The interaction between community and markets remains a central theme in the social sciences. The empirical evidence is rich: in some instances, markets strengthen social ties, while in others they undermine them. The impact of markets on inequality and welfare also varies widely. This paper develops a model where individuals in a social network choose whether to participate in their network and whether to participate in the market. We show that individual behavior is defined by the $q$-core of the network and the key to understanding the conflicting evidence is whether the market and the network are complements or substitutes. (JEL D63, D85, J15, L82, O15, Z13, Z31)

The relationship between community and markets remains a central theme in the social sciences. The empirical evidence on this subject is wide ranging and mixed. In some instances, markets are associated with the erosion of social relations, while in other contexts markets appear to be crucial for their preservation. As further discussed below, empirical research also shows that the impact of markets on welfare and inequality varies greatly. In this paper, our goal is to understand the economic mechanisms that can account for these empirical patterns.

We develop a model where individuals located within a social structure choose a network action ($x$) and a market action ($y$). Payoffs to action $x$ are increasing in the number of their “neighbors” who adopt the same action: this captures the personalized and possibly reciprocal nature of network exchange. In contrast, market exchange is anonymous and short term, and agents are price-takers: payoffs to action $y$ are independent of the decisions of others. The final ingredient is the
relationship between the returns to the network and market actions: we allow for both a complements and a substitutes relation. We study who adopts the network and market actions, respectively, and how this choice affects aggregate welfare and inequality.

Consider the trade-offs an individual faces. To fix ideas, suppose first that the two activities are substitutes and focus on the choice between market and network action. The returns to the market action are constant and independent of others’ choices. On the other hand, the payoffs to network activity are increasing in the number of neighbors who adopt $x$: for a neighbor to adopt the network action, she must in turn have enough neighbors who adopt the network action. We are led naturally to the notion of a set of individuals who each have a threshold number of neighbors, who in turn each have this threshold number of neighbors, and so forth. The $q$-core of a network is the maximal set of individuals having strictly more than $q$ links with other individuals belonging to this set. Theorem 2 shows that behavior is characterized in terms of the $q$-core of the social network. We use this characterization to study the relation between social structure and economic outcomes.

We begin with a simple question: who participates in markets and what sorts of social structure facilitate market participation? Building on the characterization in Theorem 2, we show that the answer depends on whether markets and network exchange are substitutes or complements. In the substitutes case, it is the individuals outside of the $q$-core, who benefit the least from network exchange, who choose the market action. In the case of complements, the converse holds: individuals within the $q$-core adopt the market action. Denser networks, having a larger $q$-core, see lower market participation if the two actions are substitutes; the converse is true when the actions are complements (Proposition 1).

We then turn to welfare. Our analysis yields a clear cut prediction: the emergence of markets may lower welfare in the case of substitutes, but they always raise aggregate welfare when the actions are complements (Proposition 2). The intuition is as follows: in the substitutes case when an individual leaves the network (stops choosing $x$) and instead adopts the market action $y$, she imposes a negative externality on her neighbors who stay with $x$. This negative effect may be larger than the benefits she achieves by opting for $y$. Conversely, in the complements case, the availability of market exchange always raises the returns from network exchange and thus has a positive multiplier effect on welfare.

Finally, we examine the impact of markets on inequality. We find that markets lower inequality when the market action and network exchange are substitutes, the converse is true if they are complements (Proposition 5). In the substitutes case, the market action offers an outside option to individuals who benefit the least from the network, and therefore has the potential to reduce inequality. In the complements case, the market action enhances the payoffs of the well connected individuals, and this favors those who are already better off.

We then explore two theoretical extensions of the model. In the basic model, benefits from network action depend only on the actions of neighbors. In the first extension, we allow for indirect benefits that decay with distance in the network; this model is presented and fully analyzed in online Appendix B2. The case of full decay yields the basic model above, while the case of zero decay reflects a setting
where, roughly speaking, network externalities depend on group size. We develop a new characterization result: equilibrium behavior is defined in terms of the \( q \)-central core, a generalization of \( q \)-core that brings together the decay parameter and the network topology. We also illustrate the importance of structural holes in shaping behavior.

In the basic model, individuals are identical except for differences in network connections. In the second extension, we allow for heterogeneity along other dimensions, such as human capital or initial wealth (see online Appendix B3). We show that our main results on the relation between markets, networks, and inequality continue to hold if heterogeneity in other dimensions is positively correlated to network connections.

We illustrate the scope of our model with a detailed discussion of several empirical applications. The first two applications are about markets and traditional cultures. In one example, relating to the impact of economic liberalization on working-class networks in Bombay, networks and markets are substitutes. Here, markets crowd out networks by offering an outside option to poorly connected individuals. In so doing, they reduce inequality, but also lower welfare for those who keep exchanging in the network. Our second example discusses how tourism markets can facilitate the preservation of indigenous languages. In contrast to the first example, here markets and networks are complements.

The next two applications are about information technology. We start by discussing how the rise of online social networks has eroded the market for traditional media. This is an instance where markets and social networks are substitutes. The interest of this application comes from the observation that it reverses the normal sequence: here a disruptive technology weakens the market and strengthens social networks. The final example is on the impact of mobile telephony on fishermen in Kerala, India. Here, networks and markets complement each other as social connections provide fishermen with information that raises their payoffs from market exchange. Better connected fishermen are more able to take advantage of markets, which ultimately raise welfare but also lead to greater inequality.

In online Appendix B5, we briefly discuss another application: the relation between informal risk-sharing and formal insurance. Building on recent empirical research, we show that formal insurance can be either a substitute or a complement to network exchange, depending on the type of risks that are covered by informal insurance. Observed patterns of adoption of formal insurance are in line with the predictions of our model.

To summarize, our paper offers a parsimonious framework for the study of the relation between networks and markets. Two ingredients of our model—complementarity in network exchange and the strategic relationship between networks and market activity—are central to the understanding of the conflicting evidence on the impact of markets on social networks. The third ingredient—social networks—is key to understanding the evidence on market participation and inequality.

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2 The general model thus accommodates the well-known connections model in the networks literature (Jackson and Wolinsky 1996; Bala and Goyal 2000), the network externalities literature in industrial organization (Katz and Shapiro 1985; Farrell and Saloner 1986), and the large empirical literature in development economics that focuses on the effects of group size (Munshi 2014).

3 For an introduction to the concept of structural holes, see Burt (1992).
addition, our analysis yields clear predictions with respect to the impact of network structure on behavior. These predictions should guide future empirical research that uses more fine-grained, individual-level network data.

Our paper contributes to a long-standing and distinguished literature concerned with the interaction between market and nonmarket exchange. There is, on the one hand, the classical *doux-commerce* stance, going back to the eighteenth century, which purports that markets reinforce durable and peaceful social relations (e.g., Montesquieu 1961; Paine 1951; Condorcet 1998). On the other hand, some scholars have argued that the expansion of markets, accompanied by wide-ranging changes in attitudes and institutions, can crowd out social ties and deplete welfare (e.g., Polanyi 1944; Thompson 1971; Scott 1977; for a popular recent statement of a related view, see Sandel 2012). Our paper shows that these two conflicting views can be accommodated within a simple model: markets reinforce social ties in case the two activities are complements, markets undermine social ties if the two activities are substitutes. Moreover, the effects of markets on inequality and welfare also hinge on the distinction between complements and substitutes.

We contribute to the formal study of the relation between networks and markets. There is a literature on the relation between social ties and anonymous exchange (e.g., Arnott and Stiglitz 1991; Montgomery 1991; Kranton 1996; Galeotti 2010). This work ignores the details of the network topology. There is a more recent strand of research that studies games on networks; for surveys, see Jackson and Zenou (2015) and Bramoullé and Kranton (2016). This literature focuses on actions within a network and ignores market activity. Our paper bridges these two strands of work: we propose a model that combines an anonymous market action with activity in a social network. The analysis yields novel findings on the role of the $q$-core of a network in shaping behavior which—taken together with the concepts of complements and substitutes—helps us address substantive questions on market participation, welfare and inequality.

The rest of this paper is organized as follows. Section I presents the model, while Section II provides a characterization of equilibrium. Section III presents the study of market participation, welfare, and inequality. Section IV discusses evidence from a number of empirical contexts to illustrate the scope of our model. Section V concludes.

I. The Model

Consider a group of individuals $N = \{1, 2, \ldots, n\}$ with $n \geq 3$. A link between two individuals $i$ and $j$ takes on binary values: $g_{ij} = 1$ signifies the existence of a link, while $g_{ij} = 0$ indicates the absence thereof. We assume $g_{ii} = 0$ by convention. We denote by $g$ the (undirected) network of links among individuals. The neighbors of individual $i$, in network $g$, are denoted by $N_i(g) = \{j \in N : g_{ij} = 1\}$ and the degree of $i$ by $k_i = |N_i(g)|$.

