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Charge quantization from a number operator

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ABSTRACT

We explain how an unexpected algebraic structure, the division algebras, can be seen to underlie a generation of quarks and leptons. From this new vantage point, electrons and quarks are simply excitations from the neutrino, which formally plays the role of a vacuum state. Using the ladder operators which exist within the system, we build a number operator in the usual way. It turns out that this number operator, divided by 3, mirrors the behaviour of electric charge. As a result, we see that electric charge is quantized because number operators can only take on integer values.

Finally, we show that a simple hermitian form, built from these ladder operators, results uniquely in the nine generators of $SU_c(3)$ and $U_{em}(1)$. This gives a direct route to the two unbroken gauge symmetries of the standard model.

With the recent discovery of a scalar boson, whose coupling, spin and parity properties appear to largely align with the Higgs, [1–4], there is little refuting the predictive power of the standard model of particle physics. However, amongst the standard model's impressive achievements are a number of gaps in our understanding. For one, the logic behind the particle content of the standard model is still a mystery. Furthermore, the phenomenon of electric charge quantization remains an enigma, a property which is currently put into the theory by hand. One would expect that if a deeper mathematical structure to the standard model could be found, then it would act to illuminate the voids.

Here, we propose one such mathematical structure, whose potential has largely gone unnoticed. This structure is the set of algebras known as the normed division algebras over the reals. Strikingly, there exist only four of these algebras: the real numbers, $\mathbb{R}$, the complex numbers, $\mathbb{C}$, the quaternions, $\mathbb{H}$, and the octonions, $\mathbb{O}$. It can be shown that particle physics relies heavily on the first three of these algebras.

The real numbers are used almost universally in physics; the complex numbers are central to quantum theory; the quaternions lead to the Pauli matrices, and are hence tightly entwined with the Lorentz algebra. In fact, in [5], it is shown that the complex quaternions can concisely describe all of the Lorentz representations of the standard model: scalars, spinors, four-vectors, and the field strength tensor, in terms of generalized ideals.

But what is to be said for the octonions, $\mathbb{O}$, the fourth, and final division algebra? With $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$ each undeniably etched into fundamental physics, it is hard not to wonder: is it really the case that $\mathbb{O}$ has been omitted in nature?

In earlier years, [6], Günaydin and Gürsey showed $SU_c(3)$ quark structure in the split octonions. Later, in [7], they showed anti-commuting ladder operators within that model. Our new results stem from the octonionic chromodynamic quark model of [7], and are meant to replace the provisional charges of [5]. These findings make a case in support of those who have been long advocating for the existence of a connection between non-associative algebras and particle physics, [5–20].

Using the algebra of the complex octonions, which we will introduce, we expose an intrinsic structure to a generation of quarks and leptons. This algebraic structure mimics familiar quantum systems, which have a vacuum state acted upon by raising and lowering operators. In this case, the neutrino poses as the vacuum state, and electrons and quarks pose as the excited states.

With these raising and lowering operators in hand, we are then able to construct a number operator in the usual way,

$$N = \sum_i \alpha_i^\dagger \alpha_i. \tag{1}$$

It will be seen that N has eigenvalues given by $\{0, 1, 1, 1, 2, 2, 2, 3\}$. At first sight, these eigenvalues might not look familiar, that is, until they are divided by 3. $N/3$ has eigenvalues $\{0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3},$

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$\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 1\}$, which can now be recognized as the electric charges of a neutrino (or anti-neutrino), a triplet of anti-down quarks, a triplet of up quarks, and a positron. We will then define our electric charge, Q, as

$$Q \equiv \frac{N}{3}. \tag{2}$$

As N must take on integer values, Q must be quantized.

As we will show, the remaining states within a generation are related to these particles by complex conjugation, and hence are acted upon by $-Q^*$ in the usual way.

Ours is certainly not the first instance where Günaydin and Gürsey's model has been adapted. As an extension of their model, [15,16], Dixon describes electric charge as a mix of quaternionic and octonionic objects. It would be interesting to see if a ladder system could be found, which alternately gives Dixon's Q as a number operator. Readers are encouraged to see [15,16], or other examples of his extensive work.

