LATERAL LOADING ON PILES DUE TO SIMULATED EMBANKMENT CONSTRUCTION

by Sarah M. Springman

A dissertation submitted for the degree of
Doctor of Philosophy
Cambridge University

Magdalene College

April 1989
...In memory of my dearest mama,

...to my long suffering father and brothers who have been continually embarrassed by my antics, not least of which was my interest in 'piles',

...and to all my friends who thought that studying for a doctorate was an excuse for being either an eternal student or an itinerant triathlete.
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ACKNOWLEDGEMENTS

I am deeply indebted to so many people that it is quite impossible to name them all here. I am grateful, in particular, to my two supervisors.

Mark Randolph encouraged and inspired me during the early stages of my work before escaping to Australia.

Thereafter, Malcolm Bolton gave me endless patient and enthusiastic guidance combined with valuable insight during the second phase of the research and the writing of this dissertation.

Andrew and Margaret Schofield have supported all my endeavours in so many different ways. I am indeed fortunate to have been able to join and study with the Cambridge Soil Mechanics Group.

Often the most tangible evidence of any research programme springs from the inspired craftsmanship of the technical staff. I am beholden to the following for work above and beyond the call of duty:

Chris Collison for his intensely practical efficiency,
Steve Chandler for his delicate, yet robust, strain gauging,
Jim Doherty for his infinite capacity for taking pains,
Bill Balodis for teaching me how to FLY...sometimes,
Wally Gwizdala for his cheerful skill with lathe and developer alike,
Charlie Potter for his transportational wizardry,
Ralph Ward for his steadfast generalship of the 'Q' lines,

and last but not least to Arthur Timbs, Roy Julian and Dave Pittock from the design and workshop team, and to Dave Harris, Pete Clarkson, Les Brown, Dorothy Allen, Sue Lewis and Dave Carribine from the computing group.

I have greatly enjoyed many discussions on my work with members of the staff and my colleagues, notably Arul Britto, Ryan Phillips, Richard Dean, Ian Nunez, Jimmy James, Jeffery Lewins, Wing Sun, Douglas Stewart, Mahetharan, Rev Wells and particularly Trish Hensley, with whom I burnt the midnight oil on many an occasion while engaged on this study.

Equally, a more useful focus would have been difficult to achieve without the close liaison of Myles O'Reilly and Ian Symons from the TRRL Ground Engineering Division, who also funded this work.

During my visits to Australasia, I was privileged to work at the University of Canterbury with John Berrill, Rob Davis and Graham Fairless and at the University of Western Australia with Richard Jewell, Mark Randolph, Martin Fahery and David Airey. I am also grateful for the advice and interest from many of my fellow geotechnical engineers.

Finally, I am indebted to my friends, Felix, Chris and Jess without whom I would still be finishing figures, typing corrections and collating bits of paper, and to Gill, Merle, Jo, Janet and her team.

I certify that, except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted to any other university. This thesis is the result of my own work and contains nothing which is the outcome of work done in collaboration.

Sarah Springman, Magdalene College, 1989
Lateral loading of piles due to embankment construction

Sarah Springman

ABSTRACT

The objective of this research work was to gain a better understanding of the interaction between loads applied on the surface of soils, and the behaviour of adjacent long, vertical piles, embedded at depth in a stiff substratum. The influence of lateral thrust on the piles in an upper soft clay layer, due to simulated embankment construction, was examined and soil–pile interaction mechanisms were identified both for behaviour at working load and at the ultimate lateral capacity.

The performance of a row of free headed piles and of a pile group were investigated experimentally in the geotechnical centrifuge for different pile and foundation geometries. Pile response, in terms of bending moment, deflection and lateral pressure, was determined for surcharge loads applied to the centrifuge model.

This experimental database was used to calibrate a three dimensional finite element analysis of the same, simplified, model. These investigations led to the development of an approximate formula for lateral loading, based on the differential movement between the piles and the surrounding soil, which accounted for pile spacing, relative pile–soil stiffness and the degree of soil strength mobilisation. This loading function was incorporated in a computer program, SIMPLE, which calculated the pile bending moment and deflection profiles for flexible piles and pile groups. The algorithm was checked against the centrifuge model test results and the numerical analyses, and design charts were produced for free headed piles only. Finally a design procedure was recommended for piled full–height bridge abutments and other facilities which feature passive lateral loading of piles by a nearby surcharge.
### SYMBOLS AND ABBREVIATIONS

**English**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMT</td>
<td>bending moment transducer</td>
</tr>
<tr>
<td>B</td>
<td>half–width of rectangular surface load</td>
</tr>
<tr>
<td>$c_a$</td>
<td>mobilised undrained shear strength in the active zone</td>
</tr>
<tr>
<td>$c_{mob}$</td>
<td>mobilised undrained shear strength</td>
</tr>
<tr>
<td>$c_p$</td>
<td>mobilised undrained shear strength in the passive zone</td>
</tr>
<tr>
<td>$c_u$</td>
<td>undrained shear strength</td>
</tr>
<tr>
<td>$c_v$</td>
<td>coefficient of consolidation</td>
</tr>
<tr>
<td>$D_r$</td>
<td>relative density</td>
</tr>
<tr>
<td>d</td>
<td>external pile diameter</td>
</tr>
<tr>
<td>$d_t$</td>
<td>distance between toe of embankment and front face of pile</td>
</tr>
<tr>
<td>$d_v$</td>
<td>diameter of vane</td>
</tr>
<tr>
<td>E</td>
<td>Young's Modulus of pile material</td>
</tr>
<tr>
<td>$E_p$</td>
<td>equivalent Young's Modulus of pile</td>
</tr>
<tr>
<td>$E_s$</td>
<td>elastic soil modulus</td>
</tr>
<tr>
<td>$E_w$</td>
<td>equivalent elastic pile wall modulus</td>
</tr>
<tr>
<td>ERS</td>
<td>electrical resistance strain gauge</td>
</tr>
<tr>
<td>E–L</td>
<td>extensometer</td>
</tr>
<tr>
<td>$e$</td>
<td>voids ratio (Chapter 3)</td>
</tr>
<tr>
<td>$e$</td>
<td>freestanding length of pile above mudline (Chapters 6 &amp; 7)</td>
</tr>
<tr>
<td>$e_{max}$</td>
<td>maximum voids ratio, $D_r = 0%$</td>
</tr>
<tr>
<td>$e_{min}$</td>
<td>minimum voids ratio, $D_r = 100%$</td>
</tr>
<tr>
<td>F, f</td>
<td>factors</td>
</tr>
<tr>
<td>$f_s$</td>
<td>skin friction between pile and soil</td>
</tr>
<tr>
<td>G</td>
<td>shear modulus at depth, y</td>
</tr>
<tr>
<td>$G_c$</td>
<td>characteristic shear modulus of stiff layer where, $G_c = f(G_0, m, v, \ell_c)$</td>
</tr>
<tr>
<td>m</td>
<td>gradient of shear modulus with depth, $m = dG/dy$</td>
</tr>
</tbody>
</table>
Symbols and abbreviations

$G_o$ : shear modulus at top of stiff layer
$G_{p0}$ : shear modulus via self boring pressuremeter tests at 0% volumetric strain
$G_{p2}$ : shear modulus via self boring pressuremeter tests at 2% volumetric strain
$G_{p5}$ : shear modulus via self boring pressuremeter tests at 5% volumetric strain
$G_s$ : specific gravity of soil particles
$G^*$ : shear modulus adapted to account for Poisson's ratio, $G(1+3/4v)$
g : acceleration due to earth's gravity, 9.81 m/s$^2$
g(y) : free soil displacement with depth (Chapter 2)
g$_{max}$ : maximum value of free soil displacement (Chapter 2)
H : total shear force distribution in pile
$H_e$ : height of embankment
$H_{pc}$ : additional shear force applied to pile at pile cap level
$H_s$ : shear force in pile at top of stiff layer
h : depth of lateral pressure applied to pile in the soft layer
$h_e$ : length of pile from top of stiff layer to top of freestanding section, $h_e = h_s + e$
h$_s$ : depth of soft layer
$h_u$ : unloaded length of pile in soft layer, $h_u = h_s - h$
I : second moment of area of a single pile, diameter d
$I_p$ : second moment of area of the row of piles, diameter d
$I_s$ : second moment of area of soil between piles, width d
$I_w$ : second moment of area of pile wall, width d, length 200 mm (centrifuge model scale)
$K_p$ : passive soil pressure coefficient
$K_r$ : pile flexibility factor
k : stiffness
LVDT : linear variable differential transformer
$l$ : length of pile in stiffer substratum
$l_c$ : critical length of pile in stiffer substratum for lateral loading, $l_c = f (G_c, r, E_p)$
M : bending moment distribution
$M_p$ : plastic pile bending moment
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_s$</td>
<td>bending moment at top of stiff layer</td>
</tr>
<tr>
<td>$N$</td>
<td>blow count in standard penetration test</td>
</tr>
<tr>
<td>$N_c$</td>
<td>bearing capacity factor</td>
</tr>
<tr>
<td>$n$</td>
<td>model scale factor</td>
</tr>
<tr>
<td>$n_p$</td>
<td>number of piles</td>
</tr>
<tr>
<td>OCR</td>
<td>overconsolidation ratio</td>
</tr>
<tr>
<td>PI</td>
<td>plasticity index</td>
</tr>
<tr>
<td>PPT</td>
<td>pore pressure transducer</td>
</tr>
<tr>
<td>$P$</td>
<td>total shear force in a pile per unit length</td>
</tr>
<tr>
<td>$P_\tau$</td>
<td>shear force distribution on a pile per unit length due to friction between pile and soil</td>
</tr>
<tr>
<td>$P_\sigma$</td>
<td>shear force distribution on a pile per unit length due to frontal resistance on pile</td>
</tr>
<tr>
<td>$p$</td>
<td>net lateral pressure acting on pile</td>
</tr>
<tr>
<td>$p'$</td>
<td>mean effective stress</td>
</tr>
<tr>
<td>$p_a$</td>
<td>atmospheric pressure</td>
</tr>
<tr>
<td>$p_c$</td>
<td>characteristic lateral pressure acting on a pile</td>
</tr>
<tr>
<td>$p_{ci}$</td>
<td>component of lateral pressure due to the i'th load</td>
</tr>
<tr>
<td>$p_f$</td>
<td>lateral pressure on the front pile in a group</td>
</tr>
<tr>
<td>$p_m$</td>
<td>maximum value of applied (parabolic) lateral pressure</td>
</tr>
<tr>
<td>$p_r$</td>
<td>lateral pressure on the rear pile in a group</td>
</tr>
<tr>
<td>$p_u$</td>
<td>ultimate lateral pile pressure</td>
</tr>
<tr>
<td>$q$</td>
<td>equivalent vertical uniform load for embankment simulation</td>
</tr>
<tr>
<td>$q_c$</td>
<td>measured cone resistance</td>
</tr>
<tr>
<td>$q_{crit}$</td>
<td>simulated embankment load after which elastic solutions for the foundation become inapplicable</td>
</tr>
<tr>
<td>$q_{max}$</td>
<td>maximum simulated embankment load</td>
</tr>
<tr>
<td>$r$</td>
<td>external radius of pile</td>
</tr>
<tr>
<td>SWG</td>
<td>steel wire gauge, thickness of wire/plate</td>
</tr>
<tr>
<td>$s$</td>
<td>pile spacing</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$u$</td>
<td>lateral deflection</td>
</tr>
</tbody>
</table>
Symbols and abbreviations

$u_i$: component $i$, of deflection
$u_{i1}$: corrected value of $u_i$ after pile group effects accounted for
$u_o$: deflection at ground surface, $y = 0$
$u_{pc}$: deflection at pile cap level
$u_s$: deflection at the top of the stiff layer
$v$: specific volume
$w$: moisture content
$X$: $x$ coordinate of the centreline of the single row of piles
$x$: coordinate defining longitudinal horizontal geometry
$y$: depth measured vertically downwards from top surface of the soil
$z$: coordinate defining transverse horizontal geometry

Greek

$\alpha_{ij}$: pile group interaction factors between $i$'th and $j$'th piles
$\alpha_{uH}$: pile group interaction factor for increase in deflection due to
neighbouring piles for a free headed pile under lateral load
$\alpha_{uM}$: pile group interaction factor for increase in deflection due to
neighbouring piles for a free headed pile under moment loading
$\alpha_{\theta H}$: pile group interaction factor for increase in rotation due to
neighbouring piles for a free headed pile under lateral load
$\alpha_{\theta M}$: pile group interaction factor for increase in rotation due to
neighbouring piles for a free headed pile under moment loading
$\alpha_{uf}$: pile group interaction factor for increase in deflection due to
neighbouring piles for a fixed headed pile due to lateral load
$\beta_c$: load description factor
$\Delta \sigma_n$: maximum horizontal stress change in soil due to lateral loading at $z = 0$, $x = X \pm r$
$\Delta \sigma_{nz}$: horizontal stress change in soil at soil–pile interface due to lateral loading, $-r < z < r$
$\delta u_p$: lateral pile displacement
$\delta u_s$: lateral soil displacement at centreline of piles with no pile present
Symbols and abbreviations

\[ \delta_{v_e} \] : vertical displacement due to external work
\[ \varepsilon_h \] : strain in the horizontal direction
\[ \varepsilon_v \] : strain in the vertical direction
\[ \varphi \] : angle of departure of pile loading from orientation to neighbouring pile
\[ \gamma \] : shear strain
\[ \gamma_d \] : dry unit weight of soil
\[ \gamma_c \] : bulk unit weight of embankment
\[ \gamma_s \] : saturated unit weight of soil
\[ \gamma_w \] : unit weight of water
\[ \Psi, \psi \] : factors describing length to depth ratio of deforming soft soil
\[ \mu \] : coefficient of friction
\[ v \] : Poisson's ratio
\[ \theta \] : rotation profile
\[ \theta_i \] : rotation of pile due to component i, of the loading in the soft layer
\[ \theta_o \] : rotation of pile at ground surface, \( y = 0 \)
\[ \theta_{pc} \] : rotation of pile at pile cap level
\[ \theta_s \] : rotation of pile at the top of the stiff layer
\[ \rho_c \] : factor relating homogeneity of stiffer substratum shear modulus
\[ \sigma_a \] : horizontal stress in soil under the embankment in the active zone, \( x > X \)
\[ \sigma_h, \sigma_h' \] : total and effective horizontal stress
\[ \sigma_p \] : horizontal stress in soil in passive zone, \( x < X \)
\[ \sigma_{rr} \] : radial component of horizontal stress
\[ \sigma_v, \sigma_v' \] : total and effective vertical stress
\[ \sigma_{xx} \] : horizontal stress acting in x direction
\[ \sigma_{xXn} \] : horizontal stress along \( x = X \) when there are no piles
\[ \sigma_{xxp} \] : horizontal stress at \( x = X, z = s/2 \) when piles are present
\[ \sigma_y \] : yield strength of pile material
\[ \sigma_{yy} \] : vertical stress acting in y direction
\[ \sigma_{zz} \] : horizontal stress acting in z direction
Symbols and abbreviations

\(\tau_0\) : shear stress on circumference of pile
\(\tau_{xy}\) : shear stress acting in xy plane
\(\tau_{yz}\) : shear stress acting in yz plane
\(\tau_{zx}\) : shear stress acting in zx plane

Subscripts

a : active
b : beginning of analysis
c : centrifuge
f : front (first letter) or final value (third letter)
h : factor due to shear force
i : i’th variable
m : factor due to bending moment
max : maximum
min : minimum
o : at \(y = 0\)
p : prototype (Chapters 3 & 4)
p : pile or passive (Chapters 5-7)
pc : at pile cap
r : rear
s : soil or interface between soft and stiff layer
u : denoting parameters in unloaded section of pile at base of clay layer (first letter)
u : factor due to deflection (second letter)
x,y,z : coordinate directions
\(\theta\) : factor due to rotation
1 INTRODUCTION

1.1 Background

Current British design guides for piled full-height bridge abutments lack detailed recommendations for evaluating the lateral thrust on piles caused by construction of the approach embankment. Fig: 1.1 illustrates the problem, which is compounded when there is a soft clay layer overlying a stiffer substratum. During and after construction, vertical settlement in the soft layer is accompanied by horizontal movements under the edge of the embankment, which cause lateral thrusts and hence bending moments in the piles, together with deflection and rotation of the abutment itself. Without appreciating these factors, design of both the foundation and the superstructure may be severely compromised.

Past practice has been either to suffer the consequences of damage to the bridge superstructure and abutments or to overdesign the piles to cope with this active lateral loading without examining the mechanisms controlling the surcharge–soil–pile interaction. However, the emergence of increasingly more complex projects, combined with even greater cost consciousness, has outdated these previous empirical design methods. A straightforward design procedure is needed, based on fundamental knowledge and understanding of the soil–pile interaction.

The behaviour of the embankment, soft soil and piles are closely inter-related. The embankment is necessary to provide access to the bridge deck, but construction poses several design problems, most notably foundation bearing failure and slope instability. The factor of safety against such occurrences will control the degree of immediate, undrained, shearing deformation of the foundation, and the impingement of this on the pile behaviour. Generally piles are required to bear the bridge loads and to transmit them through the soft soil to a more competent stratum below. These piles must be in place before the full-height abutment wall is built, and only then can the embankment be brought up to level behind it. The placing of this surcharge forces the soft soil layer to press against, or squeeze between, the piles, inducing bending moments and shear forces in the pile. This interaction is very poorly understood at present.
Introduction

Finally, the resulting pile head deflection and rotation will affect the bridge superstructure. A major consideration is the extent of total and differential abutment movements and the ability of the bridge to tolerate these. U.S. Department of Transportation (1985) observe that horizontal abutment movements in excess of 25–50 mm are more damaging than differential vertical settlements up to 100 mm in causing most types of structural distress. Piled foundations do not guarantee that these movements will be within acceptable limits, and structures which include full–height abutments are likely to have a higher occurrence of abutment damage although the incidence of superstructure distress and the impairment of bearings will be less. A common situation (Figs: 1.1a & b) arises when a soft fluvial deposit overlies a denser layer, into which piles can be bored or driven. It is this foundation configuration which will be examined herein. Although raking piles may be required to carry a component of horizontal thrust from the abutment, the flow of soil around the pile will not be dissimilar to that around a vertical pile, and so it was considered sufficient in the present research to adopt this simplification.

1.2 Research options

Earlier research in this area was carried out by Heyman & Boersma (1961), Leussink & Wenz (1969), and De Beer & Wallays (1972). The Laboratoire Centrale de Ponts et Chausées in France have since conducted a field study of a sand embankment with a pile at the toe (Bigot, Bourges & Frank, 1982) combined with finite element and elastic reaction modulus analyses. But the design recommendations remain largely empirical. Those existing analytical methods for laterally loaded piles which may be relevant are reviewed in Chapter 2, and some of these were adapted for further development.

Previous research workers with analogous problems have used physical models or full scale field tests which have been calibrated by numerical analyses using finite or boundary element techniques. Behavioural mechanisms were revealed and a design procedure was proposed. To date there has been no full scale field study of the particular problem modelled here, mainly due to the additional constraints of construction within tight budgets and schedules. Since field conditions vary widely, it is sometimes better to have a controlled study in which foundation
geometry, pile spacing and group effects may be varied within the framework of a known stress history and more accurate estimates of soil strength.

While this can be done at model scale in the conventional laboratory, where testing is quick and relatively cheap, there are inescapable problems. Soil behaviour is stress dependent and it is not possible to account properly for the in–situ stress conditions, despite various techniques which include surcharge loading the sample. Therefore, non–linear soil–pile interaction is incorrectly modelled.

Centrifugal modelling, in which miniature replicas simulate the same stress–strain conditions as a full scale prototype, is a more attractive research tool. Schofield (1976) discusses the advantages of this technique in which small 1/n scale samples may be rotated at sufficient speed in a centrifuge to impose an enhanced "gravity" field, ng, which recreates field conditions of stress and strain (Fig: 1.2). When dealing with the interaction of the complex strain fields surrounding a pile which is actively loaded by soil, it is vital to achieve correct modelling similarity. In addition, consolidation time is reduced by n² with the attendant advantage of saving time and money.

There are many other benefits of centrifuge model testing. Soil models with similar properties, strengths and stress histories may be reproduced and greater control may be exerted over the entire test. Choice of instrumentation is not limited by expense, site access or contractual activities, and the sizing of piles and geometry of the foundation may be predetermined to suit the problem under investigation. Models may be prepared and tested over a 3–4 week cycle at considerably lower cost than in the field, and they may be tested to the limit without the full scale consequences of danger to life and facilities.

Appropriate adjustments may be made, in stages, to the design of the centrifuge model tests to isolate particular areas of interest in a complex three dimensional problem, so that a complete understanding of the mechanisms may be reached. Inevitably there are some disadvantages, and the errors induced by testing in this way are discussed later in Chapter 3.
Introduction

1.3 Aims and objectives

The foremost aim of the research reported in this dissertation was to study the surcharge–soil–pile interaction for thrust on piles in soft clay overlying a stiffer substratum, due to nearby embankment construction. This demanded identification of the behavioural mechanisms at both the expected working load level and at failure, either for the pile response in terms of maximum applied bending moments and shear forces, or for the soil by continuous movement past the piles.

The principal objectives of this research were:

i) to design and conduct centrifuge model tests (Chapter 3), reporting the results (Chapter 4), and giving scaled–up data for an equivalent prototype with various foundation geometries, pile numbers, spacing and head fixities, by applying loading in increments up to working levels and then to failure,

ii) to calibrate the centrifuge model test performance by equivalent site specific finite element analysis (Chapter 5) and to yield more information about the surcharge–soil–pile interaction,

iii) to develop a theoretical approach (Chapter 6) which could be validated by both the centrifuge and numerical data to form the basis of a design procedure for free headed and fixed headed piles,

iv) to encode this theory into a computer program, SIMPLE, for use on a portable personal computer for general design office usage,

v) and ultimately to present a simple, workable design routine which will simplify calculation of the bending moments and deflections induced in piles embedded in this type of foundation by embankment loading adjacent to them (Chapter 7).
2.1 Introduction

There are many ways in which piles subjected to lateral loading may be analysed. As in most geotechnical design processes, the ultimate case must be examined alongside the working load situation when control of deformations takes precedence. Foundation stiffness matrices may also be determined for superstructure response calculations (Elson, 1985). The major analytical approaches will be discussed briefly here, followed by those applied directly to the problem of piles and pile groups subjected to lateral thrusts from the movement of a soft soil layer. These include field studies and case histories, numerical analyses and existing design methods.

2.2 Ultimate pile capacity in lateral loading

2.2.1 Active and passive loading

Lateral loading is described as "active" when it is applied to a pile by an external means, causing the pile to load the soil. "Passive" loading is exerted when movement of the soil subjects the pile to a lateral thrust and the associated bending stresses. In the context of this thesis, (Fig: 1.1), passive loading is applied to the piles in the soft upper layer of cohesive material, whereas active load is applied in the stiffer substratum.

2.2.2 Failure model

The failure mode of a pile subjected to active lateral loading is conditioned by the relative pile–soil stiffness and the length of the pile. For short rigid free headed piles, the pile will rotate about a point of zero deflection (Fig: 2.1a). However, most of the piles considered in this thesis are long enough to behave in the manner shown in Fig: 2.1b, when either a pile hinge will occur at the depth of the maximum bending moment or the soil will fail in front of (active loading) or around the pile (passive loading). Capped–pile failure mechanisms are shown in Figs: 2.1c–e.

2.2.3 The bearing capacity factor, $N_c$

The controlling factor in the present study is the "passive" loading due to the soft cohesive upper stratum of Fig: 1.1, so this review will be concentrated in this region. Of particular interest is the
profile of limiting lateral pressure imposed on the pile and the development of mechanisms associated with failure near the ground surface and at depth. Broms (1964) states that for active loading, a wedge will form in front of the pile up to a depth of approximately 3 pile diameters (3d), opening up a gap behind the pile. Below this level, the soil will flow past the pile, applying a limiting pressure, \( p_u' \), which depends on a bearing capacity factor, \( N_c \), and undrained shear strength, \( c_u' \), so that:

\[
p_u = N_c c_u'
\]

(2.1)

He suggested that the limiting pressure, \( p_u' \), could be taken as (a) \( p_u = 0 \) from ground surface to a depth of 1.5d, and (b) \( p_u = 9c_u' \) below a depth of 1.5d.

2.2.4 Plasticity solutions

Randolph & Houlsby (1984) introduced an exact plasticity solution for the flow of soil around a cylinder, with allowance for limiting friction at the pile–soil interface. They showed that the limiting pressure varied between \( 9.14c_u \leq p_u \leq 11.94c_u' \). This compared with estimates from Poulos & Davis (1980) of \( 7.7c_u \leq p_u \leq 10.85c_u' \), based on using a rhomboid shaped pile of diagonal length, \( d \).

2.2.5 A pressuremeter analogy

Fleming, Weltman, Randolph & Elson (1985) discuss the application of a pressuremeter analogy to the determination of the ultimate lateral capacity as follows. When the ultimate capacity is reached, the soil will flow completely around the pile, but below this upper limit, deformations will occur radially outwards in front of the pile and radially inwards behind it. This deformation mode was observed by Swain (1976) in tests on deeply embedded strip ground anchors. Therefore, the radial deformation field imposed by the pressuremeter might be sufficiently similar to pile behaviour to give an indication of the limiting pressure, \( p_u' \), acting on the pile. If friction on the side of the pile is estimated at \( c_u d \), combined with the limiting pressure from the pressuremeter test, for common values of \( G/c_u \) and making allowance for either a suction of up to minus atmospheric pressure (\(-p_a\)) behind the pile or a gap filled with the ambient head of water, the limiting pressure will lie between:
\[(\sigma_h' + 7c_u) \leq p_u \leq (\sigma_h + p_a + 7c_u)\]

(2.2)

In nearly all calculations of the limiting active lateral pressure on a pile, it is important to consider the lowest ultimate soil resistance available. However, for a pile loaded passively by the soil, the largest ultimate pressure that may be exerted on the pile is critical, for this will define the maximum loading on the pile and hence the maximum pile bending moment. The magnitude of the limiting lateral pressures and the profile adopted will be based on the work by Randolph & Houlsby (1984) and will be discussed in greater detail later (Section 6.5.2).

2.2.6 Viggiani's calculations

But, Viggiani (1981) examines the case of lateral loading on piles used to stabilise landslides. He proposes that \(N_c\) will vary, depending on whether the pile is actively \((6.26 \leq N_c \leq 12.56)\) or passively \((2.8 \leq N_c \leq 4)\) loaded, and presents some case histories to support his argument. However, it is physically inconsistent to assume that the pressure on a pile will vary if the pile is moved through the soil or vice versa. For the stability of potential landslides, the piles must be assumed to carry the minimum conceivable pressure, whereas for problems entailing lateral thrust on piles (Fig: 1.1), the upper bound pressure which will cause the maximum pile bending moment is more important.

Lateral pressure, \(p\), was either deduced by triple differentiation of inclinometer data, or from measurement by pressure cells at two sites on each pile, ignoring frictional resistance around the circumference of the pile, which may be significant (Baguelin, Frank & Said, 1977). Since all these back analyses (Viggiani, 1981) were characterised by failure of the piles through the formation of a hinge, it seems likely that \(p_u\) was not reached because the peak values of \(c_u\) were not mobilised before this happened, or that \(p_u\) was not mobilised at the same time over the entire depth, and that the large movements entailed in landslide flow had caused \(c_u\) to drop to residual values in places. In consequence, the values of \(N_c\) quoted by Viggiani (1981) for passive loading were too low for use in this research work.
2.2.7 Pile groups

The ultimate lateral capacity of a pile group may be considered as the sum of the individual resistances or, for closely spaced piles, the failure of an entire block. Elson (1985) recommends that the single pile approach is relevant for \( s/d > 8 \) where \( s \) is the pile spacing, and that group failure is prevalent for \( s/d < 3 \). Fleming et al (1985), on the other hand allowed the development of \( c_u \) along the sides of the block, stating that for ultimate pile pressure of \( 9c_u d \), block failure will occur at pile spacings less than 4.5d.

2.3 Deformations of laterally loaded piles

2.3.1 Subgrade reaction methods

Winkler (1867) described one of the original ways in which the response of piles to lateral loading was modelled. He replaced the horizontal resistance of the soil by a series of springs of appropriate stiffness, where \( p = ku \), with subgrade reaction modulus, \( k \), and lateral pile movement, \( u \). Analytical solutions were used by Matlock & Reese (1960) to give the bending moment, shear force and deflection down the pile for constant \( k \) with depth (homogeneous foundation). They also suggested a linear variation of \( k \) with depth (non–homogeneous foundation) since the secant shear modulus was generally lower at the ground surface where pile displacement and local strain fields were the greatest, and considerably larger at depth where the strains were less (Reese & Matlock, 1956). For both situations, there is a critical pile length, \( l_c \), dependent upon pile bending rigidity, \( (EI)_p \), and \( k \), below which the lateral loading has no influence.

In layered soils such as the prototype case considered in this dissertation, Davisson & Gill (1963) suggested that the soil between the surface and a depth of a few pile diameters controls the pile behaviour, and that site investigations should be most thorough in this region. Seasonal moisture content variations, surface scour and other hazards which may reduce the stiffness in this critical zone should be considered.

Experience and empirical correlations are readily available for the subgrade reaction method and so it remains popular. Non–linearities in foundation strength, stiffness and composition are
accounted for by adjusting the value or gradient of \( k \) with depth. However, there are a number of disadvantages. Transfer of shear between soil and pile is not possible, and the discrete springs fail to allow for additional displacements caused by stress cycling elsewhere, mitigating against this method for pile group design. Generally, linear elastic soil models are employed, and \( k \) is not a fundamental property of the soil, but relies on subjective assessments and empiricism.

However, lateral pile response is markedly non-linear, and the choice of subgrade reaction modulus is largely dependent on the magnitude of the pile deflection and the applied load level. Reese (1977) describes a series of complete load-transfer curves, \( p-u \), (in the notation used in this dissertation) attributed to specific pile depths, and the finite difference solution of the pile bending equation. These numerical techniques are powerful but convoluted, and the accuracy of the method is highly sensitive to the choice of load-transfer curve. Empirical correlations are generally unsatisfactory, but \( p-u \) values derived from full scale instrumented pile load tests can give better approximation to pile behaviour. Frank (1985) discusses the use of the Menard or self-boring pressuremeter to establish values of subgrade reaction modulus based on correlations between instrumented pile performance and in-situ testing on numerous French sites.

### 2.3.2 Elastic continuum approaches

For more complex problems, the elastic continuum approach (Elson, 1985) offers an internally consistent method, which can be modified to allow for group effects, battered piles and yielding of soil at a specified limit pressure. Soil modulus may be varied with depth to give a complete picture of the pile behaviour. This approach may be particularly useful for cases in which the stress range may be described as elastic and for which an appropriate secant modulus can be selected. It is recommended (Randolph, 1981a; Elson, 1985) for calculations of behaviour at working load provided a reasonable assessment of soil stiffness can be made.

Poulos (1971) describes a method in which the horizontal displacements in an isotropic elastic continuum created by a horizontal point load are calculated according to Mindlin's (1936) solution. The piles are modelled in elements as thin strips of constant stiffness, and the resulting integral equations based on the flexural bending of a thin beam are solved numerically for the
relevant boundary conditions using finite difference techniques. Equations relating pile displacement and rotation to the lateral load or bending moment applied at the head of the pile are written in terms of influence factors which are determined from the numerous design charts presented for different configurations of the pile flexibility factor:

\[ K_r = \frac{(EI)_p}{E_s \ell^4} \]  

(2.3)

and \( \ell/d \), where \((EI)_p\) is pile bending rigidity, \(E_s\) is soil stiffness, \(\ell\) is total pile length. Therefore, the values of \(K_r\) will depend on \(\ell\), and not the significant length of a laterally loaded pile, \(\ell_c\). Overestimating this effective length will unnecessarily reduce the accuracy of the numerical solution because the pile elements will be larger. This may be ameliorated by using smaller elements at the top of the pile, and increasing the size with depth (Evangelista & Viggiani, 1976). A further shortcoming is the neglect of horizontal shear along the sides of the piles.

Banerjee & Driscoll (1976) proposed a development of the boundary element model by adding a uniform distribution of shear stress around the pile for homogeneous soils. Banerjee & Davies (1978) constructed an approximate point force solution for non–homogeneous soils in which soil modulus increases linearly with depth.

A promising solution was described by Randolph (1981a) who used finite elements to model the long pile and soil continuum. The degree of foundation homogeneity is allowed for by any combination of uniform modulus (homogeneous) and linear variation of modulus (non–homogeneous) with depth. The effects of Poisson’s ratio are modelled by a modified shear modulus, and a characteristic modulus, \(G_c\), is ascribed to the pile over the active length of the pile. For an equivalent solid pile of radius, \(r\), Young’s modulus, \(E_p\), and critical pile length, \(\ell_c\):

\[ \ell_c = 2r \left( \frac{E_p}{G_c} \right)^{2/7} \]  

(2.4)

\[ G_c = G_y = \ell_c f/2 \]  

(2.5)
where the terminology, a flexible pile, may be thought of as one for which the relative pile–soil stiffness is small enough so that the pile is longer than the critical length for lateral loading, $\ell_c$. Clearly, this implies zero rotation and deflection below this depth. Stiffer, more rigid piles, however, will rotate about a point of zero deflection at some depth.

Eqns: 2.4, 2.5 may be solved by iteration before input into equations similar to those quoted by Reese & Matlock (1956), Poulos (1971), and Banerjee & Driscoll (1976) relating pile displacements and rotations at ground level to applied head load and bending moment. These equations were obtained by fitting an empirical power law to results from many finite element analyses.

This method was employed for the calculation of pile behaviour in the stiffer substratum, and will be described in greater detail later on. Having conducted the majority of the numerical work in the finite element study, this method was computationally less expensive than those which required solution of a stiffness matrix for each analysis. The savings in time and expense were not prejudiced by loss of accuracy: agreement with the more rigorous of the boundary element methods of Banerjee & Davies, (1978) was good (Randolph, 1981a; Elson, 1985).

An extension of these solutions to allow for the interaction between deformation fields around closely spaced piles, and thus the increase in pile displacement, was proposed by Poulos (1971) for head loading. Randolph (1977) extended this work by considering the relative lateral pile and soil movements, and suggested alternative expressions for interaction factors derived from a finite element study. These will be discussed in Chapter 6 below. The analysis of lateral loading on a pile group was incorporated in a computer program, PIGLET (Randolph, 1983), using the algorithms described in Randolph (1981a), some of which were transferred to computer programs written during this research work.

2.3.3 Comparison with centrifuge model tests

Centrifuge model tests conducted on laterally loaded piles in sand by Barton (1982) confirmed that the elastic solutions obtained using elastic continuum models reproduced the general
characteristics of deflected pile shape and induced bending moment very well. The location of
the maximum bending moment, the point of zero deflection, and the critical pile length were well
predicted, as was the magnitude of pile head displacement. However, the theory underestimated
pile head rotation by 25% and the maximum bending moment by 15% in the suite of tests
reported, and agreement was generally better for moment loading than force loading because local
yielding at the surface of the soil was more prevalent in the latter case.

Barton (1982) suggested that non-linear soil behaviour around closely spaced piles in groups
caused local tensile yielding in the soil. In consequence, elastic analyses underestimated the
interaction and an improved prediction of pile response was obtained by assuming that the load
on each pile was the same as that on the leading pile in each group. At larger pile spacings, the
elastic analysis was found to overestimate the pile interaction as the local soil yielding reduced
the magnitude of the far field displacements. Since elastic interaction is symmetric, each pile
would be expected to carry the same load under elastic loading conditions, but experience shows
that the front pile carries more. For pairs of piles at a spacing of $s/d = 2$, the front pile carried
60% of the applied load (Barton, 1982).

2.4 Passive lateral loading on piles due to surcharge

2.4.1 Analytical methods

There are few analytical methods which may be applied to the problem of passive lateral loading
of piles due to the placement of a nearby surcharge. One of the earlier attempts was presented by
De Beer & Wallays (1972), who suggested an empirical method for estimating the lateral pressure
acting on the pile based on the factor of safety of the embankment against collapse without
allowance for reinforcement from the piles, and the geometry and unit weight of the embankment.
Only the maximum bending moment was calculated, without knowledge of the variation of
moment and displacement with depth. The method is unable to deal with deep soft layers and
makes no allowance for the strength of the soft clay. The analysis was based on field data from
Heyman & Boersma (1961) and Leussink & Wenz (1969) amongst others, which was also
examined by Marche & Lacroix (1972) who found that the ratio of horizontal pile displacement
to settlement of the embankment grew with increasing pile flexibility.
Ito & Matsui (1975) reported a calculation method based on the extrusion of free draining soil between two piles by applying either the theory of plastic deformations or viscous flow. However, this analysis gave extremely high values of pressure, greatly exceeding the limiting pressure calculated by other means. As expected from the original assumptions, as pile spacing approached zero, the forces acting on the pile became infinite.

Marché (1974) introduced a more fundamental approach by extending the analyses introduced earlier (Poulos, 1971) to account for displacement compatibility between the pile and the surrounding soil. Application of the embankment load would cause a "free" lateral soil displacement if there were no piles present. Young's modulus for the soil and a limiting value of lateral pressure between soil and pile, both of which could vary with depth, were allocated and the equation of flexure for a thin beam was solved as before. However, the problems entailed in this analytical method, which were discussed earlier (Section 2.3.2), were still relevant and were combined with difficulties in choosing soil displacement and limiting pressure.

Bourges & Mieussens (1979), Bourges, Frank & Mieussens (1980) and Frank (1981, 1983) describe the Laboratoire Centrale de Ponts et Chausées (LCPC) approach that also uses the concept of a free soil displacement, g, varying with depth, \( g(y) \). Both the LCPC and the Poulos method are based on a relationship between lateral pressure at the pile–soil interface and the differential movement between the pile and the soil. However, the LCPC approach uses the subgrade reaction method in a non-linear form in the solution of the bending equation, for the appropriate boundary conditions, by assigning a modulus of subgrade reaction which can be related empirically to results from pressuremeter tests (Frank, 1985). For a constant value of soil modulus, solution of the equation is straightforward, but for non-linear analysis a computer program PILATE (Baguelin, Frank & Guegan, 1976; Frank, 1981, 1984) was written. The soil was divided into layers and the governing equation solved for continuity of displacement, slope, bending moment and shear force between layers.

The accuracy of this method is highly dependent on the selection of \( g(y) \) and this can be determined from an inclinometer on site for the particular embankment loading, at a distance far
Background theory and literature review

enough removed from the influence of any piles, but in the same location relative to the embankment. Bourges & Mieussens (1979) describe an alternative statistical evaluation of \( g(y)/g_{\text{max}} \) from data of a large number of instrumented embankments on sites around France, and recommend profiles depending on the previous stress history of the site.