Every individual $i$ chooses whether to participate in the network and whether to participate in the market. We denote individual $i$’s actions by $a_i = (x_i, y_i)$,
where $x_i \in \{0, 1\}$ is the network action and $y_i \in \{0, 1\}$ is the market action. Individuals make their choices simultaneously. Let $a = (a_1, a_2, \ldots, a_n)$ be the action profile; in line with convention, we will on occasion use the alternative compact notation, $a = (a_i, a_{-i})$. The number of $i$’s neighbors who choose $x = 1$ in action profile $a$ is given by

$$
\chi_i(a) = \sum_{j \in N_i(g)} x_j.
$$

The term $\chi_i(a)$ captures the level of network activity in $i$’s neighborhood at $a$. Let $\Phi_i(a | g)$ denote individual $i$’s payoffs under action profile $a$ in a network $g$. We suppose that if an individual abstains from network exchange, then the network does not affect her payoffs: choosing $x_i = 0$ may be thus interpreted as leaving the network. We normalize the payoffs from inactivity, i.e., $a_i = (0, 0)$, to 0. The payoffs to the market action only, i.e., $a_i = (0, 1)$, are given by $\pi_y \in \mathbb{R}$. Finally, in any strategy profile $a$, individual $i$’s payoffs from $a_i = (1, 0)$ and $a_i = (1, 1)$ are given by $\phi_0(\chi_i(a))$ and $\phi_1(\chi_i(a))$, respectively, with $\phi_0 : \mathbb{N}_+ \rightarrow \mathbb{R}$ and $\phi_1 : \mathbb{N}_+ \rightarrow \mathbb{R}$. To summarize,

$$
\Phi_i(a | g) = \begin{cases} 
0 & \text{if } a_i = (0, 0) \\
\pi_y & \text{if } a_i = (0, 1) \\
\phi_0(\chi_i(a)) & \text{if } a_i = (1, 0) \\
\phi_1(\chi_i(a)) & \text{if } a_i = (1, 1)
\end{cases}
$$

Following Kranton (1996), we suppose that (bilateral) reciprocal exchange between two individuals takes place only if both individuals chose $x = 1$. Hence, we assume that the payoffs from the network action $x$ displays local complementarity.

**ASSUMPTION 1:** Both $\phi_0(\cdot)$ and $\phi_1(\cdot)$ are strictly increasing in $\chi_i(a)$.

We turn now to the relation between the network and market actions. The key idea here is that the market action $y$ affects the marginal returns from network action, $x$. To develop this formally, we define

$$
\xi(\chi_i(a)) = \phi_1(\chi_i(a)) - \phi_0(\chi_i(a)) - \pi_y.
$$

Observe that $\xi(\cdot)$ is the difference between the marginal returns to $x$ when $y_i = 1$, $\phi_1(\cdot) - \pi_y$, and the marginal returns to $x$ when $y_i = 0$, $\phi_0(\cdot) - 0$. Network and market actions are said to be substitutes if $\xi(\cdot)$ is negative and (weakly) decreasing in $\chi_i(a)$. They are said to be complements if $\xi(\cdot)$ is positive and (weakly) increasing in $\chi_i(a)$. Thus, our notion of substitutes combines a substitutes relation between an individual’s network action and market action and a strategic substitutes relation between the network action of her neighbors and her own market action. Similarly, our definition of complements subsumes a complements relation between an individual’s network and market action and a strategic substitutes relation between an individual’s network action and market action.
assumption 2: $\xi(0) = 0$. Network and market actions are either substitutes or complements.

We view $\xi(0) = 0$ as a simplifying normalization: if no one in the neighborhood adopts action $x$, then the network is not functioning, and so the action $y$ does not affect the marginal payoffs from action $x$.

We now present two examples of payoff functions to illustrate our assumptions.

Example 1: Individual $i$’s payoffs are given by

$$
\Phi_i(a | g) = (1 + \theta y_i) x_i \chi_i(a) + y_i - p_x x_i - p_y y_i,
$$

where $p_x \geq 0$ and $p_y \geq 0$, respectively, are the prices of actions $x$ and $y$. We note that $x$ and $y$ are substitutes for $\theta \in [-1, 0]$ and complements for any $\theta \geq 0$.

Example 2: Individual $i$’s payoffs are given by

$$
\Phi_i(a | g) = (x_i \chi_i^\alpha(a) + y_i) \theta - p_x x_i - p_y y_i,
$$

where $p_x \geq 0$ and $p_y \geq 0$, respectively, are the prices of actions $x$ and $y$. It may be verified that $x$ and $y$ are substitutes for any $\theta \in (0, 1]$, and complements for any $\theta \geq 1$.

In Example 1, the returns to network action $x$ are linear in $\chi_i(a)$. In Example 2, returns to $x$ may be concave or convex in $\chi_i(a)$, depending on the value of the parameters $\alpha$ and $\theta$. Note that in both examples, $\pi_y$ is given implicitly by $1 - p_y$.

We study the Nash equilibrium of the game above. For a network $g$, an action profile $a^*$ is a (Nash) equilibrium if for every $i \in N$, $a_i^*$ maximizes $i$’s payoffs given $a_{-i}$.

The local complementarity in $x$ creates the potential for coordination failure. To see this, observe that if $\phi_0(0) < 0$ or $\phi_1(0) < \pi_y$, then $x_i = 0$ for all $i \in N$ is an equilibrium. This is true even if some (or all) individuals strictly prefer to coordinate on $x = 1$. As our focus is on the interaction between markets and networks, we will, for most of this paper, abstract from the coordination problem in the network activity. We say that an action profile $a$ Pareto-dominates another profile $a'$ if $i$’s payoffs are (weakly) larger under $a$ than under $a'$ for all $i \in N$, with a strict inequality for at least one $j \in N$. An equilibrium $a^*$ is said to be maximal if there does not exist another equilibrium that Pareto-dominates it.

We now comment on some of the restrictive features of payoffs. First, observe that individual payoffs depend only on own actions and on the number of neighbors who choose to participate in the network; they do not depend on market choices.
of others. In other words, we abstract from market thickness considerations. This is the natural benchmark, as we wish to contrast network-based exchange with large anonymous markets. It is possible to extend our model to incorporate market size effects. Our main insights, reflected in Theorem 2 and Propositions 1–5, continue to obtain in the general setting. In addition, there arises the possibility of coordination failure in markets and this can give rise to multiple Pareto-optimal equilibria. We comment further on coordination problems and equilibrium selection in Section II.

Second, observe that the returns to network exchange depend only on the number of (direct) neighbors who choose to participate in the network. In principle, an individual could benefit from her neighbors’ neighbors (and so forth) who choose the network action. A simple and natural way to incorporate indirect advantages is to suppose that benefits to individual \( i \) from the network actions of another individual \( j \) depend on the (geodesic) distance between them. The decay in the flow of benefits is captured by a parameter \( \delta \in [0, 1] \). We can replace \( \chi_i(a) \) with a suitably generalized version and study behavior as we vary the decay factor. Observe that if \( \delta = 0 \) then we obtain our benchmark model, and if we set \( \delta = 1 \) then benefits depend on component size. Online Appendix B2 presents and analyzes a model with these features.

Finally, observe that we implicitly assume that all individuals are the same, except for differences in network position. Online Appendix B3 shows how our insights generalize when individuals are heterogeneous along other dimensions.

II. Social Structure and Behavior

This section establishes existence and provides a characterization of maximal equilibrium. The first step in the analysis is the following result.

THEOREM 1: Suppose that Assumptions 1 and 2 hold. For a given network \( g \), a maximal equilibrium exists and is generically unique.

The proofs are provided in the Appendix. In the complements case, existence follows from standard arguments: start from a profile with everyone choosing \( a_i = (0, 0) \). Iterate through best responses: noting that actions are complements, any increase in action \( x \) by one individual provokes a further increase (weakly) in others’ actions. As the action set is binary, the process must converge and the limit is an equilibrium. In the substitutes case, the argument is a little more involved and exploits the payoff structure more directly to construct different types of equilibrium in the cases where the market action alone is attractive and where it isn’t. Existence of maximal equilibrium follows from noting that the set of strategies, and hence the set of equilibria, is finite.

Turning to the (generic) uniqueness of maximal equilibrium, the argument proceeds as follows. Start with two maximal equilibria and construct a larger maximal equilibrium in terms of number of individuals choosing action \( x \). Exploiting the local complementarity in payoffs for the network action, we then show that this new equilibrium Pareto-dominates the two initial maximal equilibria. This contradiction completes the proof of Theorem 1.
We now turn to the characterization of behavior and its relation to network structure. It is useful to examine the simple choice between \( a_i = (1, 0) \) and \( a_i = (0, 0) \). Note that if \( \phi_0(0) < 0 \), an individual \( i \) will choose \( x_i = 1 \) if and only if \( \chi_i \) is high enough (from Assumption 1, we know that \( \phi_0(\cdot) \) increases with \( \chi_i \)). Similarly, her neighbors will choose \( x = 1 \) if a sufficient number of their own neighbors choose \( x = 1 \). This motivates the following definition.

**Definition 1** (\( q \)-core (Bollobás 1984)): Given network \( g \) and \( q \in \mathbb{R}_+ \), the \( q \)-core of \( g \), \( g^q \), is the set of nodes forming the largest subgraph of \( g \) such that all individuals in \( g^q \) have more than \( q \) links to other individuals in \( g^q \).

Informally then, the \( q \)-core is the largest set of individuals who have at least \( q \) connections to one another. Observe that if a set of individuals all have more than \( q + 1 \) links to one another, then trivially they must have more than \( q \) links with one another: the \( (q + 1) \)-core is contained in the \( q \)-core. Following the same logic, the \( q \)-core is contained in the \( (q - 1) \)-core, and so on. We thus have that the \( (q + m) \)-core is always contained in the \( q \)-core, for any \( q, m \geq 0 \). We also note that for any \( q \in \mathbb{R}_+ \), the \( q \)-core of a network \( g \) always exists and is unique, although it can be disconnected or empty. For concreteness, we next provide an algorithm to obtain the \( q \)-core in any network \( g \).

