar separation’)
- Separate, transition to turbulence and then reattach

Separation in particular can have a big impact outside the boundary layer, on the global flow field and consequently on machine aeroacoustic performance.

Turbulence, in the free-stream and in boundary layers, leads to flow perturbations and can be a source of sound [9]. It can also affect the aerodynamic performance due to many accompanying factors such as the dependence of drag or boundary layer development on turbulence [10]. The size of the smallest scales of turbulent motion is determined by the Reynolds number since this is the scale where viscous forces are most influential.
In the following two sections, the relation of the test case mixed-flow fan to other types of fan is discussed to see what existing knowledge can be applied. Aerodynamic considerations which determine the overall operation are given first, followed by the interlinked acoustic considerations.

### 1.2.2.1 Aerodynamic considerations

*Velocity triangles*

Figure 1.5a shows a meridional view of the impeller, connecting flow passage, and guide vanes with the mid-span location marked with a dash line. The design impeller inlet and outlet velocity triangles at this radius are shown in Figure 1.5b (based on the CFD data referred to in Chapter 4). It is apparent that incidences are low for the flow onto the sections. The impeller section camber appears irregular due to the shape of the mid-span streamsurface which is mapped from a 3D surface shown in Figure 1.4 to this 2D cascade representation (see [11]).
1.2 Aeroacoustics of low-speed fans

Figure 1.5 – Impeller (a) meridional (axial-radial) view, including mid-span line (a) velocity triangles (meridional-circumferential directions) on the mid-span streamsurface
Dimensional analysis

The performance of a turbomachine for an incompressible fluid depends on several key variables related to the machine and the fluid. In terms of the machine, these are its rotational speed $\Omega$, impeller shroud max diameter $D_{s2}$, and volumetric flow rate $Q$. In terms of the fluid, density $\rho$ and viscosity are the key variables. Simple dimensional analysis indicates that two dimensionless groups define the performance. To distinguish them from related definitions, these will be called the ‘global’ flow coefficient and Reynolds number, respectively:

$$\phi_g = \frac{Q}{\Omega D_{s2}^3} \quad (1.2)$$

$$Re_g = \frac{\rho \Omega D_{s2}^2}{\mu} \quad (1.3)$$

Other parameters such as the pressure rise coefficient and aerodynamic efficiency depend only on these two groups:

$$\psi = \frac{\Delta p_0}{\rho \Omega^2 D_{s2}^2} = f_1(\phi_g, Re_g) \quad (1.4)$$

$$\eta_{aero} = \frac{Q \Delta p_0}{W_c} = f_2(\phi_g, Re_g) \quad (1.5)$$

where $\Delta p_0$ is the stagnation pressure rise across the fan and $W_c$ is the input rotor power. The parameter definitions given in equations (1.2)-(1.5) aid comparisons between different machine types. Specific speed $\Omega_{spec} = \phi_g^{1/2}/\psi^{3/4}$ and specific diameter $D_{spec} = \psi^{3/4}/\phi_g^{1/2}$ are dimensionless parameters often used to indicate what type of machine is most appropriate. The Cordier diagram amalgamates past experience of creating optimum machines of a range of different types. It is derived by assuming that efficiency depends only on flow coefficient (i.e. Reynolds number effects are negligible) and that the optimum efficiency of a given machine occurs at a unique value of flow coefficient [6]. As shown in Figure 1.6, it is plotted as a single curve but represents a broad spread of results on either side which still achieve high performance.
1.2 Aeroacoustics of low-speed fans

Using the operation point data shown in Table 1.1 (with the restriction equal to the required fan pressure rise), the test case fan is found to lie close to the line in Figure 1.6 within the mixed-flow range as indicated. The shape of the curve at the highest and lowest $\Omega_{spec}$ is caused by variations in centrifugal effects for the different compressor types. Axial machines are found at high $\Omega_{spec}$ in which centrifugal effects are missing, while for low $\Omega_{spec}$ radial machines, the pressure change is mainly achieved by the change in the flow mean radius [12]. Mixed-flow machines lie between these two extremes and therefore have similarities to both types of machines. For the regime of interest in this study, Reynolds number is an important factor. Its effect is neglected in the Cordier method which is based upon machines where the performance parameters depend only on $\phi$ as the Reynolds number is high. The effect of low Reynolds number on performance is discussed further below.

Performance characteristics

The effect of the change in mean radius can be seen directly from the Euler work equation [11] applied along a streamtube:
\[ \Delta W_c = \tau_A \Omega = \Omega (r_2 c_{\theta 2} - r_1 c_{\theta 1}) + \Omega^2 (r_2^2 - r_1^2) \]  

(1.6)

where \( \Delta W_c \) is the work per unit mass, \( \tau_A \) is the total moment of external forces, and \( c_{\theta} \) is the swirl component of the absolute velocity \( c \) shown in Figure 1.5. The first term on the right side of equation (1.6) is work related to flow turning by the blades and to flow passage area changes. The second term, the centrifugal effect, accounts for the ability of centrifugal compressors to achieve a much higher pressure rise per stage than axial flow machines [13, Sec. Radial Turbomachinery Design]. In axial flow machines, the radius change of the majority of the flow is zero \((r_2 = r_1)\) hence all of the energy transfer/pressure rise must be achieved by flow turning/diffusion. This turning is limited by how much of an adverse pressure gradient the boundary layer can tolerate before major separation (at which point the aeroacoustic performance deteriorates rapidly). The main design feature of centrifugal turbomachinery is that the centrifugal effect contributes \( \sim 50\% \) of the overall energy transfer/pressure rise, and this happens even with poor blade design. With a poorly-designed axial flow machine, major separation would mean the machine hardly functions at all.

Table 1.2 – Reynolds number magnitude comparison on design at the datum operating speed of 8200 RPM.

<table>
<thead>
<tr>
<th>Type</th>
<th>Characteristic length</th>
<th>Characteristic velocity</th>
<th>Value (datum operating speed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>Impeller shroud max diameter, ( D_{s2} = 2R_{s2} )</td>
<td>( \Omega D_{s2} )</td>
<td>( Re_g = \frac{\rho \Omega D_{s2}^2}{\mu} ) = 600,000</td>
</tr>
<tr>
<td>Local - impeller characteristics</td>
<td>Impeller mid-span streamwise blade chord, ( l ) (Figure 1.4)</td>
<td>Impeller inlet relative velocity, ( w_1 )</td>
<td>( Re_l = \frac{\rho w_1 l}{\mu} ) = 100,000</td>
</tr>
</tbody>
</table>

Compressor characteristics show how the design operating point relates to off-design conditions. In the Dyson desk fan product, the flow coefficient stays the same as the rotational speed is varied – large operating point deviations are not encountered during
operation. This means on-design (i.e. constant $\phi$) conditions are the primary interest in this thesis. In many other applications, such as the Dyson vacuum cleaners, the effective flow coefficient can vary significantly.

Table 1.3 – Design flow coefficient based on the global definition (§1.2.2.1) and the impeller inlet velocity triangle at mid-span (Figure 1.4b)

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition, $\phi$</th>
<th>Design value, $\phi_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>$\phi_g = \frac{Q}{\Omega D_s^2}$</td>
<td>0.0330</td>
</tr>
<tr>
<td>Local – impeller inlet</td>
<td>$\phi_l = \frac{c_m}{U} = \frac{V}{U}$</td>
<td>0.91</td>
</tr>
</tbody>
</table>

The experimentally-measured pressure rise coefficient curve for the production mixed-flow fan is shown in Figure 1.7 for the datum $Re_g$ in Table 1.2 (the measurement method is detailed in Chapter 2).

![Figure 1.7 – Production mixed-flow fan pressure-rise coefficient characteristic](image)

The characteristic is fairly straight - typical of centrifugal machines - as the outlet flow angles of the impeller/guide vanes remain quite constant as the flow rate is varied [13, Sec. Radial]
Turbomachinery Design]. Away from the design point at peak efficiency, the efficiency falls off in both directions resulting in the typical curvature of the characteristic away from the design point.

Experimental determination of the input compressor power $\dot{W}_c$ to find an aerodynamic efficiency $\eta_{aero}$ can be approached in several ways. One approach is to use the Euler equation (1.6) to relate the torque $T$ applied to the rotor to the increase in circumferential velocity across the rotor i.e. conservation of angular momentum. Then:

$$\dot{W}_c = T \Omega$$  \hspace{1cm} (1.7)

For each flow rate and speed $\Omega$, this requires a traverse at the rotor inlet (skipped and taken to be zero for a single stage) and outlet with a probe capable of measuring several velocity components with good resolution/accuracy. For large axial flow machines in which the flow is ordered and two-dimensional this is perfectly feasible, however with compact mixed-flow machines there are several complications:

- The flow is fully three-dimensional, and the annular flow passage after the impeller is narrow and inclined (see Figure 1.5a) which makes instrumentation difficult
- The spatial resolution of 3D velocity probes can be inadequate at this scale
- Friction on the hub and shroud causes local changes in angular momentum not accounted for in the Euler equation [6]

When interested in measuring a range of geometries and operating points, a more time-efficient approach is to use a torque sensor. In situ torque measurement requires a sensor to be integrated into the rig which does not load the shaft. Contactless sensors, such as those based on the magnetostrictive effect, are under development for automotive turbocharger applications [14]. For the very low torques encountered here ($\sim 0.01$ Nm) accurate measurement becomes difficult.
Since it is particularly easy to measure the input electrical power $\dot{W}_e$ to the motor controller, an overall efficiency “from plug to flow” can be defined as:

$$\eta = \frac{Q \Delta p_0}{\dot{W}_e} = \frac{Q \Delta p_0}{\dot{W}_c} = \eta_{\text{aero}} \eta_e$$

(1.8)

where $\eta_e < 1$ due to power losses between plug and the shaft. The overall efficiency is plotted in Figure 1.8 for the same $Re_g$ as in Figure 1.7. This efficiency includes the aerodynamic losses as well as losses in the motor and motor controller. At the datum rotational speed, the loss in the motor and controller (measured by $\eta_e$) is fairly constant as $\phi$ is varied by changing the flow rate. Therefore the changes in the overall efficiency along the characteristic in Figure 1.8 can be mainly attributed to aerodynamic efficiency changes. This is confirmed in Appendix A by characterising the motor with a dynamometer which generates a known range of torques (and output shaft powers) and correlating this to the input power $\dot{W}_e$.

The reduction in efficiency in Figure 1.8 due to aerodynamic losses is discussed in the next section.
Loss sources

In many applications such as gas turbines, efficiency is the most important parameter. For small appliances, the financial incentives to have high aerodynamic efficiency are not nearly as great. However, increased unsteadiness and disorder associated with poor aerodynamic performance can lead to much greater noise from the product.

There is a huge body of research on loss sources in axial flow compressors related to gas turbines. Denton notes in his comprehensive paper on loss mechanisms that many of the ideas developed for axial flow machines are applicable to machines with significant radial flow but the relative importance for each mechanism is different [15].

"Profile loss" is generated on the blade surfaces, away from the endwall secondary flows, due to the boundary layers, separation and dissipation in the wake behind a blade trailing edge. If the flow remains attached the profile loss is related to, and can be predicted with, dissipation/skin coefficients from 2D boundary layer theory. The loss scales with the surface velocity and so peaks in regions where velocities are highest i.e. on the suction surface. It also depends on the surface area which leads to a pitch-chord ratio trade-off. Increase in the pitch-chord leads to added loss from higher velocities, and subtracted loss from lower surface area. Optimal pitch-chord ratios for minimal profile loss can therefore be found [15]. If the flow separates there will be additional loss and unsteadiness from mixing after separation.

An impeller shroud, indicated in Figure 1.9, is used to prevent tip leakage from the pressure side to the suction side of the blades. However, a leakage path exists between the impeller inlet and outlet around the outside of the shroud. The flow through this path is likely to be highly unsteady as it is tightly bound between a stationary and rotating surface. The mixing of this flow with the main flow generates loss. Parasitic losses also occur due to friction between leakage and shroud outer surface, and on the impeller backplate [13, Sec. Radial Turbomachinery Design].
Secondary flows from the endwalls are of particular importance to aeroacoustic performance, and are even more complex in centrifugal/mixed-flow machines. It is known that the outlet from a centrifugal impeller is often highly non-uniform (spanwise and circumferentially) due to the so-called “jet-wake” nature of the flow [16], [17]. Dean attributed 9% total efficiency loss to the mixing of this “jet-wake” flow for a stage with 10:1 pressure ratio [18]. According to Zangeneh et al. [16], there are two main mechanisms for non-uniformity in shrouded centrifugal impellers. The first is secondary flows that move low-momentum fluids on the blade surface and endwall boundary layers. Secondly, the presence of boundary layer separation, often on the suction surface at the shroud. The development of secondary flow in the axial-radial bend of an impeller is controlled by a combination of Coriolis and curvature effects, leading to streamwise components of vorticity.

Numerical prediction using 3D CFD simulation is becoming more reliable however it is very challenging to simulate all the complex leaks, interactions and loss sources discussed here. The approach of solving the RANS equations with mixing planes between components and a suitable turbulence model is common. According to Casey [13, Sec. Radial Turbomachinery
Design] this approach is still not able to predict efficiency better than ± 2% for typical (high Re) applications and is notoriously weak at stall prediction.

**Effect of low Reynolds number**

There are two distinct ways in which Reynolds number could affect performance. Most directly, the dissipation/skin coefficients from 2D boundary layer theory are a function of Reynolds number so frictional losses increase at low Reynolds number as viscous forces are stronger. Casey [19] notes that the mechanisms leading to inefficiency in centrifugal compressors are poorly understood but found that those dependent on Re are frictional in nature at moderate Re. The approach taken by him and other authors [20] has been to develop empirical correlations for the overall efficiency reduction as a function of Reynolds number [19].

The second and less direct way is due to the ‘knock-on’ effect of boundary layer separation on the bulk flow and thus performance. In CFD simulations and experiments related to applications such as gas turbines, many simplifying assumptions are made to be able to handle the true complexity of the flow. For well-designed machines operating efficiently, the flow field is envisaged to consist of a steady bulk flow which ‘neatly’ follows the curvature of the blades, bordered by confined regions of higher complexity such as the boundary layers and secondary flows. The high Reynolds number is taken to imply that the predominant flow pattern is turbulent (i.e. minimal laminar flow) and the boundary layers are thin and resistant to separation (compared to a laminar boundary layer). Models for turbulence, transition and other complex interactions are widely used in CFD to achieve a reasonable balance between accuracy versus runtime. However, these simulations and models are matched to experimental data biased towards the high Re regime.

It is clear from the previous section that the influence of boundary layers is not restricted to surfaces in centrifugal machines where secondary flows are more prominent/less understood compared to axial machines. Secondary flows and separations are intrinsically linked to the boundary layer state, which can be highly varied and dependent on Re as discussed in §1.2.2. The interaction and mixing of these phenomena can cause significant non-uniformity, loss [16] and noise-inducing unsteadiness.
Design

In industry, including at Dyson, the preliminary design of mixed-flow turbomachinery is generally carried out using commercial packages. These utilise meanline, throughflow and blade-to-blade codes to predict and optimise performance. Empirical correlations for loss, deviation and blockage are built-in. Dyson use various tools by Concepts NREC which integrate with the 3D CFD tools for design refinement that they use such as ANSYS CFX. Some of the experimental validations for the empirical correlations have been published, for example the unified slip model for axial, radial, and mixed-flow impellers by Concepts NREC [21].

Casey notes that the final design optimisation of a radial compressor still requires a development programme on an experimental rig [13, Sec. Radial Turbomachinery Design]. Experimental testing of prototypes is routinely performed at Dyson using 3D printing techniques for producing components.

1.2.2.2 Acoustic considerations

The aeroacoustic noise sources within fans or compressors can be classified according to the frequencies at which they occur. In a typical, well-resolved spectrum, discrete-frequency tones appear superimposed on a continuous range of broadband frequencies. The nature and generation mechanisms for each class are different; tones are deterministic while broadband is random. This section gives an overview of the noise sources most important for low-speed turbomachinery and possible low Re effects. Reviews of the practicalities of measuring an aeroacoustic source, fan noise trends and fan noise prediction are left for sections 2.1, 3.1 and 4.1, respectively.

Tonal noise

Tonal noise at multiples of the shaft or blade passing frequency is generated as a result of the rotor spinning. The components of noise that would occur with no stationary obstructions in the vicinity of the rotor are termed rotor-alone tones. Additional noise is expected with
obstructions such as a stator or outlet guide vanes, and these are termed rotor-stator interaction tones.

*Rotor-alone tonal noise* can come from an isolated rotor with a steady pressure field attached to the blades spinning at the same rate as the rotor. However at subsonic speeds, the pressure fluctuations felt close to this rotor are not able to propagate away as sound waves. Rotor-alone tones can be generated at subsonic flow speeds by inlet flow distortions which give a circumferential non-uniformity that is steady in the stationary reference frame. This could be caused by, for example, small variations in the incoming boundary layer thickness on surfaces leading to the rotor or intake imperfections [22].

*Rotor-stator interaction tones* are generated by periodic impingement of rotor outflow asymmetry, e.g. blade wakes, on a downstream stationary blade row, and by the pressure field potential interaction when one steadily-loaded blade row rotates relative to another.

The flow field can be viewed as the sum of a mean (irrotational) component and a small-amplitude disturbance (“vortical gust”) which convects with the mean flow. Based on this view, Tyler and Sofrin [23] model the generation and propagation of interaction tones for axial flow machines i.e. constant radius. They consider a rotor alongside an array of identical obstructions which are equally spaced (such as a set of guide vanes) and prove the existence of spinning harmonic pressure patterns which can rotate much faster than the rotor and hence propagate along the duct. Analytically they show which duct modes are permissible with azimuthal dependence $e^{im\theta}$ at multiples of the blade passing frequency $NB\Omega$ for a rotor in close proximity to $S$ obstructions [23]. The value $m$ is restricted to:

$$m = NB + kS, \quad k = \cdots, -1, 0, 1, \cdots$$ (1.9)

which leads to a pressure pattern that spins at $NB\Omega/m$ radians per second. Typically a large value of $m$ and hence a subsonic pattern rotation rate is ensured for the lower harmonics of BPF (low $N$) by choosing the number of vanes $S$ to be more than twice the number of blades $B$. Furthermore, the plane-wave mode ($m = 0$) is not excited. This is the case for the test case fan with $B = 9, S = 22$. Physically a reduction in tonal noise is achieved with certain combinations of blade/vane numbers from destructive interference between the acoustic
1.2 Aeroacoustics of low-speed fans

fields of neighbouring blades. Peake et al. have shown that relaxing the assumption of a constant radius duct can significantly alter the resultant sound field [24], [25].

**Broadband noise**

Turbomachinery produces random, broadband noise associated with the random velocity fluctuations from turbulent eddies created/dissipated in the fluid at a range of scales. As mentioned in §1.2.2, viscous forces dissipate turbulent kinetic energy and the importance of viscosity is gauged by Reynolds number. For this fundamental reason and a number of associated reasons discussed later, low Re is expected to influence the broadband noise produced and lead to differences compared to the well-researched, high Re regime.

When considering the (reduction of) overall sound power, broadband noise generally contributes significantly more than tones because the integral over the frequency band of a tone to give its power contribution is relatively small. Due to the randomness associated with broadband noise it is difficult to contrive a situation where sources from different locations “cancel each other out” as for tonal noise.

*Rotor-alone broadband noise* can come from sources which occur when a viscous fluid flows over a surface, and by the interaction of blades with a turbulent stream. This leads to the concept of the minimum noise that an ideal spinning rotor would produce assuming steady conditions at the inlet and outlet, first suggested by Wright [26]. Blade self-noise, broadband in nature, is *always* generated at the trailing edge of blades due to the presence of a turbulent boundary layer which creates an unsteady loading on the blades and scattering [27]. Moreau et al. have recently shown that the flat plate boundary layer state at low-to-moderate Re, which varies greatly in that range, has a large impact on trailing-edge noise [28]. Another Re-dependent source which can be categorised as blade self-noise is when the boundary layer separates and becomes unstable, unsteady and noisy.

Similar to the equivalent tonal source, broadband rotor-alone noise can also be created by inflow distortion. This could come from detailed casing boundary layer unsteadiness or inlet free-stream turbulence. If the time scale associated with turbulent eddies being ingested by a rotor is much shorter than the blade-passing period, each blade experiences uncorrelated,
random velocity fluctuations from passing eddies. Short-term, small-scale variations in the blade loading then leads to broadband noise [22].

**Rotor-stator broadband interaction noise** is well-researched due to its significant contribution to the noise of modern turbofan engines [29]. Similarly to tonal interaction, it is caused by the impingement of rotor outflow asymmetry on a downstream blade row or by potential interaction. Unlike the deterministic tonal source, rotor outflow turbulence (from blade wakes, separations, boundary layers etc.) produces random fluctuations in loading as it impinges on the blade row and hence broadband noise.

**Narrow-band, random noise**

In reality, a ‘middle ground’ exists between the two extremes of (harmonic) tonal and (random) broadband sources which can be described as narrow-band, random noise [30]. This is a type of rotor-alone noise that can occur when turbulence becomes distorted, for example by a strong streamtube contraction, such that eddies are ‘stretched’ in the streamwise direction and the forces on adjacent blades become correlated. The width/height of such a narrow-band peak depends on how well-correlated the blade-to-blade events are – an infinitely long turbulent eddy would produce a high, discrete tonal spike while real eddies produce lower peaks with non-zero width. Paterson and Amiet note that it is therefore the area under each peak which is of most interest [31].

### 1.3 Noise annoyance

The loudness of sound is a subjective measure which depends on the response of the listener [32]. Human hearing has evolved to be more sensitive to some frequencies than others. The annoyance caused by noise is quantified using various subjective decibel scales. The most common method involves weighting the pressure level in frequency bands by a factor linked to hearing sensitivity in that range. The international standard ‘A-weighting’ curve, shown in Figure 1.10, is used by Dyson and many other industries. It highlights the high sensitivity in the range 1 to 5 kHz.
There are several limitations in relying on this to quantify fan noise annoyance for a source which contains pure tones superimposed on broadband over a range of frequencies. The ‘A-weighting’ curve is derived [33], and only strictly valid, for pure single tones. The judged noisiness [34] and annoyance [32] of sound containing a whine or audible tones is substantially greater than the judged noisiness of just random noise. To this end, tonal noise and its influence on the sound quality is a key interest of Dyson [5], and therefore of this thesis.