However, Frank (1981) comments that further research is required on both the interpretation of the pressuremeter tests to give an appropriate reaction law, and likewise the determination of \( g(y) \). The differential movement between pile and soil is generally a small value, and so gross errors may be induced if a reasonable estimate of the individual components is not possible or the soil profiles are not compatible. If the analysis can be calibrated by results from pressuremeter tests and inclinometer readings, and PILATE is available, this method may give reasonable results for single piles. Currently, the adaptation of \( g(y) \) to account for interaction effects between piles has not been reported.

2.4.2 A case history

A case history of a site in Provins was described by Bigot, Bourges & Frank (1982). An instrumented pile was installed and loaded laterally. The measured bending moment and pile displacement profiles are shown in Figs: 2.2a, 2.2b together with the loading configuration and values calculated using the subgrade reaction method based on reaction laws derived from nearby Menard (1955) and self boring (PAF) pressuremeter tests. Then an embankment was built in stages (Fig: 2.3), with a slope of 1 on 2. The pile was located at the toe. Measurements were taken of the lateral free soil and pile displacements and the pile bending moment immediately after embankment construction, and these were compared with the values calculated using the subgrade reaction modulus determined from the different type of pressuremeter tests. The bending moment and displacement were in quite good agreement for the correlations with modulus of subgrade reaction obtained from the self boring (PAF) pressuremeter results (Figs: 2.4a & b).
2.4.3 Finite element analyses

2.4.3.1 Introduction

The more rigorous analytical approaches use the finite element method to discretise the problem and apply an appropriate stress–strain law to the elements. In some cases, mechanisms are defined and new behavioural models are discovered. Based on observations from the calculated results, generalised parametric studies may be carried out, from which design charts are produced. Occasionally, site specific analyses are implemented. Generally these are more expensive and time consuming, but, provided the mesh is sufficiently fine and the soil model is appropriate for the analysis intended, excellent results may be obtained.

2.4.3.2 Plane strain models

Randolph (1981b) and Naylor (1982) described a plane strain adaptation of a row of piles which attempted to account for the number of piles and the spacing between them, by replacing them with a sheet pile wall of equivalent bending rigidity. Naylor (1982) considered plane strain and included link elements at the pile–soil interface to permit relative movement between the nodes at this boundary. He observed minimal soil squeeze–through between the piles, justifying the wall analogy. However Springman & Randolph (1985) commented that the prediction of individual pile bending moments and displacements using this method did not compare well with results from centrifuge model tests.

In particular, the Naylor (1982) approach was not supported by work described by Baguelin, Frank & Said (1977) who conducted plane strain finite element analyses on a section through a pile, allowing for either isotropic elastic soil or a remoulded cylinder of soil around the pile with a constant lower stiffness. This analysis drew attention to the two components of lateral pressure, \( p \), acting on the pile (Fig: 2.5a), due to the friction (Fig: 2.5b) and the frontal reaction (Fig: 2.5c) on the pile–soil interface. The pile–soil interaction was an overriding factor in determining the pile response, and the lateral pressure acting on the pile was dependent on the relative displacement between the pile, \( \delta u_p \), and soil, \( \delta u_s \). During the early stages of loading before \( p \) reached \( 2\pi c_u \) (Fleming et al, 1985), the relationship between \( p \) and \( \delta u_s - \delta u_p \) was linear. Considerable reference is made to this work (Baguelin et al, 1977) during this dissertation and the
Background theory and literature review

theory is described further in Chapters 5 and 6 below.

Briaud, Smith & Tucker (1985) describe a laterally loaded pile test (Fig: 2.6a & b) in which pressure cells were inserted in the face of the pile at locations A, B, and C, (Fig: 2.6c) to measure the frontal reaction curve across the pile diameter. The data confirmed the theory initially proposed by Baguelin et al (1977), (Fig: 2.6c). Agreement between the predicted and measured frontal reaction was good. Pressuremeter tests were conducted near to the pile test, and the results from the pressuremeter expansion curve (Fig: 2.7a) were successfully used to predict the frontal reaction (Fig: 2.7b). On the basis of these results and those from Baguelin et al (1977), it is noted that the limiting value of frictional resistance is developed at lower lateral pile displacements, typically around \( u/d \approx 1\% \) (Fig: 2.7c), whereas the frontal reaction requires \( u/d \approx 4\% \) before approaching full resistance.

2.4.3.3 Axisymmetric modelling

Carter (1981) employed an existing numerical method to solve a three dimensional problem of unsymmetric loading near a single pile in an elastic soil. He considered both the socketed and the floating pile in a homogeneous stratum and presented solutions for both cases from semi-analytical techniques. The complex three dimensional analysis was reduced to one of axisymmetry for a single pile using Fourier expansions, and this was solved by the finite element method. This approach was limited to single piles in elastic soil, with no allowance for layering.

2.4.3.4 Specific site studies

Magnan, Lepidas & Frank (1987) reported a finite element analysis conducted on a prototype sand embankment with no rigidity, imposing a normal load of 60 kPa on a foundation of depth 10 m, with the intention of calibrating the assumptions made about free soil displacement, \( g(y) \) in the solution of the LCPC reaction modulus analysis. A single pile, encastre at the base of the soft layer, was positioned at the toe of the slope in which the normal load had decayed linearly to zero over a horizontal distance of 4 m. A soil model (MELANIE) developed by LCPC since 1980 was allocated to the foundation and a parametric plane strain finite element study was carried out to establish the effect of variations of these parameters on the relationship \( g(y)/g_{\max} \). The value of
$g_{\text{max}}$ was determined by reference to the ratio of $g_{\text{max}}/s_{\text{max}}$ derived from empirical relationships, where $s$ was the vertical settlement.

These geometric and soil parameters were adjusted to discover the effect on $g(y)/g_{\text{max}}$ and the greatest variation of $g(y)/g_{\text{max}}$ from the average value was 10%. When contrasted with the profiles of $g(y)/g_{\text{max}}$ reported by Bourges & Mieussens (1979), the average curve lay between the extremes for an overconsolidated and a normally consolidated stratum. Thereafter, the bending moments and shear forces were calculated using PILATE for different pile slenderness ratios. The analysis confirmed that the choice of free soil displacement was important when this reaction modulus approach was employed, and that the most significant factor in determining $g(y)$ was the value of the foundation modulus.

Siriwardene, Moulton & Chen (1985) executed a combination of parametric and site specific finite element studies on perched, piled bridge abutments. They noted that for abutments built on deep soft compressible foundations the most frequent deformation mechanism was a combination of rotation and lateral movement away from the bridge superstructure. The most important factors were the relative stiffness between the embankment and the foundation, the depth of compressible foundation compared to the embankment height, and the type of pile support.

This research was sponsored by the US Federal Highways Authority whose findings on tolerable movement of highway bridges were reported by US Department of Transportation (1985). They found that while many bridges can suffer significant total and differential vertical settlements (up to 100 mm) without undue distress, horizontal abutment movements should be less than 25 mm to prevent damage to the abutment, superstructure and bearings. These limits provided a useful guide to an appropriate serviceability criteria in the design of piled bridge abutments.
3 CENTRIFUGE MODELLING – EQUIPMENT AND EXPERIMENTAL PROCEDURES

3.1 Introduction

3.1.1 Aim

The aim of the centrifuge model tests was to replicate field behaviour in evaluating the effects of lateral thrust on piles and pile groups when an embankment was constructed nearby. An experimental parametric study was conducted and the results are described in Chapter 4 below. The centrifuge data are compared with numerical analyses and proposed design procedures in Chapters 5 and 6. In this chapter, the theory of centrifuge scale modelling is briefly reviewed and the equipment and experimental procedures used on these centrifuge tests are described.

3.1.2 Scaling relationships in centrifuge modelling

The 10m balanced beam centrifuge (Fig: 3.1) was used for a series of tests on 1/100 scale models of idealised prototypes. Schofield (1980) describes the advantages of this approach over full scale testing. It is essential in modelling the behaviour of soil that the stress state in both model and prototype is the same. Centrifugal modelling offers this facility in addition to the advantages of smaller size, ease of management and control, shorter time scale, lower costs and the option to continue the test to failure. The disadvantages with respect to incorrect modelling of particle size, size problems relating to the instrumentation and other errors due to the distortion in acceleration in a radial field (Schofield, 1980) are discussed in Section 3.3.1 below.

The scaling relationships (Table: 3.1) illustrate the principles of centrifugal modelling by showing that stress and strain are similar in model and prototype. In contrast, time (diffusion) is reduced by a factor of 1/(100)^2 (Taylor, 1984); this allows 27 years of prototype diffusion to be modelled in 27 x 365 + 100^2 \approx 1 \text{ day}.

3.2 Testing programme

Centrifuge model tests were conducted using:

i) a single row of free headed piles (Fig: 3.2a) during the first phase of the investigation (tests 3–7),
ii) a capped pile group with two rows of piles (Fig: 3.2b) during phase two (tests 8–10).

The geometries of all models were the same with the exception of pile spacing and the composition of the foundation. The experimental methods, proprietary names and design of some equipment are described in detail by Springman, (1987a & 1987b). The main features are summarised below and are tabulated in Table: 3.2.

A reference to model tests 1 and 2, which were carried out during earlier research (Springman, 1984), is included in Table: 3.2. In these tests, the embankment loading was simulated by a sand embankment which was poured in-flight using the hopper arrangement described by Davies (1981), Horner (1982) and Almeida (1984). However, it was difficult to determine the loading characteristics of the sand embankment and to control the placement of the sand, which spilt through between the piles (Fig: 3.3). Therefore a simplified loading system was adopted for tests 3–10, which approximated a uniform normal load, and only these results are discussed here.

Consolidation of the sample commenced 2–3 weeks before the testing period. The test period itself covered five days for model making, package assembly, loading onto the centrifuge arm, the test run, unloading, post-test site investigations and X-ray analysis.

3.3 Equipment and instrumentation

The general arrangement of the models during testing was as shown in Figs: 3.2a, 3.2b. The strongbox, vane shear and cone penetrometer apparatuses were the same as those used by Almeida (1984) in his investigations of sand embankments constructed in stages on soft clay. In the series of tests reported here, the foundations were similar to those of Almeida (1984) but the loading was applied by an air-filled rubber pressure bag.

The piles were externally strain gauged for the purpose of measuring bending moments. Signals from the strain gauges were passed through half bridge circuits on two instrumented free headed piles, but these signals were multiplexed for the pile group tests to provide an extra 21 channels, and to allow 4 of the 6 piles in the group to be instrumented. Pore pressure transducers, linear variable differential transformers and the site investigation load cells and potentiometers completed
the array of instrumentation.

The model piles were installed at 1g before the loading apparatus and site investigation gear was assembled. This did not replicate the exact stress–strain conditions experienced in the field (Craig, 1983). However, using the same installation procedure, Barton (1982) achieved good agreement when using different centrifugal accelerations, pile sizes and foundation geometries to model one prototype. Craig (1985) reports variations approaching 50% in measured axial capacity of single displacement model piles pushed into medium dense sound at either 1g, or a low centrifugal acceleration, and tested under a higher gravity field, whereas lateral capacity varied by less than 10% for piles installed at 1g and loaded at 52g in the centrifuge. During lateral loading, the region of high strain at the top of the pile governs pile behaviour and there is less variation between stresses at installation at 1g and under enhanced gravity, whereas axial capacity is more dependent on soil conditions at depth. Nunez, Phillips, Randolph & Wesselink, (1988) suggest that a more important factor is disturbance due to unintentional lateral pile movement between installation and controlled testing in the centrifuge, because this significantly reduces the soil stiffness at small lateral displacements. Great care was taken to minimise this uncontrolled disturbance.

3.3.1 10 m balanced beam centrifuge

The principles, geometry and working practices pertaining to this centrifuge are described by Schofield (1980). For these tests, the beam rotated at 151.5 rpm to impose a radial acceleration of 100 gravities at a radius of 3.975 m (Fig: 3.4), which coincided with a point at 1/3 of the total depth of the foundation from the top surface of the model, which corresponds approximately to the level of the bottom of the soft clay layer.

However, the radial acceleration is linearly dependent on radius. When the depth of the model is ignored by assuming a constant radius, there is a difference between the calculated and the actual centrifuge stress field. This is minimised by taking the nominal radius at 1/3 depth from the top surface, inducing understress in the top 2/3 of the model and overstress at greater depths (Taylor, 1984). For a ratio of model depth to radius of less than 5%, the error is less than ± 1%.
Another inaccuracy is due to the curvature of the stress field. The radial direction of centrifuge acceleration is aligned with the vertical axis of the model only at the centre. Furthermore, there is additional (total stress) loading at the edge of the model due to a curved ground water surface. However, the area of most significance was located in the central 200 mm of the model, adjacent to the piles, in the clay and upper part of the sand layer, in which this effect is negligible. The related velocities of the piles and soil, with respect to the package rotating on the centrifuge, were very small, and inaccuracies due to the Coriolis effect are considered to be negligible.

The swinging platforms at the end of the arm were designed to hang at a slope of 1 on 7 (Fig: 3.4), therefore the model and water table were initially subjected to a 1/7 tilt. As the centrifuge was accelerated, the platform rotated radially outwards until it seated against the stops on the arm at around 7g. Thus the model was lying horizontally with the perspex face uppermost and the centrifugal force field acting radially as required for the modelling. An inclined mirror was bolted to the platform to allow visual inspection of the front face of the model via a closed circuit television once the strongbox had swung up. The component of body force due to the earth's gravity field introduced a small error, causing a resultant acceleration of 100.005g at an angle of \(-0.57^\circ\). This effect was considered to be negligible. Of more significance was the operator error in setting the r.p.m., which was estimated as \(\pm 1\%\).

The behaviour of the test sample was monitored by the closed circuit television (Fig: 3.4). Still photographs were taken using a Hasselblad camera which recorded the view through the perspex window on either 70 mm cartridge or Polaroid film.

The instrumentation readings were transmitted to the data acquisition system by means of the sliprings located on the central spindle of the centrifuge. Power for the site investigation tools, water and compressed air were supplied to the model by this route during the tests.

3.3.2 Strongbox and consolidometer

The equipment used for these tests was commissioned by Kings College, London for plane strain modelling (Horner, 1982). There were three parts to this package:
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i) a liner in which the sample was prepared,

ii) a one dimensional consolidometer which held the liner during consolidation (Fig: 3.5),

iii) a strongbox into which the liner fitted post-consolidation in preparation for the 100g test (Fig: 3.6).

The maximum size of the model was constrained to 200 x 675 mm in plan. Its depth was limited to 170 mm. This size of model tested at 100g (centrifuge gravity) is equivalent to a prototype of 20 x 67.5 x 17 m at 1g (earth's gravity). For the foundation cases when clay overlaid sand it was necessary to add two sheet sides (perspex or Dural) along the bottom edges of the liner to restrain the cohesionless material during transfer of the liner from the consolidometer to the strongbox.

The vertical pre-consolidation pressure was limited to 254 kPa by the capacity of the consolidometer pneumatic jack. Following consolidation, the liner was placed in the strongbox for the model preparation, and a perspex window was fitted to the front face of the strongbox so that it was possible to observe the pile and foundation movements in-flight (Fig: 3.7).

A digital clock and a manometer were bolted to the front of the box. The manometer was connected to the plumbing system so that movement of the water table could be viewed during the centrifuge run. The clock and manometer were visible on the television and on the photographs taken by the Hasselblad camera for post-test analysis (Fig: 3.7). The water table was located at the top of the sand layer at the midpoint of the package. The curvature of the centrifuge equivalent gravity field caused the water table to be 15 mm higher at the edges of the sample than in the middle (Fig: 3.7).

3.3.3 Loading system

In test 3, a latex bag was filled with water to simulate normal loading, however the design was unsatisfactory and the bag punctured during the test. In the succeeding tests, the foundation was loaded by pressurising an airtight rubber loading bag as shown in Fig: 3.8. The bag was tailor made using a liquid latex solution and a plywood mould, as described by Timbs (1986) and
Springman (1987a). A restraining box was used to resist lateral and upward deformation of the bag and to hold the bag in contact with the foundation (Fig: 3.8).

A Druck PDCR510 pressure transducer was mounted on the air supply line to the rubber bag to monitor the compressed air pressure. The arrangement was considered to apply a uniform normal load to the foundation, without permitting the build up of shear stress between the greased bottom of the bag and the top of the foundation. An initial air pressure of 7 to 14 kPa was applied after swing-up at 7g to prevent the bag from detaching from the air supply lines under self weight at increased gravity. The pressure values quoted in the results are the increments above this base level, the net load, as measured by the PDCR510, which was accurate to ±1.2% for linearity, hysteresis and temperature effects and was recalibrated to within ±2.5% during the test series. The weight of the bag was carried by the air supply line and by the soil, which, if the air line support is ignored was equivalent to a pressure of 15 kPa on the foundation.

3.3.4 Vane shear testing apparatus and cone penetrometer

The vane shear testing apparatus (Fig: 3.9a) and cone penetrometer (Fig: 3.9b) used in these tests were designed and are described by Almeida & Parry (1983). They also reported results of parametric studies and comparisons of tests carried out in-flight and post-flight, which showed that the stress relief during swing-down and the associated ingress of water was sufficient to soften the overconsolidated kaolin.

Shaft friction, peak and residual values of shearing resistance were determined by the vane apparatus, using a torsion load cell at a succession of depths in the clay layer. Rotational and vertical movements were monitored by potentiometers. Following experience from earlier tests (Almeida & Parry, 1983), the vane selected for shear testing the soft clay was 14 mm deep by 18 mm diameter. The aspect ratio of the centrifuge vane was different from the standard dimensions used in the field; this permitted more tests to be carried out in a given section of soil.

The cone penetrometer was mounted in line with the radial acceleration field at an angle of 2° to the pile axis (Fig: 3.2a & b). This ensured that the load cell was measuring resistance directly due
to the soil, with no bending component. As the penetrometer was driven through the clay and, in
some tests, into the sand layer, the axial load and pore pressure at the tip were measured and a
potentiometer recorded the vertical displacement. Possible errors in the point resistance, $q_c$, caused
by water pressure acting on the silicon seal behind the cone tip have been described by Almeida
(1984), however, malfunction of the pore pressure sensing device meant that the interpretation of
data was limited to the resistance in the sand layer, where the pore pressure effect was less than
1% of the maximum value of $q_c$.

The measurement of cone resistance was considered to be accurate to ±20%. The vane load cell
recalibrated to within ±3%. The displacement and rotary potentiometers were accurate to
±0.5 mm and ±4° respectively. The site investigation was assumed to have taken place outside
the area of influence of the embankment loading, showing only the variations within the samples.

3.4 Instrumentation

3.4.1 Piles

During the tests on a single row of piles, only 15 channels were available for transferring signals
from the piles to the data acquisition system, and so two piles were instrumented with 8 strain
gauge transducers. The top gauge on pile A (BMT 1) was left disconnected (Fig: 3.10). For the
pile group tests (Fig: 3.11), it was possible to strain–gauge four of the six piles in the group for
each test, of which the signals from three piles were multiplexed to provide more readings. Each
pile had a total of 8 bending moment transducers. The piles were made from 12.7 mm diameter,
18 SWG Dural tubing with a 10 mm long end cone at the tip. The bending rigidity of the pile was
comparable to a solid reinforced concrete pile of the same diameter.

Tinsley (120 Ω – 3/120/PC23) strain gauges were glued externally to the tubing, to avoid
difficulties associated with internal application. More gauges supplied data than for tests 1 or 2
when bonding between the gauge and the Dural had been unreliable. The gauges were painted
with an acrylic moisture barrier and further protected by 2 layers of plastic shrinkfit tubing. The
power supply to the piles was adjusted from the control room during the tests to 3 V. Strain gauge
readings under zero load varied throughout the life of the piles by 30 Nmm.
3.4.1.1 Single row of piles

The piles were 300 mm long and embedded to 170 mm in the foundation, and are shown in Fig: 3.10. The strain gauges were connected in a half bridge circuit and calibrated to measure bending moment in the piles. Common dummy resistors were stored in the junction box to complete the bridge. The small output signals were amplified there by a factor of 100, before transference via the slip rings on the rotor arm to the data acquisition system.

Comparison of the calibrations of the instrumented piles before and after centrifuge testing showed a difference in calibration constants up to 8% in some cases. Installation of the pile out of the plane in which the strain gauges had been calibrated would lead to errors in the bending moment. However, this rotation error would have been less than 2°, and considering the 1.5 mm width of the strain gauges, the effect would have been negligible.

3.4.1.2 Pile group

The piles which comprised the pile group (Fig: 3.11) were shorter, measuring 190 mm in length with a 10 mm end cone, embedded to the same depth (170 mm) in the soil. A solid cylinder of Dural was pushfitted and glued into the top 30 mm of the pile to increase the bending rigidity over this length. This allowed the pile cap to be fixed to the top of the piles by bolting through the cap into a tapped hole in the cylinder. The cylinder length was determined to ensure that the pile was essentially fixed at the top surface of the sand.

Two pairs of strain gauges were glued alongside each other, with the pairs opposite each other across the diameter (Fig: 3.11) and these were connected in a full bridge circuit to measure pile bending moments. This reduced the drift in the transducer readings at zero load due to temperature differences between the components of the Wheatstone bridge, from 30 Nmm to 4 Nmm. The variations in calibration constant were also reduced to less than 5%. The signals were amplified by a factor of 100 in the junction box.

3.4.2 Pore pressure transducers

Four Druck (PDCR81–350kPa) transducers (PPTs), (Mair, 1979) were used to measure pore
pressure in the clay layer. De-aired porous stones were fitted to protect the diaphragm and ensure that pore pressure and not total stress was measured. In general these PPTs were reliable and were linear (to ± 0.4%) over the range of pressures and temperatures experienced in the tests. However, the diameter was 6 mm at model scale, hence 600 mm at prototype scale, and it was necessary to assume that the measured pore pressure corresponded to the middle of the stone. The calibration constants were found to agree to within ± 5% during recalibration. Another source of error was due to a difference in calibration power supply and the voltage supplied to the junction box on the arm. Both voltages were measured and there was a variation of ± 1% which was compensated for when the data was digitised.

Kutter (1983) has discussed errors due to water flow into or out of the porous stone, which were necessary for the PPT to register a change in pressure. Kutter et al (1988) and White (1987) have both used finite element analysis to examine the behaviour of a clay sample with stiffer elements modelling the PPTs. On loading, differing strain rates around the PPT caused local suction, but these were short−lasting. These inaccuracies were considered to have a negligible effect here.

The PPTs settled vertically with the clay as the sample consolidated, firstly in the press and later under the centrifugal force field. The initial positions of the transducers were accurate to ± 5mm and it was possible to estimate their locations prior to the centrifuge test from the consolidation data. On application of the load, the elastic and plastic deformations created further movements, with both horizontal and vertical components. The final positions of the transducers after the package had been unloaded from the centrifuge were determined by X−ray techniques and excavation, and were measured to within ± 1 mm.

3.4.3 Linear variable differential transformers

Four Sangamo linear variable differential transformers (LVDTs) (Taylor, 1984) were positioned to record pile deflections at two levels above the foundation surface, and were set up so that the instruments read between ± 2.5 V with a stroke of 15−20 mm. This combined the required range with the best accuracy. Within the rated stroke, the instrument was linear to ± 0.3%. If the stroke was increased from these limits by 20%, non−linearity was observed and the best−fit calibration
constant differed by up to 8% from that determined for the linear range. The constants measured for each test were within ± 8%. The power supply differed by ± 0.5%, which was corrected for. The friction acting on the LVDT spindle was assumed to be negligible.

Butterfly shaped plates were fitted to the end of the probes. The LVDTs were fixed to the pile by two small rubber bands to ensure that they maintained contact with the pile at all times during the test. However, for tests 8–10, the upper LVDT was fixed in the same way to a plate which was bolted onto the side of the pile cap. The demodulated DC output from the LVDTs were filtered in the junction box for smoothing the signals before they were passed through the sliprings.

3.5 Model preparation

The methods of model preparation depended on the type of foundation. There were two cases:

i) clay-on-sand,

ii) all-clay foundation.

The soils used in the experiments were standard commercially available soils rather than those specific to a particular site. This allowed better comparison of soil behaviour between different models. Existing knowledge of soil properties was used. Stress history for the samples is shown in Figs: 3.12a, 3.12b.

Although it is possible to scale most operations in the centrifuge, the correct particle sizes are harder to model, while retaining the relevant engineering properties. However, this is only significant when the ratio of model dimension to particle size, $\chi$, is small. For cone penetrometer testing (Phillips & Valsangkar, 1987), a minimum value for $\chi$ is 10, while $\chi_{\text{min}} > 20$ is preferable. In the sand layer, median particle diameter was approximately 400 µm. Since the pile and the penetrometer were of similar diameter, $\chi = 32$, which was acceptable.

3.5.1 Consolidation procedures

3.5.1.1 Clay–on–sand foundation

Filter paper was placed on the bottom of the liner to cover the Vyon porous plastic rectangles which permitted water passage from the bottom of the sample. The liner was weighed.
Leighton Buzzard sand of B.S. 30/52 grading, with maximum and minimum voids ratio of $e_{\text{max}} = 0.810$ and $e_{\text{min}} = 0.538$ respectively (Almeida, 1984), was used for the model making. The specific gravity of the particles was $G_s = 2.65$ (Almeida, 1984). The sand was poured to form a loose deposit which was nominally either 100 or 80 mm thick. The mass of the sand used was recorded. At two or three levels, lines of black marker bullets were positioned at 25 mm spacing against the perspex side sheet to allow a visual check of movement within the sand layer.

The sample was saturated slowly from the base upwards with de-ionised water to reduce the likelihood of trapping air. The water level was kept at the top of the sand layer while the liner and sample were weighed. The dry and saturated soil unit weights were determined. The internal faces of the consolidometer were well lubricated with silicon grease before it was bolted together in readiness for transferring the liner and then adding the clay.

The voids ratio and relative density of each of the sand samples just after pouring are shown in Table: 3.3. The models were assumed to be 100% saturated and the voids ratio, $e$, and relative density, $D_r$, were calculated from:

\[
e = G_s \left( \frac{\gamma_w}{\gamma_d} \right) - 1
\]

\[
D_r = 100 \left[ \frac{(e_{\text{max}} - e)}{(e_{\text{max}} - e_{\text{min}})} \right] \%
\]

with unit weight of water, $\gamma_w$, and dry unit weight of soil, $\gamma_d$. The voids ratio and hence relative density of the sand changed throughout the test period as the sample was handled and loaded. The accuracy of the values quoted above was dependent on the measurement of the sand mass and calculated volume. This was known to be within ±2% initially. It was estimated thereafter by measurement from and comparison between in-flight photographs and fiducial marks on the perspex. It was harder to measure the saturated unit weight of the sand because some of the water supplied to the sample filled the voids in the liner drainage system. This value was therefore calculated from knowledge of the voids ratio and estimates of the specific gravity. The construction method was the same for all sand samples, but variations in relative density within each model were not explored.
Speswhite kaolin clay powder was mixed mechanically with de-ionised water under vacuum to form a slurry at 120% moisture content. After a minimum mixing period of 2 hours, the slurry was carefully placed on top of the sand layer by scoop, avoiding inclusion of air bubbles, to a depth of roughly three times the intended depth of soft clay. Samples were taken for moisture content determination.

Filter paper was placed on top of the clay before the piston was lowered into position. A nitrogen cylinder was connected up to the pneumatic jack. Load was applied in stages for total vertical consolidation pressures equalling 3, 5, 11, 21, 43 and 86 kPa. Consolidation was complete under 43 kPa after about 1 week. The load was removed, the piston supported from above and the back panel of the consolidometer unbolted for installation of the de-aired pore pressure transducers. This procedure was described in detail in Springman (1987a). Thereafter, the consolidometer was re-assembled and the sample was re-loaded up to 43 kPa until primary consolidation was complete, when the final pre-consolidation pressure of 86 kPa was applied. The total time taken by the consolidation process was a minimum of 10 days.

Vertical displacement of the piston, load and time were recorded during consolidation. After the pore pressure transducers were installed and the sample had reconsolidated under 43 kPa, dial gauges were mounted on top of two telltale rods to record accurately any further movement. Coefficients of consolidation and permeability were calculated to give an estimate of the time required for consolidation in the centrifuge (Table: 3.4).

3.5.1.2 Clay foundation

There were only two tests, 5 and 10, in which the sand substratum was replaced by a stiff clay layer. These were intended to investigate the deep clay layer case. The laboratory techniques were identical to those described in the preceding section unless stated otherwise.

The liner was put in the consolidometer and clay slurry was placed directly on top of the base filter paper. No side retaining walls were necessary. The loads were applied as before, but up to 254 kPa consolidation pressure. This process took 2–3 weeks. Once consolidation was complete, the
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consolidometer side panels were removed and the sample was trimmed to a depth of 100 mm. The excess was tested for moisture content. The side panels were replaced and more clay slurry was poured on top of the relatively stiff layer, to a depth of around 200 mm. The loading system was re-assembled and consolidation of the soft layer commenced as before with a pause at 43 kPa consolidation pressure for the installation of the PPTs in both layers of clay. The stress history is shown in Fig: 3.12b.

Moisture contents were taken from samples before and after consolidation to 86 kPa, and the saturated unit weight, $\gamma$, was calculated (Table: 3.4):

i) by reference to the kaolin soil properties ($G_s = 2.61$, Airey, 1984) and the moisture contents, $w$, where:

$$\gamma = (G_s^e + e) \frac{\gamma_w}{(1+e)}$$  

and the clay was assumed to be fully saturated so that:

$$e = w G_s$$  

ii) by weighing the clay and measuring the volume of the clay in the liner where

$$\gamma = \frac{\text{weight of clay in liner}}{\text{volume of clay}}.$$  

The accuracy of method (ii) was dependent on the estimates of the volume and weights of the soft clay, stiff clay and water. The weighing scale was accurate to $\pm 0.5$ kg which corresponded to an error of $\pm 3\%$. Thus the results for method (ii) were accurate to $\pm 13\%$. This implied that method (i) was more realistic.

3.5.2 Model making

The model making procedures were the same for both types of test sample, and commenced three days before the centrifuge flight when the liner was removed from the consolidometer for the last time. The sample was trimmed to a total depth of 160 mm and the remaining clay was tested for moisture content. The liner and sample were weighed for recording on centrifuge balance calculations and for the alternative determination of the density of the clay using Eqn: 3.5.

The internal faces of the strongbox were lubricated with Molykote 33 silicone grease to minimise
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friction before the liner was slid into it. Hambly (1969) measured friction coefficients of 0.01 < \( \mu \) < 0.19 for a greased kaolin–aluminium surface. The perspex window was sprayed with Adsil lubricating oil before it was bolted on and the pile–positioning template rig was assembled. The plane strain assumption requires the boundaries to be rigid and frictionless. Waggett (1989) reports values of \( \mu \) for kaolin–perspex interfaces lubricated by Adsil and silicone grease (Table: 3.5), which show that the latter reduced the friction the most. There will be a small out of plane strain due to deflection of the perspex as the centrifuge acceleration increases. Calculations showed that this deflection was minimal in these tests.

For the single row of piles, it was necessary to ensure that verticality, correct pile spacing and alignment of strain gauges was achieved before the piles were pushed into the foundation until their tips reached the base of the liner. This was made easier by the pile cap in later tests.

The perspex window was removed and preparations made for inserting horizontal lead threads through a template, parallel to the rows of piles, in the manner described by Phillips (1987). After the test had finished, it was possible to observe the internal deformations using radiography.

The front face of the model was marked for a visual inspection of the foundation deformation and spot–chasing if required. Black plastic bullets were placed in the front face of the clay at 15 mm centres horizontally and vertically, and a series of thin horizontal lines were painted between the black bullets using brown oxide paint.

The perspex window was refitted and a digital clock and the manometer were bolted into position on the window frame. The manometer was plumbed into the water control system. A 10 mm layer of B.S. 30/52 Leighton Buzzard sand was placed on top of the soft clay and saturated with de–ionised water. The liner drainage system was adjusted to ensure that the water table did not exceed the design level. The LVDT rig was installed and the instruments were checked to ensure they were in the correct range when they were touching the piles. The vane shear testing apparatus was bolted through the 6 mm thick covering plate which formed the detachable strongbox lid before this was fitted to the strongbox.
The loading system was assembled with the rubber pressure bag inside the restraining box and then lowered into position on the foundation. In test 4, two loading cases were examined. The bag was placed at 4 pile diameters from the row of piles, and later moved against the piles during a break in the centrifuge flight. In subsequent tests only the latter position was used. The Druck PDCR510 pressure transducer was connected into the airline from the bag.

The porous stone in the tip of the cone penetrometer was de-aired by boiling in de-ionised water, before a protective water-filled cap was fitted. This ensured that the porous stone remained saturated until the penetrometer was driven through the thin polythene membrane at the base of the cap and then into the soil. Finally the penetrometer and junction box were screwed in position on the top plate. The instruments were plugged into the junction boxes and final checks made before the whole package was weighed.

3.6 Centrifuge flight

3.6.1 Data acquisition

Analogue signals from the instrumentation were sent via the sliprings on the centrifuge spindle to the data acquisition console in the control room where they were recorded in both analogue and digital form. Some of these were amplified before passing through the sliprings, others were amplified later in the control room. Two 14 track Racal magnetic tape recorders, running at 0.9375 inches/second, were used to obtain a continuous record of the analogue signals. Then, segments of the tape were played back for each event, and data was digitised by an Alpha LSI 2 mini-computer with twin 8 inch disk drives using the FLY14 program (Dean & Edgcombe, 1988) initially and then FLY55X (Balodis, 1986) for the last three tests. Data was stored on the floppy disks and plotted out on the terminal and the HP7470 plotter in the form of time records.

Simultaneously, FLY31 (Stewart, 1984) employed the Alpha directly during the test to control the sampling and digitisation rate on all 31 channels, plotted up to six channels in engineering units on the Westward 1015 graphics terminal, and printed out raw voltage readings as required. The program took a 10 milli-second integration of the signals from the instruments, in contrast to the continuous spot sampling adopted by FLY14. Data was stored on floppy disks and hard copy plots...
were obtained as required. The FLY31 system was effective for events which did not require scanning at less than 5 second intervals, but not for the vane and penetrometer tests which needed constant logging. FLY31 has therefore been used as a back up for these fast events and only the FLY14 time records were considered relevant. As a back up, hand readings were taken using a digital voltmeter.

3.6.2 Stages of the test
The centrifuge tests were divided into several distinct phases during which the model was subjected to different load regimes.

3.6.2.1 Swing–up
As the centrifuge speed was increased gradually, the package swung upwards about the pivot so that the line joining the pivot to the centre of mass was in the same direction as the resultant gravity field (Fig: 3.4). The components of this were the centrifugal acceleration field (≈ 7g at swing–up) and the earth's gravity field, and the angle was at 82° to the base of the platform. Thereafter as the centrifuge acceleration field changed from 7g to 100g, stops on the arm prevented the package from swinging up further, and the angle of the resultant gravity field approached 90°. So the model experienced an increasing gravity field, which was also changing in direction.

An additional load experienced by the foundation shortly after swing–up was due to the application of a small pressure in the loading bag. This prevented the bag from pulling away from the supply lead as the self–weight of the bag increased.

3.6.2.2 Consolidation
When the package was subjected to a constant acceleration field of 100g, the rise in pore pressure measured by the PPTs was equivalent to the loading above them, due to the increased weight of the fluids in the soil mass together with the initial loading bag pressure. Consolidation time was required while the pore pressures dissipated to their new static levels. The 60 mm model foundation layer reached steady state in about 2 hours, whilst the 160 mm model layer took 6–7 hours. While direct comparison of the square of the drainage paths would indicate that the
160 mm layer might have needed 14 hours consolidation, the different stress histories and hence consolidation properties of the soft and stiff clay layers reduced the time required. The stiff lower clay layer, which was 100 mm deep, was overconsolidated at all times during this phase, with the OCR ranging from 5 at the top to 2 at the bottom of the layer (Fig: 3.12b).

Pore pressures were monitored and plotted, both against root time, and as isochrones for the consolidating clay layer or layers. The in–flight plotting mode of FLY31 was used to display the pore pressures, gravity level and loading bag pressure during this phase. The pore pressure data was used only to indicate the extent of consolidation rather than to support a comprehensive analysis of the effects of stage construction of embankments on soft clay.

3.6.2.3 Loading

When pore pressures had dissipated to the new static levels, the loading bag pressure was increased to 25 kPa and then by nominal 25 kPa intervals up to 100 kPa. An unload–reload loop was completed from 100 kPa. These values were controlled by reference to the gauge on the air supply line, but the readings used in the analysis were taken from the Druck PDCR510 pressure transducer. Occasionally it was possible to reach only 50 or 75 kPa before the foundation approached failure. At each loading stage a photograph was taken of the front face of the model to define overall patterns rather than magnitudes of soil movement. Further loading was delayed for about 2 minutes \( (t_p = 14 \text{ days}) \) to allow a noticeable plateau in the signal readings to appear for simplified back analysis of each loading increment.

A vane test was carried out while the foundation consolidated for a period of between 30 minutes to two hours. Then loading was continued in 25 kPa increments with unload–reload loops every 50 kPa and occasional pauses to permit further consolidation, until the clay foundation failed by squeezing under the edge of the loading box and past the piles.

3.6.2.4 Vane shear test

The vane shear tests were conducted during the first consolidation period under load. Three horizons were investigated in the soft clay layer and three were tested in the deeper stiffer layer.
Monitoring readings from the vertical potentiometer, the vane was driven down to the required depth. During the shear test, load cell readings were recorded as the shaft friction was overcome, and then later at the peak and residual values.

In general the driving rate was 6 mm/minute and the rotation rate was 72°/minute. At the end of the shear testing, the vane was withdrawn from the clay. The individual shear test at each horizon took about 10 minutes for driving, rotating, unwinding and withdrawal.

3.6.2.5 Cone penetrometer test

In the first series of tests, 3–7, it was necessary to stop the centrifuge, remove the vane apparatus plugs from the junction box and replace them by the penetrometer plugs, allowing a period of 30 minutes for further reconsolidation. But the signal multiplexing arrangement for test 8–10 allowed sufficient signal channels to eliminate this procedure. The entire loading programme was completed before the penetrometer test was carried out. At this time, the foundation was consolidating for between 30–45 minutes after reloading to maximum test pressure.

The vertical potentiometer reading was observed to ensure that the penetrometer was stopped before it reached the limit of travel on the guide rods, although the potentiometer failed to work during tests 8 and 10, and depth was estimated from average driving rates, which varied from 4–10 mm/second for the clay—on—sand foundation, to 2 mm/second for the deep clay layer.

