Assessing Interbank Connectedness Using Transmission Decomposition Techniques: an Application to Eurozone SIFIs

C.A.F. Muijssen
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Abstract

This paper aims to disentangle the impact of multiple transmission channels in interbank connectedness. We use the identification properties of a structural vector autoregression with a multivariate GARCH-in-mean structure (SVAR-MGARCH-M) to model the dynamics in equity returns of the Eurozone systemically important financial institutions (SIFIs). We can identify the impacts of multiple transmission channels such as asset communality, interbank deposits, and information contagion. Asset communality is both a factor in the transmission of shocks and a means of diversification: heightened connectedness results in an increase in shock spillovers, which are countered by risk sharing benefits. We show that connectedness increased during the financial crisis with a major role for common exposures. Institutions move from being a net recipient to a net transmitter of shocks when the asset communality channel is taken into account, suggesting that we need to evaluate the systemic importance of an institution using all transmission components.

Keywords: Banking Integration; Financial Connectedness; Systemic Risk.

JEL: C320; F360; F370; G210.

1 Introduction

With the on-going financial turmoil, the study of the interaction structures among banks and financial institutions in transmitting shocks is becoming more prevalent and significant. Connectedness among banks is not limited to periods of distress but can be exacerbated during these times: the financial crisis of the late 2000s provides a natural illustration. We observe that the transmission was not exclusively among

*PhD Student, Faculty of Economics, University of Cambridge, CB3 9DD Cambridge, United Kingdom. cafm3@cam.ac.uk, +44 (0)7585216589.
banks with strong balance sheet linkages: those that did not diversify their assets to the afflicted markets were also impacted by the same negative shocks.

The main reason for this phenomenon is that most banks only have limited interbank assets as most of their activities are with other non-financial institutions, with the possibility of common exposures. However, this only partly explains the strong correlation that we find among bank returns and the resulting comovement of volatility. Therefore, it appears that it is not sufficient to study balance sheet channels in isolation as they only partly representative on a global or area-wide scale. This is particularly true for globally systemically important financial institutions (SIFIs). Another channel of possible connectedness is provided by informational spillovers, which are driven by the expectations of investors on the volatility underlying the stock return.

Hence, we identify three main channels of interbank contagion: interbank deposit linkages, common balance sheet exposures ("asset communality") and investor behaviour ("information contagion"). In this paper we aim to disentangle these three channels by combining two broad streams of literature: network theory and multivariate time series modelling. We show that we are able to identify information contagion and interbank deposit linkages separately from asset communality while relying solely on publicly available stock return data, with the added benefit of increased transparency and replicability.

We propose a SVAR-MGARCH-M model on bank equity returns to identify three transmission channels: asset communality, interbank deposits, and information contagion through the effects of volatility in the mean equation in the form of risk premium spillovers. The advantage of market based measures is that they directly incorporate differences in valuation and are observed frequently, which allows us to use the market valued bank risk premia as a proxy for information contagion. We use identification properties of structural VARs allowing for correlated GARCH errors to disentangle the transmission channels. The approach is a marriage of the methodologies proposed by Elder (2003) and Milunovich and Yang (2013), who study use the identification properties of a multivariate GARCH in structural models to derive impulse responses. This paper combines both approaches to model information contagion effects through the risk premia and proves that the identification properties still hold in a SVAR-MGARCH-M case.

This paper contributes to the literature in several ways. First, we are able to disentangle the transmission channels using a unique identification property of the SVAR-GARCH-M model. Another contribution is the propagation of shocks originating from multiple banks and disentangling the transmission channels in the forecast error variance decompositions. One advantage is that we can directly infer the effect of bank or market specific shocks on other institutions, which has value for systemic risk analysis.

We find that there are differences in the transmission mechanism for the country and area-wide levels. We see that liquidity shocks are propagated through the
interbank market even when orthogonal to asset communality, and that information contagion has a minor role relative to asset communality and interbank deposit transmission. However, the risk sharing benefits observed on the country level are not replicated on the Eurozone scale. Also, we find that some institutions can move from being a net recipient to a net transmitter of shocks when we allow for asset communality. When evaluating the systemic impact of an institution we should not only analyse their net connectivity but also their affection through common assets, as it shows us whether the connectivity is caused by shocks from the system or shocks originating from the institution. Policy should focus on creating transparency to combat connectedness through investor dynamics as well as to foster trust in the interbank market. Monolining institutions would hinder the risk sharing properties of common assets in both a country as a Eurozone perspective.

2 Literature review

Many efforts have been made to model direct and indirect connectedness, for instance network models, vector autoregressions (VAR), correlation models and state-space specifications. Transmission and connectedness do not have a one dimensional character: there are many possible channels through which agents in a system are linked. On the one hand we have the existence and persistence of common shocks, whether from the market or from shared balance sheet items (e.g. real estate market). On the other hand, institutions are connected through the interbank market and volatility spillovers.

Previous research on interbank contagion using network models shows that banks are exposed to negative shocks of other institutions in the system and are more susceptible to systemic risk, although imperfect information on the exposures on defaulting banks makes the role of connectivity uncertain in some cases (see Allen and Gale, 2000, and Battison et al., 2012). Models disentangling interdependence from contagion have been studied extensively, see for instance Forbes and Rigobon (2002) and Corsetti et al. (2005). Forbes and Rigobon (2002) find no significant increase in stock market return correlations in times of crisis when they correct for heteroskedasticity. Corsetti et al. (2005) however find evidence of the existence of an increase in contagion in times of crisis, which is partly caused by contagion and partly through interdependence. Other approaches include Chiang et al (2007) who look at stock market contagion in times of the Asian financial crisis and use a DCC-MGARCH specification to model comovement. They make a distinction between contagion and investor herding and find that there is a significant volatility spillover effect, which can partly be explained by herding behaviour.

Diebold and Yilmaz (2012) make use of a multivariate model to distinguish between total spillovers which are caused by common shocks and directional spillovers which are related to interbank contagion, thereby linking connectivity from network
theory to time series analysis. They propose a generalised VAR system for the volatility of banks and makes a distinction between total spillovers (which can be attributed to common shocks) and directional spillovers (which are more related to contagion effects). They use a moving average representation of a VAR(1) system for the volatility of N banks, with variance decompositions that allow us to assess a fraction of the H-step ahead error variance in forecasting the volatility of one bank given shocks to the volatility of another bank in the system. They propose a generalised VAR as they do not want to orthogonalise the model as the innovations are contemporaneously correlated. Hence, they allow for correlated shocks that are accounted for using the historically observed distribution of errors. The variance decomposition then shows the relative impact of other agents volatility on the volatility of the specific bank and allows for impulse response analysis.

Elder (2003) proposes a VAR with a multivariate GARCH in mean process for the error terms. Using his model it is possible to derive a moving average representation of a VAR-MGARCH-M model where disturbances are correlated with each other. He continues to derive an analytical expression for an impulse response function for this model. Hence, the model by Elder (2003) includes a common factor (the lag of the market return) and an indirect spillover (the volatility of one bank to another, by the GARCH in mean term). However, the information channel is not directly modelled in his paper. Another method to look at impulse response functions of specific shocks is a state space representation of a model. Chua, Suardi and Tsiaplias (CST, 2012) propose an extension of the model introduced by Elder (2003) by assuming that information is an unobserved component in the model. Unobserved components have been used before in this literature but were focused to unobserved volatility (the stochastic volatility models, which are a state space version of the familiar GARCH type models). GARCH effects are difficult to include in a state space model as the Kalman filter requires a linear model for the state equations, although there is some scope for nonlinearity. Diebold and Nerlove (1989) look at common movements in volatility ascribed to a single unobserved latent factor subject to ARCH effects. Harvey, Ruiz and Shephard (1994) discuss multivariate versions of stochastic variance models. Koopman and Uspensky (2002) propose a stochastic volatility model in mean, as a state space equivalent.

The model by Elder (2003) is closely related to the model proposed by Milunovich and Yang (2013) in that they aim to derive impulse responses from a SVAR-GARCH-M model. The differences between the papers are that Elder (2003) uses a GARCH-in-mean specification, while Milunovich and Yang (2013) focus on the unique result on the identifying restrictions of the structural model with multivariate GARCH errors. This paper proposes to combine both approaches to have the advantage of the information contagion effects through the risk premia and proves that the result with respect to the identifying restrictions still holds in a SVAR-MGARCH-M case.
3 The Transmission Mechanism

We have to distinguish two types of transmission: first, there is the bank asset based contagion and second the herding behaviour of investors based on signalling.

The literature on banking contagion differentiates types: causal contagion, also defined as interbank credit channels, are the main objective of network models. A survey of network models for systemic risk and contagion is provided by Chinazzi and Fagiolo (2013). In network models one can assume a certain structure of the market in which agents operate, with an interaction structure that allows the response to be dependent on the underlying web of relationships linking banks (Allen and Gale, 2000). However, most of the networks are incomplete with the added problem of incomplete information, which may exacerbate contagion by inducing bank runs and fire sales that also change the effects of connectivity on contagion. Battison et al. (2012) find that imperfect information on exposures of defaulting banks can lead to bank runs and the role played by connectivity is uncertain in such cases. Also, network models studying the credit interbank linkages have a very narrow focus and do not take return effects into account.