**Algorithm 1:** Consider any network \( g \). In step 1, delete every node \( i \) (and its links) in \( g \) for which \( k_i \leq q \). Label the residual graph \( g_1 \). In step 2, delete every node \( i \) (and its links) in \( g_1 \) for which \( k_i \leq q \). Iterate until no node with \( k_i \leq q \) remains (i.e., when \( g_t = g_{t+1} \)). The nodes, and the corresponding links, forming the graph in this last step constitute the \( q \)-core.

By way of illustration, consider the network on Figure 1. Suppose that we want to find the \( 4 \)-core. First, find all the nodes with \( k \leq 4 \), and delete them and their links. In step 2, delete the nodes with 4 or less links in the residual network from step 1. Proceed likewise unless no node with \( k \leq 4 \) remains. The remaining nodes form the \( 4 \)-core.

An individual chooses between four possible actions, namely \((0, 0), (1, 0), (0, 1), \) and \((1, 1)\). We now develop threshold values of \( \chi_i \) that will help us characterize optimal choice. First, recall from Assumption 1 that both \( \phi_0(\cdot) \) and \( \phi_1(\cdot) \) are strictly increasing. This means that there exist \( q_1 \geq 0 \) and \( q_2 \geq 0 \) such that

\[
\phi_0(\chi_i) > \max\{0, \pi_y\} \text{ if and only if } \chi_i > q_1;
\]

\[
\phi_1(\chi_i) > \max\{0, \pi_y\} \text{ if and only if } \chi_i > q_2.
\]

Observe that, under Assumption 1, the thresholds may never be reached; in that case, we shall use the convention that they are infinite.
Next, recall from Assumption 2 that in the case of substitutes, $\phi_0(\cdot)$ increases faster than $\phi_1(\cdot)$ with respect to $\chi_i \in \mathbb{N}_+$. This means that there exists a $q_3 \geq 0$ such that

$$
\phi_0(\chi_i) > \phi_1(\chi_i) \text{ if and only if } \chi_i \geq q_3.
$$

Similarly, in the case of complements, there exists a $q_4 \geq 0$, such that

$$
\phi_1(\chi_i) > \phi_0(\chi_i) \text{ if and only if } \chi_i \geq q_4.
$$

Equipped with these thresholds, we can provide a complete characterization of (maximal) equilibrium. For expositional simplicity, we first state this result for a special class of payoff functions where the substitute and complement relation is strong. Theorem 3, in Appendix A1, presents the general result.

We say that the network and market actions are strong substitutes if they are substitutes and $q_3 < q_1$. They are strong complements if they are complements and $q_4 < q_2$. Observe that strong substitutes rule out cases where $a_i = (1, 1)$ is optimal for any $\chi_i \in \mathbb{N}_+$. Following a similar logic, we note that strong complements rules out action $a_i = (1, 0)$ being optimal for any $\chi_i \in \mathbb{N}_+$. 

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*Panel A: initial network; panel B: delete nodes with $k \leq 4$; panel C: delete remaining nodes with $k \leq 4$; panel D: 4-core: no further deletions.*

**Figure 1. The 4-Core**

Note: Panel A: initial network; panel B: delete nodes with $k \leq 4$; panel C: delete remaining nodes with $k \leq 4$; panel D: 4-core: no further deletions.

---

6 Given Assumption 2, the thresholds may never be reached; we then use the convention that they are infinite.

7 Indeed, if $q_3 < q_1$, then individuals prefer either $a_i = (0, 1)$ or $a_i = (0, 0)$ to $a_i = (1, 1)$ when $\chi_i \leq q_1$, while they prefer $a_i = (1, 0)$ when $\chi_i > q_1$.

8 It can be verified that in the payoff function presented in Example 1, $x$ and $y$ are strong substitutes if $\theta \in [-1, 0)$ and $\theta(n-1) < p_y - 1$; they are strong complements if $\theta > 0$ and $\theta > p_y - 1$. 

---
THEOREM 2: Suppose that Assumptions 1 and 2 hold. Let $a^*$ be the maximal equilibrium.

(i) Strong Substitutes: $a_i^* = (1, 0)$ if and only if $i \in g^{|x|}$. If $i \notin g^{|x|}$, then $a_i^* = (0, 0)$ in case $\pi_y \leq 0$, and $a_i^* = (0, 1)$ in case $\pi_y > 0$.

(ii) Strong Complements: $a_i^* = (1, 1)$ if and only if $i \in g^{|y|}$. If $i \notin g^{|y|}$, then $a_i^* = (0, 0)$ in case $\pi_y \leq 0$, and $a_i^* = (0, 1)$ in case $\pi_y > 0$.

In the case of strong substitutes, only individuals in the $q_1$-core will choose $a_i = (1, 0)$, while those outside will either choose $a_i = (0, 1)$ (if $\pi_y > 0$) or $a_i = (0, 0)$ (if $\pi_y \leq 0$). To develop some intuition, consider the case where $\pi_y \leq 0$. Individuals therefore choose between $a_i = (1, 0)$ and $a_i = (0, 0)$. They prefer the network action if they have at least $q_1$ neighbors who choose $x = 1$. Observe that if all individuals in the $q_1$-core do choose $x = 1$, then it follows that condition (6) is satisfied for all individuals in the $q_1$-core. Hence, players in the $q_1$-core all obtain larger payoffs by playing $(1, 0)$ than by remaining inactive—the converse is true for players outside the $q_1$-core.

In the case of strong complements, only individuals in the $q_2$-core choose $a_i = (1, 1)$, while individuals outside the $q_2$-core choose either $a_i = (0, 1)$ (if $\pi_y > 0$) or $a_i = (0, 0)$ (if $\pi_y \leq 0$). Consider the case where $\pi_y > 0$. Individuals choose between $a_i = (1, 1)$ and $a_i = (0, 1)$ and prefer the former if and only if condition (7) is satisfied. If all individuals in the $q_2$-core choose $x = 1$, then (7) is satisfied for individuals in the $q_2$-core, and so all individuals in the $q_2$-core (and these individuals only) must strictly prefer $a_i = (1, 1)$ to $a_i = (0, 1)$.

Figure 2 illustrates the results of Theorem 2 for the payoff function in Example 1 and for the network introduced in Figure 1. The left network of Figure 2 shows equilibrium strategies when $x$ and $y$ are (strong) substitutes: since $q_1 = 4.5$ and $\pi_y > 0$, individuals in the 4.5-core choose $a_i = (1, 0)$, while all other individuals choose $a_i = (0, 1)$. The right network represents the case of (strong)
complements: given the prices, $q_2 = 4$ and $\pi_y < 0$, individuals in the 4-core choose $a_i = (1, 1)$ while all others choose $a_i = (0, 0)$.

To develop a better understanding of the uses of Theorem 2, we present equilibrium outcomes in two familiar networks on [Figure 3] the regular network with degree 3 and a core-periphery (CP) network (with an equal number of nodes and links). In the case of strong substitutes (top panels), peripheral individuals, who benefit the least from network exchange, choose the market action, while all other individuals choose the network action. Everyone chooses the network action in the regular network. In the case of complements, the opposite holds: for the given prices, only the best-connected individuals (in the core of the CP network) can afford to choose the market (and the network) action: all other individuals choose inaction. In the regular network, no one has sufficient connections: inaction is pervasive.
Figures 2 and 3 help us understand the role of the topology of networks and the strategic relation between market and network activity in shaping behavior. In the case of substitutes, the first guess would be that highly connected nodes should adopt the network action while less connected nodes adopt the market action. Our analysis goes beyond this intuition. Consider the first network on Figure 2: node 9 has higher degree than node 10, and yet it chooses the market action while the latter chooses the network action. This is because node 10 forms part of the 4.5-core, while node 9 does not. Turning next to the impact of the strategic relation between network and market actions, let us compare behavior in the top panels (substitutes) and the bottom panels (complements) of Figure 3. In the substitutes case, it is the nodes lying outside the relevant q-core that choose market action, while in the complements case it is the nodes within the relevant q-core that choose this action!

So far we have focused on maximal equilibrium; we now briefly comment on nonmaximal equilibria. Observe that in every equilibrium, the set of individuals who choose action $x$ can be characterized using the appropriate $q$ level of connectivity. Further, in all nonmaximal equilibria, the set of individuals choosing $x$ is a subset of the relevant $q$-core. Different equilibria reflect different levels of coordination in terms of network activity and this has implications for welfare. We further comment on this in Section IIIB. Lastly, observe that there is a form of consistency in maximal equilibria: suppose individuals have coordinated on a maximal equilibrium in a premarket situation, when $a_i = x_i$. The introduction of the market action always leads through best responses to the new unique maximal equilibrium. This follows from an application of Theorem 1 and the algorithm for $q$-core.

We next relate the $q$-core to the idea of $q$-cohesiveness, discussed in Morris (2000). Morris (2000) defines a subset of individuals $S$ as $q$-cohesive if all individuals in $S$ have at least a fraction $q$ of their neighbors in $S$. The notion of $q$-core is similar to cohesiveness in that it requires recursive connectivity, but there is one important difference: the $q$-core relies on an absolute number of links, while the cohesive set is defined in terms of the proportion of links. This difference has a substantive content in our context: the $q$-core will refer to well connected individuals. There is no such presumption in a $q$-cohesive set.