3.6.2.6 Swing—down

During deceleration and swing—down, the model experienced the reverse process to that which occurred during swing—up and acceleration of the centrifuge to 100g. As a result of the decrease in total stresses, water was sucked into the clay, which swelled and softened as the centrifugal force reduced.

3.6.3 Post—test site investigation

Post—test site investigations were carried out after the centrifuge had stopped, commencing with Pilcon hand vane shear tests for models 5–10. Photographs of the deformed foundation and final
pile and pile group positions were taken. On the following day, the liner was extracted from the strongbox and the sample was X-rayed (tests 4, 6–10) and then sectioned (all tests).

3.6.3.1 Pilcon vane shear tests
Both the 29 mm deep by 19 mm diameter and 50 mm deep by 38 mm diameter vanes were used to give a measurement of shear strength immediately after the centrifuge was stopped. Between 2–4 horizons were investigated by the smaller vane and 1–2 were tested by the larger vane. These numbers were dependent on the depth of soft clay. Peak and residual values were noted.

3.6.3.2 X-ray procedures
After the liner was removed from the strongbox, the top 10 mm of sand was removed and the sample was X-rayed in two directions. Initially the film was placed along the rear face of the model, overlapping the area which was loaded and the piles. X-rays were directed in a horizontal plane, penetrating 200 mm of soil. The aim was to reveal the location of the pore pressure transducers.

However, it was more important to observe the deformed positions of the horizontal lead threads and locate any slips or breaks. The X-rays were transmitted vertically downwards through 160 mm of soil and 25 mm of aluminium. In general, about 25 milli–ampere–minutes of exposure at 150 kV were required for the 200 mm of soil and 60 milli–ampere–minutes at 150 kV for the latter case. Film to focus distance varied between 1 and 1.3 m and Kodak Crystallex X-ray film was used.

3.6.3.3 Excavation
Once the X-ray analysis was complete, the sample was excavated to find the positions of the pore pressure transducers and, in some cases, to uncover the lead threads.
4. **CENTRIFUGE MODELLING – PRESENTATION OF TEST DATA**

4.1 **Introduction**

This chapter presents the results relevant to the analyses of Chapters 5 and 6 below, and highlights the major observations from the centrifuge model tests. More detailed listings of data records are given by Springman (1987a, 1987b). Priority was given to the information obtained from the piles and the interpretation of the soil–pile interaction.

The treatment of the bending moments, which were derived from data from calibrated strain gauge circuits, and the deduction of pile deflections and lateral pressures, are considered first before a brief description of each test is included with a discussion of the pile behaviour. Strength test results and post–test investigations are also examined.

4.2 **Presentation of data**

Bending moment and LVDT deflection data were extracted from the time records and the best–fit polynomial was drawn through the bending moment data points, according to the sign convention shown in Fig: 4.2a. The program incorporated Randolph's (1985) algorithm, BPOLY, and was extended to suit the experimental data with appropriate boundary conditions at the pile head and tip. The root mean square error was evaluated for a range of 4th to 9th order polynomials and the curve with the smallest error was selected. The zero values of the bending moments and deflections were taken as the readings after full consolidation at 100g, just prior to commencing the first loading stage. An air pressure load of between 7 and 14 kPa was applied to the foundation after swing–up in addition to the self–weight of the rubber pressure bag, so it was not possible to deduce the bending moments and deflections at true zero load. Therefore, this combination of initial loading was defined as zero net load. The loading applied in increments following consolidation at 100g constituted the net load. This terminology is used below.

By double differentiation and double integration of the bending moment polynomial, the lateral pressure acting on the piles and the relative pile deflections were calculated. The lateral pressures
were not considered to be accurate because they were based on taking the gradient of another
gradient of a polynomial fitted to a maximum of eight data points where:

\[ 2r p = \frac{d^2M}{dy^2} \]  \hspace{1cm} (4.1)

In some cases, when the strain gauge readings appeared to have drifted or when one set of
bending moment data at a particular level was negative, the deduced lateral pressure curve was
unrealistic. If the error on \( M \) is \( \Delta M \), and the vertical spacing between the gauge positions is \( \Delta y \),
then the maximum error, assuming that the polynomial passes exactly through these points is
(Fig: 4.2b):

\[ \frac{dM}{dy} = \frac{2\Delta M}{\Delta y} \hspace{1cm} \frac{d^2M}{dy^2} = \frac{4\Delta M}{\Delta y} = 2r \Delta p \]  \hspace{1cm} (4.2)

so that the maximum error in pressure is, \( \Delta p \approx 2\Delta M/(r \Delta y^2) \), which for an error of \( \Delta M = 0.1 \) Nm,
gives \( \Delta p \approx 79 \) kPa. However, there can be further (gross) error when the order of the polynomial
changes as a consequence of misplaced data points or lost data.

The mean deflection measured by LVDTs at the top of the pile was used to estimate the
deflection mid-way between them. Earlier analyses (Springman, 1987a & b) had used these
values to find the slope of the pile, but the accuracy depended on the difference between similar,
small numbers, and the extrapolated deflection of the pile tip was erratic. Therefore, for the
single row of piles, it was assumed that there was no movement at the pile tip so that for \( y = 0 \),
\( u = u_o \), \( \theta = \theta_o \) at ground surface:

\[ u = u_o - \theta_o y + \int_0^y M \, dy \, dy \]  \hspace{1cm} (4.3)

This supposition was supported by inspection of photographs of the markers on the front face of
the model. In tests 3, 4 and 7, Eqn: 2.4 showed that the laterally loaded pile exceeded the critical
length, \( l_c \), below which there was no horizontal movement, and was therefore long enough to be
considered flexible. For tests 5 and 6, the pile was not long enough to achieve adequate embedment, but the tip movements were minimal and the analysis was continued as before.

In the group, the piles were fixed rigidly to the pile cap which was assumed to remain horizontal, so that the pile cap rotation was zero. In reality, the LVDT readings indicated a small rotation, which was less than 0.5° for the pile group embedded in the sand substratum. But, the accuracy of the LVDT data compared to the implied rotations was insufficient to justify inclusion in the analysis. In consequence, the pile tips appeared to move laterally by a small margin. A more realistic assumption would remain that of zero tip deflection of long piles. However, these fixity conditions were not appropriate for the pile group in the clay foundation. The piles were shorter than the critical length and therefore behaved in a rigid manner by rotating up to 4° combined with some horizontal translation for embankment surcharges above 150 kPa.

The loading pressure (in kPa) measured by the air bag transducer was written against each set of readings: see Fig: 4.3c for a typical plot. For test 3, the load was monitored by a PPT in the water bag, and in test 8, when the air bag transducer failed, the nominal load was read off the air supply gauge, which was later calibrated and found to be accurate to ± 5%. Root mean square error in the fitting of the polynomial to the bending moment data points was also noted together with the order of the polynomial selected by the program. Bending moment transducers which failed to record were specified on the plots.

4.3 Centrifuge model test summaries

The centrifuge model tests were carried out under a centrifugal acceleration of 100g at a radius of 3.975 m (Fig: 3.4), and thus the scale factor was 100 for dimensions, which were all at model scale (subscript 'c') on the plots and in the text unless denoted otherwise by the subscript 'p'. The tests were summarised in Table: 3.2.

4.3.1 Test 3

A single row of three piles (s/d = 5.25, s_c = 66.7 mm, s_p = 6.67 m) was driven through a 60 mm
Centrifuge modelling — presentation of test data

(h_p = 6 m) layer of kaolin and embedded in a 100 mm (l_p = 10 m) layer of sand, with a surface layer of 10 mm (1 m prototype) depth of sand. The instrumentation layout was given in Fig: 4.1. The loading system relied on increasing the level of water in a rubber bag, which was held within a restraining box, to model a loaded area at prototype scale of 20 m by 27.5 m.

After 2 hours (t_p = 2.28 yrs) consolidation following an increase of centrifugal acceleration to 100g, the foundation was loaded in increments of 25 kPa. But, after 13 minutes (t_p = 90 days), at 87 kPa, as the bag pressure was being increased to 100 kPa, the rubber bag burst, and water flowed out, severely scouring and flooding the foundation, causing large pile movements. In consequence, there were no in-flight or post-test site investigations. Data was recovered only from pile A as shown in Fig: 4.2c.

4.3.2 Test 4

The geometry was identical to test 3 (Fig: 4.1), but the loading was applied by the air bag system, which was used for the remainder of the centrifuge model tests. Initially, the bag was located at 4 pile diameters behind the piles and preliminary loading was carried out up to 86 kPa. However, the pile bending moments increased only marginally and the bag was moved closer to the piles. After swing-up under these loading conditions, a7 kPa load was imposed on the foundation to prevent the rubber bag from pulling off the air supply line due to the increasing self-weight as the centrifugal acceleration reached 100g. After 2.5 hours (t_p = 2.85 yrs) consolidation, readings were taken for zero net load and loading was conducted in nominal 25 kPa stages to a net load of 86 kPa during 10 minutes (t_p = 69 days). The foundation was unloaded to 28 kPa (net) and reloaded again. This cycle was repeated on three further occasions during the test at 146 kPa, 203 kPa and 228 kPa (net).

A vane shear test was carried out while the model was consolidated for 1 hour (t_p = 1.14 yrs) under a net load of 86 kPa. The centrifuge was then stopped to move the loading bag directly behind the piles. After a further 2 hours (t_p = 2.28 yrs) of reconsolidation at 100g, loading was increased in approximately 25 kPa increments from 30 kPa to 146 kPa (net) over a period of
29 minutes \( (t_p = 0.55 \text{ yr}) \). During a 45 minute pause \( (t_p = 0.86 \text{ yr}) \) at 146 kPa net load, a cone penetrometer test was conducted. Loading was then continued to 228 kPa over 10 minutes \( (t_p = 69 \text{ days}) \), and the sample was consolidated for a further 30 minutes \( (t_p = 0.57 \text{ yr}) \). After swing down, the sample and liner were removed from the strong box, and radiographs taken before the sample was sectioned and photographed.

The bending moment data is shown for both piles in Fig: 4.3a, 4.3b, 4.3c for the bag in position, adjacent to the piles. In some cases, BMTs failed. The curve fitting technique implied that the bending moments were negative in the upper layer of soft clay (Fig: 4.3c). However, this behaviour was not observed in other tests and so it was attributed to curve fitting inaccuracies. The maximum bending moment, \( M_{\text{max}} \approx 5 \text{ Nm} \) \( (M_{\text{max}} \approx 5 \text{ MNm}) \) and occurred at a depth of 30–45 mm, which corresponds to 2.5–3.5 times the pile diameter, below the clay–sand interface.

As expected, the soil appeared to load the piles in the upper soft clay layer, while resisting movement in the stiffer substratum. The effect of the deflection fixity condition based on the LVDT readings is clearly seen in Figs: 4.3a, 4.3b, 4.3c. The magnitude of the mean deflection at the top of the pile influenced the pile rotation at the tip, whereas the assumption that the rotation at the top of the pile was defined by the slope between the LVDT readings would suggest, in some cases, considerable movement at the pile tip. The LVDT readings were deemed to be accurate to within \( \pm 10\% \) after signal processing. Further errors were entailed in double integration of a polynomial fitted to up to 8 data points.

A jump in the BMT readings occurred during the penetrometer test. For this reason, penetrometer tests were carried out at the end of construction in the remaining centrifuge model tests. There was a time lag in the recording of pore pressure build up by the penetrometer piezometer, and so it was not possible to correct the data of total cone resistance for pore pressure effects (Almeida, 1984).
4.3.3 Test 5

This test modelled the behaviour of a deep clay foundation, with a stiffer overconsolidated lower 100 mm ($\ell_p = 10$ m) layer (Fig: 4.4). The pile spacing and loading system were identical to test 4. The experimental procedure was similar. The 3 V power supply for the piles oscillated, and the first section of data was unreliable. After stabilising the supply, data was obtained over a 17 minute period ($t_p = 118$ days) for net loads between 43 and 157 kPa (Figs: 4.5a & b) after consolidation at this lower load for 2 hours ($t_p = 2.28$ yrs). Zero values of net load were deduced once the foundation was unloaded from 93 kPa (net) before reloading and continuation of the test.

Complete sets of data were not obtained for the in-flight vane and cone penetrometer tests. The X-ray apparatus was not available at the time of this test. Hand vane tests were conducted immediately after swing down.

The piles behaved rigidly, pivoting about their tips (Figs: 4.5a & b). Bending moments were smaller, with the maximum value ($M_{max} = 2.3$ MNm) at a shallower depth than in test 4. Thus, differential movement between pile and soil was also small, but overall pile deflections at ground level were larger, approaching the equivalent of 1.5 m at prototype scale.

4.3.4 Test 6

An 85 mm ($h_p = 8.5$ m) layer of clay overlay a 75 mm ($\ell_p = 7.5$ m) layer of sand (Fig: 4.4). For these pile–soil conditions, the pile would be expected to behave more as a rigid, short pile rather than a flexible pile (Eqn: 2.4). The pile configuration remained the same as in previous tests. There was an initial 4 hour ($t_p = 4.57$ yr) period of consolidation, the bag loading was then increased in nominal 25 kPa increments up to 72 kPa (net), over a 12 minute period ($t_p = 83$ days). An unload to 0 kPa (net) — reload to 72 kPa (net) cycle was completed and then a series of in-flight vane shear tests were executed at 4 depths in the clay layer during a 2 hour ($t_p = 2.28$ yr) pause in loading under 72 kPa (net). Loading was continued to 264 kPa (net), with 3 more unload—reload loops at 132 kPa, 196 kPa and 229 kPa taking a further 49 minutes ($t_p = 0.93$ yr). A penetrometer test was conducted before swing-down. Hand vane tests were carried out at 1g after swing-down, and the positions of the deformed lead threads were exposed
by X-ray and sectioning.

The maximum bending moments were observed for a net load of 196 kPa for both piles. As this was increased above 196 kPa, the bending moments reduced slightly, indicating that the clay was now shearing past the pile (Figs: 4.6a & b). The magnitude of the maximum moment was of the same order \(5.7 \leq M_{\text{max}} \leq 6.3 \text{ MNm}\) as that measured in test 4 for a similar value of \(q\), although it occurred in this test at only about 1–2 pile diameters below the clay–sand interface. The strain gauge at \(y = 130\) mm failed during the early stages of the test for pile B. With out this data, the curve fitting routine generated unlikely bending moment profiles in the stiffer substratum as shown in Fig: 4.6c.

The lateral pressure profiles suggested that this foundation case, with a deep soft layer, experienced the changeover from active to passive loading of the soil 10–30 mm above the clay–sand interface, although the pressure was roughly parabolic in the clay for the lower bag pressures. However, pile deflection increased as net load increased from 196 kPa to 264 kPa even though bending moments were reducing: the pile had clearly rotated about the tip, reflecting rigid pile behaviour.

4.3.5 Test 7

The test geometry was the same as test 4, but with \(s/d = 3.15\), and 5 piles at \(s_c = 40\) mm (Fig: 4.7). The procedure was also similar, with initial consolidation over 2 hours (\(t_p = 2.28\) yr) and subsequent loading to 93 kPa (net) over 8 minutes (\(t_p = 56\) days) before unloading to 0 kPa (net) and reloading to 93 kPa (net). Vane tests were carried out at two depths while the foundation was under 93 kPa net load for 30 minutes (\(t_p = 0.57\) yr), before the load was increased in stages up to 189 kPa (net) during a 20 minute (\(t_p = 0.38\) yr) period with two further unload–reload loops at 152 kPa and 189 kPa (net). A penetrometer test was conducted at 100g, just prior to swing–down. Post–test hand vane tests, X–ray radiography and sample dissection were executed at 1g.

Bending moments increased steadily with bag loading up to the maximum at net load 189 kPa
(2.8 < M_{max} \leq 3.5 \text{ MNm}, \text{Figs: 4.8a & b}). This was much lower than expected and it is thought that the ultimate lateral pile capacity was not achieved. Moments were smaller than those observed in test 4 (s/d = 5.25), implying that the load was being shared amongst the piles.

The lateral pressure distributions were almost parabolic and the pressures reduced to zero at 1 m above the clay–sand interface. Significantly from the design point of view, the pile deflections seemed to be roughly the same for both tests 4 and 7 for a typical net load of 113 kPa. However, a greater tip rotation was measured in test 7; this could be due to an overestimate of pile deflection by the LVDTs. Due to the closer pile spacing, the net pile deflection in the soft layer was smaller, as expected, than in test 4.

4.3.6 Test 8

This series of tests used a pile group with 2 rows of 3 piles at s_{zc} = 66.7 \text{ mm}, s_{xc} = 50 \text{ mm} connected by a stiff pile cap (Fig: 3.11). Loading was applied by the air bag system, as before, however the transducer which measured the pressure in the air bag failed and so nominal values were taken from the external gauge on the air supply line. The foundation geometry was identical to tests 3, 4 and 7 (Fig: 4.9), which modelled flexible pile behaviour.

The model was consolidated under 100g and a nominal 7 kPa bag load for 2.25 hours \( t_p = 2.56 \text{ yr} \), and then the load was incremented to 100 kPa (gross) over 8 minutes \( t_p = 56 \text{ days} \), unloaded and reloaded. During 35 minutes \( t_p = 0.67 \text{ yr} \) further consolidation, a vane test was carried out at 3 depths in the clay layer. Loading was increased in stages of 25 kPa to 250 kPa (gross) over 19 minutes \( t_p = 0.36 \text{ yr} \), with unload–reload loops at 150, 200, 250 kPa (gross). A penetrometer test was conducted during a final 30 minute \( t_p = 0.58 \text{ yr} \) consolidation period at 250kPa. Post–test site investigations using the vane and X–ray radiography were carried out.

Pile behaviour for both front and rear piles is shown in Figs: 4.10a, 4.10b, 4.11a, 4.11b for piles AF, AR, BF and BR respectively (Fig: 3.11). Fixity at the pile cap imposed a clamping moment
which was greater than the maximum moment of the opposite sense sustained by the pile in the stiffer stratum and which was larger for the rear pile. The moment profile changed sign at the clay–sand interface for the front piles (AF, BF), and up to 30 mm above it for the rear piles (AR, BR).

The strain gauges on pile AR at depths of 10 mm, 30 mm and 50 mm failed. In consequence, the lateral pressure calculated for this pile (Fig: 4.10b), from double differentiation of the bending moments, should be treated with some caution. For pile BR the readings for the strain gauges at 70 mm were thought to be in error (Fig: 4.11b). The root mean square errors (RMS) were considerably higher for this pile, implying that the polynomial fit was not so accurate. Thus, the bending moments oscillated to small negative values over the bottom 30 mm of the pile. In the other tests on this foundation geometry, these negative values were not observed, and so this was attributed to curve fitting error.

Comparison of the derived lateral pressure distributions for the front row of piles, (Figs: 4.10a & 4.11a) with those calculated for a single, free headed pile (Figs: 4.8a & b) were similar in form and magnitude, particularly for net loads of $\approx 100$ and 200 kPa. However, the application of a shear force at the top of the rear pile, and the boundary condition which allowed the soil to squeeze upwards between the piles and under the pile cap reduced the lateral pressure on these piles (AR, BR), which took a different profile (Figs: 4.10b & 4.11b).

Pile deflections were greatly reduced by fixity at the pile cap, which was assumed to remain horizontal. The largest pile cap rotation calculated from the LVDT readings was 0.1°. A rotation of 0.01° would have given zero pile tip deflection at maximum load.

4.3.7 Test 9

The foundation geometry was similar to test 6, but 80 mm ($h_p = 8$ m) of kaolin overlay 80 mm ($\ell_p = 8$ m) of sand, with 10 mm of loose sand on the surface (Fig 4.12). It was expected that the clay layer would be just deep enough to allow the pile to behave flexibly. The pile group was identical to that used in test 8, and the test procedure was similar, with 3 hours 50 minutes
(\(t_p = 4.38\) yrs) initial consolidation followed by loading in stages up to 98 kPa (net) over 6 minutes (\(t_p = 42\) days). Following an unload to 0 kPa (net) and reload, vane shear tests were then conducted at three depths in the clay layer during the subsequent 40 minute (\(t_p = 0.76\) yr) consolidation period. Loading was then increased to 242 kPa (net) with unload–reload loops at 194 kPa and 242 kPa (net) during 22 minutes (\(t_p = 0.42\) yr) and a penetrometer test was carried out during a final 45 minute (\(t_p = 0.86\) yr) consolidation period. Hand vane shear strengths were measured at 1g immediately following swing–down and the sample was X–rayed and sectioned as in previous tests.

Two strain gauges on pile AR at depths of 10 mm and 30 mm and one on pile BR at 110 mm failed (Figs: 4.13a & b, 4.14a & b). The pile behaviour was similar to the previous test, but the magnitudes of bending moment and deflection were greater throughout. The clamping moment on the rear pile remained the critical value for design, although the maximum value observed in these piles in the stiffer substratum was proportionally larger. At maximum load, the LVDTs indicated a pile group rotation of 0.3°. A total rotation of 0.33° would have given zero pile tip deflection.

4.3.8 Test 10
With a deep clay foundation similar to test 5 (Fig: 4.12), the same pile group and a comparable test procedure, the foundation was consolidated for 6.5 hours (\(t_p = 7.4\) yrs) before the load was increased in stages to 103 kPa (net) over 10 minutes (\(t_p = 69\) days). An unload (to 0 kPa, net)–reload loop was conducted and a series of vane tests were carried out at six depths during a period of 83 minutes (\(t_p = 1.58\) yrs) further consolidation under this load. Thereafter, load increments were applied up to a maximum of 199 kPa (net) during 12 minutes (\(t_p = 83\) days) with unload–reload cycles at 151 kPa and 199 kPa (net). A penetrometer test was conducted during the final 30 minute (\(t_p = 0.57\) yr) consolidation. Hand vane shear tests, radiography and sectioning were carried out following swing–down as before.

The piles behaved as short and rigid members, effectively fixed to the pile cap. The LVDT data
indicated marginal rotation about the pile tip as net load increased from 0 to 151 kPa (Figs: 4.15a & b, 4.16a & b), after which translation of the group and further rotation was observed. The greatest pile cap rotation calculated from the LVDT data was 4°. The measured bending moments supported this conclusion. The maximum bending moments occurred in the front piles at 151 kPa net load as the soil sheared past the pile. The bending moments on the rear piles increased monotonically. The maximum moment in the front pile occurred at about 40 mm depth. This was noticeably less than the clamping moment in the rear pile.

The derived lateral pressure distributions for the front and rear piles were not consistent in either shape or magnitude. It was expected that the pressures on the rear pile would be lower than those acting on the front pile. However, the high values of root mean square error observed after fitting a polynomial to the bending moment data indicated that the curve–fit was inaccurate. This was particularly true for pile AR (Fig: 4.15b).

4.4 Vane shear test results

4.4.1 Theoretical and empirical considerations

The aim of the vane shear tests was to estimate $c_u$ for the kaolin at the circumference of the pile and for bearing capacity calculations, based on the standard assumptions that $c_u$ was uniform on shearing surfaces which formed a right cylinder. This value of $c_u$ was ascribed to the mid–height of the vane, which was reasonable for a linear variation of $c_u$ with depth. Disturbance due to insertion and the subsequent time delay to commencement of testing, effects of rotation speed and vane aspect ratio contribute to the difficulty of interpretation of field tests, which combine with the scaling problems inherent in using small instruments in centrifuge model testing.

The 'standard' field vane was described by Mahmoud (1988). Differences between it and the centrifuge vane (Fig: 3.9a) at prototype scale are listed in Table: 4.1. The vane heights were similar, but the differing aspect ratios will affect the proportion of shearing resistance accounted for in the vertical and horizontal planes. The 18 mm diameter centrifuge vane with an aspect ratio of 0.77 placed greater emphasis on the latter compared with the standard vane with a ratio of 2 (Mahmoud, 1988). For shearing laterally around a vertical pile, it would be more
appropriate to revert to standard aspect ratios, but the choice of vane size was also guided by:

i) the desire to conduct several tests, to obtain as much data as possible in the clay layer,

ii) the relative influence of shaft and vane diameters on shaft friction (which should be minimised by the slip coupling, which permits measurement of torque for 15° rotation of the shaft before the vane begins to turn) and disturbance due to soil displacement,

iii) the minimum diameter for the shaft to allow for imposed torque and strain gauging,

iv) previous research on vane size and strain rate effects using this vane apparatus in the centrifuge (Almeida & Parry, 1983).

Measurement of $c_u$ is also sensitive to the dissipation of excess pore pressures generated by insertion of the vane. Recommended field practice (Mahmoud, 1988) is to wait for 5 minutes before starting the shear test, which is equivalent to $(5 \times 60)/(d_{vp}^2/d_{vc}^2) \approx 23$ seconds in these centrifuge model tests. This time delay is somewhat arbitrary, because different values of coefficient of consolidation, $c_v$, are experienced in the field, and although vane rotation was commenced immediately power supplies were switched over, consolidation was further advanced than was normal in a standard test.

During rotation at fast strain rates, viscosity will affect shearing behaviour, tending to overestimate $c_u$. Similarly, at slow strain rates $c_u$ will be increased as drainage takes place. Roy & Leblanc (1987) recorded the smallest values of $c_u$ over a range of 5–100°/minute in field tests on a low plasticity clay (6% < PI < 19%), whereas Menzies & Merrifield (1980) showed $c_u$ decreasing as strain rate was reduced for Brent Knoll clay (PI = 37%).

In the centrifuge, the scaling effects relating to time complicate the issue as follows. Time (strain rate) scales as $t_c = t_p$ (Randolph, 1979), which will be appropriate when considering the velocity at the tip of vane on the shearing surfaces. However, diffusion occurs $100^2$ times faster in these centrifuge model tests for a correctly scaled model of the prototype. This means that the rotation rate at which the minimum recording of $c_u$ is found would be expected to be higher in the
centrifuge than for comparable field tests at 1g. Thus, for a smaller vane rotating at the same speed as a larger vane, the pore pressure will dissipate faster, but the blade velocity will be slower, implying greater drainage but reduced viscosity effects. Vane rotation rates in kaolin, (PI = 31%, Airey, 1984) were investigated in the centrifuge by Almeida & Parry (1983) for the 14 mm x 18 mm diameter vane. The lowest value of $c_u$ was observed while rotating at 72°/minute, whilst the standard field rate was 6°/minute (Mahmoud, 1988).

Undrained shear strength is defined in terms of undrained behaviour, but it is also dependent on stress history (Terzaghi, 1936), sample orientation, initial effective stress states and stress paths to failure. It is not an absolute soil property (Wroth, 1984), and it is accepted that the estimation from vane shear test results may be complex. Therefore, comparison was made with a relationship between $\frac{c_u}{\sigma_v}$ and OCR quoted by Schofield & Wroth (1968):

$$\frac{c_u}{\sigma_v} = a \text{OCR}^b$$

(4.4)

where $a$ and $b$ are constants. Nunez (1989) recommended values of $a = 0.22$ and $b = 0.62$ for kaolin, based on a literature survey. Phillips (1989) proposed values of $a = 0.19$ and $b = 0.67$ after conducting laboratory and centrifuge tests using the Almeida vane. Critical state theory may be used to derive values for $a$ and $b$, but comparisons with experimental data are preferred because theory overestimates $c_u$, especially at high OCR, for resedimented clays (Randolph & Wroth, 1981) and natural clays (Almeida, 1982).

4.4.2 Results

The results of the in-flight and post-flight vane shear test results were plotted in Figs: 4.17, 4.18. Shaft friction was measured during the first 15° rotation before the slip coupling engaged the vane and this was deducted from in-flight values of peak and residual $c_u$. The post-flight tests confirmed that the kaolin had softened considerably during swing-down.
In centrifuge tests 4, 6 and 8, the deepest vane test in the soft clay layer was too close to the sand or the base of the box. These values have been reported, but ignored in the determination of the design $c_u$ profile. Although the depth of soil between each shear test was not less than half the height of the vane (0.7 m at prototype scale), this was subsequently thought to be too little to avoid influence between adjacent shear tests, causing a reduction in the shear strength on the upper horizontal shear surface. However, the measurement of the shearing resistance on the horizontal faces is less important for deformation around a cylindrical shape with a vertical axis.

The peak vane shear results were plotted on logarithmic scales as $c_u'/\sigma_{v,\text{max}}$ against 1/OCR on Fig: 4.19 and these were bracketed to give extreme values of $a$ and $b$ from Eqn: 4.4. The upper envelope shows $a = 0.22$, $b = 0.76$, whereas the lower limit almost coincides with Nunez' (1989) recommendations of $a = 0.22$, $b = 0.62$ for design of axially loaded tension piles. Phillips (1989) proposed values which are considerably lower. Testing procedures were the same for all models centrifuged by the author, so the results were an indication of the uniformity of sample preparation, and the reproducibility of data from the vane apparatus.

The bracketing values of $a$ and $b$ were used to calculate a profile of $c_u$ with depth (prototype) (Fig: 4.20a), on which the peak results were also plotted. It was the intention to find a sensible upper bound to $c_u$ to estimate the maximum lateral pressure acting on the pile in the soft layer as the soil sheared past the pile. Allowing for the inflated values due to the proximity of the sand layer, $c_u$ increased approximately linearly with depth, and the values recommended for use in the analyses were 9.5 kPa at the surface, increasing by 1.75 kPa/m to 20 kPa at 6 m depth.

Consideration should also be given to a lower bound value of $c_u$ for calculation of embankment bearing capacity and stability. Allowance may be made for the increase in strength under the embankment due to consolidation, since it is the value of $c_u$ as the embankment approaches failure which is the appropriate choice. However, Bjerrum (1973) observes that $c_u$ determined from an in-situ vane shear test is generally not equal to the average value mobilised at failure in the field. For clays of low plasticity (PI ≤ 20%), he found that the vane shear test underestimated
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c_
 back analysed from a number of embankment slope stability failures, whereas for higher plasticity clays (PI ≥ 20%) the vane shear strengths overestimated the strength available (Fig: 4.20b). Based on the field vane shear tests carried out for the cases described in Bjerrum’s study, clay with PI = 31%, such as kaolin (Airey, 1984), c_u vane would be reduced to 93% of the original value. Since Bjerrum’s (1973) empirical findings have not been confirmed in the centrifuge, and the factor is less than 10%, it was not applied to the raw data obtained in–flight.

4.5 Cone penetrometer test results

4.5.1 Theoretical and empirical considerations

A series of miniature piezo–probe penetrometer tests were carried out to observe the behaviour of the piezocone in the clay, to obtain strength parameters for comparison with vane test results and to estimate the stiffness, G, in the sand. The interpretation of these readings is largely empirical in the field, and similar problems such as penetration speed, size and scaling effects were experienced for these in–situ tests as for the vane shear tests.

The diameter of the probe was 12.7 mm, which scaled to 1.27 m at prototype level for these model tests. Since the traditional Standard Penetration Test, piezocone or cone penetrometer probes (the latter having an area of 10 cm²) are considerably smaller, it is difficult to make a comparison. But the value of the piezo–probe tests in the centrifuge lies in the verification of the consistency and repeatability of the modelmaking, rather than the explicit determination of soil strength.

The probe was driven through the top sand layer, the soft clay layer and into the top 50–70 mm of the sand substratum at an average rate of 4–10 mm/second (2 mm/second in the all–clay layer). But, there were problems in obtaining reasonable pore pressure data, largely because the porous stone at the tip of the probe was either difficult to de–air or else suffered from abrasion of the porous surface and thus a reduction in the porosity of the sintered material. Thus, a time lag may have occurred between the actual and reported values of pore pressure, preventing accurate calculation of the total penetration resistance, which should be corrected to allow for the pore
Centrifuge modelling — presentation of test data

pressure, $u_b$, acting on the silicone seal behind the apex (Almeida, 1984).

Powell, Quarterman & Lunne (1988) report implications of the position of pore pressure sensing elements on cone penetration data in different clays from the United Kingdom. Measurement of pore pressure varies greatly depending on the position of the porous band, so that $u_b$ would be overestimated by a piezo–probe with a porous sensor in the tip. The error increases with the degree of overconsolidation. As a consequence of these problems, the core penetration data was only used for predicting sand stiffness.

Phillips & Valsangkar (1987) reported a series of penetration tests in granular soils, in which gravity level, probe size, boundary effects, grain size variations, sample density and dilatancy characteristics, penetration rate and stress cycling were investigated. One finding of these tests was that the end bearing resistance measured by the probe increased when it was less than 10–12 diameters (127–152 mm) from a rigid base. This implied that the whole sand substratum, which varied between 75–100 mm deep, was affected together with most of the clay layer. The presence of the rigid base was likely to increase the end bearing resistances observed and hence overestimate the soil stiffness. However, analysis was carried out nonetheless.

4.5.2 Results

The end bearing resistance in the lower sand layer, measured by the load cell, is plotted in Fig: 4.21 for all tests, while the results from the clay layer were ignored. The penetration rate was kept as constant as possible using the 30 V supply system to drive the apparatus. However, back calculation of driving rates showed variations between 10 mm/second and 2 mm/second in the clay and clay—on—sand foundations. The relative density, $D_r$, of the sand sample was intended to be about 50%. An average of 47.9% was achieved in five tests (Table: 3.3) but this varied widely, due to the placement method and the calculation of sand void ratio, and this was demonstrated by a standard deviation of 9.7%, and was apparent in the measurement of end bearing resistance.
4.5.3 Estimation of shear modulus

In order to estimate the shear modulus from the penetrometer test results, which essentially reproduced an axial loading problem, not a lateral one, it was necessary to invoke some empirical relationships. From Meyerhof (1976):

\[ q_c \leq 4N \]  \hspace{1cm} (4.5)

where \( q_c \) was the unit bearing capacity of the cone in tsf.

From Fig: 4.21, a lower bound value of \( q_c \) at mid-depth of a 10 m deep sand substratum was 3 MPa, which implied a loose soil with \( N = 7 \). Wroth et al (1979) suggested that:

\[ \frac{G_{\text{max}}}{p_a} = 120 N^{0.77} \]  \hspace{1cm} (4.6)

Therefore, \( G_{\text{max}} = 54 \) MPa at 5 m depth of sand layer for \( N = 7 \). Previous equations adapted by Wroth et al (1979) for small strains in sand gave:

\[ \frac{G}{p_a} = 710 \left( \frac{p'}{p_a} \right)^{0.435} \left[ 0.9 + \frac{D_f}{500} \right] 1.23 \]  \hspace{1cm} (4.7)

so that for the top and bottom of the 10 m deep sand layer, where \( p' \) was estimated at 70 kPa and 228 kPa respectively, and \( D_f \) was approximately 50%, then \( 75 < G < 126 \) MPa.

However, lateral strains at the clay–sand interface will be relatively large and Eqn: 4.7 would not be applicable. Shear modulus will reduce with increasing shear strain (Fig: 4.21b) and a lower value should be chosen. For tests 4 and 7, maximum pile deflection at the top of the sand layer, \( u_s \approx 0.4 \) mm. A simple assumption that a characteristic \( \gamma \) is approximately \( u_s/d \), would give \( \gamma \approx 3\% \) for a laterally loaded pile. Stroud (1971) quoted \( \tau/\sigma_{yy} \) against \( \gamma \) in the simple shear apparatus for \( \sigma_{yy} \approx 160 \) kPa for 14/25 Leighton Buzzard sand with \( e = 0.747 \), \( (e_{\text{min}} \approx 0.5 \).
\( e_{\text{max}} \approx 0.82 \) (Mak, 1983), \( D_r \approx 23\% \). Although this sand is coarser and less dense than that used in the model tests, the results may be used as a guide: at \( \gamma = 3\% \), \( G = 2.2 \text{ MPa} \), and so \( G_o = 2 \text{ MPa} \) was adopted with \( m = 10 \text{ MPa/m} \). Values of \( G \) in the sand substratum are compared in Table: 4.2.

4.6 Photography

4.6.1 Test records

Photographic records were kept of each test, including the front elevation before and after the test and a close-up view of the row of piles. Typical examples for tests 6 and 10 are shown in Figs: 4.22a, 4.22b, 4.23a, 4.23b. Black and white photographs were taken in-flight using a Hasselblad camera mounted above the centrifuge pit.

4.6.2 Radiography

4.6.2.1 Single row of piles

Radiographic techniques (James, 1973a) were used to reveal the location of the lead threads which were inserted into the clay during model making. After the test had finished, X-ray photographs were taken to permit investigation of the soil–pile interaction.

Although previous researchers (James, 1973b) have described methods of estimating local strains using the radiographs, together with the associated errors, the purpose of these investigations was to examine continuity of deformation and the existence of clay squeeze—through between the piles. Clear evidence of soil shearing close to the piles was seen in Fig: 4.24 for test 4, with the localised displacements confined to an area between 1–2d around the edge of each pile. Ultimate lateral pile capacity was reached in this test, and the soil between the piles was deforming uniformly outside this region. Test 6 (Fig: 4.25) shows similar conditions for the deep soft clay layer, with pronounced ruptures adjacent to each pile, whereas the results from test 7 (Fig: 4.26) relate to conditions in which there was less soil deformation.
4.6.2.2 Pile group

The radiographs of the lead threads inserted close to the pile group for test 8 confirmed that the front row carried the majority of the lateral pressure loading (Fig: 4.27a). Here, soil sheared past the piles. Considerably less displacement was noticeable adjacent to the rear row. For undrained foundation behaviour, with no volume change, this discrepancy could be blamed on the free boundary condition at the top surface. An excavation around the piles (test 8) shows the relative displacements more clearly (Fig: 4.27b). Patterns for the deeper clay layer (Fig: 4.28) and the all-clay (Fig: 4.29) models were similar. A slight frictional restraint was imposed at the edge of the models by the strongbox and perspex.

4.7 Summary

Results were presented for eight centrifuge model tests for simulated embankment construction adjacent to either a row of free headed piles or a pile group. The performance of piles at different pile spacings and foundation geometries was examined. Pile bending moment data was fitted by a polynomial, which was integrated twice to give pile deflections and differentiated twice to estimate the lateral pressure acting on the pile. By reducing the pile length in the stiff substratum beyond the critical length, pile behaviour changed from flexible to rigid.

Site investigations were conducted in-flight and post-flight, and recommendations were made for values of strength and stiffness parameters to be used in later analyses. X-ray radiography was carried out to reveal the continuity of lateral soil deformation around the piles following application of maximum load and completion of the centrifuge model test.
5.1 Introduction

Numerical analyses using finite element techniques have been particularly popular in recent years for verifying or refining design procedures. Based on the method described by Zienkiewicz (1977), a variety of finite element computer programs have been presented, with different facilities to suit a number of needs. But it is rare that problems are examined in such detail other than for research and development, owing to the cost entailed in preparing and running the analyses.

Since the behaviour of soil can be approximated by the use of an appropriate stress–strain law applied to discrete elements, the finite element method provides a valuable analytical tool for the interpretation of cases where unusual geometry or three dimensional effects are significant, and where realistic simplified models can be specified. It is particularly relevant when it is possible to compare or back analyse the performance of a well instrumented prototype, either full scale in the field or at model scale in the centrifuge. In calibrating these tests, design procedures may be developed and proven. This was the purpose of the analyses executed during this research programme.