While banks are directly linked through their balance sheet exposures, part of their comovement cannot be explained by these relatively small exposures on each other. Transmission is not exclusively observed in asset movements: bank equity is also affected by the behaviour of investors and their expectations on the stock returns. Scharfstein and Stein (1990) show that investors tend to trade excessively in the direction of the trades of other investors through the effects of the unresolved uncertainty on future stock returns. Dasgupta et al (2010) proceed by introducing a theoretical model linking investor herding to the return impact, and replicate the empirical result that institutional herding positively predicts short term returns but negatively predicts long term returns.

There is no real consensus on the interbank transmission mechanism and no proper distinction is made between interbank asset contagion and the impact of herding behaviour of investors. We propose a model that reconciles these two approaches intuitively and with flexibility for future application. We assume that the market prices capture all publicly and privately available information on the banks and adapts with changes in underlying asset valuations. Stock prices are both responsive to changes in the underlying balance sheet of the banks and to trader and institutional investor behaviour. Therefore, we identify three main channels of interbank transmission:

1. Asset Communality
2. Interbank Deposits
3. Investor Herding and Information Contagion
The selection of banks within a system is not arbitrary: particularly in the networks literature one assumes that there is randomisation in the selection of agents within the network when estimation of the full sample is infeasible. However, when modelling a financial system, the agents are rarely a random selection of individuals in a sector and it is unlikely that a random graph will lead to a good characterisation of actual linkages. Upper (2007) notes that the maximum entropy assumption, which is commonplace to identify the network, leads to an underestimation of the frequency of contagion but an overestimation of the impact. Sachs (2013) first estimates the connectivity of interbank exposures based on the full sample, and then proceeds by looking at the implications in a random setting. Finally, she looks at connectivity in a money center model.

The modelling of a subsample of a large network has been discussed extensively in the literature (see for instance Boss et al. (2004) and Soramaki et al. (2006) on scale free networks, and Craig and Von Peter (2010) on money center systems). In the case of a money center model, a few core banks are strongly interconnected while a larger number of smaller banks are only connected to one of these core banks. We can see this result in a matrix setup:

<table>
<thead>
<tr>
<th>Core Banks</th>
<th>Zero</th>
<th>Small Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Banks</td>
<td>Nonzero</td>
<td>Small Banks</td>
</tr>
</tbody>
</table>

Diagram 1: Money Center with non-connected small banks, and connected banks

Sachs (2013) finds that the stability of the system increases when the number of core banks increases due to the effect of a lower concentration index and the accompanying decrease in the size of the interbank asset linkage. Sachs also shows that a random system is on average more stable than the money center system, mainly caused by the lack of diversification in a money center system.

These results are relevant from a policy perspective as the amount and direction of interlinkages affect the relative risk and size of contagion in the system. It is necessary to test for the sample stability in a system to check whether all connections have been incorporated in the model structure, as a failure to account for specific nodes would give a biased picture of interconnectedness in the system. Particularly a preselected sample such as Eurozone SIFIs is suspected to have more connections than a money center model would prescribe. For instance, one would expect that US SIFI banks are correlated with some of the Eurozone SIFIs, however, Japanese banks are expected to be uncorrelated. The estimation procedure has to take into account the underlying network structure and endogeneity concerns.

Off-diagonal elements are the covariances between a specific small bank and the core system.
4 Method

We introduce a model which allows us to disentangle these effects in a structural model where volatility explicitly affects the mean equity returns. To do so, we make a distinction between asset effects and investor herding.

Our model, using a \((k \times 1)\) vector of returns, \(R_t\), is defined as:

\[
\Phi_B(L)R_t = \varphi_0 + \Lambda g_t + \varepsilon_t
\]  

(1)

where \(\Phi_B(L) = B_0\Phi(L)\) is a lag polynomial and \(B_0\) a non singular \(k \times k\) matrix, \(\varphi_0\) the intercept, and \(\Lambda\) the matrix of parameters of size \(k \times (k + 1)/2\) for the volatility parameters in \(g_t\) \((k(k + 1)/2 \times 1)\), where \(g_t = vech(G_t)\) from the MGARCH process. Here, \(G_t(k\times k)\) is the estimated time varying conditional covariance matrix:

\[
G_t = \begin{pmatrix}
    g_{11} & g_{12} & g_{1k} \\
    g_{21} & g_{22} & g_{2k} \\
    g_{k1} & g_{k2} & g_{kk}
\end{pmatrix}
\]

We define the MGARCH process:

\[
\varepsilon_t|F_{t-1} \sim Q(0, G_t)
\]

\[
G_t = \omega \omega' + \beta G_{t-1}\beta' + \alpha diag(\varepsilon_{t-1}) diag(\varepsilon_{t-1})' \alpha'
\]

\(G_t, \omega, \alpha\) and \(\beta\) are all \((k \times k)\) diagonal matrices. As can be seen from equation 1, the model is very similar to a structural VAR with GARCH errors.

As we assume that the market prices capture underlying movement in balance sheet exposures, any changes in assets and interbank deposits will be priced through a lagged return effect (ceteris paribus on other factors). The other channel is through the herding of investors: if the uncertainty over an asset increases, investors expect to be compensated with a higher risk premium. Investor behaviour can be measured by the expectation of the underlying uncertainty, in this case the stock price volatility. An increase in the expected volatility \((h_t)\) affects the return of the bank directly (ceteris paribus on other factors). Contemporaneous asset communality is captured by the structural \(B_0\) matrix.

Both asset effects and investor herding have a within bank as an interbank contagion component, as illustrated in diagram 2.

Contagion can arise through asset communality and interbank exposures as well as information contagion. For instance, the expectation that volatility increases in one bank changes the expectation of volatility in other banks, which is priced directly through the volatility in mean effect. Hence, the main difference between balance sheet contagion and information contagion is through the effect of expectations: the returns capture the movements of underlying assets directly while investor behaviour is captured by the expectation of uncertainty.
4.1 Reduced Form Specification

To estimate the model we first need to specify the model in its reduced form:

\[ B_0^{-1} \Phi_B(L) R_t = B_0^{-1} \varphi_0 + B_0^{-1} \Lambda g_t + B_0^{-1} \epsilon_t \] (2)

We know that \( h_t = vech(H_t) \) is the time varying conditional covariance matrix of the reduced form. We can rewrite this:

\[ \Phi(L) R_t = \theta_0 + \Pi g_t + u_t \] (3)

Where \( B_0^{-1} \varphi_0 = \theta_0 \), \( B_0^{-1} \Lambda = \Pi \), \( B_0^{-1} \Phi_B(L) = \Phi(L) \) and \( \epsilon_t = B_0 u_t \) as in Milunovich and Yang (2013) with the added conditional covariance terms in the mean equation. One can see that we need additional mapping from \( g_t \) to \( h_t \) at this stage as well: they need to be estimated and identified simultaneously with other structural parameters. In order to identify the parameters in the contemporaneous coefficient matrix \( B_0 \), the conditional covariance matrix of the reduced form has to have more elements than the structural form matrix. Therefore, \( H_t \) is specified as:

\[ u_t | F_{t-1} \sim Q(0, H_t) \]

\[ H_t = \psi \psi' + \rho H_{t-1} \rho' + \phi(I_k \otimes u_{t-1})(I_k \otimes u_{t-1})' \phi' \]

and \( \phi = diag(\phi_1, ..., \phi_k) \) (size \( k \times k \) and block diagonal with vector elements \( \phi_k \) ) to ensure the similarity to the BEKK (Bollerslev, Engle, Kraft and Kroner, 1993) specification.

To establish whether the model is indeed identified, we need to perform two exercises. The first exercise is to prove the identification of the parameters in the reduced form (3) as in Milonovich and Yang (2013). Thereafter we need to establish the mapping of the reduced form conditional covariance matrix \( H_t \) into the structural matrix \( G_t \). As Milunovich and Yang (2013) show that this method allows us to impose fewer restrictions on the contemporaneous coefficient matrix \( B_0 \), we can make use of this property in the definition of our structural form.
Proposition 1. The reduced form parameters $(\psi, \phi, \rho, \Pi)$ are identifiable if (1) $\psi$ is lower triangular with positive diagonals; (2) $\text{diag}(\phi_1, ..., \phi_k) = \phi$, where $\phi_i$ are $(1 \times k)$ vectors and the $(1 \times 1)$ element of $\phi$ is positive; (3) the $(1,1)$ element of $\rho$ is positive; and (4) $\Pi$ is invertible.

Proof: see appendix.

When proposition one holds, $H_t$ is of full rank and exists because of the stability in the parameters. Then, given that $H_t$ is of full rank, the parameters in $\Pi$ can be identified.