1.4 Aims and objectives

In order to achieve a silent fan, one first needs to identify the main sources of noise. There is very little experimental data on the main sources for fans of this type - most previous work is based on assumptions applicable to gas turbines. One such example of this would be that boundary layers are taken to be turbulent, thin and attached because Re is high. In particular, there is limited previous research into the effect of low Re on localised phenomena that can produce noise and inefficiency. There are also few publications on the aerodynamics or aeroacoustics of mixed-flow fans. Significantly more is known about purely axial and
centrifugal flow compressor behaviours – aspects from both of these being important – however it is not always clear which behaviour is most relevant to mixed-flow fans.

There is a lack of accurate measurement techniques needed to quantify the noise sources of small-scale, low-speed fans. Therefore to measure noise, a modular aeroacoustic characterisation rig will be developed which can measure narrow (i.e. tones) and broadband sources accurately at a range of operating points.

A parametric study of the noise sources in the mixed-flow fan will then be undertaken. This will identify which mechanisms are dominating the acoustic signature and indicate which components should be optimised for low noise. A range of test fans will be built from both production and 3D printed parts.

A simple model for predicting the noise from the mixed-flow fan would be an extremely useful design tool. Low-order modelling of the sources will be undertaken to link noise trends to a physical understanding of the generation mechanisms. This will involve prediction of the unsteady blade loading which produces noise, as this is the basis of most of the mechanisms discussed in §1.2.2.2. Model predictions will be compared directly to experimental measurements for a range of different Reynolds numbers.

The work in this thesis will contain several key aspects which are novel. The amenable measurement technique will be extended relative to existing methods, and optimised to characterise an aeroacoustic source with tonal and broadband components. A systematic study of the noise sources in a realistic mixed-flow configuration has not been carried out previously – with reference to the fundamental effects of low Mach/Re number. Low-order modelling has not previously been applied to this configuration in this regime.

1.5 Approach

Although fan aerodynamic efficiency is important for small domestic products, annoyance from noise can make the product unusable. Reduction at the source is considered the most efficient way to find ways to reduce fan noise. Due to their disproportionate impact on sound quality compared to broadband, high priority will be given to tonal fan noise sources. A
1.6 Road map to the rest of the thesis

primarily experimental approach is taken when quantifying noise sources as this is not limited by the assumptions of numerical methods. The size and mechanical simplicity of the fans being considered means that it is convenient to rapid-prototype full-scale test cases.

1.6 Road map to the rest of the thesis

In Chapter 2 the new aeroacoustic characterisation rig design is outlined alongside the acoustic measurement technique. This rig is optimised for the measurement of tones, and comparisons are made with a rig at Dyson which is designed to measure sound power in third-octave bands to understand when each method is most appropriate. A parametric study of tonal and broadband sources for the mixed-flow test case is performed in Chapter 3. This sheds light on which components or interactions between components are dominating the noise spectra. In Chapter 4 low-order modelling is performed using a cascade response model to predict unsteady blade loading and noise from a given input excitation. The effect of Reynolds number on the excitation is explored by hotwire measurements at the same operating point but different Re. The conclusions from each chapter are drawn together in Chapter 5, and the possible ways in which these can be utilised or further explored are suggested.
Chapter 2 Fan aeroacoustic characterisation methods

2.1 Introduction

This chapter describes the design and implementation of a new experimental rig suitable for acoustical characterisation of a fan unit - a source of flow and sound. The rig is designed to allow for flow measurements linked to both aerodynamic performance and acoustic measurements simultaneously as most of the noise is aerodynamically-generated [5] (and therefore a function of operating point). The flow field pattern of a low-speed turbomachine can be very diverse when operating off-design due to phenomena such as boundary-layer separation. The intrinsic link between hydrodynamic and acoustic fluctuations means that off-design operation leads to significant changes in both the tonal and broadband components of the acoustic signature. Since trends in both of these components are of interest, the rig must give accurate control over a range of operating point settings, and the acoustic measurement technique should be practical, and give insight into generation mechanisms.

There is a large body of literature on fan noise measurement and characterisation. In terms of gas turbine-related research, Tyler & Sofrin’s seminal paper on compressor tonal rotor-stator interaction noise [23] links blade/vane numbers to the azimuthal order of the duct modes generated. They validate their theory by azimuthally traversing a microphone at some fixed radius – the microphone being inserted on a cantilever at the open inlet of the rotor. Individual azimuthal modes are identified by effectively inducing a Doppler shift between the traversing microphone and the azimuthal mode of interest which is spinning at a unique rate. Measurements based on this concept have been performed widely, including by NASA and the DLR. In order to speed up the measurement process, a rake of microphones has been used to measure radial and azimuthal modes simultaneously. The system developed at NASA
Glenn in the 1990s [35] has been used in a variety of turbofan tests from low-speed concept rigs to full-scale production engines (see Figure 2.1).

Figure 2.1 – NASA’s (a) rotating inlet rake ring assembly on a concept rig in the wing tunnel (b) inlet (top) and exhaust (bottom) duct rake on a test stand for a full-scale Honeywell turbofan engine. Pictures from [35]

DLR researchers have used a rotating rake system to measure a variety of axial-flow components [36], for example a three-stage low-pressure turbine [37], and a single-stage low speed fan rig outlet [38].

There are several potential disadvantages of using an in-flow measurement rake:

- The rotation mechanism must be robust and allow measurements which are accurate in terms of spatial location and unaffected by vibration
- Inlet measurements can produce additional noise sources from interaction (i.e. an obstruction like \( S = 1 \) can excite all modes) which may be very prominent in low-speed applications
- Outlet measurements with the pressure transducers facing the oncoming flow are most likely to be contaminated by flow noise
- Anechoic terminations are often required at the duct ends. These eliminate reflections and prevent additional reflection at/transmission through the fan which would be measured along with the waves coming directly from the source.

Furthermore, the difference between gas turbine applications and the small-scale regime of interest in the present thesis means that this kind of technique is less applicable:

- The scale is much smaller so the duct diameter is lower and fewer modes are cut-on
- The frequencies of interest are lower because rotational speeds and velocities are lower
- The lower harmonics of BPF are the most significant
- Amplitudes are much lower so more sensitive transducers are required which are generally larger. Consequently a measurement rake may have a greater disturbing effect

The International Organization for Standardization (ISO) has developed standard 5136 for the “Determination of sound power radiated into a duct by fans and other air-moving devices” [39]. This method is widely used in academia and industry [5], [40]–[42] to characterise fans, in particular for HVAC applications. The aeroacoustic rig, used extensively by Dyson, is built to this acoustic standard and the associated standard for measuring overall fan performance [43].
The set-up, pictured in Figure 2.2, consists of a long, straight duct with the test case located in the middle. Anechoic terminations at each end serve several purposes. Principally they stop reflections of the sound power emitted by the source which would lead to axial standing waves. They also minimise extraneous noise from the surrounding environment, the inlet orifice plate and the outlet throttle which controls the fan operating point. A single microphone is positioned in the flow on the inlet and outlet sides of the fan. To account for the variation in sound pressure/intensity across the duct section, the standard relies on the theory that there is a radial position which will give sound pressure readings representative of the whole section. A simplified higher-order mode model gives this optimum radial position as midway between the duct axis and wall [44]. The model assumes the first 10 cut-on modes within each band carry equal power. Since the measurements may be influenced by flow noise, particularly at the outlet where the microphone points into the flow, there are requirements on using flow conditioners and specialised microphone shields depending on the outlet velocity magnitude and direction (i.e. swirl).

The existing ISO standard rig set-up has several advantages and disadvantages. The main advantage of the method (for industry in general) is that it allows quick, standardised measurements of overall sound power level based on third-octave bands. However, the suitability of the rig to study the narrow-band tones of interest in this work is still to be
determined. A narrow-band tone superimposed on a broadband spectrum of frequencies can contribute little to the band power. Tyler & Sofrin’s theory for tonal interaction noise shows that specific azimuthal modes are generated depending on rotor/stator numbers – one expects the excited azimuthal modes to carry more power than the others - so the equal mode power model does not apply. The in-duct method is derived and then validated against [45] third-octave measurements made according to the (standardised) free-field SPL survey method, ISO 3746 [46]. Narrow-band spectra are rarely analysed and compared in any depth as this is not what the method is designed for. On one occasion the standard authors do present a narrow-band comparison of averaged spectra, reproduced in Figure 2.3. Significant deviations between the methods are seen between 270 Hz and 402 Hz which correspond to the cut-on frequencies for higher-order modes in the fan annulus and measurement duct, respectively. Smaller deviations are seen at other shaft harmonics.

![Figure 2.3 – Narrow-band sound pressure spectra on the outlet side of an axial fan using in-duct (solid line) and free-field (dash line) ISO methods. The free-field spectrum is averaged over the measurement surface and corrected for the area difference with the duct (reproduced from [45])](image)

A hindrance of the ISO method is that there are many aspects of the rig specification and measurement procedure which are given without detailed explanation. This makes it difficult to understand the rig’s full capabilities or limitations. The approach in this work is to design a new facility for measuring broad and narrow-band noise emitted by fan openings. This
measurement capability is independent from Dyson’s facilities and is developed based on a different acoustic method. Comparisons can therefore be made between the two rigs to increase confidence that the fan is being characterised accurately.

The new rig is designed to be short enough to fit into the anechoic chamber in Cambridge and by far the most expensive components are the microphones. Mode decomposition of the underlying sound field isolates reflections in the duct and removes the need for anechoic terminations (which increase the length of the rig and are not effective at all frequencies). On each side of the fan, at least one independent sound field measurement per mode is required to find its amplitude. These measurements are stationary and located on the duct surface which causes minimal disturbance or flow noise contamination. The well-known ‘two-port’ method [47] is extended to include characterisation of the transmissive and reflective properties of the fan, including higher-order modes. This mode-based appraisal of the sound field and source has several advantages over the in-duct approach, including:

- Passive and active properties of the source are quantified
- Equally-accurate for tones and broadband in principle
- No reliance on the assumption of the equal power mode model
- Comparison of in-duct measurements against a different method

The next section of this chapter gives further details on the rig at Dyson which is used for measurements in other chapters. The design of the new rig is then presented in §2.3. This rig implements the acoustic mode decomposition and two-port methods outlined in sections 2.4 and 2.6. Comparisons are made between the measurements taken in both rigs to investigate the significance of the points outlined above.

### 2.2 Dyson ISO rig

The main components of the Dyson ISO rig are shown in a schematic in Figure 2.4.
Air enters the rig via the inlet orifice plate which is geometrically similar to the ISO specification [43]. In accordance with the standard, the volume flow rate $Q$ is calculated based on the pressure difference $\Delta p_{op}$ across the orifice:

$$Q = \alpha \varepsilon \frac{\pi D_{op}^2}{4} \sqrt{2 \rho_a \Delta p_{op}}$$

(2.1)

where $D_{op}$ is the orifice diameter, $\alpha$ is a flow rate coefficient and $\varepsilon$ is an expansibility factor given as a function of $\Delta p_{op}/(p_a - \Delta p_{op})$. The coefficient $\alpha = 0.609$ is found by calibration against a reference Venturi flow meter in accordance with ISO 5167 [48] over a range of typical flow rates with an uncertainty of 0.53%. The minimum valid Reynolds number (based on conditions at the restriction) for the reference flow meter is 5000 which corresponds to a flow rate of $\sim$3 litres per second. At the lowest rotational speed (3000 RPM) this is equivalent to a global flow coefficient of $\sim$0.01.

The orifice pressure difference $\Delta p_{op}$ is measured using a Furness Controls FCO318 manometer with an uncertainty $\leq \pm 0.5\%$ of reading. The static pressure increase across the fan is measured between axial stations upstream of the fan and downstream of the flow straightener. This is taken to equal the fan stagnation pressure increase $\Delta p_0$ since the areas are the same and the flow is fully-developed. This increase is measured using a second Furness Controls FCO318 manometer with an uncertainty $\leq \pm 0.5\%$ of reading. At each axial station, the pressure is determined from a pneumatic average of four surface pressure taps equally-spaced around the circumference.

The operating point is controlled using a throttle located downstream of the outlet anechoic termination to prevent contamination of the noise measurements (pictured in Figure 2.5a).
Fan aeroacoustic characterisation methods

Figure 2.5 – Dyson ISO rig (a) outlet throttle (b) in-flow microphone arrangement

The microphones are located some distance upstream and downstream to allow evanescent modes to become attenuated. The microphones used are B&K 4189 ½-inch free-field microphones with a dynamic range of 15 - 146 dB. Each is fitted with a nose cone shield to minimise flow noise effects on the measurement (see Figure 2.5b). According to the standard, a sampling tube would be required for mean flow velocities above 20 m/s [7] – the level encountered in the current context is around 2 m/s. The strut, which holds the microphone midway between the duct axis and wall, has a streamlined shape to minimise disturbance. The flow straightener at the outlet of the fan removes swirl which would otherwise affect the stagnation pressure increase and microphone readings.

The inner diameter of the rig is 0.14m and the length is ~14 m. The considerable length of the rig is mainly due to the required spaces between components and the microphones, and to a lesser extent, the length of the anechoic terminations (~2 m each). The large spacing from the
2.3 New rig design

In this section an overview of the new rig (Figure 2.6) is presented. The main components are pictured in Figure 2.7. Key dimensions of both rigs are given in §2.4 while details of the acoustic methods are shown in the remaining sections of the chapter.

A bellmouth inlet flow meter is used to measure the flow rate as it produces less noise and unsteadiness compared to an orifice plate. This is made to the ISO standard geometry [39] with the throat diameter $D_{bm}$ chosen to give a Reynolds number (based on this diameter; given in Table 2.1) greater than 20,000 at the flow rates of interest. In accordance with the standard, the volume flow rate $Q$ is found without calibration from:

$$Q = \frac{\pi D_{bm}^2}{4} \sqrt{2\rho_a \Delta p_{bm}}$$

where $\Delta p_{bm}$ is the pneumatically-averaged pressure difference across the bellmouth - measured using a Betz micromanometer with an uncertainty of $\pm 1$ Pa.

The fan stagnation pressure increase $\Delta p_0$, based on surface pressure taps before the fan and after the straightener, is found in the same manner as in §2.2 using a Digitron 2001P
manometer with an uncertainty ≈0.1% of full-scale. Flow conditioning devices are used downstream of the bellmouth and the fan duct to remove non-uniformity. The operating point of the fan is controlled by using an iris valve at the outlet to throttle the flow, similar to an orifice plate of variable area. The iris valve is connected to a stepper motor to obtain precise throttle settings.

Acoustic measurements are made upstream and downstream of the fan using two arrays of three, flush-mounted microphones on each side. A seventh, so-called verification, microphone is also mounted on each side, 10 cm in the axial direction away from the first ring of microphones. The microphones used are G.R.A.S. 40DD ⅛-inch pressure microphones with a dynamic range of 40 - 175 dB. Signals are acquired using a GBM Viper multichannel system capable of simultaneous data acquisition, signal conditioning for optimal ADC resolution and anti-aliasing filtering, and sensor power supply. The microphone arrangement is explained in more detail in §2.4.

As described in §2.6, the transmissive and reflective properties of the fan are determined using external loudspeaker sources. These are housed in speaker boxes (see Figure 2.7), each containing three speakers, which emit sound into the duct surface through perforations.
Figure 2.7 – Picture of the new aeroacoustic (a) whole rig and key components: (b) bellmouth inlet (c) microphone array with three (equally-spaced) microphones per axial location and seventh verification microphone (d) honeycomb flow straightener (e) iris valve outlet throttle

### 2.4 Test conditions

The same fan is measured in both rigs in datum operation (see Table 1.3 and Table 1.2). The average Mach number of the mean flow in the duct at this operating point is ~0.02. A comparison of the characteristics is shown in Figure 1.7. The uncertainty with which nominally identical operating points can be set in each rig is shown in Appendix D. A comparison of the rig set-ups is shown in Table 2.1.
Table 2.1 – Test set-up details

<table>
<thead>
<tr>
<th></th>
<th>Dyson ISO rig</th>
<th>New rig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duct inner diameter, ( R_d ) (m)</td>
<td>0.14</td>
<td>0.145</td>
</tr>
<tr>
<td>Rig length (m)</td>
<td>14</td>
<td>2.5</td>
</tr>
<tr>
<td>Flowmeter throat diameter, ( D_{fm} ) (m)</td>
<td>0.058</td>
<td>0.050</td>
</tr>
<tr>
<td>Motor control</td>
<td>Variable power, constant RPM ± 0.1%</td>
<td></td>
</tr>
</tbody>
</table>

2.5 Mode decomposition

Broadband mode decomposition techniques based on auto- and cross-spectra between stationary microphones date back to the work of Seybert and Ross [49]. In their paper they determine an alternative to the impedance tube method for measuring acoustic impedance. Broadband excitation produces incident and reflected plane-waves; the amplitudes of each are related to the spectra measured at two flush-mounted microphones.

Above a well-defined frequency, the contribution from non-plane-wave modes becomes significant. Mode decomposition of higher-order modes has been performed experimentally for a square-section duct which approximates the thin annulus of a high bypass-ratio engine [50]. For a circular-section duct, Lavrentjev et al. include higher-order modes in their analysis of the fan inlet noise spectrum [51].

The mode decomposition theory is presented next in a new compact form amenable to optimisation of the locations of the measurements. Subsequently a novel location optimisation procedure is implemented which gives the set-up shown in §2.3.

Within a duct of circular cross-section of radius \( R_d \) with no mean flow (for simplicity), the acoustic pressure \( p \) satisfies the wave equation:

\[
\frac{1}{a_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0
\]  

(2.3)
2.5 Mode decomposition

By considering a solution with separable functions in cylindrical coordinates \((r, x, \theta)\) and applying appropriate boundary conditions, the acoustic pressure due to perturbations of frequency \(\omega\) is:

\[
p(r, x, \theta, t) = C_{m,n} e^{i(\omega t + m\theta)} J_m \left( z_{m,n} \frac{r}{R_d} \right) \left( A_{m,n}^+ e^{-ik_{m,n}x} + A_{m,n}^- e^{ik_{m,n}x} \right)
\]

where \((m, n)\) are integers that represent azimuthal and radial mode number respectively, Bessel functions of order \(m\) are denoted \(J_m\) and the zeros of its derivative \(z_{m,n}\), and \(A_{m,n}^\pm\) are the wave amplitudes of mode \((m, n)\) travelling in the positive, and negative \(x\) direction. The axial wavenumber \(k_{m,n}\) is given by:

\[
k_{m,n} = \sqrt{k_0^2 - \frac{z_{m,n}^2}{R_d^2}} = \sqrt{\frac{\omega^2}{a_0^2} - \frac{z_{m,n}^2}{R_d^2}}
\]

The normalisation factor \(C_{m,n}\) [52, Sec. 9.2] is set such that:

\[
C_{m,n}^2 \iint f_m^2 \left( z_{m,n} \frac{r}{R_d} \right) dS = S
\]

to make the squared amplitude magnitude proportional to the mode power once the function is integrated over the cross-section (when finding the power). This expression can be simplified (see Appendix B) by performing the integration over a duct cross-section to give:

\[
C_{m,n} = \frac{z_{m,n}}{J_m(z_{m,n}) \sqrt{(z_{m,n}^2 - m^2)}}
\]

Assuming a uniform flow in the positive \(x\) direction gives a similar result except that the axial wavenumber now depends on \(M\) (the mean-flow Mach number) and on the direction of wave propagation, either with (+) or against (-) the mean-flow:

\[
k_{m,n}^\pm = \sqrt{\frac{k_0^2 - (1 - M^2) \frac{z_{m,n}^2}{R_d^2} \mp Mk_0}{1 - M^2}}
\]
Since the mean-flow Mach number in the duct is so low, typically ~0.02, its effect on wave propagation can be neglected so the axial wavenumber equation (2.5) is used throughout instead of (2.8). For frequencies below the cut-on frequency of a given mode, the axial wavenumber is purely imaginary and the amplitude of the wave decays exponentially along the duct axis. Conversely, above cut-on the wave can propagate without decay.

Taking the Fourier transform in time of (2.4) and summing over all possible modes gives:

\[
\hat{p}(r, x, \theta, \omega) = \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} C_{m,n} e^{im\theta} J_m \left( z_{m,n} \frac{r}{R_d} \right) \left( A_{m,n}^+ (\omega) e^{-ik_{m,n}x} + A_{m,n}^- (\omega) e^{ik_{m,n}x} \right)
\]

(2.9)

In the present work the three modes (0,0) & (±1,0) are measured experimentally. However, the scheme can be extended in a straightforward manner to additional modes.

From equation (2.9), the frequency-domain summation over the first three cut-on modes at a measurement location \((R_d, x_i, \theta_i)\) on the duct surface is given by:

\[
\hat{p}_i(R_d, x_i, \theta_i, \omega) = \sum_{m=-1}^{m=1} C_{m,0} e^{im\theta_i} J_m (z_{m,0}) \left( A_{m,0}^+ (\omega) e^{-ik_{m,0}x_i} + A_{m,0}^- (\omega) e^{ik_{m,0}x_i} \right)
\]

(2.10)

For each mode, there are two unknown amplitudes \(A_{m,n}^+\) and \(A_{m,n}^-\). This implies that a minimum of two independent measurements are required, giving two equations, for each mode present. For the modes of interest, there are two unknown amplitudes for each of \(m = -1, 0, 1\) giving a minimum of six independent measurements (\(N\) modes implies \(2N\) unknowns). Denoting the six measurements required \(\hat{p}_I, \hat{p}_{II}, ..., \hat{p}_{VI}\), the system of six simultaneous equations can be written in matrix form as:
This set of simultaneous equations is solved at each frequency $\omega$ for the unknown amplitudes:

$$\{A^\pm\} = M^{-1}\{p\} \quad (2.12)$$

Note that equation (2.12) works only for deterministic signals. For a signal with noise, average statistical quantities should be used. The formulation is therefore recast in terms of quantities to be averaged over a significant measurement period, valid for both periodic and random signals. Dividing equation (2.11) by a coherent reference pressure (or voltage) signal $r$ and using the inverse of $M$:

$$\begin{bmatrix}
\hat{A}_{0,0}^+ & \cdot & \cdot & \cdots & \hat{A}_{0,0}^- \\
\hat{A}_r & \cdot & \cdot & \cdots & \hat{A}_r
\end{bmatrix} = M^{-1}\begin{bmatrix}
\hat{H}_rI_l \\
\vdots \\
\hat{H}_rV_l
\end{bmatrix} \quad (2.13)$$

where for $i = I, II, ..., Vl$

$$\frac{\hat{p}}{\hat{r}} = \frac{\hat{p}}{\hat{r}} \frac{\hat{r}^*}{\hat{r}^*} = \frac{S_{ir}}{S_{rr}} = \hat{H}_{ri} \quad (2.14)$$

in which $\hat{S}_{ir}$ and $\hat{S}_{rr}$ represent cross- and auto-spectra, respectively. This approach works only when the coherence is good between the pairs of signals [53].