For analyses using critical state models, Phillips (1986) and Kusakabe (1982) have shown that provided the mesh is fine enough in the areas of high strain gradient, and the loading increments were small enough, good agreement has been reached between numerical and experimental results. Clearly, the choice of element and the mesh design has to reflect a compromise between an acceptable degree of accuracy and computing costs. For the research described here, linear strain elements with a quadratic variation of displacement were considered to be sufficient. 6 noded triangles with 12 degrees of freedom (Springman, 1984), and 20 noded brick elements with linearly varying pore pressure and 68 degrees of freedom were used.

5.2 Background

Initially, the behaviour of a single row of piles was examined in two dimensions using a sheet pile
wall of equivalent bending rigidity to the individual soil and pile bending rigidities (Fig: 5.1). Wall bending moments, deflections and soil deformations were calculated as an embankment was built, in sand, adjacent to the wall (Figs: 5.2a & b). Replacing the soil–pile structure by a plane strain wall was first proposed by Randolph (1981b) and Naylor (1982). They stated that this approach might be valid provided the soil did not squeeze through between the piles, and provided that the soil and piles effectively moved together as a wall. However, it was clear from subsequent research (Springman, 1987a & b) that the soil–pile interaction was controlled almost entirely by three dimensional effects, and therefore, that this approach was unsatisfactory. In consequence, the earlier plane strain analyses have been mentioned only briefly below.

A three dimensional analysis was conducted on a longitudinal section through a single row of piles, over a width of half the pile spacing, from the pile centreline to the midpoint between the piles. Mathematical analysis techniques which consider a half space rather than a full section have been examined by Smith & Hobbs (1974), who stated that the differences between these types of analyses were small. The geometry of the centrifuge model tests was reproduced and the loading was applied using a normal surcharge over the appropriate area (Fig: 5.3).

The objectives were:

i) to examine the lateral pressure acting on the piles in the soft clay layer, assuming that the clay actively loads the pile, which in turn loads and is resisted by the sand,

ii) to evaluate pile bending moments and deflections,

iii) to observe and analyse the soil–pile interaction.

The ultimate pressure on the piles in the clay layer can be estimated using upper and lower bound calculations (Randolph & Houlsby, 1984). These will be discussed in Chapter 6. This condition dictates the maximum pressure on the pile and hence the largest values of pile bending moment and deflection. Numerical proof of this case was not required at this stage. However, a method for determining the working pressure distribution on the pile in the soft, clay layer was more ambiguous. Linear elastic soil was used in these analyses in order to simplify conditions in
accordance with the design approach to be proposed in this thesis. At this stage, the analyses concentrated solely on a single row of piles, without considering the effect of a parallel row of piles connected by a pile cap.

5.3 Program details

The CRISP (CRItical State Programs) finite element suite was used for the analysis. It was initiated by Zytynski (1976) and developed further by Britto & Gunn (1987). Stress–strain soil models, notably the critical state and elastic perfectly plastic models, are available, although only the linear elastic option was employed for the three dimensional analyses. Undrained, drained, or fully coupled (Biot) consolidation analyses of axisymmetric, two or three dimensional solid bodies were on offer, with computation using double precision.

The input of geometrical data and the analysis of the results used the FEMGEN (1985) pre– and the FEMVIEW (1986) post–processors. These proprietary packages were installed together with an earlier version of CRISP on the Engineering Department IBM 4341 main frame computer (Springman & Britto, 1988). Three dimensional analyses were carried out on data transferred to the University IBM 3084 main frame computer (Britto, 1988), whilst the two dimensional work was conducted entirely on the old University IBM 3081 main frame computer (Britto, 1983). The present study was the first to employ 3D CRISP.

5.4 Two dimensional analyses

Randolph (1981b) and Naylor (1982) suggested that the three dimensional problem concerning the interaction between soil and a row of piles could be modelled using a two dimensional sheet pile wall. The bending rigidity of the piles and soil were summed and replaced by an equivalent value for a wall of the same external dimensions (Fig: 5.1) so that:

\[ E_s I_s + E_p I_p = E_w I_w \]  
(5.1)

This assumption was acceptable if the piles and soil moved together, however, differential
movement and hence soil squeeze—through would invalidate it. Later three dimensional analyses confirmed that differential movement occurred between the pile and the soil, even for elastic soil behaviour, and that there was insufficient arching of the load onto the piles to ensure plane strain.

A combination of different embankment and foundation stiffnesses were examined for two types of substratum, namely soft clay or soft clay overlying a stiffer sand layer. They were based on the results of centrifuge model tests 1 and 2 (Springman, 1984) in which a sand embankment was poured in-flight to surcharge one side of a row of piles. In practice, it was difficult to control the positioning of the centrifuge model embankment, which spilt through between the piles above foundation level (Fig: 3.3). Furthermore, the stiffness of the embankment relative to the clay would imply that, for compatibility of (large) strains at the interface between sand and clay, the embankment would become plastic. Currently the constitutive models available for use with CRISP are not capable of modelling the rupture surfaces that would develop. In the next centrifuge test series, the embankment loading was replaced by a normal load, which was generated by varying the pressure of compressed air inside an airtight rubber bag, held inside a restraining box.

When the wall bending moments were factored to give equivalent values for the individual piles, agreement between centrifuge results and a linear elastic analysis at working load, was quite good. However, by virtue of the initial plane strain assumption which required piles and soil to move together, it was inconsistent to factor the deflections in the same way, so the results were an average value of the lesser movement of the piles and the greater displacement of the soil. As expected, these numerical results heavily overpredicted the experimental pile deflections. Fig: 4.26 showed the X-ray of the internal pile—soil deformations for test 7. In this case the bag loading had not reached the ultimate level for this pile configuration, and lateral pressure on each pile was still increasing, but the soil was already squeezing between them.

The effect of varying the embankment stiffness in the numerical analyses was quite marked. The
lateral movement of the top of the clay layer, and hence the pile, at the toe of the embankment was smaller as the embankment stiffness increased, and this affected pile bending moments. Also, the vertical effective stresses at the base of the embankment were less than expected by computation based on unit weights and height of embankment, probably due to longitudinal arching within the embankment (Fig: 5.4), and so the vertical deformations were also less. Therefore, the embankment described by a normal load would give a conservative model of pile behaviour and soil deformation. However, the sand embankment generates shearing at the top of the clay layer, creating an inclined load on the foundation (Fig: 5.5) which implies that the bearing capacity aspect will be less conservatively modelled by adopting an equivalent normal load. Further comparison between centrifuge model test data and finite element analyses for foundation loading by means of a sand embankment was curtailed because experimental difficulties in controlling the in-flight construction led to complicated pile–embankment boundary conditions.

As the modulus, $E_s$, of the base layer of sand was increased, the maximum bending moment increased and was located at a shallower depth in that layer. In general, for the complete range of stiffnesses examined (average values of $E_s$ at mid–depth from 8 MPa – 417 MPa), this was found to occur between 0.5–3d below the clay–sand interface.

5.5 Three dimensional analyses

5.5.1 Mesh

The pile and foundation were modelled in four horizontal layers using 120 elements comprising 20 noded linear strain bricks with a total of 220 corner nodes (Fig: 5.3). The boundary position, pile diameter and spacing, foundation and loading geometries were set up to reproduce the centrifuge model tests which used the air bag loading system, and in particular, the tests (4 & 7) which modelled a long pile. All the dimensions were scaled up to the prototype values.

The foundation was 6 m of clay, modelled by layers of 2 m and 4 m in depth, overlying 10 m of sand. The only difference between the two tests was the number of piles in the row and hence the pile spacing to diameter ratio. In test 4 there were 3 piles with an s/d = 5.25, and in test 7 there
were 5 piles with an s/d = 3.15. A longitudinal section over a width of half a pile spacing was used. In areas of high strain around the piles, and at shallow depths under the load, the size of the linear strain elements was reduced. This was necessary to cope with the extreme variation of strain over a small distance.

The purpose of the analysis was to examine performance at working loads. An elastic model was adopted: it was also considered permissible to model total adhesion between the soil and the pile. Although link or slip elements, which allow differential movement on a soil-structure boundary, are available for two dimensional versions of CRISP, they were not yet mounted for the three dimensional option. Such elements would have been necessary if exploration of the non-linear soil behaviour at loads approaching the ultimate limit had been desired.

In some laterally loaded pile cases, when the pile is absorbing load from a structure and transmitting it to the soil, it is common to find a gap opening up behind the pile. Research by Baguenin et al (1977) was analysed by Fleming et al (1985) who quoted an approximate formula relating the change in normal stress or frontal reaction (Fig: 5.6) in front of and behind the pile to the differential horizontal movement between the pile, \( \delta u_p \), and the adjacent soil, \( \delta u_s \), so that the local stress change, \( \Delta \sigma_n \), along the pile centreline at \( z = 0 \):

\[
\Delta \sigma_n \approx G(\delta u_s - \delta u_p)/2r \tag{5.2}
\]

They comment that it is relevant up to values of \( \Delta \sigma_n = 2c_u \), at which point a gap may begin to open. Ultimate failure, when soil shears past the pile, would be expected at values of net lateral pressure across the pile of 10.5\( c_u \) (Randolph & Houlmsby, 1984). For example, when \( G/c_u = 100 \), \( (\delta u_s - \delta u_p)/d = 2\% \). However, for the embankment surcharge problem, the soil was actively loading the pile which in turn was passively loading the soil on the other side. So it was unlikely that a mechanism, which allowed the formation of a gap, would evolve.
5.5.2 Pile

The 1.27 m diameter Dural pile with 0.122 m wall thickness was represented as an isotropic elastic solid pile of the same external dimension, with an elastic modulus and second moment of area which gave the same bending rigidity as that of the tubular pile. The parameters are listed in Table: 5.1, and the pile is the order of $10^4$ stiffer than the upper, clay layer, which confirmed the need for double precision computation.

The sides of the pile were curved, and a semi–pointed tip was defined, decreasing the cross sectional area from a circle of 1.27 m diameter, to a square of side 0.2 m at the tip, over the bottom 1 m of the 16 m length of pile as shown in Fig: 5.3. This refinement was adopted after experience with the two dimensional sheet pile wall, which exhibited a fixing moment at the base of the wall before a pin–jointed tip was prescribed (Springman, 1984). Unfortunately, the number of elements which would define the mesh geometry were limited by computing restrictions, and so the relative importance of minimising this boundary effect mitigated against the choice of two layers of sand of almost equal depth. In consequence, the choice of a single linear strain semi–cylindrical pile element in the sand layer produced a coarse mesh which was less accurate, but the behaviour which was of most concern was that in the upper soft clay layer. So, the two elements which described the pile in the lower sand layer were divided into sections of 9 m and 1 m depth to give the best reproduction of the test conditions.

5.5.3 Soil models

Pile driving will cause local disturbance around the pile shaft resulting in an increase in pore pressure for soils with $1 \leq \text{OCR} \leq 10$, (Nunez, 1989) and a change in soil properties during the subsequent consolidation. Baguelin et al (1977) introduced a cylindrical segment around the pile, in which $E$ and $c_u$ were decreased by the same factor, $\beta$, and examined pile performance under these conditions. However, $c_u$ is likely to fall immediately following installation and increase above its initial value with time (Randolph, Carter & Wroth, 1979), and this generally occurs before the pile is loaded laterally. Since the parameters quoted for use in pile design are usually based on far field values, and the selection of reduction factors will be subjective without actual
measurement, it was assumed that the chosen soil properties and in-situ stresses were representative of the entire foundation.

5.5.3.1 Clay

Initially, an undrained linear elastic soil model was adopted for the clay layer. However, there was some instability in the calculation of pore pressure because of the values chosen for the bulk modulus of the water. In consequence, a drained consolidation analysis was performed with a drainage period of 1 second per increment. Approximate values of permeability were chosen for the kaolin based on the values determined from the centrifuge model consolidation.

The stiffness parameters assigned to the clay were guided at first by limits of:

\[100c_u < G < 200c_u \quad \text{or} \quad 200(1+\nu)c_u < E < 400(1+\nu)c_u\]  \hspace{1cm} (5.3)

Wroth et al (1979) recommended these values for relatively small strains, but for rather larger strains created by pile displacements of \(\delta u/d \approx 3.5\%\) at the surface of the clay and \(\delta u/d \approx 1.5\%\) at the interface between the clay and sand layers, it was thought necessary to reduce these extremes. Comparison with past analyses (Springman & Randolph, 1985), and consideration of the observations by Wroth et al (1979) that the shear modulus and \(p'\) were, in general, described by:

\[50p' < G < 100p'\]  \hspace{1cm} (5.4)

led to the choice of an elastic modulus, \(E\), of 2 MPa (for \(\nu' = 0.33\)) at the surface increasing at a rate of 533 kPa/m depth, to give a value of 5.2 MPa at the interface between the clay and sand layers. This conveniently matched the stiffness selected to represent the top of the sand, and taking \(c_{uo} = 9.5\) kPa, \(dc_u/dy = 1.75\) kPa/m (Section 4.4.2) implied that:

\[75c_u < G < 100c_u\]  \hspace{1cm} (5.5)
where the undrained shear strength was taken to be that measured by the vane shear test apparatus, in-flight, during centrifugal modelling. The choice of drained properties did not preclude the clay from behaving in an undrained fashion for short time steps of the order of 1 second. The parameters are listed in Table: 5.2.

5.5.3.2 Sand

This layer was also modelled as an isotropic elastic medium, with shear modulus increasing with depth. However, the sand was permitted to drain. The results from the in-flight penetrometer tests from the centrifuge modelling led to a choice of shear modulus, \( G \) of 2 MPa at the top of the sand layer, increasing at a rate of 10 MPa/m depth. This modelled the softening at the interface between the layers in a region where the pile was imposing high strain levels on the sand. The parameters used for this layer are listed in Table: 5.3.

5.5.4 Boundary conditions

The boundary conditions chosen for the model were intended to represent those imposed by the rigid centrifuge strongbox with frictionless internal surfaces. Although the strongbox walls were greased, it was clear from the post-test X-rays (Fig: 4.24) that the large lateral strains around the piles in the clay layer were not matched by those at the junction between soil and strongbox, and that there was some friction which restrained the soil from moving freely.

Whether the plane strain loading assumption defines the worst loading case on the piles for a prototype embankment, will depend on the positioning of piles across the abutment. Allowance should be made for loading from the embankment side slopes. However, it is likely that replacing rigid and frictionless boundaries by the three dimensional conditions pertaining in the field will allow some movement and stress relaxation in a transverse direction, which would reduce lateral pressure on, and hence displacement of, the piles.

The soil and tip of the pile were restrained in all directions at the base of the model. This was thought to be realistic since there was no visible sign of horizontal displacement at the base of the
sand layer in the centrifuge model tests. Thus the pile tip was pinned at the base. The difference between a pinned or a free tip fixity condition was examined and the effect on pile bending moments, lateral pressure on the pile, and differential pile–soil displacements was minimal. The soil was permitted to slide in the plane of the walls, but not to separate from them.

5.5.5 Loading

A 1 m layer (prototype scale) of loose sand had been placed over the top of the clay layer in the centrifuge model tests to simplify model making and to ensure that the top of the clay layer was not at zero effective vertical stress. The sand was assumed to have no strength and was replaced in the numerical analyses by a normal load of 15 kPa which acted over the top of the foundation. This measure was an economic way of reducing the number of elements.

The water table was level with the top of the clay layer. This was the case in the middle of the centrifuge models close to the piles. Errors due to curvature of either the stress field or the water surface, which was 1.5 m higher (prototype scale) at the edge of the strongbox, have been discussed in Chapter 3, and were disregarded here.

The embankment loading was modelled by placing a normal load, \( q \), over the same area as that covered by the air loading bag in the centrifuge model tests. The complicating effects of shear on the base of, or arching within, a sand embankment were ignored, as was the case in–flight.

In a bridge abutment in the field, the configuration of the piles and pile cap would be different from the case examined in both the centrifuge model tests and the numerical studies. In reality, the pile cap would be in contact with the top surface of the foundation, quite possibly applying some vertical load. So, any soil displacement at this interface would be mainly horizontal, or vertically downwards due to settlement. Since the early finite element results confirmed that there was significant upward movement of the soil around the pile because the front edge of the load was offset 1.2d back from the front of the row of piles, an alternative surcharge loading scheme was used in which the load was almost anti–symmetrical about the row of piles. This ensured that the
soil deformation profile in the vicinity of the pile was more realistic, and allowed an assessment to be made of the importance of this.

5.5.6 Details of three dimensional analyses

Analyses were conducted to investigate the behaviour of the numerical model under this loading. The pile spacing, and hence the width (z coordinate) of the model, and the stiffness of the pile were the only parameters to be adjusted. In the former case, the width of the test section, over half the pile spacing, was either 3.33 m or 2.00 m, representing pile s/d ratios of 5.25 and 3.15. The comparable in-situ stresses in the soil for the same loading case were calculated when there was no pile by replacing the material properties in the pile elements by those of the adjacent soil.

The results of these computations are discussed with respect to the lateral pressure on the piles, the pile bending moments and deflections, and the soil–pile interaction. However these analyses were complicated by the free soil surface around the piles.

5.5.7 Horizontal pressure on the pile

5.5.7.1 Theory

Baguelin et al (1977) discussed the components of horizontal pressure acting on a pile for several conditions of which Fig: 5.7 shows those considered here for:

i) a rigid disc pushed through an elastic, perfectly plastic medium,

ii) application of force by a uniform load, P, (kN/m) down the length of the pile,

iii) plane strain conditions,

iv) intact soil, therefore there were no areas of reduced E or \( c_u \),

v) perfect adhesion between soil and pile.

Soil stresses were derived from single Airy functions, in polar coordinates so that the total resistance was due to a combination of the frontal reaction and tangential or frictional resistance (Fig: 5.6). Hence, the maximum frontal reaction occurs when \( \theta = 0^\circ \), and is equal to \( \Delta \sigma_n \), whilst the tangential reaction is zero. At \( \theta = 90^\circ \), the roles are reversed and the frontal reaction is zero and the tangential reaction is a maximum, but also equal to \( \Delta \sigma_n \). At \( \theta = 180^\circ \) the lateral stress increment is tensile (\( -\Delta \sigma_n \)). The radial stress acting at the pile–soil interface was:
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\[ \sigma_{rr} = \frac{P}{2\pi r} \cos\theta \]  
(5.6)

The shear stress was:

\[ \tau_{r\theta} = \frac{P}{2\pi r} \sin\theta \]  
(5.7)

so that the lateral resistance acting on one side of the pile was uniformly distributed over the surface area of the section, and can be written as:

\[ \Delta\sigma_n = \sigma_{rr}\cos\theta + \tau_{r\theta}\sin\theta = \frac{P}{2\pi r} \]  
(5.8)

The average lateral pile pressure across the central cross-section of the pile, \( p \), is described by:

\[ p = \frac{P}{2r} \]  
(5.9)

so that (Eqns: 5.8 & 5.9):

\[ \Delta\sigma_n = \frac{p}{\pi} \]  
(5.10)

The lateral pressure acting on the pile can be re-interpreted in the coordinate system employed for the finite element analyses as:

\[ p = \int_0^r \Delta\sigma_{xx} \, dz + \int_{X-r}^{X+r} \tau_{zx} \, dx \]  
(5.11a)

where:

\[ \Delta\sigma_{xx} = \left[ \frac{\sigma_{xx}}{X+\sqrt{r^2-z^2}} - \frac{\sigma_{xx}}{X-\sqrt{r^2-z^2}} \right] \]  
(5.11b)

where \( X \) is the \( x \) coordinate at the centre of the pile, see Fig: 5.6.

5.5.7.2 Evaluation

The horizontal effective stress, \( \sigma_{xx}' \), and pore pressure at the integration points closest to the pile, were used to calculate the profile of net horizontal total stress, \( \Delta\sigma_{xx} \) (Eqn: 5.11b), acting across the pile in the direction of the loading. The shear stress close to the pile—soil interface at these same
integration points was also established, and then these two components were summed (Eqn: 5.11a) to give a single value for an average lateral pile pressure across the diameter of the pile. The linear strain elements have 21 integration points, which are spread over 3 layers. Therefore, the pressure can be defined at these depths, namely, 0.23, 1.0, 1.77, 2.46, 4.0, 5.54 m in the 6 m layer.

5.5.7.3 Results

The components of horizontal pressure due to the normal stress and shear stress are shown on Figs: 5.8a, 5.8b for the two different pile spacings where $\Delta \sigma_{xx}$ was calculated by subtracting $\sigma_{xx}$ at the back of the pile from $\sigma_{xx}$ on the front for a common value of $z$. Instead of the sinusoidal distribution of $\Delta \sigma_{xx}$ (Fig: 5.6) predicted by Baguelin et al (1977), for plane strain behaviour with uniform loading per metre length of pile, the profiles varied with depth in the soft clay layer. The distributions of $\Delta \sigma_{xx}$ at the top and bottom of the clay layer, were fairly uniform across the pile from $\theta = 0$ to $\theta = \pi/3$, dropping off rapidly to zero at the side of the pile. In the middle of the layer, where the loading was greater, the distribution was more like Baguelin's, although the three dimensional effects and the positioning of an element boundary may be responsible for the rather higher values along the $z = 0.605$ m ordinate.

The shear stress, $\tau_{zx}$, was zero when $\theta = 0$ and $\pi$, building up to a maximum value at $\theta = \pi/2$, and dropping off rather sooner on the unsurcharged side of the pile. The component of pressure acting on the pile contributed by the shear stress was about 15% of the total compared with 50% obtained by Baguelin et al (1977).

The average lateral pressure distributions were roughly parabolic with depth (Figs: 5.9b & 5.10b), with the maximum value at $y = 2.5$ m. At $y = 5.2$ m, there was no net pressure on the pile, and below this depth, the pile applied load to the soil. As pile spacing increased from $s = 4.0$ m ($s/d = 3.15$) to $s = 6.67$ m ($s/d = 5.25$), the lateral pressure increased by up to 10%.

However, the lateral pressures measured in the centrifuge model tests were rather smaller for $s/d = 3.15$ for normal load $q = 100$ kPa (Fig: 5.10b). [The centrifuge model test data for
q = 93 kPa was factored up for a direct comparison with these linear elastic numerical analyses]. The accuracy of the calculation of experimental lateral pressures by double differentiation of the bending moment data was discussed in Chapter 4. They were heavily dependent on the curve fitting procedure, and inaccuracies in the pressure distributions were observed for test 4, s/d = 5.25 (Figs: 4.3a, b & c), and so they are omitted here.

5.5.7.4 Total horizontal stress distribution

The maximum frontal reaction due to a rigid disc moving, in plane strain, through an elastic material under a uniform load per unit length, describes the change, $\Delta \sigma_{nz}$, in the normal horizontal stress in the soil at the pile circumference where $z = 0$. The value of frontal resistance, $\Delta \sigma_{nz}$, varies across the width of the pile in the $z$ direction so that at $z = 0$, $\Delta \sigma_{nz} = \Delta \sigma_n$ and at $z = r$, $\Delta \sigma_{nz} = 0$. By superposition, this change must be equal and opposite in front of and behind the pile (Fig: 5.11). However, this was not found to be the case in these analyses when the front edge of the normal load was situated 1.2$d$ away from the front of the piles. The soil was permitted to squeeze vertically upwards, and this three dimensional effect created a larger value of $\Delta \sigma_{nz}$ on the surcharge side of the pile and a smaller value of $-\Delta \sigma_{nz}$ on the unsurcharged side of the pile.

Fig: 5.12a shows the contours of $\Delta \sigma_{nz}$ based on two analyses for $s/d = 3.15$ at a depth of $y = 2.45$ m, which coincided with the maximum lateral pile pressure. To calculate the values of $\Delta \sigma_{nz}$ at the integration points, a base value of $\sigma_{xx} = \sigma_{xxn}$ was required, with exactly the same loading, geometry and material properties, but no pile. Therefore, if $\sigma_{xxp}$ is the total horizontal stress when the pile is present:

$$\Delta \sigma_{nz} = \sigma_{xxp} - \sigma_{xxn} \quad (5.12)$$

From Fig: 5.12a, the maximum value of $\Delta \sigma_{nz}$ on the surcharged side of the pile was greater than 90 kPa, whereas the value was around $-60$ kPa on the unsurcharged side.

To confirm that this discrepancy was due entirely to the three dimensional nature of the problem, an additional analysis was conducted with an almost anti-symmetrical load. Fig: 5.12b shows the
contours of $\Delta \sigma_{nz}$ at $y = 2.45$ m for a normal load of 100 kPa. In this instance, the change in normal stress was almost equal with values of ± (60–70) kPa along the pile centreline at $z = 0$. So the net value of $\Delta \sigma_{xx}$ is about the same, allowing for the difference in positioning of the load.

Values of $\Delta \sigma_{nz}$ are presented for the integration points closest to the pile at all the integration point depths, for both pile spacing ratios and for the centrifuge model test loading case (Figs: 5.13a & b). Comparisons showed that up to 20% more pressure was exerted on the pile when $s/d$ was increased from 3.15 to 5.25, and that $\Delta \sigma_{n}$ at $z = 0$ on the surcharge side of the pile was roughly twice the drop observed on the other side, from depths $y = 1–4$ m (Figs: 5.13a & b).

5.5.8 Pile bending moments

The pile bending moments may be calculated from the expression:

$$\frac{M}{I_p} = \frac{\sigma_{yy}}{r \cos \theta}$$  \hspace{1cm} (5.13)

at the integration point depths in the pile zone. The bending moments are plotted on Figs: 5.9a & 5.10a for both pile spacing ratios. It is noticeable that the $y$ dimension of the elements (Fig: 5.3) has affected the spot values of bending moment, and that those at mid–depth in the element are the most realistic. The worst case is for the largest element, over the top 9 m in the sand layer, when the value just below the interface between the clay and the sand layers was underestimated whilst the value at the bottom of the element was overestimated. Clearly, the bending moment profile is continuous, and the best fit curve should be constructed through these numerical data points.

The bending moment generated in the pile was marginally greater as the pile spacing increased. This reflected the larger lateral pressure acting on each pile when the soil was relatively less reinforced, although this effect would become less obvious as the pile spacing increased beyond some limit. Comparison with bending moments obtained in the experimental tests showed that the numerical analyses overpredicted by 5–25% for $s/d = 3.15$ and underpredicted for $s/d = 5.25$ by 20% (Figs: 5.9a & 5.10a).
5.5.9 Deformations

The assumption that the three dimensional pile–soil interaction could be modelled using a plane strain sheet pile wall was disproved by Fig: 5.14. This showed the clay squeezing past the pile in the upper clay layers at depths of \( y = 0 \) and \( y = 2 \) m, while the pile was imposing load and causing the sand to deform further at \( y = 6 \) m, with virtually no lateral movement at \( y = 15 \) m.

A general view of the displaced mesh is shown in Fig: 5.15, with an enlargement of the region where the clay has heaved up around the pile. The \( z \) component of deformation in the region close to the pile is illustrated by Figs: 5.16a, 5.16b for \( s/d = 5.25 \), and indicates that there is some flow around the pile.

A clearer picture of the behaviour of the clay layer under load was achieved by isolating the net displacement in the layer, by subtracting the vertical deformation of the clay–sand interface. Figs: 5.17a, 5.17b show the pattern at different values of the \( z \) ordinate. The net strain in the \( x \) direction is shown in Figs: 5.18a, 5.18b and the shear strain in the \( z–x \) plane in Fig: 5.19.

The deflection of the pile, \( \delta u_p \), is plotted on Figs: 5.9c, 5.10c. As the pile spacing increased, the numerical pile deflection was only marginally larger. Also shown is the displacement of the soil at the same location along the pile centreline (\( x = X \)) if the pile was replaced by soil, which was therefore unaffected by the width of section and \( s/d \) ratio.

The centrifuge model test data is also plotted, but after factoring to an equivalent \( q = 100 \) kPa. This was permissible for comparison purposes since the numerical analyses only included linear elastic soil models. However, these numerical analyses were conducted for a consolidation time of 1 second at prototype scale, which modelled almost undrained behaviour. But, it took from 8–12 minutes in centrifuge model tests 4 and 7 to reach the value of \( q \) which was factored up to 100 kPa for direct comparison. This was equivalent to between 55–83 days at prototype scale. The data agreed extremely well for test 7 (Fig: 5.10c) but the centrifuge pile deflections were 50% less than those predicted for test 4 (Fig: 5.9c).
It was not possible to compare in-flight internal deformations with those indicated by the lead threads (Figs: 4.24–4.29) injected into the clay model, because the radiographs were taken after $q_{max}$ had been applied. At this stage there were significant plastic deformations in the region around the pile. Furthermore, the stress state of the sample changed after swing down.

5.5.10 Soil–pile interaction

The true rationale for the development of pressure on, and bending moments in, the pile can be demonstrated simply by considering the differential displacement between the pile and the soil. When the soil is flowing past the pile, load will be applied by the soil to the pile, and vice versa when the pile deflection is greater than that of the surrounding soil. Therefore, it is logical that the magnitude of the pressure acting on the pile is a function of this differential displacement, $(\delta u_s - \delta u_p)$, which is shown in Figs: 5.20a, 5.20b for both pile spacings. The horizontal soil movement was modelled by $\delta u_s$ when there was no pile adjacent to the embankment load at $x = X$.

5.5.10.1 Pressure–differential displacement equation

Eqn: 5.2 assumed $\nu = 0.3$, and was proposed by Fleming et al (1985) from work by Baguelin et al (1977). From this, a relationship was developed for the pressure on a pile in a soft layer and the differential movement between the pile and the soil. From Eqns: 5.2 & 5.10:

$$p \approx 1.57G(\delta u_s - \delta u_p)/r \text{ for } \nu = 0.3 \quad (5.14)$$

For different values of $\nu$:

$$p \approx 2.66G(\delta u_s - \delta u_p)/r \text{ for } \nu = 0.5 \quad (5.15)$$

$$p \approx 1.33G(\delta u_s - \delta u_p)/r \text{ for } \nu = 0.1 \quad (5.16)$$

However, this work (Baguelin et al, 1977) was relevant for plane strain conditions, where the soil at a distance of 30 pile radii was assumed to provide a rigid, no–strain boundary. The examples examined by these three dimensional finite element analyses were definitely not behaving two dimensionally, and so there was an increase in pressure at the mid–depth of the soft layer,
decaying to zero at the upper and lower boundaries. At the surface of the clay layer, there was no restriction against vertical movement, and thus any tendency to exhibit a net pressure at this level was removed. At the base of the clay layer, the pile was deflecting further than the surrounding soil due to the loading at shallower depths and the friction between the clay and sand layers. In consequence, the pile was applying load to the soil at this horizon.

5.5.10.2 Three dimensional variations

The results for the numerical analyses are plotted as a function of $G(\delta u_s - \delta u_p)/r$ against depth in Fig: 5.21a, 5.21b, and are compared against the lateral pressures obtained from the same analyses. A relationship between $p$ and $G(\delta u_s - \delta u_p)/r$ may be seen, such that:

$$p \approx F \frac{G(\delta u_s - \delta u_p)}{r}$$

(5.17)

where $F$ may be constant or a parabolic function, which allows for the three dimensional effects (Fig: 5.22). $F$ varies from 1.8–2.4, for $s/d = 3.15$, and from 2–2.7 for $s/d = 5.25$. This agrees well with Eqn: 5.15 for $\nu = 0.5$, which is almost asymptotic to this upper limit. In engineering terms, an average value of 2 over the depth of the soft layer might be appropriate.

5.6 Concluding remarks

Finite element analyses may be used to develop design procedures and calibrate prototype events, where simplified forms of modelling soil and geometry are acceptable. However, the problem examined here was too complex for an accurate representation in plane strain, so three dimensional modelling was required.

The predictions of pile behaviour, based on elastic soil models, were quite realistic when compared with the centrifuge model test results at working loads. It is possible to relate pressure on a pile to the differential movement between the pile and the soil, whilst allowing for three dimensional behaviour. However, designers of piled bridge abutments should be aware of differences between their own structure and these analyses; namely the boundary conditions at the soil surface around the piles and the two dimensional normal loading, which simulated the embankment.
6 ANALYSIS USING THE SIMPLE AND SINPILE PROGRAMS

6.1 Introduction

This chapter describes the development of a simple theory whose purpose is to predict the bending moments and deflections of piles subjected to active lateral loading by soil. The first stage of the analysis considers a single vertical pile which has been driven through a soft layer of soil and is embedded in a stiffer substratum. The objective is to capture the form of the soil–pile interaction in an alternative fashion to the more costly and time consuming finite element method (2 and 3 dimensional). In the second stage, a simple group containing two vertical piles and a rigid pile cap is analysed. The theoretical predictions are compared with results from the centrifuge model tests.

A computer program, SINPILE (SINgle PILE), was encoded in Fortran 77 and installed on the Cambridge University Engineering Department IBM 4341 computer. A graphics package was used to simplify interactive data input and inspection of results for the single vertical pile case, and an upgraded, interactive version, SIMPLE (SIMulated PiLE group), was also developed. This program runs on an IBM PC and will analyse a group of two vertical piles, which are connected by a stiff pile cap, under similar lateral loading conditions. Hard copy plots of the bending moment and deflection of the pile are produced for single piles, otherwise the output is presented in numerical form.

Vertical loading on the abutment structure was not considered at this stage. Most analyses of pile behaviour treat the lateral and axial loading cases for a vertical pile separately, and superimpose the results to give the complete picture. This approach was followed here and so it was only necessary to predict soil properties in response to lateral loading.

6.2 Pile modelling

For simplicity, all piles are assumed to be circular. For those with rectangular, I or H cross sections, a cylindrical pile is defined such that the width of the pile, which will be loaded by the horizontal pressures generated by the embankment loading, becomes the equivalent pile diameter.
SIMPLE analysis

The Young’s modulus and second moment of area are adjusted to ensure that the bending rigidity for this equivalent solid cylindrical pile about the axis parallel to the front row of piles remains the same as the original pile, so that:

\[ E_p = \frac{(EI)}{\pi r^4} \]  \hspace{1cm} (6.1)

6.3 Foundation parameters

Assuming the piles were installed (driven, jacked or bored) through a soft layer of soil and embedded in a stiffer substratum, the lengths of the pile in each layer together with some strength parameters must be known. In the soft upper stratum, a profile of \( c_u \) with depth is required. For the stiffer substratum, values for \( G \) at the top of the layer, \( G_{o'} \), and the rate of increase with depth, \( G_m \), must be stated. These are estimated from self boring pressuremeter tests or empirical equations relating relative density, \( D_r \), and mean effective stress, \( p' \), or standard penetration test blow count, \( N \) (Appendix: 1). The SINPILE graphical input screen is shown in Fig: 6.1.

6.4 Pile response

When a soft upper stratum is loaded by an embankment, noticeable horizontal stresses are transmitted through the clay to apply lateral pressure to the piles. From a prediction of these pressures, the designer will evaluate the magnitude of the pile bending moments and deflections.

The pile response is considered in two complementary parts:

i) Upper section (AB in Fig: 6.1) of the pile in the soft soil, which receives the horizontal load from the clay, and which cantilevers out of the soft—stiff soil interface, at depth \( y = h_s' \).

ii) Lower section (BC in Fig: 6.1) of the pile embedded in the stiff substratum which resists the lateral loading from the upper layer.

6.5 Lateral pressure exerted on a pile in the soft stratum

Estimating the lateral pressures applied to the top section of the pile by the soil is of major importance in making an assessment of pile behaviour. There are many factors which affect the
magnitude and shape of this profile, and these are considered briefly herein.

The geometry and characteristics of the embankment are significant. In general, a bridge abutment features either a slope with a pile group at the toe (Fig: 1.1a) or a retaining wall founded on a pile group (Fig: 1.1b). The distance from the edge of the load to the piles, \(d_l\), and the height and plan dimensions of the embankment must be taken into account. Usually the width of the road is finite, but the length of the embankment is assumed to be infinite unless the road bends as it approaches the bridge (Fig: 1.1c). It is initially assumed, therefore, that loads are applied over half the surface area of an infinite half space.

It is well known that the inclination of the resultant load on a foundation by 15° from the vertical is enough to reduce the ultimate bearing capacity by 50% (Bolton, 1979). This effect can, similarly, reduce the bearing capacity of embankments. It may be necessary, therefore, to build the embankment on a geotextile mat or to place some reinforcement at the base, to carry the outward shear forces which could otherwise destabilise the underlying soil. The embankment material and these differing construction methods are also of great interest with respect to the potential for arching, both longitudinally and transversely, of the load carried by the foundation.

The stability and resistance to bearing capacity failure of the embankment structure should be considered separately, without allowing for additional strengthening resulting from the row of piles, which will only tend to prevent longitudinal (Section AA, Fig: 1.1c), but not lateral (Section BB, Fig: 1.1c) movement. In this way, the embankment and foundation will be designed to avoid failure during working life, whilst limiting lateral deformations to tolerable levels.

In view of this, it is common to choose a secant shear modulus which permits the foundation behaviour at or below working loads, to be described as elastic (Fig: 6.2). For situations when the soil is overconsolidated, this assumption is quite acceptable.

Baguelin et al (1977) showed that linear behaviour was observed in a plot of force per unit length, \(P\), against differential pile–soil displacement normalised by pile radius, \(\delta u/r\), for a laterally
loaded rigid cylinder moving horizontally through an elastic perfectly plastic material under plane strain conditions up to \( \Delta \sigma_n = 2c_u \), which from Eqn: 5.10 gives \( p = 2\pi c_u \) (Fig: 6.3).

Thereafter, yielding or gapping might occur. Plastic failure in the upper clay layer in the centrifuge models began locally around the front edge of the load, and within 0.5d of the piles, as a simple bearing capacity failure (Figs: 4.22b, 4.23b). As the embankment load increased, this plastic region spread towards the middle of the soft layer as the soil started to squeeze past the piles.

An elastic analysis will now be outlined, which assumes that the lateral pressure exerted on the pile in the soft layer will follow a parabolic distribution: this should cover the small-deformation behaviour. Thereafter, the ultimate loading case for the piles will be defined. These analyses are then superimposed in order to gain an overview of the whole behaviour of the surcharge-soil-pile system.

**6.5.1 Lateral pressure distribution at working load**

There are several ways in which an estimate of these lateral pressures may be made. Originally, elastic stress distributions in the foundation were calculated due to a load of a specified geometry, spread over the surface of the soil (Springman, 1988). Horizontal stress profiles were adapted according to local conditions before imposing these on the piles as a lateral pressure distribution. This approach fitted the data very well after application of some factors to these loadings to give an upper and lower limit (Springman, 1988), but failed to account for the variation of lateral pressure on a pile due to the differential movement between the pile and the soil. In this case, the soil displacements and not the in-situ stresses control the behaviour of the piles, and the effects of relative pile-soil stiffness were not allowed for. The results from the finite element analyses underlined the importance of these points.