We conclude this section with a brief discussion of indirect network benefits. Online Appendix B2 provides an analysis of the model where benefits decay in distance. We show there that behavior in a (maximal) equilibrium in this model can be described by generalizing the notion of $q$-core. The $q$-centrality of a node is simply the weighted connections with others in the network (where the weights reflect distance). The $q$-central core of $g$, denoted by $c_q(g)$, is the set of individuals with at least $q$ decay centrality in the subgraph formed only by the individuals in $c_q(g)$. The algorithm to obtain $c_q(g)$ is analogous to Algorithm 1 described above. The analysis also brings out the role of individuals who span structural holes in sustaining high levels of network activity.

### III. Market Participation, Welfare, and Inequality

This section explores the factors that affect the rate of market participation, aggregate welfare, and payoff inequality across individuals. We show that social connectedness facilitates the adoption of the market action if the two activities are
complements; the converse is true in the case of substitutes. Markets always raise aggregate welfare if the two activities are complements, but this is not necessarily the case when the two activities are substitutes. Inequality in a network is typically reinforced by the market in case the two actions are complements; the converse holds true if they are substitutes.

We say that a network $g'$ is denser than another network $g$ if $g'_{ij} \leq g_{ij}$ for any pair $i, j \in N$, and the inequality is strict for at least one such pair. We say that an individual is well connected if she lies in the appropriate $q$-core (e.g., $q_1$-core in the case of strong substitutes, $q_2$-core for strong complements). We say that the returns to $x$ are larger under $\varphi_0'$ than under $\varphi_0$ if $\varphi_0'(m) \geq \varphi_0(m)$ for all $m$ with a strict inequality for some $m$. Similarly, we say that the degree of complementarity between $x$ and $y$ is larger under $\xi'$ than under $\xi$ if $\xi'(m) \geq \xi(m)$ for all $m$, with a strict inequality for some $m$.

A. Market Participation

We examine the receptivity of social structures to market activity: are sparse or dense social structures more receptive to markets? Within a society, are highly connected and central individuals or poorly connected and marginalized individuals more receptive to markets?

Recall that given a network $g$, generically, there exists a unique maximal equilibrium $a^\ast$. Market participation in network $g$ is defined as

$$\mathcal{M}(g) \equiv \frac{\sum_{i \in N} y_i^+(a^\ast)}{N},$$

where $y^+$ is the market action chosen in the maximal equilibrium $a^\ast$.

Theorem 2 tells us that the key to market participation is the size of $g^q$ and the value of $\pi_y$. For instance, in the case of strong substitutes, if $\pi_y > 0$, then the set of market participants is simply the complement of the set $g^q$. Similarly, in case of strong complements if $\pi_y < 0$ then every individual in $g^q$ adopts the market action. This suggests that, loosely speaking, market participation is falling in the size of the core set in case of substitutes, while the converse is true in the case of complements. This observation has a number of implications, which are summarized in the next result.

**PROPOSITION 1:** Suppose that Assumptions 1 and 2 hold.

(i) Market participation is (weakly) lower in denser networks when $x$ and $y$ are substitutes, and (weakly) larger when they are complements.

(ii) The market action $y$ is adopted by less connected individuals in the case of substitutes, and by the well connected in the case of complements.

(iii) Market participation (weakly) increases with $\pi_y$; (weakly) increases with the returns to $x$ and with the degree of complementarity between $x$ and $y$ in the case of complements; and (weakly) decreases with the returns to $x$ and is
nonmonotonic in the degree of complementarity between \(x\) and \(y\) in the case of substitutes.

Denser networks imply a (weak) increase in the size of the relevant \(q\)-core: this raises individuals’ benefits from network exchange as the number of partners (weakly) increases. Increasing profitability of network exchange, in turn, increases (decreases) the profitability of market exchange in the case of complements (substitutes). Furthermore, in the case of substitutes, it is the individuals who benefit the least from network exchange (i.e., less connected individuals) who find in market exchange a valuable outside option. Conversely, in the case of complements, typically only well connected individuals, who benefit more from network exchange, find the market action \(y\) attractive.

Turning to the impact of the payoff function, market participation naturally increases with the payoffs to market action \(y\), \(\pi_y\). However, whether market participation increases with the returns to \(x\) depends on whether \(x\) and \(y\) are substitutes or complements. When they are substitutes, increasing returns to \(x\) make it more attractive relative to \(y\), which weakly reduces market participation. When they are complements, increasing returns to \(x\) makes \(y\) more appealing, which increases market participation.

When \(x\) and \(y\) are complements, an increase in the degree of complementarity between \(x\) and \(y\) increases the benefits of \(y\) when adopted jointly with \(x\), which (weakly) increases market participation. When \(x\) and \(y\) are substitutes, however, an increase in the degree of complementarity between \(x\) and \(y\) has two opposing effects. The first effect is that it reduces the cost of \(y\) in terms of foregone payoffs to \(x\), which fosters market participation. The second effect is that it fosters the adoption of \(x\) as the cost of \(x\) in terms of payoffs to \(y\) decreases, which weakly expands the \(q_2\)-core. This expansion weakly reduces market participation as it increases the payoffs to \(x\) for other individuals and, thus, their cost of adopting \(y\). These conflicting effects lead to a nonmonotonic overall impact.

We conclude this section with a brief discussion of coordination failure. Proposition 1 analyzes market penetration under maximal equilibrium. It is worth noting that coordination failure can have a powerful effect on market participation levels and that these effects vary in interesting ways depending on whether the two activities are complements or substitutes. So, for instance, it is easy to see that coordination failure in the network activity fosters market participation when \(x\) and \(y\) are substitutes, but hampers it in case they are complements.

B. Aggregate Welfare

An important and long-standing concern has been the potentially deleterious effects of markets on welfare. Our framework allows for an examination of the circumstances under which the introduction of markets is welfare-enhancing. To do so, we compare welfare in a society before and after the arrival of market action \(y\). Given a network \(g\) and action profile \(a\), aggregate welfare is given by

\[
W(a \mid g) = \sum_{i \in N} \Phi_i(a_i, a_{-i} \mid g). 
\]
Let $\mathcal{W}(g)$ denote the aggregate welfare in the maximal equilibrium $a^*$, with

$$W(g) = W(a^* | g).$$

Given $g$, we say that an outcome $a$ is efficient if $W(a | g) \geq W(a' | g)$, for any feasible $a'$. The following result summarizes our analysis of the impact of markets on aggregate welfare.

**Proposition 2:** Suppose that Assumptions 1 and 2 hold. In the case of substitutes, the introduction of markets may lower aggregate welfare. In the case of complements, the introduction of markets (weakly) increases aggregate welfare.

Observe first that in the case of complements, the introduction of $y$ (weakly) facilitates the adoption of the network action $x$. The introduction of $y$ implies (weakly) larger individual payoffs, and hence a larger aggregate welfare. However, if $x$ and $y$ are substitutes, the introduction of $y$ is not generally welfare-enhancing. A switch from the network action to the market action by some individuals leads to a fall in the payoffs of the individuals who remain with the network action. This externality can dominate any gains the market participants enjoy. The following example illustrates this point.

**Example 3 (Markets and Welfare):** Consider the core-periphery network in Figure 3, and suppose that the payoff function is as in Example 1. Fix $\theta = -0.9$ and $p < 1$. Prior to the introduction of $y$, all individuals choose $x = 1$.

Suppose now that the market action $y$ becomes available. If $0 < 0.1 < p_y \leq p_x < 1$, then all periphery individuals choose $y = 1$, while core individuals stick to $x = 1$. Periphery individuals increase their payoffs by $0 < p_x - p_y < 1$ following their switch. On the other hand, a periphery individual’s switch entails a decrease in the benefits of the core individual she is connected to of exactly 1. The net effect of the introduction of the market action is thus strictly negative.

This example motivates a closer examination of the question: are there networks for which markets always raise welfare even when markets are a substitute for network actions? Proposition 3 provides a response.

**Proposition 3:** Suppose that Assumptions 1 and 2 hold. The maximal equilibrium is efficient in regular networks; it may be inefficient in nonregular networks.

Since the network action displays local complementarities, individuals do not fully internalize the positive (social) payoffs of choosing $x = 1$. There is thus a risk of underprovision of $x$ compared to the social optimum. In regular networks,
however, underprovision is avoided. Indeed, suppose that \( x_i = 1 \) for all \( i \in N \) is the efficient outcome. Consider individual \( j \) with \( k \) neighbors who are all choosing \( x = 1 \). If \( j \) decides to choose \( x = 0 \), it is necessarily because \( x \) is not profitable to her even if she fully enjoys the local benefits of her neighbors choosing \( x = 1 \).

Since the network is regular, what is true for \( j \) is true for all other individuals, which means that the actions \( x \) and \( y \) are undertaken in regular networks if and only if they maximize the welfare of all individuals. As markets expand opportunities, it follows that in a regular (or egalitarian) society the maximum attainable welfare increases. As the maximal equilibrium is efficient in such networks, this also means that markets always raise welfare.

Proposition 4 summarizes our analysis of the effects of networks and the payoff function on individuals' earnings and on aggregate welfare.

**PROPOSITION 4:** Suppose that Assumptions 1 and 2 hold.

(i) Aggregate welfare is (weakly) larger in denser networks.

(ii) Well connected individuals have higher payoffs than less connected individuals. Furthermore, individuals' payoffs are (weakly) increasing in the number of their connections to other individuals in the relevant \( q \)-core.

(iii) Aggregate welfare weakly increases in the returns to \( x \) and in the degree of complementarity between \( x \) and \( y \); weakly increases with \( \pi_y \) in the case of complements; and is nonmonotonic in \( \pi_y \) in the case of substitutes.