Since the time of first writing, more octonionic chromo-electrodynamic models have been found. Most noteworthy of all were two papers written in the late 1970s, [9] and [10], which could also be considered as extensions of Günaydin and Gürsey's model, [7]. In these papers, the authors use two separate ladder systems: system (a) fits with the octonionic ladder operators of [7], and system (b) is introduced as quaternionic. By combining the two systems, they describe the electric charge generator not as a number operator, but as the difference between the number operators of the two systems. References [9] and [10] are both important papers, worth careful reconsideration by the community.

Our results differ from earlier versions in that we will be constructing a generation of quarks and leptons explicitly as *minimal left ideals* of a Clifford algebra, generated by the complex octonions. In doing so, we will use just a single octonionic ladder system, with its complex conjugate. This in turn allows us to (1) define electric charge more simply as $Q = N/3$, and (2) expose a more direct route to the two unbroken gauge symmetries of the standard model. Furthermore, our formalism naturally relates particles and anti-particles using only the complex conjugate, $i \mapsto -i$, which is not a feature of these earlier models. Finally, as our generation of quarks and leptons will be constructed from Clifford algebra elements, not column vectors, we will then be free to model mass and weak isospin, using right multiplication of this same Clifford algebra onto these minimal left ideals.

1. Acquaintance with $\mathbb{C} \otimes \mathbb{O}$

The complex octonions are not a tool commonly used in physics, so we introduce them here.

The generic element of $\mathbb{C} \otimes \mathbb{O}$ is written $\sum_{n=0}^{7} A_n e_n$, where the A_n are complex coefficients. The e_n are octonionic imaginary units ($e_n^2 = -1$), apart from $e_0 = 1$, which multiply according to Fig. 1. The complex imaginary unit i commutes with the octonionic e_n.

Any three imaginary units on a directed line segment in Fig. 1 act as if they were a triplet of Pauli matrices, σ_m. (More precisely, they behave as $-i\sigma_m$.) For example, $e_6 e_1 = -e_1 e_6 = e_5$, $e_1 e_5 = -e_5 e_1 = e_6$, $e_5 e_6 = -e_6 e_5 = e_1$, $e_4 e_1 = -e_1 e_4 = e_2$, etc. It is indeed true that the octonions form a non-associative algebra, meaning that the relation $(ab)c = a(bc)$ does not always hold. The reader can check this by finding three imaginary units, which are not all on the same line segment, and substituting them as in a, b, and c. For a more thorough introduction of $\mathbb{O}$ see [18–20].

Finally, we define three notions of conjugation on an element a in $\mathbb{C} \otimes \mathbb{O}$. The *complex conjugate* of a, denoted a^*, maps the complex $i \mapsto -i$, as would be expected. The *octonionic parity conjugate* of a, denoted $\tilde{a}$, takes each of the octonionic imaginary units

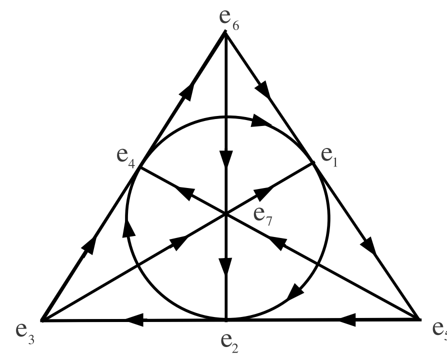

Fig. 1. Multiplication of octonionic imaginary units.

$e_n \mapsto -e_n$ for $n = 1, \ldots, 7$. That which we will call the *hermitian conjugate* of a, denoted $a^\dagger$, performs both of these maps simultaneously, $i \mapsto -i$ and $e_n \mapsto -e_n$ for $n = 1, \ldots, 7$. The parity conjugate and the hermitian conjugate each reverse the order of multiplication, as is familiar from the hermitian conjugate of a product of matrices.

2. A system of ladders

Upon some exploration, one finds a system of ladder operators within the complex octonions. Consider $\alpha_1 \equiv \frac{1}{2}(-e_5 + ie_4)$, $\alpha_2 \equiv \frac{1}{2}(-e_3 + ie_1)$, and $\alpha_3 \equiv \frac{1}{2}(-e_6 + ie_2)$, similar to that defined in [6]. For all f in $\mathbb{C} \otimes \mathbb{O}$, and assuming right-to-left multiplication, these three lowering operators obey the anticommutation relations

$$\{\alpha_i, \alpha_j\} f = \alpha_i(\alpha_j f) + \alpha_j(\alpha_i f) = 0 \tag{3}$$

for all $i, j = 1, 2, 3$. The above can be seen as a generalization of the system in [7]. That is, [7] is recovered by restricting the general f in $\mathbb{C} \otimes \mathbb{O}$ to $f = 1$.

