Using the Sunyaev-Zel’dovich effect

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February 13, 2001

A dissertation submitted for the degree of
Doctor of Philosophy in the University of
Cambridge
Preface

This dissertation is the result of work I have undertaken as a research student in the Cavendish Astrophysics Group, University of Cambridge between October 1997 and February 2001. Except where otherwise stated, it includes nothing which is the outcome of work done in collaboration, and neither it nor any similar dissertation has been submitted for a degree, diploma or other qualification at this or any other university. This dissertation does not exceed 60 000 words in length.

William F. Grainger

13 Feb 2001
Acknowledgements

I gratefully acknowledge a research studentship from PPARC which allowed the majority of this work to take place. I am also grateful to the Cavendish Laboratory and Fitzwilliam College for financial support.

I would like to thank the many people who have contributed to this work in different ways. Mike Jones, Keith Grainge, Rüdiger Kneissl and Garret Cotter have all been around to answer academic questions and make useful suggestions for interesting things to do. The "lunchtime cosmology" session members have all helped my overall understanding of the CMB.

This work would never have happened without the constant support of my supervisor, Richard Saunders. His advice and guidance have improved my work, understanding and writing style immensely, as well as helping me to enjoy my time in the department.

Thanks also to Ben Rusholme, Eric de Silva, Jo Baker, Garret Cotter and Malcolm Bremer for assistance observing, in particular to Ben and Jo for facing the Y2K bug head-on whilst up a mountain. Dave Green and George Gilbert deserve honourable mentions for help with \LaTeX, and Dave Titterington for keeping the computers working. Katherine Inskip, Helen Thomas and Douglas Pierce-Price have assisted with timely proof-reading. Everyone already mentioned as well as Sarah Bridle and Antony Lewis and have all become good friends and have helped make the past 3\frac{1}{2} years fun.

Most of all I'd like to thank Claire for just about everything.
Summary

This work is concerned with the Sunyaev-Zel’dovich (S-Z) effect in clusters of galaxies and its use in cosmology.

In Chapter 2, I consider possible causes of error in the determination of \( H_0 \) from the uncertain geometry of clusters and complications such as lack of virialisation. From numerical simulations, I find that with 20 clusters, the resultant error is 8%. Chapter 3 details and tests a new maximum likelihood algorithm for the removal of contaminating radio sources. Two new estimates of \( H_0 \) are presented, \( 78^{+20}_{-20} \text{ km s}^{-1} \text{ Mpc}^{-1} \) for 0016+16 and \( 48^{+14}_{-11} \text{ km s}^{-1} \text{ Mpc}^{-1} \) for A611.

In chapters 4 and 5 I describe a search for new clusters via extended steep-spectrum radio emission due to merger events or head-tail sources. A candidate sample of possible high-redshift clusters is produced based on \( S_{326\text{MHz}} > 65 \text{ mJy} \) and \( a_{0.3\text{GHz}}^{1.4\text{GHz}} > 1.5 \). Six of these were then used as targets for S-Z observations; no unequivocal detection was found implying each cluster has a total mass \( \lesssim 10^{15} \text{ M}_\odot \). Optical \( V \), \( R_S \) and \( I \) follow-up in two of the fields shows two galaxies in each field with \( V - R_S \approx 3 \) and \( R_S \approx 20 \); similar observations of blank fields show no such galaxies and hence these are good candidate cluster members at \( z = 0.5-0.6 \).
# Contents

1 Introduction
  1.1 The S-Z effect .................................................. 2
  1.2 Detecting the S-Z effect ........................................ 4
    1.2.1 The Ryle Telescope ........................................ 5
  1.3 Measuring $H_0$ .................................................. 5
    1.3.1 Measuring $H_0$ with the S-Z and X-ray route ........... 5
    1.3.2 Other methods of determining $H_0$ ....................... 6
  1.4 Searching for distant clusters ................................. 10
  1.5 The structure of this thesis .................................. 11

2 Systematic effects in measuring $H_0$ ............................... 13
  2.1 Introduction .................................................... 13
  2.2 Determining $H_0$ with the RT ................................ 13
  2.3 X-ray fitting and parameter degeneracy ....................... 14
  2.4 Sphericality ...................................................... 18
    2.4.1 Axial ratios of real clusters .............................. 19
    2.4.2 Simulations of elliptical clusters ....................... 22
    2.4.3 Measuring the real line-of-sight ......................... 26
    2.4.4 How many clusters are required? .......................... 26
  2.5 Cosmology ......................................................... 27
  2.6 Noise and cluster positions .................................... 28
  2.7 Isothermality .................................................... 30
  2.8 N–body simulations .............................................. 33
    2.8.1 Temperature profiles ..................................... 36
    2.8.2 Virialisation .............................................. 39
  2.9 Conclusions ...................................................... 44
3 New measurements of $H_0$

3.1 Introduction .................................................. 47
3.2 The 15-GHz sky .................................................. 47
3.3 Processing S-Z data and removing radio sources ................. 48
  3.3.1 The test data .............................................. 50
  3.3.2 The CLEAN method .......................................... 51
  3.3.3 The matrix method .......................................... 54
  3.3.4 The FLUXFITTER method ..................................... 56
  3.3.5 Possible improvements ....................................... 59
3.4 Summary of methods ............................................ 60
3.5 New Ryle Telescope observations ................................ 60
  3.5.1 X-ray observations .......................................... 61
3.6 Abell 611 ....................................................... 63
  3.6.1 $H_0$ determination .......................................... 63
3.7 0016+16 .......................................................... 66
  3.7.1 $H_0$ determination .......................................... 68
  3.7.2 Other $H_0$ determinations ................................... 69
3.8 Abell 990 .......................................................... 71
  3.8.1 WSRT ........................................................... 72
3.9 Error budget ..................................................... 78
  3.9.1 Primary flux calibration ...................................... 78
  3.9.2 X-ray temperature ........................................... 78
  3.9.3 Fitting degeneracy ........................................... 78
  3.9.4 Source subtraction residuals .................................. 78
  3.9.5 Kinetic S-Z effect ........................................... 78
  3.9.6 Rees-Sciama effect .......................................... 79
3.10 Combining $H_0$ values ........................................ 79
3.11 Conclusions .................................................... 81

4 Searching for clusters: selection and S-Z follow-up .............. 83

4.1 Introduction .................................................... 83
4.2 Cluster radio halos ............................................. 83
  4.2.1 Aging ......................................................... 84
4.3 Obtaining candidate clusters ..................................... 85
4.4 Detection efficiency ............................................ 92
4.5 Optical images .................................................. 92
4.6 CRH RT observations ............................................ 92
  4.6.1 CRH 0741 ..................................................... 102
  4.6.2 CRH 0800 ..................................................... 103
  4.6.3 CRH 0833 ..................................................... 104

A Determining the line of sight .................................. 133

B The angular size-redshift relation ................................ 135

C "Extreme" simulated clusters .................................... 137

D Determining $H_0$ from a set of clusters ......................... 143

References ......................................................... 150
Modern cosmology is based on a number of important observations. Two critical ones are the Cosmic Microwave Background (CMB), and the recession of galaxies. The latter was observed by Edwin Hubble (1929). He found that the recessional velocity of galaxies, $v$, as measured by their redshift, is proportional to their distance from the Earth, $r$. That is, for low redshifts,

$$r = H_0 v,$$

where $H_0$ is the Hubble constant.

In a big-bang cosmology, the universe is initially dense and hot. After expanding and cooling so that $k_B T$ is less than the subatomic particle binding energy, it is filled with protons, neutrons, electrons, photons and dark matter. The protons and neutrons form nuclei, and then any free neutrons decay. After $\sim 300,000$ years the universe has cooled sufficiently so that atoms can form. As atomic hydrogen forms, charged particles are removed from the plasma, the universe becomes transparent to photons and the matter and radiation decouple. This epoch is known as recombination. Recombination occurs at a redshift of $1070$ over an interval of about $\Delta z \simeq 80$. The photon radiation field continues to expand and cool and retains the information from the last scattering events which occurred at recombination. This radiation field, which was first discovered by Penzias & Wilson (1965), is very close to isotropic and homogeneous. The best-fit temperature to the spectrum is $2.728 \pm 0.004$ K (Bennett et al., 1996). Small anisotropies in the CMB exist due to our local motion with respect to it, from the beginnings of structure formation imprinted at the surface of last scattering, and from ionised material that lies between the last scattering surface and us, notably the gas in clusters of galaxies. This thesis is concerned with this last anisotropy.
1.1 The S-Z effect

Shortly after the CMB had been discovered, Sunyaev & Zel'dovich (1970, 1972) predicted what has come to be known as the Sunyaev-Zel'dovich (S-Z) effect. Birkinshaw (1999) gives a useful review of the S-Z effect and Sarazin (1988) a review of clusters in general. When CMB photons are scattered by the ionised gas lying in the potential well of a cluster of galaxies, they are more likely to scatter to a higher energy. Averaging over scattering angle, the shift in energy $\Delta \varepsilon$ from the original energy $\varepsilon$ is given by

$$\langle \Delta \varepsilon / \varepsilon \rangle \approx 4k_B T_e / m_e c^2,$$

where $T_e$ is the temperature of the scattering electrons. The fraction of photons scattered is of course proportional to the number of scatterings. The resultant energy shift can be properly derived from the Kompaneets equation

$$\frac{\partial n}{\partial y} = \frac{1}{x^2 \delta x} \left( \frac{\partial n}{\partial x} + n + n^2 \right),$$

where

$$x = \frac{h \nu}{k_B T_{\text{CMB}}},$$

$$y = \sigma_T \int n_e \frac{k_B T_e}{m_e c^2} d\ell,$$

and $n$ is the photon occupation function in terms of frequency and time. By solving equation 1.3, the change in intensity of the CMB can be calculated:

$$\Delta I = \frac{2h}{c \delta} \left( \frac{k_B T_{\text{CMB}}}{\hbar} \right)^3 \frac{x^4 e^x}{(e^x - 1)^2} \left( \coth \frac{x}{2} - \frac{1}{2} \right).$$

This is plotted in figure 1.1 and figure 1.2 illustrates this shift with an unphysically large $y$-parameter of 0.075. A physically reasonable value ($y \approx 0.0002$) would not be visible in figure 1.2. Note that the resultant spectrum is not a Planck distribution and that there no change in the intensity at a frequency of 217 GHz. It is worth noting that the Kompaneets equation is only valid in the non-relativistic limit; Challinor & Lasenby (1998) find that the fully relativistic equation introduces a correction of around 2.5% for an 8-keV cluster of galaxies when observed at 15 GHz. In the low-frequency limit, the observed temperature shift of the CMB is given by

$$\frac{\Delta T}{T} = -2y.$$

Thus the surface brightness of the S-Z effect is independent of the distance to the cluster. This is an extremely important result. Provided a cluster is massive enough to produce a measurable $\Delta T$, it can be detected at any distance. Thus, potentially, one can search for clusters at distances beyond the ranges of X-ray and optical telescopes.

![Figure 1.1: The change in intensity of the CMB due to the S-Z effect. The $\Delta I$ scale on the y-axis is plotted linearly in arbitrary units.](image1)

![Figure 1.2: The Planckian CMB distribution (dashed) and resultant intensity (solid) that would be measured after a S-Z effect with $y = 0.075$ (which is unphysically large; a physically reasonable value would not be visible on the graph).](image2)
1.2 Detecting the S-Z effect

The S-Z effect has been successfully detected recently in a number of ways: with radiometers (e.g., Herbig et al. 1995 and Myers et al. 1997 using the OVRO 5.5m dish at 32 GHz); with bolometers (e.g., Holzapfel et al. 1995a using SuZIE, a bolometer on the CSO); and interferometers (e.g., Grainge et al. 2001 using the Ryle Telescope at 15 GHz and Carlstrom et al. 1996 with the OVRO array at 30 GHz, and more recently, Rusholme 2001 with the VSA at 30 GHz). A more comprehensive list of instruments and detections is given in Birkinshaw (1999). The first two methods are similar in their observing strategies but the detection methods differ.

Bolometers detect heat; each photon falling on the detector transfers its energy to a block of material. The change in resistance of the block is then detected. The thermal noise can be dramatically reduced by using so-called “spider-web” bolometers; the central receiving material is suspended with a web of silicon nitride fibres. They have three advantages; the cross-section to cosmic rays is reduced, the lowest phonon modes can be frozen out and the low heat capacity means that a small energy change gives a large temperature change. Radiometers detect the electric field of the incoming radiation. This is then down-converted with a local oscillator and filter-mixer; these define the observing frequency and allow subsequent signal processing to be done at a lower frequency.

Both bolometers and radiometers are used with single-dish telescopes as “total power” detectors. The atmospheric signal is removed by beam-switching. This necessarily involves a change in the spillover into the dish of ground emission which then undesirably appears as a signal. This difficulty can be countered by drift-scan observations and also by the use of arrays of receivers in the focal plane. Because total power is measured, receiver stability has to be very good.

The third method is to use an interferometer. The signals from two radiometers are multiplied together and integrated (correlated); the resultant signal is proportional to the Fourier component of the sky on the scale \( \frac{\theta}{D} \) where \( D \) is the separation between dishes and \( \lambda \) is the observing wavelength. By using more dishes and correlating all the possible pairs, many scales can be measured and an image of the sky brightness, convolved with the synthesised beam, is produced by Fourier transforming the data. The deconvolution cannot be done analytically (as the aperture plane contains zeros) but a number of numerical techniques exist, e.g., CLEAN as described by Högbom (1974) and MEM as described by Cornwell & Evans (1984).

There are many advantages of interferometers for CMB observations. Anything received that is not varying at the celestial fringe rate is filtered. The path compensation system decouples and further attenuates groundspill and interference. Most of the atmospheric emission is filtered out as each baseline is sensitive to essentially one Fourier component of the sky brightness. As only correlated power is detected, the requirement for receiver stability is far less stringent than for total power systems. And if long baselines are used as well as the short ones required to detect the S-Z signal from clusters of galaxies, subtraction of contaminating radio sources can readily be done at the same frequency and time as the S-Z observation. These are all features of the Ryle Telescope.

### 1.3 Measuring \( H_0 \)

#### 1.3.1 Measuring \( H_0 \) with the S-Z and X-ray route

Cluster gas emits thermal bremsstrahlung, with an integrated X-ray luminosity \( L_X \), given approximately by

\[
L_X \propto n_e^2 T_e^{\frac{3}{2}}. \tag{1.8}
\]

Thus, with X-ray and S-Z observations, there are two independent ways of observing the same
gas, with different dependencies on $n_e$, $T_e$, and the size of the cluster. The gas temperature can be determined from the X-ray spectrum. By combining the S-Z and X-ray measurements it is possible to solve for the line-of-sight distance through the cluster and the gas density distribution. Knowing the angular size of the cluster (from the X-ray observations) and the redshift, the angular-size-distance relation can be calibrated, thus $H_0$ can be estimated. Cavaliere et al. (1978) seems to have been the first to suggest this method. For an isothermal cube of gas, subtending an angle $\theta$, at a redshift $z$, with an X-ray surface brightness $\Sigma_{\text{SB}}$, then

$$H_0 = \frac{8 (T_e g_0^2)}{n_e^2 c^3} \left( \frac{T_0}{\Delta T_{\text{bol}}} \right)^2 \theta \Sigma_{\text{SB}} \left( 1 + z \right)^2 - \left( 1 + z \right)^{3/2} .$$

This equation is useful to give a feel for the technique, but the method generally used is as follows. A model of gas — usually an isothermal King model — is fitted to the X-ray image of the cluster. The King model,

$$n_e = n_0 \left( 1 + \left( \frac{r}{r_c} \right)^2 \right)^{-\beta} ,$$

describes the radial variation of the electron density $n_e$ as a function of radius $r$ (King, 1972; Cavaliere & Fusco-Femiano, 1976); $r_c$ is known as the core radius, $\beta$ is a dimensionless constant and $n_0$ is the central electron density. A value for $H_0 - H_{0,\text{act}}$ — has to be assumed at this point.

The model of the gas is then used to predict the S-Z flux $S_{\text{pred}}$ — that a particular group’s telescope will observe. This is then compared with the measured S-Z flux, $S_{\text{abs}}$. Then $H_0$ is calculated from

$$H_0 = H_{0,\text{act}} \left( \frac{S_{\text{pred}}}{S_{\text{abs}}} \right)^2 .$$

### 1.3.2 Other methods of determining $H_0$

There are also many other methods of determining $H_0$. Some are stand-alone methods while the distance-ladder route relies on standard candles. The distance-ladder is the route that has been followed by the Hubble Space Telescope Key Project team. A few of the more important steps in the distance ladder are as follows.

The distance ladder

There exist a number of objects for which the physics is thought to be approximately understood. The majority of the HST Key Project work has been to measure the magnitudes of various objects, and then compare these to the absolute magnitudes, thus deriving a distance and hence $H_0$. The absolute magnitude is determined by a physical relationship with some other observable. All of these hinge on the Cepheid distance scale, which is called the “primary distance indicator”, and the other methods are known as the “secondary distance indicators”. Figure 1.3 shows the ranges over which different distance indicators are used.
CHAPTER 1. INTRODUCTION

<table>
<thead>
<tr>
<th>Method</th>
<th>Value / km s(^{-1}) Mpc(^{-1})</th>
<th>Error random</th>
<th>Error systematic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tully-Fisher</td>
<td>71</td>
<td>±4</td>
<td>±7</td>
</tr>
<tr>
<td>SN1A</td>
<td>68</td>
<td>±2</td>
<td>±5</td>
</tr>
<tr>
<td>Fundamental plane</td>
<td>78</td>
<td>±5</td>
<td>±9</td>
</tr>
<tr>
<td>Overall</td>
<td>71</td>
<td>±6</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1.2: Secondary distance indicator derived values of \(H_0\) from the HST Key Project paper. The "Overall" value includes other indicators such as surface brightness fluctuations and luminosity functions.

- **Cepheid variables**
  Cepheids are stars which undergo periodic brightness fluctuations. There exists a tight relation between the period and the luminosity, known as the PL relation. Thus if the period and magnitude of a Cepheid can be measured, then its distance can be inferred. The HST has measured Cepheid distances to many nearby galaxies, each containing a good number of Cepheids (in order to reduce statistical uncertainties). By setting a distance to the Large Magellanic Cloud, and also measuring a Cepheid distance to the LMC, distances to many nearby objects can be found.

- **Tully-Fisher distance scale**
  It is known empirically that the absolute magnitude of a late type spiral galaxy is correlated with its maximum rotational velocity. This is the Tully-Fisher relation (Tully & Fisher, 1977). By using late-type spirals that contain Cepheid variables, then the distance scale can be extended to greater distances than Cepheids can be observed at. The HST project has found 15 suitable galaxies with Cepheids to extend the scale.

- **Supernovae**
  Studies of the light curves of Type 1A supernovae have shown a correlation between the peak luminosity and light-curve shape. The physical basis for this correlation is currently unknown. However, a measurement of the received flux and light curve shape, with calibration with the Cepheid scale, allows \(H_0\) to be estimated out to redshifts of \(\geq 1\). Indeed, many high redshift SN1As have been found, and seem to support a A-dominated cosmology (White, 1998). However, only 10 intermediate and good quality light curves exist for SN1As in galaxies with Cepheids, and so the calibration for \(H_0\) is poor.

The last of the HST Key Project papers (Mould et al., 2000) summarised the values of \(H_0\) from each of these secondary techniques. The values are presented in table 1.2. It is important to note that these are all built on the Cepheid distance scale, which in turn is based on 32 Cepheids in the LMC, and that the distance to the LMC is assumed to be \(50 \pm 3\) kpc. The overall figure includes some other secondary distance calibrators, such as globular cluster luminosities.

### 1.3. MEASURING \(H_0\)

#### Physical methods

The fundamental problem of the HST Key Project method for determining \(H_0\) is that it all rests on one particular number, the distance to the LMC. Error propagation is also a problem: for accurate determinations of the distances of high-redshift objects, the low redshift object distances must be calibrated very accurately. As such it is important to have independent methods that are based on different physics to estimate \(H_0\).

- **Gravitational time delays**
  Refsdal (1964) discussed the possibility of estimating \(H_0\) from multiply lensed objects. If a gravitational lens produces multiple images of the same object, then the light paths to the lensed object are of different lengths. If the lensed object were variable in some way, then it is possible to determine a time delay between the light paths, and, knowing the redshifts of both lens and lensed object, determine \(H_0\). Refsdal initially discussed the possibility of using a lensed supernova, and detecting the sharp changes in magnitude. This method would be confused by the different levels of magnification that a multiply imaged system produces. However, the JVAS (Wrobleski et al., 1998) and CLASS (Myers et al., 1995) surveys, both radio based, have found many multiply imaged lenses. A small number of the lensed objects are variable, and observing campaigns have been undertaken to determine the time delays between different images. In all cases the shape of the light curve is the same between different images of the same lensed object, which is confirmation that the images are of the same region of the object.

Table 1.3 lists the published \(H_0\) values from gravitational time delays. It should be noted that the values of \(H_0\) are dependent on the models used for the lens. Indeed, Williams & Saha (2000) use a different model for the lens and find a average \(H_0\) of \(61 \pm 11\) km s\(^{-1}\) Mpc\(^{-1}\) using PKS1830−211 and PG1115+080.

- **Accretion disk time delays**
  This method also relies on a variable source. Instead of being a lensed system, it requires a variable high energy photon source to be illuminating an accretion disk. The accretion disk is modelled as having a temperature variation with radius. When the source "bursts" it locally heats the disk. The heat flows through the disk, and two points of the disk become brighter (at a given wavelength) as they enter a certain temperature region (and thus a certain radius). The
time delay between the two regions of the disk is then related to a physical distance at the source, and so $H_0$ can be determined. This technique is still in its infancy: only one determination has been performed so far, and there is a strong dependence on disk orientation. However, Collier et al. (1999) report $H_0(\cos i/0.7)^{1/2} = 42 \pm 9$ km s$^{-1}$ Mpc$^{-1}$ from multiv wavelength observations of NGC 7469.

- Expanding photospheres
  Kirshner & Kwan (1974) discussed another supernova-based method of determining $H_0$. The expanding photosphere of a type II supernova is approximately a standard candle, and with sufficient observations separated in time, the physical parameters of the explosion can be estimated including the absolute luminosity. $H_0$ can then be determined. Schmidt et al. (1992) have applied this method to 18 type II supernovae, and find $H_0 = 73 \pm 6$ (statistical) $\pm 7$ (systematic) km s$^{-1}$ Mpc$^{-1}$.

1.4 Searching for distant clusters

Clusters are the largest virialized objects in the universe and so contain a significant fraction of the baryonic matter of the universe. The virial mass of a rich cluster is close to the mean mass enclosed within an $8h^{-1}$ Mpc radius sphere, with a dependence on the density parameter $\Omega_M$. Thus the current abundance of rich clusters allows the measurement of the amplitude of density fluctuations on a scale of $8h^{-1}$ Mpc as a function of $\Omega_M$.

The evolution of the abundance of clusters is a function of the amount of matter in the universe, as measured by $\Omega_M$. In a low-density universe, fluctuations stop growing at a redshift $1 + z \approx \left( \frac{1 - \Omega_M}{\Omega_M} \right)^{1/3}$ (see Longair, 1998), whereas they continue to grow in a high density universe. As such, finding even moderate redshift clusters gives a diagnostic on $\Omega_M$. Indeed, Bahcall et al. (1997) showed that the detection of one or two Coma-like clusters at $z \approx 0.5$ (after observing $10^5$ deg$^2$ of sky) would rule out $\Omega = 1$, $\sigma_8 = 0.5$ as only $10^{-2}$ clusters would be expected.

The two most common ways to detect clusters, galaxy overdensities and X-ray emission, are both strongly redshift dependent. Haynes (1998) has shown that even with deep ground-based optical images in five colours, a cluster is completely confused with the field by $z \approx 1$. X-ray detections are also limited to $z \approx 1$, and even the new X-ray telescopes Chandra and XMM do not have sufficient field-of-view and sensitivity to perform a deep survey of a significant fraction of the sky in a reasonable time.

The redshift independence of the S-Z effect is clearly a way to get an unbiased sample of clusters: the only limit will be the mass of the cluster. However, performing a sufficiently deep survey with the Ryle Telescope is impossible due to the small field-of-view and because even the shortest baselines are too long so that most of the S-Z signal is resolved out. To overcome this, various techniques have been used to as "signposts" to point to high-redshift clusters. The Cavendish group have used a number of signposts.

1.5 The structure of this thesis

In Chapter 2, I examine possible sources of error in determining $H_0$ from the S-Z & X-ray route.

Chapter 3 describes an improved method of source subtraction for Ryle Telescope data, and then determines $H_0$ for the clusters A611 and 0016+16. The problems involved in determining $H_0$ from A990 are also discussed.

In Chapter 4, a sample of candidate distant clusters is produced by examining the NVSS, WENSS and FIRST radio survey catalogues, which are then observed with the Ryle Telescope.