The first statement in Proposition 4 rests on the observation that denser networks have larger \( q \)-cores, and thus entail higher benefits from network exchange. Denser networks thus also raise the benefits from market exchange in the case of complements, and leave them unchanged in the case of substitutes. Hence, denser networks always (weakly) raise individual payoffs.

The second point follows from the observation that the payoffs of individuals outside the \( q \)-core are independent of other individuals’ actions. Thus, individuals inside the \( q \)-core could earn the same payoffs by adopting the action of individuals outside the \( q \)-core. Since they do not, this means that they must earn higher payoffs as compared to those individuals outside of the \( q \)-core. Finally, note that an additional link to an individual inside the relevant \( q \)-core leaves unchanged the payoffs to an individual outside the \( q \)-core, while it increases the payoffs to an individual inside the \( q \)-core.

The last part of Proposition 4 describes the impact of the payoff function on aggregate welfare. Individual utility is weakly increasing in the number of neighbors who choose \( x = 1 \). Therefore, increasing returns to \( x \) and/or increasing the degree of complementarity between \( x \) and \( y \) always increases individuals’ utility both directly (since payoffs to \( x \) increase) or indirectly (if more of their neighbors decide to engage in \( x \)). The effect of \( \pi_y \) on welfare is, however, more intricate. Clearly, when \( x \) and \( y \) are complements, welfare increases when \( \pi_y \) increases. However, when they are substitutes, there are two effects that oppose each other. On the one hand, an increase in \( \pi_y \) entails a direct increase in the payoffs of individuals who choose \( y = 1 \). On the other hand, it may decrease the \( q_1 \)-core as it may push
certain individuals to switch from \( x \) to \( y \), this may entail a fall in the payoffs of their neighbors who remain with \( x = 1 \). The net effect of an increase in \( \pi_y \) on welfare may thus be nonmonotonic.

**C. Inequality**

We now turn to the impact of markets on inequality. The measurement of inequality is a vast subject (see, e.g., Atkinson 1970; Sen 1992). In order to appreciate the key factors at work, we start by examining the ratio of the highest payoffs to the lowest payoffs. Given network \( g \), this ratio is denoted by \( \mathcal{R}(g) \):

\[
\mathcal{R}(g) \equiv \frac{1 + \max \{ \Phi_i(a^*_i, a^*_{-i} \mid g) \}_{i \in N}}{1 + \min \{ \Phi_i(a^*_i, a^*_{-i} \mid g) \}_{i \in N}},
\]

where \( a^* \) is the maximal equilibrium in network \( g \).

\( \mathcal{R}(g) \) is close in spirit to other traditional metrics of inequality, including the *range*, the 20:20 *ratio*, or the *Palma ratio*. The range is the difference between the payoffs of the wealthiest and the poorest individuals of a population. The 20:20 *ratio* and the *Palma ratio* reflect the payoff ratio of the wealthiest 20 percent to the poorest 20 percent, and the payoff ratio of the wealthiest 10 percent to the poorest 40 percent, respectively. While \( \mathcal{R}(g) \) has the same structure as these two measures, it requires less information about the payoff distribution and, thus, about the network structure.

Let \( \mathcal{R}_0(g) \) denote the inequality prior to the introduction of market action, \( y \), and \( \mathcal{R}_1(g) \) its level after. Note that a rising \( \mathcal{R}(g) \) implies increasing inequality. The following proposition summarizes our analysis of the effects of markets on inequality.

**Proposition 5:** Suppose that Assumptions 1 and 2 hold. In the case of substitutes, the introduction of the market (weakly) decreases inequality. In the case of complements, if \( \mathcal{M}(g) \in (0, 1) \), then markets strictly increase inequality, while if \( \mathcal{M}(g) = 1 \), then the effects on inequality are ambiguous.

We sketch the intuition for the case of strong substitutes and complements. In the case of strong substitutes, it is easiest to see the argument when we start from a premarket situation where well connected individuals choose \( x \). The introduction of the market clearly offers the less connected individuals a potentially better option. Their switch to the market action can only lower the payoffs of the best connected who remain with the network action. Hence, the minimum payoffs must (weakly) rise and the maximum payoffs must (weakly) fall, with the introduction of the market. Putting together these observations yields us the insight that markets unambiguously lower inequality in the case where the market and the network actions are substitutes.

When \( x \) and \( y \) are strong complements, we can focus on two action profiles \( (0, 0) \) and \( (1, 1) \). Individuals who benefit the most from network exchange will also benefit the most from markets. When market penetration is incomplete, markets will thus unambiguously raise inequality. When market penetration is complete (\( \mathcal{M}(g) = 1 \),...
the worst-off individuals may benefit relatively more (or less) as compared to the best-off individuals from the newly available market $y$, depending on the social structure and the payoffs to the two actions. The following example elaborates on this point.

**Example 4 (Markets and Inequality):** Consider the network in Figure 4 and the payoff function in Example 1. Fix $p_x = 4.1$. In such case, the best-off individuals before the introduction of $y$ are individuals 1 to 6 with payoffs of 0.9, while all other individuals have payoffs 0. This means that inequality is given by $R_0(g) = 1.9$. Now suppose that $y$ is introduced at a price $p_y = 1.05$. Then, the earnings of individuals 1, 7, and 3 through 6 are 5.85, while those of individuals 2 and 8 through 11, respectively, are 7.85 and 3.85. Consequently, $R_1(g) = 1.825$, which indicates falling inequality.

Next suppose that $p_y = 2$. Then, the payoffs to individuals 1, 7, and 3 through 6 is 4.9, while those of individuals 2 and 8 through 11, respectively, amount to 6.9 and 2.9. Consequently, inequality is given by $R_1(g) = 2.026$; there is thus an increase in inequality with the arrival of a market.

Proposition 5 provides a clear-cut prediction with regard to the impact of markets on the ratio of the highest payoffs to the lowest payoffs. In online Appendix B4, we also discuss the impact of markets on the Gini coefficient. The Gini coefficient, in addition to taking into consideration the poorest and the wealthiest individuals, fully accounts for those in between. We show that our analysis of the effect of the introduction markets on inequality is mostly robust to this change of index.

**IV. Applications**

In this section, we discuss a number of empirical phenomena on the relation between community and markets. We argue that two ingredients of our model—complementarity in network exchange and the strategic relationship between network and market activity—are necessary to account for the conflicting evidence on the impact of markets on social networks. The third ingredient—social networks—is important for understanding the evidence on market participation and inequality. The discussion also clarifies the ways in which our model goes beyond the existing
theoretical work to generate new predictions that should be studied using more fine-grained data on individual connections.

A. Language, Local Culture, and Markets

In this subsection, we discuss the relation between markets, culture, and languages. The empirical record is mixed: markets and globalization are associated both with cultural change and as well as with the persistence of traditions.

Caste Networks, Globalization, and English Language Schooling.—Munshi and Rosenzweig (2006)—henceforth, MR—explore the impact of market forces on traditional institutions. The economic liberalization of the Indian economy in the 1990s entailed a shift toward the corporate and finance sectors, which increased the returns to white-collar jobs for which knowledge of English was necessary. MR estimate that in the city of Mumbai, the liberalization of the economy significantly increased the premium to English education (compared to education in Marathi) during the 1990s. However, they find that boys of working-class and heavily networked subcastes took much less advantage of the opportunities of the new economy than their female counterparts. As a result, while the gap in English education between girls of high and low castes shrank, the gap for boys remained (roughly) intact.

We map the empirical context to our model. Parents choose to send their child to a Marathi school (action $x$) or an English school (action $y$). MR report that 68 percent of the men in working-class jobs found employment through a relative or a member of the community and that a high majority of parents who chose education in Marathi reported “closer community ties” as a factor for their decision. There is thus a positive externality associated with choice of Marathi schooling. In contrast, the returns to English education are simply a function of the child’s ability and the (exogenously given) market premium to education in English. Following the discussion in MR, we postulate that boys can expect to be part of large working-class networks, while girls do not have access to any network. Finally, note that the choices of English or Marathi education are mutually exclusive; this corresponds to a situation of perfect substitutes (viz. $\theta = -1$ in Example 1).

Proposition 1 predicts that the adoption of $y$ should, ceteris paribus, be higher for girls than for boys in working-class subcastes. This prediction is consistent with MR’s findings. Proposition 5 predicts that income inequality between girls and boys in working-class subcastes should decrease. This is consistent with MR (pp. 1250–51) who find “that a previously disadvantaged group (girls) might surpass boys in educational attainment” in the most heavily networked subcastes.10

We now relate our framework to MR’s theoretical model. MR focus on the role of historical occupational choices in shaping education and occupation choices of the current generation. They show how members of otherwise identical castes may

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10 Proposition 4 predicts that while the payoffs of girls should increase, the payoffs of boys who choose education in Marathi should (weakly) decrease due to boys “leaving” the network. While no direct evidence is provided on this, MR (p. 1230) recognize this possibility and suggest that caste networks “might place tacit restrictions on the occupational mobility of their members to preserve the integrity of the network.”
make very different education choices due to different sizes of historically determined professional networks.\footnote{MR argue that the boys of subcaste networks who did adopt English education are those who were more talented. Online Appendix B3 studies a model where individuals differ in connections and also along other dimensions. There we show that variations in adoption of market action can be related to heterogeneity in these dimensions.} In our model, the existence of multiple equilibria with varying market participation rates is consistent with their work. MR emphasize the potentially negative impact of large community networks: boys who are in these networks choose to stay away from new opportunities. By contrast, girls, who are disadvantaged by the network take up the market action in larger numbers. This is in line with the predictions of our model: those outside the $q$-core of the network take up market activities, while those inside the $q$-core opt for network activity.