4.2 Structural Form Parameters

After proving that the $H_t$ matrix is indeed positive definite and the parameters are identified given the assumptions, we need to establish the mapping from the reduced form to the structural form parameters. Without GARCH effects we need sufficient restrictions for identification. Mihunovich and Yang (2013) prove with theorem 6 of Rothenberg (1971) that the inclusion of GARCH effects means that we need fewer restrictions on the contemporaneous coefficient matrix $B_0$ when it is jointly identified with the parameters in $\varepsilon_t$ and that the system is uniquely identified in this case.

This result holds globally when $\psi_t$ and $h_t$ are linear. Mihunovich and Yang (2013) use Theorem 6 of Rothenberg (1971) to show that the model is identified by looking at the Jacobian of the mapping to derive conditions for local identification for the non-linear system.

With the GARCH-in-mean specification, we need to establish additional mapping between the estimates of the conditional covariance matrices ($H_t$ to $G_t$), as $G_t$ depends on the structural form parameters.

$$u_t | F_{t-1} \sim N(0, H_t)$$

$$\varepsilon_t = B_0 u_t$$

$$\varepsilon_t | F_{t-1} \sim N(0, G_t)$$

The mapping is established as:

$$\varepsilon_t = R_t - \mu_t$$

$$\mu_t = \Phi_B(L) R_{t-1} - \psi_0 - \Lambda g_t$$

$$\varepsilon_t | F_{t-1} \sim N(0, B_0 H_t B_0')$$

$$\varepsilon_t | F_{t-1} \sim N(R_t - B_0 \Phi(L) R_{t-1} - B_0 \theta_0 - B_0 \Pi g_t, B_0 H_t B_0')$$
As \( g_t \) is the structural form conditional variance in VECH form, we need to establish additional mapping from \( h_t \) to \( g_t \) for the mean equation. This means that the mapping established in Milunovich and Yang (2013) still holds; however, we need additional mapping to form the \( G_t \) matrix in the structural form.

\[
H_t = \psi \psi' + \rho H_{t-1} \rho' + \phi(I_k \otimes u_{t-1})(I_k \otimes u_{t-1}')\phi'
= B_0^{-1}[\omega \omega' + a(I_k \otimes B_0)(I_k \otimes u_{t-1})(I_k \otimes u_{t-1}')a' + \beta G_{t-1} \beta']B_0^{-1}
\]

Establish the mapping into the mean equation:

\[
R_t = B_0 \Phi(L) R_{t-1} + B_0 \theta_0 + B_0 \Pi h_t + B_0 u_t
\]

Rewrite \( g_t = \text{vech}(G_t), \ h_t = \text{vech}(H_t) \). We need to rewrite the mean equation:

\[
\text{vech}(H_t) = (D_k^+ (B_0' \otimes B_0') D_k)^{-1} \text{vech}(G_t)
\]

where \( D_k = (k^2 x k(k+1)/2) \) is a duplication matrix with \( D_k^+ = (D_k' D_k)^{-1} D_k' \), and \( D_k^+ D_k = I_{k(k+1)/2} \).

The mapping from the reduced form to the structural form parameters is as in Milunovich and Yang (2013):

\[
\psi \psi' = B_0^{-1} \omega \omega' B_0^{-1'},
\phi = B_0^{-1} a(I_k \otimes B_0),
\rho = B_0^{-1} \beta B_0
\]

where there are \((k^2+2k)\) parameters in \((B_0, \omega, a, \beta)\) and \((k(k+1)/2+2k^2)\) parameters in \((\psi, \phi, \rho)\).

**Proposition 2.** Suppose that \((B_0, \omega, a, \beta)\) is a regular point, all elements of \( \omega \) are non-zero, and \( B_0 \) is invertible with unit diagonals. Then, \((B_0, \omega, a, \beta)\) is locally identifiable if the number of zeros in \((a_1, ..., a_k)\), the free parameters in \( \alpha \), is less or equal to 1.

Proof: see appendix.

The result only holds in a multivariate specification (see Bollerslev, Engle and Nelson 1994) as only then there is no observationally equivalent reduced form in the case that the constant and GARCH parameters are not a fixed constant (Milunovich and Yang, 2013). We find the estimates of \( B_0 \) by minimising the following likelihood function:

\[
L(B_0, G_t, \Pi) = -(Tk/2)\log(2\pi) - (T/2)\log|B_0^{-1} G_t B_0^{-1'}| - (1/2) \sum_{t=1}^{\epsilon_t} (B_0^{-1} G_t B_0^{-1'}) \epsilon_t
\]
4.3 Impulse Responses and Generalised Forecast Error Variance Decompositions

We can shock the system using idiosyncratic or common shocks. We can link the idiosyncratic shocks in $\varepsilon_t$ so that the shock will propagate to the system: $\varepsilon_t = B_0 u_t$, hence, when $B_0$ has nonzero off-diagonal elements banks will face exposure to common shocks or asset communality.

To illustrate the transmission mechanism, consider a system consisting of two banks where we face an asset side shock $\varepsilon_{2,t}$. Then, when the off-diagonal elements in $B_0$ are zero (idiosyncratic shock), we see an immediate rise in the return through the pricing effect as well as an increase in the volatility of bank 2 as investors change their expectations. The increase in the volatility changes the expectation on the volatility on bank 1 as well through information contagion, and increases the return of bank 1. In the next period, the previous period returns of both banks impact the current returns through the effect of the exposures. This continues until the shock has been fully absorbed by the system.

![Diagram 3: Transmission Channels of Shock on Bank 2](image)

The effect of $E(\varepsilon_{2,t})$ on $E(\sigma_{1,t+1})$ is zero as the shocks are diagonal. The effect of $R_{2,t}$ on $E(R_{1,t+1})$ is asset communality, the effect of $E(\sigma_{2,t+1})$ on $E(R_{1,t+1})$ is information contagion. Then, $R_{2,t}$ on $E(R_{2,t+1})$ is the asset price updating effect and $E(\sigma_{2,t+1})$ on $E(R_{2,t+1})$ is investor herding. With a diagonal $B_0$ matrix the effect of asset communality and interbank deposits is zero as only the bank specific return impacts the expected future return. When the $B_0$ matrix has nonzero off-diagonal elements, asset communality plays a role and contagion exists through both channels.

In order to pinpoint the degree of contagion, we follow the approach by Diebold and Yilmaz (2014) by computing the forecast error variance decompositions (FEVD) using the generalised variance decomposition (GVD) framework of Koop et al. (1996) and Pesaran and Shin (1998). The pairwise directional connectedness is equal to the normalised forecast error variance of the specific element:

$$C_{it \leftarrow j}^H = d_{ij}^H.$$
The spillovers of the return of institution $i$ to all other institutions, the transmitted return, is measured by:

$$C_{H_i} = \sum_{j=1, j \neq i}^k d_{ij}(H)$$

The spillovers of the return from other institutions to institution $i$, the received return, is measured similarly:

$$C_{H_i} = \sum_{j=1, j \neq i}^k d_{ij}(H)$$

Finally, the total spillover is defined as:

$$C_{H} = \sum_{i,j=1}^k d_{ij}(H)$$

We find the $H$-step ahead FEVD as:

$$d_{ij}^H = \frac{\sigma_{ij}^{-1} \sum_{h=0}^{H-1} (e_i'^{'} \Theta_h \bar{G} e_j) \times (e_i'^{'} \Theta_h \bar{G} e_i)}{\sum_{h=0}^{H-1} (e_i'^{'} \Theta_h \bar{G} e_i ^2)}$$

where $e_i$ is a $k \times 1$ selection vector with one at the $i^{th}$ and $k(i - 1) + i$ elements and zeros elsewhere, and $\Theta_h$ is the moving average coefficient of the model. The generalised variance decompositions have to be normalised as they do not sum to unity: hence, we weight them as in Diebold and Yilmaz (2014) by dividing each element in $d_{ij}$ by its row-summation. As our model has a time varying conditional covariance matrix, we need to rewrite it in the MA form as derived by Elder (2003):

$$E(R_{t+h}|F_{t-1}) = \sum_{h=0}^{H-1} B_0^{-1} \Phi_h (\psi_0 + \Lambda (g_{t+h-h}F_t)) + \sum_{h=t}^{\infty} \Phi_h (\psi_0 + \Lambda g_{t+h-h} + B_0^{-1} \varepsilon_{t+h-h})$$

By the law of iterated expectations:

$$E(g_{t+H-h}|F_t) = \sum_{s=0}^{H-h-2} ((A + B)^s \omega') + (A + B)^{H-h-1} g_{t+1}$$

Hence, the revision in the forecast of a shock $\varepsilon_{it}$ is:

$$\frac{\partial E(R_{j,t+h}|\varepsilon_{it},F_{t-1})}{\partial \varepsilon_{it}} = \sum_{h=0}^{H-1} B_0^{-1} (\Phi_h \Lambda (A + B)^{H-h-1} A) i_1 + (\Phi_H B_0^{-1}) i_0$$
Where \( i_1 \) and \( i_0 \) are \( k \times 1 \) and \( k \times 1 \) selection matrices with \( 2 \varepsilon_{i1} \) on the \( i^{th} \) and \( k(i-1)+i^{th} \) places, respectively. As the structural model is estimated with restrictions on the arch-parameters, the selection vector for the variance components of the MA version of the model is identical to the original vector. Matrices \( A \) and \( B \) are both of size \( k \times k \) in the structural form and \( G_t \) is a diagonal matrix; therefore, the coefficient matrix \( \Lambda \) contains \( k \times k \) elements. Hence, the coefficient term \( \Theta_h \) in the generalised forecast variance decomposition is:

\[
\Theta_h = B_0^{-1}\Phi^H(I_k + \Lambda((A + B)^{H-h-1}A))
\]

Thus, the variance decompositions will reflect both the contagion effects of asset communality, interbank deposits, and information herding. Using this tool set we are able to both identify the source of the transmission as well as the relative degree of contagion, where contagion from institution \( i \) to \( j \) is not equal to the analogue in the opposite case. Even in the absence of common shocks, the information transmission mechanism is still in place which ensures that we can distinguish and identify balance sheet contagion separately from the information contagion component.