In another slight deviation from [7], we define raising operators as $\alpha_1^\dagger = \frac{1}{2}(e_5 + ie_4)$, $\alpha_2^\dagger = \frac{1}{2}(e_3 + ie_1)$, and $\alpha_3^\dagger = \frac{1}{2}(e_6 + ie_2)$, which obey

$$\{\alpha_i^\dagger, \alpha_j^\dagger\} f = 0 \quad \text{for all } i, j = 1, 2, 3. \tag{4}$$

We finally also have

$$\{\alpha_i, \alpha_j^\dagger\} f = \delta_{ij} f \quad \text{for all } i, j = 1, 2, 3. \tag{5}$$

With the purpose simplifying notation, we will now implicitly assume all multiplication to be carried out starting at the right, and moving to the left, as was shown in Eq. (3). That is, we will now not write these brackets in explicitly. Furthermore, we will now be concerned only with operators, such as the α_i, as opposed to the object f. This being the case, it will now be understood that all equations will hold over all f in $\mathbb{C} \otimes \mathbb{O}$, even though f will not be mentioned explicitly. For example, we will now write Eq. (3) simply as

$$\{\alpha_i, \alpha_j\} = \alpha_i \alpha_j + \alpha_j \alpha_i = 0 \quad \text{for all } i, j = 1, 2, 3. \tag{6}$$

Incidentally, these operators acting on f may be viewed as 8×8 complex matrices acting on f, an eight-complex-dimensional column vector. Taking into account the above paragraph, our equations from here on in can be considered as relations only between the matrices.

3. Complex conjugation's analogue

Under complex conjugation, we find an analogous ladder system. Consider $\alpha_1^* = \frac{1}{2}(-e_5 - ie_4)$, $\alpha_2^* = \frac{1}{2}(-e_3 - ie_1)$, and $\alpha_3^* =$

$\frac{1}{2}(-e_6 - ie_2)$. These three lowering operators obey the anticommutation relations

$$\left\{\alpha_i^*, \alpha_j^*\right\} = 0 \quad \text{for all } i, j = 1, 2, 3. \tag{7}$$

We define raising operators as $\tilde{\alpha}_1 = \frac{1}{2}(e_5 - ie_4)$, $\tilde{\alpha}_2 = \frac{1}{2}(e_3 - ie_1)$, and $\tilde{\alpha}_3 = \frac{1}{2}(e_6 - ie_2)$, which obey

$$\{\tilde{\alpha}_i, \tilde{\alpha}_j\} = 0 \quad \text{for all } i, j = 1, 2, 3. \tag{8}$$

Finally, we have also

$$\left\{\alpha_i^*, \tilde{\alpha}_j\right\} = \delta_{ij} \quad \text{for all } i, j = 1, 2, 3. \tag{9}$$

Using these ladder operators, we will now build *minimal left ideals*, which can be seen to mimic the set of quarks and leptons of the standard model.

4. Minimal left ideals

Intuitively speaking, an *ideal* is a special subspace of an algebra because it is robust under multiplication. For this reason, ideals are well suited to describe particles persisting under evolution and transformation.

Given an algebra, A, a *left ideal*, B, is a subalgebra of A whereby ab is in B for all b in B, and for any a in A. That is, no matter which a we multiply onto b, the new product, $b' \equiv ab$, cannot leave the subspace B. It is easy to see how $b' \equiv ab$ could easily describe, for example, a particle b undergoing a transformation a.

A *minimal left ideal* is a left ideal which contains no left ideals other than $\{0\}$ and itself. In other words, it has no non-trivial ideals inside it.

In this article, we are proposing to represent quarks and leptons using minimal left ideals within our space of octonionic operators: that is, within the space of the α_i, $\alpha_j^\dagger$, and their products. A pair of these ideals, S^u and S^d, will be introduced below. Readers wishing to confirm the construction may consult [11] for an explanation of how left multiplication of $\mathbb{C} \otimes \mathbb{O}$ on itself gives a representation of the 64-complex-dimensional Clifford algebra $\mathbb{C}l(6)$. The review, [21], then lucidly describes the construction of minimal left ideals in Clifford algebras via *Witt decomposition*. (For an alternate phase space perspective on the real Clifford algebra $Cl(6)$, see [22].)