The matrix $M$ is singular or ill-conditioned if the six measurements are not fully independent. The approach of equation (2.12) differs from that of other authors [51] as $M$ and $A^\pm$ contain terms corresponding to waves travelling in both directions, as opposed to separate matrices for each direction. This makes the decomposition scheme straightforward and amenable for
optimisation. Surplus measurements can be used to form additional entries in the $p$ vector and matrix $M$ to give an overdetermined system of equations to improve mode decomposition results [54]. However, the minimum number of measurement locations (in this case six) is optimised here, as published by Newman et al. [55].

In order to optimise the locations of the six pressure measurements over a wide frequency range, an optimisation scheme has been developed in MATLAB. This tests every possible combination of azimuthal angles with six measurement locations equally split between planes at two axial locations $x_A, x_B$ (for ease of construction). The aim of the optimisation is to find the azimuthal configuration which minimises the condition number $\kappa[M]$ at the high end of the frequency range of interest (2 kHz). The condition number quantifies how independent the six measurements are; a lower condition number implies a higher independence of the measurement set. During the first phase of the optimisation, the axial spacing between the planes is fixed i.e.

$$x_i = \begin{cases} x_A, & i = I - III \\ x_B, & i = IV - VI \end{cases} \quad (2.15)$$

In this case the optimisation process for the azimuthal locations can be represented mathematically as:

$$\min_{\theta_i \in [0, 2\pi]} \kappa[M(x_B - x_A, \theta_i, \omega)] \quad (2.16)$$

assuming that the ideal axial spacing would be close to the theoretical optimum for the plane-wave only case corresponding to a quarter of the wavelength [56] i.e. $x_B - x_A = \lambda_{2kHz}/4$. Subsequently, this assumption is relaxed by fixing the ideal azimuthal distribution and then varying the axial spacing between zero and half wavelength spacing:

$$\min_{x_B - x_A \in [0, \lambda_{2kHz}/2]} \kappa[M(x_B - x_A, \theta_i, \omega)] \quad (2.17)$$

The ideal azimuthal configuration is found to be equal angular spacing. However the optimum axial spacing is found to be higher than the quarter wavelength ideal for the plane-wave region (see Figure 2.8 (a)). The condition number for this ideal arrangement at lower frequencies is shown in Figure 2.8 (b).
Figure 2.8 – (a) Variation of condition number $\kappa$ of modal matrix $M$ with axial spacing normalised by wavelength at 2 kHz, minimum indicated (b) Variation of condition number $\kappa$ for the ideal axial and azimuthal arrangement at the range of frequencies of interest

The peak in condition number visible in Figure 2.8(b) around 1.3 kHz corresponds to the cut-off frequency for the first azimuthal mode and is due to the axial wavenumber being close to zero. There is a frequency at which the matrix is theoretically singular (and the condition number infinite) however this hasn’t caused problems in practice. The localised peak condition number in the figure is not significant enough to lead to deviations in the predictions, as shown later.

The theory and optimised microphone set-up (Figure 2.7c) requires experimental verification. On each side a seventh microphone is positioned at some arbitrary far-field location $(R_d, x_{VII}, \theta_{VII})$. The spectrum is calculable based on the modal amplitudes from the six-microphone array:

$$\hat{S}_{VII} = |\hat{p}_{VII}|^2 = \left|\frac{\hat{p}_{VII}}{\hat{\bar{p}}}\right|^2 \hat{S}_{rr}$$  \hspace{1cm} (2.18)
where $p_{VII}$ is given by (2.10) as a function of location, mode amplitudes and reference signal $\hat{r}$. If the mode amplitudes are correct this should match the actual measurement at the verification location.

Initial tests carried out using a loudspeaker excited with random noise instead of a fan show excellent agreement at the verification location. The first microphone is used as the reference signal in (2.18). When measuring the fan, a problem of higher flow noise on the outlet side due to flow unsteadiness is alleviated with the flow straightener in place – its effect on the outlet source power is discussed in §2.6.1.3. A comparison of predicted (equation (2.18)) and actual measurements on both sides of the production fan in datum operation (§2.4) is shown in Figure 2.9. On the inlet side, the agreement (for tones and broadband) is excellent apart from at amplitudes below the lower end of the dynamic range of the microphones (40 dB). Furthermore, the importance of including non-plane-wave modes is highlighted as the plane-wave only prediction deviates massively from the measured value above 1.3 kHz. The agreement on the outlet side is even better as the amplitudes are higher.

![Figure 2.9 - Comparison between predictions (incl. all cut-on modes) and actual measurements for fan at the verification location on the (a) inlet side with prediction assuming only plane-waves (p-w) for reference (b) outlet side (plane-wave prediction omitted)](image-url)
The first significant tone above the plane-wave frequencies occurs at around 1.7 kHz. This frequency is investigated in detail in §2.6, including a comparison to the level measured in the Dyson ISO rig in §2.6.2.

2.5.1 Power spectrum and power spectral density

The solution equation (2.14) is based on cross- and auto-spectra of signals, for example for one of the microphones:

\[ \hat{S}_{i,j} = \hat{p}_i \hat{p}_j \]  

(2.19)

Depending on the frequency of interest, the quantity \( \hat{S} \) is substituted for the (cross) power spectrum or power spectral density using Welch’s method of averaging over many (overlapping) periodograms [57] in MATLAB.

The power spectral density \( (A^2/\Delta f) \) with units of dB/Hz is used for broadband frequencies where the amplitude is corrected by dividing by a factor to account for the power of the windowing function and the frequency bin \( \Delta f \). For specific tones, the power spectrum \( (A^2) \) with units of dB is used whereby the amplitude is weighted to give the correct mean-square level by using the corresponding correction for window power.

2.6 Two port source analysis with higher-order modes

Any linear source such as a fan within a duct with openings or “ports” can be modelled as a two-port source using a system of equations which relate its input and output states. This situation is illustrated in Figure 2.10 for a given mode. The so-called “scattering matrix”
formulation relates the pressure wave amplitudes on the inlet and outlet sides of the source, and was introduced by Davies [58]. It allows the waves due to the source alone (denoted by superscript ‘s’ in the figure) to be discerned from the total wave amplitudes travelling each way on either side of the source (denoted \( A_1^\pm & A_2^\pm \)). At lower frequencies, only the plane-wave mode can propagate on each side of the source which results in 2 simultaneous equations:

\[
\begin{bmatrix}
A_1^{+0,0} \\
A_2^{+0,0}
\end{bmatrix}
= 
\begin{bmatrix}
\rho_1 & \tau 21 \\
\tau 12 & \rho_2
\end{bmatrix}
\begin{bmatrix}
A_1^{-0,0} \\
A_2^{-0,0}
\end{bmatrix}
+ 
\begin{bmatrix}
A_1^{s0,0} \\
A_2^{s0,0}
\end{bmatrix}
\]

(2.20)

where \( S \) is the scattering matrix, \( A^s \) is a source vector and the sign convention for the wave directions of travel is shown in Figure 2.10. This scattering matrix contains four parameters characterising the reflective (\( \rho \)) and transmissive (\( \tau \)) properties of the source duct element. Extension to the first three modes gives 6 simultaneous equations and 36 unknowns in the scattering matrix:

\[
\begin{bmatrix}
A_1^{+0,0} \\
A_1^{+1,0} \\
A_1^{+1,-1,0} \\
A_2^{+0,0} \\
A_2^{+1,0} \\
A_2^{+1,-1,0}
\end{bmatrix}
= 
\begin{bmatrix}
\rho_{1,0,0-0,0} & \cdots & \tau 21_{-1,0-0,0} \\
\vdots & \ddots & \vdots \\
\tau 12_{0,0-1,0} & \cdots & \rho 2_{-1,0-1,0}
\end{bmatrix}
\begin{bmatrix}
A_1^{-0,0} \\
A_1^{-1,0} \\
A_1^{-1,-1,0} \\
A_2^{-0,0} \\
A_2^{-1,0} \\
A_2^{-1,-1,0}
\end{bmatrix}
+ 
\begin{bmatrix}
A_1^{s0,0} \\
A_1^{s1,0} \\
A_1^{s1,-1,0} \\
A_2^{s0,0} \\
A_2^{s1,0} \\
A_2^{s1,-1,0}
\end{bmatrix}
\]

(2.21)

where the terms have a degree of symmetry about the leading diagonal coefficients (corresponding to a proportion of reflection without a change in the mode number). For example, the coefficient \( \tau 12_{0,0-1,0} \) corresponds to transmission of a wave from side 1 to 2 in which the mode number changes (from 0 to -1). Dividing through by a coherent reference signal \( \hat{r} \) gives a form where the amplitudes follow from mode decomposition on sides 1 and 2 using (2.13), valid for random signals:
Two port source analysis with higher-order modes

\[
\begin{bmatrix}
A_{1,0,0}^+ \\
\vdots \\
A_{2,-1,0}^+
\end{bmatrix}
\hat{\rho} =
\begin{bmatrix}
A_{1,0,0}^- \\
\vdots \\
A_{2,-1,0}^-
\end{bmatrix}
\hat{\rho} +
\begin{bmatrix}
A_{1,0,0}^s \\
\vdots \\
A_{2,-1,0}^s
\end{bmatrix}
\hat{\rho}
\]

(2.22)

Note that as in (2.13) the vector components of the above equation represent averaged statistical quantities. Using an external source such as a loudspeaker, the scattering matrix \( S \) is found experimentally by producing sound fields in the duct of a much higher amplitude and uncorrelated with the source vector \( A^s \), which becomes effectively zero. Once \( S \) is known, the external sources are deactivated and the source vector can be found based on mode decomposition to give \( A^\pm \) since all other quantities are known in (2.22) - this is done in §2.6.2.

The number of sound fields required increases with the number of cut-on modes. The simpler plane-wave frequency two-port source data is shown next (as done previously by other authors [54], [59]) followed by the extension to higher-order modes.

### 2.6.1 Scattering matrix measurement

#### 2.6.1.1 Plane-wave frequency range

For the plane-wave frequencies, modelled using equation (2.20), the four unknown parameters which make up the scattering matrix are found with two independent sound fields \( I, II \) which each give two equations:

\[
\begin{bmatrix}
\uparrow A^+ \\
\downarrow \hat{\rho}_I \\
\uparrow A^+ \\
\downarrow \hat{\rho}_{II}
\end{bmatrix}
\begin{bmatrix}
\rho_1 & \tau_{21} \\
\tau_{12} & \rho_2 \\
\end{bmatrix}
\begin{bmatrix}
\uparrow A^- \\
\downarrow \hat{\rho}_I \\
\uparrow A^- \\
\downarrow \hat{\rho}_{II}
\end{bmatrix}
\]

(2.23)

as each sound field gives a vector of amplitudes. The external loudspeakers are driven using random noise to cover a range of frequencies at high enough amplitude to give high coherence between the microphones and the reference loudspeaker voltage signal. The duct surface was perforated to enable sound to be emitted into the duct without disturbing the flow. On iteration \( I \), only the speakers on the inlet side are activated and a measurement is
taken. For iteration \( \mathcal{I} \), this is repeated but with only the outlet side activated. The scattering matrix is then determined by inverting the matrix to the right of \( S \) in equation (2.23).

Figure 2.11 shows the measured plane-wave reflection and transmission parameters for the production fan in datum operation. In the absence of flow, the reciprocity principle implies that the transmission coefficients should be equal. Due to the low Mach number of the flow, it is evident from the figure that there are only small deviations from this principle. Transmission through, and reflection at the source, clearly occurs and contributes to the measured waves travelling away from the fan. Furthermore, it can be seen that transmission decreases smoothly with increasing frequency to a low level.

![Figure 2.11 – Scattering matrix data at plane-wave frequencies for the production fan in datum operation: (a) reflection (b) transmission](image)

Calculation of the source power once the scattering matrix is known is shown in §2.6.2.1 for these frequencies.

### 2.6.1.2 Fan tone frequency above the plane-wave range

When including the first three modes for which there are six unknown modal amplitudes contained in the vectors \( A^\pm \) (see equation (2.21)), the scattering matrix \( S \) contains thirty six
2.6 Two port source analysis with higher-order modes

parameters in a six-by-six matrix. At least six independent sound fields are required to solve for the matrix $\mathbf{S}$ (i.e. $2N$ sound fields for $N$ modes). By varying the location and phases of the active external speakers, several independent sound fields can be generated in the duct. The uncorrelated parts of the measured pressure signals are suppressed using the loudspeaker driving voltage as the reference signal and averaging over many spectra using Welch’s method [57].

The use of single frequency sine wave excitation gives a high coherence with all microphones. This is particularly important when measuring a source with very low transmission for which it is difficult to achieve high coherence between the loudspeaker signal and a microphone measuring on the opposite side of the source. The disadvantage of exciting one frequency at a time is that it takes longer to cover a broad range, although a frequency step of $5$ Hz has given sufficient accuracy/execution time in the past [54].

![Figure 2.12 – Illustration of the azimuthal arrangement of the speakers where the speakers are driven with a phase shift between each to (preferentially) excite an azimuthal mode](image)

Figure 2.12 shows the $90^\circ$ azimuthal spacing of the array speakers. To preferentially excite the first azimuthal mode, the speakers were excited with a $90^\circ$ phase shift between the driving signals as summarised in combination 1 in Table 2.2 along with the two other combinations. Six sound fields were generated by exciting the inlet and outlet arrays in turn.
Table 2.2 – Phase settings of the speaker excitation voltages to preferentially excite different modes

<table>
<thead>
<tr>
<th>Combination</th>
<th>( \hat{\nu}_1 )</th>
<th>( \hat{\nu}_2 )</th>
<th>( \hat{\nu}_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>+90</td>
<td>+180</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-90</td>
<td>-180</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The mode amplitudes from each excitation set form a vector in the matrices of mode amplitudes (from equation (2.22) with \( A^s/\hat{r} = 0 \)):

\[
\begin{bmatrix}
\uparrow A^+ \\
\downarrow \hat{r}_1 \\
\vdots
\downarrow \hat{r}_{vl}
\end{bmatrix}
= S
\begin{bmatrix}
\uparrow A^- \\
\downarrow \hat{r}_1 \\
\vdots
\downarrow \hat{r}_{vl}
\end{bmatrix}
\]

from which \( S \) can be found through an inversion as before. It is difficult to excite modes independently using the speaker installation described above. This is due to several reasons:

- The sound from the speakers enters the duct through perforations. This yields a distributed source and so we do not have excitation at discrete point sources that are 90° out of phase required for excitation of a pure spinning mode.
- For simplicity, there is no fourth speaker at the bottom needed to complete the symmetry of the excitation.

This resulted in some level of dependence between sets. This problem is solved by using an additional speaker at a different axial location to give another data set, and equation (2.24) is solved as an overdetermined system.
2.6 Two port source analysis with higher-order modes

Table 2.3 – Scattering matrix $S$ data at 1.7 kHz for production fan in datum operation

<table>
<thead>
<tr>
<th></th>
<th>Reflected as:</th>
<th>Transmitted as:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0,0)</td>
<td>(1,0)</td>
</tr>
<tr>
<td>Incoming (0,0) to Side 1</td>
<td>0.75</td>
<td>0.20</td>
</tr>
<tr>
<td>Incoming (1,0) to Side 1</td>
<td>0.12</td>
<td>0.84</td>
</tr>
<tr>
<td>Incoming (-1,0) to Side 1</td>
<td>0.10</td>
<td>0.04</td>
</tr>
<tr>
<td>Incoming (0,0) to Side 2</td>
<td>0.69</td>
<td>0.08</td>
</tr>
<tr>
<td>Incoming (1,0) to Side 2</td>
<td>0.06</td>
<td>0.57</td>
</tr>
<tr>
<td>Incoming (-1,0) to Side 2</td>
<td>0.04</td>
<td>0.13</td>
</tr>
</tbody>
</table>

At the tone frequency of interest, the scattering matrix is determined for the production fan operating at its design point. The magnitudes of the reflection and transmission coefficients in the scattering matrix are given in Table 2.3. As at plane-wave frequencies, in general the transmission through the fan is low while reflection is relatively high, particularly at the inlet. In each of the four quadrants of the table the largest values are seen on the leading diagonal which corresponds to reflection/transmission without mode number change.

Ideally the flow straightener located between the fan outlet and microphone array should not affect the measured fan characteristics. It is expected that the presence of the flow straightener may produce additional noise (as noted in the standard [39]) or have an effect on higher-order modes. This is analysed in the next section.

Calculation of source power once the scattering matrix is known is shown in §2.6.2.2 for the tone at this frequency.

2.6.1.3 Effect of flow straightener

The acoustic effect of the honeycomb flow straightener (located as in Figure 2.6) is investigated by characterising its passive properties in isolation without the fan or flow. Ideally this should have very low reflection and high transmission. Measurements at plane-
wave frequency scattering matrix show almost complete transmission and are omitted here. One would expect that any effect of the straightener will be more pronounced when higher-order modes begin to propagate. This is because the honeycomb passages are effectively ducts of much smaller diameter through which only plane-waves will propagate unattenuated at modest frequencies. For the tone frequency of interest, the scattering matrix data is shown in Table 2.4.

Table 2.4 – Scattering matrix $S$ data for the honeycomb flow straightener at 1.7 kHz

<table>
<thead>
<tr>
<th>Reflected as:</th>
<th>Transmitted as:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>(1,0)</td>
<td>(1,0)</td>
</tr>
<tr>
<td>(-1,0)</td>
<td>(-1,0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Incoming (0,0) to Side 1</th>
<th>0.12</th>
<th>0.03</th>
<th>0.23</th>
<th>0.90</th>
<th>0.18</th>
<th>0.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incoming (1,0) to Side 1</td>
<td>0.03</td>
<td>0.30</td>
<td>0.04</td>
<td>0.05</td>
<td>0.84</td>
<td>0.02</td>
</tr>
<tr>
<td>Incoming (-1,0) to Side 1</td>
<td>0.05</td>
<td>0.01</td>
<td>0.33</td>
<td>0.03</td>
<td>0.03</td>
<td>0.79</td>
</tr>
<tr>
<td>Incoming (0,0) to Side 2</td>
<td>0.06</td>
<td>0.11</td>
<td>0.02</td>
<td>0.95</td>
<td>0.02</td>
<td>0.11</td>
</tr>
<tr>
<td>Incoming (1,0) to Side 2</td>
<td>0.06</td>
<td>0.40</td>
<td>0.05</td>
<td>0.03</td>
<td>0.83</td>
<td>0.03</td>
</tr>
<tr>
<td>Incoming (-1,0) to Side 2</td>
<td>0.01</td>
<td>0.02</td>
<td>0.36</td>
<td>0.04</td>
<td>0.05</td>
<td>0.80</td>
</tr>
</tbody>
</table>

It is clear from Table 2.4 that azimuthal mode ($\pm 1,0$) transmission is marginally less complete than that of the plane-wave mode. This is coupled with the higher reflection coefficients for the azimuthal modes relative to plane-wave. By considering the sum of the square of the magnitudes of the reflection coefficients, it can be seen that only a very small proportion of the acoustic power is reflected by the honeycomb. For the plane-wave mode which contributes most to the power, at most 7% is reflected and this would only change the power prediction for this mode on the order of 0.3 dB. For the azimuthal modes, at most 17% is reflected and this would have an effect on the order of 0.8 dB for these modes. Consequently, the outlet side sound power at this frequency is expected to be accurate to around $\pm$ 1 dB due to the presence of the honeycomb flow straightener.
2.6 Two port source analysis with higher-order modes

2.6.2 Fan narrow-band comparison

Once the scattering matrix is known for a given frequency or frequency range, the source vector in equation (2.22) is calculable. The quantities of interest, such as the magnitudes of the mode amplitudes, are found using averaged quantities for example:

$$|A_{0,0}^s|^2 = \left| \frac{A_{0,0}^s}{\hat{p}} \right|^2 \hat{S}_{rr}$$  \hspace{1cm} (2.25)

where $\hat{S}_{rr}$ is the auto-spectra of the reference signal. Calculation of the source sound power, taking into account higher-order modes, requires integration of the axial intensity field over a cross-section. The time-averaged axial intensity as a function of the acoustic pressure $\hat{p}$ (from (2.9) with $A_{m,n}^+ = A_{m,n}^s$, $A_{m,n}^- = 0$) and axial acoustic velocity $\hat{u}$ (related to $\hat{p}$ from momentum equation) is given by:

$$\bar{I}_x(r, x, \theta, \omega) = \frac{1}{2} Re(\hat{p}\hat{u}^*)$$  \hspace{1cm} (2.26)

This is integrated over the cross-section area $S$ to find the sound power:

$$P^s = \iint \bar{I}_x dS$$  \hspace{1cm} (2.27)

It is explained in appendix §B.2 that the ‘cross-terms’ in the integration involving products of the amplitudes of different modes do not contribute to sound power. Equation (2.27) can therefore be written for a given mode as:

$$P^s_{m,n} = \frac{Sk_{m,n}}{2k_0\rho_0a_0} |A_{m,n}^s|^2$$  \hspace{1cm} (2.28)

where the overall net sound power is found by adding the contribution from each mode i.e. $P^s = \sum_{m,n} P^s_{m,n}$.