5 Data Description

All data needed is publicly available on Bloomberg and consists of daily price series of individual banks from the 4th of January 2005 until August 2013.

The G-SIFIs are identified by the Financial Stability Board (FSB) on an annual basis. The current list of SIFIs was published in November 2012 and the ranking is based on the assessment methodology provided by the Basel Committee on Banking Supervision (BIS, 2013). In order to be characterised as a G-SIFI, banks are ranked according to five indicators which are weighted into a single factor: cross-jurisdictional activity, size, interconnectedness, substitutability and complexity. In this case, interconnectedness among banks is measured by the exposures on their balance sheet and their interbank assets rather than measures of contagion. Therefore, it is likely that institutions identified as G-SIFIs, which have a great amount of balance sheet interconnectedness, also exhibit substantial information contagion.

The Eurozone SIFIs are Deutsche Bank (DBK), BNP Paribas (BNP), Credit Agricole (ACA), ING Bank (INGA), Santander (SAN), Societe Generale (GLE), BBVA Group (BBVA) and Unicredit Group (UCG). The market capitalisations of the Eurozone G-SIFIs (in millions of Euros) ranges between 20,683.61 (Credit Agricole) and 67,318.02 (Santander). To put these numbers in perspective: the largest bank (also G-SIFI) in the European area, HSBC Group, has a market capitalisation of 125,432.45 million pounds.

Figure 1 shows the evolution of the bank specific stock prices over the sample period. The banks have experienced one major drawdown during the sample period from September 2007 until January 2009.
This in line with the period of the subprime crisis and shows that the Eurozone market was indeed affected by these world market events.

Table 1 presents the summary statistics of the continuously compounded returns with average returns around zero. We observe that especially ING has a significant variance and exhibits far from normal returns. All banks show a distribution with fat tails.

<table>
<thead>
<tr>
<th></th>
<th>BNP</th>
<th>ACA</th>
<th>ING</th>
<th>GLE</th>
<th>UCG</th>
<th>BBVA</th>
<th>SAN</th>
<th>DBK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
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<td>-0.019</td>
<td>-0.015</td>
<td>-0.014</td>
<td>-0.033</td>
<td>-0.011</td>
<td>-0.009</td>
<td>-0.012</td>
</tr>
<tr>
<td>Variance</td>
<td>1.458</td>
<td>1.701</td>
<td>2.220</td>
<td>1.815</td>
<td>1.803</td>
<td>0.964</td>
<td>0.993</td>
<td>1.353</td>
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<tr>
<td>Skewness</td>
<td>0.353</td>
<td>0.319</td>
<td>-0.016</td>
<td>0.079</td>
<td>-0.129</td>
<td>0.383</td>
<td>0.367</td>
<td>0.305</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.567</td>
<td>5.440</td>
<td>12.891</td>
<td>5.868</td>
<td>5.578</td>
<td>6.296</td>
<td>7.309</td>
<td>8.975</td>
</tr>
</tbody>
</table>

Table 1: Summary Statistics for Bank Returns

We have an estimation window of 2175 days, which encompasses the period of January 2005 until September 2013. This enables us to analyse the transmission mechanism of SIFIs before, during and after the financial crisis. The estimates of the ARCH and GARCH parameters ensure that the conditional covariance matrix is positive definite and invertible. First we present the coefficient estimates of the reduced form model, which gives an indication of the significance and magnitude of the interbank deposit channel and information contagion respectively. In the next section we present the estimates of the structural $B_0$ matrix and the presence of asset communality shocks. Network subsets are considered in the followed section as well as time variation in connectedness estimates.
6 Results

The estimates of the one lag VAR returns and volatility for the Eurozone SIFIs are presented in tables 2 and 3. We observe that the lagged returns have a significant impact in a few cases, most notable BNP Paribas on the other French banks and Unicredit (UCG). Interestingly, the effect of the previous period return of BNP on the identified banks is negative: an increase in the return of BNP decreases the next period return, but it does not affect the current return of BNP directly. This result suggests that the direct transmission mechanism, the price updating and interbank channel of contagion, have a significant impact on the equilibrium returns and validates the focus on the interbank deposit channel of contagion.

<table>
<thead>
<tr>
<th></th>
<th>BNP</th>
<th>ACA</th>
<th>GLE</th>
<th>UCG</th>
<th>BBVA</th>
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<th>DBK</th>
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<tbody>
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<td>0.02</td>
<td>-0.02</td>
<td>0.12</td>
<td>-0.06</td>
<td>0.09</td>
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</tr>
<tr>
<td>ACA</td>
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<td>-0.03</td>
<td>0.02</td>
<td>-0.04</td>
<td>0.16</td>
<td>-0.13</td>
<td>0.17**</td>
<td>0.03</td>
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<tr>
<td>GLE</td>
<td>-0.16*</td>
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<td>0.04</td>
</tr>
<tr>
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<td>-0.03</td>
<td>0.09</td>
<td>0.06</td>
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<tr>
<td>BBVA</td>
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<td>-0.01</td>
<td>-0.02</td>
<td>-0.02</td>
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<td>-0.04</td>
<td>0.10</td>
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<tr>
<td>SAN</td>
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<td>-0.01</td>
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<td>-0.07</td>
<td>0.09</td>
<td>-0.01</td>
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<tr>
<td>DBK</td>
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<td>0.05</td>
<td>0.02</td>
<td>-0.04</td>
<td>-0.01</td>
<td>0.03</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>ING</td>
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<td>0.03</td>
<td>-0.02</td>
<td>0.11</td>
<td>-0.05</td>
<td>0.14</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

Table 2: Eurozone Banks: Mean Equation Lagged Return Phi Estimates

When we turn to the the estimates of the volatility in mean parameters lambda (presented in table 3), we observe that there is evidence of information contagion among a few banks in the SIFI groups. We see that the return of ING is negatively affected by changes in the volatility of Societe Generale (GLE). The returns of Groupe Agricole (ACA) are negatively affected by their own volatility. Possible differences in signs suggest that investors update their portfolio choice depending on the perceived relationship between the banks. For instance, an increase in the uncertainty on the stock of Societe Generale causes investors to perceive the stock of ING more negatively, while the opposite is true for BNP Paribas by changes in the uncertainty of Deutsche Bank (although this effect is not significant). This result can be explained by diversification across countries by investors. Hence, the sign of information contagion is dependent on the market link. Checking for the impact of excess rather than raw returns only emphasises the importance of the interbank channel.

We can conclude from the reduced form estimates that the interbank channel is the main form of connectedness between banks in the Eurozone system. Moreover, the result does not hold for all banks within the system: particularly the French banks are connected to each other, while the connectedness of other institutions
Table 3: Eurozone Banks: Mean Equation Volatility Pi Estimates

<table>
<thead>
<tr>
<th></th>
<th>BNP</th>
<th>ACA</th>
<th>GLE</th>
<th>UCG</th>
<th>BBVA</th>
<th>SAN</th>
<th>DBK</th>
<th>ING</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP</td>
<td>-21.69</td>
<td>-16.11</td>
<td>-6.22</td>
<td>-1.50</td>
<td>-68.32</td>
<td>-63.56</td>
<td>20.98</td>
<td>-1.35</td>
</tr>
<tr>
<td>ACA</td>
<td>-2.81</td>
<td>-16.38*</td>
<td>-8.40</td>
<td>0.58</td>
<td>-50.75</td>
<td>-55.66</td>
<td>5.75</td>
<td>-3.87</td>
</tr>
<tr>
<td>GLE</td>
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<td>-9.71</td>
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<td>-68.89</td>
<td>7.80</td>
<td>-0.91</td>
</tr>
<tr>
<td>UCG</td>
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<td>-70.04</td>
<td>-5.72</td>
<td>-7.68</td>
<td>-5.93</td>
</tr>
<tr>
<td>BBVA</td>
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<td>-2.57</td>
<td>-63.17</td>
<td>-47.81</td>
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<td>-5.61</td>
</tr>
<tr>
<td>SAN</td>
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<td>0.08</td>
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<td>-41.91</td>
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</tr>
<tr>
<td>DBK</td>
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<td>-24.16</td>
<td>-6.47</td>
<td>-6.29</td>
</tr>
<tr>
<td>ING</td>
<td>-5.43</td>
<td>-16.11</td>
<td>-8.27*</td>
<td>-2.28</td>
<td>-103.75</td>
<td>-73.71</td>
<td>5.13</td>
<td>-3.93</td>
</tr>
</tbody>
</table>

The main implication to our framework is that we cannot disentangle the contribution of balance sheet components to total connectedness: as the interbank deposits significantly affect the returns, asset communality as captured by the structural $B_0$ matrix can only be identified as the difference between the connectivity captured by both reduced form transmission channels. This implies that we can identify the effect of asset communality on total connectedness with respect to the combined impact of information contagion and interbank deposits, but not from each channel separately.