But our theoretical analysis goes beyond the group size effect highlighted in MR: we predict that market participation will be shaped by the $q$-core (and its generalization, the $q$-central core). And we establish that the impact of the network structure on behavior and on welfare and inequality can be large: this should motivate new empirical work.

Tourism and the Preservation of Indigenous Cultures.—The revival and preservation of endangered local cultures and languages is a major theme in social anthropology. Medina’s (2003) ethnographic work in the village of Succotz, situated next to the Mayan ruins of Xunantunich in Belize, illustrates how tourism markets may be instrumental in revitalizing and preserving local cultures and languages. She argues that since archaeological work made the site of Xunantunich available, tourism has presented new possibilities for Succotzenos to reclaim Maya identity and culture (Medina 2003, p. 361). Similarly, De Azeredo Grünewald’s (2002) anthropological study of the Pataxó Indians in Porto Seguro, Brazil, suggests that growth in tourism has facilitated a cultural revival. In addition to handicraft, this cultural revitalization is particularly visible in the use of indigenous languages.

We now analyze this evidence in light of our model. Let the market exchange action $y$ stand for tourism activity (e.g., touring, selling handicrafts) and the network exchange action $x$ stand for cultural activity (e.g., learning the indigenous language). The returns to $x$ depend on the number of neighbors who adopt it: for instance, the returns to learning a local language depend on the number of people in one’s network one can speak the local language with. In contrast, the returns to $y$ depend on (exogenous) market opportunities. As discussed above, $x$ and $y$ may reinforce each other: cultural activities expand tourism activities (due to tourists’ cultural demand), and tourism activities increase the benefits of engaging in cultural activities (e.g., through increased status of local cultures, increased business opportunities). This corresponds to a case of $x$ and $y$ being complements.

First, Proposition 1 predicts that an increase in market opportunities should raise the returns and hence the viability of local culture and language. This is consistent with the evidence. Furthermore, research in sociolinguistics points to the importance of social structure in the preservation and revitalization of local cultures and especially languages. In particular, close-knit social networks are necessary for language revitalization, while the loosening of social networks is an important factor in language erosion (see, e.g., Milroy and Milroy 1985, 1999; Fishman 1990, 1991;
These findings are consistent with our predictions. Second, our model (Propositions 2 and 5) also predicts an increase in both welfare and inequality, as those who can benefit from both \( x \) and \( y \) gain larger payoffs than those who cannot. This is consistent with the view, widely held in the literature on tourism and culture. That while tourism can generate opportunities for economic development, this process can be highly uneven in nature and may foster inequality (Cole and Morgan 2010, p. xv). This is especially true in the case of tourist enclaves (see, e.g., Mbaiwa 2005).

To the best of our knowledge, the model presented in this paper is the first to relate tourism markets, social networks, and the revival of local cultures. In particular, our model provides clear predictions with respect to the conditions under which tourism markets could have such an impact (i.e., where social networks have enough closure and density, as captured by the \( q \)-core). This should help guide future empirical work in this field.\(^{12}\)

### B. Information Technology

The development and spread of modern information and communication technologies has had large economic effects. We now illustrate how our model helps to explain important empirical phenomena associated with these technologies.

**The Digital Provide: Networks and Mobile Phones.**—The widespread adoption of mobile telephones in developing countries has been extensively studied (see, e.g., Donner 2006; Aker and Mbiti 2010). Jensen (2007) studies their economic effects on fishermen in Kerala, India. Prior to the introduction of mobile phones in 1997, fishermen fished and sold their catch almost exclusively within their local catchment zone, which led to high levels of waste and price discrepancies between different markets on the coast. The introduction of mobile phones changed this state of affairs.\(^{13}\) Fishermen use their phone to share information about prices, demand and supply with friends and relatives, potential buyers, and auctioneers they are connected to (Srinivasan and Burrell 2013). By 2001, more than 65 percent of all fishing boats in Kerala owned a mobile phone. Mobile phones increased profits across the board, but they also increased economic inequality among the fishermen.

We map our model onto this empirical context. Let the network action \( x \) stand for “information sharing” with neighbors (e.g., information about prices, local demand, or fishing sites). The payoffs to \( x \) depends positively on the number of neighbors (e.g., buyers, auctioneers, and friends) who exchange information. Let the market action \( y \) stand for selling fish. We interpret the introduction of mobile phones as a decrease in the cost of sharing information with neighbors (i.e., a decrease in \( p_x \), in Example 1). Exchanging information via the use of a mobile phone increases the

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\(^{12}\) Growth in tourism will generate demand for traditional culture goods and products. This offers a simple route through which tourism may strengthen traditional cultures. We thank an anonymous referee for pointing this out. But we would like to note that, in our model, this demand effect can be accommodated through a change in the price of market activity (\( p_y \), in Example 1). The mechanisms identified in our model complement this demand effect. In particular, our results predict that a community that has a larger \( q \)-core would respond more to tourism markets as compared to a community with a small \( q \)-core.

\(^{13}\) Jensen (2007, p. 891) notes that fishermen with mobile phones “often carry lists with the numbers of dozens or even hundreds of potential buyers” whom they are connected to.
returns to \( y \): this mirrors Jensen’s model. So the incentives to buy mobile phones will be larger for those with more connections: we are in the world where network-based activity and market activity are complements.

Theorem 2 and Proposition 1 predict that a decrease in \( p_x \) may make \( x \) profitable when combined with \( y \) for better connected fishermen. Jensen (2007) demonstrates that bigger boats adopted mobile phones the most, while Srinivasan and Burrell (2013) present evidence on a positive correlation between boat size and connections (especially with auctioneers). The prediction of our extended model (with heterogeneous individuals) is thus in line with the evidence. Further, Proposition 2 predicts an unambiguous increase in aggregate welfare following a decrease in \( p_x \). This is in line with Jensen’s (2007, p. 913) finding of “net welfare gains for both buyers and sellers, due to more efficient allocation of fish.” Finally, Proposition 5 suggests that the introduction of mobile phones should lead to an increase in inequality: this again is consistent with the evidence.

We comment briefly on the relation between our model and the theoretical model in Jensen (2007). Jensen (2007) explains the greater adoption of mobile phones by larger boats in terms of higher returns to more accurate price information for boats that have a larger catch. The extended model presented in online Appendix B3 incorporates individual-level heterogeneity: there we show that all our main results extend if this heterogeneity is positively correlated with differences in network connections. Thus, the main empirical finding in Jensen (2007) can be accommodated within our framework. Similar observations hold for findings on welfare and inequality in Jensen (2007): they mirror Propositions 1 and 5 above.

However, the focus of our paper on the relation between social networks and markets is quite different from Jensen (2007): our model predicts that the adoption of mobile telephones by (suitably connected) fishermen, by enabling greater social communication, will strengthen social networks. This finding addresses a substantive question on the relation between markets and social networks, and clearly goes beyond the findings in the Jensen (2007) paper.

**Online Social Networks and the Decline of Traditional Media.**—The explosive growth of online social networks is one of the defining features of the last decade. Some of these networks (e.g., Facebook) have, over time, become prominent platforms for news sharing. The Reuters Institute for the Study of Journalism (RISJ) reports that more than one-half of the population of many countries (e.g., Brazil, Spain, Italy, and Finland) use Facebook for news purposes. The use of online social networks is strongly related to age: in the countries it surveyed, the RISJ reports that roughly 40 percent of 18–24 year-olds find news via online social networks, as opposed to only 17 percent for people aged over 55. This rise of online news exchange is mirrored in a sharp decline of traditional media such as print newspapers (Newman 2009; Currah 2009).

These sweeping changes in the media can be analyzed through the lens of our model. Let action \( x \) refer to information exchange among neighbors, while action \( y \) refers to purchase of print newspapers (or a subscription to TV channels). The returns to \( x \) clearly depend on how many friends exchange and discuss news in the network. In line with the payoff function in Example 1 we denote by \( p_x \) the time cost of sharing information with friends (e.g., sharing articles or posting information...
on Facebook), while \( p_y \) represents the price of marketed media. Since the same news can be accessed from both sources, traditional media and social networks are (imperfect) substitutes. Indeed, online news consumption and traditional media consumption (i.e., print and TV) are strongly negatively correlated (RISJ 2014, p. 45).

The key to the changes observed in the last decade or so is the dramatic fall in the price of exchanging information within networks, due to technologies such as Facebook. Theorem 2 and Proposition 1 predict that with a fall in \( p_x \), better connected individuals (i.e., individuals in the \( q_1 \)-core) will switch from the market action \( y \) to the network action. Thus, our model predicts that exchange in social networks should be particularly popular among well connected individuals. Empirical work shows that connections decline in age. Thus, our model predicts that online news engagement must decline with age. This is consistent with the data (see, e.g., RJIS 2014).

To the best of our knowledge, the present paper offers the first formal model of social networks that studies the relationship between online engagement and traditional print newspapers. As in the previous applications, here again our theory offers tractable predictions on the role of a specific topological property of networks in understanding behavior. Moreover, Propositions 4 and 5 predict that a rise in aggregate welfare and inequality should accompany a decrease in \( p_c \). Here the available evidence is suggestive: online social networks users have access to a much wider range of news and information sources than individuals using only traditional media (Currah 2009; RISJ 2014). Given the size of the online community, aggregate welfare (measured by access to information and news) has probably gone up, but the disparities in participation on online media also suggest that inequality in information access has probably increased.