**6.5.1.1 Soil stresses**

The piles, soft foundation layer and load are shown in Fig: 6.4. If, the properties of the soft layer are described by elastic, perfectly plastic parameters \( G \) and \( c_u \), so that the additional horizontal
stresses are $\sigma_a$ on the embankment side of the pile and $\sigma_p$ on the other, and likewise the
mobilised shear strengths are $c_a$ and $c_p$ in these active and passive zones, neglecting shear on
vertical planes so that the principal planes are horizontal and vertical:

$$c_a = (q - \sigma_a)/2$$  \hspace{1cm} (6.2)

$$c_p = \sigma_p/2$$  \hspace{1cm} (6.3)

Zone A behaves like a triaxial compression sample whilst zone P resembles a triaxial extension:
both samples have deformations linked to the displacement of their common interface, while the
difference in horizontal stress between the two zones relates to the net force on the row of piles
between them. If the piles are replaced by a uniform deep frictionless wound (Bolton, 1979),
then $\sigma_a = \sigma_p$, and this is the case when pile spacing, $s$, is very large and the piles are not
contribute to the stability of the foundation. But if the thrust acting on the piles for unit height
is $pd$ (Fig: 6.5), then Fig: 6.6 defines Mohr’s circle, and for equilibrium of the soil block ABCD
enclosing the piles:

$$pd = (\sigma_a - \sigma_p)s$$  \hspace{1cm} (6.4)

Substituting Eqns: 6.2 & 6.3 into 6.4:

$$p = \frac{s}{d} \{q - 2c_a - 2c_p\}$$  \hspace{1cm} (6.5)

If $c_a = c_p = c_{mob'}$ which assumes that the $c_{mob} - \gamma$ relationship is the same for compression and
extension, Eqn: 6.5 becomes:

$$p = \frac{s}{d} \{q - 4c_{mob}\}$$

or:

$$c_{mob} = \frac{1}{4} \left\{ q - \frac{pd}{s} \right\}$$  \hspace{1cm} (6.6)

From Eqn: 6.6, when $p = 2\pi c_u$ and the soil begins to yield around the pile:
\[ q = c_u \left( 4 + \frac{2\pi d}{s} \right) \]  \hspace{1cm} (6.7)

which for \( s/d = 4 \) (and \( p/c_u = 2\pi \)) implies that the limit to pseudo-elastic behaviour has been reached when \( q/c_u = 5.6 \).

However, it is unlikely that clay on both the active and passive sides of the piles will reach failure simultaneously. The stress paths to failure will be different, following a longer route for the passive zone which could be assumed to fail in extension. Soil in the active zone beneath the surcharge would be expected to fail first in compression.

This analysis employs the deep frictionless wound assumption (Bolton, 1979), which, without the influence of the piles, implies bearing failure at \( q = 4c_u' \), in comparison with \( q = 5.14c_u \) calculated assuming a 90° radial fan between active and passive zones. Therefore, Eqn: 6.6 may be thought of as a safe bearing line for pseudo-elastic behaviour.

### 6.5.1.2 Soil deformations

Consider the possible soil deformations, assuming plane strain, undrained conditions, with rigid, frictionless boundaries. If two constant strain triangles (Bolton & Powrie, 1988), are chosen to represent the behaviour either side of the row of piles (Fig: 6.7a), and then, the sides of the triangles which represent the ground surfaces and the pile rotate through \( \theta \), the lateral soil movement, \( \delta u_s \), at the location of the piles is:

\[ \delta u_s = e_h (h - y) \]  \hspace{1cm} (6.8)

If, however, the effect of the embankment load is such that the deformations extend over a larger area, consider two rectangular areas of constant strain (Fig: 6.7b), such that the horizontal dimension, \( \psi h \), is a factor, \( \psi \), of the depth of the layer, \( h \) and \( \delta u_s \) is constant with depth. For constant volume deformation:
\[ \delta u_s = \varepsilon_h \psi h \]  
(6.9)

But in both cases, the Mohr's circle of strain for undrained behaviour (Fig: 6.8) shows:

\[ \varepsilon_v = -\varepsilon_h = \gamma/2 \]  
(6.10)

and for:

\[ \tau = G\gamma = c_{mob} \]  
(6.11)

combining Eqns: 6.10 & 6.11:

\[ -\varepsilon_h = c_{mob}/(2G) \]  
(6.12)

for triangular deformation profile, from Eqns: 6.8 & 6.12:

\[ \delta u_s = c_{mob}(h - y)/2G \]  
(6.13)

from Eqns: 6.13 & 6.6:

\[ \delta u_s = \frac{(sq - pd)(h - y)}{8sG} \]  
(6.14)

and similarly, for rectangular deformation profile, with dimension characteristic, \( \psi \):

\[ \delta u_s = (c_{mob}\psi h)/2G \]  
(6.15)

and:

\[ \delta u_s = \frac{(sq - pd) \psi h}{8sG} \]  
(6.16)

6.5.1.3 Pile behaviour

If the pile is fixed at the interface between the soft and stiff layers, and the pressure distribution is assumed to be parabolic with a maximum value, \( p_m \), at mid-depth of the layer (Fig: 6.9), considering the bending moments for the pile:

\[ M = \int_0^y pd \, dy \, dy \]  
(6.17)
then, integrating twice:

\[ M = p_m d \left[ \frac{y^2}{2} - \frac{(y - h/2)^4}{3h^2} - \frac{hy + h^2}{6} \right] \]
\[ (6.18) \]

but:

\[ M = -E I \frac{d^2 u}{dy^2} \]
\[ (6.19) \]

combining Eqns: 6.18 & 6.19 and integrating twice:

\[ \delta u_p = \frac{p_m}{EI} \frac{dh^4}{5760} \left[ \frac{1}{h} \left( \frac{2y}{h} - 1 \right)^6 + \frac{1}{24} \left( \frac{y}{h} \right)^4 - \frac{1}{36} \left( \frac{y}{h} \right)^3 + \frac{1}{96} \left( \frac{y}{h} \right)^2 - \frac{49}{480} \left( \frac{y}{h} \right) + \frac{449}{5760} \right] \]
\[ (6.20) \]

which for \( y = 0 \) at the top of the pile or \( y = h/2 \) at mid-depth gives:

\[ \delta u_p = (7dp_m h^4)/(90EI) \text{ or } \delta u_p(h/2) = (11dp_m h^4)/(384EI) \]
\[ (6.21) \]

6.5.1.4 Soil–pile interaction

Adopting the relationship based on work by Baguelin et al (1977) and Fleming et al (1985) from Eqn: 5.17, it is possible to relate the pressure on the pile to the differential displacement between the pile and the surrounding soil:

\[ p = F \frac{2G(\delta u_s - \delta u_p)}{d} \]

Hence from Eqns: 6.14 & 6.20:

\[ p = \frac{2FG}{d} \left[ \frac{(sq - pd)(h - y)}{8sG} - \frac{p_m dh^4}{EI} \left[ - \frac{1}{5760} \left( \frac{2y}{h} - 1 \right)^6 + \frac{1}{24} \left( \frac{y}{h} \right)^4 - \frac{1}{36} \left( \frac{y}{h} \right)^3 + \frac{1}{96} \left( \frac{y}{h} \right)^2 \right. \right. \]
\[ - \frac{49}{480} \left( \frac{y}{h} \right) + \frac{449}{5760} \left] \right) \]
\[ (6.22) \]
so for $y = h/2$, when $p = p_m$:

$$p_m = \frac{q h}{8d \left[ 1 + \frac{h}{F s} + \frac{11 \ G h^4}{192 E I} \right]}$$  \hspace{1cm} (6.23)$$

Similarly, for the rectangular deformation profile, an equation relating the lateral pressure and deflection can be defined using Eqns: 5.17, 6.16 & 6.20 from which, for $y = h/2$:

$$p_m = \frac{q \psi h}{4d \left[ 1 + \frac{\psi h}{F s} + \frac{11 \ G h^4}{192 E I} \right]}$$  \hspace{1cm} (6.24)$$

Note that Eqn: 6.23 is identical to Eqn: 6.24 when $\psi = 1/2$ and $y = h/2$, and this will be used to describe the triangular deformation profile. Thus it is possible to predict an envelope for the lateral pressure acting on the pile for a variety of pile configurations and soil strengths, based on different soil deformation profiles.

6.5.2 Ultimate lateral pile capacity

The ultimate lateral load capacity of the pile must be considered since this defines the absolute upper bound to the pile bending moments and deflections. As the loading increases with construction of the embankment, so will the lateral pressures approach the level at which yielding commences ($p = 2\pi c_u$) when it is no longer adequate to describe the foundation behaviour as pseudo-elastic, and before reaching the ultimate case when the soil would shear past the pile. When the soil is moving plastically past the pile over the entire depth of the soft layer, the pile has received the maximum possible lateral thrust. Naturally this assumes that the pile is capable of sustaining such moments and shear forces, and having determined this upper bound to the pile bending moment, a check should be made to ensure that the pile cross section is adequate.

Randolph & Houlsby (1984) calculated the limiting load on a cylindrical pile moving through an infinite medium of homogeneous, perfectly plastic soil, using classical plasticity theory. They stated that for a smooth pile, the ultimate resistance per unit length was:
(6 + π)c_u d = 9.14c_u d \hspace{1cm} (6.25a)

while for a rough pile the value was:

(4\sqrt{2} + 2π)c_u d = 11.94c_u d \hspace{1cm} (6.25b)

Classical plasticity theory depends on two different approaches for the estimation of collapse loads. When both of these methods, the upper and lower bounds, agree, then an exact solution has been found, and this is the case for the Randolph analysis. This confirmed the approximate rules of thumb proposed by:

i) Broms (1964), that the maximum resistance was 9c_u d at a depth below 1.5d,

ii) Poulos & Davis (1980), that the variation was 7.7c_u d to 10.85c_u d, where these figures were the average between an upper bound which is approximately 10–15% higher than the lower bound. At the surface, the limiting resistance was reduced to 2c_u d to account for upward movement of the soil around the pile and the formation of a gap behind the pile. The full resistance was developed below a depth of 3d.

If the limiting friction, f_s, at the pile–soil interface varies between 0 for a smooth pile and c_u for a completely rough pile, a friction ratio, f_s/c_u may be used (Randolph & Houlsby, 1984). Fig 6.10 shows the flow mechanism around a laterally loaded pile for the intermediate case when f_s/c_u = 0.5. It is a similar case which is most likely to pertain to field conditions with a pile that is neither completely smooth nor completely rough, so an average of the Randolph & Houlsby values for limiting lateral pressure (Eqns: 6.25a & b) is recommended here:

p_u = 10.5c_u \hspace{1cm} (6.25c)

Whenever the lateral pile pressures due to a given embankment load, q_{crit}', approached the 2πc_u criterion at any depth in the layer, the soil was deemed to have passed into a plastic domain which extended upwards and downwards from that critical depth as more embankment loading was applied. At this stage an elasto–plastic zone must exist, and assumptions based on elastic foundation behaviour become invalid.
SIMPLE analysis

Considering the maximum embankment load before failure, $q_{\text{max}}$, an upper bound calculation was made for a local undrained failure of a weightless foundation with uniform $c_u$, which allowed for some reinforcement by the piles due to the energy dissipated by the soil shearing past the pile. Fig: 6.11a shows the active and passive zones marked by two $45^\circ$ isosceles triangles, with a radial fan in between. The displacements of each wedge are recorded in Fig: 6.11b. Conservation of energy for a unit width dictates that:

$$q_{\text{max}} h \sqrt{2} \delta v_e = 10.5c_u \frac{d}{s} h \delta u_s + (2 + \pi) c_u h \sqrt{2} \delta v_e$$

but $\delta u_s = \delta v_e \sqrt{2}$, so:

$$q_{\text{max}} = (2 + \pi)c_u + 10.5c_u \frac{d}{s}$$

$$q_{\text{max}} \approx (2 + \pi)c_u \left(1 + \frac{2d}{s}\right)$$

(6.26)

and for $s/d = 4$, $q_{\text{max}}$ is increased by 50% to $7.71c_u$.

6.5.3 Determination of elasto–plastic boundaries

6.5.3.1 Construction of plasticity plot

The elastic zone is limited by $p = 2\pi c_u$ and the safe bearing line, Eqn: 6.6. Completely plastic flow occurs when $p = 10.5c_u$ (Eqn: 6.25c) or when $q_{\text{max}}$ from Eqn: 6.26 is reached. These areas are plotted on Fig: 6.12, with ordinate $p/c_u$ and abscissa $q/c_u$. The failure condition for out of plane bearing capacity collapse at the side of the embankment, ignoring the side slope, depends on the nature of the transverse shear stresses and whether there is sufficient movement between embankment and foundation to generate them (Jewell, 1987). For a smooth footing, with uniform $c_u$, $q = (2 + \pi)c_u = 5.14c_u$, whereas $q \approx 2.3c_u$ when $\tau = c_u$ (Fig: 6.13a). Alternatively, for a foundation in which $c_u$ increases with depth from $c_{uo}$ at the surface, and for a smooth footing, Fig: 6.13b (Davis & Booker, 1973) may be applied. The out of plane stability for the surcharge load acting on the centrifuge models was not considered because the presence of the strongbox prevented deformation in this plane. However, if the strongbox walls were ignored, and road width, $2B = 20$ m (at prototype scale), for $dc_u/dy = 1.75$ kPa/m and $c_{uo} = 9.5$ kPa, this would
give an improved bearing capacity of \( q \approx 7.5c_{u0} \) (Fig: 6.13b). This could also be shown on the appropriate plasticity plot as \( q/c_u \approx 7.5c_{u0}/c_u \approx 4.75 \).

In between these regions (Fig: 6.12), the behaviour is elasto–plastic, and deformations will become increasingly more critical. It is useful to discover at what embankment load, \( q_{\text{crit}} \), the foundation ceases to behave pseudo–elastically. For \( 2\pi < p/c_u < 10.5 \) there will be general yield of the soil around the pile. For \( q/c_u \) falling between the safe bearing line, (Eqn: 6.6) and the bearing capacity failure line (Eqn: 6.26) there will be general yield of the soil mass. Theoretically, as the surcharge load increases, the gradual failure will progress towards the intersection of \( p/c_u = 10.5 \) (Eqn: 6.25c) and Eqn: 6.26, when there will be ultimate plastic failure of the soil mass and the soil around the pile.

It is difficult to quantify the effect of the elasto–plastic zone. It is safer to examine the absolute upper bound, at which the lateral pressure reaches \( 10.5c_u \) over the entire depth of the soft stratum. This can be adjusted later on for a deep soft layer, by keeping \( p_u = 10.5c_u \) over an appropriate depth of loading, \( h_u \), decreasing to zero over the remainder of the depth, \( h_u \) (Fig: 6.14).

In general, the design values of \( p/c_u \), \( q/c_u \) should be prevented from encroaching into the elasto–plastic zone, in view of the excessive deformations that would result from this level of loading. Therefore an acceptable serviceability criterion is to restrict the behaviour of the foundation and hence the lateral pressures imposed on the pile to the pseudo–elastic design zone.

6.5.3.2 Comparison with centrifuge model test results

Comparing results for centrifuge model tests 3, 4 and 7, the maximum pressure has been listed for each level of embankment loading, together with the depth at which \( p_m \) occurred (Table: 6.1). Taking an average \( c_u = 15 \text{ kPa} \) at \( y \approx 3 \text{ m} \), the non–dimensional groups \( p_m/c_u \) and \( q/c_u \) were calculated and the results plotted on two plasticity plots for \( s/d = 3.15 \) and 5.25 (Figs: 6.15a & b).

Also included were the expressions relating \( p_m \) to \( q \) (Eqn: 6.24) up to the limits of elasticity, for
three soil deformation profiles, triangular ($\psi = 1/2$), square ($\psi = 1$) and rectangular with $\psi = 2$
(Figs: 6.7a & b) which were valid up to $p = 2\pi c_u$. For the centrifuge tests in which the surcharge
working load was built up over a period of months, the recommendations of Fleming et al, (1985)
that $F = 1.57$ (Eqn: 5.14) were adopted. The remaining parameters matched the centrifuge model
test data, with $h = h_s = 6$ m.

Within the elastic zone, the model test results (Figs: 6.15a & b) almost fall between the $\psi = 1/2$
and the $\psi = 1$ deformation characteristics. Thereafter, the experimental data rises more steeply
until levelling off at around the $10.5c_u$ limit. So, it appears that this analysis predicts reasonable
values of lateral pile pressure for both an elastic phase and the ultimate plastic limit.

However, it is probably more realistic to compare the SIMPLE analysis with that using finite
elements, since all the parameters were identical for the pile and soil. If Figs: 5.17a, 5.17b are
examined to determine the strain distribution under the embankment according to the procedure
laid out in Section 6.5.1.2, setting $\psi \approx 2$ for $s/d = 5.25$, and $\psi \approx 1.5$ ($s/d = 3.15$) would give a fair
approximation to the numerical deformations. Referring to the plasticity plots (Figs: 6.15a & b),
the finite element results lie at equivalent SIMPLE values of $\psi \approx 1.7$ ($s/d = 5.25$) and $\psi \approx 1.4$ ($s/d$
$= 3.15$), which is reasonable agreement given the small differences in the two analyses.

6.5.4 Net effect of lateral pressure

Once the profile of the lateral pressure acting on the pile has been determined, the net effect on
the pile section in the stiffer substratum may be calculated. By integrating the lateral pressure to
give the shear forces, which are in turn integrated to yield the moment diagram for the upper
section of the pile, the net bending moment, $M_s$, and shear force, $H_s$, which will be applied to the
pile at the soft–stiff interface may be determined.

Figs: 6.16a, 6.16b show the graphical input screens for SINPILE, in which the pressure
distribution may be varied linearly, parabolically or using a cubic spline fit to selected data
points. Assuming at this stage that the pile is clamped at the interface between the two layers,
where $y = h_s$, the maximum bending moment, $M_s'$, and shear force, $H_s'$ for the section of the pile
in the soft layer will be found at this horizon. The absolute maximum value of pile bending moment generally occurs at a depth of 2–3d into the stiffer substratum.

6.6 Behaviour of the pile in the stiff substratum

The Randolph (1981a) solutions for the deflection and rotation at the head of a pile, and pile bending moment and deflection under either a head force or moment loading were used to predict the behaviour of the lower section of the pile in the stiffer substratum. They were based on equations derived originally by Hetenyi (1946) for a pile that was long. Therefore, the critical length of pile, \( \ell_c \), was shorter than the length of the pile, \( \ell \), (Fig: 6.17) and thus the pile was considered to be flexible rather than rigid.

The equations were obtained by fitting those of a similar form to data from a parametric finite element study of the problem. This technique provided a semi–rigorous analysis which is generally considered superior to p–u curves and the boundary element methods for regular design office usage (Elson, 1985), and the solution is presented below in simple chart form.

Several parameters were defined, based on the shear modulus of the stiff layer, which was \( G_o \) at the top \((y = h_s)\), increasing \( m \) per metre with depth, where, for \( y > h_s \):

\[
G = G_o + m (y - h_s)
\]

(6.27)

and the shear modulus was adjusted to include the effects of Poisson's ratio so that:

\[
G^* = G (1 + 3v/4)
\]

(6.28)

and thus a characteristic shear modulus was described as (Fig: 6.17):

\[
G_c = G^* \ell_c / 2
\]

(6.29)
and a soil homogeneity factor, which lay between 0.5 and 1, as:

$$\rho_c = \frac{G^*_{\ell_c/4}}{G^*_{\ell_c/2}}$$  \hspace{1cm} (6.30)

and critical slenderness ratio of the pile:

$$\ell_c/r = 2\left(\frac{E_P}{G_c}\right)^{2/7}$$  \hspace{1cm} (6.31)

These equations were used to determine the values of $u_s$ and $\theta_s$ at the soft–stiff interface, $y = h_s$, of the soil:

$$u_s = \frac{(E_P/G_c)^{1/7}}{\rho_c G_c} \left[ 0.27H_s(\ell_c/2)^{-1} + 0.3M_s(\ell_c/2)^{-2} \right]$$  \hspace{1cm} (6.32)

$$\theta_s = \frac{(E_P/G_c)^{1/7}}{\rho_c G_c} \left[ 0.3H_s(\ell_c/2)^{-2} + 0.8\sqrt{\rho_c M_s(\ell_c/2)^{-3}} \right]$$  \hspace{1cm} (6.33)

and were incorporated into the curves which showed non–dimensionalised deflection (Figs: 6.18a & 6.19a) and moment (Figs: 6.18b & 6.19b) versus depth normalised by the critical pile length, for either a lateral force, $H_s$, or a moment, $M_s$, acting at the nominal ground surface, $y = 0$, and for different values of soil homogeneity, $\rho_c = 0.5, 0.75, 1.0$ (Randolph, 1981a).

This approach gave a simple elastic solution for the behaviour of the pile in the stiffer substratum, which was sufficiently accurate for the majority of engineering problems where soil working stresses were much lower than the ultimate load condition and an appropriate secant modulus could be selected. The main source of error lay in allotting values to $G_o$, $G_m$ and $v$. However, the bending moment profile was far more sensitive to changes in the choice of lateral loading in the soft layer and hence the values of $H_s$ and $M_s$ at the top of this stiffer layer, than to variations in the shear modulus for the lower layer.
6.7 Pile bending moments and deformation profiles

6.7.1 Prediction by SIMPLE

Once the bending moment profile was determined for both sections of the pile, together with the displacement and rotation of the pile at the interface, \( y = h_s \), the additional displacement in the upper section of the pile was calculated. Allowance was made for the rigid body displacement due to base rotation (\( \Delta u = \theta_s (h_s - y) \)) and the deflection due to the bending moment, which was found by double integration, and these were included in the SINPILE and SIMPLE algorithms. A typical plot is attached as Fig: 6.20, and a copy of a numerical output is located in Table: 6.2.

6.7.2 Effects due to piles nearby

Neither the SINPILE nor the SIMPLE algorithms allowed for the presence of other piles in the immediate vicinity, so, the interaction between them and the cumulative effect on deformation and rotation should be accounted for separately at this stage. Poulos (1971) pioneered the use of interaction factors in this area, and wrote the expression for a group of \( n_p \) piles:

\[
 u_i = \frac{1}{k} \sum_{j=1}^{n_p} \alpha_{ij} H_j
\]  

(6.34)

where \( \alpha \) was the interaction between the \( i \)th and \( j \)th piles, \( k \) was the stiffness of a single isolated pile, and \( H \) was the lateral load. Thus, interaction factors were defined depending on the spacing, angle and type of loading, and pile head fixity (Fig: 6.21). For free headed piles subject to a lateral head load, \( H \), or moment, \( M \), the factors were \( \alpha_{uH} \) and \( \alpha_{uM} \) for deflection and \( \alpha_{\theta H} \) and \( \alpha_{\theta M} \) for rotation.

However, since most pile groups are fixed against rotation at their head by a stiff pile cap, the only relevant factor is \( \alpha_{uf} \). Randolph (1981a) conducted finite element analyses on laterally loaded fixed headed piles and concluded that \( \alpha_{uf} \) was approximated by:

\[
 \alpha_{uf} = 0.6 \rho_c (E_p / G_c)^{1/7} (t/s)(1 + \cos^2 \varphi)
\]  

(6.35)
SIMPLE analysis

unless $\alpha_{uf}$ exceeded 0.33 at close pile spacings, when the value was replaced by:

$$\alpha_{uf} = 1 - 2/(27\alpha_{uf})^{1/2}$$

(6.36)

Poulos (1971) proposed that interaction factors for fixed headed piles were larger than for free headed piles, and Randolph (1983) suggested that 0.6 should be replaced by 0.4 in Eqn: 6.35 to give:

$$\alpha_{uH} = 0.4 \rho_c (E_p/G_c)^{1/2}(r/s)(1 + \cos^2\varphi)$$

(6.37)

For $\alpha_{uH} > 0.333$, Eqn: 6.36 was adopted with the subscript 'uf' replaced by 'uH'. The other interaction factors were considerably smaller than $\alpha_{uH}$ and were taken as (Randolph, 1983):

$$\alpha_{uM} = \alpha_{\theta H} \simeq \alpha_{uH}^2$$

(6.38)

$$\alpha_{\theta M} \simeq \alpha_{uH}^3$$

(6.39)

Thus, the individual values of the interaction factors are determined for each pile in relation to its neighbours, and summed to give the total effect on the pile displacements. For plane strain cases where the load and stiffness are nominally equal, the deflections can simply be factored up to account for the interaction between the group or row of piles.

6.7.3 Comparison with centrifuge model tests

The centrifuge model tests were conducted on a variety of foundation geometries and pile spacings. Of the five tests carried out for a single row of piles, the three most relevant for the purposes of this comparison are as follows:

Test 4 with 3 piles at 5d spacing in 6 m soft clay overlying 10 m sand,

Test 6 with 3 piles at 5d spacing in 8.5 m soft clay overlying 7.5 m sand,

Test 7 with 5 piles at 3d spacing in 6 m soft clay overlying 10 m sand.

The substratum properties were assumed to be the same as those specified for the numerical analyses (Tables: 5.1, 5.2, 5.3).
6.7.3.1 Scaling factors

Scaling factors should be applied to the values of centrifuge model test experimental data shown in the figures to convert them to a prototype equivalent:

<table>
<thead>
<tr>
<th>Scale factor</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending moment</td>
<td>$100^3$</td>
</tr>
<tr>
<td>Lateral pressure</td>
<td>1</td>
</tr>
<tr>
<td>Deflection</td>
<td>100</td>
</tr>
</tbody>
</table>

6.7.3.2 Experimental data

The extraction and reduction of experimental data was described in Chapter 4. During the comparisons, allowance must be made for the inaccuracies entailed in double differentiation of the polynomial, which was fitted to the bending moment data points, to give the lateral pile pressure. Further problems arose due to the choice of end conditions for the pile displacement profile. It was assumed that the pile did not move at the base, and that the pile head deflection was defined by the average of two LVDT readings at the mid–height between the instruments. For a flexible pile, there is a critical length, $l_c$, below which there will be no deflection or rotation (Eqn: 2.4). Therefore, if the LVDT data implied a tip rotation greater than zero, for piles longer than $l_c$, either Eqn: 2.4 was not correct or the LVDT readings were too large.

The estimates of the interaction effects on displacements due to neighbouring piles have not been included in the predictive calculations, and should be considered separately since they are valid for either head force or moment loading rather than lateral thrust. Table: 6.3 lists the factors for free headed piles with the same geometry as those in the centrifuge model tests, assuming that each pile exhibits the same stiffness and receives the same lateral load. Therefore the SIMPLE deflections should underestimate those from the experimental data.

6.7.3.3 Validation of design criteria

A review of the results (Figs: 4.3, 4.6, 4.8) supports the basic premise that the pile can be split into two sections for the purpose of the analysis. The loading by the upper soft soil layer is clearly demonstrated by the lateral pressure diagrams, whilst the resistance of the stiff substratum
is also apparent below the interface between the soft and stiff layers.

Values of $p_m$ for the assumed parabolic profile were determined for the three soil deformation profiles (triangular, $\psi = 1/2$, and rectangular with $\psi = 1$ & $\psi = 2$) at different embankment loads within the elastic zone where $q < q_{crit}'$ and a SIMPLE analysis was run. In previous comparisons of the prediction of the lateral pressure profile within the elastic region, it was found that the experimental data fell between the $\psi = 1/2$ and $\psi = 1$ cases for an assumed parabolic form (Figs: 6.15a & b). Therefore it was of interest to see whether the experimental bending moment and deflection profiles also lay within this envelope.

6.7.3.4 Elastic loading case for a shallow clay layer

During test 4, there was a drift in bending moment data from strain gauges on the upper section of the pile, leading to a higher root mean square error in the polynomial fit to these data points. In consequence, the double differentiation of the bending moments to find the lateral pressures on the pile was less accurate and did not follow the expected parabolic format, which had been observed in other tests. (Fig: 6.22). Nonetheless, the form of the bending moment profile was similar to $\psi = 1$ case and was almost bracketed by the predicted upper and lower elastic limits, $1/2 \leq \psi \leq 2$.

However, in test 7, for which it was possible to consider three such load cases below or very close to $q_{crit}'$, both the lateral pressure and the bending moments bisected the $\psi = 1/2$ and $\psi = 1$ cases, moving gradually towards the upper value of $\psi$ as $q$ increased (Figs: 6.23 – 6.25). But the experimental displacement, which was within the predicted range for $q = 53$ kPa, was considerably larger than expected for $q = 72$ and 93 kPa. This can be explained in two ways. Firstly, the interaction amongst the row of piles has not been taken into account in plotting the predicted displacement envelope. Table: 6.3 lists the interaction factors for the pile geometry used in the tests. Secondly, the method of fitting the doubly integrated bending moments through the average of the LVDT displacement readings has implied a rotation at the base of the pile. This is inconsistent with the assumption that piles are only affected by lateral loading above their
critical length, and also with observations from the data. By removing the rotation at the base of
the pile, the displacement profile would fall within the predicted envelope. Although the
LVDT data were estimated to be accurate to \(\pm 10\%\), adjustment by this amount was insufficient
to explain the discrepancy.

6.7.3.5 Elastic loading case for a deep clay layer

Test 6 had the maximum possible depth of clay in the centrifuge strongbox (equivalent to 8 m at
prototype scale) whilst ensuring that the lower section of the pile, which was embedded in the
sand (8 m), was just long enough to be considered flexible. However, the sand layer settled
0.5 m more than anticipated during the sample preparation phase. So, the lower pile section was
0.5 m less than critical length and behaved in a semi–rigid fashion. Nonetheless, the analysis was
carried out to give a comparison.

For this deeper (8.5 m) clay layer the point at which the clay ceased to load the pile occurred
between 1.5–3.5 m above the clay–sand interface (Figs: 6.26 & 6.27). So the initial assumption
that the pile might be loaded over an 8 m depth, which allowed a \(q_{\text{crit}} = 72 \text{ kPa}\), for values of \(c_u\)
larger than 13 kPa (Eqn: 6.6), was rather conservative. However, the analysis was conducted and
the results were reported in Figs: 6.26 and 6.27. Since the strain gauges in the upper section of
one pile were showing very small negative values, probably due to drift, the double
differentiation to give the lateral pressures resulted in an overestimation. However the \(\psi = 1/2\)
and \(\psi = 1\) cases describe the upper and lower limits of the experimental bending moments very
effectively, although the semi–rigid behaviour of the pile contributed to the rotation of the pile
about the tip and hence the larger values of displacement than those estimated.

6.7.3.6 Ultimate load case for a shallow clay layer

In this section, the maximum loads applied by the embankment are compared against the
postulated \(10.5c_u\) upper bound. In test 4, the magnitude of the \(10.5c_u\) bending moment profile
was slightly below the moments seen at maximum embankment load in the centrifuge model test,
and the predicted maximum moment occurred at a shallower depth (Fig: 6.28). In the
experimental tests, the bending moment increased marginally between the embankment loads of
203 and 228 kPa (Fig: 4.3c) for pile B, suggesting that the lateral loading had approached the extreme case whereby further embankment loading merely caused the soil to shear past the pile. Although this would define the maximum bending moment for pile B, it was noted that the deflections were still increasing and that this upper limit had not been reached for pile A. Neither bending moment curve was well fitted by the polynomial, but particularly so for pile B. In consequence, the lateral pressure profile was not so realistic and it was difficult to see from this whether the 10.5c_u criterion was appropriate. Comparison of bending moments resulting from each case, experimental and analytical, showed closer agreement.

In test 7, the foundation geometry was the same as test 4, but there were 5 piles, implying that the soil was more reinforced. Therefore a higher embankment load would be required before the soil reached the 10.5c_u condition, and squeezed through between the piles. In fact, this ultimate case was not approached except at the middle of the soft layer, and the moments were still increasing steadily as the load was applied (Figs: 4.8a, 4.8b). Test 4 was taken to a maximum embankment load of 228 kPa, whereas this test was stopped at 189 kPa. So the 10.5c_u criterion was expected to overpredict the experimental moments (Fig: 6.29), although the pile tip rotations obtained from curve fitting the bending moment data points meant that the experimental deflections were considerably greater than predicted.

6.7.3.7 Ultimate load case for a deep clay layer

In test 6, the clay reached the 10.5c_u condition over the majority of the soft layer. The maximum moments were achieved before the last two embankment loads were added (Figs: 4.6a & 4.6b). However, it was clear that the soil was not shearing past the pile over the whole of the 8 m soft layer since the calculated bending moment profile overestimated the observed maximum (Fig: 6.30). A second calculation was carried out assuming that the soil sheared past the pile over the top 6 m, with the lateral pile pressure dropping off from the 10.5c_u condition to zero at 8 m (Fig: 6.31). Although there is not much difference for this case because the top of the pressure profile has more impact on the moment calculated at the soft–stiff interface, this approach will be important when the soft layer is deeper. However, this still overestimated the maximum moment.
The displacements were greatly underpredicted due to the rigid behaviour of the pile which rotated about the pile tip. In part, the difference was due to the assumptions made about the pile displacement profile and the choice of head displacement, but Eqn: 2.4, which defines the critical pile length $l_c$, indicated that the pile was not quite long enough to be considered flexible, and this was confirmed by the experimental results. Therefore, the pile's rotation reduced the relative pile–soil displacement at the ground surface and the pile behaviour was not only short and rigid, but the stiff layer was approaching failure as well.

Fleming et al (1985) quote the limiting pressure with depth for laterally loaded piles in cohesionless soils as lying between $K_p \sigma'_v d$ at the soil surface and $K_p^2 \sigma'_v d$ with depth, which was supported by Barton's (1982) centrifuge model testing on laterally loaded piles embedded in a uniform dense sand. This shows that the piles would be expected to fail in the sand layer for an ultimate pressure of around $20\sigma'_v$, which gives a range of 0.9 – 3 MPa for a 10 m deep sand layer. Clearly these values are far greater than the deduced lateral pressures and so elastic behaviour was assumed for the sand substratum.

6.8 Pile group analysis

6.8.1 Introduction

The SIMPLE analysis was adapted to deal with a pile group containing two rows of vertical piles fully fixed into a stiff pile cap, which was positioned at any height above ground level. The foundation conditions were those pertaining to the single pile solution, with a soft soil layer overlying a stiffer substratum. Calculations were made for the lateral pressures acting on the front and rear rows of piles, and then the single pile solution was used to solve the problem for two independent free headed piles, to calculate the rotation and deflection at the top of both of the piles. It was also possible to apply an additional horizontal shearing force at the pile cap. Finally, a stiffness matrix was constructed, relating moment and lateral load to rotation and deflection at pile cap level, for the piles embedded in the sand layer, with the end conditions imposed by the pile cap:
i) deflection equal,
ii) zero rotation,
iii) equal and opposite shear forces.

6.8.2 SIMPLE pile group algorithm

Initially, the front and rear piles were treated separately, as unconnected, free headed piles. Referring to Fig: 6.32a, the typical case shows a group with a length of pile, \( h_s \), in the soft stratum, with a freestanding length, \( e \), and an external horizontal thrust on the pile cap, \( H_{pc} \), which is divided equally in its effect between both piles.

Estimation of the lateral pressures, \( p_f \), acting on the front piles and \( p_r \) on the rear piles in the soft layer were calculated by the methods described in Section 6.5 using appropriate values of deformation shape characteristic, \( \psi \). Clearly, the pile spacing and ground surface boundary conditions will affect the magnitude and shape of \( p_r \), which is generally less than \( p_f \). X-rays of lead threads inserted in the centrifuge models of pile groups showed considerably less differential movement between soil and pile for the rear row (Fig: 4.27–4.29). In this case, a reduced value of \( p_m \) calculated using \( \psi = 1/2 \) could be used, since upward deformation of soil around the pile was not restricted by a pile cap. Then, \( p_f \) and \( p_r \) were applied together with \( H_{pc}/2 \) at pile cap level for each pile. Using the single pile solution, the head rotations (\( \theta_{fb}, \theta_{rb} \)) and deflections (\( u_{fb}, u_{rb} \)) were found for both piles, assuming the two piles were free headed and independent of each other.

Next, a force and moment were applied at pile cap level for each pile (\( H_{fc}, H_{rc}, M_{fc}, M_{rc} \)) such that the rotation at the top of each pile was \(- \theta_{fb} \) and \(- \theta_{rb} \) and the deflections, \( u_{ff} \) and \( u_{rf} \) resulting from this were determined (Fig: 6.32b). An intermediate step required the transference of the effect of \( H_{fc}, H_{rc}, M_{fc}, M_{rc} \) at the pile cap, to \( H_{fs}, H_{rs}, M_{fs}, M_{rs} \) at the interface between the soft layer and the stiff substratum so that if \( h_e = h_s + e \):

\[
H_{fs} = H_{fc} \quad M_{fs} = M_{fc} + h_e H_{fc}
\]  

(6.40)
Rotation and deflection at the interface, \( y = h_s \), \( \theta_{fs} \), \( \theta_{rs} \), \( u_{fs} \) and \( u_{rs} \), were calculated from Eqns: 6.32 & 6.33:

\[
\begin{align*}
  u_{fs} &= f_{huf} H_{fs} + f_{muf} M_{fs} \\
  \theta_{fs} &= f_{h\theta f} H_{fs} + f_{m\theta f} M_{fs}
\end{align*}
\]

where \( f_{huf} \), \( f_{muf} \), \( f_{h\theta f} \) and \( f_{m\theta f} \) were factors:

\[
\begin{align*}
  f_{huf} &= \frac{(E_p/G_c)^{1/7}}{\rho_c G_c} 0.27 (l_c/2)^{-1} \\
  f_{muf} &= f_{h\theta f} = \frac{(E_p/G_c)^{1/7}}{\rho_c G_c} 0.3 (l_c/2)^{-2} \\
  f_{m\theta f} &= \frac{(E_p/G_c)^{1/7}}{\rho_c G_c} 0.8 \sqrt{\rho_c} (l_c/2)^{-3}
\end{align*}
\] (6.41) (6.42) (6.43)

Knowing \( u_{fs} \) and \( u_{rs} \) at the interface, \( y = h_s \), the additional rotation and deflection in the upper section of the pile \((-c \leq y \leq h_s)\) due to moment were calculated so that:

\[
\begin{align*}
  \Delta \theta_f &= \int_{h_s}^{c} M_f \, dy \\
  \Delta u_f &= \int \int_{h_s}^{c} M_f \, dy \, dy
\end{align*}
\] (6.44)

and:

\[
\begin{align*}
  \theta_{fb} &= \theta_{fs} + \Delta \theta_f \\
  u_{ff} &= u_{fs} + \Delta u_f + \theta_{fs} h_c
\end{align*}
\] (6.45)

If Eqns: 6.40 – 6.45 are repeated for the rear pile with a different subscript, 'r', replacing the subscript 'f', then the deflection and rotation at pile cap level can be described in terms of the moment and lateral force at this level:
\[ \theta_{rb} = \frac{H_{rc}}{k_{hr}} + \frac{M_{rc}}{k_{mr}} \]

\[ u_{rb} = \frac{H_{rc}}{k_{hr}} + \frac{M_{rc}}{k_{mr}} \]  

\[ \theta_{fb} = \frac{H_{fc}}{k_{hfr}} + \frac{M_{fc}}{k_{mfr}} \]

\[ u_{ff} = \frac{H_{fc}}{k_{hfr}} + \frac{M_{fc}}{k_{mfr}} \]  

where \( k \) is a stiffness factor:

\[ \frac{1}{k_{hfr}} = f_{hfr} + f_{mfr} h_e + h_e^2/(2EI) \]  

\[ \frac{1}{k_{mfr}} = f_{mfr} + h_e/(EI) \]  

\[ \frac{1}{k_{huf}} = f_{huf} + (f_{muf} + f_{huf}) h_e + f_{muf} h_e^2 + h_e^3/(3EI) \]  

\[ \frac{1}{k_{muf}} = f_{muf} + f_{muf} h_e + h_e^2/(2EI) \]

and Eqns: 6.48 - 6.51 are repeated for the rear pile with the subscript 'r' replacing 'f'. These equations were re-arranged with the end conditions:

\[ u_f = u_r = u_{fb} - u_{ff} = u_{rb} - u_{rf} \]

\[ \theta_f = \theta_r = 0 \]

\[ H_{fc} + H_{rc} = 0 \]

so that the stiffness matrix became:

\[
\begin{bmatrix}
0 & \frac{1}{k_{mfr}} & -\frac{1}{k_{hfr}} \\
\frac{1}{k_{mfr}} & 0 & \frac{1}{k_{hfr}} \\
-\frac{1}{k_{muf}} & \frac{1}{k_{mur}} & -\frac{k_{huf} + k_{huf}}{k_{huf}} \\
\end{bmatrix}
\begin{bmatrix}
M_{fc} \\
M_{rc} \\
H_{fc} \\
\end{bmatrix}
= 
\begin{bmatrix}
\theta_{rb} \\
\theta_{fb} \\
u_{rb} - u_{rb} \\
\end{bmatrix}
\]
and was solved to give the values of $M$, $H$, $\theta$ and $u$ at the top of both piles (e.g. $M_p$, $H_p$, $\theta_p$, $u_p$ for the front pile). Thence, the single pile solution was employed directly to give the bending moment and deflection profile down each pile for the vertical pile group with the appropriate fixity conditions (Fig: 6.32c).