6.1 Structural Form and Spillovers

We further estimate the structural coefficient matrices by maximising the auxiliary likelihood function. It is clear that the $B_0$ matrix does not have nonzero off-diagonal elements, which means that some shocks are common and correlated. The transmission mechanism is not only dependent on whether shocks have a correlated structure, but also on the estimated parameters. We can illustrate the mechanism by rewriting the model in its MA form and analysing the generalised FEVDs.

The connectedness table for the Eurozone banks with the full $B_0$ matrix is presented below. The entries in the table show the percentage in variance resulting from a shock from a particular institution (‘pairwise connectedness’), as well as the variance due to own shocks. Total connectedness and the ‘from’, ‘to’ and ‘net’ categories explain the total variance due to shocks from other institutions, variance of shocks due to a shock from one particular institution, total shocks received from other institutions, and the difference between the ‘from’ and ‘to’ categories, respectively. A positive value for net connectedness identifies a net transmitter of shocks, while negative values identify a net recipient. The spread in the ‘net’ category is an indicator of the degree of risk sharing in the sector: whenever we see a decrease in
the spread under communality versus orthogonality, common exposures have risk sharing benefits as the network becomes more homogeneous.

Table 5 presents the Eurozone connectedness table with the full $B_0$ matrix. We find that the contagion in returns, measured by the total spillover, is higher than in the case of volatility spillovers in the US institutions (Diebold and Yilmaz, 2014). We see that the pairwise connectedness measures have a magnitude similar to own connectedness (diagonal elements). The spread in the 'to' category is higher than the spread in the 'from' category, in line with the result in Diebold and Yilmaz (2014). We see that BNP Paribas, Groupe Agricole and Unicredit are great sources of transmission in the system, but they are also great recipients of shocks. The net impact tells us whether the particular institution is mostly a transmitter or a recipient of shocks. GLE and UCG are notable net transmitters, while BNP, BBVA and Deutsche Bank (DBK) are notable recipients.

When we focus on idiosyncratic shocks by forcing the $B_0$ matrix to be orthogonal (thereby switching the asset communality channel off), the connectivity changes. Table 6 shows the net results in both cases. We see that total connectivity decreases, however, we see a decrease in the spread of net spillovers in the orthogonal case: this suggests that the system is less homogenous when common shocks are taken into account, and we see a dispersion in the network. The implications of this are clear: when common shocks are taken into account, some institutions become much more important in their transmission than others and cause significantly more instability in the network when they are faced with negative shocks. This result shows that risk sharing through common assets is not strong on the Eurozone level as of the asymmetry in common asset holdings.

Another observation is that some institutions change their status of a net transmitter to a net recipient, and vice versa. A bank that transmits more when we take common shocks into account has assets that most of the system holds in common. The opposite holds too: when a bank receives more shocks with communality but turns into a transmitter without it, it holds assets in common with the system while by itself it is not very communality-systemic important.

Let us look more closely at the result for GLE. From table 5 we see that GLE does not transmit many shocks compared to other institutions, but receives fewer shocks from others. When we compare the results for orthogonality and communality, we observe that GLE becomes a net transmitter when common shocks are taken into account, giving it the status of a communality-systemic important bank. We obtain a similar result for UCG: most institutions in the network hold assets in common with this institution. The opposite holds for BNP: under communality, BNP is a net recipient while it is a net transmitter when shocks are orthogonal. The connectivity of BNP is different from GLE and UCG: BNP holds assets in common with the system and is more affected by systemic shocks, and hence is not a communality-systemic important institution.
Table 4: Inverse of Contemporaneous Coefficient Matrix $B_0$ of Eurozone SIFIs

<table>
<thead>
<tr>
<th></th>
<th>BNP</th>
<th>ACA</th>
<th>GLE</th>
<th>UCG</th>
<th>BBVA</th>
<th>SAN</th>
<th>DBK</th>
<th>ING</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP</td>
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<td>0.966</td>
<td>0.967</td>
<td>1.593</td>
<td>-12.872</td>
<td>-2.687</td>
<td>1.634</td>
<td>4.219</td>
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<tr>
<td>ACA</td>
<td>-7.517</td>
<td>0.533</td>
<td>0.534</td>
<td>0.879</td>
<td>0.920</td>
<td>-1.483</td>
<td>0.902</td>
<td>2.329</td>
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<tr>
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<td>0.925</td>
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<td>1.593</td>
<td>-2.568</td>
<td>1.562</td>
<td>4.032</td>
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<tr>
<td>UCG</td>
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<td>SAN</td>
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<td>2.687</td>
<td>19.111</td>
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<td>0.949</td>
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<td>1.634</td>
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</table>

Table 5: Eurozone SIFI Connectedness Table

<table>
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<th>UCG</th>
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<td>BNP</td>
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<td>11.27</td>
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<td>99.99</td>
<td>90.16</td>
<td>90.52</td>
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Table 6: Net Connectedness With Communality (top) and Orthogonal Shocks

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<th>SAN</th>
<th>DBK</th>
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<td>87.0</td>
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<tr>
<td>Received</td>
<td>100.0</td>
<td>99.9</td>
<td>64.1</td>
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<td>94.6</td>
<td>99.9</td>
<td>90.2</td>
<td></td>
</tr>
<tr>
<td>Net</td>
<td>-4.2</td>
<td>-4.5</td>
<td>22.9</td>
<td>13.6</td>
<td>-9.9</td>
<td>-7.9</td>
<td>-11.7</td>
<td>3.0</td>
<td>90.52</td>
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<table>
<thead>
<tr>
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<th>GLE</th>
<th>UCG</th>
<th>BBVA</th>
<th>SAN</th>
<th>DBK</th>
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<tr>
<td>Transmitted</td>
<td>89.3</td>
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<td>87.0</td>
<td>84.8</td>
<td>87.8</td>
<td>90.0</td>
<td>85.2</td>
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<tr>
<td>Received</td>
<td>76.2</td>
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<td>87.7</td>
<td>90.9</td>
<td>89.3</td>
<td>92.</td>
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<td>-2.2</td>
<td>-9.4</td>
<td>3.9</td>
<td>89.75</td>
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</table>
For policy implications we need to consider the net status of either a transmitter or recipient of shocks, as well as the impact of asset communality separately to evaluate the complete systemic importance of a particular institution. Also, the change in the spread of the net transmittor status over the two cases is indicative of the benefits of risk sharing in terms of system stability.

In the Eurozone case, transmission of shocks through interbank deposits is significant. Hence, diagonalising $B_0$ means that we focus on the interbank market and not on asset communality. We see from our results that asset communality is indeed present, and changes the spread of the net transmitter status. When we evaluate the systemic impact of an institution we should not only analyse their net connectivity but also their relative affectedness to asset communality, as it shows us whether the connectivity is caused by holding assets similar to the other institutions or that it caused by other institutions holding similar assets to the bank. The implications are clear: in the Eurozone network as it is currently defined, exposure to common shocks leads to an increase in the spread of net transmitter status and hence an increase in system instability in contrast to the expected benefits of risk sharing.

6.2 Network Subset: Country Level Transmission

The results for Eurozone connectedness suggest that there are differences in the relative connectedness between institutions based on the country linkages. To illustrate the effect among French banks, we zoom in on the select subsample of SIFIs containing only the French banks over the sample period. In table 4 we see that ACA and BNP negatively affect each other through the lagged returns, albeit small: changes in the returns of ACA affect the future return of BNP, which suggests that underlying movements of the bilateral interbank deposits ACA and BNP affect the pricing of the next period return of the respective banks. For instance, if ACA were to face a illiquidity shock they might decide to remove their interbank deposits at BNP to fill their funding gap. As the interbank deposit is removed at the current period, BNP will face a propagated funding shock from ACA which is priced in their future return. A similar mechanism is present for asset-based shocks. Notice that the volatility in mean parameters are not significant for most of the banks: this is in consistent with the result for the Eurozone, where the main effect between the French banks was through interbank deposit linkages.

