Low-density superhard materials: Computational study of Li-inserted B-substituted closo-carboranes LiBC\textsubscript{11} and Li\textsubscript{2}B\textsubscript{2}C\textsubscript{10}

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Insertion of Li atoms into a B-substituted carbon cage produces two superhard compounds with relatively low density: LiBC\textsubscript{11} and Li\textsubscript{2}B\textsubscript{2}C\textsubscript{10}. For each structure, phonon frequencies across the whole Brillouin zone are positive, indicating dynamical stability. Electronic structure calculations indicate that they are semiconductors under ambient conditions. Estimates of the Vickers hardness, based on a semi-empirical model, highlight the incompressible nature of these two compounds. We then performed calculations on the ideal strengths of these two structures to confirm the hardness and investigate origins of the mechanical properties. Strikingly, both LiBC\textsubscript{11} and Li\textsubscript{2}B\textsubscript{2}C\textsubscript{10} can be classed as superhard materials, with hardness of 49 GPa and 41 GPa, respectively. The current results shed light on the properties of new superhard carbon cage structures more generally.

Introduction

The search for superhard materials with Vickers hardness, \(H \geq 40\) GPa has been an important focus for some time in materials science and technology. A well-known family of superhard materials is that comprising light elements (such as C\textsubscript{60}, B\textsubscript{12}C\textsubscript{60}, B\textsubscript{14}C\textsubscript{60}, diamond, B\textsubscript{C}\textsubscript{60}, \textsuperscript{7} B\textsubscript{C}_\textsuperscript{6}\textsuperscript{8}, \textsuperscript{9} B\textsubscript{C}_\textsuperscript{10}, \textsuperscript{10} B\textsubscript{C}_\textsuperscript{11}, \textsuperscript{11} pnnm-CN\textsuperscript{12}, \textsuperscript{13} CN\textsuperscript{13}, \textsuperscript{14} B\textsubscript{C}_\textsuperscript{14}\textsuperscript{15}, and c-BN\textsuperscript{16}), where strong covalent bonding between light elements often leads to the formation of rigid three-dimensional crystalline networks with extreme resistance against stresses across a wide range of loading conditions. The low thermal stability of diamond in oxidizing environments and the high synthetic cost of these traditional superhard materials, have stimulated the search for novel superhard materials exhibiting improved stability over a wide range of conditions with good properties.

Sodality-like cages (named after the cage zeolitic oxide) formed by groups of 12 atoms are thought to be rooted in some extraordinary properties. A good example is a new clathrate sodalite-like structure of BN, which has recently been predicted to be “superhard”, with a hardness of 58.4 GPa\textsuperscript{17}. Considering the important role carbon plays in the materials world, it is interesting to explore the effects of inserting metal atoms into sodalite-like C cages. With larger atomic radii, C cages display relatively smaller cavities, even for the smallest metal atom (we consider Li atoms in this work). This inevitably leads to structural destabilization: here we seek to explain the electronic origins of such destabilization. Since a closed-shell electron configuration is helpful to stabilise a compound, and the C atoms forming the cages form already have a closed-shell electronic configuration, the insertion of electropositive Li atoms donate electrons to the antibonding bands and weaken the bonding. The insertion of Li does not, of itself, lead to superior hardness, but it does stabilise superhard phases. It is necessary to maintain the strong chemical bonds by adjusting the number of electrons in the system. One possible solution is to substitute C atoms with electron deficient B atoms in the framework as proposed by Tao Zeng et al. in their recent work\textsuperscript{18}. In fact, isolated closo-carboranes, like 1,5-C\textsubscript{3}B\textsubscript{12}H\textsubscript{12}, 1,6-C\textsubscript{3}B\textsubscript{12}H\textsubscript{12}, 2,4-C\textsubscript{3}B\textsubscript{12}H\textsubscript{12}, have been synthesised experimentally\textsuperscript{19}, therefore there is a good possibility that the bulk solid closo-carboranes may also be synthesised.

We explored the possibility of stabilising the sodalite-like C cage by two strategies of coupled Li-insertion – B-substitution and proposed two compounds (LiBC\textsubscript{11} and Li\textsubscript{2}B\textsubscript{2}C\textsubscript{10}), stable at ambient conditions. Electronic structure calculations suggest that both compounds are semiconductors with band gaps of 0.6-1.3 eV. Subsequent first-principles study of their mechanical properties indicates that both compounds are superhard. Moreover, they are also the lightest compounds among the family of known light element superhard materials. The predicted stable, superhard, Li/B/C ternary compounds, with remarkably low density, may have great potential importance for technological application and shed light on the general principles on the rational design of superhard structures.