Chapter 5 reports the results from optical follow-up of this sample of objects.

Throughout this thesis, all coordinates are J2000. As a shorthand, I occasionally use $H_0 = 50h_{50}$ km s$^{-1}$ Mpc$^{-1}$. I take $\Omega_M = 1$ and $\Omega_{\Lambda} = 0$ unless otherwise stated.
Chapter 2

Systematic effects in measuring $H_0$

2.1 Introduction

Many estimates of $H_0$ via the S-Z and X-ray route have been made, e.g. Herbig et al. (1995); Birkinshaw & Hughes (1994); Patel et al. (2000); Grainge et al. (2001). Various assumptions have to be made. In this chapter I consider a number of these.

2.2 Determining $H_0$ with the Ryle Telescope

There are a number of modifications to the basic technique described in section 1.3.1. Firstly, a spherical King profile is not a good model. High resolution X-ray images show that clusters are not spherically symmetric. Figure 2.1 is a high signal-to-noise image of A2218, a rich cluster and one of the first with a detected S-Z effect (see Jones et al., 1992).

We model the cluster as being ellipsoidal rather than spherical. The $(r/r_c)^2$ term in the King model (equation 1.10) is replaced with

$$\left(\frac{x'}{r_{c1}}\right)^2 + \left(\frac{y'}{r_{c2}}\right)^2 + \left(\frac{z'}{r_{c3}}\right)^2,$$

(2.1)

where $(x', y', z')$ is a point in the cluster, measured along the principle axes of the ellipsoid. The core radii $(r_{c1}, r_{c2}, r_{c3})$ are also measured along these axes and are known as the intrinsic core radii. As discussed in section 2.4, the true orientation of the cluster is unknown from X-ray measurements, and so the intrinsic core radii are unknown. The projected core radii, however, can be found and we use these. The core radius along the line-of-sight is set to be the geometric mean of the projected
core radii. The problem is illustrated in figure 2.2. Adapting equation 1.9 shows that

\[ H_{0,\text{calc}} \propto H_0 \frac{l_4}{l_\|} \]  

(2.3)

where \( l_4 \) is the distance across the cluster in the plane of the sky and \( l_\| \) is the distance along the line of sight. By assuming a set of randomly orientated clusters then the geometric mean of the \( H_{0,\text{calc}} \) values will be equal to the true value \( H_0 \).

The comparison between predicted and measured S-Z flux depends on the telescope used. In the case of the Ryle Telescope, a \( \chi^2 \) minimisation is performed between the measured and predicted visibilities, because these are what are measured with an interferometer, and so the noise properties are well understood. Groups using single-dish telescopes necessarily do this in the map plane. The likelihood, \( L \), for a range of \( S_{\text{model}} \) values (which depend directly on \( H_0 \)) is calculated, i.e.

\[ L = e^{-\frac{(S_{\text{model}} - S_{\text{data}})^2}{2}} \]  

(2.3)

and then the \( H_0 \) is given by the putting the most likely \( S_{\text{model}} \) into equation 1.11.

2.3 X-ray fitting and parameter degeneracy

X-ray image data for moderate redshift clusters is Poisson-noise dominated, i.e. the probability of detecting \( r \) events given a mean event number \( \lambda \) is

\[ P(r|\lambda) = \frac{\lambda^r e^{-\lambda}}{r!} \]  

(2.4)

\footnote{Data taken directly from the ROSAT archive at http://ledas-www.star.le.ac.uk/annieV4/}

Identifying \( r \) with the number of counts received by the telescope, \( y_i \) at a position \( x_i \), and \( \lambda \) with the number of counts predicted by the model, \( f(a, b, c, \ldots) \), this becomes

\[ P(y_i|a, b, c, \ldots) \propto e^{-f(x_i; a, b, c, \ldots)} f(x_i; a, b, c, \ldots)^{y_i} \]  

(2.5)

where \( a, b, c, \ldots \) are the parameters of the model. The overall likelihood is then the product of these probabilities over the entire receiver

\[ P(y|a, b, c, \ldots) = \prod_i P(y_i|a, b, c, \ldots) \]  

(2.6)

The product can be turned into a sum by taking logs, which is computationally both more efficient and less prone to error. The correct expression to maximise is then the log of the likelihood,

\[ \log L = \sum_i \left[-f(x_i; a, b, c, \ldots) + y_i \log(f(x_i; a, b, c, \ldots)) - \log(y_i! \right] \]  

(2.7)

In order to perform the summation, a model of the X-ray sky must be generated. This is done using code developed by Keith Grainge. To begin, a King distribution of densities on a grid is determined for a particular \( \beta \) and core radii. The X-ray emission is then be calculated by summing \( n_e^2 \) along each column, and then multiplying by suitable prefactors (accounting for temperature, detector response, etc.), assuming that they are constant over the entire map.

The \( \gamma \)-parameter can also be calculated by a summation of \( n_e \), and then a multiplication by the relevant parameters, such as \( T_e \) and \( n_e \). The visibilities observed by the Ryle Telescope are calculated by multiplying the S-Z sky by the primary beam response, \( B \), of the telescope, and then performing
a Fourier Transform. The aperture plane, \( V(u, v) \), is then sampled at points corresponding to the shortest baselines of the Ryle Telescope:

\[
V(u, v) = \text{FFT}(y(x', y') \times B(x', y')),
\]

(2.8)

where \((x', y')\) represents a position on the sky.

Figure 2.3 shows the likelihoods, i.e. \( L \) values, for a fit to a ROSAT HRI observation of Abell 773. The plot was produced by varying one core radius, \( \beta \) and the central density. The axial ratio and inclination of the model cluster were kept constant. The likelihoods were then marginalised over the central density. The contours enclose the regions that have likelihoods of 68% and 95%. A naïve application of these contour plots would be to quote the error on \( \beta \) to be \( 0.82^{+0.13}_{-0.06} \). This is incorrect due to the degeneracy between \( \beta \) and core radius; the data must be marginalised over other parameters. Figure 2.4 shows the fully marginalised plots for the three parameters. The correct error to quote for \( \beta \) is then \( 0.79^{+0.09}_{-0.08} \).

One might imagine that this degeneracy between parameters would have a serious effect on \( H_0 \) determinations. In fact, for interferometers, it does not. Reese et al. (2000) has shown that for a spherical isothermal King model, the lines of derived angular-diameter distance (which is equivalent to lines of observed flux) are close to parallel to the degeneracy direction. Figure 2.5 shows the probability contours overlaid on contours of equal flux as would be observed by the Ryle Telescope for each model. Here I have used an elliptical King model, and held the ratio of core-radii to be constant. The most likely value of the central density is used for calculating the flux at each \((\beta, \text{core radius})\) point. The plot shows that there is a 40 \( \mu \)Jy variation, corresponding to 35% in the observed flux, between the two extreme points on the 68% contour of the \( \beta \)-core-radius plot. Note

Figure 2.3: Likelihood contours for a King model fit to an X-ray image of Abell 773. The contours are at the 68% and 95% confidence limits. Theta is the major axis core radius in units of arcseconds.

Marginalised likelihood for the central density.

Marginalised likelihood for the core radius.

Marginalised likelihood for \( \beta \).

Figure 2.4: Parameter likelihood distributions in the King model, as applied to Abell 773.
that this is a small (in comparison to, say, the noise on a temperature determination) variation because the lines of constant flux are close to parallel to the degeneracy direction. The degeneracy leads to a 7% error in the estimated \( H_0 \) value for this cluster. All this applies only to Ryle Telescope measurements (and those of similar interferometers); it does not apply to single dish measurements for which the problem is worse because the contours of constant observed flux are perpendicular to the \( \beta \)-core-radius degeneracy, see e.g. Birkinshaw & Hughes (1994).

2.4 Sphericality

From an X-ray observation of a cluster, the distribution of gas projected on the sky is clear. However, the distribution along the line of sight, and so the 3-dimensional gas distribution, is not. Consequently there will be an uncertainty in an \( H_0 \) estimate from any particular cluster. Numerical simulations (see section 2.3) show that at low redshift, i.e. \( z < 0.5 \), most clusters do not have extreme axial ratios. Indeed, a quick look at a random sample of cluster images from RASS showed that clusters might have axial ratios as low as 0.5, and so it is important to quantify the geometric uncertainty and investigate its effect on \( H_0 \) determinations.

A systematic bias in \( l_\| \) would cause \( H_0 \) determinations to be biased. A surface-brightness-limited sample would contain this bias: a higher surface-brightness cluster at a particular redshift will have a larger \( l_\| \) than a lower surface-brightness cluster if the temperatures and \( l_\perp \) values are the same. A luminosity or flux-based sample may still have this bias in practice, unless the surface-brightness limit of the survey used is low enough.

2.4.1 Axial ratios of real clusters

An attempt at a luminosity-limited cluster sample exists in Grainge (1995). This sample was constructed from the RASS by Edge early in the period of ROSAT data reduction as a list of candidates for \( H_0 \) determination with the Ryle Telescope. The surface brightness limit was 2.5 times greater than the RASS detection limit for clusters. Therefore, the Grainge sample is a good starting point for investigating the shape of real clusters, as the conclusions will be directly applicable to the Ryle Telescope sample.

Fig 2.6 shows the luminosities of the candidates of the sample as a function of redshift. A stricter luminosity cutoff of \( 6 \times 10^{37} \) W (0.1–2.4 keV bandpass) rather than Grainge's \( 4 \times 10^{37} \) W has been imposed. Note that Grainge's "arbitrary units" luminosities have been converted to Watts by correlating his units with those reported in Ebeling et al. (1996). A small number of clusters have also been rejected from the original Grainge sample; these are of low flux and so may introduce bias. This higher cutoff rejects 47 of the original sample of 96 clusters. The ROSAT pointed observation archive was examined, and data acquired for as many of the clusters as possible. Of the sample of 49, 36 clusters have had pointed ROSAT observations.

Each of the clusters in the new sample of 36 clusters was then fitted with an elliptical isothermal King model. If a cooling flow is obvious from the ROSAT image, then the pixels that contain the cooling flow are not used in the fitting. Including the cooling flow pixels would bias the core-radius to small values. Again, any pixels that contain bright compact regions or point sources (likely AGNs) are not used in the fitting process.

The axial ratios found are shown in table 2.1, and the distribution is shown in figure 2.7. There are only 31 clusters in table 2.1 as 5 clusters have been rejected as follows. Abell 1758 is actually two separate clumps of X-ray emitting gas and is clearly a physically complex system. As such it would never be used for \( H_0 \) determination. The other four clusters (Abell 1722, Zwicky 2701, Abell 1704 and 1224+20) all have strong cooling flows, and very little additional X-ray emission from the bulk of the cluster gas. Thus a good model for the bulk of the cluster gas could not be obtained in each case from these ROSAT observations, and such clusters would not be used for \( H_0 \) determination.

The completeness of this sample of 31 clusters is obviously in doubt; has the removal of 18 clusters from the initial luminosity-limited sample of 49 changed the completeness? The luminosity-redshift diagram in figure 2.8 shows the 31 clusters as solid hexagons and the rejected clusters as crosses. The rejected clusters seem to be reasonably randomly distributed around the plot, and so it likely that the completeness has not been affected. As a check, a Kolmogorov-Smirnov test was performed.

The Kolmogorov-Smirnov test indicates whether two distributions are identical. The NAG library implementation was used. The test was performed on the 49 luminosities of the strict cutoff sample against the luminosities of the sample of the 31 observed clusters. The test statistic was \( D = 0.171 \), with a "tail probability" of 57%. This means that there is a 57% chance of obtaining a test statistic \( D = 0.171 \) if the hypothesis that the distributions are identical is true. As such, it is possible to say
Figure 2.6: Luminosity redshift diagram for the 96 clusters in the Grainge sample. The luminosities are for the 0.1-2.4 keV bandpass.

Table 2.1: The axial ratios for the 31 clusters (from Grainge 1996) that have pointed ROSAT observations.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Axial ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A113</td>
<td>0.43</td>
</tr>
<tr>
<td>A586</td>
<td>0.98</td>
</tr>
<tr>
<td>A611</td>
<td>0.94</td>
</tr>
<tr>
<td>A621</td>
<td>0.77</td>
</tr>
<tr>
<td>A665</td>
<td>0.91</td>
</tr>
<tr>
<td>A697</td>
<td>0.72</td>
</tr>
<tr>
<td>A773</td>
<td>0.72</td>
</tr>
<tr>
<td>A959</td>
<td>0.83</td>
</tr>
<tr>
<td>A963</td>
<td>0.88</td>
</tr>
<tr>
<td>A990</td>
<td>0.76</td>
</tr>
<tr>
<td>A1246</td>
<td>0.84</td>
</tr>
<tr>
<td>A1361</td>
<td>0.65</td>
</tr>
<tr>
<td>A1413</td>
<td>0.71</td>
</tr>
<tr>
<td>A1423</td>
<td>0.82</td>
</tr>
<tr>
<td>A1682</td>
<td>0.58</td>
</tr>
<tr>
<td>A1763</td>
<td>0.67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Axial ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1914</td>
<td>0.88</td>
</tr>
<tr>
<td>A1965</td>
<td>0.80</td>
</tr>
<tr>
<td>A2034</td>
<td>0.81</td>
</tr>
<tr>
<td>A2111</td>
<td>0.70</td>
</tr>
<tr>
<td>A2146</td>
<td>0.71</td>
</tr>
<tr>
<td>A2218</td>
<td>0.79</td>
</tr>
<tr>
<td>A2219</td>
<td>0.75</td>
</tr>
<tr>
<td>A2244</td>
<td>0.91</td>
</tr>
<tr>
<td>A2254</td>
<td>0.44</td>
</tr>
<tr>
<td>A2390</td>
<td>0.75</td>
</tr>
<tr>
<td>A2390</td>
<td>0.75</td>
</tr>
<tr>
<td>Zw1370</td>
<td>0.72</td>
</tr>
<tr>
<td>Zw3916</td>
<td>0.74</td>
</tr>
<tr>
<td>Zw7160</td>
<td>0.72</td>
</tr>
<tr>
<td>0016+16</td>
<td>0.85</td>
</tr>
<tr>
<td>1910+67</td>
<td>0.74</td>
</tr>
</tbody>
</table>

2.4. SPHERICALITY

Figure 2.7: The distribution of axial ratios for the sample of 31 candidate $H_0$ clusters that have been observed with ROSAT.

Figure 2.8: Luminosity-redshift diagram for the more stringent cutoff sample. Solid hexagons represent the clusters that are included in the sample of 31 clusters, crosses for those that do not (see text for details).
that the loss of the 18 clusters appears to make little difference. The sample of 31 clusters will be referred to as “the \( H_0 \) sample” from here on.

Mohr et al. (1995) have also determined axial ratios for a sample of clusters. Their sample, which was chosen to have high signal-to-noise images from the *Einstein* X-ray observatory data archive, is a representative X-ray sample, although they argue that the sample is statistically indistinguishable from a flux-limited sample. The Mohr et al. sample is a low-redshift sample and there is no overlap between that and the sample being used here, but it is a useful cross check. It gives an axial ratio distribution, as shown in figure 2.9, that is slightly wider than the \( H_0 \) candidate sample, but its median axial ratio is closer to one. The difference in the shape of the distributions may be due to evolution, although obviously much more data would be required to investigate this.

2.4.2 Simulations of elliptical clusters

With the axial ratio distribution for the \( H_0 \) sample found, it is possible to simulate a realistic cluster sample. I simulated a set of 50 isothermal King model clusters, all at redshift \( z = 0.171 \), with a temperature of 6.7 keV, an exposure time with *ROSAT* of 44.5 ks, a central electron number density of \( 10^4 \text{ m}^{-3} \) and \( \beta = 0.65 \). I follow Grainge in using these parameters, which are a good match to A2218, as a “typical” cluster. By holding these parameters constant, any effect on \( H_0 \) determinations must be due to ellipticity. For each simulated cluster the three intrinsic core radii were chosen from a top hat probability distribution ranging from 60 to 120 arcseconds. The simulated clusters were randomly orientated by selecting three Euler angles \( \alpha, \beta \) and \( \gamma \) from the appropriate probability distributions. Euler angles completely describe the transformation between two arbitrary vector bases. The angle \( \alpha \) describes the rotation around the \( x \)-axis, \( \beta \) around the new \( y \)-axis, and \( \gamma \) around the new \( z \)-axis. This is illustrated in figure 2.10. The following probability distributions were used:

\[
P(\alpha) = \begin{cases} 
\frac{1}{2\pi} & \text{if } 0 < \alpha < 2\pi, \\
0 & \text{otherwise,}
\end{cases}
\]

\[
P(\beta) = \begin{cases} 
\frac{1}{2} \sin(\beta) & \text{if } 0 < \beta < \pi, \\
0 & \text{otherwise,}
\end{cases}
\]

\[
P(\gamma) = \begin{cases} 
\frac{1}{2\pi} & \text{if } 0 < \gamma < 2\pi, \\
0 & \text{otherwise,}
\end{cases}
\]  

(2.9)

where \( P(\alpha) \) is the probability of a particular value of the Euler angle \( \alpha \) being selected, and so on. These choices ensure that the cluster is then orientated randomly in space.

The gas distribution was determined for a grid of \( 256 \times 256 \times 256 \) cells, each representing, in projection, a \( 15'' \times 15'' \) square on the sky. Two of axes of the grid are aligned on the sky, and the third is directed towards the observer. The centre of the grid, i.e. at \( (128,128,128) \), was taken to be centre of the cluster. So that the cluster can be orientated in any direction on the sky, the vector \( (i - 128, j - 

Figure 2.9: Distribution of axial ratios in the Mohr et al sample. The Mohr et al sample is not directly comparable to the \( H_0 \) sample as it is a low-redshift, representative sample.

Figure 2.10: The Euler angles \( \alpha, \beta \) and \( \gamma \) transform the basis set \( (u_1, u_2, u_3) \) to the basis set \( (u'_1, u'_2, u'_3) \).
128, k − 128) is multiplied by the Euler rotation matrix:

\[
\begin{pmatrix}
    x' \\
    y' \\
    z'
\end{pmatrix} =
\begin{pmatrix}
    i - 128 \\
    j - 128 \\
    k - 128
\end{pmatrix} 
\times
\begin{pmatrix}
    \cos \alpha \cos \beta \cos \gamma - \sin \gamma \sin \alpha & \sin \alpha \cos \beta \cos \gamma + \cos \alpha \sin \gamma & -\sin \beta \cos \gamma \\
    -\cos \alpha \cos \beta \sin \gamma - \sin \alpha \cos \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & \cos \gamma \sin \beta \\
    \cos \alpha \sin \beta & -\sin \alpha \sin \beta & \cos \beta
\end{pmatrix}.
\]

(2.10)

The values of the vector \((x', y', z')\) are then put into equation 2.1 to calculate the density. The \(y\) parameter, and thus the visibilities, and the X-ray emission can then be calculated in the same way as described in section 2.3. \(H_0\) was set to 50 km s\(^{-1}\) Mpc\(^{-1}\) and an Einstein-de Sitter cosmology was used.

The procedure outlined in section 2.2 was then followed to obtain a fit to the observed model of the gas distribution for each of the 50 simulated clusters. As discussed there, the axes of the ellipsoidal King model clusters were set to be aligned on the sky, i.e. the projected core radii were found, and the third core radius along the line-of-sight was set to be the geometric mean of the projected core radii. The projected axial ratio values derived from this fitting are shown in figure 2.11. \(H_0\) values were determined for each of the 50 simulated clusters. The geometric mean for this \(H_0\) distribution is 50.3 ± 1.2 km s\(^{-1}\) Mpc\(^{-1}\).

It is clear that there are differences in the distributions between this simulated set of axial ratios and both Mohr et al.’s sample of real clusters and the \(H_0\) sample; for example the medians and ranges of the distributions are different. In order to produce a simulation with a larger range of projected axial ratios than the \(H_0\) sample, the procedure was repeated for a larger range of core radii in the simulated clusters. A top hat distribution for the intrinsic core radii was still used, but the range was extended to 20 to 120 arcseconds. The axial ratio distribution was measured for this simulated data set, and is shown in figure 2.12. The axial ratio distribution in this case is clearly wider than either the \(H_0\) sample or the Mohr et al. sample. The geometric mean for the \(H_0\) distribution in this case is 48.7 ± 1.5 km s\(^{-1}\) Mpc\(^{-1}\).

In order to get closer to the observed axial ratio distribution, a final set of 50 clusters were simulated, with intrinsic core radii varying between 35 and 120 arcseconds. The projected axial ratio distribution found for this sample is shown in figure 2.13. \(H_0\) values were found for this simulated set of clusters, and the geometric mean of the \(H_0\) values was found to be 50.2 ± 1.2 km s\(^{-1}\) Mpc\(^{-1}\). Note that for all three simulations, the correct value of \(H_0\) is enclosed by the error bars of the geometric mean. Thus taking the geometric mean does indeed reduce the effect of unknown line-of-sight distance given a randomly orientated set of elliptical clusters.
2.4.3 Measuring the real line-of-sight

As the three intrinsic core radii for each cluster in each simulated sample are known, along with the orientation, it is possible to calculate the line-of-sight depth through the cluster. Equation 2.2 implies there will be a strong correlation between the error in that line-of-sight depth and the value of $H_0$ from the cluster. Appendix A details the derivation of the calculation of the line-of-sight depth through the cluster. Figure 2.15 shows the $H_0$ values determined for each cluster (in the third simulation which gave the axial ratio distribution most like that observed) plotted against a measure of how wrong the line-of-sight core radius was. As expected, this is a straight line and so there is a strong correlation between error in $H_0$ and the error in the line-of-sight core radius. This shows that if $H_0$ is known, either from having a large sample, or using a value from an alternative method, then measuring $H_0$ from the combination of S-Z and X-ray is in fact providing information about the 3-D matter distribution in each individual cluster of the sample.

2.4.4 How many clusters are required?

It is important to understand how the size of the errors on the $H_0$ value change with the number of clusters used in the determination. All other things being equal, the errors should reduce as more clusters are used. Figure 2.16 shows this variation. The final axial ratio sample (with the intrinsic core radii between 35 and 120°) is used here. The left hand plot has the clusters ordered with the largest actual line-of-sight depth first. In the right hand plot, the clusters are added to the sample in a random order. In the simulation, the clusters all have the same central density, $\beta$, temperature and redshift. The simulated clusters with a higher line-of-sight size will thus have higher X-ray flux. In these clusters, the line-of-sight distance has been underestimated, leading to an underestimate of $H_0$. In addition, it is not clear without a priori knowledge of the value of $H_0$ that the correct value has been found, as the value has not been consistent over many clusters. This is not the case with the randomly ordered sample; the derived value of $H_0$ remains constant within the error bars however many clusters are used. It is also clear that the error from an incomplete sample of clusters is ±5 km s$^{-1}$ Mpc$^{-1}$ when only 15 clusters have been observed.

As such, we can conclude that in a luminosity limited sample, only 15 clusters are required for the error from the unknown orientation to become about 10%. This is a smaller uncertainty than current S-Z measurements and X-ray temperatures.

2.5 Cosmology

The current status of the “fundamental parameters” seems encouraging (see IA1, 2000). More and more experiments are reporting values with smaller and smaller error bars, which seem to be converging on a consistent picture of the universe. Obviously this may be premature, but it seems to be that the universe might be well described as having $\Omega_{\Lambda} = 0.7$ and $\Omega_M = 0.3$, where $\Omega_M$ is the normalised matter density of the universe (normalised such that 1 is the critical density) and $\Omega_{\Lambda}$ is the vacuum energy density; Longair (1998) gives a review of these parameters. The Cavendish group currently calculate $H_0$ whilst assuming an Einstein-de Sitter universe. There are two reasons for this. Firstly, the evidence for $\Omega_{\Lambda}$, i.e. high-redshift SN1A results, as reported in Perlmutter et al. (1999),
is very recent. The other reason is that the Einstein-de Sitter cosmology is one of a small number of cosmologies for which there is an algebraic form for the distance measures. However, the integration required to calculate the angular-diameter distance for an arbitrary cosmology is not difficult on a computer, and is discussed in Appendix B. The result derived there is that angular distance measure

\[ D_A = \frac{c}{H_0} \int_0^{\sqrt{1 - (\Omega_0 + \Omega_L)}} \frac{r H_0 \sqrt{1 - (\Omega_0 + \Omega_L)}}{c} \frac{dr}{(1+z)^{1/2}} \],

(2.13)

When integrated numerically (using Simpson’s rule), these equations were accurate to within 1% when compared to the analytic situations (i.e. when either \( \Omega_L \) or \( \Omega_M = 0 \)). In order to determine the effect on \( H_0 \) determination, a cluster was simulated. The simulation was performed in the same way as described in section 2.4.2. The cosmology assumed was \( \Omega_L = 0.7 \) and \( \Omega_M = 0.3 \) with \( H_0 = 50 \) km s\(^{-1}\) Mpc\(^{-1}\), and the cluster was placed at \( z = 0.5 \). An elliptical cluster was then fitted to the simulated X-ray map, but for Einstein-de Sitter universe, \( H_0 \) was then determined for the cluster, as described in section 2.2. The reported value of \( H_0 \) was 41.7 km s\(^{-1}\) Mpc\(^{-1}\). Figure 2.17 shows the returned \( H_0 \) values for different redshifts. This shows that if an Einstein-de-Sitter universe is assumed, a low value of \( H_0 \) is returned, i.e. if the universe is different from an Einstein-de-Sitter, the derived \( H_0 \) value will be higher. This figure also shows that with low redshift clusters (i.e. \( z < 0.3 \)), the difference in \( H_0 \) values is around 10%. Thus to determine \( q_0 \), measurements of \( H_0 \) for a range of redshifts with errors smaller than 10% are required. Finally, figure 2.17 also shows that as the ratio of the distance measures is a good fit to the data points and so the ratio of distance measures being all that is required to convert between these two cosmologies. The deviation between the points and the line is less than 1% and is due to the small numerical error from the integration in equation 2.13.