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>$\Phi$</th>
<th>$\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP</td>
<td>0.0001</td>
<td>-0.111***</td>
<td>-0.050*</td>
</tr>
<tr>
<td>ACA</td>
<td>-0.0003</td>
<td>-0.120***</td>
<td>-0.078</td>
</tr>
<tr>
<td>GLE</td>
<td>0.0001</td>
<td>0.035</td>
<td>-0.068**</td>
</tr>
</tbody>
</table>

Table 7: Mean Equation Estimates of French Banks
The comparison between the analyses for the Eurozone banks and the French banks illustrates that interbank exposures dominate in both cases, while information contagion is insignificant for most connections. The sign of the exposure depends on the real connection between institutions: when investors can diversify across markets, uncertainty of one bank positively affects the return of a bank outside this market while it negatively affects a bank within this market through the effect of market specific conditions. Consider for instance a negative shock to the French economy: French banks will be directly affected and their returns will reflect the movement in their balance sheet items, while on the Eurozone level investors will reallocate their holdings to non-French banks, decreasing the return on French banks but increasing the return of non-French banks. Similar results are obtained for the Spanish system and the Dutch-German case.

From the reduced form estimates we can see that interbank deposits are the significant transmission channel while information contagion only plays a minor role. This has implications for our analysis as we cannot distinguish the impact of common shocks from interbank deposit effects in case we allow for asset communality. We observe from table 8 that the total connectedness of the system is lower than in the Eurozone case. In terms of the pairwise results, BNP affects ACA and GLE with the greatest impact. It is clear from the net table that BNP is a net transmittor, while GLE and ACA are net recipients of shocks. When we consider a state without asset communality, hence, an orthogonalised \( B_0 \) matrix, ACA and GLE show a stronger net recipient role while BNP is even more a net transmitter. When we consider asset communality effects, it is clear that the other banks have a high degree of communality with BNP while BNP is not as much affected by the communality with the other banks. This is in accordance with the results for the parameter estimates, where we find that the interbank deposit channel has the greatest impact in the French system. The behaviour of investors only emphasises the impact.

<table>
<thead>
<tr>
<th></th>
<th>BNP</th>
<th>ACA</th>
<th>GLE</th>
<th>FROM</th>
<th>BNP</th>
<th>ACA</th>
<th>GLE</th>
<th>FROM</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP</td>
<td>21.38</td>
<td>44.95</td>
<td>33.66</td>
<td>82.94</td>
<td>59.18</td>
<td>16.31</td>
<td>24.51</td>
<td>66.80</td>
</tr>
<tr>
<td>ACA</td>
<td>54.33</td>
<td>25.79</td>
<td>19.88</td>
<td>73.75</td>
<td>62.44</td>
<td>12.90</td>
<td>24.67</td>
<td>70.47</td>
</tr>
<tr>
<td>GLE</td>
<td>49.60</td>
<td>27.51</td>
<td>22.89</td>
<td>70.05</td>
<td>56.62</td>
<td>14.46</td>
<td>28.92</td>
<td>62.97</td>
</tr>
<tr>
<td>TO</td>
<td>78.62</td>
<td>74.21</td>
<td>77.11</td>
<td>76.64</td>
<td>40.82</td>
<td>87.10</td>
<td>71.08</td>
<td>66.33</td>
</tr>
<tr>
<td>NET</td>
<td>4.32</td>
<td>-0.46</td>
<td>-7.06</td>
<td>-7.06</td>
<td>25.98</td>
<td>-16.64</td>
<td>-8.11</td>
<td>-8.11</td>
</tr>
</tbody>
</table>

Table 8: French SIFI Connectedness Table, \( B_0 \) Full (left) and \( B_0 \) Diagonal (right)

The result that the net impact of institutions is exacerbated when asset communality is not considered suggests that there are substantial benefits in diversification strategies. For instance, while BNP can be considered as a communality-systemic
important institutions as the system holds assets in common with the bank and its status as a net transmitter, the other banks receive less shocks in a system with asset communality. Hence, even though BNP transmits significantly more, diversification efforts counter the receipts of shocks: the net effect is favoured over a situation without any common aspects. This is in contrast with the result obtained in the Eurozone case.

6.3 Network Subsets and Stability

We need to check whether results are robust to the specification of the network. As in the money centre model discussed in the theoretical framework, connectedness should not be affected by banks that fall outside the main scope of the network. Therefore, we expect that banks that share a common market would have an impact on the relative connectivity in the system while those that fall outside this scope will not affect the estimates significantly. In the following analysis we estimate the transmission mechanism for French banks with a Spanish bank (Santander) and a British bank (Royal Bank of Scotland), where we expect that Santander has a significant impact on system connectivity as of the Eurozone connection while we expect the opposite for the latter case. The following tables illustrate the impact for the French-Spanish case and the French-UK case.

<table>
<thead>
<tr>
<th></th>
<th>Φ</th>
<th>Λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP</td>
<td>-0.091</td>
<td>0.003</td>
</tr>
<tr>
<td>ACA</td>
<td>-0.084</td>
<td>0.001</td>
</tr>
<tr>
<td>GLE</td>
<td>-0.077</td>
<td>-0.030</td>
</tr>
<tr>
<td>SAN</td>
<td>-0.054**</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Table 9: French System with Spanish Bank Reduced Form Estimates

We see that the transmission from BNP to Santander is significant on the interbank channel, and of the same sign as in the French system. Surprisingly, Societe Generale is the main transmitter in both the interbank market as well as in terms of information contagion: it positively affects all French banks on the interbank market, while it causes negative information effects on these banks. Whenever there is a positive liquidity shock at GLE, other banks will be directly and positively affected. Investors however see an increase in volatility in GLE as a sign that the other banks are also affected by the increase in underlying uncertainty. The results for the structural form confirm the prevalence of GLE in the system, and the affectedness of Santander by the French banks.

2We convert British stock prices to their Euro counterparts and calculate the return from there. Hence, there is some degree of exchange rate risk involved.
When we turn to the estimates for the French banks with the British bank RBS, we see that there is no connectivity in the transmission between the French banks and RBS. However, the transmission of the French banks among each other is also diminished and insignificant, illustrating the need for a correct definition of the main network. In our case, focusing on the French banks ignores the full network that encompasses the subsample.

<table>
<thead>
<tr>
<th></th>
<th>Φ</th>
<th></th>
<th>Λ</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP</td>
<td>-0.064</td>
<td>-0.030</td>
<td>0.079</td>
<td>0.003</td>
</tr>
<tr>
<td>ACA</td>
<td>-0.066</td>
<td>-0.041</td>
<td>0.085</td>
<td>0.023</td>
</tr>
<tr>
<td>GLE</td>
<td>-0.067</td>
<td>-0.074</td>
<td>0.130</td>
<td>0.029</td>
</tr>
<tr>
<td>RBS</td>
<td>-0.033</td>
<td>-0.002</td>
<td>-0.007</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Table 10: French System with British Bank Reduced Form Estimates

We zoom in on the connectivity through the FEVDs. We observe that RBS hardly contributes to the variance of the French banks, which makes it a net recipient of shocks when we include it in the system. The pairwise connectivity with the French banks is almost negligible compared to the connectivity among the French banks, particularly BNP as before. When we consider a state without asset communality, we see that the impact of RBS becomes more pronounced even though there is no change in total connectivity: in turn, the net recipient status diminishes. When we consider the French banks, we see that BNP turns into a net recipient while GLE turns into a net transmitter when we do not take asset communality into account. This suggests that the system holds relatively many assets in common with BNP while GLE is more dependent on the system, identifying BNP again as a communality-systemic important institution. While diversification has a stabilising effect on a country level, the same does not hold when we consider institutions outside of the defined network.

<table>
<thead>
<tr>
<th></th>
<th>BNP</th>
<th>ACA</th>
<th>GLE</th>
<th>RBS</th>
<th>FROM</th>
<th>BNP</th>
<th>ACA</th>
<th>GLE</th>
<th>RBS</th>
<th>FROM</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP</td>
<td>29.9</td>
<td>42.9</td>
<td>26.8</td>
<td>0.3</td>
<td>82.5</td>
<td>14.4</td>
<td>28.0</td>
<td>37.9</td>
<td>19.7</td>
<td>76.3</td>
</tr>
<tr>
<td>ACA</td>
<td>69.2</td>
<td>18.5</td>
<td>12.1</td>
<td>0.2</td>
<td>86.9</td>
<td>19.3</td>
<td>24.7</td>
<td>33.2</td>
<td>22.9</td>
<td>78.0</td>
</tr>
<tr>
<td>GLE</td>
<td>44.3</td>
<td>36.6</td>
<td>18.9</td>
<td>0.3</td>
<td>78.1</td>
<td>15.9</td>
<td>28.3</td>
<td>34.9</td>
<td>20.9</td>
<td>74.4</td>
</tr>
<tr>
<td>RBS</td>
<td>28.0</td>
<td>43.3</td>
<td>28.4</td>
<td>0.3</td>
<td>72.79</td>
<td>11.3</td>
<td>31.4</td>
<td>30.3</td>
<td>26.9</td>
<td>70.2</td>
</tr>
<tr>
<td>TO</td>
<td>70.0</td>
<td>81.5</td>
<td>81.1</td>
<td>99.7</td>
<td>83.1</td>
<td>85.6</td>
<td>75.3</td>
<td>65.0</td>
<td>73.0</td>
<td>83.1</td>
</tr>
<tr>
<td>NET</td>
<td>12.5</td>
<td>5.4</td>
<td>-3.0</td>
<td>-26.9</td>
<td>85.6</td>
<td>75.3</td>
<td>65.0</td>
<td>73.0</td>
<td>83.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: French Connectedness Table With RBS, B₀ Full and B₀ Diagonal (right)
6.4 Transmission and Connectedness During the Financial Crisis

So far, we have considered the impact of a possible liquidity shock on the connectivity of institutions in a static fashion. Particularly, we are interested in whether there was an increase in the connectivity of institutions during the financial crisis. Therefore, we perform our analysis for French SIFIs on a rolling window of 500 days to get four estimation results: before (5-01-2005 until 27-12-2006), during (28-12-2006 until 17-12-2008), recovery (18-12-2008 until 9-12-2009), and after the crisis (10-12-2009 until 22-01-2013). The results for the reduced form and the structural form are presented in figure 2 and 3.

