How far from equilibrium is active matter?
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NUMERICAL SIMULATIONS

We use Euler time-stepping to simulate the dynamics of AOUPs:
\[ \dot{r}_i = -\mu \nabla_i \Phi + v_i; \quad \tau \dot{v}_i = -v_i + \sqrt{2D} \eta_i, \]
(1)

To observe the MIPS reported in Fig. 1 of the main text, we use periodic boundary conditions and the repulsive potential given by Eq.(2) of the main text. The figure is obtained by starting from a random homogeneous configuration, integrating the dynamics (1) with a time-step \( dt = 10^{-3} \) and taking a snapshot of the particle positions after a time \( t = 10^4 \).

To test the validity of our modified FDT, we consider AOUPs in \( \mathbb{R}^2 \), confined by a harmonic potential
\[ V_W(x,y) = \frac{\lambda}{2} \theta(x-L)(x-L)^2 + \frac{\lambda}{2} \theta(-x)x^2 + \frac{\lambda}{2} \theta(-y)y^2 + \frac{\lambda}{2} \theta(y-L)(y-L)^2, \]
(2)
where \( \theta(u) \) is the Heaviside function. In the simulations reported on Fig. 2 of the main text, we use \( \lambda = 10 \). We integrate the dynamics of AOUPs using \( dt = 5 \times 10^{-4} \). We first let the system relax to its steady-state by simulating its dynamics for a time 50.

To measure the correlation function \( C_{eff}(t) \), we choose a given value of \( t_0 \) and store \( x_i(t_0) \) and \( \dot{x}_i(t_0) \). We then compute \( [x_i(t_0) - x_i(t_0 + t)]x_i(t_0 + t) \) and \( [\dot{x}_i(t_0) - \dot{x}_i(t_0 + t)]\dot{x}_i(t_0 + t) \) for \( t \in [0, 2] \). We finally average over 20,000 values of \( t_0 \) to obtain the correlation function plotted in Fig. 2 of the main text.

To measure the susceptibility \( \chi(t) \), we create a copy of the system at a given time \( t_0 \). This copy evolves with a perturbed dynamics in which \( \Phi \rightarrow \Phi - f \epsilon_i x_i \) where the \( \epsilon_i \) are chosen at random in \( \{-1, 1\} \). The original system evolves with the unperturbed dynamics and we use the same noise realisations \( \eta_i \) for the two systems [1]. We then deduce the susceptibility as
\[ \chi(t) = \sum_i \epsilon_i x^c_i(t + t_0) - x_i(t + t_0) / f, \]
(3)
for \( t \in [0, 2] \), where \( x^c_i \) are the abscissa of the perturbed system. We finally average over 20,000 values of \( t_0 \) to obtain \( \chi(t) \).