2.6 Noise and cluster positions

Noise on S-Z measurements affects not only the flux level but also the position. These are related, and the correlation between them is discussed in section 3.3.4. As they are from the same gas, the X-ray and S-Z positions should agree, if the cluster is isothermal and symmetric. As the positional error from the X-ray image is almost always smaller than that from the S-Z measurement, it is reasonable to set the position to be that from the X-rays. What happens if this is not done?

As in section 2.4.2, an A2218-like cluster was simulated. Twenty visibilities corresponding to points along the shortest baseline track of the Ryle Telescope were then determined. From this noiseless set of visibilities, 500 realisations of noisy observations were produced. This was done by adding a random number drawn from a Gaussian distribution of a given width to the real and imaginary visibility points. The NAG routine c05ddf was used to produce Gaussian distributed random numbers. The 1-\( \sigma \) for each visibility point was set to 600 \( \mu \)Jy; as 20 samples were used, this gives a noise level of 135 \( \mu \)Jy when averaged along the shortest baseline track, which is typical for a reasonable length S-Z observation with the Ryle Telescope.

Naturally-weighted maps were made from the 500 realisations, and from these maps, positions and fluxes of the S-Z decrements measured. In order to allow this process to be automated, AIPS was not used. Instead, mapping code written by Mike Jones was used. The code produces a map from the visibilities by gridding the visibility points and then FFTing into the image plane. The central decrement on the map was found by considering a 3 \( \times \) 3 grid around the minimum of the map. A two-dimensional quadratic function was fitted by minimising \( \chi^2 \) to the 3 \( \times \) 3 grid. \( H_0 \) was then estimated for each of the 500 datasets, in one case setting the cluster position at the X-ray centre and in the other the S-Z centre.

From equation 1.11, if the observed flux is Gaussian distributed, then the value of \( H_0^{1/2} \) will be Gaussian distributed. Figures 2.18 and 2.19 show the normalized value of \( H_0^{1/2} \), \( (H_0/50)^{1/2} \) for both tests. For the first test, where the cluster was placed where it was known to be, the value of \( (H_0/50)^{1/2} \) is 0.998 \( \pm \) 0.007. For the second test, \( (H_0/50)^{1/2} \) is 1.030 \( \pm \) 0.007. These values correspond to \( H_0 \) values of 50.2\( \pm \)5.7 and 51.1\( \pm \)9.7 km s\(^{-1}\) Mpc\(^{-1}\) respectively.

This experiment demonstrates that it is important to use the best knowledge available for the cluster positions, i.e. the position from the X-ray measurement. Not doing so risks underestimating the value of \( H_0 \).
2.7 Isothermality

So far, the majority of $H_0$ determinations in the literature (e.g., Birkinshaw & Hughes 1994, Hobbs et al. 1997b, Myers et al. 1997 and others) have assumed the cluster in question is isothermal. One reason for this is that most X-ray observations of relevant clusters have been consistent with isothermality. The intra-cluster gas temperature measurements are derived from spectral observations, and as the number of photons received by an X-ray telescope for a cluster at $z = 0.2$ in a reasonable length observation is low, it is often only possible to measure the spectra over a large area of emission. This means that it is hard to determine any temperature structure of the cluster. However, the temperature could be a slow function of radius without having a large effect on the observed spectra. Indeed, recent observations of the Coma cluster with the XMM satellite made by Arnaud et al. (2001) show a slowly decreasing temperature profile. Despite the observational difficulties, efforts have been made to determine temperature profiles for a large sample of clusters, notably by White (2000) and Markovich et al. (1998) with archival ASCA data. The analysis is complicated by the energy and position dependence of the telescope's point-spread function. White and Markovich come to differing conclusions about the existence of an overall temperature profile in clusters; Markovich finds an overall decreasing profile whereas White does not. Taken at face value this implies that if a universal profile exists, it is a weak function of position. As shown by Maggi (1996) a temperature that decreases with radius can lead to an underestimate of $H_0$. However, that work was limited in scope: only a King model with $\beta = 1$ was considered. Obviously this needs extending to a wider range of $\beta$ values.

One can derive temperature profiles theoretically. If a cluster is close to being virialised, it is reason-

able to assume that the cluster gas is close to being pressure supported. The equation for hydrostatic equilibrium is

$$\frac{dp}{dr} = -\frac{GM_m(r) \rho(r)}{r^2},$$

(2.14)

where $M_m$ is the total enclosed mass within a radius $r$, $\rho(r)$ is the gas density and $p$ is the pressure. Solving for $p(r)$ then allows the temperature profile $T(r)$ to be calculated by treating the gas as ideal, i.e.

$$p(r) \propto \rho(r) T(r).$$

(2.15)

Note that this is more general than assuming a polytropic profile, where $p \propto \rho^\gamma$, as that assumes the gas is adiabatic.

The calculation is complicated as the gravitational potential of the cluster is due to the gas, the galaxies and the dark matter. For convenience in this discussion, the gas is assumed to follow the dark matter, which is taken to be nine times denser. The mass contribution from galaxies is ignored. Frenk et al. (1999) shows that the gas following the dark matter is a reasonable approximation over the majority of the cluster; at very small radii — almost the limit of the simulations — the gas density is slightly lower. The gas is assumed to follow a spherical King profile. The enclosed mass is found by numerical integration, and a 4th order Runge-Kutta algorithm from Press et al. (1993) is used to solve for $p(r)$. The temperature profile is then found from equation 2.15. The code was verified as correct by comparison with previous work by Maggi and by testing with a $p \propto \rho^{\frac{4}{3}}$ density profile, which analytically gives an isothermal profile, and was found to do so using the code. Figure 2.20 shows the temperature profiles for four different values of $\beta$. Following Maggi, the temperature profiles are normalised such that the average temperature observed by an X-ray telescope is the same in each case.

In order to determine the effect on $H_0$, hydrostatically supported clusters were simulated with different values of $\beta$. As before, all the other parameters in the simulation were kept the same: $z = 0.171$, $r_c = 60''$ and $n_0 = 10^4$ electrons m$^{-3}$. X-ray and Ryle Telescope observations were simulated, as discussed in section 2.4.2. A spherical isothermal King model was then fitted to the hydrostatic X-ray image, and $H_0$ determined as described in section 2.2. The results are shown in table 2.2.

For $\beta < 0.8$, the reported $H_0$ is high. Figure 2.20 shows that for radii less than 5 core radii, the average gradient of the temperature profile increases for these $\beta$ values, and decreases when $\beta > 0.8$. Also note that the fitted $\beta$ and core radius values are lower than the initial parameters of the simulation; despite X-ray emission being a weak function of temperature, it is clearly having an effect on the X-ray fitting, and therefore on $H_0$.

Figure 2.21 demonstrates the effect that temperature profiles have on $H_0$ determination via the observed S-Z profile. If the temperature profile is decreasing with radius, as in the case of $\beta = 1$,
Figure 2.20: Temperature profiles for pressure supported King model clusters. The dark matter is 9 times as dense and is followed by the gas. The profiles are normalised to give the same observed temperature inside three core radii.

Table 2.2: Results from fitting an isothermal King model to a hydrostatically supported King model.

\[
\begin{array}{cccc}
\text{set } \beta & n_0 & r_0 & \text{resultant } H_0 \\
\hline
0.5 & 9.33 & 59 & 0.52 & 137 \\
0.6 & 9.56 & 56 & 0.59 & 81.3 \\
0.7 & 9.35 & 56 & 0.68 & 60.3 \\
0.8 & 9.20 & 54 & 0.77 & 52.2 \\
0.9 & 9.16 & 54 & 0.83 & 48.8 \\
1.0 & 9.16 & 54 & 0.91 & 47.5 \\
\end{array}
\]

2.8 N-body simulations

In order to fully test the \( H_0 \)-determination technique, all the various effects need to be incorporated. To do this, I have simulated \( H_0 \) estimates using cluster hydrodynamical simulations produced by Eke et al. (1998), kindly provided to me by Vincent Eke as observed images. The simulations were performed as follows.

The positions of 64\(^3\) dark matter particles were simulated from an initial power spectrum of Bond & Efstathiou (1984) until \( \sigma_8 = 1.05 \), which was taken as the present, using \( \text{AP}^3 \)M code. The 10 most massive collections of particles were then resimulated with roughly 10 times as many dark matter particles (the number of particles varied during the simulation dependent on the resolution required) and gas particles. The simulation method was SPH and did not include radiative cooling or any energy input from galaxies (which is a small effect for massive clusters). At each of 7 redshifts, maps were made of bolometric X-ray emission, \( y \)-parameter, and emission-weighted temperature. A \( \Omega_M = 0.7, \Omega_\Lambda = 0.3 \) cosmology with \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) was used. Vincent Eke also provided the kinetic energy and potential energy of the gas in the cluster rest frame and the emission weighted
temperature integrated over the cluster.

Each of the 10 clusters was also observed from three perpendicular directions, giving a total of 210 observations. However, it is not clear that these are statistically independent, and so I used only one set of projections. As an illustration, figure 2.22 shows the X-ray, y-parameter and temperature maps for one cluster at $z = 0.2$.

The redshift of the observations and the ages of the universe at these redshifts are shown in table 2.3. As the crossing time for clusters, $\sim 10^9$ years (see Sarazin, 1988), is of the same order of magnitude as the higher redshift timesteps, then there is time for significant evolution to take place between consecutive observations of each cluster. Thus simulations of a cluster views at seven different redshifts can, to some extent, be taken to represent seven different clusters.

Ryle Telescope "observations" were then produced from the y-parameter maps by multiplying them by the primary beam, and then FFTing into visibility space, sampling to produce sets of visibilities. The standard procedure, as discussed in section 2.2, was then used to estimate $H_0$ for each cluster. Only the central region of X-ray emission maps were used for fitting the King profiles. Three clusters at $z = 1.09$, and one at $z = 0.78$, did not converge to a $\beta$-model fit, and so are not included in the following analysis.

The overall average value of $H_0$ values found was $76.5^{+4.1}_{-2.4}$ km s$^{-1}$ Mpc$^{-1}$ from 66 clusters. The quoted error is the error on the distribution of $H_0$ values and does not include any errors due to noise. Figure 2.23 shows the distribution of $H_0$ values returned with redshift. The very extreme clusters, i.e. those with very high $H_0$, are obvious. The highest $H_0$ for redshifts 0.15, 0.2 and 0.78 are the same cluster (cluster number 10), and a separate cluster (number 8) has the highest $H_0$ for $z = 0.25$, 0.38 and 0.5. The images of these extreme clusters are shown in appendix C. The majority of these clusters are clearly not well modelled by an elliptical King profile, and so would not be seen as good candidates for $H_0$ estimation. However, the X-ray surface brightness map of the cluster at $z = 0.78$ seems to be a relaxed system, i.e. a good candidate for $H_0$ estimation. Thus selecting a sample on the basis of X-ray surface brightness maps would still include a value of 137 km s$^{-1}$ Mpc$^{-1}$ as an estimate, and so would not be a good indicator of suitability.


**Table 2.4:** $H_0$ values for the three different orientations

Table 2.4 shows the values averaged over the 66 clusters in the other two orientations. This shows that the line-of-sight degeneracy has been effectively removed.

It is important to note that with these noise-less simulated observations, figure 2.23 implies the Ryle Telescope is not a good instrument with which to determine the cosmological parameters $\Omega_M$ and $\Omega_\Lambda$.

**Virial radii**

As the virial radius of each cluster is known in the simulation, it is possible to test whether the core radius is correlated with the virial radius. Figure 2.24 shows the plot. As expected there is a strong correlation. Fitting a straight line to this gives:

\[
\text{Virial radius} = (13.1 \pm 0.8) \times \text{core radius/arcsec} + (100 \pm 50) \text{ arcsec.}
\]  

(2.16)

This result is useful when comparing the results of hydrodynamic simulations, which are often quoted in terms of virial radii, with real observations, which are usually reported in terms of core radii. It also shows that all the simulations go out to $\approx 50$ core radii.

**Axial ratios**

Figure 2.25 shows the distribution of axial ratios found for the simulated “low redshift” clusters (i.e. $z < 0.5$). It is notable that it is a narrower distribution than found in section 2.4.1 but with a few more extreme outliers.

**2.8.1 Temperature profiles**

In order to allow a comparison with the discussion of section 2.7, the emission-weighted temperature maps have to be deconvolved into their three dimensional temperature profiles. In order to do this, the profile was parameterised as

\[
T(r) = T_0 + T_1 x + T_2 x^2 + T_3 x^3,
\]

(2.17)

![Figure 2.23: $H_0$ values determined from orientation 1. The plot on the left is all 66 clusters. On the right, the geometric mean of the clusters at the same redshift has been calculated, and the width of the distribution estimated.](image)

![Figure 2.24: Plot of the virial radius from the simulation against the geometric mean of the projected core radii found in the King model fitting.](image)
where \( x \) is the number of core radii from the cluster centre. The density \( n_e(r) \) profile was assumed to be a King profile, and had already been determined from the X-ray map. The emission-weighted temperature can then be determined by integrating the function

\[
T = \frac{\int n_e^2(r) T^2(r) dr}{\int n_e^2(r) T(r) dr}
\]  

(2.18)

along a line-of-sight. This gives a temperature for one pixel on the map, and must be repeated over the entire map. As the temperature profile is assumed to be a slowly varying function of radius, and the bolometric X-ray luminosity was taken to vary as \( T^{1/2} \), it is reasonable to assume that the X-ray emission does not vary greatly with radius due to temperature variations. As such, the \( n_e \) profile was not varied; this should give a close approximation to the real temperature profile. (A simultaneous fit between the X-ray emission and the weighted temperature profile would have been strictly correct.)

The fitting between the model and given temperature maps was performed by calculating \( \chi^2 \) for them within the central 5 core radii and giving equal weight to each pixel. The parameters \( T_0, \ldots, T_3 \) were then varied to minimize \( \chi^2 \); this was performed with the Numerical Recipes routine AMOEBA (see Press et al., 1993). As an illustration, figure 2.26 shows the temperature profile derived for the cluster shown in figure 2.22.

The average temperature gradient over the central 5-core radii was calculated, as this is the region to which the Ryle Telescope is sensitive. As a fourth-order polynomial has been used, the average temperature gradient out to 5-core radii can trivially be calculated as

\[
\Delta T(r) = T_1 + 5T_2 + 25T_3.
\]  

(2.19)

Figure 2.27 shows a histogram of the average temperature gradients over the central 5-core radii. The width of the distribution is clearly narrow, and the distribution is biased slightly towards decreasing temperature profiles. The mean of the measurements is 0.009 ± 0.002 keV (core radius)\(^{-1} \). If these simulations do represent reality, then it is not surprising that temperature profiles are hard to detect.

In section 2.8 \( H_0 \) has been estimated for each of the simulated clusters and figure 2.28 shows that there is no strong correlation between the returned \( H_0 \) value and the average gradient of the temperature profile. Indeed, the correlation coefficient is 30%. As the temperature gradients are so gentle, any bias is being washed out by larger effects, such as unknown line-of-sight.

### 2.8.2 Virialisation

Figures 2.29 and 2.30 show two different clusters from the simulations, both at redshift 0.2, with very different ratios of kinetic to potential energy. Figure 2.29 has K.E./P.E. = 0.2265 whereas figure 2.30 has K.E./P.E. = 0.0111. In the first case, the temperature map shows a great deal of structure and the \( y \)-parameter map shows some structure but is still quite smooth. This is not surprising given pressure support is present. In the second case the majority of the cluster is virialised and smooth.

The 15 clusters with the lowest K.E./P.E. give a geometric mean of \( H_0 = 74.3^{+3.3}_{-3.5} \) km s\(^{-1} \) Mpc\(^{-1} \). For the 15 clusters with the highest normalized KE the result is \( H_0 = 75.2^{+4.4}_{-4.6} \) km s\(^{-1} \) Mpc\(^{-1} \). Only one set of orientations was used for this selection. It is clear that the the rms is much lower in the case of low relative K.E. This is not surprising given that a high K.E./P.E. implies that the gas has not virialised, and could be in different phases or undergoing shocks.

If the cluster gas is undergoing shocks, then the X-ray emission will be raised, as will the emission

![Figure 2.26: The temperature profile deprojected from the simulated cluster shown in figure 2.22.](image)
CHAPTER 2. SYSTEMATIC EFFECTS IN MEASURING $H_0$

2.8. N-BODY SIMULATIONS

Figure 2.27: Distribution of the average temperature gradients between 0 and 5 core radii present in the hydrodynamical simulations. The y-axis units are keV (core radius)$^{-1}$.

Figure 2.28: Returned $H_0$ value against average temperature gradient.

Figure 2.29: Maps of the X-ray emission, $y$-parameter and emission weighted temperature for a cluster at $z = 0.15$ with a high K.E./P.E. The simulations were performed by Eke, Navarro and Frenk, and the maps were produced and kindly provided by Vincent Eke.
2.8. N-BODY SIMULATIONS

Figure 2.31: Left: the one-phase, two-dimensional square of gas. Right: the two-phase, two-dimensional square of gas.

weighted temperature. Assuming that the shocks remain in pressure equilibrium, the observed S-Z flux will remain constant, and so $H_0$ will be biased high (cf. equation 1.9). Note that a strong shock is also very obvious from X-ray images, and so, in real life, a strongly shocked cluster is unlikely to be used for $H_0$ estimation (see Saunders et al., 2001).

What happens with different phases? Consider the two dimensional system illustrated in the left of figure 2.31. The S-Z flux detected and X-ray luminosity are given by

$$SZ \propto n_0T_0^3f.$$  

(2.20)

$$\nu_{SB} \propto n_0^2T_0^{3/2}.$$  

(2.21)

Rearranging gives

$$\frac{1}{l} \propto \frac{\nu_{SB}}{SZ} T^{3/2},$$

(2.22)

which is proportional to $H_0$. (Note that equation 1.9 assumes that $\nu_{SB} \propto n_0^2f(T)$; here I assume that $f(T) \propto \sqrt{T}$). Now consider two phases in this medium, as shown in the right half of figure 2.31. The first phase has normalised values for $n$ and $T$. The second phase has a temperature of $\eta$ and, as it is in hydrostatic equilibrium, a density of $1/\eta$. The overall S-Z flux from the square remains the same as with an isothermal square with $n = 1$ and $T = 1$, as the size of the square has not changed. The X-ray surface-brightness is given by

$$\nu_{SB} \propto (1 - \beta) + \beta \frac{1}{\eta^{3/2}} T^{3/2}$$

(2.23)

$$\propto (1 + \beta(\eta^{-3/2} - 1)),$$

(2.24)

and the emission-weighted temperature is given by

$$T = \frac{(1 - \beta) + \beta \eta^{-3/2}}{1 + \beta(\eta^{-3/2} - 1)}.$$  

(2.25)

The combination of these into equation 2.22 gives the equation

$$\frac{1}{l} \propto (1 + \beta(\eta^{-3/2} - 1)) \left( \frac{1 + \beta(\eta^{-3/2} - 1)}{1 + \beta(\eta^{-3/2} - 1)} \right)^{3/2}.$$  

(2.26)

Figure 2.33 shows a plot of this equation. Note that $\eta$ can vary around 1, whereas $\beta$ can only vary between 0 and 1. This implies that different gas phases will always cause $H_0$ to be reported low.
However, as discussed in section 2.7, significant volumes of differing temperature are unlikely remain unnoticed in real cluster observations.

Thus shocks can increase the returned \( H_0 \), and different gas phases decrease the returned \( H_0 \). A high K.E./P.E. cluster will have a more complex gas distribution, and so more likely to have both shocks and a multi-phase medium. Thus it is not clear whether there will be any large net bias at all, but the width of the distribution will most certainly increase.

With the numerical simulation data it is possible to get a more realistic determination of the number of clusters required to obtain a reasonable estimate of \( H_0 \). Figure 2.33 shows the variation in \( H_0 \) with number of clusters used. The simulated clusters with a redshift less than 0.4 are used and have been selected in a random order. It is clear that error bars stay approximately the same and enclose the correct value of \( H_0 \) after 20 clusters have been analysed. The error at this point is 8%. Thus after 20 clusters have been observed, measurement errors have become dominant and any systematics in the overall technique are effectively unimportant.

2.9 Conclusions

1. The degeneracy in fitting an isothermal King model to an X-ray profile has been investigated, and a strong correlation between the core radius and \( \beta \) found. The shortest baselines of the Ryle Telescope — which are sensitive to the S-Z effect — probe a region of the profile that remains constant along the degeneracy direction, and so the effect on \( H_0 \) determinations is low at around 14%.

2. A sample of clusters (with redshifts from 0.144 - 0.546) suitable for \( H_0 \) determination with the Ryle Telescope has been examined, and projected axial ratios determined for the 31 clusters.

With archival ROSAT pointed observations. The axial ratios vary between 0.43 and 1; the most common value is between 0.7 and 0.75. The Mohr et al. sample was also considered (redshift range 0.01 - 0.18), the range of projected axial ratios is similar but the most common value is between 0.8 and 0.9.

3. A set of 50 clusters were simulated orientated randomly, all at \( z = 0.171 \). I found that allowing the intrinsic core radii to be chosen randomly between 35 and 120° gave an projected axial ratio distribution similar in range to that observed in the \( H_0 \)-sample above.

Ryle Telescope and X-ray observations were simulated using the 35 to 120° range of intrinsic axial ratio and \( H_0 \) estimates made, giving the following result.

4. With 50 clusters, the geometric mean \( H_0 = 50.2^{+2.2}_{-2.6} \) km s\(^{-1}\) Mpc\(^{-1}\). (The value set in the simulation was 50 km s\(^{-1}\) Mpc\(^{-1}\).) With 15 clusters, the likely error is 10%, which is less than the random noise on current estimates.

With further simulations, I have shown the following.

5. If the position of a cluster is taken from noisy observations of the S-Z, then it is possible to underestimate \( H_0 \). Thus the best positional accuracy measurement, usually the X-ray image, should be used to determine the position of a cluster when estimating \( H_0 \).
5. If a cluster is in hydrostatic equilibrium, and the dark-matter profile follows a King model, and the cluster is assumed to be isothermal during $H_0$ estimation, then a cluster with $\beta = 0.8$ will overestimate $H_0$ by 5\%, whereas a cluster with $\beta = 0.7$ will underestimate $H_0$ by 5\%.

6. Temperature increasing with radius in the cluster will bias $H_0$ determinations high, and decreasing profiles will bias them low.

Using $\gamma$-parameter maps, X-ray surface brightness maps, emission weighted temperature maps, and K.E. and P.E. values in clusters from hydrodynamic N-body simulations, I have shown following.

7. With 20 cluster simulations, all with $z < 0.4$, non-ideal effects tend to cancel, the resultant estimated $H_0$ value is little biased, and the resultant error, including unknown orientation, is 8\%.

8. Using three independent views of the same simulated clusters, the effect of line-of-sight uncertainty is effectively removed, supporting the notion that it can be removed in real observations.

9. There is indeed a tendency to get decreasing temperature profiles, but there is no significant correlation between the average gradient and estimated $H_0$ for each cluster because the gradients are small and other effects, such as shocks and multi-phase media are larger.

10. The core radius is, as expected, proportional to the virial radius.

11. The degree of virialisation in a sample of clusters affects the width of the distribution of $H_0$ values found but it seems not to bias the result. With the 15 most relaxed clusters, $H_0$ was estimated as $74.3^{+1.2}_{-1.3}$ km s$^{-1}$ Mpc$^{-1}$ whereas with the 15 clusters with highest K.E. the estimated $H_0 = 75.2^{+0.6}_{-0.6}$ km s$^{-1}$ Mpc$^{-1}$ (for an input value of 75 km s$^{-1}$ Mpc$^{-1}$).

As no significant bias has been found, this work adds weight to the validity of the S-Z + X-ray route for determining $H_0$.

Chapter 3

New measurements of $H_0$

3.1 Introduction

In chapter 2, I have shown that if good X-ray and S-Z observations exist, it is possible to measure $H_0$ with a reasonably-sized sample of clusters. However, the effects of confusing radio sources have not yet been addressed. In this chapter, I examine various subtraction strategies that have been developed for Ryle Telescope data, including my own. I then go on to determine $H_0$ from the clusters 0016+16 and A611.