Figure 2: Information and Interbank Deposit Transmission for French SIFIs

The left axis belongs to interbank estimates, while the right axis belongs to information contagion. We observe that there is a change in the relationship at the crisis period: the interbank channel estimates increase in magnitude for GLE, while the effect through information contagion differs across the three institutions. Particularly information contagion increased for GLE and ACA during the financial crisis, and only declined after the recovery period. Societe Generale sees a spike in deposit transmission after the recovery. In terms of relative magnitude, information contagion has a greater effect in the recovery time. This follows our initial results that (liquidity) shocks are transmitted more heavily through the interbank market in crises, and follows our intuition that information contagion is exacerbated.

Turning to the results for net connectivity, we see that both GLE and BNP see an inversion in net connectedness at the crisis: in the two bank system, BNP turns from a net recipient into a net transmitter of shocks while the opposite happens at GLE. Interestingly, we do not see similar movements for the orthogonal system which is much more stable over the sample. It diverges when communality is taken into account, suggesting that diversification efforts stabilise the system in times of crisis and show a movement of the underlying asset holdings of the institutions.

Figure 4 shows the evolution of total connectedness over the sample. Total connectedness stayed relatively stable with asset communality, but we see convergence of the orthogonal model towards the communality case, particulary during
the recovery period. This suggests that communality became a larger element of connectivity in the financial crisis compared to the other transmission channels, which is in line with previous findings.

In all reduced form cases, transmission through interbank deposits is the main mechanism while information contagion plays a minor role. We see that asset communality is a major source of transmission, highlighting both the benefits and drawbacks of risk sharing. We see that communality leads to more system instability in the Eurozone case while it shows significant risk sharing benefits in the French (country) case. The impact of asset communality is only enhanced in the financial crisis, showing a great increase compared to a state without common shocks. In terms of policy this implies that combatting transmission through diminishing connectedness based on asset communality should not be the main objective; monolining institutions would only hinder further risk sharing and policy should focus instead on diminishing concentration and the prevalence of money centers. To consider the full impact of an institution we need to consider whether they are a
communality-systemic important institutions (e.g., net transmitter status increases when communality is taken into account) as policy differs for these institutions. Also, promoting transparency is not limited to diminishing transmission through information contagion but also helps establish trust on the interbank market, decreasing the impact of negative liquidity shocks.

7 Concluding Remarks

We have shown that we can identify multiple transmission channels using a SVAR-MGARCH-M model for a small system of banks. We prove that the identification property of VAR-GARCH still holds in a multivariate GARCH-in-mean specification. This structure is used to estimate the structural coefficient matrix and to find the forecast error variance decompositions (FEVD’s) as in Diebold and Yilmaz (2014). Our paper adds to this work by disentangling the impact of asset communality, interbank deposits, and information contagion by using the unique property of the structural form.

We use market data in evaluating the channels of contagion. The advantage of market-based measures is that they directly incorporate differences in valuation and are observed frequently. One major advantage of this method is that we can directly infer the effect of bank or market specific shocks on other institutions, which has value in the measurement and assessment of systemic risk in a sector.

We find that the dominating transmission mechanism in the reduced form is interbank deposit linkages, while information contagion plays a minor role. Robustness checks with network subsets and stability analysis reach the same conclusion. Investors use the information on the uncertainty of stocks to update their portfolio accordingly. An estimation over a rolling window highlights the importance of the interbank channel during the financial crisis. The FEVD’s confirm our findings: when we consider a state of the world without asset communality, (when $B_0$ is diagonal), we can see a shift in the underlying connectivity of particular institutions. Those institutions that see an increase in their positive net transmission are identified as communality-systemic important. Notable is the reversal in connectivity during the financial crisis for some institutions, as well as the increased importance of common shocks: the presence of asset communality highlights both the benefits as the drawbacks of risk sharing.

For policy implications we need to identify communality-systemic important institutions, as well as assess the impact of asset communality separately to evaluate the complete systemic importance of a particular institution. Our results show that asset communality is a significant factor in the Eurozone area, however, common shocks lead to an increase in system instability as some banks become strong transmitters and increase the dispersion in spillover effects. This is contrast to the country case, where there are substantial risk sharing benefits. The impact of common shocks on system stability in the Eurozone do not warrant monolining
policies as it is likely that risk sharing benefits are only to accrue with an increase in competition in the sector.

Rather, policy should target concentration and transparency in the sector: policies fostering transparency not only diminish information contagion, but also increase interbank trust. The focus on the interbank deposit channel is warranted on the area level, particularly with the expected increase in connectedness with the implementation of SEPA.

The econometric model we have employed can be applied on other datasets, although with a greater number of institutions it might benefit from being modelled using a DCC-GARCH specification for computational feasibility. The benefit of market based measures is that they reflect market perception; even if the market may not be correct, the measures react according their perception.

This paper provides an alternative way to look at interbank contagion. Future research on the marriage of network theory with multivariate structural time series models is promising, and could deepen our understanding of the different elements of asset communality (which is far more complex in reality than the current paper assumes). An extension of our model would analyse the impact of earnings announcements on the model estimates, as this information will be captured by investor behaviour and could lead to a significant change in volatility and connectivity. An extension of the model to systemic risk measurement and stress testing will greatly contribute to our understanding of interbank relations and the impact on systemic risk.

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nal of Forecasting, no. 28(1), pp. 57-66.


A1. Reduced Form Identification

**Proposition 1.** The reduced form parameters \((\psi, \phi, \rho, \Pi)\) are identifiable if (1) \(\psi\) is lower triangular with positive diagonals; (2) \(\text{diag}(\phi_1, ..., \phi_k) = \phi\), where \(\phi_i\) are \((1 \times k)\) vectors and the \((1 \times 1)\) element of \(\phi\) is positive; (3) the \((1,1)\) element of \(\rho\) is positive; and (4) \(\Pi\) is invertible.
Proof. 1. Show that if the diagonal elements in $\psi$ are positive and that the matrix is lower triangular, $\psi$ is defined.

When the diagonal elements of matrix $\psi$ are positive, we know that $\psi\psi'$ is a positive definite matrix.

Dhrymes proposition 58, remark 34 states that a decomposition of positive definite matrix into the product of a triangular matrix and its transpose always exists and this decomposition is unique if the diagonal elements are restricted to be positive, Then, $\psi$ is the unique Cholesky factor of $\psi\psi'$.

2. Show that $\phi(1, 1) > 0$ is sufficient to identify the $\phi$ matrix.

Ignore the constant and GARCH terms; the (1,1) element of $H_t$ is:

$$h_{11,t} = \phi_1 u_{t-1} u_{t-1}' \phi_1'$$

So the coefficient attached to $u_{1,t-1}^2$ is $\phi_{11}^2$, which means that $\phi_{11}$ is identified up to its sign. Restricting it to be positive identifies this term.

Next, the coefficient attached to $u_{1,t-1} u_{j,t-1}$ is $(\phi_{11} \phi_{j1} + \phi_{j1} \phi_{11}) = 2\phi_{11} \phi_{j1}$. So, $\phi_{j1}$ is identified for all $j$ because $\phi_{11}$ is identified. Hence, the first column of $\phi$, $\phi_1$, is identified.

$$H_{12,t} = \phi_1 u_{t-1} u_{t-1}' \phi_2' = 0$$

as $\phi = diag(\phi_1, ..., \phi_k)$ and $\phi_i$ are $1 \times l$ vectors. Therefore, $\phi_1 u_{t-1} u_{t-1}' \phi_1'$ exists but $\phi_1 u_{t-1} u_{t-1}' \phi_2'$ is zero. Hence it is identified per definition. This can be continued for the other elements in $\phi$.

3. Show that if and only if $\rho(1, 1) > 0$, $\rho$ is identified.

Ignore the constant and ARCH terms:

$$H_t = \rho H_{t-1} \rho'$$

with $\rho = diag(\rho_1, ..., \rho_k)$. So the coefficient attached to $H_{11}$ is $\rho_{11}^2$:

$$H_{11,t} = \rho_1 H_{11,t-1} \rho_1'$$
which means it is again identified up to its sign, which is restricted to be positive.