From our first ladder system, we define

$$\begin{aligned} \omega &\equiv \alpha_1\alpha_2\alpha_3, \\ \omega^\dagger &\equiv \alpha_3^\dagger\alpha_2^\dagger\alpha_1^\dagger, \end{aligned} \tag{10}$$

which lead to the identies $\omega^\dagger\omega\omega^\dagger = \omega^\dagger$ and $\omega\omega^\dagger\omega\omega^\dagger = \omega\omega^\dagger$.

The eight-complex-dimensional minimal left ideal for the first ladder system is given by

$$\begin{aligned} S^u \equiv\ & \mathcal{V}\ \omega\omega^\dagger \\ & + \bar{\mathcal{D}}^{\mathrm{r}}\ \alpha_1^\dagger\omega\omega^\dagger + \bar{\mathcal{D}}^{\mathrm{g}}\ \alpha_2^\dagger\omega\omega^\dagger + \bar{\mathcal{D}}^{\mathrm{b}}\ \alpha_3^\dagger\omega\omega^\dagger \\ & + \mathcal{U}^{\mathrm{r}}\ \alpha_3^\dagger\alpha_2^\dagger\omega\omega^\dagger + \mathcal{U}^{\mathrm{g}}\ \alpha_1^\dagger\alpha_3^\dagger\omega\omega^\dagger + \mathcal{U}^{\mathrm{b}}\ \alpha_2^\dagger\alpha_1^\dagger\omega\omega^\dagger \\ & + \mathcal{E}^+\ \alpha_3^\dagger\alpha_2^\dagger\alpha_1^\dagger\omega\omega^\dagger, \end{aligned} \tag{11}$$

where $\mathcal{V}, \bar{\mathcal{D}}^{\mathrm{r}}, \ldots, \mathcal{E}^+$ are 8 suggestively named complex coefficients.

As

$$\alpha_i\ \omega\omega^\dagger = 0 \quad \forall i, \tag{12}$$

$\omega\omega^\dagger$ plays the role of the vacuum state, where the term *vacuum* is used loosely. Readers may recognize the similarity between S^u and a Fock space.

The conjugate system analogously leads to

$$\begin{aligned} S^d \equiv\ & \bar{\mathcal{V}}\ \omega^\dagger\omega \\ & + \mathcal{D}^{\mathrm{r}}\ \alpha_1\omega^\dagger\omega + \mathcal{D}^{\mathrm{g}}\ \alpha_2\omega^\dagger\omega + \mathcal{D}^{\mathrm{b}}\ \alpha_3\omega^\dagger\omega \\ & + \bar{\mathcal{U}}^{\mathrm{r}}\ \alpha_3\alpha_2\omega^\dagger\omega + \bar{\mathcal{U}}^{\mathrm{g}}\ \alpha_1\alpha_3\omega^\dagger\omega + \bar{\mathcal{U}}^{\mathrm{b}}\ \alpha_2\alpha_1\omega^\dagger\omega \\ & + \mathcal{E}^-\ \alpha_3\alpha_2\alpha_1\omega^\dagger\omega, \end{aligned} \tag{13}$$

where $\bar{\mathcal{V}}, \mathcal{D}^{\mathrm{r}}, \ldots, \mathcal{E}^-$ are eight complex coefficients.

This new ideal, (13), is linearly independent from the first, (11), in the space of octonionic operators. Clearly, the two are related via the complex conjugate, $i \mapsto -i$. In fact, the complex conjugate is *all* that is needed in order to map particles into anti-particles, and vice versa. This was a feature in the models of [7,11], and also in the context of left- and right-handed Weyl spinors in [5].

The Clifford algebra $\mathbb{C}l(6)$ is known to have just a single 8-complex-dimensional irreducible representation, as in S^u, above. In this paper, we will none-the-less be including the conjugate ideal, S^d, in anticipation of future work, which will combine S^u and S^d into a single irreducible representation under $\mathbb{C}l(6) \otimes \mathbb{C}l(2)$. (Later on, we will then consider $\mathbb{C}l(6) \otimes \mathbb{C}l(4)$, suggesting a connection to the Pati–Salam model.) Unlike in the earlier literature, this additional factor of $\mathbb{C}l(2)$ will originate from right multiplication of our octonionic operators on these ideals, as mentioned at the end of this text.