3.2 The 15-GHz sky

For a massive cluster at moderate redshift, the flux that the Ryle Telescope detects from the S-Z effect at 15 GHz is typically ~500 $\mu$Jy. This is sufficiently faint that radio sources will almost invariably be present with comparable or greater amplitudes. Therefore removing the effects of radio sources is an essential step. As the Ryle Telescope is an interferometer with a wide range of baselines (in comparison to the range of baselines for detecting the S-Z flux), it can simultaneously measure the extended S-Z flux and the fluxes and positions of the small angular size radio sources. Figure 3.1 shows the variation of S-Z flux with baseline for the Ryle Telescope. There is very little flux from the S-Z effect above 1.5 kA, and so baselines longer than this can be used to measure sources to a good approximation and remove their effect from the S-Z signal. The measurement is simultaneous and, of course, at the same frequency, and so the spectral index and variability of the sources are unimportant. If the required signal-to-noise for the S-Z decrement of a massive cluster is 10:1, then all sources down to 50 $\mu$Jy must be accurately removed. By choosing the configuration such that there are more long baselines than short, it is possible to achieve suitable signal-to-noise for both long- and short-baseline maps to optimise the observations to reach 50 $\mu$Jy sources and also achieve
good signal-to-noise for the S-Z effect. The Ryle Telescope has a dynamic range of 100:1 which implies that S-Z detections should not be attempted if a source of > 5 mJy is present in the field.

Although the S-Z central decrement is independent of redshift, the flux observed by a particular telescope will vary with redshift. Figure 3.2 shows the variation of received flux from a fixed physical core-radius cluster with redshift for an Einstein-de-Sitter universe. The drop at both very low and very high redshift is caused by the cluster subtending a large angle, narrowing the flux profile observed in visibility space, and so reducing the flux received on a particular baseline.

3.3 Processing S-Z data and removing radio sources

As the Ryle Telescope is an east–west telescope, a typical observing run of an S-Z field is 12 hours long, with no shadowing. The run is split into 32-second samples. After every 30 samples, a bright nearby point source is observed for either 5 or 10 samples (depending on its flux) for use as a phase calibrator. Primary flux calibration is achieved by observing either 3C48 or 3C286 for about 20 minutes either before or after a run. Details of the data reduction strategy are discussed in Grainge (1995). In essence, the flux- and phase-calibrator observations allow the complex gains for each subband and channel to be calculated.

Each sample is given a weight determined from the noise-injection system: noise at a known small level is injected before each front-end amplifier; this power is synchronously detected after the automatic gain control stage, thus providing relative calibration of each antenna’s system temperature and so permitting relative calibration between voltage and Jansky units whilst allowing for, for example, airmass, cloud and dew on the horn covers.

A small number of samples are then averaged together in time to produce each visibility point; the separation of subband and channels are kept. In the standard S-Z observing configuration, 10 samples are time-averaged together. However, in an extended configuration, say C9, averaging 10 samples together in time would smear out the signal on longer baselines. So only 3 samples are averaged together. Only data from airmass 1 to 5 are used as the other (unmoveable) airmass provides such long baselines that the rate of change of phase due to the fluctuating atmosphere makes these baselines unusable. In addition, sources are often resolved, making source subtraction more difficult.

Once each day’s run has been calibrated, it is loaded into the Astronomical Image Processing System (AIPS). The task HORUS (which grids, weights and Fourier transforms visibility data into a dirty map and beam) is used to produce naturally weighted maps of all the data for each 12-hour run. Naturally weighted maps are ones where the visibility data from different 12-hour runs are combined together, the visibility points are weighted by the inverse of their variance. No additional weighting is used for map production. Natural weighting is used to give the best possible signal-to-noise ratio at the expense of larger sidelobes. (The alternative, uniform weighting, takes account of the fact that there are more short baseline visibility points and so down weights them. This reduces the sidelobes of the beam but decreases the signal-to-noise.) Maps are almost always 512 by 512 pixels, with each pixel being 4 x 4". Typical noise on a map is 200 μJy for a full 12-hour run with no shadowing; any maps with substantially more noise than this (which would normally be due to poor weather) or obvious interference (which is very rare) are rejected.
Flux  ΔRA  ΔDec
/ mJy  / arcseconds
3000  10  10
1000  -35  15
300   50  -40
300   120  100
150   60  -60
150   0  0

Table 3.1: The point source fluxes and positions in the simulated data set. The convention used in this and other tables is the same as required for the task UVSUB, i.e. positive ΔRA is an increase in the RA value, so the source is to the east (to the left on conventional maps) and positive ΔDec is an increase in Dec, i.e. to the north.

The ‘days’ of data are then concatenated together. A map of the total data set is then produced and the overall noise level is checked. A long baseline map is then made so that sources can be identified and subtracted. There are three methods of source subtraction that have been applied to Ryle Telescope data:

- using **CLEAN**, which has been used for the bulk of published S-Z measurements from the Ryle Telescope (e.g. Grainge et al., 1996),
- the matrix method, which was first used by Keith Grainge (1995), and
- the **FLUXFIT** method, first used by Riju Das (1999), and modified here.

This sequence began with **CLEAN**, the classical radio-astronomy image deconvolution technique. The matrix method is a more linear method for removing the effects of sidelobes, and **FLUXFIT** addresses the problem of simultaneously fitting both point sources and S-Z in all the data. This is the first proper comparison of these methods.

### 3.3.1 The test data

The three methods can be readily compared and explained with an example of a simulated data set containing both sources and an S-Z effect. This was provided by Mike Jones, and is a simulation of a 54-day observation of a field at declination 44°. The observing used was based on a standard Cb configuration. The point source fluxes and positions are shown in table 3.1. The S-Z hole was modelled with two negative extended Gaussian sources \( S_1 \) and \( S_2 \), each positioned at 30° in RA and 20° in Dec from the pointing centre. \( S_1 \) is an elliptical Gaussian with a total flux of \(-5\) mJy and a FWHM size of 189"×129" with the major axis at PA 30°. \( S_2 \) is a circular Gaussian with a total flux of \(-3\) mJy and a FWHM of 60°. Figs 3.3 and 3.4 show maps made by **HORUS** with all the data, and with baselines longer than 2 kλ respectively.

### 3.3.2 The **CLEAN** method

The **CLEAN** algorithm, first described in Hoagman (1974), is an efficient iterative way of deconvolving images in traditional radio astronomy. It was developed to deal with “incompletely sampled visibility plane” data in 1974, and, apart from being made more efficient, has been a standard method since. It works in the following way. The brightest peak in an image is found. The location to search can be constrained with “**CLEAN boxes**”. A delta-function is placed in a model of the sky at the position of the peak with a flux that is a fraction of the flux — typically 5 or 10% — from the image. The delta-function is then convolved with the beam, and the residual image, i.e. the difference between the convolved model and the image, is then calculated. The cycle is then repeated using the residual image. The delta functions that **CLEAN** places in the model are known as “**CLEAN components**”. Finally, when the residual map is thought to be empty of sources, the **CLEAN** components are convolved with a **CLEAN**-beam and added back onto the map. Clark (1980) showed that this algorithm can be made more efficient by splitting the algorithm into major and minor cycles. A minor cycle consists of performing the standard **CLEAN** on the most extreme values of the current residual image. At the end of the minor cycle (defined as when the restricted residual map becomes smaller than a multiple — defined in Clark — of the input restricted map), the clean components are Fourier transformed, subtracted from the visibility data, re-gridded and **FITTED** back to the map plane and then subtracted. Thus small errors in the minor cycle are corrected as the subtraction is done exactly.

Why are fractions of **flux** subtracted on each loop? As the dirty map is a convolution of the sky with the beam, sources will be sitting on the sidelobes of other sources. As the sidelobes can be both positive and negative, it is possible that the flux on the dirty map is smaller than the actual flux. Thus, over-subtraction is avoided. The implementation used here, **APCLN**, is known as a visibility-based **CLEAN**. It is visibility-based in the sense that the calculation of the residual image is done in...
the uv-plane. (Some implementations do the subtraction in the image plane.)

For this method, a dirty map of all the baselines longer than 1.5 kA is produced by HORUS. CLEAN boxes are then placed around the visible sources, and ApCLN is used to deconvolve the beam from the dirty map. After deconvolution, the CLEAN-values of the fluxes are measured, using the AIPS verb MAXFIT. This verb fits a two-dimensional quadratic function to the 9 pixels around a peak in the map, and reports the peak interpolated flux and position. The task UVSUB is then used to subtract the fitted point sources.

In some fields, such as this example, there is a bright radio source. (At 15 GHz in an S-Z field, "bright" corresponds to > 1 mJy). The large sidelobes of the beam make it difficult to identify other sources in the field. In these cases, MAXFIT is used to estimate the position and flux of the source from the dirty map, and it is subtracted straight away with UVSUB. Then the above algorithm can be applied to the source-subtracted dataset. It is important to be aware that if the strong source is sitting on the positive grating ring of a weaker source, then the measurement will be an overestimate of the true flux. As a precaution, not all of the flux from the bright source is initially removed with UVSUB. This leads to a source being visible at the position where the flux had been incompletely removed, as it should. In this example, source 3 in table 3.2 was reduced by subtracting 2000 µJy from position (9,10) before looking for more sources. The remaining 1000 µJy of flux was then found to be at the position (11,9). Given the resolution of the simulated observation, 37.5 arcsec, this apparent shift is entirely consistent with noise.

The sources and the positions found in the simulated data set are listed in table 3.2, and the CLEANed map is shown in figure 3.5. The seven sources found are labelled. It should be noted that only sources 1 to 6 were initially measured from the CLEAN map and subtracted from the dataset. The 7th source was visible after mapping the post-subtraction data. Had a source been visible outside the area CLEANed after subtraction, it would have been necessary to redo the CLEAN; in this case the source had been CLEANed. The map made from baselines longer than 2 kA was consistent with noise after all the 7 sources had been subtracted. The final S-Z flux observed was -3.39 mJy, at a position (31,20) arcseconds from the point of origin. The S-Z values were measured with MAXFIT from a map made with the baselines shorter than 1.5 kA. Note that for this dataset the S-Z flux is unphysically large for 15 GHz and that the profile is not a King profile. The model used also produced more flux on longer baselines than a King profile would, and so a higher baseline cut was used in this case to avoid contaminating the point source maps.

A comparison of the sources found (table 3.2) and the sources actually in the model (table 3.1) shows the how well the CLEAN method works. The most glaring problem is that 7 sources have been detected where there are only 6. This is one example of CLEAN introducing a bias.

### Table 3.2: Fluxes and positions as measured by MAXFIT after Cleaning.

<table>
<thead>
<tr>
<th>Source number</th>
<th>Flux / µJy</th>
<th>Offset from pointing centre / arcseconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>118, 104</td>
</tr>
<tr>
<td>2</td>
<td>374</td>
<td>-48, -37</td>
</tr>
<tr>
<td>3</td>
<td>2977</td>
<td>11, 9</td>
</tr>
<tr>
<td>4</td>
<td>919</td>
<td>-35, 15</td>
</tr>
<tr>
<td>5</td>
<td>178</td>
<td>-60, -58</td>
</tr>
<tr>
<td>6</td>
<td>181</td>
<td>-100, -9</td>
</tr>
<tr>
<td>7</td>
<td>310</td>
<td>-8, 6</td>
</tr>
</tbody>
</table>

![Figure 3.5: A CLEANed map of the simulated data. The map was made using all baselines longer than 2 kA and then CLEANed with boxes around each source. Source 3 has had 2000 µJy of flux already removed to aid identification of weaker sources. The contour levels are -180, -90 µJy (dashed), 90 x 41, 41, 2, 2, 4, 4, 4 µJy beam^-1 (solid). The beam size FWHM is shown in the bottom left. The object to the north of source 3 is below the flux cut for positive detections.](image_url)
3.3. PROCESSING S-Z DATA AND REMOVING RADIO SOURCES

**Figure 3.6:** Long (i.e. greater than 2 kJ) baseline map for the test data after the sources in table 3.3 have been subtracted. The greyscale range is from −480 to 480 μJy beam$^{-1}$.

Produced by HORUS, and of course includes the effect of sidelobes. This method has the advantage over CLEAN in that sources that are measured in the map to be point sources are modelled with one flux and position. With the simulated test data, the brightest 5 sources as identified with the CLEAN method were used, i.e. the sources labelled 5 and 6 in figure 3.5 were not fitted. The resulting matrix was

$$
\begin{pmatrix}
634 \\
117 \\
2568 \\
508 \\
1010
\end{pmatrix}
= 
\begin{pmatrix}
1 & 0.084 & 0.125 & -0.071 & 0.031 \\
0.084 & 1 & -0.043 & -0.047 & -0.130 \\
0.125 & -0.043 & 1 & -0.256 & 0.324 \\
-0.071 & -0.047 & -0.256 & 1 & -0.235 \\
0.031 & -0.130 & 0.324 & -0.235 & 1
\end{pmatrix}
\begin{pmatrix}
S_{\text{sky},1} \\
S_{\text{sky},2} \\
S_{\text{sky},3} \\
S_{\text{sky},4} \\
S_{\text{sky},7}
\end{pmatrix}
$$

(3.2)

where the vector on the left side of the equation is a measurement of the dirty flux (in μJy) at each point. The value $S_{\text{sky},z}$ is the beam-attenuated flux on the sky of source $z$, using the same labelling as for sources found in the CLEAN method.

The matrix method is linear, an immediate advantage over the CLEAN method. If on the first subtraction attempt some sources are still present, then the additional terms for the matrix can then be measured and the solution recalculated. It has the disadvantage of being slow.

Table 3.3 shows the fluxes found. A comparison with the sources known to be in the model (table 3.1) shows that the fluxes are significantly different. There are two reasons for this. Firstly, not all the flux has been subtracted. The long-baseline map after subtraction is shown in figure 3.6. That not all the flux has been subtracted is however a small effect; the flux left on the map is only 150 μJy,
5% of the measured S-Z effect in this case. Secondly, the positions used have been determined from the unCLEANed map, by searching for extrema with MAXFIT. Occasionally, the relative positions of sources are such that one source is in the steepest part of a sidelobe of another, and MAXFIT does not find an extremum that corresponds to that source. This has occurred in this case with sources 3 and 7 (using the CLEAN method numbering). In these situations, MAXFIT is not used, and the position from the CLEAN method is used. The flux value from the unCLEANed map in that position is then used. Occasionally this too fails. In this situation, when the CLEAN positions are used for sources 5 and 6 — the two faintest sources — the resultant solution contains negative fluxes, which is clearly unphysical. Thus one of the 7 sources is not real, or the position used is not correct. Without additional information, for instance a lower frequency map or higher sensitivity data, it is impossible to say which source is real and which is noise. This is why the example here only uses the brightest 5 sources. The reliance on the CLEAN method and with the difficulties automating the process for large numbers of sources means that the “matrix method” has only been applied in simple cases with a few well separated sources.

### 3.3.4 The FluxFitter method

In an attempt to overcome the problems of the CLEAN and matrix methods, a piece of code known as FluxFitter was written. The first version was written by Rhiju Das (1999), which I then extended to allow all the parameters to vary and to model the S-Z effect correctly. The algorithm that it uses is straightforward.

1. An initial guess of a model of the sky is found, using a set of parameters that represent the positions and fluxes of each point source, including the S-Z hole. This model is determined from the long baseline Ryle Telescope maps (either raw or CLEANed) and, for the S-Z parameters, X-ray observations.

2. The flux that the Ryle Telescope would observe is calculated for every visibility point.

3. The misfit between the model and real visibilities is then calculated as \( \chi^2 \).

4. The parameters are then varied to minimise \( \chi^2 \).

The best-fitting parameter values are then used to subtract the point sources; this is done within AIPS. There are two advantages to this method. Firstly, it works almost entirely in the aperture plane; only the source identification and approximate position finding is done in the image plane. Working in the aperture plane is good because that is where the data are taken (and so where any problems start) and the noise distribution is known to be Gaussian. The second advantage of this method is in point 1. A simultaneous fit to the positions and fluxes of the point sources and the S-Z hole is a clear improvement over either of the previous methods. It allows full use of all visibility data, which increases the signal-to-noise ratio for point source measurements. An associated advantage is that if the S-Z flux is present on longer baselines then the correct result in obtained.

The initial parameters for the model are still estimated by using the CLEAN method. In a complex situation with both bright and faint sources, the map is CLEANed, and then positions measured. These are then subtracted and the subtracted data then mapped again. This loop can be performed many times to estimate the number of sources and their approximate positions and fluxes. In a less complex situation, CLEAN is not used, and sources are approximately subtracted and then the data remapped. Again, this is just to provide an initial guess for FluxFitter.

Currently only the flux of the S-Z hole is varied. The other parameters that describe the hole — position, core radius of the cluster and \( \beta \)-parameter for the cluster — are all fixed. This is done for a number of reasons. Firstly, the radio data do not well constrain the core-radius or \( \beta \)-parameter, due to the lack of many different-length baselines that are sensitive to the S-Z hole. The X-ray measurement constrains the core radius, \( \beta \)-parameter and position much better. In addition, varying any of the parameters of the S-Z hole, i.e. changing its shape or position, requires the S-Z hole to be recalculated and a FFT to calculate what the Ryle Telescope would observe. The minimiser generally requires a good deal of \( \chi^2 \) evaluations (the fitting for the simulated test data takes 750 iterations and roughly the same number of function evaluations), and so performing an FFT at each \( \chi^2 \) evaluation slows the entire process down a great deal. However, varying the amplitude of the S-Z hole does not require an FFT and so it is included as a parameter. The flux of the S-Z hole as reported by FluxFitter is not currently used in determining \( H_0 \). Instead the fit is done to the unCLEANed data as described in section 2.2.

**FluxFitter** was run on the simulated test data. As the simulated S-Z hole is not a King profile, attempting to fit a King profile to it would bias the point-source fitting. Thus only baselines longer than 2 k\( \lambda \)s were used. The fluxes and positions from the CLEAN method were used as an initial guess. The source fluxes and positions from fitting the simulated data are shown in table 3.4. After subtraction with UVSUB, the map of baselines greater than 2 k\( \lambda \)s was consistent with noise. A map of baselines shorter than 1.5 k\( \lambda \) and with these 7 sources subtracted shows a hole of flux \( -3.327 \) mJy at an offset (30, 20). All of these results are consistent with the results from the CLEAN method. Although this test does not show the entire advantage of FluxFitter it does show that sources can be fitted accurately in a complex environment without recourse to CLEAN.

**FluxFitter** also reports error bounds. As figure 3.7 shows, the \( \chi^2 \) contours are elliptical and oriented along the variable axes, which shows that the parameters are independent. The error on each parameter is calculated by finding the parameter values at which the reduced \( \chi^2 \) increases by 1.
### Table 3.4: Source positions and fluxes as reported by FLUXFITTER

<table>
<thead>
<tr>
<th>Source number</th>
<th>Flux / µJy</th>
<th>Offset from pointing centre / arcseconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>290 ± 34</td>
<td>117.5 ± 1.3, 103.5 ± 2.2</td>
</tr>
<tr>
<td>2</td>
<td>373 ± 34</td>
<td>46 ± 1, -36.4 ± 1.5</td>
</tr>
<tr>
<td>3</td>
<td>3017 ± 34</td>
<td>9.3 ± 0.1, 9.4 ± 0.2</td>
</tr>
<tr>
<td>4</td>
<td>992 ± 34</td>
<td>-35.1 ± 0.4, 15.7 ± 0.6</td>
</tr>
<tr>
<td>5</td>
<td>198 ± 34</td>
<td>-59.5 ± 1.7, -55.9 ± 2.9</td>
</tr>
<tr>
<td>6</td>
<td>172 ± 34</td>
<td>-106.5 ± 2.4, -10 ± 3</td>
</tr>
<tr>
<td>7</td>
<td>215 ± 34</td>
<td>-15.3 ± 1.6, -4.3 ± 3</td>
</tr>
</tbody>
</table>

#### Figure 3.7: \( \chi^2 \) contours for the flux and position in Dec for the brightest source in the CRH 1716 field (cf. chapter 4)

The spacing between contour levels in each figure is such that the reduced \( \chi^2 \) value increases by 1 between contours.

#### Figure 3.8: \( \chi^2 \) contours for position in RA and Dec for the brightest source in the CRH 1716 field (cf. chapter 4)

The error bound reporting was checked by simulating a point source with varying signal-to-noise ratios. 500 observations of a single point source were simulated; the signal-to-noise was kept constant for groups of 10 simulations, and the position was held constant for all the simulations. The visibilities were simulated with Gaussian noise. The known position was then fed to FLUXFITTER as an initial guess, and the best fitting position and flux recorded. It was found that the quoted error bar does enclose the position for 67% of the time. It was also found that the uncertainty of the position, that is the size of \( \sigma \), varies as the inverse of the signal-to-noise. This result is shown in figure 3.9. Note that this relation holds down to very low signal-to-noise ratios. The result is useful for determining whether a tentative source found with the Ryle Telescope at low signal-to-noise has a position coincident with higher significance data, for example from NVSS or DSS.

#### 3.3.5 Possible improvements

Source recognition is clearly the biggest problem that still remains. It is the only step that is still performed in the map plane rather than the aperture plane. As producing the map requires Fourier Transforms, the noise properties of the map are not straightforward; in the aperture plane the noise is known to be Gaussian on each measured point. There are computational issues involved here. Producing a map and identifying sources by hand is possible and fairly cheap in computer time. Minimising the misfit between the data and a given number of sources is also cheap; but allowing the number of sources to vary vastly increases the complexity of the problem and the time required. It is possible that more advanced minimising techniques, such as simulated annealing (see, for example, Press et al., 1993) or using maximum entropy techniques will make this possible and robust in the
3.4 Summary of methods

Tables 3.2, 3.3 and 3.4 show the results of three different methods for source fitting. The positions and fluxes put into the model are shown in table 3.1. The resultant position and depth of the S-Z holes after subtraction are shown in table 3.5. The parameters are all measured from unsmeared maps, made with baselines shorter than 1 ka. The position and flux from the matrix method are incorrect due to incomplete subtraction. The CLEAN and FLUXFIT methods give similar results, which is not surprising as they give similar results for the source fluxes and positions.

All three methods benefit from prior knowledge of the source distribution on the sky. This can be estimated from looking at lower frequency surveys such as NVSS or FIRST. Most falling-spectrum sources, i.e. with $\alpha > 0$ (where $S \propto v^{-\alpha}$, $v$ is the observing frequency and $\alpha$ is the spectral index) will be detected in NVSS and/or FIRST. However, as shown by Cooray et al. (1998) in clusters and Taylor (2000) generally, there are rising spectrum sources, i.e. with $\alpha < 0$ that are present at 15 GHz and not detected in NVSS and FIRST.

3.5 New Ryle Telescope observations

Abell 611, Abell 990 and cluster 0016+16 are three moderately high redshift clusters identified by Keith Grainge (1995) as being good candidates for S-Z observations with the Ryle Telescope. They form part of the luminosity-limited sample of clusters discussed in chapter 2 identified as being potential candidates for $H_0$ determination. Table 3.6 shows the basic properties of the clusters.

3.5.1 X-ray observations

To perform a rigorous source subtraction using FLUXFIT, a model of the gas distribution in each cluster needs to be found. ROSAT images were taken from the archive and an isothermal elliptical King model fitted, as discussed in chapter 2. The ROSAT images are shown in figure 3.10. In A990, White (2000) finds evidence for a cooling flow with a mass deposit rate of $3 \times 10^{12} M_\odot$ yr$^{-1}$. This result is from a two temperature fit to ASCA data. However, the signal-to-noise ratio of the ROSAT observation is such that a cooling flow cannot be positively identified. In order to avoid the possible bias, a central region was masked out for A990. If these pixels do contain a cooling flow and were not ignored, then the core radius may be biased low and the $\beta$ value biased high. In all the clusters, any regions which looked like a point source were also masked out and not used in the fitting. The results found for each cluster are quoted in table 3.7. 0016+16 has been observed and analysed by many groups; the results are shown in table 3.8. Note that only Hughes & Birkinshaw (1998) do not fit a spherical model; they find an elliptical ratio of $0.85 \pm 0.02$, closely comparable to my value of 0.84. The $\beta$ and core-radius values are also in good agreement.

The X-ray observation of A611 is of poor quality. The image contains a couple of bright pixels, which, on comparison with the POSS image, are coincident with a large galaxy. These pixels were removed; including them pushed the central density up and the core radii down to very small values.