6.8.3 Comparison between experimental results and theoretical predictions

Three centrifuge model tests were carried out to investigate the behaviour of a pile group containing two rows of vertical piles, which were fixed to a stiff pile cap (Fig: 3.3). Both rows of piles were instrumented and the resulting bending moment profiles for normal loading were analysed in the same way as those for the single row of piles. Figs: 4.11a, 4.11b show a pair of results for the central pile, pile B, where the additional designation F or R indicates whether the pile was in the front or the rear row.

The foundation preparation, loading bag assembly, pile stiffness and diameter were the same as before, so that the sole variant was the configuration of the piles and their fixity condition. The model test which investigated the behaviour of the group when embedded in a deep clay layer is not considered here, allowing comparison with the theory for two cases:

- Test 8 with 2 rows of 3 piles at 5d spacing in 6 m soft clay overlying 10 m sand,
- Test 9 with 2 rows of 3 piles at 5d spacing in 8 m soft clay overlying 8 m sand.

The interaction factors due to head loading are shown in Table: 6.4.

Initially, best-fit lateral pressure distributions were assumed from the back analysis of those obtained from the experimental data, for one value of embankment load and for a pair of piles that were directly in line with each other. Presented in Figs: 6.33, 6.34 are experimental results compared against values deduced from the SIMPLE analysis. These figures demonstrate that the theoretical results gave a good approximation to those observed from the centrifuge model tests.

If, however, the lateral pressure exerted by the soil on the front and rear piles was estimated from a similar analysis to that described in Section 6.5, Eqn: 6.24 will be rewritten as:
\[ p_m = \frac{q \psi h}{4d \left[ \frac{1}{F} + \psi h + \frac{25}{576EI} \right]} \] (6.56)

where the fraction 25/576 replaces 11/192 in Eqn: 6.24. This assumes that the piles were fixed at both ends of the soft layer, with zero rotation and a lateral deflection at the top equal to half that of an equivalent free headed pile under the same loading conditions (Fig: 6.9). Eqn: 6.56 was not very sensitive to the change in fixity condition for the pile, because the \( \frac{Gh^4}{EI} \) term contributed little to the value of the denominator.

A comparison was made between the centrifuge model test results and predictions based on this analysis. Both tests 8 and 9 were considered, with values of \( 1/2 \leq \psi \leq 2 \) describing the soil deformation behaviour in front of the leading row of piles. The soil applied loading over the full depth of the soft clay layer, 6 and 8 m respectively, and the other parameters were identical to those used in earlier comparisons.

By continuity and for an undrained soil the same lateral movement would be anticipated for each pile, and using the simple deformation models of Section 6.5, the same lateral pressure would be expected to act on both the front and rear rows of piles so that if:

\[ \psi_f = \psi \psi_f \] (6.57)

then \( \psi = 1 \). In practice, this will not happen, but it will give the worst possible loading case for the pile group. If the soil is permitted either to move vertically or to consolidate, then \( \psi < 1 \). Looking at the X-rays of the deformed lead threads (Figs: 4.27–4.29), these show that for test 8 (Figs: 4.27a & b), the rear row of piles experienced 20–30% of the differential displacement of the front row, so \( \psi = 0.3 \) could be adopted, to give a maximum \( \psi_f = 0.6 \), for \( \psi_f = 2.0 \). In the analyses a constant value of \( \psi_f = 0.5 \) was assumed.
From inspection of the test results (Figs: 4.10, 4.11, 4.13, 4.14), it was clear that the rear pile was loaded over a shallower depth. In most cases, this depth was about 2 m less than for the front pile. Accordingly, the value of h was reduced by 2 m for the calculation of maximum pressure on the rear pile. This is due to the pile being forced through the clay, which is undergoing less lateral displacement due to upward flow between the back piles and the embankment.

Figs: 6.35–6.38 show the results of the SIMPLE analysis on the pile group. Prediction of the lateral pressure was quite good for the front pile in test 8 for $\psi_f = 1$, although the depth of loading was smaller in the centrifuge model tests. Consequently, the experimental bending moments were slightly overestimated by SIMPLE for $\psi_f = 1$. However, the predicted deflections were considerably larger than those measured in test 8 (Fig: 6.35).

The analysis for the rear pile probably overestimated the pressure acting on it under these experimental conditions for $\Psi = 0.3$ and $\psi_f = 1$ compared with assumed $\psi_r = 0.5$, so it was expected that the pile bending moments would be overpredicted by the analysis (Fig: 6.36).

Comparison between the lateral pressure profiles was hampered by loss of data from bending moment transducers for pile BR in the clay layer, which affected the profile obtained after double differentiation of the fitted polynomial (Fig: 6.36). However, the general trend showed that the pressure was greater than zero at the top of the clay layer, and dropped below zero at about 2 m above the clay–sand interface.

The results were similar for test 9 (Figs: 6.37 & 6.38), although the differences between lateral pressures were more pronounced due to a deeper clay layer. In this analysis, the pile was fractionally too short (8 m cf 8.042 m) to be described as a flexible pile, however the analysis was continued.
6.9 **Concluding remarks**

Comparison between calculated and experimental bending moments confirmed that the lateral pressure imposed on flexible piles was created by differential movements between the pile and the soil. Initially, formulae based on elastic behaviour of the soil were used to evaluate the differential movement between pile and soil, and hence the lateral pressure profile, which was invariably parabolic for this single row of free headed piles. The analysis based on these assumptions successfully bracketed the experimental results.

Determination of a critical load, at which the soil moved into elasto–plastic behaviour, preceded an upper bound load based on a calculation of the ultimate lateral capacity of a pile. This fully plastic behaviour suggested that the pressure on the pile in the soft layer had built up to \(10.5c_u\), and it was this condition that gave a plausible upper bound to the pile bending moments.

For deeper soft layers, appropriate steps must be taken to allow for the tailing off of net lateral pressure across the pile with depth, otherwise the maximum pile bending moments are considerably overestimated.

It was also possible to predict the behaviour of the pile group based on an extension of the analysis proposed for the determination of lateral pressure on a single row of free headed flexible piles. Despite the differences between boundary conditions for the theoretical and experimental models, agreement was quite acceptable. This method would be capable of defining an upper limit to the design bending moment at working load, even if the deflections were more significantly overestimated.
7. DESIGN PROCEDURE

7.1 Introduction

The development of a theoretical analysis is only one of the many stages in the conception, design and construction of any facility. So it is with the problem of piled bridge abutments examined in this thesis. Earlier chapters have touched on the implications of the simplifications and assumptions which have led to the theoretical model which deals with lateral thrust on piles. It is now appropriate to study this in a wider context of the structure as a whole.

The sub/superstructure for a piled full–height bridge abutment should be designed as an integrated assembly. Following site investigation and field trials, geotechnical analyses will be implemented to consider bearing capacity and stability of the approach embankment, pile and pile group design including axial and lateral loading, long term total and differential settlement, lateral earth pressures, horizontal movements above and below ground level and retaining wall design.

An analysis has been proposed for the prediction of bending moments in, and deflections of, either a row of free headed piles or a pile group when an embankment is constructed adjacent to the piles. These piles were embedded in a stiff substratum, which is overlain by a soft clay layer, Design charts which assist this calculation for a free headed pile are introduced in this section. Alternatively, a computer program, SIMPLE, may be used to carry out this analysis.

Once the preliminary abutment design is completed, the effect on the bridge superstructure may be evaluated. Total and differential settlements, horizontal translation and differential movements, tilting, longitudinal and transverse distortion, and dynamic displacements will be considered. If these are within acceptable limits then the costs will be determined and the design refined only if a cheaper, serviceable alternative can be found. If the design is not within the serviceability criteria, then the foundation system, structural design or foundation will be adapted, and the optimising process continues.

Undrained behaviour of the foundation is generally more critical than when drainage is permitted,
and this was analysed herein. In the centrifuge model tests on long flexible piles, the bending moments induced by undrained loading reduced only slightly during consolidation. For short stiff piles, rotation about the tip allowed the pile displacement to increase marginally with time, decreasing the differential pile–soil movement and significantly reducing the measured bending moments. However the long term foundation consolidation will affect the displacement of the abutment and may cause tilting. Drained conditions should therefore be considered in relation to tolerable movements and the serviceability of the abutment and bridge deck.

7.2 Foundation characteristics

The first step is to investigate the ground conditions: to draw up a profile and to determine foundation strength parameters.

7.2.1 Clay

In the soft upper stratum it is necessary to have a profile of $c_u$ with depth:

$$c_u = c_u^{o} + \frac{dc_u}{dy} \cdot y$$  \hspace{1cm} (7.1)

Many factors influence the mobilised values of shear strength. Installation disturbance may combine with variability of the upper, weaker and more friable soil which lies in the critical zone for lateral resistance near the surface, due to weathering, seasonal changes in moisture content, and the possibility of scour. In this instance, there is a requirement for two values of $c_u$:

i) a lower bound strength, $c_u^{\text{min}}$ for bearing capacity calculations and for estimating the embankment load at which it is inappropriate to describe the foundation behaviour as pseudo–elastic, i.e. defining the ideal design zone (Fig: 6.12),

ii) an upper bound, $c_u^{\text{max}}$ to estimate the maximum lateral pressure which may be applied to the pile by the soft soil.

Eqn: 4.4, which predicts $c_u = f(OCR, \sigma_v')$, may also be used to generate a profile of $c_u$ with depth.
Design procedure

The stiffness of the soft clay, although required in the calculation of the lateral pressure acting on the pile, does not greatly affect the result. Selection of $G$ as a function of $c_u$ for soft clays was discussed in Section 5.5.3.1 when parameters were allocated for the finite element analyses. Evaluation of the site investigation data was discussed in Sections 4.4 and 4.5.

7.2.2 Determination of shear modulus in the stiffer substratum

The stiffness of the sand layer has been modelled using a linear profile of shear modulus which has been considered acceptable for engineering design (Randolph, 1977). Knowledge of the variation of $G$ with $\gamma$ will enable the designer to choose appropriate values of $G$ for the deformations expected in the region around the pile.

A sensitivity analysis on the magnitude of $G$ with depth is presented in Appendix: I, together with a discussion on:

i) laboratory determination,

ii) in-situ testing using a pressuremeter,

iii) empirical considerations.

In stiff clay or soft rock, the effects of softening or weathering at the surface of the layer should also be considered. Generally, $G/p' \approx 100$ for normally consolidated clays and $G/p' \approx 200$ for overconsolidated clays (Fleming et al, 1985).

7.3 Pile geometry

Once the pile material and shape have been chosen, a first guess of pile size and stiffness may be made. Equivalent pile properties $E_p$ and $d$ are calculated following the precepts outlined in Section 6.2 (Eqn: 6.1). The total length of pile required to ensure flexible behaviour under lateral loading may be decided once the critical pile length in the stiffer substratum has been determined from Eqn: 2.4.

For practical values of $s_x/d$ (e.g. > 2), the interaction between soil deformation zones around a
The stiffness of the soft clay, although required in the calculation of the lateral pressure acting on the pile, does not greatly affect the result. Selection of $G$ as a function of $c_u$ for soft clays was discussed in Section 5.5.3.1 when parameters were allocated for the finite element analyses. Evaluation of the site investigation data was discussed in Sections 4.4 and 4.5.

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For practical values of $s_x/d$ (e.g. $> 2$), the interaction between soil deformation zones around a
pile group can be dealt with during the selection of lateral pressure profiles, \( p_f \) and \( p_r \) for the front and rear piles (Section 6.8.3). Since the pile cap was assumed to be sufficiently rigid to prevent bending, the pile cap rigidity and geometry need not be entered for the SIMPLE analysis.

7.4 Lateral pressure on a pile in the soft layer

Clear recommendations on the choice of lateral pressure distribution emerged from Chapter 6:

i) parabolic profile for the pseudo-elastic working load case,

ii) linear profile for the plastic ultimate load case.

The equivalent embankment loading may be determined by taking the vertical stress due to the unit weight of the fill for the height of the embankment. Inclination of the net load on the foundation (Fig: 5.5) and the implications of this for foundation bearing capacity should be considered separately.

7.4.1 Preparation of the plasticity plot

The first stage considers the boundaries of pseudo-elastic and plastic behaviour, by preparing the plasticity plot (Section 6.5.3.1, Fig: 6.12). Plotting \( p/c_u \) (ordinate) against \( q/c_u \) (abscissa), the following zones may be distinguished:

i) the ideal design zone, which lies underneath the \( p/c_u \max = 2\pi \) line defining the point at which soil starts to yield around the pile, and to the left of the safe bearing line, Eqn: 6.6, when there is yield of the soil mass under the embankment, where \( c_{mob} = c_u\min' \)

ii) the elasto-plastic zone, which lies underneath the ultimate pile pressure defined by \( p/c_u \max = 10.5 \) and to the left of Eqn: 6.26 which defines the point at which complete bearing capacity failure will take place locally at the abutment wall along the road centreline (Section AA in Fig: 1.1c),

iii) the fully plastic boundary in which the soil shears plastically past the pile as the soil mass fails under the embankment,

The out of plane bearing capacity collapse condition (Section BB on Fig: 1.1c), will allow for the existence of any shear stress acting on the base of the embankment (Figs: 6.13a & b).
Design procedure

With these considerations in mind, it is possible to evaluate the lateral pressures acting on the pile in the soft clay layer due to the differential movement between the pile and the soil.

7.4.2 Ideal design zone: working load case

After inspection of the surcharge–soil–pile configuration, the shape of the lateral pressure profile in the soft layer may be confirmed. Relative soil–pile movement, and hence lateral pressure, at the top and bottom of the soft layer will be prevented during this loading phase by frictional restraint under the pile cap and along the soft–stiff soil interface. So, a parabolic pressure distribution may be adopted.

For deep soft layers, the depth, \( h \), over which the pile will be loaded laterally by the soil must be determined. This mechanism is complex and the design guidelines are, as yet, unresolved. However, this will be a refinement of what has been proposed here as a safe method of estimating lateral pressure. The finite element analyses showed the changeover from passive to active soil loading at \( y \approx 5.2 \text{ m} \) (Figs: 5.9b & 5.10b). In the centrifuge model tests, the passive lateral pressure decreased to zero between a depth of \( 5 < y < 6 \text{ m} \), below which \( \delta u_s - \delta u_p < 0 \). The depth at which this occurs is dependent on the embankment stiffness, shape, width and height and the relative pile–soil stiffness.

The factor, \( F \), which was used in Eqn: 6.24 to determine the parabolic lateral pile pressure characteristics will be chosen after consideration of the foundation conditions (Eqns: 5.14–5.16). Then the maximum pressure, \( p_m \) at the mid–depth of the soft layer may be determined (Eqn: 6.24) for different values of the soil displacement field factor, \( \psi \). In the centrifuge model tests, \( p_m/c_u \) was bounded by \( 1/2 \leq \psi \leq 2 \) (Figs: 6.15a & b), but \( \psi = 1 \) was a good approximation to an upper limit within the elastic zone for \( F = 1.57 \), and it is recommended here for design. This simple approach will give a conservative estimate.

7.4.3 Plastic failure: ultimate pile pressure

The ultimate lateral pressure which could act on the pile must also be considered. Defined as \( 10.5c_u \) over the entire depth of soft soil, a linear pressure distribution is usually adopted to give
an absolute upper bound. For stiffer clay layers, with higher values of \( c_u \), the embankment loading may not be sufficient to generate \( p_u = 10.5c_u \) over any or all of the depth of the layer. Eqn: 6.24 predicts the lateral loading for an equivalent pseudo–elastic interaction: a sensitivity analysis on the parameters affecting \( q/p \) should be carried out to find the worst case. A similar argument must be considered when dealing with deeper clay layers where \( p_u \) may be tailed off as shown in Fig: 6.14.

7.4.4 Design charts

If the computer program, SIMPLE, is not available, these chosen lateral pressure profiles can be fitted with any combination of the following for use with the design charts (Fig: 7.1a):

i) constant pressure with depth,

ii) pressure increasing or decreasing linearly with depth,

iii) parabolic loading, with zero pressure at the top and bottom of the layer and the maximum value at the mid–depth,

iv) parabolic loading, with zero pressure at the surface, and a maximum value either above or below the mid–depth.

The computer program allowed this distribution to be either linear, parabolic or a cubic spline fitted to data points of lateral pressure versus depth. Figs: 6.1, 6.16a, 6.16b showed the data input mode for the SINPILE version. It is possible to fit a large number of likely lateral pressure profiles using these design charts, by combining and superimposing the distributions above.

Using the chart shown in Fig: 7.1a, the value of a characteristic pressure, \( p_c \), and a load distribution factor, \( \beta_c \), may be determined, where, for linear relationships between \( p \) and depth, \( y \):

\[
p_c = p_{y=h/2}
\]

(7.2)

\[
\beta_c = \frac{p_{y=h/4}}{p_{y=h/2}}
\]

(7.3)

The charts are prepared for values of \( 0.5 \leq \beta_c \leq 1.5 \). Parabolic loading cases are also included.
Non-dimensional groups are defined such that lateral pressure $p$, force $H$, moment $M$, rotation $\theta$, and deflection $u$, and are presented as (Figs: 7.1a–c, 7.2a & b):

\[
\begin{align*}
\frac{p}{p_c}, \quad \frac{H}{p_c rh}, \quad \frac{M}{p_c r h^2}, \quad \frac{\theta EI}{p_c r h^3}, \quad \frac{u EI}{p_c r h^4} \quad \text{versus} \quad \frac{y}{h}
\end{align*}
\]

for different values of $\beta_c$, for the behaviour of the soft upper soil layer. Having established the values of $p_c$ and $\beta_c$, the pressure applied, the bending moment distribution and in particular $H_s$ and $M_s$ at the soft–stiff soil interface, $y = h$, may be determined from the charts (Figs: 7.1b & c) for all the components. These can then be applied to the bottom part of the pile which is embedded in the stiffer substratum. If, however, the pressure distribution reduces to zero above the interface (loading case (iv), Fig: 7.3), then simple structural analysis will determine the values of $H_s$ and $M_s$ at the top of the stiff layer based on moment, $M_h$, and shear force, $H_h$, at a depth $y = h$:

\[
H_s = H_h \quad (7.4)
\]

\[
M_s = M_h + (H_s h_u) \quad (7.5)
\]

### 7.5 Behaviour of stiff substratum

These are the steps in the analysis for the elastic behaviour of the lower section of the pile:

i) assume the pile is flexible if it exceeds a critical length $\{\ell_c = f (G_c, r, E_p)\}$, which is dependent on relative pile–soil stiffness,

ii) calculate a characteristic shear modulus $\{G_c = f (G_0, m, \nu, \ell_c)\}$ (Eqns: 6.27–6.29),

iii) iterate between i) and ii) (Eqns: 6.29, 6.31) to determine final values of critical pile length, $\ell_c$, and equivalent shear modulus, $G_c$; find $\rho_c$, (Eqn: 6.30),

iv) substitute these values into the algebraic expressions which relate deflection and rotation of the pile in response to a force or moment applied at the head of the pile (Eqns: 6.32, 6.33), or apply them to the charts which give normalised profiles of deflection and moment against depth (Figs: 6.18, 6.19).
7.6 Calculation of pile bending moment, rotation and deflection

Thus, the bending moment distributions in both sections of the pile are evaluated (Figs: 6.18b, 6.19b, 7.1c) and by reference to the charts of normalised moment versus depth (Figs: 6.18b, 6.19b), the maximum value can be noted, together with the deformation profile in the lower pile (Figs: 6.18a, 6.19a) and the rotation of the pile at the soft–stiff soil interface (Eqn: 6.33). The rotation and deflection components due to the loading in the top part of the pile may be found from Figs: 7.2a, 7.2b respectively. Furthermore, allowance can be made for a freestanding section of pile above the mudline and also for loading case (iv) when the pile is loaded over less than the full depth of soft clay (Fig: 7.3, 7.4), so that:

\[
\Delta \theta_u = \int_{h_s}^{h} \frac{M}{EI} \ dy = \frac{(M_h + M_s)h_u}{2EI} \tag{7.6}
\]

\[
\Delta u_u = \int_{h_s}^{h} \frac{M}{(EI)} \ dy \ dy = \frac{(M_h + M_s)h_u^2}{4EI} \tag{7.7}
\]

and:

\[
\theta_h = \theta_s + \Delta \theta_u \tag{7.8}
\]

If there is no ‘unloaded’ section, \( h_u = 0 \), and \( \Delta u_u = 0, \Delta \theta_u = 0 \) and \( \theta_h = \theta_s \). These can be added to the values from the lower part of the pile such that (Fig: 7.4):

\[
u_{pc} = u_s + h_u \tan \theta_s + \Delta u_u + h \tan \theta_h + \sum_{y=0}^{y=h_y} u/cos \theta_h + \sum \tan \theta \tag{7.9}
\]

\[
\theta_{pc} = \theta_h + \sum_{y=0}^{y=h_i} \theta \tag{7.10}
\]

Hence bending moment and deflection profiles may be predicted, and output as a plot (Fig: 6.20) or in numerical format (Table: 6.2), depending on the hardware available.

However, the most important information for the designer is the displacement and rotation at the pile head and the maximum bending moment carried by the pile, which generally occurs just
below the soft—stiff soil interface. The design charts may be used to find this information quite efficiently for simple distributions of lateral pressure in the soft layer. For a lateral load, $H$, Fleming et al (1985) estimated that the maximum moment:

$$M_{\text{max}} = (0.1/\rho_c) H \ell_c$$  \hspace{1cm} (7.11)

which occurs at approximately $\ell_c/4r$ for $\rho_c = 1.0$, and $\ell_c/3r$ for $\rho_c = 0.5$.

Increased pile deflections due to the proximity of other piles should be allowed for. However, the interaction factor methods described in Section 6.7.2 relate simply to a pile which is loaded by a single lateral load or moment, and not by the surrounding soil. This would be appropriate for the lower part of the pile, whose load has been modelled by these means. For the upper section, an interaction factor might be chosen based on the head load or moment that induced an identical displacement at ground level to that achieved through the soil's lateral thrust.

7.7 Bending moment design zones

7.7.1 Construction

A graph of maximum pile bending moment at significant nominal embankment loads can be constructed to show the design considerations for each case geometry. The aim is to guide the designer's final choice of pile size, material, and spacing, in order to keep within an ideal design area which has been described using the assumption that the foundation is behaving pseudo—elastically (Fig: 7.5). The maximum pile bending moments for the elastic solution with $\psi = 1/2$, and 2 define two points on this graph. Using the plasticity plot (Fig: 6.15a), pressures at the limit of the ideal design area (e.g. A, B) may be input into a SIMPLE analysis to calculate these maximum pile bending moments. An upper limit can be drawn through the origin to the $\psi = 2$ point, A, and a lower limit from the origin to the $\psi = 1/2$ point, B. The $\psi = 1$ line will fall between OA and OB, and its position is calculated in the same way, but was omitted from this figure for clarity.
Design procedure

In the centrifuge model tests, the air loading bag did not restrain the foundation from moving horizontally at ground level, whereas a sand embankment would be expected to do so. This would reduce the area which was straining under the embankment and so the value $\psi = 1$ which gave a good approximation to the results observed in the centrifuge model tests, should be an appropriate recommendation for an upper limit in practice (Fig: 7.7). A rough locus of $q_{\text{crit}} (AB)$ may be sketched by reference to the appropriate plasticity plot and the intersection of the $\psi$ lines with the limit to the elastic zone (Fig: 6.15a, points A & B). This area (OAB) describes the ideal design zone (Fig: 7.5).

The maximum possible induced pile bending moment is drawn as the upper bound, (line PQRS), to show the completely plastic boundary. The elasto–plastic zone is constructed by continuing the upper and lower elastic limit lines to intersect the upper bound (lines AQ, BU).

Bearing capacity failure under the embankment end wall, allowing for additional resistance from the piles will give the ultimate loading from the embankment. This should also be drawn on this graph (Eqn: 6.26, line RTV). Then it is possible to locate the design solution at a safe and efficient position within these bounds.

7.7.2 Implications of varying parameters

Fig: 7.6 shows the parameters which will change the position and size of these zones and boundaries. Measures that may be taken to ameliorate the design options include:

i) **ground improvement techniques**: pre-loading, embankment piling, excavation of selected soft material, installation of stone columns or wick drains, reinforcement,

ii) **embankment load**: reduce embankment height, use lightweight fill,

iii) **pile design**: alter pile spacing, diameter or material.

It is preferable to keep the problem within the elastic design zone to minimise yielding; either of the soil mass or of the soil around the pile, to ensure that the structure remains serviceable. It is also necessary to check that the plastic moment of the pile:
\[ M_p = Z_p \sigma_y \]  

is greater than the maximum moment imposed by the ultimate loading case for piles in soft clay, where \( \sigma_y \) is the yield strength of the pile material, \( Z_p \) is the section modulus. A different failure criterion is required for a reinforced concrete pile.

7.7.3 Shallow soft clay layer

Fig: 7.7 shows the bending moment design zones for tests 3 and 4, with \( h = 6 \) m, \( s/d = 5.25 \). The experimental data falls mainly between the \( \psi = 1/2 \) and \( \psi = 1 \) lines. The elastic design zone was small, indicating that \( q_{\text{crit}}/c_u \) lay between 4.7–5 (Fig: 6.15a), which for \( c_u = 15 \) kPa gave \( 70 < q_{\text{crit}} < 75 \) kPa.

The upper limit to the pile bending moments was well predicted by the \( 10.5c_u \) criterion. However, bearing capacity failure was expected at \( q = 106 \) kPa (Eqn: 6.26). This underestimated the strength of the foundation, which had clearly improved during consolidation and was capable of withstanding \( q_{\text{max}} = 228 \) kPa. Although this might suggest that \( p_u \) (= \( 10.5c_u \)) would increase, examination of radiographs for this test (following maximum load, Fig: 4.24) clearly showed the formation of ruptures around the pile. Despite consolidation of the clay in some locations under the embankment, this was accompanied by shearing and possibly by strain softening, which may explain the good agreement between predicted and experimental bending moments at maximum surcharge loading.

When \( s/d \) was reduced to 3.15, the elastic design zone was slightly larger (Figs: 6.15b & 7.8). In this case, \( \psi = 1/2 \) and \( \psi = 1 \) provided effective boundaries and the maximum bending moment was below the ultimate value predicted and still increasing with load at the end of the test. Bearing capacity failure was calculated to occur at \( q = 126 \) kPa but this was exceeded. At \( q_{\text{max}} = 189 \) kPa there were signs of excessive foundation movement under the edge of the load.
7.7.4 Deep soft clay layer

Fig: 7.9 shows the bending moment design plot for the semi-rigid pile behaviour examined in test 6. The \( \psi \) lines at 1/2 and 2 bounded the experimental data and the effect of the consolidation period at \( q = 72 \) kPa was clearly seen. As the pile rotated marginally, more load was carried by the soil and the pile bending moments decreased. Bearing capacity failure under the abutment end wall was anticipated at \( q = 106 \) kPa, but this assumed that the pile was securely fixed into the lower sand stratum. In fact, signs of local failure first appeared under the edge of the surcharge load for \( q = 150 \) kPa (see Fig: 4.22b for photograph taken post-test).

The clay reached the 10.5\( c_u \) condition over the majority of the soft layer, and a maximum moment was achieved before the last two surcharge loads were added. However, it was clear that the soil was not shearing past the pile over the whole of the 8 m soft layer since the ultimate bending moment overestimated the maximum observed by 35 and 63\% (Fig: 7.9). A second calculation was carried out assuming that the soil sheared past the pile over the top 6 m, with the lateral pile pressure dropping off from the 10.5\( c_u \) condition to zero at 8 m (Fig: 6.31). However, this still overestimated the maximum moment by 3 and 24\% (Fig: 7.10). The elastic pressure distribution was also changed to allow application of pressure over the top 6 m of clay (Fig: 7.3).

7.8 Example

It may be helpful to work through an example which illustrates the use of the design charts and procedures to predict ground level pile deflections and maximum pile bending moments. Consider an idealisation of Fig: 1.1b, in which a long rectangular block of fill, 8 m high, is placed adjacent to a row of free headed piles which penetrate a 6 m layer of soft clay and are embedded in a stiffer sand substratum. These piles may be, as a preliminary choice, of minimum length 16 m below ground, 1.27 m diameter reinforced concrete, with \( E = 40 \times 10^6 \) kPa, \( I = 0.1277 \) m\(^4\), installed at a spacing of 4.0 m, with \( s/d = 3.15 \). There will be no freestanding length of pile above ground level, \( y = 0 \) m.
7.8.1 Problem geometry and foundation properties

For the embankment: specify lightweight fill, $\gamma_e = 15.5$ kN/m³, $h_e = 8$ m, $q = 124$ kPa.

For the soft clay: take $c_u \text{min} = 22$ kPa (for bearing capacity calculations),

(0 ≤ y ≤ 6 m) take $c_u \text{max} = 22 + 2y$ kPa (for calculation of $p_u'$, Eqns: 6.25c, 7.1),

if $G/c_u \approx 100$, $G_y=3 = 2800$ kPa,

choose $F = 1.57$ (Eqns: 5.14, 6.24).

For the sand: $G = 2 + 10 (y - 6)$ MPa (Eqn: 6.27),

(y ≥ 6 m) $v = 0.3$, $G^* = (1 + 0.3 \times 0.75) G = 1.225 G$ (Eqn: 6.28),

assume $\ell_c = 10$ m, $G_c = 1.225 \times 52 = 63.7$ MPa (Eqn: 6.29),

$\ell_c = 1.27 (40 \times 10^{-3}/63.7)^{2/7} = 8$ m (Eqn: 6.31),

iterate so that $\ell_c = 8.4$ m, $G_c = 53.9$ MPa,

$\rho_c = (1.225 \times 23)/(1.225 \times 44) = 0.523$ (Eqn: 6.30).

Then, determine the lateral pressures acting on the pile in the clay layer, assuming that the pile will be laterally loaded over the entire 6 m depth of clay.

7.8.2 Working load case: parabolic distribution

Assume soil deformation characteristic, $\psi = 1$, and the soil is loaded over the entire depth of soft layer so $h_u = 0$ and $h = h_s$. From Eqn: 6.24:

$$p_m = \frac{124 \times 1 \times 6}{4 \times 1.27 \left[ \frac{1}{1.57} + \frac{1}{4 \times 4} + \frac{11}{192} \times \frac{2800 \times 6^4}{40 \times 10^6 \times 0.1277} \right]} = 139.5 \text{ kPa}$$

which implies that the change in pressure in front of the pile is ≈ +70 kPa, whilst the drop in pressure behind the pile is about −70 kPa.

Check the plasticity plot (Fig: 6.12) to ensure that this loading case will plot inside the ideal design area:
For $s/d = 3.15$, $h = 6$ m, $F = 1.57$, (Fig: 6.15b), this working load case will plot on the boundary of the ideal design zone, at the junction with the $\psi = 1$ line. This is acceptable. Bearing capacity failure under the abutment end wall (Eqn: 6.26) will occur at $q/c_u \approx 8.4$, so $q_{\text{max}} \approx 185$ kPa, which is $\gg 124$ kPa. Out of plane bearing capacity collapse would also require investigation.

### 7.8.3 Ultimate load case: linear distribution

From Eqns: 6.25c and 7.1:

$$p_u = 10.5 c_u = 231 + 21y \text{ kPa}$$

### 7.8.4 Calculation of pile bending moment, rotation and deflection

Using the design charts (Figs: 7.1 & 7.2) and the equations defined in Chapters 6 and 7, work through the example as follows:

<table>
<thead>
<tr>
<th>Working load case</th>
<th>Ultimate load case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Establish characteristic lateral pressure and load description factor:</td>
<td></td>
</tr>
<tr>
<td>$p_c$ (Eqn: 7.2, Fig: 7.1a)</td>
<td>139.5 kPa</td>
</tr>
<tr>
<td>$\beta_c$ (Eqn: 7.3)</td>
<td>—</td>
</tr>
<tr>
<td>Refer to design charts to find shear force and moment at the top of the sand layer:</td>
<td></td>
</tr>
<tr>
<td>$H_s/(p_c r h)$ (Fig: 7.1b)</td>
<td>1.333</td>
</tr>
<tr>
<td>$M_s/(p_c r h^2)$ (Fig: 7.1c)</td>
<td>0.667</td>
</tr>
<tr>
<td>$H_s$</td>
<td>0.71 MN</td>
</tr>
<tr>
<td>$M_s$</td>
<td>2.13 MNm</td>
</tr>
</tbody>
</table>

Establish the deflection and rotation at the top of the sand layer:
Design procedure

<table>
<thead>
<tr>
<th>Working load case</th>
<th>Ultimate load case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_h \frac{r G_c}{H_s} \left[\frac{E_p}{G_c}\right]^{1/3}$ (Fig: 6.18a)</td>
<td>0.51</td>
</tr>
<tr>
<td>$u_m \frac{r^2 G_c}{H_s} \left[\frac{E_p}{G_c}\right]^{3/7}$ (Fig: 6.19a)</td>
<td>0.575</td>
</tr>
<tr>
<td>$u_h$</td>
<td>4.11 mm</td>
</tr>
<tr>
<td>$u_m$</td>
<td>3.32 mm</td>
</tr>
<tr>
<td>$u_s = u_h + u_m$</td>
<td>7.43 mm</td>
</tr>
<tr>
<td>$\theta_s$ (Eqn: 6.33)</td>
<td>$2.618 \times 10^{-3}$rads</td>
</tr>
<tr>
<td></td>
<td>$7.93 \times 10^{-3}$rads</td>
</tr>
</tbody>
</table>

There is no unloaded section of pile in the clay layer, so $h_u = 0$ and the additional components of $u$, $\theta$ in the soft layer:

| $\Delta \theta E I (p_c r h^3)$ (Fig: 7.2a) | 0.2 | 0.30 |
| $\Delta u E I (p_c r h^4)$ (Fig: 7.2b)     | 0.156 | 0.23 |
| $\Delta \theta$                           | $7.469 \times 10^{-4}$rads | $2.37 \times 10^{-3}$rads |
| $\Delta u$                                | 3.5 mm | 10.89 mm |

and due to the rotation, $\theta_s$, at the base of the soft layer (Fig: 7.4):

- $h \tan \theta_s$ = 15.71 mm
- $47.56$ mm

so rotation and deflection at the ground surface will be:

- $\theta_o = \theta_s + \Delta \theta$ (Eqn: 7.10) = $3.365 \times 10^{-3}$rads
- $1.03 \times 10^{-3}$rads

- $u_o = u_s + \Delta u + h \tan \theta_s$ = 26.64 mm
- $80.80$ mm

- $u_o/d$ = 2.1%
- $6.4\%$

However, allowance has not been made for the interaction between piles. Factors for increasing calculated rotations and deflections were listed in Table: 6.3 for a row of 5 piles at $s/d = 3.15$, with the pile–soil stiffness ($E_p/G_c$) for the centrifuge model tests which are identical to $E_p/G_c$ from this example. The loading was assumed to be applied equally at the top of each free headed pile by means of a shear force, $H$, or a moment, $M$. For the most critical (middle) pile, $\alpha_{uH} = 0.32$, $\alpha_{uM} = \alpha_{\theta H} = 0.102$, $\alpha_{\theta M} = 0.033$. These additional interactions mean that the total lateral displacements for a free headed pile would be certain to exceed the 25 mm serviceability criterion for differential lateral displacement (US Department of Transportation, 1985). But, since the pile
configurations used for a bridge abutment generally have a fixed pile cap, this would reduce the deflections.

By inspection (Figs: 6.18b, 6.19b), to find the maximum bending moment at:

<table>
<thead>
<tr>
<th>Working load case</th>
<th>Ultimate load case</th>
</tr>
</thead>
<tbody>
<tr>
<td>((y-h_s)/\ell_c)</td>
<td>0.25</td>
</tr>
<tr>
<td>(y)</td>
<td>8.1 m</td>
</tr>
<tr>
<td>(M_h/H_{s_c} \ell_c) (Fig: 6.18b)</td>
<td>0.17</td>
</tr>
<tr>
<td>(M_m/M_s) (Fig: 6.19b)</td>
<td>0.84</td>
</tr>
<tr>
<td>(M_{\text{max}} = M_m + M_h)</td>
<td>2.80 MNm</td>
</tr>
<tr>
<td></td>
<td>8.39 MNm</td>
</tr>
</tbody>
</table>

Check \(M_{\text{max}}\) is less than the plastic moment for the pile. If not, redesign reinforcement, increase concrete strength, or increase size of pile since it is unlikely that \(s/d < 3\) will be used in practice.