As a final note, we point out that another interesting way to obtain anti-particles could be to use the conjugate †, instead of ∗. In that case, the two minimal left ideals would not be entirely linearly independent from each other. That is, we would find a special Majorana-like property unique to the neutrino: $(\omega\omega^\dagger)^\dagger = \omega\omega^\dagger$.

5. Ladders to the unbroken symmetries

Having obtained these minimal left ideals, we would now like to know how they transform, so as to justify the labels we gave to their coefficients in Eqs. (11) and (13). It so happens that a very simple form leads uniquely to the generators of the two unbroken gauge symmetries of the standard model, $SU_c(3)$ and $U_{em}(1)$. We will find these generators, and then apply them to our minimal left ideals.

Consider $\alpha \equiv c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3$ and $\alpha' \equiv c_1'\alpha_1 + c_2'\alpha_2 + c_3'\alpha_3$, where the c_i and c_j' are complex coefficients. We can then build hermitian operators, $\mathcal{H}$, of the form

$$\mathcal{H} \equiv \alpha'^\dagger\alpha + \alpha^\dagger\alpha'. \tag{14}$$

Taking the most general sum of these objects results in nine hermitian operators:

$$\sum_{\mathcal{H}} \mathcal{H} = r_0\ Q + \sum_{i=1}^{8} r_i \Lambda_i, \tag{15}$$

where r_0 and r_i are real coefficients. Q is our electromagnetic generator from Eq. (2), and the eight Λ_i can be seen to generate $SU_c(3)$. Indeed, these Λ_i coincide with those introduced in [6] (which generate a subgroup of the octonionic automorphism group, G_2).

The result of Eq. (15) is worth emphasizing. That is, the simple form, $\sum_{\mathcal{H}}\mathcal{H}$ leads *uniquely* to the generators of the two unbroken gauge symmetries of the standard model.

In terms of ladder operators, the $SU_c(3)$ generators take the form

$$\Lambda_1 = -\alpha_2^\dagger\alpha_1 - \alpha_1^\dagger\alpha_2 \qquad \Lambda_2 = i\alpha_2^\dagger\alpha_1 - i\alpha_1^\dagger\alpha_2$$
$$\Lambda_3 = \alpha_2^\dagger\alpha_2 - \alpha_1^\dagger\alpha_1 \qquad \Lambda_4 = -\alpha_1^\dagger\alpha_3 - \alpha_3^\dagger\alpha_1$$

$$\Lambda_5 = -i\alpha_1^\dagger\alpha_3 + i\alpha_3^\dagger\alpha_1 \qquad \Lambda_6 = -\alpha_3^\dagger\alpha_2 - \alpha_2^\dagger\alpha_3$$
$$\Lambda_7 = i\alpha_3^\dagger\alpha_2 - i\alpha_2^\dagger\alpha_3 \qquad \Lambda_8 = -\frac{1}{\sqrt{3}}[\alpha_1^\dagger\alpha_1 + \alpha_2^\dagger\alpha_2 - 2\alpha_3^\dagger\alpha_3], \tag{16}$$

all eight of which can be seen to commute with Q, and its conjugate.

Now, the minimal left ideal, S^u, transforms as

$$e^{i\sum\mathcal{H}} S^u e^{-i\sum\mathcal{H}} = e^{i\sum\mathcal{H}} S^u, \tag{17}$$

where the equality holds because $\omega^\dagger\alpha_i^\dagger = 0$ for all i.

We now identify the subspaces of S^u by specifying their electric charges with respect to $U_{em}(1)$, and also which irreducible representation they belong to under $SU_c(3)$. Clearly, i, j and k are meant to be distinct from each other in any given row.

$\underline{Q}$	$\underline{\boldsymbol{\Lambda}}$	$\underline{S^u}$	$\underline{\text{ID}}$
0	1	$\omega\omega^\dagger$	ν (or $\bar{\nu}$)
1/3	$\bar{3}$	$\alpha_i^\dagger\omega\omega^\dagger$	$\bar{d}_i$
2/3	3	$\alpha_i^\dagger\alpha_j^\dagger\omega\omega^\dagger$	u_k
1	1	$\alpha_i^\dagger\alpha_j^\dagger\alpha_k^\dagger\omega\omega^\dagger$	e^+

(18)

So, here we identify a neutrino, ν, (or antineutrino, $\bar{\nu}$), three anti-down type quarks, $\bar{d}_i$, three up-type quarks, u_k, and a positron, e^+.