2.6.2.1 Plane-wave frequency range

For the plane-wave frequency range for which $k_{m,n} = k_0$ equation (2.28) reduces to
\[ P^s = \frac{S}{2\rho_0a_0} |A^s|^2 \] (2.29)

with the subscripts ‘0,0’ omitted. This can be rearranged into a formula relating sound power to sound pressure level:

\[
PWL = 10\log \left( \frac{P^s}{P_{ref}} \right) = 20\log \frac{|A^s|/\sqrt{2}}{p_{ref}} + 10\log S - 10\log \left( \frac{\rho_0a_0}{(\rho a)_{ref}} \right) = SPL_{rms} + 10\log S - 10\log \left( \frac{\rho_0a_0}{(\rho a)_{ref}} \right)
\] (2.30)

since \( P_{ref} \) is 1 pW, \( p_{ref} \) is 20 μPa and \( (\rho a)_{ref} = p_{ref}^2/P_{ref} \). This is the so-called plane-wave formula relating sound power and pressure applied in the ISO standard method for all frequencies [39], [44]. In the anechoic rig, a microphone directly measures pressure fluctuation amplitude equivalent to root mean square sound pressure level \( SPL_{rms} \), independent of the transverse location. This suggests that the ISO rig measurements should be exact in terms of both broad and narrow-band of the fan spectra with the new rig.

To account for the slightly different duct areas, sound power is compared instead of sound pressure in Figure 2.13. The random nature of broadband noise means that use of power spectral density (instead of power spectrum) is appropriate to remove dependence on the frequency bin \( \Delta f \) (see §2.5.1). The comparison shows that the broadband levels are in very good agreement at most frequencies - the tonal components vary since their levels do depend on \( \Delta f \). Since no anechoic termination removes all reflections, a certain (low) level of reflection is permitted by the ISO standard [39] and this is verified by experimentally measuring the termination reflection coefficient. Above 800 Hz for the ISO rig measurements, the amplitude fluctuates by approximately ±2 dB due to some kind of reflection phenomenon. This is likely to affect the accuracy of narrow-band measurements and is included as a possible source of uncertainty in the measurements in Chapter 3 and Chapter 4 based on Appendix E.
2.6 Two port source analysis with higher-order modes

2.6.2.2 Fan tone frequency above plane-wave range

A key interest of the current chapter is to understand how best to measure a tone above the plane-wave frequency. In the standard, the plane-wave formula (2.30) is used to find the sound power based on the linear average of at least three SPL\(_{rms}\) measurements at different azimuths.

The reference signal (for equation (2.22)) which is correlated with the tone is taken from the motor controller tachometer – coherence values are shown in Table 2.5. As a guide, the ISO standard states that the turbulent pressure fluctuations should be at least 6 dB lower than the acoustic fluctuations in which case the coherence must be greater than 0.64 (for plane-wave frequencies) [39].

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Inlet</th>
<th>Outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7 kHz</td>
<td>0.94</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 2.5 – Magnitude squared coherence values with 256 spectra averages
The magnitudes of the mode amplitudes on the inlet and outlet $A_{m,n}^s$ are found using equation (2.25) and the power from (2.28). A modal breakdown of the power on the inlet and outlet sides for the tone of interest is given in Table 2.6. The data shows that the power is not equally distributed between each mode at this frequency, particularly on the outlet side where the plane-wave mode dominates.

Table 2.6 – Source data for each mode at 1.7kHz tone from production fan in datum operation. Equivalent SPL according to the plane-wave formula is calculated with equation (2.30).

<table>
<thead>
<tr>
<th>Power (dB re. 1 pW)</th>
<th>Inlet, $P_{1,m,n}^s$</th>
<th>Outlet, $P_{2,m,n}^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane-wave mode (0,0)</td>
<td>24.9 (42.9)</td>
<td>37.3 (55.3)</td>
</tr>
<tr>
<td>Azimuthal mode (1,0)</td>
<td>18.2 (38.7)</td>
<td>25.6 (46.1)</td>
</tr>
<tr>
<td>Azimuthal mode (-1,0)</td>
<td>20.6 (41.1)</td>
<td>26.9 (47.5)</td>
</tr>
<tr>
<td>Total</td>
<td>26.9</td>
<td>37.9 ± 1 dB</td>
</tr>
<tr>
<td>SPL from plane-wave formula</td>
<td>44.9</td>
<td>55.9</td>
</tr>
</tbody>
</table>

The accuracy of taking SPL measurements at $r/R_d = 0.5$ to represent the tone sound power level via the plane-wave formula (2.30) but above the plane-wave range is now explored. Based on the decomposed source modes which exist under anechoic conditions, Figure 2.14 and Figure 2.15 show a sample transverse variation of the predicted SPL on the inlet and outlet sides, respectively. This variation is dependent on axial location as the acoustic pressure magnitude contains a term which is a function of $e^{ix(k_0-k_1)}$ in the presence of the plane-wave and first azimuthal modes. Since the $x$ datum is arbitrary, the figures have been produced by taking $x = 0$ and represent a worst-case in terms the position dependency of a measurement.
2.6 Two port source analysis with higher-order modes

Figure 2.14 – Variation of the SPL that could be measured under anechoic conditions for a duct section on the inlet side

Figure 2.15 – Variation of the SPL that could be measured under anechoic conditions for a duct section on the outlet side

Comparable measurements are performed in the Dyson ISO rig (§2.2) with the microphones located mid-way between the duct axis and wall. According to the standard, an average must be taken of at least three sound pressure levels at different azimuthal locations before applying equation (2.30). This is achieved by rotating the fan duct section relative to the rig and microphones which remain stationary. The microphones (B&K 4189 ½-inch) have a diameter which is almost 20% of the duct radius and so effectively measure the area-average of sound pressure over its diaphragm.
Table 2.7 – Fan tone at 1.7 kHz measured in Dyson ISO rig for production fan in datum operation

<table>
<thead>
<tr>
<th></th>
<th>Inlet</th>
<th>Outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SPL_{rms} @ 0^\circ$</td>
<td>38.9</td>
<td>50.0</td>
</tr>
<tr>
<td>$SPL_{rms} @ 120^\circ$</td>
<td>34.0</td>
<td>53.5</td>
</tr>
<tr>
<td>$SPL_{rms} @ 240^\circ$</td>
<td>42.8</td>
<td>50.1</td>
</tr>
<tr>
<td>$SPL_{rms}$ average</td>
<td>39.3</td>
<td>51.3</td>
</tr>
<tr>
<td>$PWL_{ISO\ rig}$ from plane-wave formula (2.30)</td>
<td>21.2</td>
<td>33.2</td>
</tr>
<tr>
<td>$PWL_{ISO\ rig} - PWL_{new\ rig}$</td>
<td>-5.7</td>
<td>-4.7</td>
</tr>
</tbody>
</table>

Table 2.7 shows variations in the measured SPL at different angles with a peak of up to 8 dB difference between the largest and smallest SPL on the inlet side. These large variations can be linked to the highly position-dependent inlet sound field shown in Figure 2.14 existing under anechoic conditions. A difference of almost -6 dB is observed between the tone inlet sound power levels measured in each rig. On the outlet side, both Table 2.7 and Figure 2.15 agree that the sound field SPL varies less appreciably with traverse position which leads to better agreement between the power levels determined in each rig. This is due to a more dominant plane-wave mode which carries a greater of proportion of the power, as shown in Table 2.6.

### 2.7 Summary and conclusions

A new method and rig has been developed to measure fan broadband and tonal noise which gives deeper insight into the source acoustic characteristics than existing methods. The aeroacoustic rig at Dyson based on the ISO standard is designed to measure overall sound power levels in third-octave bands. A principal interest here is to measure deterministic (tonal) noise accurately. The ISO standard makes assumptions primarily applicable to broadband, random noise and hence assumes that it contributes most to overall sound power. The new scheme does not require these assumptions as the modal structure of the source
sound field is directly measured using mode decomposition and a two-port source formulation.

For plane-wave frequencies, the narrow-band sound powers in both rigs are in close agreement as a single radius SPL measurement in the ISO method is representative of the section area sound power. This also demonstrates that the new method in which duct terminations are not anechoic can accurately isolate waves coming directly from the source from any reflection/transmission. With the production fan in datum operation, transmission through the fan is very low for frequencies at or above the blade-passing frequency for both the plane-wave and first azimuthal modes.

The most significant tone produced by the fan in datum operation above the plane-wave frequency range is measured in both rigs. The breakdown of mode power shows that the power is not equally distributed, which is expected since only a random, broadband source is likely to excite all modes indiscriminately. When higher-order modes carry significant power the SPL becomes strongly dependent on transverse location in both rigs, and calculation of this dependence, based on source mode amplitudes, shows a complex variation pattern. The narrow-band tone sound power level is underestimated using the ISO rig method. The agreement between the two methods is better on the outlet as the plane-wave mode dominates.

Figure 2.16 – ISO flow straightener types (a) ‘star’-type present in Dyson ISO rig (b) ‘honeycomb’-type present in new rig (reproduced from [43])
The standard flow straighteners shown in Figure 2.16 should ideally have no effect on the fan noise measurement. However, it is briefly stated in the ISO standard that they can affect sound power measurements - hence the standard suggests taking an average of the reading with and without the straightener [39]. Measurements of the passive properties of a honeycomb flow straightener show how it particularly affects higher-order modes. At modest acoustic frequencies, non-planar modes get cut-off, changing the outlet sound power.

There are several key conclusions from the work in this chapter most relevant to later work:

- Transmission through the fan in datum operation is very low for the blade-passing frequency which suggests (tonal) noise generated by the guide-vanes will propagate more readily to the outlet side while the impeller will most affect the inlet side noise.
- The flow straightener introduces an uncertainty of around ± 1 dB in outlet measurements at some frequencies.
- The Dyson ISO rig is best suited to measure broadband levels. At frequencies above the plane-wave range, the inaccuracy may be large especially in the presence of a harmonic source that generates a specific azimuthal mode. In this case the new rig must be used.
Chapter 3  Low speed mixed-flow fan noise sources

3.1  Introduction

Acoustic waves are generated by many different sources within the fan unit. These waves propagate, interfere with waves from other sources, and can be scattered or absorbed by the flow or geometry. The sound eventually measured in the far-field of the fan further depends on the propagation path between the fan inlet/outlet and a rig microphone – this has been studied in Chapter 2. The aim of this chapter’s work is therefore to build an understanding of the important sources within the fan based on a range of aeroacoustic tests.

Both the tonal and broadband components are of interest, and it is difficult to alter one of these components without affecting the other. Changes to the fan geometry not only change the associated sources but also change the propagation paths of sound from other sources. By looking at the effect of changes to one aspect of the fan at a time, key interactions which generate noise can still be identified. Once the contributions from different components are established this will feed into further work to gain physical insight into, and model, the generation mechanisms.

To remove the rotor-stator interaction noise component, experiments have routinely been performed on isolated low-speed fan rotors in which downstream flow obstructions such as stators or struts are removed or located far away from the rotor. Fitzgerald & Lauchle [28], for example, reduce the tonal noise from their test case axial fans by making several geometry modifications. These include the addition of a bellmouth inlet, modified outlet support struts/shroud, and blade suction side serrations. Fukano et al. [60] measure the noise from several low-speed fan configurations (axial, mixed-flow) with different rotor tip clearance and eccentricity and show that both should be minimised to reduce noise and improve performance. Carolus et al. investigate a HVAC-type axial rotor [61] and centrifugal
impeller [62] which produce BPF tones due to inlet distortion from a nominally well-managed inflow. This is found to be due to an unstable vortex structure which varies in position relative to the inlet centreline. LES simulations accompanying the centrifugal experiments show that the characteristic frequencies associated with this structure, illustrated in Figure 3.1, are very low compared to the blade speed which leads to correlated events between each blade.

Moreau & Enghardt [63] include the effect of inlet turbulence in their experimental study of a diverse range of parameters on axial broadband noise. They find that increased inlet turbulence has a significant effect on noise at low fan loading where other mechanisms (e.g. self-noise) are less dominant. With a nominal or high loading, they find that the other mechanisms mask the noise due to increased inlet turbulence.

The operating point of a fan stage determines the flow field and blade loading, which heavily influence noise generation. In Moreau & Enghardt’s study, rotor self-noise and rotor-stator interaction compete to dominate the broadband sound field at nominal and high fan loading [63]. When operating near stall, rotating instabilities have been shown to produce broad peaks at some fraction of BPF in both axial [64] and centrifugal machines [65].
Most past studies are focused on high Reynolds number applications – the effect of low Re on the turbomachinery noise has not been studied in detail. Moreau et al. [28] show that at low-to-moderate Reynolds numbers the flow structure over an isolated aerofoil is complex and highly varied for different geometries, thus influencing the trailing-edge noise.

Broadband noise reductions through reduced numbers of stators depend upon what mechanism or mechanisms are responsible for most of the noise. In gas turbine applications, turbofan broadband noise is said to be dominated by rotor-stator interaction due to the turbulent rotor wake [66]. But Carolus et al. [41] note that in HVAC applications the main mechanism is turbulence ingestion as fan inlet conditions are often poor due to upstream components, imperfect intakes, etc. This is to be contrasted with the inlet conditions of a turbofan which at times may be very clean. As mentioned previously, however, Moreau & Enghardt find that rotor self-noise is of similar importance as rotor-stator interaction in their fan low-speed rig under nominal conditions [63]. Dominance of rotor self-noise is demonstrated in the low-speed fan experiments of Sutliff [67].

The approach of the work outlined in this chapter is to make changes to the fan datum geometry or operating point which should mainly affect certain generation mechanisms. An example of this is halving the number of vanes which should reduce rotor-stator wake interaction broadband noise. In many cases this may affect other generation mechanisms – in this case the rotor-stator wake interaction tonal noise will increase due to reduced destructive interference between vanes. The sensitivity of the overall measured noise to changes to the targeted sources will highlight which mechanisms are dominating the sound field.

The practicalities of modifying the production geometry and producing 3D printed parts are described first in §3.2. The aerodynamic operating curves of the test cases are presented in §3.3. In §3.4 the results of tests without vanes are compared to the datum configuration with vanes. To investigate the impeller off-design acoustic performance, the operating point is varied in §3.5. The effect of reducing the number of vanes is considered in §3.6. Finally in §3.7 the inlet distortion to the impeller is characterised by spatially traversing a hotwire and the inlet condition is altered by purposefully adding distortion. The chapter concludes in §3.8 by considering which sources are important in the regime considered in the present thesis.
3.2 Fan construction, modification and operation

The production fan has several features, as indicated in Figure 3.2, which complicate investigation of the sound sources. An enlarged vane carrying the cable to the electric motor makes the vanes asymmetric. This creates asymmetry in the destructive interference of the BPF acoustic fields generated by the 22 vanes interacting with the 9-bladed impeller [68]. Perforations in the hub surface allow flow to bypass the vanes and are backed with acoustic foam. Both of these features are removed in the modified configurations tested in the remaining chapters. Impellers and vanes are prototyped using stereolithography (SLA). The SLS technique is found suitable when needing parts which are rigid, tractable and difficult to break, at the expense of a rougher surface finish. The SLA technique produces components with a superior surface finish but the material is brittle so thin blades are easily broken. All impellers, production or 3D printed, require balancing to account for asymmetry which leads to noisy vibration. Blade imperfections or imbalance manifests itself clearly in the acoustic spectra at harmonics of the shaft frequency which, in the worst case, can dwarf the aeroacoustic noise at high rotational speeds.

![Figure 3.2 - Production fan features which affect the fan aeroacoustic performance and are removed in the modified configuration](image-url)
3.2 Fan construction, modification and operation

Removal of the enlarged vane requires a re-routing of the motor cabling which must cross the flow path at some location. To minimise flow disturbance and extraneous noise, the cable crosses some distance downstream of the fan unit, as shown in Figure 3.3, where the velocities are very low (~ 2 m/s).

A summary of the test cases used in this chapter and Chapter 4 are given in Table 3.1. The geometry of individual blades is kept constant as the number of impeller blades or guide vanes is varied. For each configuration, measurements are taken at the design global flow coefficient (Table 1.3) while the global Reynolds number is varied in 8 steps from 37% to 122% of the datum (Table 1.2) by changing the running speed. Furthermore, with the global Reynolds fixed to the datum level, measurements are taken at 3 off-design global flow coefficients, above and below the design point. These form the characteristics which are compared in §3.3.
Table 3.1 – Test configurations

<table>
<thead>
<tr>
<th>Housing</th>
<th>Impeller (type)</th>
<th>Guide vanes (type)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9 (production)</td>
<td>22 (SLA)‡</td>
</tr>
<tr>
<td></td>
<td>11 (SLA)</td>
<td>9 (SLA)</td>
</tr>
<tr>
<td></td>
<td>22 (SLA)</td>
<td>11 (SLA)</td>
</tr>
<tr>
<td></td>
<td>9 (production)</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>11 (SLA)</td>
<td>n/a</td>
</tr>
</tbody>
</table>

3.3 Operating curves

The performance characteristics for the 9-bladed impeller installed in both housings are shown in Figure 3.4. Overall efficiency is calculated as discussed in §1.2.2.1 and Appendix A, and therefore suitable for comparisons as long as the motor is running at the datum speed

‡ Datum configuration
with the same impeller. It is apparent that the increase in the vane space-chord ratio (and loading) with 9 vanes instead of 22 has minimal effect on the pressure rise and efficiency. This is because the vanes are only lightly loaded and the high number in the production design is for acoustic rather than aerodynamic reasons. The housing without vanes gives a more marked effect on performance with the pressure rise up to 17% lower at the design $\phi$ where the efficiency decrease also peaks at around 10% lower. The main reasons for this are thought to be down to changes to the geometry downstream of the original guide vane location (which are explained in §3.4). The housing without vanes has an extended annulus in which the velocities are much higher compared to the levels in the full duct, giving higher surface friction loss. Furthermore, the cylindrical struts which replace the vanes are comparatively bluff giving more loss. With the fan heavily throttled at low flow coefficients, the performances converge as the velocities and hence losses are lower. As analysed in more detail in §4.4.1 the impeller flow field is not thought to be heavily influenced by the vanes which are relatively far away – as previously shown in centrifugal compressor studies [69], [70].

An uncertainty for the production fan operating point at the datum Reynolds number is shown in Figure D.1 for the Dyson ISO rig.
The objective with the 11-bladed impeller is to maintain a similar flow field and aerodynamic performance to the 9-bladed impeller at the design point. Since the blade angles are unchanged, the flow angles should be approximately the same (these as measured in §4.5). Qualitatively, the performance characteristic trends, shown in Figure 3.5, are identical to the 9-bladed impeller in similar configurations (from Figure 3.4). The design flow coefficient
still coincides with the high efficiency point for the configuration with 22 vanes in Figure 3.5b. Reducing the impeller space-chord ratio decreases the loading on the blades, can reduce secondary flow at the design point and can improve off-design performance. Comparing the pressure rise curve for each impeller with 22 vanes, the 11-bladed impeller achieves a higher pressure rise across the range. Halving the number of vanes to 11 gives a small reduction in pressure rise across the range but overall the performance is, as with the 9-blade impeller tests, insensitive to this change.
3.3.1 Effect of Reynolds number on pressure rise performance

Figure 3.6 highlights the effect of reducing the rotational speed while maintaining constant flow coefficient, equivalent to reducing the fan Reynolds number. For each of the impeller – guide-vane combinations, a clear reduction in pressure rise performance at low Reynolds
number is seen - $\psi$ is up to $\sim 10\%$ lower than at the datum speed/Reynolds number. At Reynolds numbers above the datum, the values appear to tend towards a constant value which must correspond to the regime where $\psi = f(\phi)$ only. The same trend is seen for the tests without guide vanes. The flow field downstream of the impeller is measured experimentally in §4.5 with a particular focus on the unsteadiness which influences noise generation.

![Figure 3.6 – Variation of loading coefficient with Reynolds number at the design flow coefficient](image)

An uncertainty for the $B = 9, S = 22$ fan operating point at the design flow coefficient is shown in Figure D.2 for the Dyson ISO rig. This shows that decrease in $\psi$ at lower Reynolds number is not due to inaccuracies in the measurements.

In the coming sections, acoustic spectra for the range of operating conditions and geometries corresponding to the aerodynamic performance results in this section (§3.3) are compared. The Dyson ISO rig is chosen for the measurements since, with reference to the findings of Chapter 2, it provides sufficient accuracy to perform a broad range of tests in a reasonable amount of testing time. All SPL spectra are calculated using Welch’s method (see §2.5.1) with 512 averages (50% overlap), a 51.2 kHz sampling frequency for 60 seconds, and a frequency bin of 4 Hz. SPL can be linked to the sound power using equation (2.30) which introduces a constant offset. At the blade-passing harmonic frequencies of interest:
• Conclusions are drawn from the trends in a range 37-122% of the datum global Re implying a range of BPFs.
• Up to a global Re of 117% of the datum, only plane-waves are cut-on below 1BPF (9-bladed impeller).
• None of the blade-vane combinations (as listed in Table 3.1) imply a preferential excitation of non-plane-wave modes.

This means that rig gives high accuracy for the tones at the plane-wave frequencies (potentially limited by the amplitude modulation effect quantified in Appendix E of <=2.5 dB). Above the plane-wave frequencies, reasonable accuracy can be achieved still since no preferential excitation of non-plane-wave modes is expected.

### 3.4 Effect of removing vanes on BPF tones

To try to isolate the impeller tonal noise sources, the fan design is modified to enable removal of the vanes which potentially lead to interaction noise as discussed in §3.1. In the original configuration, the vanes serve as the rigid link between the hub (housing the motor) and the casing. Ideally there would be a very large distance between the impeller and any supportive flow obstruction in the annulus:

• To give time for the unsteadiness (e.g. wakes) from the impeller to mix-out
• To avoid flow obstruction potential field upstream influence on the impeller

Subject to the limitations in the rigidity of 3D-printed components, a trade-off is found between having a longer annular section at the expense of a more substantial/obstructive link between the hub and casing. Cylindrical struts are chosen as their loading is not a strong function of incidence compared to a streamlined body. In their paper on rotor-strut interaction, Lu et al. show that the rotor is the dominant tonal source due to potential interaction [71] – the struts themselves don’t contribute like an aerofoil-section strut would. The annulus extension distance (of six strut diameters) is such that the inviscid static pressure field of the cylinders has decayed. Four cylindrical struts (10mm diameter) provide sufficient
rigidity with the annulus extended by 60mm in the downstream direction, as shown in Figure 3.7.