Again, the next coefficient to $H_{t-1}$ is $\rho_1 H_{1,t-1} \rho_j = 0$ as $H_{1,t-1}$ is zero as it exhibits zero off diagonal elements. Hence the parameters can be identified for each $H_{st}$.

4. At last, we need to show that $\Pi$ is an invertible and unique matrix.

We have established from (1) to (3) that $H_t$ is of full rank and positive definite when $\psi$ is of full rank.

Now, we need to show that $\Pi$ is unique and $vech(H_t)$ is invertible.

Ignoring lag terms and errors:

$$R_t = \Pi h_t$$

$$h_t = vech(H_t)$$

$$R_t = \begin{pmatrix}
\lambda_{11} & 0 & 0 & \lambda_{12} & 0 & \lambda_{1k} \\
\lambda_{21} & 0 & 0 & \lambda_{22} & 0 & \lambda_{2k} \\
\lambda_{k1} & 0 & 0 & \lambda_{k2} & 0 & \lambda_{kk}
\end{pmatrix}
\begin{pmatrix}
h_{11t} \\
h_{12t} \\
h_{1kt} \\
h_{22t} \\
h_{2kt} \\
h_{kkt}
\end{pmatrix}$$

As $H_t$ is positive definite and has a positive first element, the matrix is invertible.

Per equation the returns are defined as follows:

$$R_{it} = \lambda_{ij} h_{it}$$

We can uniquely identify $\lambda_{ij}$: from the GARCH dynamics we can estimate:

$$\lambda_{ij} = \text{cov}(h_{it}, R_{it}) / \text{var}(h_{it})$$

As each parameter depends on a different covariance, $\Pi$ is defined.
A2. Structural Form and Mapping of Parameters

After proving that the $H_t$ matrix is indeed positive definite and the parameters are identified given the assumptions, we need to establish the mapping from the reduced form to the structural form parameters.

Without GARCH effects we need sufficient restrictions for identification. Milunovich and Yang (2013) proof with theorem 6 of Rothenberg (1971) that the inclusion of GARCH effects means that we need fewer restrictions on the conditional covariance matrix $B_0$ when it is jointly identified with the parameters in $\varepsilon_t$ and that the system is uniquely identified in this case.

Rothenberg (1971) proofs that the identification of a system of structural equations basically boils down to a question of uniqueness of the solutions to a system of equations.

**Assumption 1.** A vector $\alpha$ satisfies a set of continuously differentiable constrain equations $\psi_i(\alpha) = 0$ with $i = (1..k)$ with Jacobian matrix $\Psi(\alpha)$. The constrained structural parameter space is denoted by $A'$.

**Assumption 2.** The probability density for $Y$ depends on the structural parameter $\alpha$ only through an $r$-dimensional 'reduced form' parameter $\theta$. That is, there exist $r$ known continuously differentiable functions $\theta_i = (h_i(\alpha))$ where $i = (1..., r)$ mapping $A$ into $R^r$ and a function $f^*(y, \theta)$ such that:

$$f(y, \alpha) = f^*[y, h(\alpha)] = f^*(y, \theta)$$

for all $y \in B$ and $\alpha \in A$.

**Assumption 3.** Let $A^* \subset R^r$ be the image of $A'$ under the mapping $h$. Then every $\theta$ in $A^*$ is assumed to be globally identifiable. Hence, identification of vector $\alpha^0 \in A'$ depends solely on the properties of $h$ and $\psi$. If $\theta^0$ is the image of $\alpha^0$, then $\alpha^0$ is identifiable if and only if:

$$\theta^0_i = h_i(\alpha)$$
$$0 = \Psi_j(\alpha)$$

have a unique solution $\alpha^0$ with $j = (1..k)$ and $i = (1..r)$.

Hence the identification problem just becomes question of uniqueness of the solutions to system of equations (Rothenberg, 1971). Rothenberg defines the Jacobian matrices as:

$$H(\alpha) = [\partial h_i/\partial \alpha_j]$$
$$\Psi(\alpha) = [\partial \Psi_i/\partial \alpha_j]$$
$$\omega(\alpha) = [H(\alpha), \Psi(\alpha)]$$
Theorem 6. If $\alpha^0$ is a regular point of $\omega(\alpha)$, then $\alpha^0$ is locally identifiable if and only if $\omega(\alpha^0)$ has full rank $m$.

This result holds globally when $\psi_i$ and $h_i$ are linear. Milonovich and Yang (2013) use Theorem 6 to show that the model is identified by looking at the Jacobian of the mapping to derive conditions for local identification for the non-linear system.

With the GARCH-in-mean specification, we need to establish additional mapping between the estimates of the conditional covariance matrices ($H_t$ to $G_t$), as $G_t$ depends on the structural form parameters.

$u_t|F_{t-1} \sim N(0, H_t)$
$\varepsilon_t = B_0 u_t$
$\varepsilon_t|F_{t-1} \sim N(0, G_t)$

The mapping is established as:

$\varepsilon_t = R_t - \mu_t$
$\mu_t = \Phi_B(L)R_{t-1} - \psi_0 - \Lambda g_t$
$\varepsilon_t|F_{t-1} \sim N(0, B_0H_tB_0')$
$\varepsilon_t|F_{t-1} \sim N(R_t - B_0\Phi(L)R_{t-1} - B_0\theta_0 - B_0\Pi g_t, B_0H_tB_0')$

As $g_t$ is the structural form conditional variance in VECH form, we need to establish additional mapping from $h_t$ to $g_t$ for the mean equation. This means that the mapping established in Milonovich and Yang (2013) still holds; however, we need additional mapping to form the $G_t$ matrix in the structural form.

$H_t = \psi\psi' + \rho H_{t-1}\rho' + \phi(I_k \otimes u_{t-1})(I_k \otimes u_{t-1}')\phi'$
$= B_0^{-1}[\omega\omega' + a(I_k \otimes B_0)(I_k \otimes u_{t-1})(I_k \otimes u_{t-1}')I_k \otimes B_0]d' + \beta G_{t-1}\beta']B_0^{-1}$

Establish the mapping into the mean equation:

$R_t = B_0\Phi(L)R_{t-1} + B_0\theta_0 + B_0\Pi h_t + B_0 u_t$

Rewrite $g_t = \text{vech}(G_t)$, $h_t = \text{vech}(H_t)$. We need to rewrite the mean equation:

$\text{vech}(G_t) = \text{vech}(B_0'\Pi B_0)$
$\text{vech}(B_0'\Pi B_0) = D_k^+ \text{vec}(B_0'\Pi B_0)$

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\[ = D_k^+(B'_0 \otimes B'_0)\text{vec}(H_t) \]

\[ \text{vech}(H_t) = (D_k^+(B'_0 \otimes B'_0)D_a)^{-1}\text{vech}(G_t) \]

where \( D_k = (k^2 + k(k + 1)/2) \) is a duplication matrix with \( D_k^+ = (D_k'D_k)^{-1}D'_k \), and \( D_k'D_k = I_{k(k+1)/2} \).

The mapping from the reduced form to the structural form parameters is as in Milunovich and Yang (2013):

\[ \psi \psi' = B_0^{-1}\omega' B_0^{-1}' \]

\[ \phi = B_0^{-1}a(I_k \otimes B_0) \]

\[ \rho = B_0^{-1}\beta B_0 \]

where there are \((k^2 + 2k)\) parameters in \((B_0, \omega, a, \beta)\) and \((k(k + 1)/2 + 2k^2)\) parameters in \((\psi, \phi, \rho)\). The Jacobian of the mapping is given as follows:

\[
R_t = \begin{pmatrix}
J^\psi_\omega & J^\psi_b & 0 & 0 \\
0 & J^\phi_b & J^\phi_a & 0 \\
0 & J^\rho_b & 0 & J^\rho_\beta
\end{pmatrix}
\]

Where \((b_{21}, \ldots, b_{-1,k}), (\omega_1, \ldots, \omega_k), (a_1, \ldots, a_k), (\beta_1, \ldots, \beta_k)\) are free parameters in \((B_0, \omega, a, \beta)\). They proceed by applying Theorem 6 of Rothenberg (1971) to proof the identification of the system:

**Proposition 2.** Suppose that \((B_0, \omega, a, \beta)\) is a regular point, all elements of \(\omega\) are non-zero, and \(B_0\) is invertible with unit diagonals. Then, \((B_0, \omega, a, \beta)\) is locally identifiable if the number of zeros in \((a_1, \ldots, a_k)\), the free parameters in \(\alpha\), is less or equal to 1.

The Jacobian is of full rank if and only if:

\[ J^\psi_\omega v_1 + J^\psi_b v_2 = 0 \]

\[ J^\phi_b v_2 + J^\phi_a v_3 = 0 \]

\[ J^\rho_b v_1 + J^\rho_\beta v_4 = 0 \]

Then, \(v = [v'_1, v'_2, v'_3, v'_4]\) is a unique point. The rest of the proof can be found in Milunovich and Yang (2013).