The value of \(M_{\text{max}}\) at working load should plot onto Fig: 7.5 at the junction of the \(\psi = 1\) line and the locus of \(q_{\text{crit}}\'. Table: 7.1 summarises the calculation at working load.

When the analysis was repeated using the SIMPLE program for a group of piles with the same attributes, with identical parabolic working load case, \(p_m = 139.5\) kPa, acting on both front and rear piles, \(u_0\) was reduced by 62% to 10 mm (\(u_0/d = 0.8\%\)), with \(\theta_0 = 0\) (which is a pre-condition of the program). The maximum moments at working load were -1.78 MNm at \(y = 0\) m, and +1.24 MNm at \(y = 9.15\) m which are 64% and 44% of \(|M_{\text{max}}|\) for the single free headed pile at working load respectively. Similar reductions obtain for the ultimate load case when the maximum moments were -5.45 MNm at \(y = 0\), and +3.73 MNm at \(y = 9.15\) m. These improvements in lateral pile performance, under both serviceability and collapse conditions, demonstrate the advantages of using a pile group with a fixed pile cap.
7.9 Other design aspects

While a satisfactory pile design is being conceived, analysed and approved for the lateral loading case, other local considerations such as axial loading capacity, total or differential settlement, must be investigated together with the impact on the rest of the structure of the pile behaviour. US Department of Transportation (1985) comment that horizontal differential movements are far more damaging to abutments and bridge decks than differential vertical settlements. They recommend that the combined tolerable movement criteria are 100 mm vertical and 25 mm lateral movement.

The effect of time on the behaviour of the piled foundation depended on the relative length of the pile and hence the displacement mechanism. For a deep soft layer, the pile acted rigidly by rotating about the tip. As the surrounding soil consolidated under a constant load, the pile bending moments reduced noticeably (Fig: 7.9) accompanied by a slight increase in pile displacement. However there was little change in either M or u for long flexible piles.

Retaining wall and pile cap design, embankment bearing capacity and stability must also be examined. A complete breakdown of costs, availability of materials, site conditions and location, transportation and environmental impact will all be factors in the final choice of design.

7.10 Concluding remarks

Any design is only as good as the weakest link. All aspects of the design problem must be investigated so that the engineer develops a feel for the variables, and so that modifications and improvements may be implemented to give the safest and most serviceable, cost-efficient structure.
8 CONCLUSIONS

8.1 Concluding remarks

The performance of a row of free headed piles embedded in a stiff stratum overlain by a shallow soft clay layer was investigated, and the factors influencing the pile response due to lateral thrust in the soft layer were determined. Some advances were also made in discerning modes of pile group behaviour.

Two mechanisms were identified; elastic interaction during the early loading phase, progressing to complete plastic flow when the surcharged foundation reached failure. An ultimate failure mechanism was proposed, and this allowed the maximum values of lateral pile pressure, bending moment, deflection and rotation to be deduced. But, in general, this would not be the controlling factor in a design which must satisfy serviceability criteria, and so the analyses developed within this dissertation concentrated on the treatment of pile and soil deformations at working load. This approach is consistent with the design methodology applied to almost all soil–structure interaction problems.

Three methods were employed. Experimental work was carried out using a geotechnical centrifuge. This provided a database on free headed piles and pile groups. In parallel, a site specific finite element study for single piles was calibrated against this data. Finally, a design procedure was developed and this was compared with the results from the centrifuge model tests and the numerical analyses.

The centrifuge model tests were designed and conducted successfully for prototypes constructed to working load levels over periods of 2–3 months, before the load was increased to failure over a longer time span. The simplification introduced by replacing the sand embankment with an apparatus which imposed a normal load on the foundation removed the doubt about the magnitude of this load, and enabled testing conditions to be more controlled. Therefore, the stiffness of the embankment and the existence of any shear stresses which might act at the base of
it were not modelled. In practice, the embankment stiffness tends to reduce the load transmitted
to the foundation, and so the normal loading model was conservative. For configurations which
permit relative lateral movement between the soil foundation and the base of the embankment,
the net load will be inclined to the vertical and the bearing capacity will be reduced.
Consideration of this type of failure was included in the design procedure, although a bridge
abutment incorporating a full-height retaining wall will be less susceptible to this behaviour.

Soil beneath a corner of an abutment can spread in both orthogonal horizontal directions, which
should tend to reduce the relative soil–pile displacement, and therefore the lateral soil pressure.
It is suggested that the plane strain approach will be conservative in practice. The layout of piles
below the edge of the road should be positioned so that they are not liable to be subjected to
greater lateral thrusts than those along the centreline of the approach embankment. However, this
is unlikely to cause problems in the practical design for pile groups supporting bridge abutments.

The critical pile length which determined whether the pile behaved in a flexible or rigid fashion
during lateral loading was investigated. The pile length and relative pile–soil stiffness were
important comparators and Eqn: 2.4 (Randolph, 1981a) was employed with good results when
calibrated against the experimental data. Since the mode of pile deformation and hence the gross
pile movements and rotation at ground level were paramount in the evaluation of tolerable
abutment movement for serviceability, this confirmation is useful.

The bending moment data were processed numerically to give the pile deflection and a guide to
the pressure acting on the piles. The shape of the profiles was generally as expected, although
double differentiation of the polynomial fitted to the bending moment data points deduced from
the strain gauge readings gave lateral pressures that were less accurate. On the few occasions
when this error was combined with curve fitting to suspect data points, an entirely different
pressure distribution was observed.

In general, the derived experimental lateral pressure distributions were parabolic, tailing off to
zero at ground level and at around 5–6 m below the surface for the prototype tests conducted. This vindicated the key assumption of the SIMPLE analysis that the pile was loaded passively in the soft layer, while applying load to the soil in the stiffer substratum.

In the centrifuge model tests on pile groups, the pile cap was located above ground level so that the strain gauge instrumentation could be properly installed. However, this changed the boundary conditions at the upper surface so that soil was able to move vertically. In consequence, the lateral pressure acting on the front row of piles was greater than that exerted on the rear row. The maximum bending moments occurred at the junction with the pile cap for the rear piles. However, the magnitude of these moments and the deflection of the group were less than those for free headed piles under an equivalent load. In practice, there would be continuity of plastic flow between each row of piles in undrained conditions, and the behaviour of successive rows of piles would be similar to that of the first row.

The site investigation conducted during the centrifuge model tests measured vane shear strength and cone penetration resistance. The accuracy and interpretation of these tests affected the values selected for the constitutive properties to be used in the numerical modelling. Since the analysis proposed in this dissertation is dependent on values of $c_u$ in the soft clay layer and $G$ in the stiffer substratum, more fundamental relationships would be helpful.

Baguelin et al (1977) showed that lateral pressure, $p$, acting on a pile was proportional to the relative displacement between the soil and the pile, $(\delta u_s - \delta u_p)$, for an elastic soil model and plane strain conditions. Earlier two dimensional finite element studies, which replaced a row of piles by a sheet pile wall, severely misrepresented the pile–soft soil interaction by assuming no relative movement. Thus previous hypotheses which proposed that a plane strain analogue could model this three dimensional problem during loading at pseudo–elastic levels were disproved. A three dimensional analysis was carried out using CRISP to find the lateral pressure acting on the piles, the relative pile–soil displacement and the bending moments in the pile.
Baguelin et al. (1977) explored the relationship between \( p \) and \( (\delta u_s - \delta u_p) \) for a linear elastic soil flowing in plane strain around a single pile. A three dimensional elastic finite element study, although restricted in the number of elements, confirmed the qualitative relationship but demonstrated that simulated embankment loading causes pile behaviour which cannot be represented in plane strain. Strain softening of the soil around the pile was not modelled at this stage, but the results from the numerical analysis were verified by the experimental data obtained up to the working load.

The SIMPLE program predicted bending moment and deflection profiles down the pile due to the application of a lateral thrust on the pile in the soft layer under the embankment, which was resisted by the section of pile embedded in the stiffer substratum. This method was developed and programmed in Fortran for interactive analysis of flexible piles, and was extended to allow for pile groups with a rigid pile cap, fixity at the pile–pile cap junction and no pile cap rotation.

Assumed displacement fields for the soft soil under the surcharge were proposed, and used to estimate the lateral pressure experienced by the pile in this layer for elastic soil behaviour. These bounds to applied pressure, and the resulting pile response, bracketed the centrifuge model test results.

For abutments founded on either a deep soft layer or a stiffer clay, a reduction in the design ultimate lateral pressure acting on the pile was recommended based on the centrifuge model test results. Otherwise the design maximum pile bending moment would be unreasonably large for the embankment loading applied.

The lateral pressure at which the soil began to yield around the pile \( (p = 2\pi c_u) \) was defined on a plasticity plot together with the ultimate limiting pressure \( (p_u = 10.5c_u) \) at which the soil flowed plastically past the pile. Embankment failure by bearing capacity collapse around a row of piles and a forecast of the onset of general plastic yielding of the soil mass were also found. This latter condition and the expression, \( p = 2\pi c_u \), bounded an area within which behaviour could be
modelled as pseudo–elastic. The intersection between the limiting pile pressure and bearing capacity failure described the final state of plastic collapse where failure occurred simultaneously in the soil mass and around the pile.

Design charts were presented to assist quick hand calculation of the SIMPLE analysis. Randolph's (1981a) charts for lateral loading of flexible piles were used for the behaviour of the pile in the stiff substratum. Additional charts were developed to account for the pile response in the soft layer, and these were constructed in a similar manner. This provided a powerful way of examining the effect on the piles of all the variables when the computer program was not available.

Finally, a design methodology for piled full–height bridge abutments was discussed with a view to producing a cost efficient structure, with minimal maintenance requirements due to long term abutment movements. Safety and serviceability criteria were combined by including this analysis in the general design plan.

This research has contributed to the general understanding of the performance of piles subjected to lateral soil movement. A design procedure for piled full–height bridge abutments, and other problems which include active lateral loading of piles by a nearby surcharge, has been suggested.

8.2 Recommendations for future work

The three dimensional site specific finite element study for a single pile could be continued using either Cam Clay (Schofield & Wroth, 1968) or a strain hardening or softening soil model to allow for disturbance due to pile installation, subsequent consolidation and the reduction in soil strength due to the high strains in the immediate vicinity of the pile. Parametric studies might also be useful, based on a typical prototype, to illustrate the effect on pile performance of the variation of the foundation geometry, pile spacing and pile–soil stiffness.
Conclusions

Further thought is required on the behaviour of pile groups and the pressure distributions imposed on the piles. The three dimensional finite element analyses might be extended to investigate this, using both elastic and elasto–plastic soil models. In the past, interaction between the piles has been described (Poulos, 1971; Randolph, 1983) in terms of a pile loaded at the head by either a force or a bending moment and not a lateral thrust. Based on numerical analysis of a pile group, alternative expressions could be developed to allow for passive loading of the pile.

The properties of the soils used in the centrifuge model tests were not derived satisfactorily from the site investigation data. Further research is necessary in the area of small scale in–situ testing in the centrifuge and the empirical relationships quoted in the literature between measured resistances and soil strengths and stiffnesses.

Integrated design of the piles, pile cap and abutment retaining wall loaded by a sand embankment has not been examined in detail. Future centrifuge model tests might include elements from those described herein, the facility for in–flight construction of the embankment, and the option to apply a combination of dead and live loads to the abutment (Sun & Bolton, 1988). This would encompass the complete boundary value problem. A further extension to this work might include an investigation into the advantages of embankment piling. Almeida (1984) describes the installation of stone columns and their use in improving the performance of embankments constructed on soft clay.
Determination of shear modulus in the sand layer

APPENDIX 1

Determination of shear modulus in the sand layer

A.1 Introduction

Most granular soil deposits will have experienced sufficient cycles of loading to have reached a stable hysteretic state, although this process takes longer for increasing amplitudes of stress and for looser soils (Bolton, 1988). However for completely virgin soil, the initial loading cycle will show plastic shear strains and a lower value of shear modulus. Duncan & Chang (1970), Bolton (1988) estimate that this reduction is of the order of 2 for dense and 5 for loose deposits.

A.2 Impact of shear modulus profile on pile bending moments

Considering a stiff layer with shear modulus described by Eqn: 6.27, the approach outlined in Section 6.6 may be used to produce bending moment profiles for \( \frac{p_u}{c_u} = 10.5c_u \) in the soft clay, where \( c_u \) is constant with depth. Non-dimensionalised graphs (Figs: A.1a & b) illustrate the effect of maintaining soil homogeneity factor, \( \rho_c = 0.5 \) (linear variation of G with depth) and \( \rho_c = 1.0 \) (G constant with depth). Large lateral strains are expected in the soil at the ground surface around a laterally loaded pile, and a smaller secant shear modulus is appropriate at this horizon. Lateral strains will decrease to zero at depths below the critical pile length, where a higher value of shear modulus should be selected, and some interpolation between 0.5 < \( \rho_c < 1.0 \) is realistic.

A.3 Laboratory determination

Powrie (1987) conducted triaxial and plane strain (in a triaxial apparatus) testing on two almost identical overconsolidated (OCR \( \approx 2.3 \)) samples of kaolin, which were prepared in a similar fashion to the Author's samples. He investigated the effect of stress reversals on the variation of \( \frac{E_u}{c_u} \) with axial strain, where \( E_u \) was the secant value of undrained elastic modulus and the strain was reset to zero after each reversal (Fig: A.2). Notably, it is the modulus obtained just following stress reversal, which gives the stiffest response and, as expected, the plane strain samples showed greater stiffness and strength. For undrained samples such as these, a direct conversion to G may be made using \( G = \frac{E_u}{3} \).
In the past, it was common practice to measure the G–γ relationship from small scale laboratory tests in the triaxial or simple shear apparatuses after restoring the sample to the presumed in-situ stress state. However, there are inherent problems in ensuring that the sample remains undisturbed during insertion of the sampling tube, transportation and subsequent storage, and finally, during extrusion and preparation for testing. Choosing a small volume of soil to depict the behaviour of the whole mass, once outside the confines of controlled sample preparation for centrifugal modelling, may also lead to misinterpretation of properties if the presence of fissures, larger fragments and soil anisotropy are ignored.

Nowadays, laboratory determination of parameters may be combined with in-situ testing, which has become more popular through the development of the self boring pressuremeter (Jezequel, 1968; Wroth & Hughes, 1973; Mair & Wood, 1987), other types of cone penetrometer (Meigh, 1987) and flat plate dilatometers (Marchetti, 1980). Of these options, the self boring pressuremeter is thought to offer the least disturbance to the soil fabric and the in-situ stress state (Wroth & Hughes, 1973), and it is used to measure $G = f(\gamma)$ directly without recourse to empirical correlations.

A.4 Self boring pressuremeter

In-situ testing using a self boring pressuremeter offers horizontal pressure–deformation characteristics from which the appropriate secant shear modulus may be evaluated, at a variety of depths, and for the range of stresses and strains that will be experienced during the life of the foundation. French research was summarised by Baguelin, Bustamante & Frank (1986), who defined values of shear modulus at 0, 2% and 5% volumetric strain as $G_{p_0}$, $G_{p_2}$ and $G_{p_5}$.

Lateral loading effects are likely to dissipate over the critical pile length, with zero lateral strain below this, so $G_{p_0}$ would be an appropriate value at $y = \ell_c$. At the surface, where larger strains are expected, a smaller modulus, perhaps $G_{p_5}$, would be applicable. However, the disturbance caused by pile installation may indicate that $G_{p_2}$ is a better choice at $y = \ell_c$. To investigate this suggestion, full scale laterally loaded pile tests were analysed. These were conducted in France
and are reported below, together with nearby self boring pressuremeter tests.

A full scale pile test at Provins (Bigot, Bourges & Frank, 1982) was carried out on a steel pipe pile which was 0.926 m diameter, 0.015 m thick and 11.55 m long (Fig: A.3a). The pile was instrumented to give bending moment and deflection data. A horizontal load of 120 kN was applied 0.2 m above the ground level and the resulting bending moments were measured. Using Eqn: 6.27 to define $G$, values of $G_0$ and $m$ were applied to the analysis described in Section 6.6 (Figs: 6.18 & 6.19), to give a best fit to the observed profiles of pile bending moment and deflection from Provins (Fig: A.3b). The agreement between the best fit values of $G$ obtained from these sources would have been better if the stiffening effect of the extensometer tubes, which had been welded to the pile, had been included in the calculation of pile bending rigidity.

A pressuremeter test was conducted in the vicinity of the Provins pile. Despite observing an almost constant average shear modulus below an overconsolidated, stiffer crust, this did not represent the values of $G_0$ and $m$ back analysed from the site bending moment and deflection data because $G$ is a function of $\gamma$. $G_{p2}$ and $G_{p5}$ were plotted against depth (Frank, 1988), together with the best fit values of modulus (Fig: A.4). $G_0 = 0.8$ MPa, $m = 0.35$ MPa/m gave the best fit to the deflection curve whilst $G_0 = 0.48$ MPa, $m = 0.26$ MPa/m gave the best approximation to the experimental pile bending moment data. $G_{p5}$ overestimated the shear modulus at the surface and $G_{p2}$ underestimated the value at depth.

Another lateral loading test (Plancoet) was described by Price, Wardle, Frank & Jezequel (1987) (Fig: A.5a). A combination of extensometers (E–Ls) and strain gauges (ERS) were used to provide data from which pile bending moments were determined (Fig: A.5b). These are presented in Fig: A.5c together with the best fit data as before. Predicted and experimental data agreed well for $G_0 = 0$ MPa, $m = 0.8$ MPa/m. This profile is plotted on results from a pressuremeter test which was conducted near the pile (Fig: A.6, Frank, 1988).

With greater strains expected at the surface than at depth, it was not surprising that $G_0$ was close to the value of $G_{p5}$ and the shear modulus was between $G_0$ and $G_{p2}$ at depth, as for the Provins
pile. Fleming et al (1985) recommended that $G_o$ was either half the value taken for axial loading at ground level, or zero, increasing to the full value taken for axial loading, at the critical depth.

A.5 Empirical considerations

Robertson & Campanella (1983) obtained correlations between dynamic shear modulus, cone resistance and vertical effective stress for uncedmented, normally consolidated sands under small strains for standard cone penetration tests (Fig: A.7). For the mid-depth of the 10 m sand layer in the centrifuge model tests, where a lower bound $q_c = 3$ MPa, their estimate of $G_{max} \approx 50$ MPa compared with the assumed value of 52 MPa (Table: 4.2).

Wroth et al (1979) conducted a literature survey investigating ways of estimating $G$. Often $G$ may be proportional to $p'$, and it is usually realistic to allow a linear distribution of $G$ with depth for sands under high strains. However, at small strains, Eqn: 4.7 was suggested, based on curve fitting dynamic laboratory test data on sands, whilst the following expression accounted for the effect of strain:

$$\frac{G}{p_a} = \frac{710}{1 + 1600\gamma \frac{p'}{p_a}} (0.765 - 0.33 \exp^{-3000\gamma} \left[0.9 + \frac{D_r}{500}\right]^{1.23} \quad (A.1)$$

These equations were valid for a range of $10^{-6} < \gamma < 10^{-2}$; $0.25 < p'/p_a > 2$; $20 < D_r > 100$, and imply that $300 < G/p' > 600$, which is generally applicable for lateral loading at working levels.

Relationships between standard penetration test (SPT) data and $G$ were also reviewed by Wroth et al (1979) who recommended Eqn: 4.6, based on data between $60 \ N^{0.71} < G_{max}/p_a > 300 \ N^{0.8}$. However, the value of blow count, $N$ determined by SPT depends on the type of hammer and method of initiating its fall. Frydman (1970) conducted field trials which showed variations in $N$ of up to 40%. More recent work by Seed et al (1985) compared international testing methods and recommended correction factors to align these with a standard.
References


References


References


* see end of references
References


Randolph, M.F., (1983). PIGLET — A computer program for the analysis and design of pile groups under general loading conditions.


Reese, L.C., Matlock, H., (1956). Non-dimensional solutions for laterally loaded piles with soil modulus assumed proportional to depth. Proc. 8th Texas Conf. SMFE, Austin, Texas.


References


### TABLE: 3.1

Centrifuge scaling relationships

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Scaling relationship Model:Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity</td>
<td>m/s²</td>
<td>n</td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>1/n</td>
</tr>
<tr>
<td>Area</td>
<td>m²</td>
<td>1/n²</td>
</tr>
<tr>
<td>Volume</td>
<td>m³</td>
<td>1/n³</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>1</td>
</tr>
<tr>
<td>Mass</td>
<td>kg</td>
<td>1/n³</td>
</tr>
<tr>
<td>Stress</td>
<td>kPa</td>
<td>1</td>
</tr>
<tr>
<td>Strain</td>
<td>%</td>
<td>1</td>
</tr>
<tr>
<td>Force</td>
<td>N</td>
<td>1/n²</td>
</tr>
<tr>
<td>Deflection</td>
<td>m</td>
<td>1/n</td>
</tr>
<tr>
<td>Bending moments</td>
<td>Nm</td>
<td>1/n³</td>
</tr>
<tr>
<td>Time (diffusion)</td>
<td>s</td>
<td>1/n²</td>
</tr>
<tr>
<td>Time (strain rate)</td>
<td>s</td>
<td>1</td>
</tr>
</tbody>
</table>

### TABLE: 3.2

Details of centrifuge model tests

<table>
<thead>
<tr>
<th>Date</th>
<th>Test</th>
<th>No of pile rows</th>
<th>No of piles per row</th>
<th>Pile fixity</th>
<th>Loading system</th>
<th>Prototype foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb</td>
<td>KP/SMS1</td>
<td>1</td>
<td>3</td>
<td>Free</td>
<td>Sand</td>
<td>16 m clay</td>
</tr>
<tr>
<td>Mar</td>
<td>KP/SMS2</td>
<td>1</td>
<td>3</td>
<td>Free</td>
<td>Sand</td>
<td>6 m clay on 10 m sand</td>
</tr>
<tr>
<td>Dec</td>
<td>KP/SMS3</td>
<td>1</td>
<td>3</td>
<td>Free</td>
<td>Water</td>
<td>6 m clay on 10 m sand</td>
</tr>
<tr>
<td>Feb</td>
<td>KP/SMS4</td>
<td>1</td>
<td>3</td>
<td>Free</td>
<td>Air</td>
<td>6 m clay on 10 m sand</td>
</tr>
<tr>
<td>Apr</td>
<td>KP/SMS5</td>
<td>1</td>
<td>3</td>
<td>Free</td>
<td>Air</td>
<td>6/10 m soft/stiff clay</td>
</tr>
<tr>
<td>May</td>
<td>KP/SMS6</td>
<td>1</td>
<td>3</td>
<td>Free</td>
<td>Air</td>
<td>8.5 m clay on 7.5 m sand</td>
</tr>
<tr>
<td>Aug</td>
<td>KP/SMS7</td>
<td>1</td>
<td>5</td>
<td>Free</td>
<td>Air</td>
<td>6 m clay on 10 m sand</td>
</tr>
<tr>
<td>Oct</td>
<td>KP/SMS8</td>
<td>2</td>
<td>3</td>
<td>Fixed</td>
<td>Air</td>
<td>6 m clay on 10 m sand</td>
</tr>
<tr>
<td>Nov</td>
<td>KP/SMS9</td>
<td>2</td>
<td>3</td>
<td>Fixed</td>
<td>Air</td>
<td>8 m clay on 8 m sand</td>
</tr>
<tr>
<td>Jan</td>
<td>KP/SMS10</td>
<td>2</td>
<td>3</td>
<td>Fixed</td>
<td>Air</td>
<td>6/10 m soft/stiff clay</td>
</tr>
</tbody>
</table>
TABLE: 3.3

Voids ratio and relative density of sand samples in centrifuge model tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Depth of sand, mm</th>
<th>Dry unit weight, kN/m³</th>
<th>Voids ratio</th>
<th>Relative density, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>KP/SMS3</td>
<td>100</td>
<td>15.82</td>
<td>0.643</td>
<td>61.2</td>
</tr>
<tr>
<td>KP/SMS4</td>
<td>100</td>
<td>15.10</td>
<td>0.721</td>
<td>33.5</td>
</tr>
<tr>
<td>KP/SMS6</td>
<td>75</td>
<td>15.68</td>
<td>0.658</td>
<td>55.9</td>
</tr>
<tr>
<td>KP/SMS7</td>
<td>100</td>
<td>15.78</td>
<td>0.647</td>
<td>59.8</td>
</tr>
<tr>
<td>KP/SMS8</td>
<td>100</td>
<td>15.32</td>
<td>0.698</td>
<td>41.2</td>
</tr>
<tr>
<td>KP/SMS9</td>
<td>80</td>
<td>15.50</td>
<td>0.677</td>
<td>48.9</td>
</tr>
</tbody>
</table>

Tests KP/SMS5 and KP/SMS10 were all–clay models.

TABLE: 3.4

Moisture content and saturated unit weight of clay samples

<table>
<thead>
<tr>
<th>Test</th>
<th>Depth of clay, mm</th>
<th>Moisture content % Pre</th>
<th>Moisture content % Post</th>
<th>Saturated unit weight kN/m³ Pre (i)</th>
<th>Saturated unit weight kN/m³ Post (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KP/SMS3</td>
<td>60</td>
<td>118.3</td>
<td>58.1</td>
<td>13.9</td>
<td>16.4</td>
</tr>
<tr>
<td>KP/SMS4</td>
<td>60</td>
<td>121.9</td>
<td>58.7</td>
<td>13.8</td>
<td>16.4</td>
</tr>
<tr>
<td>KP/SMS5</td>
<td>60*</td>
<td>120.0</td>
<td>59.9</td>
<td>13.9</td>
<td>16.3</td>
</tr>
<tr>
<td></td>
<td>100#</td>
<td>119.4</td>
<td>49.0</td>
<td>13.9</td>
<td>17.1</td>
</tr>
<tr>
<td>KP/SMS6</td>
<td>85</td>
<td>118.5</td>
<td>58.8</td>
<td>13.9</td>
<td>16.4</td>
</tr>
<tr>
<td>KP/SMS7</td>
<td>60</td>
<td>117.6</td>
<td>58.4</td>
<td>14.0</td>
<td>16.4</td>
</tr>
<tr>
<td>KP/SMS8</td>
<td>60</td>
<td>118.1</td>
<td>57.9</td>
<td>13.9</td>
<td>16.4</td>
</tr>
<tr>
<td>KP/SMS9</td>
<td>80</td>
<td>119.3</td>
<td>59.5</td>
<td>13.9</td>
<td>16.4</td>
</tr>
<tr>
<td>KP/SMS10</td>
<td>60*</td>
<td>120.8</td>
<td>58.6</td>
<td>13.9</td>
<td>16.4</td>
</tr>
<tr>
<td></td>
<td>100#</td>
<td>121.8</td>
<td>48.6</td>
<td>13.9</td>
<td>17.1</td>
</tr>
</tbody>
</table>

(i) and (ii) refer to the method of saturated unit weight determination described in section 3.5.1.2. Pre and post refer to pre–consolidation and post–consolidation. * = soft, # = stiff clay layer.

For a vertical stress increase from 43kPa up to 86kPa in the soft layer of clay:

vertical permeability was \( \approx 3 \times 10^{-8} \) m/second,

coefficient of consolidation was \( \approx 16 \text{ mm}^2/\text{minute} \).

TABLE: 3.5

Interfacial friction between kaolin and perspex

<table>
<thead>
<tr>
<th>Interface</th>
<th>Lubricant</th>
<th>OCR</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>kaolin/kaolin</td>
<td>Nil</td>
<td>1</td>
<td>0.34</td>
</tr>
<tr>
<td>kaolin/perspex</td>
<td>Nil</td>
<td>8</td>
<td>0.54</td>
</tr>
<tr>
<td>kaolin/perspex</td>
<td>Adsil</td>
<td>1</td>
<td>0.21</td>
</tr>
<tr>
<td>kaolin/perspex</td>
<td>Silicone grease</td>
<td>8</td>
<td>0.34</td>
</tr>
<tr>
<td>kaolin/perspex</td>
<td></td>
<td>1</td>
<td>0.11</td>
</tr>
<tr>
<td>kaolin/perspex</td>
<td></td>
<td>8</td>
<td>0.25</td>
</tr>
<tr>
<td>kaolin/perspex</td>
<td></td>
<td>1</td>
<td>0.04</td>
</tr>
<tr>
<td>kaolin/perspex</td>
<td></td>
<td>8</td>
<td>0.09</td>
</tr>
</tbody>
</table>
**TABLE: 4.1**

Properties of vane shear test apparatus

<table>
<thead>
<tr>
<th>Property</th>
<th>Standard field</th>
<th>model scale</th>
<th>Centrifuge prototype scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>height/diameter</td>
<td>2</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>diameter</td>
<td>65 mm</td>
<td>18 mm</td>
<td>1.8 m</td>
</tr>
<tr>
<td>time post insertion</td>
<td>5'</td>
<td>&lt; 1'</td>
<td>&lt; 7 days</td>
</tr>
<tr>
<td>calculation of $c_u$</td>
<td>$0.86T/\pi \ d^3$</td>
<td>$0.55T/\pi \ d^3$</td>
<td>$0.55T/\pi \ d^3$</td>
</tr>
</tbody>
</table>

where $T$ is the torque measured by the vane torsion cell

**TABLE: 4.2**

Determination of shear modulus in the sand substratum

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>G (MPa)</th>
<th>Assumed</th>
<th>$N = 7$</th>
<th>$D_r = 50%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>52</td>
<td>82</td>
<td>102</td>
</tr>
<tr>
<td>5</td>
<td>54</td>
<td></td>
<td></td>
<td>126</td>
</tr>
</tbody>
</table>

(reduce to nearly 0)

**TABLE: 5.1**

Pile properties for finite element analyses

All properties are quoted for prototype scale.

for $15 \ m > y > 0 \ m$

$r = 1.27 \ m$ \hspace{1cm} $E_p = 40.0 \ 10^3 \ MPa$ \hspace{1cm} $\nu = 0.33$ \hspace{1cm} $\gamma = 16.1 \ kN/m^3$

for $16 \ m > y > 15 \ m$

at the tip, the pile dimensions are a square of side 0.2 m, with the same $E_p$ & $\nu$
### TABLE: 5.2

Clay properties for finite element analyses

All properties are quoted for prototype scale

\[
E_o = 2 \text{ MPa} \quad dE/dy = 0.533 \text{ MPa/m} \quad y_o = 0 \text{ m} \quad v = 0.33 \quad \gamma = 16.1 \text{ kN/m}^3
\]

\[k_x = k_y = k_z = 10^{-8} \text{ m/s}\]

### TABLE: 5.3

Sand properties for finite element analyses

All properties are quoted for prototype scale.

\[
E_o = 5.2 \text{ MPa} \quad dE/dy = 26 \text{ MPa/m} \quad y_o = 6 \text{ m} \quad v = 0.3 \quad \gamma = 16.1 \text{ kN/m}^3
\]

### TABLE: 6.1

Centrifuge model test results: embankment load v maximum lateral pile pressure

<table>
<thead>
<tr>
<th>q (kPa)</th>
<th>(q/c_u)</th>
<th>(p_m) (kPa)</th>
<th>(p_m/c_u)</th>
<th>(p_m) (kPa)</th>
<th>(p_m/c_u)</th>
<th>depth to (p_m) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 3</td>
<td>s/d = 5.25, (h_s = 6) m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1.67</td>
<td>25</td>
<td>1.67</td>
<td>–</td>
<td>–</td>
<td>4.2</td>
</tr>
<tr>
<td>52</td>
<td>3.47</td>
<td>45</td>
<td>3.0</td>
<td>–</td>
<td>–</td>
<td>2.9</td>
</tr>
<tr>
<td>75</td>
<td>5.0</td>
<td>87</td>
<td>5.8</td>
<td>–</td>
<td>–</td>
<td>2.2</td>
</tr>
<tr>
<td>Test 4</td>
<td>s/d = 5.25, (h_s = 6) m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>3.87</td>
<td>80</td>
<td>5.33</td>
<td>–</td>
<td>–</td>
<td>0.5</td>
</tr>
<tr>
<td>86</td>
<td>5.73</td>
<td>100</td>
<td>6.67</td>
<td>–</td>
<td>–</td>
<td>0.75</td>
</tr>
<tr>
<td>113</td>
<td>7.53</td>
<td>140</td>
<td>9.33</td>
<td>–</td>
<td>–</td>
<td>1.0</td>
</tr>
<tr>
<td>146</td>
<td>9.73</td>
<td>158</td>
<td>10.53</td>
<td>–</td>
<td>–</td>
<td>1.25</td>
</tr>
<tr>
<td>180</td>
<td>12.0</td>
<td>163</td>
<td>10.87</td>
<td>–</td>
<td>–</td>
<td>1.5</td>
</tr>
<tr>
<td>203</td>
<td>13.5</td>
<td>181</td>
<td>12.07</td>
<td>–</td>
<td>–</td>
<td>0.5</td>
</tr>
<tr>
<td>228</td>
<td>15.2</td>
<td>160</td>
<td>10.67</td>
<td>–</td>
<td>–</td>
<td>2.0</td>
</tr>
<tr>
<td>Test 7</td>
<td>s/d = 3.15, (h_s = 6) m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>3.53</td>
<td>42</td>
<td>2.8</td>
<td>47</td>
<td>3.13</td>
<td>3.2</td>
</tr>
<tr>
<td>72</td>
<td>4.8</td>
<td>56</td>
<td>3.73</td>
<td>59</td>
<td>3.93</td>
<td>2.9</td>
</tr>
<tr>
<td>93</td>
<td>6.2</td>
<td>80</td>
<td>5.33</td>
<td>115</td>
<td>7.67</td>
<td>2.6</td>
</tr>
<tr>
<td>113</td>
<td>7.53</td>
<td>120</td>
<td>8.0</td>
<td>115</td>
<td>7.67</td>
<td>2.85</td>
</tr>
<tr>
<td>133</td>
<td>8.87</td>
<td>125</td>
<td>8.33</td>
<td>150</td>
<td>10.0</td>
<td>2.55</td>
</tr>
<tr>
<td>152</td>
<td>10.13</td>
<td>127</td>
<td>8.47</td>
<td>142</td>
<td>9.47</td>
<td>2.85</td>
</tr>
<tr>
<td>172</td>
<td>11.47</td>
<td>177</td>
<td>11.8</td>
<td>171</td>
<td>11.4</td>
<td>2.4</td>
</tr>
<tr>
<td>189</td>
<td>12.6</td>
<td>155</td>
<td>10.33</td>
<td>158</td>
<td>10.53</td>
<td>2.5</td>
</tr>
</tbody>
</table>

\(c_u\) was assumed to be 15 kPa at the mid-depth of the clay layer (Chapter 4)
Front Pile

Pile Properties

Total effective length of pile for lateral loading (m) = 17.000
Total length of pile (m) = 17.000
Includes...
Freestanding: length of pile (m) = 1.000
Radius of pile (m) = 0.635
Young's Modulus of pile (*10^6 kPa) = 79.000
Moment of Inertia (m^4) = 3.073250

Soil Properties

Poisson's ratio = 0.302
Shear modulus at top of stiff layer (kPa) = 2500.000
Gradient of shear modulus with depth, 15/1Z (kPa/m) = 10000.000
Characteristic modulus (kPa) = 53911.338
Shoc = 0.523

Loading Details

Loading is applied with a parabolic distribution
Lateral stress at top of soft layer (kPa) = 0.100
Lateral stress at bottom of soft layer (kPa) = 0.100
Maximum value of lateral stress (kPa) = 41.700
At a depth of (m) = 1.300

Results

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Bending Moment (kNm)</th>
<th>Deflection (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.000</td>
<td>0.000</td>
<td>0.02089</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.01113</td>
</tr>
<tr>
<td>0.400</td>
<td>0.285</td>
<td>0.01735</td>
</tr>
<tr>
<td>0.800</td>
<td>2.182</td>
<td>0.01657</td>
</tr>
<tr>
<td>1.200</td>
<td>7.120</td>
<td>0.01580</td>
</tr>
<tr>
<td>1.600</td>
<td>16.392</td>
<td>0.01502</td>
</tr>
<tr>
<td>2.000</td>
<td>31.073</td>
<td>0.01424</td>
</tr>
<tr>
<td>2.400</td>
<td>52.099</td>
<td>0.01346</td>
</tr>
<tr>
<td>2.800</td>
<td>80.232</td>
<td>0.01269</td>
</tr>
<tr>
<td>3.200</td>
<td>116.063</td>
<td>0.01193</td>
</tr>
<tr>
<td>3.600</td>
<td>160.016</td>
<td>0.01115</td>
</tr>
<tr>
<td>4.000</td>
<td>212.324</td>
<td>0.01038</td>
</tr>
<tr>
<td>4.400</td>
<td>273.130</td>
<td>0.00962</td>
</tr>
<tr>
<td>4.800</td>
<td>342.292</td>
<td>0.00895</td>
</tr>
<tr>
<td>5.200</td>
<td>419.375</td>
<td>0.00814</td>
</tr>
<tr>
<td>5.600</td>
<td>504.555</td>
<td>0.00741</td>
</tr>
<tr>
<td>6.000</td>
<td>596.685</td>
<td>0.00670</td>
</tr>
<tr>
<td>6.400</td>
<td>695.078</td>
<td>0.00601</td>
</tr>
<tr>
<td>6.800</td>
<td>799.929</td>
<td>0.00534</td>
</tr>
<tr>
<td>7.200</td>
<td>907.096</td>
<td>0.00470</td>
</tr>
<tr>
<td>7.600</td>
<td>1018.314</td>
<td>0.00409</td>
</tr>
<tr>
<td>8.000</td>
<td>1131.145</td>
<td>0.00350</td>
</tr>
<tr>
<td>9.000</td>
<td>1295.111</td>
<td>0.00242</td>
</tr>
<tr>
<td>10.000</td>
<td>1421.335</td>
<td>0.00148</td>
</tr>
<tr>
<td>11.000</td>
<td>1511.261</td>
<td>0.00075</td>
</tr>
<tr>
<td>12.000</td>
<td>1607.344</td>
<td>0.00027</td>
</tr>
<tr>
<td>13.000</td>
<td>1704.317</td>
<td>0.00027</td>
</tr>
<tr>
<td>14.000</td>
<td>1804.367</td>
<td>-0.00006</td>
</tr>
<tr>
<td>15.000</td>
<td>1904.348</td>
<td>-0.00083</td>
</tr>
<tr>
<td>16.000</td>
<td>0.000</td>
<td>0.00000</td>
</tr>
<tr>
<td>16.000</td>
<td>0.000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Rotation at top of pile = 0.001947 radians
### TABLE: 6.3