Figure 3.7 – CAD design comparison between housings for tests with and without vanes

Given that the objective is to make comparisons between the fans with and without vanes, the following limitations are acknowledged. In terms of fan aerodynamics:

- The overall stagnation pressure rise is reduced due to additional losses
- Upstream influence of the vanes on the impeller would mean that matching $\phi$ and $Re$ is insufficient to give identical impeller loading. This influence is assumed to be small.

In terms of the fan acoustic signature:

- The struts are expected to give additional broadband noise, and potentially low frequency vortex-shedding
- The passive properties of the fan may vary with the vanes removed (e.g. one might expect lower transmission through the fan in the absence of vanes)

For these reasons it is not possible to compare the broadband levels – broad spectra are included in the results below to show the tone level relative to the broadband level.
Tests are performed on the 9-bladed production impeller at a range of global Reynolds numbers above and below the datum (Table 1.2). A comparison between the 22-vane and vaneless configurations at the datum operating speed is shown in Figure 3.8. Previous experiments to characterise the passive properties of this test case using the new rig (see Chapter 2) show that the transmission through the fan is slightly reduced for the vaneless configuration around BPF [68]. This suggests that removal of the vanes is not having a large effect on the propagation of sound through the fan annulus; one would expect an increase in transmission in the absence of vanes if that was the case.
3.4 Effect of removing vanes on BPF tones

Figure 3.8 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at datum global Reynolds number (a) inlet (b) outlet

The comparison in Figure 3.8a shows that the tonal noise is not propagating to the inlet side at this operating speed and hence it does not change when the vanes are removed. Conversely on the outlet side, Figure 3.8b, there is a narrow peak at BPF and a broader peak at 2BPF. The peak at 2BPF, which is of equal amplitude with and without the vanes, must be due to
the impeller and this mechanism is investigated in §3.7. At BPF the vaneless configuration produces a tone which must also be due to the impeller. Somewhat surprisingly, this tone level is enhanced by the addition of the vanes even though the interaction modes (for $B = 9, S = 22$) are spinning subsonically. This can be attributed to scattering/interaction of these evanescent modes with the flow exhausting into the full duct just after the vanes. Scattering gives a plane-wave mode which can propagate at that frequency. This does not occur at 2BPF as there is no excitation at this frequency from the impeller (see §4.5 and Figure 4.15).
3.4 Effect of removing vanes on BPF tones

Figure 3.9 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at 37% of datum global Reynolds number (a) inlet (b) outlet

Similar trends are seen at other operating speeds in the test range. At the lowest Reynolds number, Figure 3.9a shows that the BPF tone is perceptible on the inlet side and is slightly higher with vanes. On the outlet, Figure 3.9b, tone amplitude increases with vanes most
significantly at the BPF, again where there is significant excitation and scattering of the evanescent interaction modes.

Figure 3.10 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at 122% of datum global Reynolds number (a) inlet (b) outlet
3.5 Effect of changing operating point

At the highest Reynolds number, Figure 3.10a shows a BPF tone which is generated primarily by the impeller on the inlet. On the outlet, Figure 3.10b, the BPF tone is again much higher due to interaction - dissimilarly the 2BPF tone is also slightly higher. The comparisons for other global Reynolds numbers have been provided in §C.1 (Appendix C).

The key findings from these results are that the impeller is producing tonal noise in the absence of interaction – the origin of these tones is investigated in §3.7. Furthermore in the experimental set-up, tonal interaction noise is produced at the outlet even with the correctly chosen number of vanes.

3.5 Effect of changing operating point

März et al. reported on tests on a low-speed axial fan, which exhibits a characteristic hump at roughly half of the blade passing frequency at low flow coefficients where the $\psi$-$\phi$ characteristic levels-off due to the onset of stall [64]. Similar humps at this frequency have been identified by other authors for centrifugal turbomachinery [65], [72]. In §3.3 it is seen that the mixed-flow fan pressure rise performance at low $\phi$ does not diminish as it does in axial machines. Empirically additional noise is known to be produced when operating in this regime. This can be due to increased self-noise but also due to interaction of the disordered impeller outflow with downstream components e.g. outlet guide vanes.

To understand the changes in impeller self-noise, measurements are taken using the vaneless configuration at the four flow coefficients shown in Figure 3.4 (9-blade) and Figure 3.5 (11-blade). Figure 3.11 contains four parts corresponding to the inlet and outlet spectra of the 9 and 11-bladed impellers. Both impellers exhibit the same behaviour as the flow coefficient is reduced below the design point in two steps:

- A high-amplitude, broad ‘hump’ grows at around half the blade-passing frequency by up to 15 dB and 13 dB for the 9 and 11-bladed impellers respectively (on the inlet side)
- The blade-passing frequency tone levels reduce until indistinguishable from the broadband level
At the flow coefficient above design, broadband levels reduce marginally while the BPF tone levels increase slightly.

Figure 3.11 – Effect of changing operating point on spectra measured in the vaneless configuration at the datum global Reynolds number: (a)/(b) 9-bladed impeller, (c)/(d) 11-bladed impeller
As in axial machines, off-design impeller operation implies that the flow incidence at the leading-edge is higher or lower than the datum, leading to non-ideal behaviour. Illustrative velocity triangles at the impeller mid-span are shown in Figure 3.12 for the three off-design flow coefficients based on a simple scaling of the datum velocity triangle. Since the blade speed $u_1$ is kept constant, flow coefficient change implies a proportionate change in the axial velocity component which causes the incidence to become negative (for $\phi/\phi_d > 1$) or highly positive (for $\phi/\phi_d < 1$).

In axial turbomachinery, stall cells (azimuthal sectors of stalled flow) begin to occur at the stability limit where the characteristic levels-off. Part-span stall cells, whose effect is confined to the tip region, produce a relatively small pressure loss compared to full-span cells. As summarised by Day [73, Sec. Stall and Surge], part-span cells are usually observed for low hub-tip ratio stages but also occur at high hub-tip ratios when the loading is very low. These rotate at 70 – 80% of the rotor speed which is faster than is common for full-span cells. März et al. explains the characteristic ‘hump’ around half the blade-passing frequency as...
being due to a rotating instability similar to rotating cells associated with their unshrouded axial compressor operating near stall with a large tip clearance [64]. This disturbance propagates in a pitchwise direction at roughly half the rotor speed. More recently, Young & Day have undertaken a detailed investigation of tip clearance effects on instabilities which give irregularity in the BPF signature and broadband pressure perturbations around 30% of BPF.

For the highly-positive incidence cases, the increasing ‘hump’ in the acoustic spectra (Figure 3.11) at around half the blade passing frequency must be associated with development of asymmetric flow distribution in the impeller. The degradation in the $2\pi/B$ periodicity of the flow is further demonstrated by the disappearance of the blade passing frequency tones associated with perturbations occurring in a repetitive fashion as each blade passes. This suggests that high-incidence operation is leading to instability occurring on some of the blades (at a given instance in time) resulting in unsteadiness and noise. It also indicates that the BPF tones are generated by unsteady loading on the impeller since they change as the impeller operating point is varied.

A feature in the configuration considered in this work is that the impeller is shrouded which prevents the influential tip leakage flows. Furthermore, since the pressure rise performance of the mixed-flow configuration is undiminished at low flow coefficients, it is not clear if the asymmetry in the impeller flow distribution is due to conventional stall cells forming. This one would expect to start to affect performance; however it may be unaffected because the pressure rise is being achieved by virtue of the increase in radius and thus not heavily affected by flow separation. Alternatively, the flow may be remaining in a near-stall state under the influence of a rotating instability leading to the characteristic ‘hump’ like in the work of März et al. [64].

### 3.6 Effect of reducing number of vanes giving a ‘cut-on’ mode

There are a number of recent experimental investigations on turbofan aeroacoustics whereby broadband interaction noise is reduced by having fewer vanes [66], [74]. These are often referred to as ‘cut-on’ designs as the number of vanes is not high enough to give only
3.6 Effect of reducing number of vanes giving a ‘cut-on’ mode

Evanescent interaction tone modes (due to destructive vane interference). The increased propensity to generate strong tones can be alleviated by re-designing the blades such that the acoustic fields generated along the span of a single vane interfere destructively [75]–[77].

To investigate the effect of reducing the number of vanes in the small-scale regime of interest in the present thesis, tests are performed using the 9-bladed production impeller at a range of global Reynolds numbers above and below the datum (Table 1.2). The number of vanes is reduced by more than half to 9, which Figure 3.4 shows has minimal impact on on-design performance. Acoustically this represents the worst possible case as an interaction mode at each harmonic $N$ of BPF is always cut-on since, from (1.9), for $k = -N$:

$$m = 9N + 9k = 0$$  \hfill (3.1)

i.e. a plane-wave mode is excited. For the first few harmonics of BPF, this is the only mode that is cut-on but it can produce sound very effectively.
Figure 3.13 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and 9 vanes at the datum global Reynolds number (a) inlet (b) outlet (c) outlet – extended frequency range

Figure 3.13 shows a comparison between the 22-vane and 9-vane configurations at the datum global Reynolds number and flow coefficient. On the inlet side at this speed the tones are not significantly above the broadband level for either configuration, while the broadband is slightly lower around the BPF by a few dB. On the outlet side, there is a strong interaction tone at BPF in the reference case which is enhanced by 2.4 dB – a modest amount compared
to the other speeds explored next – while the broadband level is lower around the BPF by a few dB as before. At 2BPF, the outlet tone is broad and the level is fairly invariant with the number of vanes. Figure 3.13c shows the outlet spectra comparison at much higher frequencies where the differences between the two are minimal.
At Reynolds numbers other than the datum a significant BPF tone is seen on the inlet side, as shown in Figure 3.14a for the lowest Re. With 9 vanes, this tone is enhanced by 6 dB and 7 dB on the inlet and outlet, respectively. The broadband level around BPF is again lower by a few dB. At the other end of the Reynolds number range, Figure 3.15 shows that the
broadband decrease is slightly larger in the presence of higher Mach numbers associated with operating at the highest global Re. Furthermore, the tones are enhanced by 6 dB and 7 dB on the inlet and outlet, respectively. The comparisons for other global Reynolds numbers have been provided in §C.2 (Appendix C).
These trends can be explained as follows. In terms of the tones, the source of higher BPF amplitude produced in the 9 vane case is due to the periodic loading on the vanes from the impeller wakes (not from the impeller itself). In both the reference 22-vane and 9-vane cases,
3.6 Effect of reducing number of vanes giving a ‘cut-on’ mode

the impeller is operating at the same flow coefficient and Reynolds number, and hence the wakes are nominally identical. This excitation is primarily at BPF (see Figure 4.15) and hence produces sound at this frequency, while the 2BPF level is largely unaffected even though, from equation (3.1), the plane-wave mode is cut-on. The tone level increase is plotted in Figure 3.16 for 37% to 122% of the datum global Reynolds number (as rotation speed increases). The BPF outlet tone increases by up to ~7 dB while 2BPF outlet tone only fluctuates by approximately ±1-2 dB. This insensitivity to the change in vane acoustic excitation further substantiates the finding in §3.4 and §3.5 that the 2BPF tone is mainly produced by the impeller alone. It is clear from Figure 3.16 that the vanes radiate sound more effectively to the outlet side as the increase in the BPF tone is consistently higher on the outlet compared to the inlet. Nonetheless the tone does transmit from the guide vanes to the inlet giving an increased level, particularly at the lowest frequencies. The increase in the inlet tone due to the vanes drops off more quickly than that of the outlet up to 1.2kHz which is consistent with the declining fan transmission coefficient measured up to this frequency (Figure 2.11b). Above 1.2 kHz the vane interaction tone enhancement becomes greater again.

Figure 3.16 – Tone level increases for the \( B = 9 \) impeller interacting with \( S = 9 \) vanes relative to the datum \( S = 22 \) vane case as global Reynolds number is varied
The repeatability of the harmonic BPF excitation occurring at adjacent vanes means that the resulting amplitude is lower with more vanes due to destructive interference between the acoustic fields of each vane. Conversely, the random nature of excitation from turbulence impinging on the vanes means that destructive interference between the fields from adjacent vanes is not possible at these broadband frequencies. It is for this reason that an increase is seen at the broadband frequencies around the BPF with 22 vanes compared to 9 vanes (Figure 3.13 - Figure 3.15). Figure 3.13c highlights that at the other significant broadband frequencies the noise signature is remarkably unaffected. This is likely to be due to the fact that the noise at these frequencies is coming from the impeller. Noise source reduction targeted at these frequencies should therefore focus on the impeller. Low-order modelling of the guide-vane interaction mechanism is undertaken in Chapter 4.

3.7 Origin of tones in the absence of vanes

It has been seen in §3.4 that noise at blade-passing frequency harmonics is generated even in the absence of guide vanes. As discussed in §1.2.2.2, the generation mechanism for rotor-alone noise with subsonic flow is normally inflow distortion. This could come from the flow upstream of the impeller but also from the leakage path from the impeller outlet to the inlet via the outside of the shroud (see Figure 1.9). In this section the former source of distortion is investigated experimentally to understand the relative importance of both sources, in view of the fact that the latter source is more difficult to examine.

A schematic of the flow entering the impeller from upstream is shown in Figure 3.17. Since the diameter of the bellmouth is considerably smaller than the diameter of the duct, the flow must separate from the walls and the streamtube contract. This raises the prospect of elongation of turbulent eddies to produce a correlation between the forces on adjacent blades.
3.7 Origin of tones in the absence of vanes

To investigate the spatial distribution and unsteadiness of the impeller inlet flow, a one-dimensional straight hotwire probe (Dantec type 55P11) is traversed in a plane immediately upstream of the bellmouth (see Figure 3.17). This is calibrated using the same calibrator as in §4.4.1 for which the velocity sample uncertainty is quantified in Appendix F. An automated traverse system designed by Dyson [78] and integrated by them into the Dyson ISO rig (§2.2) moves the probe radially and azimuthally, and is pictured in Figure 3.18a. The wire, oriented perpendicular to the duct axis as shown in Figure 3.18b, is moved radially using the stepper motor attached to a lead screw. The azimuth of the probe is varied using a worm gear driven by a stepper motor which accurately rotates the duct section. In the measurements presented next, a circular plane is explored in radial steps of 5mm and azimuthal steps of 11.25°. The mean velocity is calculated based on 5 second samples at 10kHz for each point.
Figure 3.18 – (a) External, side view of components of automated traverse system integrated into Dyson ISO rig (reproduced from [78]) (b) view along duct axis with hotwire probe inserted radially with wire oriented perpendicular to axis.

At the design flow coefficient, the effect of the streamtube contraction is clear from Figure 3.20a which shows the mean velocity (normalised by $u_{ref} = \Omega R_{s2}$) is reasonably
3.7 Origin of tones in the absence of vanes

axisymmetric and drops off quickly outside the radius of the impeller shroud $R_{s1}$. The turbulence intensity (TI), calculated as the rms of velocity fluctuations over the mean velocity, is low (~1.5% or less) within $r \leq R_{s1}$, after which regions of apparent high intensity appear due to the fluid being almost stationary.

The flow region considered relevant to noise generation is that contained within the radius defining the shear layer outer radius. Within this radius, velocities are high and the effect of the shear layer is expected to be most significant due to high radial velocity gradients. A series of experiments to assess the sensitivity of the noise signature to the flow condition are detailed next.

To distinguish the effect of turbulence on tonal and broadband noise, a grid with 2x2 cm square holes, pictured in Figure 3.19a, is initially inserted 10 cm upstream of the impeller. This is designed to generate turbulence “without a change in the mean velocity profile” [78]. Hopper et al. [78] show that this achieves only a modest increase in turbulence intensity (to TI ≈ 5%) at the impeller inlet plane due to the steadying effect of the streamtube contraction. The broadband level in Figure 3.19b thus increases slightly while the BPF tone remains
almost constant. The 2BPF tone reduces with higher TI which suggests that the source of this tone is most sensitive to inflow turbulence. This is consistent with the findings in previous sections that the 2BPF tone comes from the impeller.

Figure 3.20 – Characterisation of inlet flow condition (a) datum condition - mean velocity distribution (b) datum condition - TI distribution (c) distortion condition - mean velocity distribution (d) distortion condition - TI distribution

To investigate the tone level sensitivity to azimuthal inlet flow distortion, a flow obstruction is introduced at the impeller inlet plane. At the blade-passing frequency, only plane-waves propagate, and this mode is excited strongly if

\[ m = B + kS = 0 \]  (3.2)
where the number of blades $B = 9 \ (k \in \mathbb{Z})$ - this occurs for interaction with 1, 3 or 9 equally-spaced obstructions. Flow obstructions spaced this way introduce a periodicity in mean velocity which repeats once, three, or nine times around the circumference.

A rectangular flow obstruction introduced close to the impeller inlet gives a clear region of low velocity as shown in Figure 3.20c. The effect of the blockage appears quite localised – the mean velocity is reasonably independent of angle in the semi-circular region opposite the obstruction. Figure 3.20d shows that the TI increase in the vicinity of the obstruction is even more localised, and appears primarily in the regions where the flow is almost stationary. For these reasons, the influence of the azimuthal distortion is expected to outweigh that of increased turbulence.

At each radius, the variation of mean velocity with azimuth $\varphi$ is decomposable into Fourier components of the form $e^{im' \varphi}$. For the datum condition of Figure 3.20a, the decomposition in Figure 3.21a shows the mean flow has a non-axisymmetric component which peaks at $r/R_{s1} = 1.3$ around the shear layer. This leads to the generation of a plane-wave mode at blade-passing frequency since the key Fourier components of order 1 (in particular), 3 and 9 are all present. The datum condition outlet acoustic spectrum for the vaneless geometry operating at the design flow coefficient, Figure 3.22, shows a strong tone at the BPF attributable to the impeller through this mechanism.
The distorted mean velocity of Figure 3.20c corresponds to the decomposition shown in Figure 3.21b. The non-axisymmetric components peak around $r/R_{s1} = 0.6$ and again contain the key components - order 1 in particular – which lead to plane-wave generation at the BPF. Due to the much higher magnitude of these components compared to the undisturbed case, the BPF increases by over 20 dB in Figure 3.22. Mainly at frequencies below 2BPF, the broadband levels also increase but to a lesser extent (typically by 5 - 10 dB).
While it is difficult to independently alter the level of ingested turbulence or steady azimuthal distortion to isolate each effect, the experimental data presented in this section suggests that the latter is the dominant factor. The tonal noise from the obstructed impeller increases markedly due to strong harmonic changes in the inlet velocity vector and hence loading on the impeller blades. Even with a nominally clean inlet flow, the shear layer region of the streamtube entering the impeller is not perfectly axisymmetric. This is a source of tonal noise measured in the absence of vane interaction as seen in §3.4.

The region of higher turbulence intensity behind the obstruction is localised and does not significantly alter the average TI within $r/R_{s1} \leq 1$. The increase in broadband noise is therefore due to increased impeller self-noise (from steady azimuthal distortion) rather than from turbulence ingestion. As a blade passes through the region of low axial velocity, the incidence in the impeller frame becomes too high for the flow to follow the blade curvature. This leads to broadband noise related to that seen in §3.5 at very low overall flow coefficient.

When the fan is installed in-product, the flow enters radially through vents and is turned sharply toward the axial direction. This gives a significant possibility for imperfections in the flow distribution across the inlet in terms of mean velocity and turbulence intensity. The
results in this section show that the inlet, including the vents, must be designed to optimally condition the flow.

### 3.8 Summary and conclusions

A broad study has been undertaken to understand parameters that affect low-speed fan noise. The relative importance of different noise sources for the regime of interest depends on a range of geometric, flow, and acoustic-related factors.

Fan aerodynamic performance is not infringed when more than halving the number of outlet guide vanes since they are lightly-loaded/low camber – the large number of vanes in the production fan is primarily for acoustic reasons. The configuration without any vanes has reduced performance due to geometry modifications (struts, annulus extension, etc.), which introduce additional loss. A Reynolds number effect is seen at low operating speeds, with and without vanes, as the pressure-rise coefficient decreases up to ~10% relative to the datum speed with the same $\phi$. The effect of Re on noise-inducing flow perturbations is studied in Chapter 4.

The impeller in isolation is found to produce noticeable BPF tones. The BPF tone is greater still with a vane number that should produce cut-off modes. This is thought to be due to imperfect vane field cancellation immediately downstream of the vanes where the flow is unsteady and abruptly changes radius. In the product, the convoluted flow path after the guide vanes is also very different to the constant-radius annulus model of Tyler and Sofrin’s theory (see §1.2.2.2). As such, similar imperfect cancellation can be expected.

At low flow coefficients the impeller produces high, stall-related self-noise centred on a frequency of approximately half the BPF. This behaviour resembles that of low-speed axial fans. Off-design performance at very low $\phi$ is not considered a key design parameter since it should not be encountered during normal operation of the product.

When selecting a low number of guide-vanes which produces a strong cut-on mode, the wake-vane interaction tone noise increases markedly on the outlet, and propagates to the inlet, at low frequency in particular. Only a small drop in broadband noise is observed at
some frequencies around BPF. It is apparent that other broadband frequencies – unaffected by the change in vane numbers - are most likely from the impeller.

Hotwire measurements at the impeller inlet show that the inflow turbulence level is low in the rig however small azimuthal distortions in the mean velocity produce BPF tones due to harmonic impeller blade loading. Azimuthal inflow distortion is unavoidable when the fan is installed in the product as the flow must enter radially through vents and turn to the axial direction. These results highlight the importance of conditioning the flow to minimise tonal noise.
Chapter 4  Low-order aeroacoustic modelling

4.1  Introduction

It has been seen in Chapter 3 that a significant source of noise is caused by interaction between the impeller and the guide vanes (rotor-stator interaction). Most affected are the tones which occur in a frequency range that is particularly sensitive to humans and therefore has a significant impact on sound quality. To a lesser extent broadband noise levels are reduced by decreasing the number of vanes. A proven low-order prediction scheme should help to explain noise trends in the design space and facilitate noise reduction at the source.