V. Concluding Remarks

Personal relationships provide opportunities for economic exchange. Individuals also obtain goods and services through anonymous exchange, namely, markets. The interaction between communities and markets remains a central theme in the social sciences. Some social scientists and philosophers have argued that the expansion of markets, accompanied by wide-ranging changes in attitudes and institutions, can crowd out social ties. Others have asserted that markets raise the returns from reciprocity and thereby strengthen social ties. These conflicting views are reflected in the wide variety of empirical evidence.

We develop a model where individuals located in a social network choose a network action and a market action. The key to our results and to understanding the empirical patterns is the relation between the two activities, i.e., whether they are (strategic) complements or substitutes. Our first result provides a characterization of individual behavior in terms of the \( q \)-core of the social network. We show that in the case of substitutes, it is the individuals who benefit the least from network exchange (i.e., individuals outside the \( q \)-core) who adopt markets. Conversely, in the case of complements, only well connected individuals find markets attractive. Markets always raise aggregate welfare if the two activities are complements, but may lower welfare when the two activities are substitutes. Inequality in social networks is reinforced by markets in case of complements, but lowered in the case of substitutes.
Our model provides a parsimonious framework to organize evidence on a broad range of empirical phenomena. It also yields novel predictions with respect to the relationship between a measure of networks—the $q$-core—and behavior. These predictions should help guide future empirical research that uses more fine-grained, individual-level network data.

We conclude with a few remarks on some limitations of our model and avenues for future research. First, heterogeneity in connections plays a key role in our analysis. However, other forms of heterogeneity might matter and mitigate or overturn the effects of network heterogeneity. Second, we assume the network and market actions to be either complements or substitutes for all individuals. However, there may be situations where markets and networks are substitutes for certain individuals and complements for others. Our arguments will need to be modified to study behavior in such situations. Third, a key assumption of our model is that market activity yields exogenous payoffs. There are a number of reasons why this assumption may fail: we have already discussed the role of market thickness. Another consideration is that the performance of markets may be affected by institutions—such as law enforcement agencies or the state—whose quality may depend on the network structure.\footnote{See Putnam (1993) for a study of the relation between social ties and the performance of the state.} Finally, we should add that the network is taken as fixed: the study of how social networks evolve as they interact with markets is an important subject for future work.

APPENDIX

A. Main Proofs

PROOF OF THEOREM 1:

The argument for existence of equilibrium in case of complements is standard and is sketched in the main text. Details are omitted. We next take up the case of substitutes.

(i) $\pi_y \leq 0$: If $\phi_0(0) > 0$, then $a_i^* = (1, 0)$ for all $i \in N$ is an equilibrium. If $\phi_0(0) \leq 0$, then $a_i^* = (0, 0)$ for all $i \in N$ is an equilibrium.

(ii) $\pi_y > 0$: If $\phi_1(0) \leq \pi_y$, then $a_i^* = (0, 1)$ for all $i \in N$ is an equilibrium. Lastly, if $\phi_1(0) > \pi_y$, then $a_i^* = (1, 1)$ for all $i \in N$ is an equilibrium (due to complementarity in returns from action $x$ across individuals).

Uniqueness.—Suppose that there exist two distinct profiles $a$ and $a'$ that are both maximal equilibria. This means that there exist individuals $i$ and $j$ such that $i$ does strictly better under $a$, while $j$ fares strictly better under $a'$.

Consider first the case of complements. Define a new profile $\hat{a}$, with $\hat{x}_i = \max\{x_i, x'_i\}$ and $\hat{y}_i = \max\{y_i, y'_i\}$ for all $i$. If $\hat{a}$ constitutes an equilibrium, then it follows that $\hat{a}$ Pareto-dominates $a$ and $a'$ as there is a strict inequality for at least a pair of agents. This contradicts the hypothesis that $a$ and $a'$ are maximal.
equilibria. If \( \hat{a} \) does not constitute an equilibrium, then iterate using best responses starting from \( \hat{a} \). Observe that all actions are complements, so best responses can only lead to an increasing number of individuals choosing \( x = 1 \) and/or \( y = 1 \). As in the existence proof, this process converges and the limit is an equilibrium. Note that at every iteration stage, the payoffs of every individual are weakly rising relative to \( \hat{a} \), which again contradicts the hypothesis that \( a \) and \( a' \) are maximal.

Finally, consider the case of substitutes. Construct a profile \( \hat{a} \), where \( \hat{x}_i = \max\{x_i, x_i'\} \) and \( \hat{y}_i = \min\{y_i, y_i'\} \) for all \( i \). Suppose that \( \hat{a} \) constitutes an equilibrium. Clearly, the payoffs of all individuals choosing \( x = 1 \) under either \( a \) or \( a' \) must be weakly larger in \( \hat{a} \) (due to local complementarity in \( x \)). Note also that an individual \( k \) switches from \( y_k = 1 \) (or \( y_k' = 1 \)) to \( \hat{y}_k = 0 \) only if \( \min\{y_k, y_k'\} = 0 \). As the payoffs from \( y \) are independent of others’ choices, this must entail a weak increase in individual \( k \)'s payoffs. Hence, \( \hat{a} \) Pareto-dominates \( a \) and \( a' \), a contradiction to the hypothesis that \( a \) and \( a' \) are maximal equilibria. The case where \( \hat{a} \) does not constitute an equilibrium can be studied by iteration as in the complements case above: details are omitted. ■

**THEOREM 3:** Suppose that Assumptions 1 and 2 hold. Let \( a^* \) be the maximal equilibrium

(i) Suppose that \( q_3 < q_1 \) and \( q_4 < q_2 \).

• **Strong substitutes:** \( a^*_i = (1, 0) \) if and only if \( i \in g^{q_1} \). If \( i \not\in g^{q_1} \), then \( a^*_i = (0, 0) \) in case \( \pi_y \leq 0 \), and \( a^*_i = (0, 1) \) in case \( \pi_y > 0 \).

• **Strong complements:** \( a^*_i = (1, 1) \) if and only if \( i \in g^{q_2} \). If \( i \not\in g^{q_2} \), then \( a^*_i = (0, 0) \) in case \( \pi_y \leq 0 \), and \( a^*_i = (0, 1) \) in case \( \pi_y > 0 \).

(ii) Suppose that \( q_3 \geq q_1 \) and \( q_4 \geq q_2 \).

• **Substitutes:** \( a^*_i = (1, 0) \) if and only if \( i \in g^{q_2} \) and \( k_i(g^{q_2}) > q_3 \); and \( a^*_i = (1, 1) \) if and only if \( i \in g^{q_2} \) and \( k_i(g^{q_2}) \leq q_3 \). If \( i \not\in g^{q_2} \), then \( a^*_i = (0, 1) \).

• **Complements:** \( a^*_i = (1, 1) \) if and only if \( i \in g^{q_1} \) and \( k_i(g^{q_1}) > q_4 \); and \( a^*_i = (1, 0) \) if and only if \( i \in g^{q_1} \) and \( k_i(g^{q_1}) \leq q_4 \). If \( i \not\in g^{q_1} \), then \( a^*_i = (0, 0) \).
The proof to Theorem 3 follows from Theorem 1 and from the definition of the thresholds developed in the main text. We illustrate the difference between strong and nonstrong substitutes with Figure A1: the case for complements is analogous. Players’ payoffs are as in Example 1. In panel A, \( \theta = -\frac{3}{5}, p_y = 0.1, p_x = 2 \); it can be easily shown that \( q_1 = 2.9 \) and \( q_3 = 1.5 \) (where \( q_1 \) corresponds to the value of \( \chi_i \) when \( \phi_0( \cdot ) \) crosses \( \pi_y \), while \( q_3 \) corresponds to the point where \( \phi_0( \cdot ) \) crosses \( \phi_1( \cdot ) \)). Observe that for any \( \chi_i \in [0, 2] \), player \( i \) strictly prefers to play \( a_i = (0, 1) \). Since \( q_3 < q_1 \), she strictly prefers \( a_i = (1, 0) \) for any \( \chi_i \geq 3 \); hence, \( a_i = (1, 1) \) is always dominated by another action. In panel B, \( \theta = -\frac{1}{3}, p_x = -1.5, p_y = 2 \); hence, \( q_1 = 3.5 \) and \( q_3 = 4.5 \). In this case, actions \( x \) and \( y \) are not strong substitutes. Moreover, since \( q_3 > q_1 \), observe that \( a_i = (1, 1) \) is optimal for any \( \chi_i = 4 \).

The proofs of Propositions 1–4 (and related lemmas) are presented in the online Appendix to the paper.

PROOF OF PROPOSITION 5:
Throughout this proof, we use the subscripts 0 and 1 to refer to outcomes prior to and after the introduction of the market action \( y \), respectively.

Substitutes.—First note that if \( \pi_y \leq 0 \) and \( x \) and \( y \) are substitutes, then \( y_i^* = 0 \) at equilibrium for all \( i \in N \), which leaves \( \mathcal{R}(g) \) unchanged. We thus restrict attention to cases where \( \pi_y > 0 \).