3.6 Abell 111

A611 was observed for 16 sets of 12 hours between November 1994 and January 1995 with the Ryle Telescope in configuration Cb. I rejected three days of data taken in bad weather after examining the 1-day maps and noise levels. A map of the combined 13 days of data using baselines longer than 1.5 kA had a noise level of 70 μJy beam\(^{-1}\), and only one source was visible, with a flux of 299 μJy beam\(^{-1}\) at RA 8\(^{h}\) 0\(^{m}\) 57\(^{s}\) 1 Dec. +36° 3° 40′. This source was removed with UVSUB, using the flux and position from the dirty map. A long-baseline map of the subtracted data was consistent with noise, with no other sources in the field. A map of the baselines shorter than 1 kA shows a decrement of –624 ± 157 μJy at RA 8\(^{h}\) 0\(^{m}\) 57′ 4 Dec. +36° 3° 5′. This is 9′ from the X-ray position (2.4′ in RA and 8.6′ in Dec). The S-Z and X-ray positions are fully consistent given the CLEAN beam of 97′ by 291′.

Table 3.9 lists the sources found in the FIRST catalogue around the pointing centre for A611. The NVSS catalogue does not contain any sources for this region. Neither of the two FIRST sources are detected at 15 GHz and the source that is present at 15 GHz is not detected at lower frequencies. Figure 3.11 shows a dirty map made with short baselines without any source subtraction, whereas figure 3.12 shows the same thing with the source subtracted. It is clear that source subtraction is important and even a ≈ 300 μJy source can mean the difference between a positive detection and a null result.

FLUXFITTER was then run using the X-ray data to provide a model of the S-Z decrement, using all the baselines. Again, the initial guess was defined by the 1.5 kA-only fitting. The source was found to be at RA 8\(^{h}\) 0\(^{m}\) 57′ 1 ±0.9 Dec. +36° 3′ 35′ ±8 with a flux of 275 ± 85 μJy, which is in good agreement with the long-baseline only values. Figure 3.13 shows a CLEANed map of baselines shorter than 1 kA after this source has been subtracted. The decrement (as measured from the map) is –610 ±100 μJy at RA 8\(^{h}\) 0\(^{m}\) 57′ 4 Dec. +36° 3′ 8′. The extension to the south is not significant.

3.6.1 \(H_0\) determination

Using the X-ray parameters to generate a model of the cluster, model visibilities can be calculated, as described in chapter 2. I assumed the same gas temperature as White (2000), 7.9±0.96 × 10^7 K. The model visibilities were then directly compared with the measured visibilities. Figure 3.14 shows the likelihoods calculated for different \(H_0\) values using equation 2.3. The best-fit \(H_0\) for A611 is then

---

Table 3.9: The radio sources in the FIRST catalogue within 400′ of the Ryle Telescope pointing centre for A611.

<table>
<thead>
<tr>
<th>Position</th>
<th>Flux /mJy</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>Dec</td>
</tr>
<tr>
<td>8 1 20.248</td>
<td>36 5 9.3</td>
</tr>
<tr>
<td>8 0 54.948</td>
<td>36 9 6.1</td>
</tr>
</tbody>
</table>

Table 3.8: Values from the literature for X-ray analysis of the shape of 0016+16.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>(\beta)</th>
<th>(r_e)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSPC</td>
<td>0.4±0.04</td>
<td>50.5±4.5</td>
<td>Neumann &amp; Bohringer (1997)</td>
</tr>
<tr>
<td>HRI</td>
<td>0.6±0.01</td>
<td>38.1±4.2</td>
<td>Neumann &amp; Bohringer (1997)</td>
</tr>
<tr>
<td>PSPC</td>
<td>0.728±0.025</td>
<td>40.7±2.7</td>
<td>Hughes &amp; Birkinshaw (1998)</td>
</tr>
<tr>
<td>joint PSPC &amp; HRI</td>
<td>0.749±0.024</td>
<td>42.9±2.4</td>
<td>Rene et al. (2000)</td>
</tr>
</tbody>
</table>

Figure 3.10: ROSAT HRI image of A611. The exposure time is 17ks. The greyscale range is 0 to 32 (total) counts.

Figure 3.11: ROSAT HRI image of A909. The exposure time is 67ks. The greyscale range is 0 to 10 (total) counts.

Figure 3.12: ROSAT images for the clusters discussed in this thesis.
CHAPTER 3. NEW MEASUREMENTS OF $H_0$

3.6. ABEll 611

The gray-scale range for both images is $-620$ (dark) to $620$ (dark) $\mu$Jy beam$^{-1}$.

Figure 3.11: Dirty short baseline map of A611 with no source subtraction.

Figure 3.12: Dirty short baseline map of A611 with source subtracted. The S-Z effect in A611. The contour levels are $-520$, $-300$, $-260$ and $-130$ (solid), $130$ and $260$ $\mu$Jy beam$^{-1}$ (dashed). The map has been CLEANed, the restoring beam, which is $201''$ by $97''$ FWHM at a position angle $4.8^\circ$, is shown in the bottom left.

Figure 3.13: The S-Z effect in A611.

Figure 3.14: Likelihood plot for different $H_0$ values from fitting to the source-subtracted S-Z data from A611.

$33.3^{+7.1}_{-5.0}$ km s$^{-1}$ Mpc$^{-1}$. The error quoted is due to the noise in the S-Z measurement, and does not include any of the other sources of error in the determination.
3.7 0016+16

0016+16 is at a low declination for the Ryle Telescope, and so suffers a good deal of shadowing. This obviously reduces the amount of useful data, and so this cluster has been observed for more days than either A990 or A611. The increased integration time is also due to the fact that 0016+16 is a high-redshift cluster and so deemed interesting. 0016+16 was observed for 13 sets of 10 hours in July and August 1993 in configuration Cb, for 32 sets of 12 hours in December 1993 and January 1994 in configuration Cd and 31 sets of 10 hours between July 1994 and January 1995 in configuration Cb. I have considered the two different data sets from the Cb configuration and the Cd configuration data separately because of source variability.

Two sources were found in long-baseline maps, both of which were variable. The fluxes and positions as reported by FluxFitter are shown in table 3.10. Only baselines longer than 2 kA were used in these fits, as the X-ray map looked relatively compact and so the S-Z flux as observed by the Ryle Telescope might be detected on relatively longer baselines. Figures 3.15 and 3.16 show the flux measured for each source, averaged over sets of 4 days. Note that sampling is not uniform. The variability is clearly real, and must be dealt with.

There are two regimes of variability to worry about:

![Figure 3.15: The flux of the southern source in the 0016+16 field as a function of time.](image1)

![Figure 3.16: The flux of the second source in the 0016+16 field as a function of time.](image2)

For each plot, the data is binned into sets of 4 days of observation to reduce error bars.

<table>
<thead>
<tr>
<th>Position</th>
<th>Flux / µJy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 18 31.41 +16 20 45</td>
<td>308 ± 27</td>
</tr>
<tr>
<td>0 18 24.59 +16 26 55</td>
<td>0 ± 72</td>
</tr>
</tbody>
</table>

Table 3.10: Fluxes for the sources found in the 0016+16 field in the different telescope configurations

Short-timescale variability

Consider a source varying during a 12-hour observation. A subtraction of the average flux over the 12-hour period will leave residuals that depend on time.

However, if many 12-hour observations are performed, and the variability is random, then it is equivalent to having the uv data points for a uniform source but with higher variance. Thus a simple point-source subtraction will be sufficient. Non-random “source variability” could be due to, for example, the pointing model for the telescope not being correct and the source moving up and down the beam in a consistent way for each observation.

Long-timescale variability

Consider a source that varies slowly over the course of a year. Each 12-hour observation will see an effectively constant source, and so a straightforward point source subtraction is again sufficient, again with a slightly higher noise.

But consider the case where the uv coverage of the telescope is changed during the observation period, say from Cb to Cc, and the source has been slowly decreasing in luminosity. The average flux from the Cb data will be higher than for the Cc data. Combining the two different data sets would then cause the flux to vary across the uv plane in a complex way. A simple source subtraction would oversubtract from some uv points and undersubtract from others, not necessarily averaging out. Thus data from different telescope configurations must be analysed separately.

The NVSS source catalogue was examined. Three sources within 400" of the pointing centre were found; they are listed in table 3.11. This area is not currently covered by the FIRST survey. The first bright source listed is also seen at 15 GHz. The beam amplitude of the Ryle Telescope is 75% of its peak value for this source, which means that the source could be as much as 14 mJy beam⁻¹ at 15 GHz, giving a spectral index of \( \alpha = -0.8 \). Both the spectral index and flux have large errors due to the variability of the source, and the possible pointing error of the Ryle Telescope. If each dish pointing is not accurate, then the measured value will vary. This is important for bright sources near the null as the percentage change in flux caused by a small pointing offset is large. If the pointing model were slightly wrong then the flux measurement could vary in a consistent way for each 12-hour observation period. There is no evidence that this is the case.
68

CHAPTER 3. NEW MEASUREMENTS OF \( H_0 \)

Position | Flux / mJy
---|---
Integrated | Peak
0 18 31.3 | 16 20 42 | 289.3 \( \pm \) 8 | 269.3 \( \pm \) 8
0 18 56.3 | 16 28 52 | 3.7 \( \pm \) 0.5 | 3.7 \( \pm \) 0.5
0 18 58.9 | 16 27 28 | 2.9 \( \pm \) 0.5 | 2.9 \( \pm \) 0.5

Table 3.11: NVSS sources within 400\( ^\circ \) of the pointing centre of the 0016+16 S-Z measurement.

| Telescope configuration | Source position | Source flux / \( \mu \)Jy | S-Z
---|---|---|---
Cl | 0 18 31.4 \( \pm \) 0.1 | 16 20 57.5 \( \pm \) 0.02 | 195 \( \pm \) 25 | -904 \( \pm \) 200
 | 0 18 24.6 \( \pm \) 0.1 | 16 25 55.1 \( \pm \) 0.1 | 140 \( \pm \) 26
CBlady | 0 18 31.6 \( \pm \) 0.2 | 16 20 29.4 \( \pm \) 0.15 | 280 \( \pm \) 50 | -530 \( \pm \) 140
CBrady | 0 18 31.5 \( \pm \) 0.1 | 16 20 41 \( \pm \) 0.5 | 426 \( \pm \) 33 | -503 \( \pm \) 91
 | 0 18 24.5 \( \pm \) 0.2 | 16 26 59 \( \pm \) 0.8 | 248 \( \pm \) 33

Table 3.12: Source positions and fluxes found in the 0016+16 field from a complete analysis including the X-ray data with FLUXFITTER

Once the datasets had the point sources subtracted, they were concatenated to give a new dataset that has only the S-Z signal present. As the configuration changes, the number of samples averaged together also changes. In the concatenation, the different datasets were weighted by their variance, which is directly proportional to the number of samples that are averaged together. The hole from the CLEANed map is -730 \( \pm \) 90 \( \mu \)Jy at a position of RA 0\( ^\circ \) 18\( ^m \) 32\( ^s \) 41 Dec. +16\( ^\circ \) 26\( ^m \) 12\( ^s \) 24.

As the X-ray parameters have been determined, it is also possible to run FLUXFITTER using all the baselines. As before, the different configurations were considered separately, and the sources listed in table 3.10 were used as the initial guesses. The source positions and fluxes found are shown in table 3.12 and agree well with the values found with baselines longer than 1.5 k\( \lambda \). Figure 3.17 shows a CLEANed map of the S-Z hole made using all the data (weighted as before) and using all baselines up to 1.0 k\( \lambda \). The decrement measured from the map is -593 \( \pm \) 47 \( \mu \)Jy at RA 0\( ^\circ \) 18\( ^m \) 33\( ^s \) 3 Dec. +16\( ^\circ \) 27\( ^m \) 14\( ^s \), which is 17\( ^\circ \) in RA and 76\( ^\circ \) in Dec from the X-ray centroid. The beam is 90\( ^\prime \) (in RA) by 310\( ^\prime \) (in Dec) so this is in good agreement. The lowest two contours show a hint of extension both to the south and west.

3.7.1 \( H_0 \) determination

In order to determine \( H_0 \) from this cluster, a measurement of the gas temperature is required. There are four determinations of the gas temperature in 0016+16 in the literature.

- Neumann & Bohringer (1997) attempt to use PSPC and HRI data from ROSAT. They do not constrain the hydrogen column density well and so quote values of 8.6\( ^\pm \)0.9 and 5.5\( ^\pm \)0.5 keV (1-\( \sigma \) errors) within 100\( ^\circ \), for column densities of 5 and 6 \( \times \) 10\(^{20} \) cm\(^{-2} \).
- Furuzawa et al. (1998). They use ASCA observations, and find a value of 8.0\( ^\pm \)1.0 (95% confidence) using data within 6\( ^\circ \) of the cluster position. The average temperature drops as the radius of data included increases.
- Hughes & Birkinshaw (1998) use the same ASCA observations of Furuzawa and determine a column density from PSPC observations. They also note that there is an AGN 196\( ^\circ \) north of the cluster centre and perform a simultaneous fit to the cluster temperature and the AGN emission. They find a temperature of 7.55\( ^\pm \)0.72 keV (1-\( \sigma \) errors) with a column density of 5.59\( ^{0.31}_{\pm 0.36} \) \( \times \) 10\(^{20} \) cm\(^{-2} \).
- White (2000) also uses the ASCA observations and finds an average temperature of 8.03 \( \pm \) 1.03 keV (1-\( \sigma \) errors). White does not find any temperature variation with radius.

I take the Hughes & Birkinshaw value as it has the smallest errors and they take account of the AGN emission. Using this temperature, and following the same procedure as before, I find \( H_0 = 78.1_{\pm 1} \) km s\(^{-1} \) Mpc\(^{-1} \) from 0016+16. Figure 3.18 shows the likelihoods. Again, the error is due only to the noise on the S-Z decrement.

3.7.2 Other \( H_0 \) determinations

Both Hughes & Birkinshaw (1998) and Reese et al. (2000) have determined \( H_0 \) via the S-Z & X-ray route using 0016+16. Reese et al. use OVRO and BIMA at 28.5 GHz. The only source they find is the bright southern source; the positional agreement is good and the flux is 9 \( \pm \) 2 mJy at 28.5
GHZ. They quote a distance of 2041.4±64 Mpc to 0016+16 for an Einstein-de-Sitter universe, which corresponds to an $H_0$ of $60.5^{+5}_{-4}$ statistic uncertainties only. This is slightly lower than my value, which could be explained by their incomplete source subtraction. Indeed, mapping my un-source-subtracted data shows that the flux of the S-Z hole is 7% larger than in the source-subtracted data. Thus as Reese et al. do not subtract both sources, their S-Z flux will be larger, and so the $H_0$ value lower.

Hughes & Birkinshaw (1999) use the OVRO telescope, operating at 20.3 GHz using beam- and position-switching techniques. They do not produce a map of the S-Z effect, but instead produce a scan along a north-south line through the cluster. Also, they do not detail the corrections they make for radio sources, but it is reasonable to assume that they use the data collected by Moffet & Birkinshaw (1989) and attempted a source subtraction from that. Within the Ryle Telescope field-of-view, Moffet & Birkinshaw find two sources which have good positional agreement with the two sources that I find. However, the observations were not simultaneous, making the subtraction of the variable source effectively impossible. The final value they quote before considering the effect of sphericity is $H_0 = 47^{+2.4}_{-2.1} \pm 2.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ where the second error is due to an uncertainty in the zero level due to possible ground-spill. Again, as the X-ray fitting has given substantially similar results, the error is almost certainly in the S-Z value, and probably in the source subtraction.

![Figure 3.18: Likelihood plot from fitting the source-subtracted S-Z data from 0016+16.](image)

### 3.8 Abell 990

Abell 990 was observed for 25 days between March 1995 and October 1995, again with the Ryle Telescope in configuration B. Seven days were rejected due to high noise on the 1-day maps. One source of $\approx 600 \mu$Jy was clear on long-baseline maps and was subtracted. A number of fainter sources were then visible. The positions and fluxes of all of these sources as measured with MAXFIT were used as an initial model for FLUXFITTER, using all baselines longer than 1.5 kλ in the unsubtracted data set. The sources found are shown in table 3.13.

After subtraction with UVSPUR, the long-baseline map is consistent with noise, and the map of baselines less than 1 kλ shows a decrement of $-395 \pm 120 \mu$Jy at a position of RA $10^h 23^m 45^s 3$ Dec. $+49^\circ 9^\prime 6^\prime\prime$. This is $36^\circ$ in RA and $48^\circ$ in Dec away from the X-ray position. The synthesized beam is 116$^\prime$ by 162$^\prime$ so this offset is, in percentage terms, the worst agreement of the clusters considered here, but just consistent.

Using the X-ray data, FLUXFITTER was run using all the baselines. When run using the X-ray position for the cluster, the agreement between the previous values (for the long baselines only) and the combined fit is not good. It is also poor when the cluster position is set to the S-Z position reported above. In both cases, the long-baseline map does not show any significant sources after the subtraction. There does remain the possibility of a weak extended source, slightly offset from the cluster centre. This would be resolved out of the long-baseline data, but would still contaminate the short baseline S-Z map, altering the flux and position of the observed-S-Z decrement.

<table>
<thead>
<tr>
<th>Flux</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>155 ± 6</td>
<td>10 23 47.8 ± 0.4</td>
</tr>
<tr>
<td>181 ± 6</td>
<td>10 23 48.9 ± 0.4</td>
</tr>
<tr>
<td>146 ± 6</td>
<td>10 23 50.1 ± 0.4</td>
</tr>
<tr>
<td>491 ± 6</td>
<td>10 24 2.4 ± 0.1</td>
</tr>
</tbody>
</table>

Table 3.13: Fluxes and positions found by FLUXFITTER for A990 using baselines longer than 1.5 kλ

<table>
<thead>
<tr>
<th>Position</th>
<th>FIRST Flux</th>
<th>NVSS Flux</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>Integrated</td>
<td>Integrated</td>
</tr>
<tr>
<td></td>
<td>±0.13 / mJy beam$^{-1}$</td>
<td>/ mJy</td>
</tr>
<tr>
<td>10 23 29.7</td>
<td>49 2 31.2</td>
<td>1.7</td>
</tr>
<tr>
<td>10 23 37.1</td>
<td>49 6 40.0</td>
<td>-</td>
</tr>
<tr>
<td>10 23 47.8</td>
<td>49 11 28.5</td>
<td>1</td>
</tr>
<tr>
<td>10 23 54.2</td>
<td>49 2 34.5</td>
<td>1.6</td>
</tr>
<tr>
<td>10 23 58.6</td>
<td>49 13 47.4</td>
<td>2.1</td>
</tr>
<tr>
<td>10 24 2.3</td>
<td>49 6 51.6</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3.14: NVSS and FIRST sources found in the A990 field at 1.4 GHz. Dashes indicate no source in the catalogue.
The FIRST and NVSS source catalogues were then investigated. Searches were performed in each catalogue within a radius of 400" from the Ryle Telescope pointing centre. The sources found are in table 3.14. As the resolution of the Ryle Telescope is similar to that in the NVSS survey, the sixth source — which is extended at FIRST resolution but pointlike in NVSS — is not a problem. Indeed, a comparison with table 3.13 shows that it is present at 15 GHz, at 491 ± 60 μJy. Taking into account the primary beam attenuation (which is a multiplicative factor of 0.26 at this point), the spectral index, α, of this object is α_{15GHz} = -1.26 ± 0.3. The FIRST image of the source hints at a head-tail morphology. However, the second object in the list is more troublesome. The catalogue limit for the FIRST survey is 1 mJy. This implies that the object is extended at 1.4 GHz and is being resolved out by the FIRST survey. There is a danger that it will be resolved out from the long baseline maps made with the Ryle Telescope at 15 GHz, and so not identified as a source and not subtracted. This depends on its spectral index; in order to attempt to determine the spectral index, WSRT observations of A990 were obtained.

3.8.1 WSRT

The Westerbork Synthesis Radio Telescope (WSRT) is a 14-element East-West interferometer situated near Dwingeloo, The Netherlands. Each antenna is 25 m in diameter. Ten elements are fixed with a separation of 144 m; the further four elements are mounted on a rail track. WSRT has good low frequency sensitivity and the range of baselines means that resolution is well matched to the expected angular sizes of extended sources in clusters.

On 26 Oct 1997 and 5 December 1997, WSRT observed 8 clusters for 19 hours in the UHFlow and UHFhigh bands; these correspond to 321–379 MHz split into 9 channels and 800–880 MHz split into 8 channels. Eight clusters were observed; A697, A611, A665, A773, A990, A1413, A1914 and A2034. Overall flux and phase calibration was achieved by observing 3C48 and 3C147 both at the beginning and end of the 19-hour observing run. In order to obtain useful observations of all of the 8 clusters in as short a time as possible, long snap-shots were used. That is, 20 minutes were spent on A611, then another 20 on A665, etc. After all 8 clusters had been observed, the sequence was repeated. This then allowed all the observations to be completed in two 19-hour runs rather than 16 12-hour runs. The snap-shotting was not done completely evenly; figure 3.19 shows the uc-coverage for A990.

Unfortunately, some of the visibility points were swamped by interference. Figure 3.20 shows an amplitude-baseline plot for A990. It is clear that there is a good deal of interference on the shorter baselines. In order to eliminate it, all visibility points with a flux higher than 10 Jy were removed. The clipped data are shown in figure 3.21.

After clipping, total intensity maps of A990 were made. A cellsize of 20"×20" with a 1024×1024 grid were used for the UHFlow data, covering 5.7" × 5.7". As FFTs are being performed, the map size in pixels must be a power of 2. This provides a good compromise between ensuring that the sky is Nyquist sampled, mapping the entire field of view, and computation time. The longest available baseline is 2.7 km for WSRT, which corresponds to 3 kλ at UHFlow and spatial sizes of 0.8". Thus 20"
is sampling comfortably faster than Nyquist. The antenna diameter of 25 m, or 27.8 Å at UHFlow, corresponds to a field-of-view of 2.1°. As the signal-to-noise was expected to be good, the field of view was large and many sources were expected, uniform weighting was used. This reduces the sidelobes of the synthesised beam (in comparison to natural weighting) making CLEAN more reliable.

I used a cellsize of 8° with the UHFlow data and kept the grid size as 1624 x 1024. As the frequency has risen by a factor of $\approx 2.4$, dropping the cellsize by the same factor keeps all the criteria satisfied.

The mapping and CLEANing were performed by the task IMAGR. The CLEAN is visibility based for the subtraction stages; the peak identification still occurs in the map plane. The CLEAN box used covered the entire field of view and the CLEAN was stopped when the residual image no longer showed structure.

Figures 3.22 and 3.23 show the short and long baseline UHFlow maps. The cutoff between long and short was 1.5 kÅ, which corresponds to 0.45 h$^{-1}$ Mpc at the redshift of 0.9. There is some extended emission at the centre of figure 3.22, labelled “B”. There is still some flux in the long-baseline map which indicates that the source has structure on scales smaller than 0.45 h$^{-1}$ Mpc.

Table 3.15 shows the fluxes measured for the UHFlow, UHFhigh and NVSS maps. Note that the $uv$ ranges quoted are, in all cases bar the NVSS value, a subset of the data taken. If all the data were used, then the UHFhigh data would include baselines up to 7 kÅ and so would be sensitive to structure on shorter spatial scales. By keeping the $uv$ coverages the same, the flux is measured on the same spatial scale at different frequencies, and so the same spatial region. This is clear for the two measurements at 840 MHz; there is a 20% drop in the flux as $uv$ coverage does not include the very short baselines.

The 6C image frame that includes this field (figure 3.24) shows extended structure. However, the source to the east (labelled “A” in figures 3.22 and 3.23) if sufficiently close that it will confuse any flux measurement made from the 6C image.

These low frequency observations imply that there is an extended source in the field. It adds substantial weight to the notion that, at 15 GHz, the lack of agreement between FLUXFITTER source fluxes for long-baselines and when using the X-ray derived model for the cluster implies there is an extended source present. In order to further test this hypothesis, the X-ray model parameters were used to produce a model of the S-Z decrement if $H_0=50$ km s$^{-1}$ Mpc$^{-1}$. The visibilities were calculated and subtracted from the observed, point-source-subtracted, visibilities. A map made with these calculated visibilities is shown in figure 3.25. This is clearly a low signal-to-noise map (the noise is 120 mJy beam$^{-1}$ and the brightest points are 350 mJy beam$^{-1}$), and it has not been CLEANed. The two brightest points of the map are in coincident positions with the sources labelled “A” and “B” in figure 3.22. Figure 3.26 includes this data point along with the lower frequency observations.

Although the flux shown in figure 3.26 is not a rigorous value and certainly seems to be an over estimate, the radio source environment is clearly complex for A990 and so a reliable $H_0$ estimate would be impossible with current observations.
CHAPTER 3. NEW MEASUREMENTS OF $H_0$

3.8. ABELL 990

Figure 3.24: Image from the 6C survey at 151 MHz. The dashed box in the centre of the image indicates the region shown in figures 3.22 and 3.23.

Figure 3.25: Raw map of the Ryle Telescope data with a cluster subtracted from the X-ray position, assuming $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$. The range of greyscale is from $-580$ (light) to $580$ µJy beam$^{-1}$. The two peaks are coincident with points labelled “A” and “B” in figures 3.22 and 3.23.