Low-order modelling has been a very successful approach in gas turbine applications to help designers produce quieter designs. It has allowed them to gain a deeper understanding of the physical mechanisms governing sound production, make predictions based on efficient schemes, and thus optimise designs to reduce noise at the source - all of which are key interests in the current context. However the applicability of these methods to the fans considered in this work is not precisely known. There are important differences in terms of the aerodynamics (e.g. incompressibility, much lower Reynolds number), acoustics (e.g. very low subsonic Mach numbers) and manufacturing precision which should be taken into consideration.

Conventional response models often applied to gas turbines estimate the unsteady loading on an aerofoil or cascade due to periodic or random ‘gusts’ that convect with the mean flow, and broadly lead to tonal or broadband noise respectively. In the case of a rotor, the incident disturbances could be due to different types of inflow distortion [79], while for stationary blades the disturbances are often due to unsteadiness from an upstream rotor.

The earliest and simplest analytical response model was developed by Sears [80] for a single, thin aerofoil of infinite span in an incompressible flow and is commonly referred to as the
Sears function. Compressibility effects were then included by Goldstein [81] and Amiet [82] who derived Sears functions for high and low frequency limits respectively. Departing from a purely linearised analysis, Goldstein and Atassi [83] examined the effect of thickness, camber and angle of attack for a single aerofoil in an incompressible flow. Extension of the response models of a single thin aerofoil to a linear cascade of finite span was carried out by Goldstein [81] for a compressible flow. Although it appears as if there are a range of these models for a variety of practical situations, care must be taken in selecting an appropriate approach as some clearly offer advances in a particular direction (e.g. more realistic geometry) at the sacrifice of making approximations in other areas. Furthermore, as noted by recently by Mish and Devenport [84] in their work on noise from aerofoils in turbulence, some models are lacking in experimental validation.

Modelling a blade row as a series of two-dimensional cascades stacked radially is referred to as the ‘strip’ approximation for the unsteady aerodynamics (Figure 4.1). Computational prediction codes based on this stem from the cascade response models developed in the 1960/70s by Whitehead [85] for incompressible flows, and later by Smith [86] and Whitehead [87] for subsonic flows. These were validated experimentally and shown to be very accurate for a lightly-loaded rotor with high hub-to-tip ratio at low Mach number. Huff noted that their subsequent widespread application and extension has led to the understanding that source non-compactness effects are important for tone prediction of turbofan engine fans [88], which have lower hub-to-tip ratios. The validity of using strip theory was questioned by Kobayashi [89] who included three-dimensional effects with a lifting surface theory developed by Namba [90]. It was found that the two-dimensional calculations are “reasonably adequate” as the tone duct mode acoustic power agreed to an accuracy of ±2 dB.
Although derived for periodic excitation, Smith’s theory [86] has been widely built upon to predict broadband noise created by a set of blades (rotor or stator) interacting with random turbulence [92]–[95]. For example, Figure 4.2 shows a comparison by Cheong et al. [92] of their predicted and measured (at NASA Lewis [96]) spectra for turbofan outlet guide vanes interacting with turbulence. Although the turbulence length scale and intensity inputs were selected to best-fit the experimental levels, the trends are clearly well captured with their 2D model for a wide range of frequencies. They also note that similar levels of agreement are obtained using a three-dimensional theory [97] and hence that three-dimensional effects are comparatively weak.
There is a comparatively small body of published research where low-order modelling has been applied to predict fan noise for non-gas turbine applications which may operate in a regime close to that being considered here. The main examples include the work of Gerard et al. [98], [99] who modelled and passively reduced the tonal noise from industrial ventilation fans, and Huang who extended the compressor noise theory of Lowson [100] and applied it to computer cooling fans [101]. However, the possibility of fundamental differences between the regimes has not been considered in these analyses.

As discussed in §1.2.2 the aeroacoustics of centrifugal fans has been studied in some detail using experiments and CFD [17], [42], [62], [65], [102], [103], however the fully three-dimensional nature of the flow means low-order modelling is difficult. Empirical models for broadband noise based on past designs, for example Mugridge’s correlation [102], are widely used in industry as they are extremely quick and only depend on fan overall performance parameters (i.e. $\phi$, $\psi$, $\eta$ etc.). There is little or no published research on mixed-flow fan aeroacoustic modelling.
The aim of the work outlined in this chapter is to develop a novel low-order prediction scheme for the tonal interaction noise of mixed-flow fans and make comparisons with experimental data. An important requirement is that efficient predictions can be made for a variety of operating points to support parameter studies and aid designers. The intrinsic link between sound generation and the flow field means that an experimental approach is appropriate to be sure that those important influences, such as increased viscous effects, are included. Although the focus is on tonal sources, the scheme, if proven, should be extendable to broadband sources. The relative mechanical simplicity of these types of fans means that geometry modifications are made easily and quickly using 3D printing techniques.

Figure 4.3 shows several key features about the design distribution of mean flow across the span as it enters the guide vanes. Figure 4.4 shows the guide vane geometry in terms of non-dimensional parameters, and the linear stagger variation to give more constant incidence across the span. Flow angle is based on a RANS simulation (with SST turbulence model) performed by Dyson at the design flow coefficient and datum speed. From this information:

1. The radial velocity component is negligible because the flow passages turn the flow towards the axial direction
2. The absolute velocity is reasonably constant across the central 50% of the span of the high hub-to-tip ratio blades
3. The blade sections are thin and lightly cambered
4. The incidence of the flow onto the blades is low and reasonably constant
These points suggest the following approximations should be valid, respectively:

1. A quasi-three-dimensional acoustic model whereby the flow at a given radius is considered two-dimensional
2. A mean flow represented by the mean-line conditions or by strip theory across the span
3. A flat plate blade model
4. A minimal steady loading effect on unsteady loading which generates the sound
Figure 4.4 – Guide vanes design for the production fan (a) 3D view of geometry (b) geometric parameters and swirl angle of mean flow entering the guide vanes
The sections in this chapter first detail the derivation of Smith’s cascade response model which approximates the blades as flat plates to predict the unsteady loading due to velocity perturbations from upstream. The model is applied to the case of the fan guide vanes which experience an upwash perturbation from the impeller – this upwash is found using a two-dimensional hotwire inserted into an adapted version of the fan. Hotwire and CFD data on the mean-flow velocities are compared to show that the two-dimensional approximation holds. Rapid-prototyped impellers and guide vanes with the same blade geometry but different blade numbers are characterised experimentally. The measured interaction BPF tone is compared to predictions at a range of rotational speeds (i.e. global Reynolds number) but with equal flow coefficient. This highlights differences which can be attributed to changes in Reynolds number.

4.2 Cascade response model

Smith [86] starts with the basic equations: linearised momentum and continuity, where the flow is assumed to be isentropic. The small perturbation quantities \( u, v, p \) etc. are specified to have harmonic space and time dependence of the form
\[ u = \bar{u} \exp i(\omega t + \alpha x + \beta y) \]  

where \( \bar{u} \) is an amplitude constant of the perturbation, \( \alpha, \beta \) are wavenumbers and the directions are shown in Figure 4.5. The condition for a non-trivial solution to the linearised equations implies two different physical phenomena: a pressure wave travelling upstream and downstream, and a spatial vorticity distribution convected by the mean flow. Both of these contribute to the solution.

The blades are modelled as flat plates with a distribution of singularities (vorticity) along the chord line of each blade in the two-dimensional infinite cascade. These fluctuate in strength at frequency \( \omega \) and events occurring at any particular blade are duplicated at all other blades with a constant phase angle \( \sigma \) between each blade and its neighbour. This requires that the wavenumber in the \( y \) direction is given by

\[ \beta = \frac{\sigma - 2\pi r_k}{s} \]  

where \( r_k \) is an integer which is linked to the azimuthal order of the mode.

The pressure wavenumber in the \( x \) direction, dependent on \( \beta \) and the frequency \( \omega \), follows from the non-trivial solution condition and is given by

\[ \alpha = \frac{U(\omega + V\beta) \pm a_0 \sqrt{(\omega + V\beta)^2 - (a_0^2 - U^2)\beta^2}}{a_0^2 - U^2} \]  

This leads to the typical ‘cut-on’ or ‘cut-off’ behaviour which is dependent upon the sign of the expression under the square root. For

\[ (\omega + V\beta)^2 - (a_0^2 - U^2)\beta^2 > 0 \]  

the quantity \( \alpha \) is real and the disturbances propagate as acoustic waves i.e. ‘cut-on’. While for

\[ (\omega + V\beta)^2 - (a_0^2 - U^2)\beta^2 < 0 \]  

the quantity \( \alpha \) has an imaginary part so the disturbances decay exponentially with distance from the source i.e. ‘cut-off’. Acoustic resonance occurs when the expression equals zero at the cut-off frequency giving a single, real value of \( \alpha \). This corresponds to a single wave being
generated which propagates in the tangential direction [104] and the effect on the amplitude near this condition can be seen in Figure 4.6.

Since the blades are flat and aligned with the mean-flow, the vorticity distribution (and unsteady lift) depends only on the incoming velocity perturbations perpendicular to the chord-line i.e. the upwash component. The physical condition of zero net upwash to the blade surface at all points $\hat{z} = z/c$ along the chord is used to relate the input upwash perturbation $w(\hat{z})$ to the sum of the contributions from chordal elements $d\hat{z}_0$ of vorticity at $z_0$ of (normalised) strength $\Gamma_W = \Gamma/W$

$$\frac{w(\hat{z})}{W} = \int_0^1 \Gamma_W(\hat{z}_0)K(\hat{z} - \hat{z}_0)\, d\hat{z}_0 \quad (4.6)$$

The kernel function $K(\hat{z} - \hat{z}_0)$ in equation (4.6) contains two different types of infinite series corresponding to the contributions from convected vorticity (which is summed analytically) and from the pressure waves (summed numerically). This integral equation is converted to a matrix equation to solve numerically for the vorticity distribution which is then related to response of the cascade in terms of acoustic waves, lift coefficient, moment coefficient and shed vorticity.

The acoustic response function output $F^\pm$ of the model is shown to be only a function of the non-dimensional parameters summarised in Table 4.1, where the superscripts ‘+’ and ‘-’ indicate the upstream and downstream-propagating waves, respectively.
4.2 Cascade response model

Table 4.1 – Input variables to the cascade response model of Smith

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacing-to-chord ratio</td>
<td>geometric</td>
<td>$s \over c$</td>
</tr>
<tr>
<td>Stagger angle</td>
<td>geometric</td>
<td>$\theta = \tan^{-1} \frac{V}{U}$</td>
</tr>
<tr>
<td>Mach number</td>
<td>mean flow</td>
<td>$M = \frac{W}{a_0}$</td>
</tr>
<tr>
<td>Reduced frequency</td>
<td>perturbation</td>
<td>$\lambda = \frac{\omega c}{W}$</td>
</tr>
<tr>
<td>Inter-blade phase angle</td>
<td>perturbation</td>
<td>$\sigma$</td>
</tr>
</tbody>
</table>

The amplitude of the acoustic waves at frequency $\omega$ are then given by

$$A^\pm = \rho_0 \bar{w} W F^\pm \left( \frac{s}{c}, \theta, \frac{W}{a_0}, \frac{\omega c}{W}, \sigma \right)$$  \hspace{1cm} (4.7)

where the leading-edge upwash perturbation at frequency $\omega$ is harmonic and of the form $\bar{w} \exp(i\omega t)$ with constant magnitude $\bar{w}$.

Whitehead wrote the LINearized SUBsonic (LINSUB) program in Fortran to implement Smith’s theory with the vorticity distribution comprising a discrete number of points along the chord line. As shown in Figure 4.6, results from a MATLAB version of this code\(^8\), used in this work, match well with the computational results presented in Smith’s original paper.

\(^8\)Originally written by V. Jurdic at the Institute of Sound and Vibration Research, University of Southampton
A description of the prediction scheme that has been implemented to apply the cascade response model is given in the following section.

### 4.3 Prediction scheme

The acoustic modes determined by the cascade response model are equivalent to the interaction patterns described by Tyler & Sofrin [23]. The key parameter from Table 4.1 is the inter-blade (or inter-vane) phase angle which is specific to the interaction pattern being considered.

This can be seen by considering $B$ blades interacting with $S$ vanes for which several modes at each frequency of azimuthal order $m = NB + kS$ are generated for different values of $k \in \mathbb{Z}$ of the form $e^{im\phi}$ (as discussed in §1.2.2.2). The phase difference between events occurring at adjacent vanes, located at angles $\phi_1, \phi_2$, is simply $m(\phi_2 - \phi_1)$. Since the angular spacing $\phi_2 - \phi_1 = 2\pi/S$, the input inter-vane phase angle to the cascade model for the mode is

$$\sigma = m(\phi_2 - \phi_1) = m \frac{2\pi}{S} \quad (4.8)$$
A routine to call the cascade response model function has been written which first determines the cut-on modes for a given fan configuration. The condition in equation (4.4) can be shown to imply a range of ‘cut-on’ phase angles as a function of the other input parameters:

\[
M \sin \theta - \sqrt{1 - M^2 \cos^2 \theta} \leq M \sin \theta + \frac{\sigma}{c} \leq \frac{\sigma}{s M \lambda} \\
\frac{1}{c} \sqrt{1 - M^2} \leq M \sin \theta + \sqrt{1 - M^2 \cos^2 \theta}
\]  

(4.9)

where the vane spacing at the radius \( R \) is given by \( s = 2\pi R / S \). This cut-on condition is closely related to Tyler & Sofrin’s [23] requirement for a supersonic pattern tip speed. For each mode which is found to give a phase angle in the cut-on range, the cascade response model function is evaluated. This process is repeated at each strip across the span for which the geometric \( (\theta, s/c = 2\pi R / Sc) \), mean flow \( (M) \) and perturbation parameters \( (\sigma, \lambda = 2\pi NB\Omega c / W) \) may vary. For a given strip, the process to create an equivalent flat-plate cascade is illustrated in Figure 4.7.

![Figure 4.7 – Steps to go from annular blade row strip to infinite flat-plate cascade (reproduced from [91])](image)

**4.4 Experimental upwash determination**

The leading-edge upwash perturbations induced by the unsteady flow from the impeller impinging on the guide vanes are difficult to measure in a fully representative way using hotwire anemometry. Since the net upwash is by definition zero on the surface, it goes without saying that one cannot choose a measurement location very close to the vane leading-
edge as the normal velocity will tend to zero. Measurement locations close to the impeller outlet will also not account for the evolution of the flow as it is turned \( \sim 45^\circ \) by the flow passages – a particularly significant effect in the mixed-flow configuration compared to axial-flow. Furthermore, the small scale of the fans, coupled with the limited robustness/compactness of hotwires means that it is difficult to accurately position the probe with potentially only a small spacing between each vane.

The approach taken in this work is to make modifications to the fan design to enable removal of the vanes and positioning of a radial hotwire traverse measurement where the vane leading-edge was originally. By removing the vanes (as described in §3.2), the upstream influence of the vanes on the impeller and on the wake evolution is neglected. The performance curves of §3.3 suggest that even halving the number of vanes has a limited effect on the performance which implies that the flow angles are largely unaffected. This approach is akin to the method of Smith in his original rotor tonal noise experimental validation [86], and to Jenkins’ turbofan OGV broadband noise predictions [91]. The resulting CAD rig integration design is shown in Figure 4.8a and the actual printed housing in Figure 4.8b. This is integrated into the new rig (§2.3) to control the operating point. The uncertainty with which the same operating point can be achieved in both rigs with the 3D printed setup is discussed in Appendix D (Figure D.3).
In the original configuration, included again for comparison, the vanes serve as the rigid link between the hub (housing the motor) and the casing. In place of this link are four cylindrical struts which provide sufficient rigidity with the annulus extended by 60mm in the downstream direction. This extension distance (of six strut diameters) has been chosen so that the inviscid static pressure field of the cylinders has a negligible influence on the measurement location. Furthermore, the wakes have a greater distance to mix out before interacting with flow passage obstruction.

To minimise problems due to prong interference and high turbulence intensities, Dantec recommends the use of sensor wires with gold-plated ends for which the prong spacing is larger while maintaining the same active length as the conventional type. Accordingly a two-dimensional X-probe gold-plated hotwire (Dantec cross-flow type 55P52) is used here which is designed for radial insertion such that the sensor plane, perpendicular to the probe axis, coincides with a plane through the mid-radius of the strip being considered.

Figure 4.8 – Guide vane upwash determination set-up as part of the new rig (a) CAD design (b) picture
The process to convert the voltage time series from each wire $E_1, E_2$ to an upwash velocity perturbation $\bar{w}(\omega)$ is as follows:

1. Apply predetermined polynomial linearisation functions $f_1(E_1), f_2(E_2)$ to give calibration velocities $W_{cat1}(t), W_{cat2}(t)$. Sample fits are shown in Figure 4.10a.
2. Decompose calibration velocities into 2D velocity components $U(t), V(t)$ according to the following equations [105]:

$$U = \frac{1}{\sqrt{2}} \sqrt{(1 + k_2^2) W_{cat2}^2 - k_2^2 W_{cat1}^2}$$  \hspace{1cm} (4.10)$$

$$V = \frac{1}{\sqrt{2}} \sqrt{(1 + k_1^2) W_{cat1}^2 - k_1^2 W_{cat2}^2}$$  \hspace{1cm} (4.11)$$

where $k_1, k_2$ are the yaw factors for each wire from a directional calibration.

3. Resolve to find upwash component $w(t)$ by taking the dot product of velocity vector \{U, V\} with unit normal vector to the vane chord line \{−sin $\theta$, cos $\theta$\} and take the Fourier Transform of this to get $\bar{w}(\omega)$.

### 4.4.1 Calibration of cross-wire probe

The velocity and directional calibrations are performed using a jet calibrator (Dantec type 54H10), shown in Figure 4.9a which produces a constant, low-turbulence jet of known velocity. The angle of the probe is varied accurately using a pitch/yaw/roll manipulator (Dantec type 90H03).
4.4 Experimental upwash determination

Velocity calibrations are carried out with the probe rotated such that the angle $\theta$ between the flow direction and Wire 2 is $45^\circ$ (see Figure 4.9b). These calibrations cannot be carried out with the flow perpendicular to each wire in turn, as one would normally with a single wire probe. This is because when $\theta \approx 0^\circ$ the active length of Wire 1 is in the wake of a prong from Wire 2 and a falsely low cooling velocity is measured. Equations (4.10) and (4.11) are therefore derived by stipulating that the calibration velocities $W_{cal1}, W_{cal2}$ are determined as functions of the voltages $E_1, E_2$ with the flow direction set at $\theta = 45^\circ$. The effective cooling velocity $W_e$ dependence on $\theta$ is modelled using a yaw dependence equation [106] which for Wire 2 in Figure 4.9b is given by

$$\frac{W_{e2}}{W} = (\cos^2 \theta + k_2^2 \sin^2 \theta)^{\frac{1}{2}}$$

(4.12)

where the subscript 2 denotes Wire 2, and $k_2$ is its yaw factor. The velocity calibration is carried out simultaneously for both wires for a range of jet velocities which cover the expected range. Figure 4.10a shows polynomial fits to a 10-point velocity calibration for both wires which differ slightly due to wire resistance variations.
Figure 4.10b shows that the measurement points from the direction yaw calibration line up very well with the equation for the yaw dependence of Wire 2. It is clear that for angles of less than 25° the equation fit for Wire 1 is poor. This is due to the fact that Wire 1 is affected by the wake of the prong of Wire 2 at those angles. From the CFD velocity data presented in Figure 4.4b it can be seen that the expected mean circumferential velocity at the measurement location is greater than the mean axial velocity given with a flow angle of 57° at mid-span. The cross-wire is therefore inserted such that Wire 2 was aligned with the axial direction giving flow angles relative to the probe greater than 45°.

### 4.4.2 Comparison with RANS simulation

To validate the two-dimensional approximation based on cross-wire measurements of the modified geometry, comparisons have been made with data from the RANS simulation described previously. The RANS simulation utilises a mixing plane between the rotating impeller domain and the stationary domain of the guide vanes whereupon a circumferential mean is taken. This implies it is possible to extract time-averaged conditions at the guide vane inlet (only) for comparison with the time-average of the experimentally-measured data. Figure 4.11 shows that the agreement in terms of the velocity magnitude at mid-span is
excellent while the angle is slightly lower in the experiment (by 3°). This is potentially due to the presence of the guide vanes in the simulation (not present in the experiments) which have begun to turn the flow slightly by the time it reaches the leading-edge. The effect on the magnitude of the velocity being measured with this level of deviation is minimal.

![Graph showing comparison between experimental and RANS mean velocity vector](image)

Figure 4.11 – Comparison at the same flow coefficient between experimental and RANS mean velocity vector (normalised by $u_{ref} = \Omega R_s^2$) at the guide vane inlet velocity (black) and angle (red)

Furthermore, this demonstrates that the approach of simplifying the fan geometry to exclude the vanes for the purposes of finding the input to the acoustic model can be applied using CFD methods. Fast, steady simulations to find the leading-edge upwash can be performed (e.g. RANS) whereby the impeller’s rotating domain (for a single blade passage) is extended downstream past the original vane location with an appropriate boundary condition for the hub/casing surfaces (not stationary in the rotating frame). This fully accounts for the evolution of the impeller outflow as it is turned by the flow passages, while minimising computation time.
4.5 Measurements on printed geometries

It has been discussed in §1.2.2.2 and §4.3 that at a given multiple of BPF, several propagating modes can be generated and this depends on the inter-blade phase $\sigma$. The most simple of these is the plane-wave mode which corresponds to neighbouring blades being perturbed at the same instance ($\sigma = 0$). This type of mode is selected in the following experiments to produce a strong fundamental blade-passing tone by having an equal number of blades and vanes. All other modes are cut-off at this frequency. A strong interaction ensures that the tone produced is easy to distinguish experimentally from the other tonal sources.