Consider the case where \( \min \{ \Phi_i(a_{i,0}, a_{i,0}^- | g) \} \in N \geq \pi_y \). Since \( \pi_y > 0 \), then clearly \( \min \{ \Phi_i(a_{i,0}, a_{i,0}^- | g) \} \in N > 0 \). This means that the individual with minimum payoffs before the introduction of \( y \), say individual \( j \), chooses \( x_{j,0} = 1 \). If this is true for \( j \), this is true for all individuals, and so \( x_{i,0} = 1 \) for all \( i \in N \). Hence, the payoffs of any individual \( i \) before \( y \) can be written as

\[
\Phi_i(a_{i,0}, a_{i,0}^- | g) = \phi_0(k_j).
\]
It follows that $\mathcal{R}_0(g)$ can be written as follows:

$$\mathcal{R}_0(g) = \frac{1 + \phi_0(k)}{1 + \phi_0(k)},$$

where $k = \max\{k_i\}_{i \in N}$ and $k = k_j = \min\{k_i\}_{i \in N}$. Since $\phi_0(k_j) > \pi_y$, clearly $x_{i,1} = 1$; individual $j$ has no incentive to stop choosing $x_j = 1$, and if this is true for $j$, this is true for all other individuals. Hence, $x_{i,1} = 1$ for all $i \in N$, which means that $\mathcal{R}_1(g)$ can be written as follows:

$$\mathcal{R}_1(g) = \frac{1 + \phi_0(k) + \max\{0, \pi_y + \xi(k)\}}{1 + \phi_0(k) + \max\{0, \pi_y + \xi(k)\}}.$$

We conclude by noting that since $\xi(m)$ is decreasing in $m \in \mathbb{N}_+$, clearly $\mathcal{R}_1(g) < \mathcal{R}_0(g)$.

Consider next the case where $\min\{\Phi_i(a_{i,0}^*, a_{i,-0}^* | g)\}_{i \in N} < \pi_y$. Note that this immediately entails that $\min\{\Phi_i(a_{i,0}^*, a_{i,-1}^* | g)\}_{i \in N} < \min\{\Phi_i(a_{i,1}^*, a_{i,-1}^* | g)\}_{i \in N}$. Recall that action $y$, in the case of substitutes, never fosters the adoption of $x$; hence, $x_{i,0}^* = x_{i,1}^*$ for all $i \in N$. Consider now individual $j$, who has the highest payoffs after the introduction of $y$. If $a_{j,1}^* = (1, 0)$, then clearly max $\{\Phi_i(a_{i,0}^*, a_{i,-0}^* | g)\}_{i \in N} \geq \Phi_j(a_{j,1}^*, a_{j,-1}^* | g)$ since $j$ has (weakly) fewer neighbors choosing $x = 1$ after the introduction of $y$. Since $\min\{\Phi_i(a_{i,0}^*, a_{i,-0}^* | g)\}_{i \in N} < \min\{\Phi_i(a_{i,1}^*, a_{i,-1}^* | g)\}_{i \in N}$ and $\max\{\Phi_i(a_{i,0}^*, a_{i,-0}^* | g)\}_{i \in N} \geq \Phi_j(a_{j,1}^*, a_{j,-1}^* | g)$, then clearly $\mathcal{R}_1(g) < \mathcal{R}_0(g)$. Now if $a_{j,1}^* = (0, 1)$, then it follows immediately that $\mathcal{R}_0(g) = 1$, and so again $\mathcal{R}_1(g) < \mathcal{R}_0(g)$.

Suppose last that $a_{j,1}^* = (1, 1)$. If $\Phi_j(a_{j,1}^*, a_{j,-1}^* | g) \leq \max\{\Phi_i(a_{i,0}^*, a_{i,-0}^* | g)\}_{i \in N}$, then clearly again $\mathcal{R}_1(g) < \mathcal{R}_0(g)$. If $\Phi_j(a_{j,1}^*, a_{j,-1}^* | g) > \max\{\Phi_i(a_{i,0}^*, a_{i,-0}^* | g)\}_{i \in N}$, then note that $\mathcal{R}_0(g)$ and $\mathcal{R}_1(g)$, respectively, can be written as follows:

$$\mathcal{R}_0(g) = \frac{1 + \max\{\Phi_i(a_{i,0}^*, a_{i,-0}^* | g)\}_{i \in N}}{1 + \max\{0, \phi_0(k)\}};$$

$$\mathcal{R}_1(g) = \frac{1 + \pi_y + \phi_0(\chi_j^*) + \xi(\chi_j^*)}{1 + \pi_y + \max\{0, \phi_0(k) + \xi(k)\}}.$$
Complements.—We first prove that whenever $x$ and $y$ are complements and $\mathcal{M}(g) \in (0, 1), \mathcal{R}_1(g) > \mathcal{R}_0(g)$. First, note that if $\mathcal{M}(g) \in (0, 1)$, then there exists an individual $i$ and an individual $j$ such that $y^*_i = 1$ and $y^*_j = 0$. From Theorem 3, we know that $y_i^* \leq 0$, and if $y^*_j = 1$, then $x^*_i = 1$. Suppose without loss of generality that individuals $i$ and $j$ have respectively the highest and lowest payoffs in $N$ after the introduction of $y$. Recall that in the case of complements, $x^*_i \geq x^*_j$ for all $i \in N$, and so $\Phi_i(a^*_i, a^*_j) \leq \Phi_i(a^*_i, a^*_j) \leq \Phi_i(a^*_i, a^*_j) \leq \Phi_i(a^*_i, a^*_j)$ for all $i \in N$.

We begin with an important result.

**Lemma 3:** If $\pi_y \leq 0$ and $x$ and $y$ are not strong complements, then $x^*_i = x^*_i$ for all $i \in N$.

**Proof:**

A contrario, suppose that there is an $i$ such that $x^*_i = 0$ and $x^*_i = 1$. Since $x^*_i = 1$, then it follows from Theorem 3 that $i \in g_0$. Since $\pi_y \leq 0$, it follows from equation (6) that $q_1$ is left unchanged by $y$: hence, prior to the introduction of markets, $i \in g_0$. However, since $i \in g_0$ before the introduction of $y$, then $x^*_i = 1$, which is a contradiction. 

Our proof then proceeds in two steps.

**Step 1:** We first show that 

$$\min \{\Phi_i(a^*_i, a^*_j) \mid g\} = \min \{\Phi_i(a^*_i, a^*_j) \mid g\}.$$ 

Suppose first that $a^*_i = (0,0)$. Then, $\Phi_i(a^*_i, a^*_{i,j}) = 0$, and it must be the case that $j$ was also choosing $a^*_j = (0,0)$ before the introduction of $y$. Hence, clearly, 

$$\min \{\Phi_i(a^*_i, a^*_j) \mid g\} = \min \{\Phi_i(a^*_i, a^*_j) \mid g\}.$$ 

Suppose second that $a^*_i = (1,0)$. Since $j$ is the worst-off individual, then clearly $x^*_j = 1$ for all $i \in N$, and from Lemma 3, it must also be the case that $x^*_i = 1$ for all $i \in N$. It follows that $\Phi_i(a^*_i, a^*_j) = \Phi_i(a^*_i, a^*_j) = \Phi_i(a^*_i, a^*_j) = \Phi_i(a^*_i, a^*_j)$. Lastly, note that $j$ must have been the worst-off individual before $y$: indeed, since $\Phi_i(a^*_i, a^*_j) = \Phi_i(a^*_i, a^*_j) = \Phi_i(a^*_i, a^*_j)$, it follows that $\min \{\Phi_i(a^*_i, a^*_j) \mid g\} = \min \{\Phi_i(a^*_i, a^*_j) \mid g\}$. Hence, with complements, whenever $\mathcal{M}(g) \in (0, 1)$, 

$$\max \{\Phi_i(a^*_i, a^*_j) \mid g\} = \max \{\Phi_i(a^*_i, a^*_j) \mid g\}.$$ 

Suppose that individual $i$ was also the best-off individual before the introduction of $y$. Since $y^*_i = 1$, then $\Phi_i(a^*_i, a^*_j) < \Phi_i(a^*_i, a^*_j)$. Suppose then that individual $i$ was not the best-off individual prior to the introduction of $y$, and that individual $l$ was. Clearly, this entails that $a^*_i = (1,0)$, and so $a^*_i = (1,1)$.

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15 Indeed, suppose a contrario that $a^*_i = (1,0)$. We then know from Lemma 3 that $x^*_i = x^*_i$ for all $i \in N$. Since $\Phi_i(a^*_i, a^*_j) < \Phi_i(a^*_i, a^*_j)$, then we know that $k_i(g^*_i) < k_i(g^*_i)$. From Theorem 2, we thus know that $y^*_i \geq y^*_i$, which is a contradiction since $y^*_i = 1$ and $y^*_i = 0$. Hence, $a^*_i = (1,1)$. 

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If \( a^*_l = (1, 1) \), then we know that \( \Phi_i(a^*_{l,0}, a^*_{l-1,0} \mid g) < \Phi_i(a^*_l, a^*_{l-1} \mid g) \), and since \( \Phi_i(a^*_l, a^*_{l-1} \mid g) = \Phi_i(a^*_{l,1}, a^*_{l-1,1} \mid g) \) by assumption, then clearly \( \max \{ \Phi_i(a^*_{l,0}, a^*_{l-1,0} \mid g) \}_{i \in \mathcal{N}} < \max \{ \Phi_i(a^*_{l,1}, a^*_{l-1,1} \mid g) \}_{i \in \mathcal{N}} \).

Steps 1 and 2, together, show that in the case of complements, whenever \( \mathcal{M}(g) \in (0, 1) \), \( \mathcal{R}_1(g) > \mathcal{R}_0(g) \). Lastly, note that the example in the main text (Example 4) constitutes a proof by construction that when \( \mathcal{M}(g) = 1 \), \( \mathcal{R}(g) \) can both increase or decrease following the introduction of \( y \).

REFERENCES