Figure 3.26: Fluxes measured in AIPS for the extended object. Note that the 15-GHz flux has not been measured directly; see text for details.
3.9 Error budget

The errors on $H_0$ quoted for A611 and 0016+16 are the errors purely due to noise on the S-Z measurement. There are additional sources of error.

3.9.1 Primary flux calibration

The Ryle Telescope is flux-calibrated daily. As discussed by Grainge (1995), measurements with the VLA show that the flux of 3C 48 and 3C 286 show slight variations with time. As such, it is reasonable to estimate the overall flux calibration level of the Ryle as 5%. This leads to an overall uncertainty in $H_0$ of 10%.

3.9.2 X-ray temperature

The gas temperature is put into the $H_0$ determination as $T^2$, and so any errors will double for $H_0$. The errors on the temperature measurement are $\pm 7.7\%$ for 0016+16 and $\pm 6.6\%$ for A611.

3.9.3 Fitting degeneracy

The fitting degeneracy is fully discussed in section 2.3. I estimate that the variation in the S-Z flux measured perpendicular to the degeneracy direction is similar to that in A773, i.e. $15\%$. Thus the error in $H_0$ is $7\%$ for 0016+16. However, the X-ray observation of A611 is of low signal-to-noise, and strongly dependent on the inclusion or otherwise of the bright pixels. Thus I estimate that the X-ray fitting adds a larger error for A611 than for 0016+16.

3.9.4 Source subtraction residuals

The residual effects of unsubtracted sources, which are distributed randomly in the map plane, is to increase the rms noise. Since this is measured directly from the visibilities, the quoted S-Z error includes the contribution from unsubtracted sources.

3.9.5 Kinetic S-Z effect

As well as the second-order thermal S-Z effect, the first-order kinetic S-Z effect exists. The ratio of thermal S-Z to kinetic S-Z is given by

$$\frac{\Delta T_{\text{thermal}}}{\Delta T_{\text{KE}}} = \frac{\frac{2h_0}{m_e c^2} \int n_e \, dv}{\frac{T}{2} \int n_e \, dv} \sim 1.17 \frac{T}{\text{keV}} \left( \frac{v_p}{1000 \text{ km s}^{-1}} \right)^{-1}$$

(3.3)

where $v_p$ is the radial peculiar velocity of the cluster. It is due to the bulk motion of the cluster gas and the consequent Doppler shift. It is an order of magnitude smaller than the thermal S-Z effect even for a large peculiar velocity of 1000 km s$^{-1}$. Measurements with SuZIE by Holzapfel et al. (1997a) of A2163 (z=0.201) and A1689 (z=0.181), give peculiar velocities less than 500 km s$^{-1}$ for both these clusters, giving deviations from the Hubble flow of $< 1\%$. Assuming peculiar velocities of 500 km s$^{-1}$ for A611 and 0016+16 puts the ratio of thermal to kinetic S-Z to be 15.9 and 15.7 respectively, i.e. the kinetic S-Z is up to 6% of the thermal S-Z. The peculiar velocities can reasonably be assumed to be orientated randomly, and so this does not introduce a bias into the $H_0$ value. I assume that the error it introduces adds in quadrature with the noise in the S-Z measurement.

3.9.6 Rees-Sciama effect.

An additional distortion to the CMB exists; the Rees-Sciama effect, which is first described in Rees & Sciama (1968). As a cluster collapses, the CMB photon must climb out of a larger gravitational well than it fell into, causing a net shift. Obviously this will be important for clusters that are just collapsing rather than virialised systems, e.g. Dabrowski et al. (1999). As all the clusters in the Ryle Telescope sample are of relatively low redshift, then it is reasonable to assume that this effect is unimportant for $H_0$ determination.

3.10 Combining $H_0$ values

The overall “error budget” is shown in table 3.16. In order to determine a final $H_0$ value, the values from each cluster need to be combined as a weighted geometric mean. The details of this are shown in appendix D. This will reduce the line-of-sight degeneracy error. The $H_0$ values that have been determined by the Cavendish group are listed in table 3.17, and plotted in figure 3.27.

The weighted geometric mean of these values is $52.4^{+2.7}_{-2.4}$ km s$^{-1}$ Mpc$^{-1}$ if the cosmology is Einstein-de-Sitter. (If $\Omega_M = 0.3$ and $\Omega_{\Lambda} = 0.7$, then $H_0 = 54.5^{+2.2}_{-2.1}$ km s$^{-1}$ Mpc$^{-1}$.) The $\chi^2$ value for this distribution is 10.3, with 3 degrees of freedom. These values do not include the error from line-of-sight uncertainty which, for 5 clusters, adds an error of 12%. Thus the “final answer” for 5 clusters is $H_0 = 48.7^{+3.2}_{-3.1} \pm 6$ km s$^{-1}$ Mpc$^{-1}$ (Einstein-de-Sitter universe).
CHAPTER 3. NEW MEASUREMENTS OF $H_0$

### Table 3.16: The error budget table for 0016+16 and A611. The errors are all shown as percentages on the quantities, not how they impact $H_0$.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0016+16</td>
<td>74.814$^{+0.17}_{-0.25}$</td>
<td>This work</td>
</tr>
<tr>
<td>A611</td>
<td>48.14$^{+0.03}_{-0.03}$</td>
<td>This work</td>
</tr>
<tr>
<td>A1413</td>
<td>57.23$^{+0.03}_{-0.03}$</td>
<td>Grainge et al. (2001)</td>
</tr>
<tr>
<td>A773</td>
<td>55.71$^{+0.03}_{-0.03}$</td>
<td>Jones et al. (2001)</td>
</tr>
<tr>
<td>A2218</td>
<td>33.41$^{+0.03}_{-0.03}$</td>
<td>Grainge (1995)</td>
</tr>
</tbody>
</table>

### Table 3.17: The published $H_0$ values from the Cavendish group.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$H_0$ [km s$^{-1}$ Mpc$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0016+16</td>
<td>78.3$^{+1.0}_{-0.8}$</td>
</tr>
<tr>
<td>A611</td>
<td>78.3$^{+0.9}_{-0.9}$</td>
</tr>
<tr>
<td>A990</td>
<td>78.3$^{+1.0}_{-1.0}$</td>
</tr>
</tbody>
</table>

### Figure 3.27: $H_0$ values determined in Cambridge.

#### 3.11. CONCLUSIONS

The problem of source subtraction and methods to counter it have been investigated using a simulated S-Z observation. This has demonstrated the following.

1. The **CLEAN** method can work well in simple cases, but does not use all the available information and can introduce bias.
2. The matrix method fails in typical situations such as the one simulated here. The failure is mainly due to the high sidelobes from the Ryle Telescope; they make it difficult to determine accurate positions, and so fluxes, for sources close to each other on a map.
3. **FLUXFITTER**, first used by Riju Das and extended and improved here, works well, uses all the available information and produces the simplest model. A key advantage of it is that it works with the visibilities, where noise is known to be Gaussian, rather than in the map plane where the noise is correlated. Another very important advantage is that if there is S-Z flux present on the longer baselines, the methods gets the right result.
4. All three techniques suffer from the problems of source identification, which is currently performed in the image plane where the noise characteristics are complex.

I have observed three clusters with the Ryle Telescope in order to estimate $H_0$.

5. Two clusters, 0016+16 and A611, have good S-Z detections. A611 gives $H_0 = 32_{-3}^{+11}$ km s$^{-1}$ Mpc$^{-1}$. 0016+16 gives $H_0 = 78_{-10}^{+17}$ km s$^{-1}$ Mpc$^{-1}$. These errors include the errors on the S-Z measurement, X-ray temperature determination, X-ray shape fitting degeneracy, and a number of smaller effects.
6. Combining these values with previously published $H_0$ estimates from the Cavendish group, I find $48.7_{-4.1}^{+4.2}$ km s$^{-1}$ Mpc$^{-1}$ assuming an Einstein-de-Sitter universe. The spread of $H_0$ values found is consistent with noise, indicating that all sources of error have been correctly estimated.
7. No S-Z effect was found in observations of A990. However, lower frequency observations of the radio source environment all support the hypothesis that there is some extended emission which is contaminating the 15-GHz measurement.
Chapter 4

Searching for distant clusters: radio selection and S-Z follow-up

4.1 Introduction

Here I describe a further technique for finding high-redshift clusters, based on radio halos. There are many different radio surveys, and almost without exception, the most-distant objects that are detected are of higher redshift than those detected in optical surveys. Thus, by using a suitable combination of low and high frequency surveys, steep spectrally-aged radio halos can be found. The emphasis is on finding high redshift clusters, i.e. those not detectable from, for example, POSS plates.

4.2 Cluster radio halos

It has been known for some time that clusters of galaxies contain a significant fraction of the radio source population at low frequencies (see e.g. Willson (1970) and references therein). Some clusters have diffuse emission that is spread over a substantial fraction of the cluster but is not associated with any particular galaxy as well as emission associated with jet-producing central engines; e.g. Coma (Cordey, 1985) and A2256 (Rottgering et al., 1994); for a review see e.g. Hanisch (1982). The diffuse synchrotron emission is usually substantially spectrally aged.
4.2.1 Aging

There are two main energy loss mechanisms for high energy electrons: synchrotron and inverse-Compton (see, for example, Longair, 1994). Both have similar energy loss rates:

\[
\frac{dE}{dt} = -\frac{4}{3}\gamma^2 e^2 U, \tag{4.1}
\]

where \(\gamma\) is the electron Lorentz factor and \(U\) is the energy density of the magnetic field (for synchrotron) or the energy density of the radiation field (for inverse-Compton). Electrons with an energy \(E\) (\(\propto \gamma\)) radiate synchrotron energy at frequency \(\nu\), where \(\nu\) is given by

\[
\nu \approx \left(\frac{E}{m_e c^2}\right)^2. \tag{4.2}
\]

Given a power-law distribution of electron energies \(N(E)dE \propto E^{-\alpha}dE\), the flux emitted is then given by

\[
S \propto \nu^{-(\alpha-1)/2}. \tag{4.3}
\]

A spectrum which initially has constant spectral index will then "age". As the energy loss rate is proportional to frequency, the flux from higher frequencies will drop, and the age is related to a characteristic "break" frequency in the spectrum.

In practice, the precise form of the resultant spectrum is calculated numerically. Figure 4.1 shows the spectrum of synchrotron emission with an injection spectrum of 0.5, i.e. \(n = 2\), that has been aged in a magnetic field of 0.27 mT for 10\(^6\) years and so has a break frequency \(\sim 1\) GHz, as calculated by ANMAP, a piece of software written by Paul Alexander.

One issue is to explain the existence of relativistic electrons that are distributed throughout a substantial fraction of the cluster, rather than being associated with an AGN. Two models have been advanced to explain the existence of these high energy electrons:

- Harris & Miley (1978) suggest that the halo is caused by a relic radio galaxy;
- an alternative is the cluster merger hypothesis (e.g. Harris et al., 1980).

In the dead AGN hypothesis, the AGN’s jet is turned off. The hot spot, not being powered anymore, fades. The lobe material, which was originally back-flow from the hot-spot, continues to expand adiabatically. The lobe continues to emit synchrotron radiation and ages.

In the second hypothesis, two sub-clusters are merging. The relative in-fall velocity of the sub-clusters is sufficiently high to cause a shock front in the gas. When a shock front passes over particles, they can gain energy (Landau & Lifshitz, 1987). A Fermi acceleration process then allows very high energies to be obtained. As the shock moves through the cluster, this boosting of energy can happen over a large volume. These particles then emit synchrotron radiation, and age once the shock as passed by and they are no longer being accelerated. As discussed in e.g. Longair (1994), Fermi acceleration processes naturally produce a power-law electron energy distribution with \(n = 2\).

Rottgering et al. (1994) shows that A2256 contains both diffuse emission and emission from jet-producing central engines. Saunders (1982) shows that a double radio source takes \(\geq 10^8\) years to fade from adiabatic losses. However, the diffuse cluster emission does not look remotely like the structure of double radio sources, or what might be expected if the jets had been turned off. In addition, the majority of double radio sources are not found at the centre of clusters, and if the energy supply were suddenly removed, the lobe would float away from the cluster by buoyancy. But diffuse cluster emission is seen in the centre of clusters, so the cluster merger hypothesis must be at least important, if not dominant. However, I stress that steep-spectrum emission can indicate clusters whether the emission is from cluster mergers or central engines producing, for example, twin-jets. Of course such emission can also be produced by high luminosity (hence high synchrotron-ageing) and/or high redshift (hence high inverse-Compton-ageing) double radio sources.

4.3 Obtaining candidate clusters

I used the 326-MHz WENSS and 1.4-GHz NVSS surveys as the basic catalogues to find clusters: they have the frequencies and sensitivities to pick up aged halos, have similar beam sizes (which is necessary to give meaningful spectral indices) which are comparable to the angular sizes of distant clusters and have wide, useful sky coverage.
4.3. OBTAINING CANDIDATE CLUSTERS

The Westerbork Northern Sky Survey (WENSS, Rengelink et al., 1997) is a survey of the sky north of Dec. +30° at 326 MHz, with a resolution of 54" × 54" cosec(δ). It was made with the WSRT and the catalogue produced from the maps has a flux limit of 18 mJy (5σ).

Other relevant radio surveys exist, e.g. Green Bank (Condon et al., 1989), and the Cambridge surveys (i.e. 6C and 7C), but WENSS and NVSS have the relevant resolutions and by far the best sensitivity at these resolutions. WENSS is the best for my purpose here as it is of relatively good resolution at a relatively low frequency. The factor of four in frequency between WENSS and NVSS, in combination with the low noise in each survey, means that spectral indices can be well determined.

These three surveys are detailed in table 4.1. The different sky coverages are shown in figure 4.2. Only the northern part of the FIRST survey is shown, the southern region is around RA 0 and Dec. 0. (Note that this work was done before the FIRST survey extended their sky coverage up to Dec. +58°.) Combining the NVSS and WENSS surveys allows steep-spectra objects to be found, and then correlating NVSS with FIRST shows which are extended.

The overall positional accuracy of the WENSS and NVSS surveys is claimed to be around 1" for "strong sources". This uncertainty varies as the inverse of the signal-to-noise ratio (cf. section 3.3.4).

Initially, the positions of sources in the WENSS and NVSS catalogues were compared. Sources were labelled as coincident between the surveys if their separation was less than 40", and their "spectral indices" calculated. Figure 4.3 shows the distribution found. Only "objects" with "spectral indices" > 1.5 and WENSS fluxes greater than 65 mJy are plotted. The catalogue limit for WENSS is 5 mJy. If an 15 mJy source in WENSS has a spectral index of 1.2, then its flux at 1.4 GHz is 35 mJy; the NVSS completeness limit. Thus an object at the catalogue limit in WENSS with a spectral index > 1.2 will not be detected in the NVSS survey. It is important to detect objects in FIRST and NVSS so that it can be determined if they are extended or not, so the WENSS flux limit was set to 65 mJy, allowing objects with a spectral index of 2 to be present in the NVSS survey.

There are two components to the distribution. The first, which dominates at large separations, is "objects" which are not actually the same object in the two surveys. The second dominates at low separations and contains the candidates sought. I took 16" as the divide between the two components. With this limit, there are 1079 objects in the low separation component in the sky area covered by both NVSS and WENSS.

---

Table 4.1: Parameters of the surveys used.

<table>
<thead>
<tr>
<th></th>
<th>WENSS</th>
<th>NVSS</th>
<th>FIRST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (MHz)</td>
<td>326</td>
<td>1400</td>
<td>1400</td>
</tr>
<tr>
<td>Sky area (sr)</td>
<td>3.1</td>
<td>10.3</td>
<td>3.1</td>
</tr>
<tr>
<td>Limiting flux density (5σ, mJy)</td>
<td>15</td>
<td>2.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Resolution</td>
<td>54&quot; × 54&quot; cosec(δ)</td>
<td>45&quot;</td>
<td>5&quot;</td>
</tr>
</tbody>
</table>

---

Figure 4.2: The different sky coverages for the three surveys. The lower limit for the NVSS survey is shown. The light hatched area shows the WENSS survey, and the trapezoid represents the state of the FIRST survey when this work was done. Lines of δ = -10, 0 and 10 are also shown to indicate the position of the Galactic plane.

---

Figure 4.3: The distribution of separations for the combination of NVSS and WENSS catalogues over the (catalogued) entire sky, with α > 1.5, integrated flux at 326 MHz > 65 mJy.
The catalogue was then trimmed to remove objects that were outside the FIRST survey area. The extension was then considered. As the resolution of FIRST is much better than NVSS, objects in FIRST were considered to match NVSS objects if they are within $16''$. This combination generated three sets of objects.

- Objects with no FIRST counterpart: there were 79 of these.
- Objects with a single FIRST counterpart: 102 of these.
- Objects with many FIRST counterparts: 70 of these.

In the objects in the first two sets, there is an extended source which has some flux resolved out by the more extended VLA configuration. The largest scale visible in the FIRST survey is $\approx 1'$ (VLA calibration manual, 1990). Thus the amount of flux lost gives an estimate of the angular size of the object.

In the third set, there are usually two sources sufficiently close on the sky that both NVSS and WENSS confuse them and only report a single source. The FIRST catalogue was produced by fitting Gaussians to the maps and putting each Gaussian into the catalogue as a "source" and so objects with structure are, according to FIRST, many sources.

Due to noise, it can be difficult to determine purely from the NVSS and FIRST fluxes whether or not an object is extended. Indeed, it is possible for the catalogue values from FIRST to be numerically larger than the flux value in NVSS. Thus automated rejection is difficult. So the FIRST and NVSS images of the 251 objects were then examined by eye. Objects were then rejected for a number of reasons. Occasionally the NVSS region is unusually noisy and the catalogue contains noise peaks. In addition, the FIRST coverage is patchy at the edges; thus some objects are identified as not having a FIRST counterpart when the FIRST survey does not in fact cover that region.

Objects were rejected on morphology; figure 4.4 shows a typical object that was rejected for being a (slightly bent) classical double radio source. Table 4.2 details the number rejected for each morphology. Unknown includes the morphologies which are not clear, such as possible classical double sources. Additional resolution would be required to classify the objects fully. They are being selected because they have steep-spectrum, and hence aged, emission. Figure 4.5 shows a point source that was removed from the sample. In this case, the FIRST integrated flux is $10.42 \pm 0.145$ mJy and the NVSS flux is $9.8 \pm 0.6$ mJy. Figure 4.6 shows an object, CRH 0741, that was left in the sample. The arrowed object is probably a classical double radio source (a higher resolution map would be required to be definite). In the centre of the figure is a region of extended emission — a possible cluster halo.

After the culling, 26 objects remained. They are listed in table 4.3. Figure 4.7 shows the distribution of ratios of peak and integrated fluxes for both the NVSS and WENSS fluxes. These figures show the importance of combining with the FIRST survey. There are objects whose peak and integrated fluxes are equal — i.e. they are unresolved in both WENSS and NVSS — yet some of them are resolved, or are not even visible in the FIRST survey.

![Figure 4.4](image-url) A classical double as observed with FIRST from the candidate list. The grey-scale runs from $-0.6$ (white to 20 mJy beam$^{-1}$) (black). This object was rejected as it not a radio halo.

![Figure 4.5](image-url) An obvious point source from the FIRST survey. The greyscale runs from $-0.6$ (white) to 10 mJy beam$^{-1}$ (black). This object was also rejected.

![Figure 4.6](image-url) The region CRH 0741. An obvious radio galaxy (arrowed) and a region of diffuse emission at the centre of the map. The image is from the FIRST survey, and the greyscale runs from $-0.63$ (white) to 2.6 mJy beam$^{-1}$ (black).
Table 4.3: The Candidate Radio Halos (CRHs).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CRH 0716</td>
<td>17 16 17.67</td>
<td>45 14 42.37</td>
<td>2.13</td>
<td>8.40</td>
<td>105</td>
<td>105</td>
<td>6.00</td>
<td>3.20</td>
</tr>
<tr>
<td>CRH 0720</td>
<td>17 16 17.45</td>
<td>45 14 42.37</td>
<td>2.13</td>
<td>8.40</td>
<td>105</td>
<td>105</td>
<td>6.00</td>
<td>3.20</td>
</tr>
<tr>
<td>CRH 0741</td>
<td>17 16 17.45</td>
<td>45 14 42.37</td>
<td>2.13</td>
<td>8.40</td>
<td>105</td>
<td>105</td>
<td>6.00</td>
<td>3.20</td>
</tr>
<tr>
<td>CRH 1440</td>
<td>17 16 17.45</td>
<td>45 14 42.37</td>
<td>2.13</td>
<td>8.40</td>
<td>105</td>
<td>105</td>
<td>6.00</td>
<td>3.20</td>
</tr>
</tbody>
</table>

The analysis of the candidate radio halos and cluster centers, along with their radii, are given in Table 4.3. The analysis includes: RA and Dec for each halo, integrated flux in mJy, peak flux in mJy, and WENSS and NVSS frequencies. The CRHs are denoted by the corresponding numbers.

The analysis of the candidate radio halos and cluster centers, along with their radii, are given in Table 4.3. The analysis includes: RA and Dec for each halo, integrated flux in mJy, peak flux in mJy, and WENSS and NVSS frequencies. The CRHs are denoted by the corresponding numbers.

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The analysis of the candidate radio halos and cluster centers, along with their radii, are given in Table 4.3. The analysis includes: RA and Dec for each halo, integrated flux in mJy, peak flux in mJy, and WENSS and NVSS frequencies. The CRHs are denoted by the corresponding numbers.
### Table 4.4: CRHs identified with known clusters. The references for redshift are 1 — Struble & Rood (1999); 2 — Gubanov & Reshetnikov (1999).

<table>
<thead>
<tr>
<th>Name</th>
<th>Cluster</th>
<th>Separation (arcmin)</th>
<th>Redshift</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRH 0827</td>
<td>Abell 668</td>
<td>1.6</td>
<td>0.159</td>
<td>1</td>
</tr>
<tr>
<td>CRH 0854</td>
<td>Abell 713</td>
<td>2.9</td>
<td>0.109</td>
<td>2</td>
</tr>
<tr>
<td>CRH 0901</td>
<td>Abell 733</td>
<td>2.6</td>
<td>0.116</td>
<td>1</td>
</tr>
<tr>
<td>CRH 1012</td>
<td>Abell 943</td>
<td>3.3</td>
<td>0.149</td>
<td>2</td>
</tr>
<tr>
<td>CRH 1245</td>
<td>Abell 1608</td>
<td>4.7</td>
<td>0.13</td>
<td>1</td>
</tr>
<tr>
<td>CRH 1418</td>
<td>Abell 1896</td>
<td>1.7</td>
<td>distance class 5</td>
<td></td>
</tr>
</tbody>
</table>

### 4.4 Detection efficiency

A somewhat similar piece of work, although more limited in scope, has been performed by Giovannini et al. (1999). They examined the NVSS images around the 295 X-ray-brightest Abell-type clusters from Ebeling et al. (1996) for extended emission, and found 29 “halos”. Thus $\sim 14\%$ of Abell-type clusters are inferred to have radio halos. They also claim to confirm that none of the clusters with a halo contain a cooling flow, which is further evidence that halos are produced by cluster merger events.

Where their search areas have overlapped with the CRH search area, they find five clusters, only one of which I identified as a CRH. However, on examining the other four Abell clusters, it is clear that these have been excluded from the CRH sample by virtue of insufficiently steep spectral indices. Thus I estimate that this search method detects $\sim 20\%$ of halos in Abell-type clusters.

### 4.5 Optical images

In order to find high-$z$ clusters, the POSS plates for each of the CRHs were examined. The following images are all scanned versions of Polaroids that I took of copies of the $E$-band optical plates that were held at the RGO. For two CRHs, CRH 0917 and CRH 1419, POSS plates were not available. DSS images of these are included instead. The cross marks are centred on the positions in table 4.3, and also include scale: the distance between the cross hairs is 4'. At $z = 0.3$, 4' corresponds to 1.32 $h^{-1}$ Mpc which is a typical size of a cluster.
4. SEARCHING FOR CLUSTERS: SELECTION AND S-Z FOLLOW-UP

4.5. OPTICAL IMAGES

CRH 0800. A collection of faint objects to the N-W, roughly 1' away. A candidate distant cluster.

CRH 0853. The radio sources are faint, and there is nothing on the optical plate. A candidate distant cluster.

CRH 0917. DSS image; the entire image is 4' x 4'. No galaxies visible, so a candidate distant cluster.

CRH 1036. At least two bright galaxies at the centre, and so not a good high-z system.

CRH 1141. Five possible galaxies near and out to 30'' from the centre, so probably not a high-z candidate.

CRH 1152. Again, this is not a high-z cluster as there are two obvious galaxies close to the centre.
CRH 1303. Three candidate galaxies ≈ 45" from the centre, so not a high-z candidate.

CRH 1310. Two galaxies close to the cross hair position, so a likely low-z system.