At rotational frequencies up to 10,000 RPM (122% of datum global Re), the highest BPF is 1.8 kHz corresponding to a wavelength $\lambda = 0.2$ $m$. Therefore, unlike the high-speed case of a turbofan OGV mentioned in the Introduction of this chapter, the guide vanes are acoustically compact both in terms of chord and span. Radial effects on the acoustic perturbation are minimal and only the variation of the mean-flow parameters need be considered.

The sound from the production 9-bladed impeller interacting with 9 vanes has been measured using the Dyson ISO rig described in §2.2. For this test case the blade spacing is very large in both the impeller and guide vanes. Tests on an 11-bladed impeller interacting with 11 vanes have been carried out to explore what blade spacing is accurately modelled as a cascade (as opposed to a series of isolated aerofoils). The printed parts pictured in Figure 4.12 are produced using the stereolithography (SLA) technique which has high accuracy and smooth surface finish.
4.5 Measurements on printed geometries

4.5.1 Acoustic effect of sudden expansion in the experiments

The in-duct measurements made using single microphone measurements are accurate as the sound is travelling as a plane-wave. In the experiment the outlet acoustic waves generated by the guide vanes encounter a sudden expansion from the annulus to the full width of the duct, spreading the emitted power over a much larger area. This situation is shown schematically in Figure 4.13.
Following the method described by Dowling & Ffowes-Williams [32, Sec. 3.1], the amplitude of the plane wave travelling in the annulus towards the expansion is denoted $A^{gv}$, any reflection $A^{ar}$, and the transmitted wave amplitude that is eventually measured some distance downstream under anechoic conditions $A^s$. The mean-flow is very low Mach number and can be neglected. By conservation of mass at the junction (indicated by a dash line), and from the plane-wave relation between acoustic velocity and acoustic pressure:

$$\frac{S_a}{\rho_0 a_0} (A^{gv} - A^{ar}) = \frac{S_d}{\rho_0 a_0} A^s \tag{4.13}$$

The junction is of negligible volume and has negligible energy storing capacity. Thus the energy flux into the function must equal the flux out, which leads to a condition where the pressure is continuous and

$$A^{gv} + A^{ar} = A^s \tag{4.14}$$

Equations (4.13) and (4.14) are solvable to estimate the change in amplitude due to the sudden expansion

$$A^s = \frac{2S_a}{S_a + S_d} A^{gv} \tag{4.15}$$
4.5.2 Dimensional considerations for the mean-flow

For a given strip, the downstream-propagating amplitude $A^+$ repeated here from equation (4.7) is a function of several dimensionless parameters defined in Table 4.1

$$A^+ = \rho_0 \bar{\omega} W F^+ \left( \frac{S}{c}, \theta, \frac{W}{a_0}, \frac{\omega c}{W}, \sigma \right)$$

(4.16)

where $\omega = N B \Omega$ with $N = 1$ as the fundamental BPF is being considered.

Table 4.2 – Test ranges for each configuration

<table>
<thead>
<tr>
<th>Impeller type</th>
<th>$B$</th>
<th>Guide vanes type</th>
<th>$S$</th>
<th>Relative global flow coefficient, $\phi/\phi_d$</th>
<th>Global Re relative to datum</th>
<th>RPM</th>
<th>Vanes spacing-to-chord, $s/c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>9</td>
<td>SLA</td>
<td>9</td>
<td>1</td>
<td>37 - 122%</td>
<td>3000 - 10000</td>
<td>2.91 (mid-span)</td>
</tr>
<tr>
<td>SLA</td>
<td>11</td>
<td>SLA</td>
<td>11</td>
<td>1</td>
<td>37 - 122%</td>
<td>3000 - 10000</td>
<td>2.38 (mid-span)</td>
</tr>
</tbody>
</table>

With the flow coefficient $\phi$ kept constant as the rotational speed is changed (in the range shown in Table 4.2), and in the absence of Reynolds number effects, the mean-flow velocity triangles should scale with the impeller blade rotational speed. The mid-span data of the mean flow in Figure 4.14 shows that $W$ and $\omega$ are both scaling linearly with $\Omega$ as the reduced frequency $\omega c/W$ remains approximately constant, as does the flow direction $\theta$. Similarly, the other parameters $\sigma$ and $s/c$ are constant because they depend on the type of mode (always plane-wave, $\sigma = 0$) and blade geometry respectively. Therefore the dependence in (4.16) effectively simplifies to

$$A^+ \big|_{\phi = \text{const.}} = \rho_0 \bar{\omega} W F^+ \left( \frac{W}{a_0} \right)$$

(4.17)

i.e. the cascade response function $F^+$ only depends on the vane Mach number $M = W/a_0$ like in Smith’s experiments (Figure 4.6, [86]).
In the next section, dimensional arguments are applied to the unsteady flow quantity $\bar{w}$.

![Graph](image.png)

Figure 4.14 – Variation of acoustic model input parameters measured at the mid-span guide-vane LE location downstream of the (a) 9-bladed impeller (b) 11-bladed impeller. The design flow coefficient is kept constant as the global Reynolds number is varied (by changing $\Omega$).
4.5.3 Effect of Reynolds number on velocity perturbation

As with the non-dimensional mean-flow parameters, the upwash perturbation intensity $\bar{w}/W$ should only depend on flow coefficient $\phi$ if Reynolds number effects are small.

The velocity spectra in Figure 4.15 correspond to the mean-flow data points in Figure 4.14 for each impeller. For each global Reynolds number, a strong peak can be seen at the BPF along with smoothly-varying broadband perturbations from turbulence. Excitation at the second harmonic of BPF is minimal which seems to explain why there is no increase in this tone due to rotor-stator interaction (see §3.6).
Figure 4.15 – Variation of upwash velocity perturbation intensity spectra measured at the mid-span guide-vane inlet location downstream of the (a) 9-bladed impeller (b) 11-bladed impeller. The design flow coefficient is kept constant as the global Reynolds number is varied 37 – 122%

The trend in BPF level in Figure 4.15b demonstrates a significant effect of reducing Reynolds number on the periodic upwash perturbations which determine the tone amplitude. At the five highest Reynolds numbers, $\bar{w}/W$ is nearly a constant with an approximate value of 1% i.e.
independent of Re (and therefore dependent only on flow coefficient). As the Reynolds number is decreased further, the normalised perturbation increases by up to 60% corresponding to a 4 dB higher acoustic amplitude. Simultaneously the broadband perturbation intensity rises noticeably at frequencies below 1 kHz for the three lowest Reynolds number cases. This undoubtedly leads to increased broadband noise levels – an effect which can be examined in the future by adapting the present approach, as done previously for gas turbine applications [91], [94].

The trend in BPF level in Figure 4.15a for the 9-bladed impeller demonstrates the same Re effect, however the level consistency is not as exact at the five highest Reynolds numbers. The emergences of small-amplitude shaft harmonics show that there are stronger flow patterns in this impeller which only repeat once per revolution. This is thought to be due to the blade spacing being too high in the 9-bladed impeller to maintain a perfectly periodic flow from blade to blade. The 11-bladed impeller will therefore be taken as the preferred test case.

Taking the results of Figure 4.14 and Figure 4.15 together, they show that viscosity plays a more important role in the unsteady flow processes compared to the steady flow. The strong periodic component is due to blade wakes and secondary flows occurring inside the impeller. Increased boundary layer thickness or separation at low Reynolds number is a likely explanation for its increased relative importance in the global flow field.

4.5.4 Comparison of acoustic predictions and measurements

In this section the low-order modelling approach is applied to make predictions that can be compared against experiments.

To account for variations across the span, the cascade acoustic amplitude $A^+$ is calculated from (4.16) at four equally-spaced locations spanwise. The parameters $\theta$, $W/a_0$ and $\omega c/W$ are measured by traversing the hotwire radially (with the operating point kept constant). These amplitudes are averaged and corrected as discussed in §4.5.1 to account for the area change and give a prediction for the measured plane-wave amplitude in the duct:
Low-order aeroacoustic modelling

\[
A^s = \frac{2S_a}{S_a + S_d} \left( 1 - \frac{1}{p} \sum_{i=1}^{l} \frac{A^+}{A^{gv}} \right)
\]

(4.18)

where \( l = 4 \). Figure 4.16 shows how the calculated amplitude \( A^+ \) varies as the cascade model parameters change across the span. Four-equispaced locations at 20, 40, 60 and 80 percent span are used in (4.18) and their average \( A^{gv} \) is indicated. In terms of dB, the predictions based on the average are less than 1 dB lower than the mid-span value. Thus good agreement could be attained by only considering the conditions at mid-span, as done in Smith’s experiments [86], when the hub-to-tip ratio is high.

![Figure 4.16](image)

Figure 4.16 – Radial variation of cascade acoustic amplitude relative to the mid-span value for the 11-bladed impeller at the design operating point and datum Reynolds number.

Acoustic measurements are made using the anechoic rig described in §2.2. Upwash measurements taken separately using the vaneless set-up (Figure 4.8) are at the same flow coefficient and rotational speeds (i.e. the same global Reynolds number). With these matched, the impeller flow field is the same (dimensionally) in both cases. The two test configurations are summarised in Table 4.2.

The comparison for the 9-bladed impeller interacting with 9 guide vanes is shown in Figure 4.17 while the results for the 11-bladed impeller are shown in Figure 4.18. The best
agreement can be seen for the 11-bladed impeller for which the trend with increasing BPF is very well represented. Differences in the level are around 5 dB or less.

Figure 4.17 – Comparison for 9-bladed impeller operating at the design flow coefficient

Figure 4.18 – Comparison for 11-bladed impeller operating at the design flow coefficient

The trend of the 9-bladed experimental levels is less clear, particularly at frequencies below 1.2 kHz where the tone levels seem to remain fairly constant. This is thought to be because
the blade spacing in both the impeller and guide vanes is too high. As discussed in §4.5.3, high blade spacing in the impeller causes poor flow periodicity from one blade passage to the next. Furthermore, the acoustic coupling assumed in the guide-vane cascade response model may not be valid when the vane spacing reaches a certain level. The agreement at frequencies above 1.2 kHz is better but the trend is difficult to identify clearly without access to more data points.

### 4.5.5 Uncertainty analysis

In this section the potential sources of uncertainty in both the ‘Prediction’ and ‘Experiment’ data from Figure 4.17 and Figure 4.18 are discussed. The most significant sources are quantified to produce the uncertainty bars shown in the figures above.

**Prediction**

According to equation (4.17), the main parameters which affect the estimation of $A^{gv}$ are $\rho_0 = p_a/RT_a$, $\bar{w}$ and $W$ - the response function $F^+$ is relatively insensitive to small differences in $M = W/a$ (no nearby resonance like in Figure 4.6). Atmospheric temperature $T_a$ is known for the new rig set-up with a bias uncertainty of $<\pm1\%$ with an influence factor of 1 on $A^{gv}$ (defined in Appendix D) equivalent to an uncertainty of $<\pm0.1$ dB – this is neglected. Similarly, atmospheric pressure $p_a$ is known with a bias uncertainty of $<\pm0.1\%$ with an influence factor of 1 on $A^{gv}$ equivalent to an uncertainty of $<\pm0.01$ dB – again this is neglected. The mean velocity vector magnitude $W$ measured using the 2D hotwire can be estimated to be accurate to $\pm5\%$ since it is difficult to calculate how its total uncertainty relates to the uncertainty of a single velocity sample for a single wire (calculated in Appendix E to be 1.4%). With an influence factor of 1 on $A^{gv}$, the velocity uncertainty of 5% is equivalent to $\pm0.4$ dB. Assuming a similar uncertainty for $\bar{w}$ and adding the two gives a total uncertainty in $A^{gv}$ of $\pm0.8$ dB.

The simple 1D treatment to relate $A^{gv}$ and the final prediction $A^s$ in equation (4.15) is another potential source of uncertainty which may give a total greater than $\pm0.8$ dB. However, this order of magnitude is considered representative.


Experiment

The tone amplitudes measured in the Dyson ISO rig on the outlet side are subject to the limitations investigated in Chapter 2, namely:

- Inaccuracy when measuring tone frequencies with strong higher-order modes
- Lower transmission of the flow straightener for higher-order modes
- Amplitude modulation due to reflection phenomenon

The first two of the above are not applicable since a dominant plane-wave mode is being generated by the fan test cases. The effect of the reflection phenomenon (which is dependent on frequency) is estimated in Appendix E and is used to give the uncertainty bars in the previous section.

4.6 Summary and conclusions

A detailed study has been carried out to understand and efficiently model the tonal interaction noise from a mixed-flow fan operating at low speed and Reynolds numbers. Very few previous publications explore mixed-flow fan aeroacoustics in general, and acoustic modelling is limited to empirical correlations.

Low-order methods developed for gas turbine applications have been applied to the regime of this work. The influence of low Reynolds number on aeroacoustics is investigated. The size and mechanical simplicity of the production fans warrants an experimental approach based on rapid-prototyping (3D printing) of a variety of unscaled test case configurations. Complex boundary-layer effects such as transition are thus captured accurately without the reliance on turbulence/transition models of CFD methods.

The approximate two-dimensional quality of the guide vane flow is demonstrated using steady CFD data. The novel prediction scheme utilises a computational 2D cascade response model to link guide vane inlet flow perturbations to the noise generated. A modified, vaneless version of the production geometry is printed to allow cross-wire measurements of the noise-generating velocity perturbations originating in the impeller. The measured mean-flow is
compared with CFD data to validate the 2D approximation and the effect of removing the vanes. Dimensional analysis is used to illustrate the Reynolds number influence on the measured mean-flow and perturbations. The periodic BPF perturbation amplitude is used to make tone predictions which are compared to measurements on several printed impellers and guide vanes.

There are several key conclusions from the work in this chapter:

- Neglecting the upstream flow influence of the vanes by removing them makes only a small difference to the mean flow. This suggests that equivalent steady RANS simulations could also be used to calculate the input perturbations. A mixing plane would not be required so the rotor domain (for only a single flow passage) could be extended downstream to the guide vane location – with appropriate boundary conditions for the stationary flow passage sections. Analytical wake models have also been applied in axial-flow machines and may be useful to reduce dependence on experiments/CFD [95].
- Rapid-prototyping techniques are sufficiently practical and accurate to support experimental studies that can be compared against CFD.
- Noise-inducing normalised flow perturbations increase significantly at lower Reynolds numbers as viscosity governs the parts of the impeller flow field, such as the boundary layers, which are closely linked to periodic unsteadiness. Simultaneously, the global flow field in terms of the mean flow remains less affected.
- Acoustic predictions based on the mean-line flow conditions only differ from the ‘strip’ approach predictions by less than 1 dB as the hub-to-tip ratio is high.
- The low-order approach gives insight into and predictions of tone levels and trends in an efficient manner.
Chapter 5  Conclusions and future work

In the present thesis, a variety of experiments have been performed, and a range of methods and schemes developed. A summary of what has been done is given in this chapter, along with conclusions that can be drawn. Based on these, proposals for future work are made.

Chapter 2

A new rig and method has been developed to characterise the modes produced by a ducted aeroacoustic source and its passive acoustic properties. The method is formulated in terms of transfer functions between flush-mounted microphones and a reference microphone/signal, allowing isolation of the acoustic signal from localised flow noise. The rig is used to measure the first three modes but the framework can be extended to many more modes. The small-scale and low-speed character of the test case fans means that the number of modes/microphones is more practical than in gas turbine applications. Unlike in the ISO rig, microphones are flush-mounted thus giving low disturbance, minimising the settling length required between the fan and microphones, and hence the overall rig length. Conversely, the ISO rig is extremely long to give undisturbed conditions around the inflow microphones.

A location optimisation procedure applied to the six microphones required to decompose three modes shows that the optimum axial spacing is different to that of the simpler two-microphone technique. Likewise, the use of external sources to characterise the response of the source duct element to reflection/transmission is shown to be more complex. An automated system can be developed to excite external speakers at different axial/azimuthal locations to speed up the scattering matrix determination process. Different strategies can be tested utilising external excitation of multiple frequencies simultaneously, different frequency steps and extra speakers giving an overdetermined system of equations. These would speed up the process of characterising a fan spectrum over its operating range, while requiring minimal human input once the optimised process is automated.
Comparisons are made between the new and ISO rigs to understand any limitations of each rig. At plane-wave frequencies, an off-centre microphone in the ISO rig is expected to directly measure the source plane-wave mode amplitude since it is uniform across a section, and the duct is anechoic. The narrow-band comparison between the rigs is good from the point of view of how the third-octave band levels would compare, however the reflection phenomenon appearing to be only present at these frequencies in the ISO rig has the potential to affect tone level determination slightly.

Above the plane-wave frequencies, the ISO method assumes that each cut-on mode carries equal power such that third-octave sound power can be estimated from the SPL at mid-radius. Unlike for tones, broadband sources are not expected to preferentially excite a specific mode, and since broadband sources often contribute most to a band’s power, the equal mode power assumption may work. At the tones, depending on the high-order mode power relative to the plane-wave, measurements at mid-radius are found to under predict by up to ~6 dB. This may call into question conclusions drawn from narrow-band spectra measured in this way – as found in the work of many authors [5], [40]–[42]. If tone frequencies containing strong non-plane wave modes are of interest, mode decomposition techniques should be used.

Using the existing experimental set-up, the modal power distribution can be measured at sample broadband frequencies to quantify how well the assumption of equal mode power represents reality. These sorts of results can then feed into the development of measurement methods or computational models to ensure that the assumptions being made are backed-up by experimental data.

Measurements of the passive properties of a standard honeycomb flow straightener show how it particularly affects higher-order modes as the passages act like small ducts with width much less than the sound wavelength. Thus at modest acoustic frequencies, non-planar modes get cut-off, changing the outlet sound power by ~1 dB. The other ‘star’-type flow straightener, present in the Dyson ISO rig, can be tested in a similar way to see if it has a smaller effect due to its wider channel width. Ideally, the effect of a mean swirling flow should be included as a potential source in itself.
Chapter 3

The noise sources of a mixed-flow fan test case have been investigated by performing experiments with a series of modified components. Fan components are manufactured at Dyson using 3D printing techniques. Some level of manufacturing imperfection/asymmetry always exists in the end product used by the consumer. It would therefore be interesting to undertake a study which accounts for variations in, for example, the impeller flow periodicity due to imperfections, and its effect on the repeatability of noise measurements.

The majority of the tests focus on performance at the design flow coefficient. Rotor-alone sources are studied by comparing the reference configuration to the housing without vanes. Only changes in the tone levels are traceable to the absence of vanes as the cylindrical struts produce additional broadband noise. These tests show the impeller produces BPF tones even though its rotation speed is low subsonic. In order to isolate the broadband noise produced by the impeller, a new rig could be designed such that there are no flow obstructions in the annulus. This would require:

- A long extension to the annulus to give time for the flow to settle (illustrated in Figure 5.2)
- Flush-mounting of the microphones on the casing surface
- Re-formulation and application of the mode decomposition equations of Chapter 2 with a hard-wall boundary condition on the hub - the Mach no of the swirling flow should still be low enough to neglect
- A very stiff structure to hold the hub at the end of the annulus since it is effectively a very long cantilever i.e. a set-up resembling that of Smith [86, Fig. 5].

A particularly large amplification of the ‘cut-off’ BPF tone is seen at the outlet with 22 vanes due to imperfect cancellation of the acoustic field of each vane. The most abrupt change in the flow radius occurs in the rig immediately after the vanes – a situation substantially different from that assumed in Tyler and Sofrin’s theory [23]. This suggests that the abrupt radius changes that exist in the product – i.e. the flow entering the impeller axially from an initially radial direction, and immediately after the guide vanes – must be carefully designed. The transitions should be made as smooth/symmetric as possible to approximate the fixed-
Conclusions and future work

radius duct which would lead to mode attenuation. The strength of the interaction mode can also be reduced by finding ways to minimise the excitation from the impeller. For example, splitter vanes in the impeller (Figure 5.1) can be used to change the blade passing frequency to less annoying frequencies, and reduce the impeller loading towards the trailing edge which could reduce wakes and secondary flow. An extension to the spacing between the impeller and guide vanes would give wakes etc. more time to mix-out [107].

![Figure 5.1 – Mixed-flow impeller splitter vanes illustration (by Venter Consulting Engineers)](image)

When the impeller is operating at low flow coefficients below design, a characteristic hump grows at around half the BPF while, unlike in an unshrouded axial rotor subject to rotational instabilities, the BPF tone reduces until indistinguishable from the original broadband level. The disappearance of the BPF tone suggests that the rotating instability disrupts the $2\pi/B$ circumferential periodicity of the impeller and affirms that the tone at design is sourced in the impeller. The change to the flow field can be characterised using the existing hotwire set-up to measure velocity spectra downstream of the impeller - one expects the BPF to be diminished and the harmonics of the shaft increased. Complex and noisy interactions are expected between the asymmetric flow and downstream vanes – an effect measurable by repeating the experiments with vanes in place. Further investigation would be required to understand the mechanism of the rotating instability since, for an axial rotor, it is said to be linked/driven by tip vortex-blade interactions. However these are not present in this shrouded impeller.
Reduction of the number of vanes by more than half, producing a strong tonal interaction, gives a small decrease in broadband around the BPF while higher frequencies are unaffected. This suggests that relatively small overall broadband reductions can be made by reducing rotor-stator interaction. The other broadband sources, namely impeller self-noise and inflow distortion from turbulence, should be taken to be most significant, particularly at the frequencies which have been found to be unaffected by changes in the number of vanes. Complex methods to reduce interaction broadband, such as the trailing edge blowing attempted by Sutliff to reduce rotor wake turbulence [67], are likely to be similarly ineffective at reducing overall noise.