Interaction factors for a single row of piles

Centrifuge model tests 4 – 7

<table>
<thead>
<tr>
<th>No. of piles in row:</th>
<th>3 (s/d = 5.25)</th>
<th>5 (s/d = 3.15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designation:</td>
<td>middle</td>
<td>outer</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>A/C</td>
</tr>
<tr>
<td>(\alpha_{\text{uH}})</td>
<td>0.128</td>
<td>0.096</td>
</tr>
<tr>
<td>(\alpha_{\text{uM}} = \alpha_{\theta H})</td>
<td>0.016</td>
<td>0.009</td>
</tr>
<tr>
<td>(\alpha_{\theta M})</td>
<td>0.002</td>
<td>0.001</td>
</tr>
</tbody>
</table>

### TABLE: 6.4

Interaction factors for a pile group

Centrifuge model tests 8 – 10

Pile group containing 2 rows of 3 piles:

<table>
<thead>
<tr>
<th>Designation:</th>
<th>middle</th>
<th>outer</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF/BR</td>
<td>AF/AR/CF/CR</td>
<td></td>
</tr>
</tbody>
</table>

| \(\alpha_{\text{uf}}\) | 0.401 | 0.354 |

Loading is in the same direction as that in the centrifuge model tests
(normal to the front and rear row of piles)
### TABLE: 7.1

**Determination of pile maximum bending moment, rotation & deflection**

**Case:** Test example: Working load  
**Ref No:** 21/1  
**Date:** 1/2/88

<table>
<thead>
<tr>
<th>Load No &amp; Shape</th>
<th>$p_c$ kPa</th>
<th>$h_u$ m</th>
<th>$h$ m</th>
<th>$\beta_c$ or Parab</th>
<th>$y/h = 1$</th>
<th>$y/h = 0$</th>
<th>$uEI$ $p_c$ rh</th>
<th>$\theta$ radians</th>
<th>$u$ mm</th>
<th>$uEI$ $p_c$ rh$^4$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>139.5</td>
<td>0</td>
<td>6</td>
<td>Para</td>
<td>1.333</td>
<td>0.71</td>
<td>0.667</td>
<td>2.13</td>
<td>0.2</td>
<td>0.000747</td>
</tr>
</tbody>
</table>

Additional $H$ at pile cap level:  

If unloaded soft layer ($h_u > 0$): $H_s = H_y=h_s = H_y=h = \frac{1}{\rho_c} M_s = M_y=h + h_u (H_y=h_u) = 0$

Establish $u_s$ & $\theta_s$ by taking $H_s$ & $M_s$ & using charts (Figs: 6.18 & 6.19) or equations:

$$u_s = \frac{(E_p/G_c)^{1/7}}{\rho_c G_c} \left[ 0.27 H_s (\ell_c/2)^{-1} + 0.3 M_s (\ell_c/2)^{-2} \right] \quad \theta_s = \frac{(E_p/G_c)^{1/7}}{\rho_c G_c} \left[ 0.3 H_s (\ell_c/2)^{-2} + 0.8 \sqrt{\rho_c} M_s (\ell_c/2)^{-3} \right]$$

For lateral force, $H_s$, $u_s = \frac{r G_c (E_p/G_c)^{1/7}}{M_s}$, moment, $M_s$, $\frac{u_m r^2 G_c (E_p/G_c)^{3/7}}{M_s} \approx 0.575$, $u_s = u_h + u_m$

Then, using charts (Figs: 7.2a & b), calculate rotation & deflection:

<table>
<thead>
<tr>
<th>y (m)</th>
<th>$u$ (mm)</th>
<th>$\theta$ (rads)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_s$</td>
<td>$u_h = 4.11 u_m = 3.32$; $u_s = 7.43$</td>
<td>0.00262</td>
</tr>
<tr>
<td>h</td>
<td>$\Delta u = h \tan \theta_s = 0$</td>
<td>0; $\Delta \theta = 0$</td>
</tr>
<tr>
<td>0</td>
<td>$\Delta u = h \tan \theta_s + \Sigma u = 19.21$; $\Delta \theta = \Sigma \theta = 0$</td>
<td>0.00075</td>
</tr>
<tr>
<td>$-\epsilon$</td>
<td>$\Delta u = \epsilon \tan \theta_s$</td>
<td></td>
</tr>
<tr>
<td>$-\epsilon$</td>
<td>$\Sigma u = 26.64$; $\Sigma \theta = 0.00337$</td>
<td></td>
</tr>
</tbody>
</table>

Determination of maximum moment by inspection, $M_{max}/H_s \ell_c \approx 0.84$, $M_{max}/H_s \ell_c \approx 0.17$ so

$$M_{max} \approx 2.8 \text{ MNm}$$

at depth, $y = \ell_c \left( y/\ell_c \right) + h_s = 8.4 (0.25) + 6 = 8.1 \text{ m}$
SECTION
DIFFERENT TYPES OF BRIDGE ABUTMENT AND ASSOCIATED EMBANKMENT LOADING

Fig: 1.1 BRIDGE ABUTMENT & APPROACH EMBANKMENT

Fig: 1.2 Weight: gravity effects in a prototype are identical to inertial effects in a centrifugal model (after Schofield, 1980)
Fig: 2.1 Failure modes for laterally loaded piles (after Fleming et al, 1985)

(a) Short pile    (b) Long pile    (c) No hinge    (d) 1 hinge    (e) 2 hinges

Fig: 2.2a Measured and calculated pile bending moment

Fig: 2.2b Measured and calculated pile deflection

Experimental pile at Provins: Head load test

(after Bigot, Bourges & Frank, 1982)
Fig: 2.3  Pile and embankment at Provins  
(after Bigot, Bourges & Frank, 1982)

Fig: 2.4a  Measured and calculated pile bending moment

Fig: 2.4b  Measured and calculated pile deflection

Experimental pile at Provins: Completion of embankment construction

(after Bigot, Bourges & Frank, 1982)
Fig: 2.5

Fig: 2.6

Fig: 2.7

(after Briaud, Smith, Tucker, 1985)
1. Tunnel
2. Air duct
3. Air vent
4. Louvre
5. Air vent
6. Shaft
7. Corridor
8. Observation room
9. Crane
10. Specimen box
11. Spoke
12. Flange plates
13. Bridging structures
14. Mounting plate
15. Bearing shaft
16. Horizontal plate
17. Bearing plate
18. Sliding stack
19. Drive motor
20. Electromagnetic brake
21. Eddy current coupling
22. Electromagnetic brake
23. Gearbox
24. Water tower (position of)
25. Swinging carriage
26. Platform
27. Swing arm pivot
28. Torsion bar

C.U.E.D. 10m. Centrifuge

Fig. 3.1: Cambridge Geotechnical Centrifuge
Fig: 3.3 Test 2: Sand embankment spilling through between piles

Fig: 3.4 General arrangement of centrifuge arm during testing (after Almeida, 1984)
Fig: 3.5 Consolidometer apparatus (after Almeida, 1984)

Fig: 3.6 Strong box & liner
Fig: 3.7 In-flight photograph showing centrifuge model, curvature of water table, manometer and clock

Fig: 3.8 Airtight rubber loading bag and restraining box
Fig: 3.9a  Vane apparatus Mark II (after Almeida, 1984)

Fig: 3.9b  Piezocone penetrometer (after Almeida, 1984)
INSTRUMENTED PILE

THE PILES ARE 12.2m DIAMETER DUAL TUBING, 1/63G, WHICH ARE INSTRUMENTED EXTERNALLY BY PAIRS OF TINLEY STRAIN GAuges 1132 OHM 31/120/4/02B. 2 LAYERS OF SHAMFIT TUBING WERE APPLIED TO THE PILE TO WATERPROOF THE GAuges. 10 = 70 KVAM, 1 = 733 MN, 1 = 1.219 MN

INSTRUMENTED PILE USED FOR SINGLE PILE TESTS

FIG NO. 3.10

INSTRUMENTED PILE GROUP WITH 2 ROWS OF 3 PILES

FIG NO. 3.11

STRAIN GAUGE CIRCUIT

A ACTIVE GAuges
B COMON DUMP RESISTORS IN JUNCTION BOX

STRAIN GAUGE POSITIONS
FULL BRIDGE CIRCUIT
(Scale = 2)

BENDING MOMENT TRANSUCERS

PILE BF PILE AF PILE BR PILE AR
1 9 17 25
2 10 18 26
3 11 19 27
4 12 20 28
5 13 21 29
6 14 22 30
7 15 23 31
8 16 24 32

THE PILES ARE 12.2m DIAMETER DUAL TUBING, 1/63G, WHICH ARE INSTRUMENTED EXTERNALLY BY 2 PAIRS OF 12OHM TINLEY STRAIN GAuges 31/120/4/02B. THESE WERE CONNECTED IN A FULL BRIDGE CIRCUIT.

2 LAYERS OF SHAMFIT TUBING WERE APPLIED TO EACH PILE TO WATERPROOF THE GAuges.

E = 70 KVAM, 1 = 733 MN, 1 = 1.219 MN
Fig: 3.12a Stress history for tests 3, 4, 6-9

Fig: 3.12b Stress history for tests 5, 10
PLAN ON TYPICAL LAYOUT OF INSTRUMENTATION

CROSS SECTION THROUGH INSTRUMENTATION
TEST KP/SMS3&4

LEGEND

○ — — LEAD THREADS  ○ PPT  PORE PRESSURE TRANSDUCER  ○ VANE TEST

-—— LVOT  × STRAIN GAUGE

GENERAL ARRANGEMENT OF MODEL INSTRUMENTATION
Fig. 4.2a Definition and sign conventions for calculation of forces on piles

Fig. 4.2b Error in polynomial fit to datapoints

Legend:
- Data points (symbol) are fitted by polynomials (line), of order N_poly & root mean square error, RMS, as shown in the legend.

CENTRIFUGE TEST KP/SMS 3
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

FILE NO. 4.2c
CENTRIFUGE TEST KP/SMS 4  PILE A
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT LOAD 1 2 3 4 5 6 7 8
10.0 58.0 86.0 113.0 146.0 180.0 203.0 226.0
LINE SYMBOL O A X O X Z
RMS 0.095 0.165 0.229 0.289 0.337 0.362 0.342 0.330
NPOLY 6 6 6 6 6 6 6 6
DATA POINTS (SYMBOL) ARE FITTED BY POLYNOMIALS (LINE) OF ORDER, NPOLY
& ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND

CENTRIFUGE TEST KP/SMS 4  PILE B
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT LOAD 1 2 3 4 5 6 7 8
10.0 58.0 86.0 113.0 146.0
LINE SYMBOL O A X O X Z
RMS 0.095 0.143 0.209 0.202 0.223
NPOLY 6 6 6 6 6
DATA POINTS (SYMBOL) ARE FITTED BY POLYNOMIALS (LINE) OF ORDER, NPOLY
& ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND
CENTRIFUGE TEST KP/SMS 4  PILE B
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT  6  7  8
LOAD  180.0  203.0  226.0
LINE  ——— 
SYMBOL  ▲  X  Z
RMS  0.027  0.038  0.124
NPOLY  5  8  5

DATA POINTS (SYMBOL), ARE FITTED BY POLYNOMIALS (LINE), OF ORDER, NPOLY & ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND.
TEST KP/SMS5

TEST KP/SMS6

LEGEND

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>o</td>
<td>Lead Threads</td>
</tr>
<tr>
<td>@</td>
<td>PPT</td>
</tr>
<tr>
<td>⊕</td>
<td>Pore Pressure Transducer</td>
</tr>
<tr>
<td>✩</td>
<td>Vane Test (Inflight)</td>
</tr>
<tr>
<td>ℒ</td>
<td>Penetrometer Test</td>
</tr>
<tr>
<td>x</td>
<td>Strain Gauge</td>
</tr>
<tr>
<td>v</td>
<td>LVDT</td>
</tr>
</tbody>
</table>

CROSS SECTION THROUGH INSTRUMENTATION (KP/SMS5&6) FIG NO: 4.4
CENTRIFUGE TEST KP/SMS 5  PILE A
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND

CENTRIFUGE TEST KP/SMS 5  PILE B
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
CENTRIFUGE TEST KP/SMS 6  PILE A
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS 6  PILE B
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
CENTRIFUGE TEST KP/SMS 6  PILE B
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
PLAN ON TYPICAL LAYOUT OF INSTRUMENTATION

CROSS SECTION THROUGH INSTRUMENTATION

LEGEND

○ LEAD THREADS
○ PPT
○ PORE PRESSURE TRANSDUCER
+ VANE TEST
▼ PENETROMETER TEST

INSTRUMENTATION FOR 5 PILE MODEL (KP/SMS7)
CENTRIFUGE TEST KP/SMS 7
PILE A
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS 7
PILE B
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
PLAN ON TYPICAL LAYOUT OF INSTRUMENTATION

CROSS SECTION THROUGH INSTRUMENTATION
TEST KP/SMS 8

LEGEND

- - - - - LEAD THREADS  O PPT  PORE PRESSURE TRANSDUCER  O VANE TEST

\[ \text{LVOT} \]  \[ \times \]  \text{STRAIN GAUGE}  \[ \$ \]  PENTAMETER TEST

GENERAL ARRANGEMENT OF MODEL INSTRUMENTATION
CENTRIFUGE TEST KP/SMS 8  PILE AF
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND

CENTRIFUGE TEST KP/SMS 8  PILE AR
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
CENTRIFUGE TEST KP/SMS 8 PILE BF
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT  LOAD  SYMBOL  RMS  NPOLY
1    25.0  0.026  7
2    50.0  0.081  5
3    75.0  0.016  6
4    100.0  0.037  6
5    125.0  0.047  6
6    150.0  0.062  6
7    175.0  0.077  6
8    200.0  0.081  6
9    225.0  0.097  6
10   250.0  0.056  6

DATA POINTS (SYMBOL), ARE FITTED BY POLYNOMIALS (LINE) OF ORDER, NPOLY & ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND.

CENTRIFUGE TEST KP/SMS 8 PILE BR
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT  LOAD  SYMBOL  RMS  NPOLY
1    25.0  0.035  6
2    50.0  0.072  6
3    75.0  0.140  6
4    100.0  0.301  6
5    125.0  0.283  6
6    150.0  0.146  6
7    175.0  0.133  6
8    200.0  0.173  6
9    225.0  0.187  6
10   250.0  0.188  6

BENDING MOMENT TRANSDUCER AT DEPTH OF 0.11m FAILED TO RECORD FOR ALL OF THE INCREMENTS.
DATA POINTS (SYMBOL), ARE FITTED BY POLYNOMIALS (LINE) OF ORDER, NPOLY & ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND.
TEST KP/SMS10

TEST KP/SMS9

LEGEND

- LEAD THREADS
- PPT
- PORE PRESSURE TRANSDUCER
- VANE TEST (INFLATION)
- LVOT
- STRAIN GAUGE
- PENETROMETER TEST

CROSS SECTION THROUGH INSTRUMENTATION (KP/SMS9 & 10) FIG NO: 4.12
CENTRIFUGE TEST KP/SMS 9  PILE AF
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS 9  PILE AR
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
CENTRIFUGE TEST KP/SM 9 PILE BF
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT LOAD LINE SYMBOL RMS H.POLY
25.0 49.0 71.0 98.0 124.0 146.0 169.0 194.0 217.0 242.0
0.015 0.051 0.063 0.058 0.087 0.087 0.108 0.128 0.087 0.123
7 6 6 6 7 7 7 7 7 6
DATA POINTS (SYMBOL) ARE FITTED BY POLYNOMIALS (LINE) OF ORDER HPOLY & ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND

CENTRIFUGE TEST KP/SM 9 PILE BR
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT LOAD LINE SYMBOL RMS H.POLY
25.0 49.0 71.0 98.0 124.0 146.0 169.0 194.0 217.0 242.0
0.033 0.053 0.065 0.054 0.028 0.063 0.086 0.121 0.106 0.114
6 6 6 6 6 6 6 6 6 6
BENDING MOMENT TRANSDUCER AT DEPTH OF 0.128 FAILED TO RECORD FOR ALL OF THE INCREMENTS.
DATA POINTS (SYMBOL) ARE FITTED BY POLYNOMIALS (LINE) OF ORDER HPOLY & ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND.
CENTRIFUGE TEST KP/SMS10  PILE AF
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS10  PILE AR
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
CENTRIFUGE TEST KP/SMS10  PILE BF
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS10  PILE BR
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
VANE SHEAR STRENGTH TESTS

TEST KP/SMS4  6.0 M CLAY ON 10.0 M SAND

TEST KP/SMS5  16.0 M CLAY ON 0.0 M SAND

TEST KP/SMS6  8.5 M CLAY ON 7.5 M SAND

TEST KP/SMS7  6.0 M CLAY ON 10.0 M SAND

VANE SHEAR STRENGTHS HAVE BEEN MEASURED INFLIGHT (1.5MM BY 1MMDIA) AND IMMEDIATELY AFTER THE RUN WAS FINISHED (2.5MM BY 1MMDIA & 5MM BY 3MMDIA). THE SHEAR STRENGTH VALUES ARE PLOTTED AT MID DEPTH OF THE VANE.
VANE SHEAR STRENGTH TESTS

TEST KP/SM8 6.0M CLAY ON 10.0M SAND

TEST KP/SM9 8.0M CLAY ON 8.0M SAND

14MM (INFLIGHT) 29MM (POSTFLIGHT) 50MM (POSTFLIGHT)

PEAK
RESIDUAL

VANE SHEAR STRENGTHS HAVE BEEN MEASURED INFLIGHT (14MM BY IMPROA) AND IMMEDIATELY AFTER THE RUN WAS FINISHED (29MM BY IMPROA & 50MM BY IMPROA). THE SHEAR STRENGTH VALUES ARE PLOTTED AT MID DEPTH OF THE VANE.

FIG NO: 4.18a

TEST KP/SM10 16.0M CLAY ON 0.0M SAND

14MM (INFLIGHT) 29MM (POSTFLIGHT) 50MM (POSTFLIGHT)

PEAK
RESIDUAL

VANE SHEAR STRENGTHS HAVE BEEN MEASURED INFLIGHT (14MM BY IMPROA) AND IMMEDIATELY AFTER THE RUN WAS FINISHED (29MM BY IMPROA & 50MM BY IMPROA). THE SHEAR STRENGTH VALUES ARE PLOTTED AT MID DEPTH OF THE VANE.

FIG NO: 4.18b
PEAK VANE SHEAR STRENGTH & PAST STRESS HISTORY

\[
\log\left(\frac{c_u}{\sigma_{v,max}}\right) = a(1/OCR)^{1-b}
\]

Test No. Valid Tests
4  2
6  2
7  2
8  2
9  3
10 5

Best fit boundaries to equation

\[
\begin{align*}
\text{upper} & : a \approx 0.766, b \approx 0.22 \\
\text{lower} & : a \approx 0.632, b \approx 0.22 \\
\text{mean} & : a \approx 0.706, b \approx 0.22 \\
\text{Núñez(89)} & : a \approx 0.62, b \approx 0.22 \\
\text{Phillips(80)} & : a \approx 0.67, b \approx 0.22
\end{align*}
\]

Fig: 4.19

PEAK VANE SHEAR STRENGTHS CENTRIFUGE MODEL TESTS

Undrained shear strength, Cu, kPa

LEGEND

- Test 4
+ Test 6
\(\Diamond\) Test 7
\(\Box\) Test 8
\(\times\) Test 9
\(\square\) Test 10

- Design line
- Upper limit
- Lower limit

PEAK values
Vane 18mm diameter
14mm long

Results calculated from in-flight vane shear tests @ 100g

? - test too close to top of sand layer

Fig: 4.20a
Fig: 4.20b Theoretical factor of safety at failure of embankments on soft clay plotted against the plasticity index of the clay (after Bjerrum, 1973)

Fig: 4.21a Cone penetrometer results
Centrifuge model tests
Cone resistance MPa

Fig: 4.21b Shear modulus as a function of shear strain
Fig: 4.22a  Test 6: Pre-test

Fig: 4.22b  Test 6: Post-test
Fig: 4.23a  Test 10 in the centrifuge, just after reaching 100g

Fig: 4.23b  Test 10 in the centrifuge, at the end of the test
Photographs from X-rays (taken vertically downwards through sample), showing post-flight deformation of lead threads around the piles (The sample was loaded on the right hand side of the piles)
Photographs from X-rays (taken vertically downwards through sample), showing post-flight deformation of lead threads around the piles (The sample was loaded on the right hand side of the piles)

Fig: 4.27b  Test 8: Excavation of clay sample to expose lead threads
Fig: 4.28  Test 9

Fig: 4.29  Test 10

Photographs from X-rays (taken vertically downwards through sample), showing post-flight deformation of lead threads around the piles (The sample was loaded on the right hand side of the piles)
Fig: 5.1 Finite element analysis of equivalent sheet pile wall (model scale)

Fig: 5.2a General arrangement of the model for 2-dimensional finite element analysis

Fig: 5.2b 2-dimensional finite element mesh geometry, delineating different material zones and boundary conditions
Fig: 5.3 3-dimensional finite element mesh geometry, delineating different material zones and boundary conditions
Fig: 5.4 Schematic representation of longitudinal arching in an embankment subjected to foundation settlement

Fig: 5.5 Inclined loading on foundation due to embankment
Fig: 5.6 Distribution of horizontal components of reaction (after Baguelin, Frank & Said, 1977)

Fig: 5.7 Model of pile, soil and boundary conditions (after Baguelin, Frank & Said, 1977)
Fig: 5.8a  Distribution of horizontal components of reaction due to surcharge loading (s/d = 5.25)
Fig: 5.8b  Distribution of horizontal components of reaction due to surcharge loading (s/d = 3.15)
COMPARE PILE BEHAVIOUR, S/D = 5.25, 3 PILES
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
TEST/PILE LOAD 100.0 CENTRIFUGE DATA WAS FACTORED UP FROM RESULTS FOR Q = 50 KPA TO 100 KPA, PILE A
SYMBOL 0 X FINITE ELEMENT + WITH PILE  
+ FINITE ELEMENT + NO PILE SIMPLE FOR Q = 100 KPA

COMPARE PILE BEHAVIOUR, S/D = 3.15, 5 PILES
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
TEST/PILE LOAD 100.0 CENTRIFUGE DATA WAS FACTORED UP FROM RESULTS FOR Q = 93 KPA TO 100 KPA, PILE A
SYMBOL 0 0.038 0.028 0.018 0.008 0.000 SIMPLE
0.38 0.28 0.18 0.08 0.0 prototypes SIMPLE
0.8 SIMPLE

FIG NO: 5.9
FIG NO: 5.10
Fig: 5.11  Normal stress change in the soil along the pile centreline in the direction of the lateral line load

Net Total Horizontal Stress Acting on Pile

\[ \sigma_{xx} = \sigma_{x,xn} \, (\text{kPa}) \]

Fig: 5.12a

Net Total Horizontal Stress Acting on Pile

\[ \sigma_{xx} - \sigma_{x,xn} \, (\text{kPa}) \]

Fig: 5.12b
Fig: 5.13 Change in total stresses at integration points = \( (\sigma_{xx}\text{ with pile}) - (\sigma_{xx}\text{ no pile}) \)

- **s/d = 5.25 (3 pile case)**
- **s/d = 3.15 (5 pile case)**

Pile

- \( y=0.23\text{m} \)
- \( y=1.0\text{m} \)
- \( y=1.78\text{m} \)
- \( y=2.46\text{m} \)
- \( y=4.0\text{m} \)
- \( y=5.55\text{m} \)

Numbers represent total stresses at the adjacent locations on the dimension curve.
Fig: 5.14  Soil-pile interaction, showing clay squeezing past the pile at y=0 and y=2m (s/d = 5.25)

Fig: 5.15  Deformed shape of mesh due to surcharge load (s/d = 3.15)
Fig: 5.16a  Deformed mesh showing z component of displacement around pile (s/d = 5.25)

Fig: 5.16b  Contours of z component displacement (s/d = 5.25)
Fig: 5.17a  Net displacements in clay layer due to 100kPa load (s/d = 5.25)

Fig: 5.17b  Net displacements in clay layer due to 100kPa load (s/d = 3.15)
Fig: 5.18a  Contours of x component of strain (s/d = 5.25)
Fig. 5.18b  Contours of x component of strain (sd = 3.15)
Fig. 5.19 Contours of shear strain in zx plane (s/d = 5.25)
Fig. 5.20 Comparing differential soil-pile deformation
Fig. 5.21 Variation of pressure function $G (\delta u_s - \delta u_p)/r$ with depth

Fig. 5.22 Graph showing variation of factor relating net pressure acting on a pile due to an embankment nearby and the pressure function, $G (\delta u_s - \delta u_p)/r$

Net pressure, $p = F \cdot \text{pressure function}$

$F$ constant with depth = 2.67 for $V = 0.5$

- $s/d = 5.25$
- $s/d = 3.15$
SCREEN 1

Fig: 6.1

Shear stress \( \tau \) vs Shear strain \( \gamma \)

Fig: 6.2 Relationship between shear stress, shear strain, and secant shear modulus
For model:
(see Fig: 5.7)
\( r = 0.01 \) m
\( E_s = 13 \) MPa
\( c_u = 30 \) kPa

Fig: 6.3 Loading curves for intact model (after Baguelin, Frank & Said, 1977)

Fig: 6.4 Section through equivalent embankment load, pile, and active and passive soil zones

Fig: 6.5 Plan on pile loaded by active and passive soil zones
**Fig: 6.6 Mohr’s circle of stress for active and passive soil zones**

**Fig: 6.7a Triangular soil displacement field**

**Fig: 6.7b Rectangular soil displacement field**

**Fig: 6.8 Mohr’s circle of strain for undrained soil**
Fig: 6.9 Calculation of maximum pressure acting on pile in soft layer due to fixity condition at ground surface.

Fig: 6.10 Flow mechanism for soil around laterally loaded pile, $f_s/c_u = 0.5$ (after Fleming et al, 1985)
Fig: 6.11a Bearing capacity calculations allowing for reinforcement by pile

Passive

$\delta u_s = \sqrt{2} \delta v_e$

Active

Fig: 6.11b Displacement diagram

$P/c_u$

Plastic zone

Ultimate plastic failure of soil mass and soil around pile

$q = (2+\pi)(1+ \frac{2d}{s})$ $c_u$

Pseudo-elastic zone

General yield of soil mass

$\frac{dp}{sc_u} = \frac{q}{c_u} - 4$

Range of $q_{crit}$

Fig: 6.12 Plasticity plot for soft layer
Fig: 6.13a The influence of outward shear stress in reducing bearing capacity for a surface footing (after Jewell, 1987)

Fig: 6.13b The influence of strength increasing with depth on bearing capacity, and the improvement resulting from restraining shear stress at the foundation surface (after Davis and Booker, 1973; Houlbsy and Wroth, 1983; Jewell, 1987)
Fig 6.15 Plasticity plots for a single row of piles in a 6m layer of soft clay
UPPER PILE LOADING DISTRIBUTION

Please enter type of loading distribution

STRAIGHT LINE = 1
PARABOLIC = 2
CURSOR INPUT = 3

Loading distribution = ?

Enter
Lateral pressure at top of layer. \( p_0 = ? \)
Lateral pressure at bottom of layer. \( p_h = ? \)

SCREEN 2

Fig: 6.16a

ALTERNATIVE LOADING DISTRIBUTION (CURSOR INPUT = 3)

Lateral pressure (x scale factor) kPa

A CUBIC SPLINE HAS BEEN FITTED TO YOUR DATA POINTS

SCREEN 2A

Fig: 6.16b

PLEASE ENTER:
PRESSURE SCALING FACTOR = ?
PRESSURE AT TOP = ?
NO OF PTS DEFINING PRESSURE (<18) = ?
PRESSURE AT BOTTOM = ?
Fig: 6.17 Definition of $\rho_c$ and $G_C$

\[
l_c = 2r \left( \frac{E_D}{G_C} \right)^{2/7}
\]

Fig: 6.18 Generalized curves of lateral deflection and bending moment profile for force loading

Fig: 6.19 Generalized curves of lateral deflection and bending moment profile for moment

(after Fleming et al, 1985)
ANALYSIS FOR A SINGLE PILE: predict L/sqy6

Bending Moment kNm

0

5,000

Pile Depth m

0

10

20

-10

-20

Soft

Stiff

Pile Deflection mm

0

20

40

60

80

100

Pile Depth m

0

10

20

-10

-20

Radius of pile (a) = 0.63
Effective length (a) = 15.63
Equivalent E (MN/m²) = 39596.99
\((4E\pi D/3.142\pi^3)\)

Poisson's ratio = 0.30
Gc (MN/m²) = 72.53
Gm (MN/m²) = 15.00
Gc (MN/m²) = 2.00

DIRECTION OF LATERAL LOADING

PILE, diameter, d

PILE GROUP EFFECTS & INFLUENCE FACTORS

FIG NO: 6.21
CENTRIFUGE TEST KP/SMS 7
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

LEGEND
LIFT/PILE 5A 5B
LOAD 93.0 93.0
LINE 0 0
RMS 0.036 0.028
NPOLY 5 6

SIMPLE ANALYSIS
WITH SOIL DEFORMATION PROFILE
--- TRIANGULAR, \( \psi = 1/2 \)
--- SQUARE, \( \psi = 1 \)
--- RECTANGULAR, \( \psi = 2 \)

BENDING MOMENT TRANSDUCER AT DEPTH OF 0.04M FAILED TO RECORD FOR ALL OF THE INCREMENTS (PILE A)
DATA POINTS (SYMBOLS) ARE FITTED BY POLYNOMIALS (LINE), OF ORDER: NPOLY & ROOT MEAN SQUARE ERROR, RMS, AS SHOWN IN THE LEGEND

CENTRIFUGE TEST KP/SMS 7
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

FIG NO: 6.24
CENTRIFUGE TEST KP/SMS 4
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS 7
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
CENTRIFUGE TEST KP/SMS 6
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

FIG NO: 6.30

LEGEND
LIFT/PILE 10A 10B
LOAD 264,0 264,0
SYMBOL O A
RMS 0.205 0.087
NPOLY 5 6

BENDING MOMENT TRANSDUCER AT
DEPTH OF 0.067 FAILED TO RECORD
FOR ALL OF THE INCREMENTS (FILE A)
DATA POINTS (SYMBOLS) ARE FITTED BY
POLYNOMIALS (LINE), ORDER NPOLY
& ROOT MEAN SQUARE ERROR, RMS,
AS SHOWN IN THE LEGEND

CENTRIFUGE TEST KP/SMS 6
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

FIG NO: 6.31

LEGEND
LIFT/PILE 10A 10B 10.5CM FOR TOP 6M
LOAD 264,0 264,0 OF SOFT LAYER
SYMBOL O A
RMS 0.205 0.087
NPOLY 5 6

BENDING MOMENT TRANSDUCER AT
DEPTH OF 0.067 FAILED TO RECORD
FOR ALL OF THE INCREMENTS (FILE A)
DATA POINTS (SYMBOLS) ARE FITTED BY
POLYNOMIALS (LINE), ORDER NPOLY
& ROOT MEAN SQUARE ERROR, RMS,
AS SHOWN IN THE LEGEND
PARAMETERS USED IN THE ANALYSIS OF PILE GROUP BEHAVIOUR

(a) Equivalent free headed piles
(b) Moments & forces applied to give equal & opposite rotations
(c) Pile group behaviour & parameters
CENTRIFUGE TEST KP/SMS 8  PILE AF
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS 8  PILE AR
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
CENTRIFUGE TEST KP/SMS 8
FRONT PILE
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION

CENTRIFUGE TEST KP/SMS 8
REAR PILE
BENDING MOMENT, LATERAL PRESSURE & DEFLECTION
LATERAL LOADING ON PILES IN A SOFT LAYER UNDER AN EMBANKMENT

LATERAL PRESSURE ACTING ON PILE WITH DEPTH

HORIZONTAL FORCE DISTRIBUTION ON PILE WITH DEPTH

BENDING MOMENT DISTRIBUTION IN PILE WITH DEPTH

LEGEND

<table>
<thead>
<tr>
<th>β_c</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>Parabolic</td>
</tr>
<tr>
<td>1.0</td>
<td>β_c = 1.25</td>
</tr>
<tr>
<td>0.5</td>
<td>β_c = 0.75</td>
</tr>
</tbody>
</table>

h = Depth of layer over which loading is applied
r = Radius of pile
E = Young's modulus of pile material
I = Second moment of area of pile
P_c = Lateral pressure on pile at y = h/2
P_c = Lateral pressure on pile at y = h/4 / P_c
ROTATION & DEFLECTION PROFILES DUE TO LOADING DEFINED BY CHARACTERISTIC LOAD, $p_c$, & LOAD DESCRIPTION FACTOR, $\beta_c$

**ROTATION PROFILE OF PILE WITH DEPTH**

\[
\frac{\theta (E I)}{(p_c r h^3)}
\]

**DEFLECTION PROFILE OF PILE WITH DEPTH**

\[
\frac{u (E I)}{(p_c r h^4)}
\]

---

**LEGEND**

- $\beta_c = 1.5$ — Parabolic
- $\beta_c = 1.0$ — $\beta_c = 1.25$
- $\beta_c = 0.5$ — $\beta_c = 0.75$

$h = \text{Depth of layer over which loading is applied}$

$r = \text{Radius of pile}$

$E = \text{Young's modulus of pile material}$

$I = \text{Second moment of area of pile}$

$p_c = \text{Lateral pressure on pile at } y = h/2$

$\beta_c = \text{Lateral pressure on pile at } y = h/4 / p_c$

---

**Fig: 7.2a**

**Fig: 7.2b**
Fig: 7.3 Lateral pressure on a pile in a deep soft layer at working load (parabolic distribution)
LIMITING VALUES OF MAXIMUM PILE BENDING MOMENTS & DESIGN ZONES

Plastic zone: 10.5 Cu condition throughout layer - UPPER BOUND

Elastoplastic zones: soil begins to yield at soil-pile interface

Load on pile reaches limit to elastic zone

Nominal Embankment Load  kPa

Fig: 7.5

FACTORS AFFECTING POSITION OF THE DESIGN ZONE FOR MAXIMUM PILE BENDING MOMENTS

Check pile geometry & material is able to carry this moment

Plastic zone:
- Increase h or Cu or d
- Decrease h or Cu or d

Elastoplastic zone:
- Soil begins to yield at soil-pile interface
- Decrease s/d, increase Cu
- Increase s/d, decrease Cu
- Decrease Cu, increase He, xe
- Increase Cu, decrease He, xe

Elastic zone:
- Ideal design area
- Decrease h or s/d or d
- Increase h or s/d or d

Nominal Embankment Load  kPa

Fig: 7.6
COMPARISON BETWEEN EXPERIMENTAL MAXIMUM PILE BENDING MOMENTS & DESIGN ZONES

Tests KP/SMS3 & 4

Fig: 7.7

LEGEND

--- Stability
--- 10.5 Cu
SMS4 Pile B
SMS4 Pile A
SMS3 Pile A
--- $\psi = 2$
--- $\psi = 1$
--- $\psi = 1/2$

Zone limits

3 Piles in 6m soft layer overlying 10m stiff layer

COMPARISON BETWEEN EXPERIMENTAL MAXIMUM PILE BENDING MOMENTS & DESIGN ZONES

Test KP/SMS7

Fig: 7.8

LEGEND

--- Stability
--- 10.5 Cu
SMS7 Pile B
SMS7 Pile A
--- $\psi = 2$
--- $\psi = 1$
--- $\psi = 1/2$

Zone limits

5 Piles in 6m soft layer overlying 10m stiff layer
COMPARISON BETWEEN EXPERIMENTAL MAXIMUM PILE BENDING MOMENTS & DESIGN ZONES
LOADING OVER TOP 8M OF SOFT LAYER
Test KP/SMS6

LEGEND
--- Stability
--- 10.5 Cu
*SMS6 Pile B
*SMS6 Pile A
--- $\psi = 1/2$
--- $\psi = 1$
--- $\psi = 2$
--- Zone Limits
3 Piles in 8.5m soft layer overlying 7.5m stiff layer
Fig: 7.9

COMPARISON BETWEEN EXPERIMENTAL MAXIMUM PILE BENDING MOMENTS & DESIGN ZONES
LOADING OVER TOP 6M OF SOFT LAYER
Test KP/SMS6

LEGEND
--- Stability
--- 10.5 Cu
*SMS6 Pile B
*SMS6 Pile A
--- $\psi = 1/2$
--- $\psi = 1$
--- $\psi = 2$
--- Zone Limits
3 Piles in 8.5m soft layer overlying 7.5m stiff layer
Fig: 7.10
PILE BENDING MOMENT IN STIFFER SUBSTRATUM

SHEAR MODULUS (G) CONSTANT WITH DEPTH \((dG/dy = 0)\)

SHEAR MODULUS (G) INCREASING WITH DEPTH \((G_0 = 0)\)

LEGEND

- \(1,000\)
- \(2,000\)
- \(5,000\)
- \(10,000\)
- \(20,000\)
- \(50,000\)
- \(100,000\)

GEOMETRY
Depth of soft layer \(h = 6\) m
Pile radius \(r = 0.635\) m
Critical pile length \(= l_c\)
Bending rigidity \(EI = 5.13\) MNm^2

LATERAL PRESSURE = 10.5Cu APPLIED IN UPPER SOFT LAYER
\(l_c (dG/dy = 0)\) is generally > \(l_c (G_0 = 0)\)

Fig: A.1
Fig. A.2  $E_u/C_u$ versus log axial strain for plane strain and triaxial tests (after Powrie, 1987)

Fig. A.3a  Pile and embankment of Provins. General scheme (after Bigot, Bourges & Frank, 1982)
Fig: A.3b  Experimental pile of Provins. Measured and calculated displacements and bending moments for the head load test (after Bigot, Bourges & Frank, 1982)

PROVINS LATERAL PILE TEST
PRESSUREMETER RESULTS

(Bigot, Bourges, Frank 1982)

Shear modulus in soil MPa

LEGEND

- - - Deflections
- - - Bending moments
- - - Gp2
- - - Gp5

The straight lines are the combination of shear modulus at y = 0 and increase with depth which give (through the RANDOLPH analysis) the best fit to the test pile data

Fig: A.4
(a) Soil profile data

(b) Details of test pile

(c) Comparison of soil reaction, bending moment and deflection profiles (at 30kN applied lateral load)

Fig: A.5 Plancoet pile test (after Price, Wardle, Frank & Jezequel, 1987)
PLANOET LATERAL PILE TEST PRESSUREMETER RESULTS

Shear modulus in soil MPa

Legend

- Randolph
- Gp5
- Gp2
- Gp0

Fig: A.6

Fig: A.7 Dynamic shear modulus for uncedmented, normally-consolidated, predominantly quartz sands - small strains (after Robertson & Campanella, 1983)