CRH 1317. A cD-like galaxy with at least two extended objects close to it, so not a high-z system.

CRH 1324. Two faint objects visible close to the centre, so not a likely distant system.

CRH 1334. Two galaxies evident close to the centre, so not a likely high-z object.

CRH 1419. DSS image; the entire image is 4' x 4'. No galaxies are clearly visible, so a high-z candidate.
4. SEARCHING FOR CLUSTERS: SELECTION AND S-Z FOLLOW-UP

4.5. OPTICAL IMAGES

CRH 1422. There is a group of galaxies 30° S-West of the centre so this is probably not a high-z candidate.

CRH 1428. Several bright galaxies 30-45° from the centre, plus hints of some closer to the centre, so this is probably not a high-z system.

CRH 1440. Two obvious galaxies at the centre, so not a high-z system.

CRH 1655. Clearly visible galaxies at the centre and ≈ 45° away, so not a high-z object.

CRH 1716. Nothing at the centre, and so a candidate high-redshift cluster.

It is clear from the optical images that this technique is detecting both Abell and non-Abell clusters at low redshift. The FIRST images of three low redshift CRHs (as identified from the POSS plates) are shown in figures 4.8, 4.9 and 4.10. These three CRHs are associated with obvious nearby groups of galaxies, and are representative of three regimes of NVSS/FIRST flux ratio. CRH 0742 (figure 4.8) has a total integrated FIRST flux of 12.4 mJy, whereas the flux in the NVSS catalogue is 12.8 mJy, i.e. the NVSS and FIRST fluxes are very similar. However, the morphology of the FIRST image is clearly extended, and the POSS plate clearly shows a group of galaxies at the centre of the field. The FIRST image of CRH 1317 (figure 4.9) shows a point-like morphology, but has clearly lost flux; FIRST reports 2.2 mJy and NVSS reports 10.7 mJy. In this case the POSS plate shows a clear cD-like galaxy with two extended objects. Likewise, CRH 1655 (figure 4.10) shows a blank FIRST field. The NVSS survey finds a 3.8-mJy source at this position. Again, the POSS plate shows a group of galaxies.

From this procedure, 6 candidate high-redshift CRHs were found; CRH 0741, 0800, 0853, 0917, 1419 and 1716. The FIRST images are shown in figures 4.11 and 4.11-4.15. Although some look point-like, all have lost flux. Indeed, CRH 1317 shows that a point-like FIRST image does not mean that there is not extended cluster emission present.
Figure 4.8: FIRST image of CRH 0742. The grey scale range is ~0.8 (light) to 1 mJy beam\(^{-1}\).

Figure 4.9: FIRST image of CRH 1317. The grey scale range is ~0.7 (light) to 2 mJy beam\(^{-1}\).

Figure 4.10: FIRST image of CRH 1655. The grey scale range is ~0.6 (light) to 1 mJy beam\(^{-1}\).

Table 4.5: Predicted worst case fluxes at 15 GHz. These all assume a constant spectral index between 326 MHz and 15 GHz.

<table>
<thead>
<tr>
<th>Name</th>
<th>Spectral index</th>
<th>Predicted 15-GHz flux / mJy</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRH 0741</td>
<td>1.65</td>
<td>0.30</td>
</tr>
<tr>
<td>CRH 0800</td>
<td>1.55</td>
<td>0.10</td>
</tr>
<tr>
<td>CRH 0853</td>
<td>1.93</td>
<td>0.05</td>
</tr>
<tr>
<td>CRH 0917</td>
<td>1.67</td>
<td>0.12</td>
</tr>
<tr>
<td>CRH 1419</td>
<td>2.16</td>
<td>0.02</td>
</tr>
<tr>
<td>CRH 1716</td>
<td>2.13</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Figure 4.11: FIRST image of CRH 0800. The grey scale range is ~0.7 (light) to 6 mJy beam\(^{-1}\).

Figure 4.12: FIRST image of CRH 0853. The grey scale range is ~0.6 (light) to 2 mJy beam\(^{-1}\).

Figure 4.13: FIRST image of CRH 0917. The grey scale range is ~0.7 (light) to 3 mJy beam\(^{-1}\).

Figure 4.14: FIRST image of CRH 1419. The grey scale range is ~1 (light) to 1 mJy beam\(^{-1}\).

Figure 4.15: FIRST image of CRH 1716. The grey scale range is ~0.6 (light) to 0.7 mJy beam\(^{-1}\).
4.6. CRH RT observations

4.6.1 CRH 0741

This candidate was observed for 20 sets of 12-hours between August 1998 and March 1999, in configuration Ce. Of the 20 sets of observations, 1 deemed 16 to be of sufficient quality (based on the maps made from each day and the noise on each day) to be used. Those days rejected were generally rejected for bad weather. The approximate positions and fluxes of sources were identified from maps made with baselines longer than 1.5 kλ and these values were then used as the initial guess for FLUXFitter (see section 3.3.4). The sources found were:

<table>
<thead>
<tr>
<th>Flux / μJy beam⁻¹</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1350 ± 60</td>
<td>7 41 33.1 ± 0.1 47 52 16.4 ± 1</td>
</tr>
<tr>
<td>291 ± 60</td>
<td>7 41 38.5 ± 0.4 47 51 58 ± 4</td>
</tr>
<tr>
<td>186 ± 60</td>
<td>7 40 34.2 ± 0.4 47 46 13 ± 5</td>
</tr>
<tr>
<td>102 ± 60</td>
<td>7 42 0.4 ± 0.8 47 47 27 ± 9</td>
</tr>
</tbody>
</table>

A short-baseline map (i.e. baselines shorter than 1 kλ) was produced with the above sources subtracted and is shown in figure 4.16. The deepest feature that could be called a decrement is −265 ± 117 μJy beam⁻¹.

![Figure 4.16: Short baseline map of the CRH0741 field. The greyscale level is −320 (light) to 320 mJy beam⁻¹.](image-url)
### Observations

#### CRH 0800

The field was observed for 18 days between September 1998 and August 1999, in configuration Ce. I judge 11 of the 18 to be of sufficient quality. Four significant sources were found:

<table>
<thead>
<tr>
<th>Survey</th>
<th>Frequency (MHz)</th>
<th>Flux of “A” (mJy)</th>
<th>Flux of “B” (mJy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WENSS</td>
<td>326</td>
<td>2.7 ± 3.3</td>
<td>24 ± 6</td>
</tr>
<tr>
<td>NVSS</td>
<td>1400</td>
<td>9.4 ± 0.15</td>
<td>6.4 ± 0.5</td>
</tr>
<tr>
<td>FIRST</td>
<td>1400</td>
<td>7.5 ± 0.3</td>
<td>3.2 ± 0.138</td>
</tr>
</tbody>
</table>

Table 4.7: Fluxes of the sources “A” and “B” in figure 4.17 in various surveys. All values are integrated fluxes.

#### 4.6.2 CRH 0800

The field was observed for 18 days between September 1998 and August 1999, in configuration Ce. I judge 11 of the 18 to be of sufficient quality. Four significant sources were found:

<table>
<thead>
<tr>
<th>Flux (µJy beam⁻¹)</th>
<th>Position (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1253 ± 100</td>
<td>35 24 38 ± 11</td>
</tr>
<tr>
<td>298 ± 100</td>
<td>35 17 46 ± 10</td>
</tr>
<tr>
<td>158 ± 100</td>
<td>35 22 14 ± 16</td>
</tr>
<tr>
<td>245 ± 100</td>
<td>35 20 26 ± 11</td>
</tr>
</tbody>
</table>

The source-subtracted short-baseline map is shown in figure 4.17, with the NVSS map shown as a contour plot. On first inspection, there is a possible S-Z hole (labelled “Possible decrement”) of 850 ±120 µJy beam⁻¹. However, this is 230° from the pointing centre, where the primary beam of the Ryle Telescope is 21%. The signal-to-noise ratio for this is 7, and so the positional accuracy is ± 40°. Thus, this decrement, if it were real, would be around 4 mJy as observed by the Ryle Telescope if it were beam-attenuated. This is unphysically large. As the figure shows, there are NVSS sources (labelled “A” and “B”) found on the sidelobes of this “S-Z” hole. Table 4.7 shows the fluxes of the objects “A” and “B” in the WENSS, NVSS and FIRST catalogues. Object “B” is actually two objects in the FIRST catalogue, and has a large error in the WENSS catalogue due to the possibility of confusion with the source to the north. Assuming that the the spectral indices remain constant, “A” has a 15-GHz flux of 1.7 ± 0.2 mJy and “B” has a 15-GHz flux of 0.7 ± 0.5 mJy. Both values are uncorrected for beam attenuation. The sidelobes of the “hole” are 890 and 700 µJy beam⁻¹. The negative sidelobes of the beam are 88% and so these are consistent with being sidelobes (with the addition of noise) of an “S-Z” hole. In addition, neither of these two sources is found at 15 GHz in the long-baseline maps. They both have a slight extension at 1.4 GHz, so the most conservative hypothesis is that these two sources are slightly extended and present at 15 GHz and their separation on the sky is sufficient to confuse the Ryle Telescope in this particular case. Due to this, an S-Z effect cannot be detected with these data.

It is worth noting that the same data have been analysed by Riju Das (Priv. comm.). He took the alternative route and attempted to correlate the Ryle Telescope map with NVSS data. In doing this he found 12 sources, three of which agree with the first three sources I identified. His conclusion for the decrement: −1.2 ± 0.5 mJy a long way down the beam agree with these results, showing that an alternative strategy for source subtraction results in the same conclusion of an unphysically large S-Z effect.

---

4.6. CRH RT Observations

![Figure 4.17: In greyscale, the source-subtracted short-baseline map of CRH0800 from the Ryle Telescope at 15 GHz. The contours levels (which are −2, −1, 1, 1.414, 2, ... mJy beam⁻¹) are from NVSS. The greyscale ranges from −880 (light) to 880 µJy beam⁻¹.](image)
4.6.3 CRH 0853

This field was observed for 17 days between December 1998 and July 1999, in configuration Ce. Each run had good weather and all the data was concatenated together. A long-baseline map has a noise level 55 $\mu$Jy beam$^{-1}$. No sources above 220 $\mu$Jy beam$^{-1}$ — the 4-$\sigma$ level — were visible. The short baseline map, shown in figure 4.18, shows a hint of decrement at RA 08$^{h}$ 53$^{m}$ 30$^{s}$ ± 5 Dec. +50° 25' 35" ± 30. The magnitude of the decrement is $-227 \pm 83$ $\mu$Jy beam$^{-1}$. In an attempt to increase the signal-to-noise ratio, the pointing centre was adjusted to RA 08$^{h}$ 53$^{m}$ 29.5$^{s}$ Dec. +50° 25' 42.15".

Integration continued for 22 days through September and October 1999. Of these, only 7 were not badly affected by weather. The noise level on the long-baseline map was 90 $\mu$Jy beam$^{-1}$. Again, no sources were found. The short-baseline map for this pointing is shown in figure 4.19. Again, only a hint of a decrement, $-345 \pm 140$ $\mu$Jy beam$^{-1}$ is visible.

Previous work with the Ryle Telescope has shown that finding no point sources is rare (Grainge, 1995; Das, 1999). In order to check the lack of sources, maps were made using a lower-baseline limit of 1 k$. This increases the number of $uv$ points used for map making and so increases the sensitivity. The maps were then averaged together, and the resultant map is shown in figure 4.20. Again, this map is consistent with noise. Thus the short-baseline maps are not contaminated with faint point sources, and this is a null detection of an S-Z decrement.

Figure 4.18: Short baseline map of CRH0853. The greyscale range is from $-270$ (light) to 270 $\mu$Jy beam$^{-1}$.

Figure 4.19: Short baseline map of CRH0853 with slightly offset pointing centre. The range is $-550$ (light) to 710 $\mu$Jy beam$^{-1}$.

Figure 4.20: Averaged long baseline map of CRH0853 with both pointings. The greyscale range is from $-180$ (light) to 190 $\mu$Jy beam$^{-1}$. 
4.6.4 CRH 0917

This field was observed for 12 hours on August 22nd 1998 and for 7 sets of 12 hours in June and July 1999, all with the Ryle Telescope in configuration Ce. Six of these sets of observations were used. Four sources were found in the long-baseline map. They are:

<table>
<thead>
<tr>
<th>Flux $/\mu$Jy beam$^{-1}$</th>
<th>Position RA Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>4020 ± 135</td>
<td>9h 17m 42.75 ± 0.03 36° 17' 34 ± 1</td>
</tr>
<tr>
<td>505 ± 135</td>
<td>9h 17m 54.9 ± 0.5  36° 17' 51 ± 7</td>
</tr>
<tr>
<td>430 ± 135</td>
<td>9h 17m 33.3 ± 0.5  36° 15' 43 ± 7</td>
</tr>
<tr>
<td>240 ± 135</td>
<td>9h 17m 41.0 ± 0.7  36° 17' 41 ± 14</td>
</tr>
</tbody>
</table>

Subtraction of these sources leaves a long-baseline map that is consistent with noise. The short-baseline map, shown in figure 4.21, shows a 1.3 mJy beam$^{-1}$ source centred on RA 9h 17m 42.4± Dec. +36° 17' 41". Thus the 4.02 mJy beam$^{-1}$ source is an extended object. With the current dataset, this is impossible to remove and so nothing can be said about the presence (or otherwise) of an S-Z decrement.

Figure 4.21: Greyscale: Short baseline map of CRH 0917 with all four listed sources subtracted. The range of the greyscale is –1 (light) to 1 mJy beam$^{-1}$. Contours: Long baseline map with the three weaker sources subtracted. The contour levels are 0.1, 0.2, 0.4, 0.8, 1.6, ... mJy beam$^{-1}$.

4.6.5 CRH 1419

This field was observed for 4 sets of 12 hours in August and September 1999. Three days of data were used, and a small section (10 minutes) was trimed from the second day. When first analysed, this section of noisy data led to the mistaken conclusion that the short baseline maps contained positive extended flux. This led to ceasing further observations of this field. However, with the noisy data removed, two point sources are visible on the long-baseline map. 1 kA was used as the cutoff to increase the sensitivity, with a risk of contamination from an S-Z decrement. Using FLUXFITTER, I find:

<table>
<thead>
<tr>
<th>Flux $/\mu$Jy beam$^{-1}$</th>
<th>Position RA Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>932 ± 180</td>
<td>14h 20m 10.5 ± 0.2 41° 54' 1 ± 5</td>
</tr>
<tr>
<td>591 ± 180</td>
<td>14h 19m 59.9 ± 0.5 41° 53' 13 ± 13</td>
</tr>
</tbody>
</table>

The short-baseline map is shown in figure 4.22. The deepest decrement near the pointing centre is $-490 ± 235 \mu$Jy beam$^{-1}$. This is consistent with a null detection of an S-Z decrement.

Figure 4.22: Greyscale of the short-baseline map of CRH1419. The range is $-550$ (light) to 640 $\mu$Jy beam$^{-1}$. 
4.6.6 CRH 1716

This field was observed between August 1998 and August 1999 for 10 12-hour sessions with the Ryle Telescope in Ce configuration. Of these, 9 sets of observations were used. 5 sources were found:

<table>
<thead>
<tr>
<th>Flux / µJy beam⁻¹</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>430 ± 100</td>
<td>17 17 4.8 ± 0.3 47 3 37.7 ± 4.3</td>
</tr>
<tr>
<td>384 ± 100</td>
<td>17 17 1.5 ± 0.3 46 59 17.8 ± 5.6</td>
</tr>
<tr>
<td>302 ± 100</td>
<td>17 16 52.8 ± 0.4 46 56 48.5 ± 7.5</td>
</tr>
<tr>
<td>287 ± 100</td>
<td>17 16 45.3 ± 0.7 46 56 46.6 ± 6.2</td>
</tr>
<tr>
<td>220 ± 100</td>
<td>17 16 43.7 ± 0.9 46 56 47.52 ± 7.2</td>
</tr>
</tbody>
</table>

The short baseline map is shown in figure 4.23. The deepest decrement near the pointing centre is, again, consistent with noise.

![Image](image.png)

**Figure 4.23:** Short baseline map of CRH1716. The greyscale range is -740 (light) to 550 µJy beam⁻¹.

4.7 Conclusions

The results of the S-Z measurements are shown in table 4.8. None of the detections are greater than 3σ. As the Ryle Telescope is only able to detect clusters with masses $\approx 10^{15} M_\odot$, which corresponds to the most masses of the luminous clusters, this implies that the CRHs found here, if they are clusters, have masses $\lesssim 10^{15} M_\odot$.

<table>
<thead>
<tr>
<th>Object</th>
<th>Deepest possible hole / µJy beam⁻¹</th>
<th>Signal-to-noise ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRH 0741</td>
<td>−265 ± 117</td>
<td>2.29</td>
</tr>
<tr>
<td>CRH 0800</td>
<td>Source confusion</td>
<td></td>
</tr>
<tr>
<td>CRH 0853</td>
<td>−227 ± 83</td>
<td>2.73</td>
</tr>
<tr>
<td>CRH 0917</td>
<td>Source confusion</td>
<td></td>
</tr>
<tr>
<td>CRH 1419</td>
<td>−489 ± 235</td>
<td>2.08</td>
</tr>
<tr>
<td>CRH 1716</td>
<td>−442 ± 170</td>
<td>2.6</td>
</tr>
</tbody>
</table>

**Table 4.8:** Summary of the Ryle Telescope observations of CRH fields.

In order to search for clusters via steep-spectrum radio emission, I have combined the NVSS, FIRST and WENSS surveys.

1. A total of 254 objects were found with $R_{NVSS} > 1.5$ and flux at 320 MHz greater than 65 mJy.

2. 69 of the objects are classical double radio sources, 17 are fat doubles, three are wide-angle tail sources, and 33 are unclassified (typically because of inadequate resolution). These were then rejected from the sample. They are being selected by having aged, hence steep-spectrum, diffuse emission.

3. Of the 26 remaining objects, six are associated with known Abell clusters, showing that this technique does detect clusters at low redshifts. In addition, the optical images of many of the fields show previously unidentified clusters.

4. The steep-spectrum method is estimated detect $\sim 20\%$ of Abell-type clusters which contain radio halos; the rest have halos without steep spectra.

5. A further six candidates had no counterpart evident on the POSS plates, and these distant candidates were then observed with the Ryle Telescope. No positive detections of the S-Z effect were made; in one case a bright extended source makes an S-Z detection impossible and in a second two slightly extended sources produce what seems to be an unphysically large S-Z effect. In the other four cases, the short-baseline maps were consistent with noise. However, the sensitivity of the Ryle is such that this only eliminates the possibility that these candidates have masses $\gtrsim 10^{15} M_\odot$. 
Chapter 5

Searching for distant clusters: optical follow-up

5.1 Introduction

This chapter presents optical imaging of the fields of the cluster radio halo candidates described in the previous chapter. The lack of S-Z detections implies that any clusters present must have total masses less than those of the biggest Abell clusters, but if they lie at redshifts less than $z \approx 1$ then it is reasonable to expect they will be detected with ground-based optical multicolour imaging (see Haynes, 1998). It turned out that problems with the instrument and weather prevented the acquisition of deep multicolour imaging, yet the results are still interesting.

5.2 Telescope and instrument

The CRH objects were observed with the 120-inch Shane telescope at the Lick Observatory, east of San Jose, California on two separate occasions. A 3-m telescope, with high throughput and a good CCD, allows galaxies with $R \approx 25$ to be detected with a signal-to-noise ratio of 5 in an hour. As such, this should allow the brightest cluster galaxies to be detected out to a redshift of $z \approx 0.8$. In addition, the Shane telescope has a spectroscopic mode as well as a direct-imaging mode, making it potentially useful for detecting the objects via imaging and then taking spectra to measure redshifts.

The Lick Observatory is located on Mount Hamilton at an altitude of 4200 feet. The instrument used with the 120-inch telescope was the Kast Dual Spectrograph, which is mounted at the Cassegrain focus. It can be used for direct imaging as well as spectroscopy. Gratings and grisms can quickly
be placed into the light path to enable the switch between imaging and spectroscopy. A dichroic splits the light into red and blue for both imaging and spectroscopy modes. In imaging mode, the field of view in both the red and blue is ≈ 2.4’. This is not ideal for my purpose as it is difficult to determine whether the field contains an overdensity of galaxies or not, but the ability to take both multicolour images and spectra may be sufficient for determining the existence or otherwise of a cluster of galaxies. The CCDs are Reticon devices with 1200×460 pixels, and each pixel corresponds to 0.78″ on the sky, which is adequate given the typical seeing of between 1.5 and 2″ at the site (Chloros and Wright).\(^1\)

For spectroscopy, a grism of 452 line mm\(^{-1}\) giving 2.8 Å pix\(^{-1}\) was selected on the blue arm, and one of 620 line mm\(^{-1}\) giving 4.6 Å pix\(^{-1}\) on the red arm. The total range is 3200–7700 Å (with a dichroic split at 5500 Å) which, given the candidate galaxies are in the range \(z = 0.5–0.8\), is good for detecting absorption lines, the 4900 Å break and likely emission lines (e.g. [OII] at 3727 Å). Unfortunately, there proved to be no time for this.

### 5.3 Observations

The observed CRH fields, along with the observers, are detailed in Table 5.1. The strategy was to take images of each field in three colours (\(V\), \(I\) and \(R_0\)) to identify, at the telescope, high-redshift objects and attempt to take spectra. Each night, before twilight, short dark exposures were taken both of the chip windowed for direct imaging and of the entire chip for spectroscopy. Some long dark exposures were also taken during the day. Dome flats were taken in both spectroscopic and direct modes. After sunset, direct and spectroscopic twilight flats were taken. After twilight, direct images of the standard stars SA 106 1024 and SA 140 345 (the former for the January observations and latter for June) and spectroscopy of the standard stars Hiltner 600 (January) and Feige 92 (June) were taken. To minimise the effects of flexure, known to be as much as 5 pixels parallel to the dispersion direction,\(^2\) spectra of the standard lamps available at Lick (neon-argon and helium-mercury-cadmium) were taken whilst leaving the telescope guiding after any spectroscopy was attempted. Observations of the CRHs were made whilst they were at low airmass (always less than 1.25), which dictated the order of observations. At dawn, more twilight flats (both direct and spectroscopic) were taken.

\(^1\)Published at http://nls-www.ucolick.org/techdocs/obsstats/avgseeing.html

\(^2\)From http://www.ucolick.org/~mountain/mhamilton/techdocs/instruments/kast/kast_index.html

### 5.4 Data reduction

When a CCD is read out, there is a bias due to DC offsets in its amplifiers. The amplifiers are at the end of the rows, and reading out involves shifting the charge along the row to the amplifier. The DC offset on each amplifier can vary with time for many reasons, e.g. temperature, which might depend on, for example, telescope position. It is usual to have an overscan region, i.e. a few unexposed pixels on each row of the chip, so that the offset can be measured. Unfortunately at Lick the bias “is taken care of”, and the subtraction is done by the instrument. The mean value of the pixel counts is 2.08 ± 0.06 for a one-second exposure, so although the bias removal is doing a reasonable job, it is not perfect. Neither chip was found to have a significant dark current. The zero point was set by subtracting the 1 second dark frame taken at the beginning of the night from all the images. This, along with all the data reduction, was performed with the standard IRAF task CCDPROC.

Dome flats were not found to provide a complete illumination correction. Thus twilight images were also used. Once determined, the overall illumination correction was applied to some bright twilight images. No residual structure was found in the image, and so the same correction was applied to the CRH observations and calibrators.

Deep observations were done in two or three separate exposures, for two reasons. Firstly, it is
5.4. DATA REDUCTION

properly. This was due to slight misalignment of an empty filter wheel in the instrument, causing some vignetting whilst taking dome and twilight flats. The filter wheel then reset itself to the correct position. Thus for CRH 1419 and 1716, the dome and twilight flats have been disregarded, and the illumination pattern determined by stacking all the available images for each band together. The frames were averaged together, and then normalised. To remove galaxies and stars, the average and standard deviation of each pixel (between the frames) were determined, and pixels which differed by more than 3- were removed. This process works well for the majority of the image; the section on which it fails to produce a flat field has been removed.

As Lick Observatory is on a relatively low site, there is a good deal of OH line emission from the atmosphere. The OH lines are bright and are of a sufficient wavelength to show strong fringing. As is obvious from, for example, figure 5.4, not all the fringe has been removed from the I-band images. As the effect is small (around 2%), no effort beyond standard flat fielding was taken to remove the fringing pattern.

In the R image of CRH 1419, shown in figure 5.4, a bright star is visible in the north of the frame. The trail is due to the well of the pixel overfilling with electrons, which can then spill over into neighbouring pixels. The charge transfer that occurs at readout stage does not then move all the electrons, leading to an obvious stripe.