As the minimal left ideal, S^d, is related to S^u by complex conjugation, we then see that it transforms as

$$e^{-i\sum\mathcal{H}^*} S^d e^{i\sum\mathcal{H}^*} = e^{-i\sum\mathcal{H}^*} S^d, \tag{19}$$

where the equality holds because $\omega\alpha_i = 0$ for all i. This leads to the table below.

$-\underline{Q}^*$	$-\underline{\boldsymbol{\Lambda}}^*$	$\underline{S^d}$	$\underline{\text{ID}}$
0	1	$\omega^\dagger\omega$	$\bar{\nu}$ (or ν)
−1/3	3	$\alpha_i\omega^\dagger\omega$	d_i
−2/3	$\bar{3}$	$\alpha_i\alpha_j\omega^\dagger\omega$	$\bar{u}_k$
−1	1	$\alpha_i\alpha_j\alpha_k\omega^\dagger\omega$	e^-

(20)

Here, we identify an antineutrino, $\bar{\nu}$ (or a neutrino, ν), three down-type quarks, d_i, three anti-up type quarks, $\bar{u}_k$, and the electron, e^-.

We have now shown a pair of conjugate ideals, which behave under $SU_c(3)$ and $U_{em}(1)$ as does a full generation of the standard model. These are summarized in Fig. 2.

6. A signal from *W* bosons

Perhaps unexpectedly, it turns out that S^u packages all of the isospin up-type states together, and S^d packages all of the down-type states together. This is of course, if one goes ahead and makes an assumption about the placement of ν into S^u and $\bar{\nu}$ into S^d.

We point out that ω is negatively charged, and converts isospin up particles into isospin down, via *right* multiplication on S^u. It thereby exhibits features of the W^- boson. Similarly, $\omega^\dagger$ is positively charged, and converts isospin down particles into isospin up, via *right* multiplication on S^d. In doing so, it exhibits features of the W^+ boson.

Other characteristics of the W bosons do not appear at the level of this article. For example, there is nothing to specify that these candidate bosons act only on left-handed particles. We also have no description here for the polarization states of these would-be bosons.

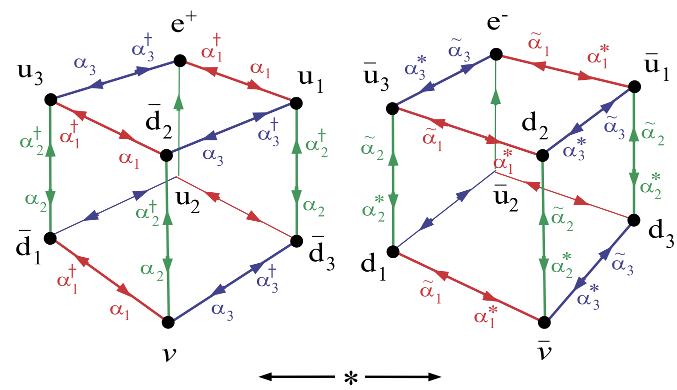

Fig. 2. A full generation represented by cubes S^u (left) and S^d (right). Quark and electron states may be viewed as excitations from the neutrino or anti-neutrino. As the "vacuum" represents the neutrino, and not the zero particle state, this model does not constitute a composite model in the usual sense.

An obvious first step in this direction is to consider the Clifford algebra, $\mathbb{C}l(8)$, which comes from the algebra $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ acting on itself. This is in analogy to how we found that $\mathbb{C}l(6)$ results from $\mathbb{C}\otimes\mathbb{O}$ acting on itself via left multiplication, as is explained in [11]. The idea, then, is to find minimal left ideals in $\mathbb{C}l(8)$, and follow the same procedure as was introduced here.

7. Conclusion

Using only the complex octonions acting on themselves, we were able to recover a number of aspects of the standard model's structure.

First of all, we found that a simple hermitian form led *uniquely* to the two unbroken gauge symmetries of the standard model, $SU_c(3)$ and $U_{em}(1)$. This new $U_{em}(1)$ generator, Q, happens to be proportional to a number operator, thereby suggesting an unexpected resolution to the question: Why is electric charge quantized?

Then, using octonionic ladder operators, we have built a pair of minimal left ideals, which is found to transform under these unbroken symmetries as does a full generation of quarks and leptons.

If the algebra of the complex octonions is *not* behind the structure of the standard model, it is then a striking coincidence that $SU_c(3)$ and $U_{em}(1)$ both follow readily from its ladder operators.

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