![Diagram of experimental set-up](image)

**Figure 5.2** – Experimental set-up specifically designed to produce ideal inflow conditions and isolate rotor-alone noise - for an axial fan in this illustration (adapted from [41])

The effect of inflow distortion on impeller noise has been characterised via inflow spatial hotwire traverses and by measuring the response to different distortions. Initial attempts to increase TI are seen to have a surprisingly small effect which is thought to be due to the streamtube contraction in the rig [78]. The impeller is shown to be highly sensitive to circumferential mean velocity distortion which helps to explain the tones produced without vanes. Even with the nominally well-managed inflow in the rig, circumferential non-uniformity in the streamtube contraction flow field is inevitable. Similarly, when installed in the product the flow enters radially through vents and turns abruptly giving significant potential for unsteadiness and asymmetry. This inlet must be carefully designed to turn and condition the flow optimally. To determine the minimum noise as a benchmark, the outlet
noise from the impeller can be measured with a hemispherical turbulence control screen fitted at the inlet (Figure 5.2).

Chapter 4

A low-order prediction scheme has been developed based on an existing model of how velocity perturbations induce unsteady loading on a 2D cascade of blades. Demonstrating the suitability of the model in a regime where viscosity plays a more important role, the scheme is used to predict rotor-stator wake interaction tones based on 2D hotwire input velocity data at a range of $Re$. The guide vanes flow of the mixed-flow fan is approximately 2D with minimal upstream influence on the impeller, while the thin vane sections can be considered flat plates. These simplifications lead to a computationally inexpensive scheme based on key geometric and perturbation parameters which are amenable to optimisation. The approach of measuring the impeller outflow in a configuration without vanes can be replicated in CFD for comparison with experiment, and to gain insight into the source of the noise-inducing perturbations (if the transition models are sufficiently accurate). As illustrated in Figure 5.3, a steady RANS solver with the rotor domain extended downstream to the original guide vane location (with appropriate boundary conditions) would give quick results.

![Rotor domain extension](image)

Figure 5.3 – Illustration of CFD rotor domain extension (with appropriate boundary conditions applied for stationary walls) for direct comparison with experimental set-up
Following the direction taken by others for turbofan rotor-stator interaction [95] (for example), a quick, low-order model could be used in place of experimental/CFD perturbation data to make the scheme even more useful for design space exploration. Based on an understanding of velocity perturbation sources, tone noise suppression techniques can be considered such as blade TE blowing to reduce momentum deficits [108], [109], or the use of lean and sweep to give acoustic cancellation along a single vane [75].

The mechanism for rotor-alone tonal noise produced by the impeller is closely related to that of the vanes: perturbations in inlet velocity in the blade frame leading to unsteady blade cascade loading. While the flow enters axially, radial components of velocity are large by the impeller trailing edge which makes simplified modelling difficult. The problem of a gust interacting with an aerofoil is often referred to as leading edge noise since most of the noise is produced at the leading edge. A possible avenue of investigation could therefore be to focus on the impeller leading edge region where the flow (entering axially in the rig) is approximately 2D and the sections are very thin.

Decreasing Reynolds number is seen to increase both the blade-passing and broadband components of perturbation intensity downstream of the impeller while the mean flow remains unchanged. This effectively means Reynolds number effects can directly increase interaction noise. It is to be expected that viscosity plays a more important role in altering the unsteady component of the flow field – which would be zero in an inviscid fluid – than it does the mean flow. One expects the boundary layers to thicken at low $Re$ but it is less clear if the increase in perturbation intensity can be linked to the greater propensity for (laminar) separation. Further experiments to determine the state of the boundary layers are required – the simplest way would be to introduce a trip and see if that affects the perturbations/noise. If the boundary layer is found to undergo laminar separation at low $Re$, surface treatments could be a simple way to improve aeroacoustic performance.

Based on the blade-passing perturbation intensity at a range of Reynolds numbers, the tone levels from an equal number of blades and vanes ($B = S$) interacting have been predicted. This combination is chosen as it produces a strong interaction (eclipsing other sources) and a single plane-wave mode whose propagation after the vane area expansion is simply modelled.
The prediction scheme matches well with experiments which demonstrate that the mechanism is well understood, and that the approach can be trusted for other designs. Using the existing rig and mode decomposition technique, predictions for a single cut-on BPF azimuthal mode can be made, e.g. with $B = 11$, $S = 12$ exciting $m = 1$, and compared to experiment (particularly around the cut-off frequency). In order to do this, the effect of the area expansion on an azimuthal mode would need to be quantified.

The broadband perturbation data can be used to study the effect of Reynolds number on broadband noise in a similar manner to the tones. The cascade response model applied in this work has been successfully adapted for a cascade interacting with turbulence in gas turbine applications [92], [95]. This mechanism is relevant to both rotor-stator interaction and rotor-alone broadband sources, the latter of which is highlighted above as a more important source in the present regime.
References


Appendix A Drive efficiency

The drive efficiency shows the losses due to the motor and motor controller. This is measured using a dynamometer attached to the shaft of the motor to measure output power vs the input electrical power to the plug of the motor controller. The production motor is selected to operate efficiently at the datum rotational speed (i.e. datum global Reynolds number). The dynamometer test data in Figure A.1 shows how the output shaft power increases monotonically with input electrical power at the datum rotational speed.

![Graph showing the relationship between output shaft power and input electrical power](image)

Figure A.1 – Dynamometer test data for the production motor and motor controller at the datum rotational speed (i.e. datum global Reynolds number)

Using the data in Figure A.1, the drive efficiency can be deduced when the motor is subsequently attached to the impeller by measuring the input electrical power for each flow coefficient point (along the datum characteristic). Figure A.2 shows that this efficiency is fairly constant for the typical flow coefficients encountered above $0.5\phi_d$. 
Figure A.2 – Variation with flow coefficient of the drive efficiency for the production fan at the datum rotational speed (i.e. datum global Reynolds number)
Appendix B Integrals related to duct sound power

B.1 Simplification of Bessel function integral

Integrating over a circular cross-section $S$ with elemental area $dS = 2\pi rd\!r$ and radius $R_d$

$$\int \int J_m^2 \left( z_{m,n} \frac{r}{R_d} \right) dS = 2\pi \int_0^{R_d} J_m^2 \left( z_{m,n} \frac{r}{R_d} \right) r\!dr$$  \hspace{1cm} (B.1)

Using the ascending series definition for $J_m$ and the recurrence relation relating $J_m$ and $J_{m-1}$ (see [110] for example) this integral can be expressed in terms of Bessel functions

$$2\pi \int_0^{R_d} J_m^2 \left( z_{m,n} \frac{r}{R_d} \right) r\!dr = S \frac{J_{m-1}^2(z_{m,n}) - 2m f_{m-1}(z_{m,n}) J_m(z_{m,n}) + z_m^2 J_m^2(z_{m,n})}{z_{m,n}}$$  \hspace{1cm} (B.2)

From the definition for the zeroes of $J_m^\prime(z)$ denoted $z_{m,n}$ and the recurrence relation for this differential quantity:

$$J_m(z_{m,n}) = J_{m-1}(z_{m,n}) = -\frac{m f_m(z_{m,n})}{z_{m,n}} = 0$$  \hspace{1cm} (B.3)

Substituting for $J_{m-1}$ in the right-hand side of equation (B.2) using this result simplifies the integral:

$$\int \int J_m^2 \left( z_{m,n} \frac{r}{R_d} \right) dS = S J_m^2(z_{m,n}) \frac{z_{m,n}^2 - m^2}{z_{m,n}^2}$$  \hspace{1cm} (B.4)

The definition for the normalisation factor, equation (2.6), can be written as
Integrals related to duct sound power

\[ C_{m,n}^2 \iint J_m \left( \frac{r}{R_d} \right) dS = S \]  \hspace{1cm} (B.5)

which leads to the final expression for \( C_mn \) by substituting the simplified expression

\[ C_{m,n} = \left| \frac{z_{m,n}}{J_m(z_{m,n})\sqrt{(z_{m,n}^2 - m^2)}} \right| \]  \hspace{1cm} (B.6)

### B.2 Source sound power integral

The acoustic pressure and velocity of the sound field are linked by the linear conservation of momentum equation. The axial component of this equation, neglecting the mean flow (as discussed in §2.5), leads to the following relations for waves travelling in the positive \( x \) direction only:

\[ \hat{\rho} = \frac{i}{\omega \rho_0} \frac{\partial \hat{p}}{\partial x} \]  \hspace{1cm} (B.7)

The source sound field is given by equation (2.9) with \( A_{m,n}^+ = A_{m,n}^s \) and \( A_{m,n}^- = 0 \)

\[ \hat{p} = \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} C_{m,n} e^{im\theta} J_m \left( \frac{z_{m,n}}{R_d} \right) A_{m,n}^s(\omega) e^{-ik_{m,n}x} \]  \hspace{1cm} (B.8)

The time-averaged axial intensity from the source, assuming the modes are cut-on (i.e. \( k_{m,n} \) is real), is given by:

\[ \bar{I}_x = \frac{1}{2} Re(\hat{\rho} \hat{\rho}^*) \]  \hspace{1cm} (B.9)

For a single cut-on source mode this quantity can be written:

\[ \frac{1}{2} Re \left( \frac{k_{m,n}}{k_0 c_0 \rho_0} \hat{p} \hat{p}^* \right) = \frac{k_{m,n}}{2k_0 c_0 \rho_0} C_{m,n}^2 J_m^2 \left( \frac{r}{R_d} \right) |A_{m,n}^s|^2 \]  \hspace{1cm} (B.10)
The source power is found by integrating the axial intensity over a duct cross-section. This leads to a simplified expression as the ‘cross-terms’ involving products of the amplitudes of different modes contain orthogonal functions which integrate to zero, thus:

$$\begin{align*}
P^s &= \iint \bar{I}_x \, dS = \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \frac{k_{m,n}}{2k_0c_0\rho_0} |A^s_{m,n}|^2 \left[ C_{m,n}^2 \iint J^2_m \left( z_{m,n} \frac{r}{R_d} \right) \, ds \right] \\
&= \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \frac{S k_{m,n}}{2k_0c_0\rho_0} |A^s_{m,n}|^2
\end{align*}$$

(B.11)

where the term in square brackets reduces to simply $S$ from equation (B.5). As expected the total power is a summation of the power contribution from each mode $P_{m,n}$, which can be written:

$$P^s_{m,n} = \frac{S k_{m,n}}{2k_0c_0\rho_0} |A^s_{m,n}|^2$$

(B.12)
Appendix C Results for other global Reynolds numbers

Global Reynolds number is varied at constant global flow coefficient by changing the rotational speed:

Table C.1 – Relation between rotational speed and global Reynolds number

<table>
<thead>
<tr>
<th>Rotational speed (RPM)</th>
<th>Global Reynolds number relative to datum (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>37</td>
</tr>
<tr>
<td>4000</td>
<td>49</td>
</tr>
<tr>
<td>5000</td>
<td>61</td>
</tr>
<tr>
<td>6000</td>
<td>73</td>
</tr>
<tr>
<td>7000</td>
<td>85</td>
</tr>
<tr>
<td>8200</td>
<td>100</td>
</tr>
<tr>
<td>9000</td>
<td>110</td>
</tr>
<tr>
<td>10000</td>
<td>122</td>
</tr>
</tbody>
</table>
C.1 Effect of removing vanes on BPF tones

Figure C.1 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at 49% of datum global Reynolds number (a) inlet (b) outlet
Figure C.2 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at 61% of datum global Reynolds number (a) inlet (b) outlet
Results for other global Reynolds numbers

Figure C.3 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at 73% of datum global Reynolds number (a) inlet (b) outlet
Figure C.4 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at 85% of datum global Reynolds number (a) inlet (b) outlet
Results for other global Reynolds numbers

Figure C.5 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and in the vaneless configuration at 110% of datum global Reynolds number (a) inlet (b) outlet
C.2 Effect of reducing number of vanes giving a ‘cut-on’ mode

Figure C.6 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and 9 vanes at 49% of the datum global Reynolds number (a) inlet (b) outlet
Figure C.7 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and 9 vanes at 61% of the datum global Reynolds number (a) inlet (b) outlet
Figure C.8 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and 9 vanes at 73% of the datum global Reynolds number (a) inlet (b) outlet
Figure C.9 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and 9 vanes at 85% of the datum global Reynolds number (a) inlet (b) outlet
C.2 Effect of reducing number of vanes giving a ‘cut-on’ mode

Figure C.10 – Acoustic spectra comparison for the 9-bladed production impeller with 22 symmetric vanes and 9 vanes at 110% of the datum global Reynolds number (a) inlet (b) outlet
Appendix D Uncertainty of fan operating point

The two key parameters relied upon to define the fan operating point are flow coefficient and pressure rise coefficient. The data reduction equations for these quantities in terms of the parameters with uncertainty from Table D.1 are

\[
\phi_g = \frac{Q}{\Omega D_{s2}^3} = \frac{\alpha \varepsilon \frac{\pi D_{fm}^2}{4} \sqrt{\frac{2 p_a}{RT_a} \Delta p_{fm}}}{\Omega D_{s2}^3}
\]

(D.1)

\[
\psi = \frac{\Delta p_0}{\rho \Omega^2 D_{s2}^2} = \frac{\Delta p_0}{\frac{p_a}{RT_a} \Omega^2 D_{s2}^2}
\]

(D.2)

where the flow rate \( Q \) is given in a generalised form from equations (2.1) and (2.2) with the subscript ‘\( fm \)’ used to signify either of the two flow meters.

Table D.1 – Sources of uncertainty in measuring the parameters which define the global flow coefficient and pressure rise coefficient

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric temperature, ( T_a )</td>
<td>Thermometer bias</td>
</tr>
<tr>
<td>Atmospheric pressure, ( p_a )</td>
<td>Barometer bias</td>
</tr>
<tr>
<td>Pressure across flow meter, ( \Delta p_{fm} )</td>
<td>Manometer precision</td>
</tr>
<tr>
<td>Stagnation pressure increase across fan, ( \Delta p_0 )</td>
<td>Manometer precision, flow straightener inefficiency bias</td>
</tr>
<tr>
<td>Rotational speed, ( \Omega )</td>
<td>Motor control precision</td>
</tr>
<tr>
<td>Compound coefficient, ( \alpha \varepsilon )</td>
<td>Calibration bias</td>
</tr>
</tbody>
</table>
The rotational speed is kept constant with a precision of ±0.1% at the speed corresponding to the datum Reynolds number, and is averaged over the time-series measurement period. The uncertainty of this parameter is therefore negligible and is neglected. The stagnation pressure increase across the fan is measured in conjunction with a low loss flow straightener according to the ISO standard [43]. The flow straightener pressure loss is approximately 25% of the dynamic head [111] which can be estimated as $0.5\rho(Q/\pi R_d^2)^2$. Since the velocities in the duct are very low, the bias uncertainty in $\Delta p_0$ this gives for the production fan at the datum $Re$ and $\phi_d$ is:

$$\frac{0.25 \times 0.5\rho(Q/\pi R_d^2)^2}{\Delta p_0} = \pm 0.1\% \quad (D.3)$$

i.e. very small – but increases somewhat at higher flow coefficients and so is included in the uncertainty calculation using equation (D.3). Manometer bias uncertainties of the differential pressures ($\Delta p_{fm}, \Delta p_0$) are removed by ‘zeroing’ the device before each measurement run.

Tests are performed in both the Dyson ISO rig and the new rig which use different pressure/temperature measurement devices (the motor controller is kept the same). It is important to ensure the operating point can be set accurately in both rigs. In the Dyson rig, the compound coefficient for the orifice plate has been calibrated by Dyson with a reference Venturi flow meter in accordance with ISO 5167 [48]. In the new rig, the bellmouth flow meter component coefficient is taken to be 1 with an uncertainty of ±1.5% in accordance with the ISO standard [43].
C.2 Effect of reducing number of vanes giving a ‘cut-on’ mode

Table D.2 – Measurement uncertainty for key parameters measured in the Dyson ISO rig and new rig

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dyson ISO rig</th>
<th>New rig</th>
<th>Influence coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>Bias</td>
<td>Precision</td>
<td>$\phi_g$</td>
</tr>
<tr>
<td>$T_a$</td>
<td>±0.05 °C</td>
<td>±0.2 °C</td>
<td>0.5</td>
</tr>
<tr>
<td>$p_a$</td>
<td>±50 Pa</td>
<td>±67 Pa</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Delta p_{fm}$</td>
<td>±0.5%</td>
<td>±0.25 Pa</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Delta p_0$</td>
<td>Eq. (D.3)</td>
<td>±0.5%</td>
<td>Eq. (D.3)</td>
</tr>
<tr>
<td>$\alpha\varepsilon$</td>
<td>±0.53%</td>
<td>n/a</td>
<td>±1.5%</td>
</tr>
</tbody>
</table>

Following the ASME partial differential method [112] based on the uncertainties and influence coefficients for $\phi_g, \psi$ given in Table D.2, total measurement uncertainty of $\phi_g, \psi$ are calculated (a root-mean-square total). For the production fan at the datum Reynolds number, Figure D.1 shows that the characteristics measured in each rig are in excellent agreement. Furthermore, the uncertainty (indicated by the error bars) is good around the key design $\phi_g$ with a value of ±0.6% in the Dyson rig, and ±1.5% in the new rig (a conservative estimate limited by the compound coefficient uncertainty).
Figure D.1 – Characteristic comparison for the production fan operating at the datum global Reynolds number with experimental uncertainty estimates.

Figure D.2 shows at the design coefficient, the uncertainty in $\psi$ as $Re_g$ is small - much smaller than the magnitude change of $\psi$ measured in the Dyson rig.

Figure D.2 – Variation of loading coefficient with global Reynolds number at the design flow coefficient $(B = 9, S = 22)$ measured in the Dyson ISO rig with experimental uncertainty estimates.
C.2 Effect of reducing number of vanes giving a ‘cut-on’ mode

In order to confirm that the operating points set in both rigs at constant $\phi_g$ with varied $Re_g$ are comparable, the uncertainty is calculated for the production 9-bladed impeller in the vaneless housing. This is most important to the results in Chapter 4 where hotwire measurements are performed in the new rig while acoustic measurements are taken in the Dyson ISO rig assuming the same operating point is set. Figure D.3 shows that the operating point is known with good accuracy and that the uncertainty increases slightly at the lowest Reynolds numbers in the new rig.

Figure D.3 – Variation of global flow coefficient uncertainty with global Reynolds number at the design flow coefficient ($B = 9, S = 0$)
Appendix E Uncertainty due to amplitude modulation phenomenon in Dyson ISO rig

The comparison between the two rigs at the plane-wave frequencies in Figure 2.13 highlights an amplitude-modulation effect in the Dyson ISO rig that does not appear in the new rig spectrum. For a typical spectrum, the fluctuating component is extracted as a function of frequency through filtering – the results are shown in Figure E.1. Note that the blade-passing frequency peak occurring at 1.3 kHz in the reference spectrum is not included.

Figure E.1 – Typical outlet spectrum containing narrow-band fluctuations alongside the extracted amplitude fluctuations with the peaks and troughs indicated

Based on this data, the uncertainty as a function of frequency is estimated as half the difference between the peaks and troughs. In Figure E.2 this is plotted as a smoothed function for the tone frequencies studied in detail in Chapter 4
Figure E.2 – Uncertainty introduced due to reflection-like phenomena causing narrow-band amplitude fluctuations in the Dyson ISO rig.
Appendix F Uncertainty of a hotwire velocity sample

The uncertainty of the data obtained using the single and X-probe are a combination of the uncertainty of the individually-acquired voltages converted into velocity, and on how well the data represents the flow.

The uncertainty of a velocity sample from a single wire depends on factors related to instrumentation, calibration equipment and experimental conditions as detailed by Jørgensen [105].

F.1 Anemometer

The constant temperature anemometers used (Dantec MiniCTA type 54T42) have low drift, low noise and good repeatability so these factors do not add significantly to the uncertainty compared to other sources [105]. The frequency characteristic is flat up to the flow frequencies of interest and hence does not increase the uncertainty.

F.2 Calibration and linearisation

Calibration, performed using the jet calibrator (Dantec type 54H10), is a major source of uncertainty. The uncertainty of the jet velocity depends on the barometer, micromanometer and thermometer used to measure atmospheric pressure, the pressure difference across the jet nozzle and the air temperature. The barometer with a full scale error of ±0.42% influences the accuracy of the velocity calculation by ±0.42% of reading on average. The micromanometer (Furness controls FCO332 or DPS VLDPT) with an error of ±0.5% of reading influences the accuracy of the velocity calculation by ±0.5% of reading. The thermometer gives the
Uncertainty of a hotwire velocity sample
temperature correct to ±0.2 K which influences the accuracy of the velocity calculation by
±0.06% of reading. The relative standard uncertainty (RSU) is therefore ±0.66% typically.

The linearisation uncertainty is calculated as the standard deviation of the curve fitting
percentage errors. For the typical 10-point calibrations of Figure 4.10a this amounts to 0.26%
relative standard uncertainty.

F.3 Experimental conditions

The positioning uncertainty relates to the alignment of the probe in the experimental rig after
calibration and can be expressed as \((1 - \cos \theta) / \sqrt{3}\). Typically the probe can be positioned
with an uncertainty of \(\Delta \theta = 1^\circ\).

Temperature variations from calibration to experiment change the sensor over-temperature.
These are minimised by performing calibrations immediately before moving the probe to the
experimental rig.

Atmospheric pressure variations from calibration to experiment also have a minimal effect.

F.4 Combined uncertainty of a velocity sample

The important sources of uncertainty discussed above are quantified in the table below.

Table F.1 – Error sources

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Typical value of input variant</th>
<th>Relative standard uncertainty (RSU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrator</td>
<td>0.66%</td>
<td>0.0066</td>
</tr>
<tr>
<td>Linearisation</td>
<td>0.26%</td>
<td>0.0026</td>
</tr>
<tr>
<td>Probe positioning</td>
<td>(1^\circ)</td>
<td>(\approx 0)</td>
</tr>
</tbody>
</table>

The relative expanded uncertainty is equal to \(2\sqrt{\sum (RSU)^2} = 1.4\%\)
About the Author

Tim graduated with BA and MEng degrees in Engineering from the University of Cambridge with an Aerospace and Aerothermal specialisation in 2010. He then worked for the European Space Agency in Madrid as part of its one-year Young Graduate Trainee programme. He returned to Cambridge in 2011 to begin doctoral research in the area of aeroacoustics in close collaboration with Dyson. Having completed his PhD in 2014, he subsequently embarked on a career as a consulting engineer with Frazer-Nash Consultancy in Bristol.

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