The limiting magnitudes have been estimated in the following way. A region of each image without any obvious objects was selected and the mean and standard deviation of the pixels determined. The number of counts per second that a 5- event would produce was then determined and converted to a magnitude. Limiting magnitudes were found to be fainter for the observations of CRH 1419 and CRH 1716 (≈ 23.5 in R, 23 in V, 22 in I) than the CRH 0853 observations (≈ 22.5 in R, 22 in V, 21 in I), consistent with the weather experienced. Note that the limiting magnitude for the I-band images is generally high; this due to the sky being brighter in I-band and the standard deviation being high due to fringing. The limiting magnitudes are not as good as predicted for a “good” 3-m telescope as the filters available at the Shane reduce the area of the collimated beam by 48%, and so limit the number of photons getting from primary mirror to CCD.

For these three CRHs, I have produced colour-magnitude plots; each is presented next to the images. The fluxes have all been determined with the Source Extractor (SExtractor) software, written by Bertin & Arnouts (1996). SExtractor identifies objects and then determines the flux within a set aperture. As none of the fields was particularly crowded and all reasonably well flattened, very similar results were found by setting the background to a constant level. The apertures determined by SExtractor were plotted over each frame and false detections removed by eye. Most false detections occurred at the edges of the image where the background measurement algorithm failed. The plotted squares are to guide the eye only; the apertures were determined by SExtractor. With the bright objects in the fields, the determination of the magnitudes was found to be consistent with magnitudes estimated from POSS plates.

5.4.1 Images

Figures 5.2, 5.4 and 5.6 show the images of CRH 0853, 1419 and 1716 respectively. North is up and east is to the left. Some comments on these now follow.

In the images of CRH 1419 and CRH 1716, the southern part of the aperture is not flattened
### Table 5.3: Magnitudes for objects in CRH 0853 shown in figure 5.2.

<table>
<thead>
<tr>
<th>Number</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.6</td>
</tr>
<tr>
<td>2</td>
<td>20.6</td>
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<tr>
<td>3</td>
<td>20.8</td>
</tr>
<tr>
<td>4</td>
<td>20.8</td>
</tr>
<tr>
<td>5</td>
<td>20.9</td>
</tr>
<tr>
<td>6</td>
<td>20.9</td>
</tr>
<tr>
<td>7</td>
<td>21.0</td>
</tr>
</tbody>
</table>

### Figure 5.2: \( R_s \) (top) \( V \) (left) and \( I \) (right) band images of CRH 0853. The two bright objects with \( R_s \approx 18.6 \) (upper right) and \( R_s \approx 18.4 \) (lower left) are visible on the original photographic prints of the POSS with estimated \( B \) magnitudes of \( 18.5 \)–19.
Figure 5.3: Magnitude-colour diagram for CRH 1419. A typical error bar is shown.

Figure 5.4: $R_s$ (top) $V$ (left) and $I$ (right) band images of CRH 1419. The bright star is clear on POSS images.
CHAPTER 5. SEARCHING FOR CLUSTERS: OPTICAL FOLLOW-UP

5.4. DATA REDUCTION

Figure 5.5: Magnitude-colour diagram for CRH 1716. A typical error bar is shown.

Figure 5.6: $R_i$ (top) $V$ (left) and $I$ (right) band images of CRH 1716. The bright object to the left is evident on the POSS plate.
5.5 Interpretation

From the colour-magnitude diagrams, it is possible to find evidence for clusters. It has been known for some time that more luminous elliptical galaxy clusters tend to have redder colours; e.g. Visvanathan & Sandage (1977) and Kodama et al. (1999). Indeed, Kodama et al. have shown that this can be used with clusters over a range of redshifts; they examine clusters between $z = 0.31$ and $z = 1.2$ and find a consistent picture. Thus in a colour-magnitude diagram, the big elliptical galaxies will then appear as a spur of objects. CRH 1419 and 1716 both have 2 galaxies of similar magnitude and colour that are brighter in $R_s$ and redder than the rest of the field. To achieve a very red object, the rest frame 4000-Å break needs to be placed between the pass bands of the $V$ and $R_s$ filters. This simple notion places the rest frame 4000-Å break between 5400–6200Å, corresponding to redshifts 0.35–0.55. Modelling by Bruzual A. & Charlot (1993) predicts that an elliptical galaxy has $(G - R)_{\text{Vega}} = 2$ if it has $z = 0.35$–0.8; see also Haynes (1998).

The brightest of these red galaxies in each field has an $R_s$ magnitude of 19.8 and 19.9 respectively, with $V - R_s \approx 3$ in each case. This appears to be evidence for a cluster in each field, but what is the chance of such galaxies appearing at random in each field? To assess this, I have plotted colour-magnitude diagrams of 10 random fields taken from the INT WFC survey (Lewis et al., 2000), which has deep multicolour imaging reaching 24.5 in $R$ and 24.6 in $G$; $G$ is very close to $V$. To perform a proper comparison with the CRH fields, the fields were chosen to look like blank POSS fields, i.e. contain no objects with $V$-band magnitudes less than 20, and the search radius was set to 1.2′. Figures 5.8 and 5.9 are the colour magnitude diagrams. None of these has $G - R$ approaching 3 until $R$ reaches 22. I conclude that the red galaxies in the two CRH fields are therefore not “field” galaxies there by chance but are cluster members. Note also that the WFC data provide a test of the magnitude scale. In the “field” objects, $G - R = 0 - 2$ for $R \sim 21$ objects, which is the case for the fainter objects in the CRH fields. The two mispointed CRH observations of 0800 and 0917, figures 5.11 and 5.10 respectively, do not have good colour information, but both have many objects with $R_s$ magnitudes around 21.

Figure 5.7 shows the Gunn-$R$ versus redshift diagram for Abell clusters calculated from the data given in Hoessel et al. (1980). Fitting a straight line to this plot gives the relation

$$\log z = (0.184 \pm 0.006)R_{\text{mag}} - (3.93 \pm 0.09),$$  

(5.2)

where $R_{\text{mag}}$ is the Gunn-$R$ magnitude of brightest cluster galaxy. The Abell cluster data necessarily extend only to moderate redshifts. The relation will change for $z > 0.5$ as the 4000-Å break enters the $R$-band wavelength range, and galaxy evolution will also cause a change at high redshifts. A flattening of the Hubble diagram can be seen in the $R$-band magnitudes of 3C radio galaxies, as shown in Eales (1985). The line is also reproduced in figure 5.7 and shows that the relation fitted to the 3C radio galaxies are approximately 0.75 magnitudes brighter than given by equation 5.2. However, Eales (1985) uses the Cousins (1976) $RI$ systems and so this magnitude shift may be partly due to the different system used. The Abell-based and 3C-based lines are close to each other in the region of the magnitudes of the CRH brightest candidate cluster members, and if the CRH galaxies have absolute magnitudes similar to the Abell or 3C galaxies, then their redshifts are $0.5 - 0.6$.

Further imaging in multiple colours and with a wide field of view is of course necessary for all the fields; multiple colours would give a better indication of the redshifts even without spectra being taken, and a wider field of view might allow the local projected galaxy density to be measured to determine whether or not there is an over-density of galaxies.

As the detection efficiency of the CRH search technique is (approximately) known (see section 4.4), and redshifts have been estimated for the detected clusters, it is possible to estimate a comoving space density for these clusters. For an Einstein-de-Sitter universe, the comoving volume $V$ out to a redshift $z$ is given by

$$V(z) = \Omega \frac{8 \pi^3}{3 H_0^2(1+z)^3} (1+z),$$  

(5.3)

where $\Omega$ is the solid angle of sky observed. However it is difficult to determine the limiting redshift of the search technique as the selection function for CRHs is complex. Instead I present comoving number densities of CRH clusters for three limiting redshifts; the approximate redshift of the detected clusters, 0.6, the approximate limit of the $R_s$ images, 0.8 and an arbitrary large redshift, 1.5. Table 5.4 lists the comoving number densities found, assuming that the selection function remains the same (i.e. 20% of the clusters with halos will be detected by this technique and 14% of clusters have halos).

As a comparison, the REFLEX cluster survey Collins et al. (2000) finds 452 clusters with an X-ray flux greater than $3 \times 10^{-15}$ W m$^{-2}$ over a sky area of 4.24 steradians. Obviously this is not luminosity-limited and so not complete at higher redshifts, but they claim to detect everything to
Redshift range  | Comoving number density 
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3-0.6</td>
<td>$5 \times 10^{-8}$</td>
</tr>
<tr>
<td>0.3-0.8</td>
<td>$3 \times 10^{-8}$</td>
</tr>
<tr>
<td>0.3-1.5</td>
<td>$8 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Table 5.4: Comoving number densities for Abell-type clusters, based on the number of CRHs found, assuming different redshift ranges, $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ and an Einstein-de-Sitter universe. The error in the three densities is dominated by the very small number statistics on the comparison between CRHs and Abell clusters in section 4.4.

$z = 0.35$. This gives a comoving number density of REFLEX clusters of $6.8 \times 10^{-8}$ Mpc$^{-3}$. This value is similar to the number densities found for Abell-type clusters. This implies that there is not a large amount of evolution in the cluster comoving number density, which is certainly consistent with the results found from X-ray surveys (e.g. Nichol et al., 1997).

5.6 Conclusions

The two fields CRH 1419 and CRH 1716 which have adequate images both have red $(V - R_2 \gtrsim 3)$ galaxies with $R_2 \approx 20$ which are consistent with being cluster members. A search of the INT WFC survey of ten random patches of sky (of the same size as my Lick images) for galaxies with these magnitudes and colours revealed no such galaxies. I conclude that the red galaxies in these two CRH fields are clusters members, with estimated redshifts of 0.5 – 0.6.

For the clusters identified here — CRH 1419 and 1716 — and also the other fields, CRH 0741, 0800, 0853 and 0917, the next step is obvious. Multicolour images of a wide field around the clusters are necessary, and then, when cluster members have been identified, spectroscopy to accurately determine redshifts. Long pointed observations with X-ray telescopes would also very useful. Finally, if a more sensitive S-Z telescope were available, then that too could be used to explore these objects in greater detail. But, of course, what is really needed is an S-Z survey.

Figure 5.8: Colour-magnitude diagrams for blank field regions from the INT WFC survey.
CHAPTER 5. SEARCHING FOR CLUSTERS: OPTICAL FOLLOW-UP

5.6. CONCLUSIONS

<table>
<thead>
<tr>
<th>Object</th>
<th>$R_s$ magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.2</td>
</tr>
<tr>
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<td>20.9</td>
</tr>
<tr>
<td>3</td>
<td>19.8</td>
</tr>
<tr>
<td>4</td>
<td>21.1</td>
</tr>
</tbody>
</table>

Table 5.5: Magnitudes for the objects labelled in figure 5.10. Typical error is ±0.2 magnitudes.

Figure 5.9: Colour-magnitude diagrams for blank field regions from the INT WFC survey.

Figure 5.10: $R_s$ band image of CRH 0917.
CHAPTER 5. SEARCHING FOR CLUSTERS: OPTICAL FOLLOW-UP

5.6. CONCLUSIONS

<table>
<thead>
<tr>
<th>Object</th>
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</thead>
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<td>12</td>
<td>22.0</td>
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</table>

Table 5.6: Magnitudes for the objects labelled in figure 5.11. Typical error is $\pm 0.2$ magnitudes

Figure 5.11: $R_s$ (top) $V$ (left) and $I$ (right) band images of CRH 0800.

There are a good deal of objects to the north of the $R_s$ image. These are not visible in the $V$ or $I$ images as they are too far north.
Appendix A

Determining the line of sight

The problem is to determine the line-of-sight depth of an arbitrary orientated ellipsoid. I shall consider the general problem of the size through an ellipsoid, as the surface where out to one core radii defines an ellipsoid.

The surface of the ellipsoid can be represented as the coordinates \((a \sin \theta \cos \phi, b \sin \theta \sin \phi, c \cos \theta)\), where \(a, b, c\) are the lengths along the principle axes, and \(\theta\) and \(\phi\) are parametric variables. The coordinates can then be rotated to an arbitrary orientation by multiplying by the matrix

\[
\begin{pmatrix}
\cos \alpha \cos \beta \cos \gamma - \sin \gamma \sin \alpha & \sin \alpha \cos \beta \cos \gamma + \cos \alpha \sin \gamma & -\sin \beta \cos \gamma \\
-\cos \alpha \cos \beta \sin \gamma - \sin \alpha \cos \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & \cos \beta \sin \gamma \\
\sin \alpha \sin \beta & \cos \alpha \sin \beta & \cos \beta 
\end{pmatrix},
\]

(A.1)

where \(\alpha, \beta, \gamma\) are the Euler angles. Thus the new \(x, y, z\) co-ordinates of the ellipsoid are given by

\[
x = A_{11}a \sin \theta \cos \phi + A_{12}b \sin \theta \sin \phi + A_{13}c \cos \theta
\]

(A.2)

\[
y = A_{21}a \sin \theta \cos \phi + A_{22}b \sin \theta \sin \phi + A_{23}c \cos \theta
\]

(A.3)

\[
z = A_{31}a \sin \theta \cos \phi + A_{32}b \sin \theta \sin \phi + A_{33}c \cos \theta
\]

(A.4)

where \(A_{ij}\) is the \(ij\)th term of the matrix A.1. The line-of-sight size of the ellipsoid is then twice the value of \(z\) where \(x = 0\) and \(y = 0\). By setting this condition in equations A.2 and A.3, \(\phi\) and \(\theta\) are then given by

\[
\tan \phi = \frac{-e A_{11}A_{23} + A_{12}A_{21}}{e A_{12}A_{23} + A_{13}A_{22}}
\]

(A.5)

\[
\tan \theta = \frac{e A_{21}A_{33} - A_{11}A_{32}}{e A_{22}A_{33} - A_{21}A_{32} \sin \phi}
\]

(A.6)

By putting the values into equation A.4, the line of sight size of the ellipsoid can be found. This equation has been tested numerically by producing a grid with the value of 1 in each cell if it inside the ellipsoid and 0 if was outside. Then summing along the central pixel gives an approximation of
the line-of-sight through the cluster. The technique described above was found to be both correct and faster.

Appendix B

The angular size-redshift relation.

I present here a derivation of the angular size relation for a general cosmology. The Friedman equation,

$$ \dot{R}^2 = \frac{8\pi\rho GR^2}{3} - \frac{c^2}{R^2} + \frac{1}{3}A R^2, \quad (B.1) $$

defines $R(t)$, the scale factor, for the universe where $\rho$ is the total inertial mass density, $A$ is the cosmological constant, $G$ is the gravitational constant and $R$ is the radius of curvature of the geometry of the universe at the current epoch. By defining the following parameters,

$$ \Lambda = 3H_0^2\Omega_{\Lambda}, \quad (B.2) $$
$$ \Omega_M = \frac{8\pi\rho G}{3H_0^2}, \quad (B.3) $$

the Friedman equation becomes

$$ \dot{R}^2 = \frac{\Omega_M H_0^2}{R} - \frac{c^2}{R^2} + \Omega_{\Lambda} H_0^2R^2. \quad (B.4) $$

The $R$ term can be removed by noting that $H_0 = \frac{\Omega_{\Lambda} H_0}{R_0}$ and $R_0 = 1$, giving

$$ H_0^2 = \Omega_M H_0^2 - \frac{c^2}{R^2} + \Omega_{\Lambda} H_0^2. \quad (B.5) $$

Rearranging this gives the result

$$ -\frac{c^2}{R^2} = H_0^2(1 - (\Omega_M + \Omega_{\Lambda})). \quad (B.6) $$

Substituting this back into the Friedman equation gives

$$ \dot{R}^2 = H_0^2 \left( \frac{\Omega_M}{R} + 1 - (\Omega_M + \Omega_{\Lambda} + \Omega_{\Lambda} R^2) \right). \quad (B.7) $$
Noting that \( R = \frac{1}{1+z} \) (and so \( \frac{dR}{dR} = -\frac{1}{(1+z)^2} \)) and that \( \frac{d}{dz} = \frac{d}{dR} \frac{dR}{dz} \), and substituting gives

\[
\frac{dz}{dr} = -(1+z)^3H_0\Omega_Mz + \Omega_L \left( \frac{1}{(1+z)^2} - 1 \right).
\]

Using the increment of radial comoving coordinate distance \( dr = c\frac{dz}{dR} = -cR(1+z) \) I find the expression

\[
dr = \frac{c\, dz}{(1+z)H_0\Omega_Mz + \Omega_L \left( \frac{1}{(1+z)^2} - 1 \right)}.
\]

Setting \( \Omega_L = 0 \) in this relation and integrating gives the solution found by Mattig (1958).

The proper size of an object, \( d, \) at a redshift \( z, \) which subtends an angle \( \Delta \theta \) is simply given by the \( d\theta \) term in the Robertson-Walker metric (see Longair, 1998), i.e.

\[
d = \frac{1}{1+z}R \sin \left( \frac{r}{R} \right) \Delta \theta,
\]

which can also be written

\[
d = \Delta \theta D_A,
\]

where \( D_A = \frac{1}{1+z}R \sin \left( \frac{r}{R} \right). \)

Integrating equation (B.9) gives the value of \( r \), which must be done numerically for a general solution. The calculation can be done whilst avoiding complex numbers if sinh is used as factors of \( i \) then cancel.

If \( \Omega_0 + \Omega_L < 1, \) then \( (1+z)D_A = \frac{c}{H_0\sqrt{1 - (\Omega_0 + \Omega_L)}} \sinh \frac{rH_0\sqrt{1 - (\Omega_0 + \Omega_L)}}{c} \)

If \( \Omega_0 + \Omega_L > 1, \) then \( (1+z)D_A = \frac{c}{H_0/\sqrt{(\Omega_0 + \Omega_L)} - 1} \sin \frac{rH_0\sqrt{1 - (\Omega_0 + \Omega_L)}}{c} \)

If \( \Omega_0 + \Omega_L = 1, \) then \( (1+z)D_A = \int_0 \frac{c\, dz}{(1+z)H_0\sqrt{\Omega_M(1+z) + \frac{\Omega_L}{(1+z)^2}}} \)

### Appendix C

"Extreme" simulated clusters

I present here the bolometric surface-brightness maps of clusters 8 and 10, the two most extreme clusters (in that they give the most extreme \( H_0 \) values) simulated by Eke et al. (1998). They were kindly provided to me by Vincent Eke as observed images. See section 2.8 for details.
Cluster 8 at $z = 0.15$. The contour levels are $(1, 2, 3, 4, 5, 6, 11, 12, 14, 16, 18, 21, 23, 30, 36, 40, 50, 60, 80, 90) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 49 km s$^{-1}$ Mpc$^{-1}$.

Cluster 8 at $z = 0.2$. The contour levels are $(1, 2, 3, 4, 5, 6, 11, 12, 14, 16, 18, 21, 23, 30, 36, 40, 50, 60, 80, 90) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 54 km s$^{-1}$ Mpc$^{-1}$.

Cluster 8 at $z = 0.23$. The contour levels are $(1, 2, 3, 4, 5, 6, 11, 12, 14, 16, 18, 21, 23, 30, 36, 40, 50, 60, 80, 90) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 97 km s$^{-1}$ Mpc$^{-1}$.

Cluster 8 at $z = 0.38$. The contour levels are $(1, 2, 3, 4, 5, 6, 11, 12, 14, 16, 18, 21, 23, 30, 36, 40, 50, 60, 80, 90) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 91 km s$^{-1}$ Mpc$^{-1}$.

Cluster 8 at $z = 0.55$. The contour levels are $(1, 1.6, 2.5, 3.9, 6.1, 9.7, 15, 24, 38) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 86 km s$^{-1}$ Mpc$^{-1}$.

Cluster 8 at $z = 0.78$. The contour levels are $(1, 1.7, 2.8, 4.8, 8.1, 14, 23, 39, 65, 110) \times 10^{-11}$ W m$^{-2}$. No $H_0$ was derived for this cluster.

Cluster 8 at $z = 1.09$. The contour levels are $(1, 1.7, 2.8, 4.6, 7.7, 13, 21, 36, 50, 99) \times 10^{-11}$ W m$^{-2}$. No $H_0$ was derived for this cluster.
Cluster 10 at $z = 0.15$. The contour levels are $(1, 3.1, 5.5, 9.7, 17, 30, 53, 93, 165) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 102 km s$^{-1}$ Mpc$^{-1}$.

Cluster 10 at $z = 0.2$. The contour levels are $(1, 2.1, 4.4, 9.2, 19, 40, 84, 180, 370, 770) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 171 km s$^{-1}$ Mpc$^{-1}$.

Cluster 10 at $z = 0.25$. The contour levels are $(1, 2.2, 4.8, 10, 23, 40, 90, 240, 510, 1100) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 52 km s$^{-1}$ Mpc$^{-1}$.

Cluster 10 at $z = 0.38$. The contour levels are $(1, 2.1, 4.4, 9.4, 20, 42, 89, 190, 400, 840) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 64 km s$^{-1}$ Mpc$^{-1}$.

Cluster 10 at $z = 0.55$. The contour levels are $(1, 1.9, 3.5, 6.7, 13, 24, 44, 84, 160) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 75 km s$^{-1}$ Mpc$^{-1}$.

Cluster 10 at $z = 0.78$. The contour levels are $(1, 1.8, 3.3, 6.1, 11, 21, 38, 70, 130, 230) \times 10^{-11}$ W m$^{-2}$. $H_0$ derived for this cluster = 107 km s$^{-1}$ Mpc$^{-1}$.

Cluster 10 at $z = 1.09$. The contour levels are $(1, 1.7, 2.7, 4.5, 7.5, 12, 21, 34, 56, 93) \times 10^{-11}$ W m$^{-2}$. No $H_0$ was derived for this cluster.
Appendix D

Determining $H_0$ from a set of clusters

As discussed in section 2.4, the weighted geometric mean is required to determine $H_0$ from a set of clusters. The “standard” geometric mean, $f$, of a set of values $y_i$ is given by

$$ f = \sqrt[\prod_{i=1}^{n} y_i]{}.$$  \hspace{1cm} (D.1)

It is clear how to extend this to the weighted case. Consider the simple case of two values, $a$ and $b$, with weights $w_a$ and $w_b$. Clearly the geometric mean is given by $\sqrt{ab}$. If, for example, the $w_a = 2$ and $w_b = 1$, then the weighted geometric mean would be equivalent to the geometric mean of $a, a, b$, i.e. $\sqrt[3]{a^2 b}$. This can also be written

$$ f = w_a \sqrt[w_b a^{w_a} b^{w_b}]{}.$$  \hspace{1cm} (D.2)

Thus the general case is

$$ f = \prod_{i=1}^{n} y_i^{m_i},$$ \hspace{1cm} (D.3)

where

$$ \sum_{i=1}^{n} m_i = 1,$$ \hspace{1cm} (D.4)

When values of “weighted geometric means” are quoted, this is what has been calculated.

The error is calculated from the general formula,

$$ df^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2.$$ \hspace{1cm} (D.5)
In this case, the function $f$ is given by equation D.3. So

$$df^2 = \left( \frac{dH}{H} dH \right)^2 + \ldots$$

(D.6)

$$= \sum_{i=1..n} \left( \frac{dH}{H} dH \right)^2$$

(D.7)

$$\frac{df^2}{f^2} = \sum_{i=1..n} \left( \frac{dH}{H} dH \right)^2$$

(D.8)

I carried out five simple tests as a check of the above; the results agree with what I intuitively would have expected.

<table>
<thead>
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<th>The test</th>
<th>The result</th>
</tr>
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<tr>
<td>2 of 50 ± 10</td>
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</tr>
<tr>
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<td>50 ± 4.47</td>
</tr>
<tr>
<td>10 of 50 ± 10</td>
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<tr>
<td>50 ± 10 and 100 ± 20</td>
<td>70.7 ± 10</td>
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</table>

REFERENCES


Birkinshaw, M. 1999, Phys. Rep., 310, 97


Cavaliere, A., Dallase, L., & de Zotti, G., eds. 1978, Hubble constant from X-ray and microwave observations of clusters of galaxies.


REFERENCES

Cousins, A. W. J. 1976, MmRAS, 81, 25
Dillon, N. 1987, PhD thesis, University of Cambridge, UK
Djorgovski, S. 1985, PASP, 97, 1119
Giovanini, G., Tordi, M., & Feretti, L. 1999, New Astronomy, 4, 141

REFERENCES

Hubble, E. 1929, Proc. NAS, 15, 168
IAU. 2000, New Cosmological Data and the Values of the Fundamental Parameters. Symposium no. 201. (International Astronomical Union.)
Jones, M. 1990, PhD thesis, University of Cambridge, UK
REFERENCES

—. 1998, Galaxy Evolution, 1st edn. (Springer)

Maggi, A. 1996, Part III project, University of Cambridge, UK


Mattig, W. 1958, Astronomische Nachrichten, 284, 109


Refsdal, S. 1964, MRAS, 128, 307


Rusholme, B. 2001, PhD thesis, University of Cambridge, UK

Sarazin, C. L. 1988, X-ray emission from clusters of galaxies (Cambridge University Press)


—. 1972, Comments Astrophys. Space Phys., 4, 173


VLA calibration manual. 1990, The VLA calibration manual (NRAO)

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