Computational methods

First-principles electronic structure calculations were based on density functional theory (DFT) and performed using the Vienna \textit{ab initio} simulation package (VASP)\textsuperscript{20}. The generalised gradient approximation (GGA) in the scheme of Perdew-Burke-Ernzerh of (PBE)\textsuperscript{21} was used to describe the electron exchange correlation.
Results and discussion

The sodalite-like carbon cage adopts a remarkable cubic configuration (Im3m, Pearson symbol c14), with all 12 carbon atoms sharing identical point symmetry (Fig. 1(c)). The calculated cubic cell parameter is in good agreement with the hypothetical structure proposed by Filipe et al.22. The small difference (0.986%) between our calculated cubic unit cell parameter and that of Filipe et al.22 can be explained by the different electron exchange correlation interactions chosen (PBE in this work and LDA in Ref. 32). To maintain the total number of electrons after inserting a Li atom into the cavity, we replaced one of the framework C atoms with a B and then the structure was fully re-optimised. The resulting LiBC11 maintains the framework topology although the cage is slightly deformed. We further placed two Li atoms into two cavities formed by a double B-substituted C cage. In this case, there are five distinct ways for the double B substitutions. We examined all the possibilities and found that the most energetically favourable structure is to replace two non-adjacent atoms of one C-C bond. This configuration is in agreement with a previous study35. The unit cells of the resulting LiBC11 and Li2B2C10 (lowest enthalpy) structures are shown in Fig. 1, and the corresponding equilibrium structural parameters and space group at ambient pressure are listed in Table 1. The bond length of B-C is 1.63 Å and C-C bond lengths range from 1.55 Å to 1.60 Å in LiBC11. In Li2B2C10 the B-C bond length is 1.65 Å while C-C bond lengths are 1.57/1.59 Å.

Table 1 The space group and calculated equilibrium structural parameters: cubic unit cell parameters and Wyckoff positions of LiBC11 and Li2B2C10 at ambient pressure.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Space group</th>
<th>Lattice parameter (Å)</th>
<th>Atomic coordinates (r/a, fractional)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C12</td>
<td>Im3m</td>
<td>a=4.383</td>
<td>C: 12d(0.000, 0.250, 0.500)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a=4.383</td>
<td></td>
</tr>
<tr>
<td>LiBC11</td>
<td>Pmna2</td>
<td>a=4.470</td>
<td>Li: 1a(0.000, 0.000, 0.058)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b=4.469</td>
<td>B: 1c(0.500, 0.000, 0.249)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c=4.441</td>
<td>C: 2e(0.240, 0.000, 0.504)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Li2B2C10</td>
<td>P43/mnm</td>
<td>a=4.578</td>
<td>Li: 2d(0.250, 0.000, 0.000)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c=4.424</td>
<td>B: 2e(0.000, 0.000, 0.250)</td>
</tr>
</tbody>
</table>

Fig. 1 (colour online) Crystal structures of (a) LiBC11, (b) Li2B2C10 and (c) C22. The Li atoms are represented as large blue spheres, while B and C atoms are represented as small spheres, pink and grey respectively.

To investigate the electron structure, the valence band structures and the corresponding density of states projected onto the atomic orbits (PDOS) were computed and the results are shown in Fig. 2. Here, the zero energy refers to the top of the valence band. LiBC11 and Li2B2C10 are both semiconductors characterised by indirect band gaps of 1.3 eV and 0.6 eV, respectively. The HSE hybrid functional usually provides a better description of the electronic band structure33,34 (especially the band positions), but is computationally costly. In view of the fact that density functional theory, especially the semi-local PBE functional we used here, tends to underestimate the band gap of this class of compounds by 30%–50%, we predict the experimental band gaps should be in the range of 1.9–2.6 eV and 0.9–1.2 eV for LiBC11 and Li2B2C10, respectively.

These values are considerably smaller than those of other light-element superhard materials (such as diamond, B4C, BC3, BN, and cubic BN), which typically have band-gaps ranging from 3.0 eV to 3.6 eV. The small band gaps may suggest potential optical applications of these materials. A smaller gap is expected as an Li atom is introduced into the system since it will occupy the bottom of the conduction band. When a second B is inserted into the structure, the bottom of the conduction band [in Fig. 2(c)] is lowered, resulting an even lower band gap compared with that in Fig. 2(a). The PDOS plots demonstrate that C-2p electrons contribute most to both the upper conduction bands and lower valence bands.

The calculated electron localization functions show both LiBC11 and Li2B2C10 are ionic with the Li atoms donating their valence electrons to the B such that the effective configuration of B becomes $s^2p^2$, as for the carbon. The ionic LiBC11 and Li2B2C10 compounds are isoelectronic with the C22 cage. Therefore, the strong covalent bonding between C-C and B-C are maintained thus preserving the stability of the rigid three-dimensional crystalline networks.
In Table 2, information on the calculated volume per unit cell, the volume per atom and the density of LiBC\(_2\) and Li\(_2\)B\(_2\)C\(_7\) are listed. Comparisons are made with the corresponding values for the empty C\(_{12}\) cage structure and for several previously proposed light-element superhard materials. It can be seen that the cage structures display larger volumes than non-cage structures. Specifically, the volume of the unit cell of Li\(_2\)B\(_2\)C\(_{10}\) (92.71 Å\(^3\)) is substantially larger than that of LiBC\(_2\) (88.71 Å\(^3\)). The volume per atom (the volume per unit cell divided by the total number of atoms in the unit cell) shows the same trend as the volume per unit cell: 6.82 Å\(^3\)/atom and 6.60 Å\(^3\)/atom for caged LiBC\(_2\) and Li\(_2\)B\(_2\)C\(_{10}\), respectively, in contrast to smaller values (ranging from 5.68 to 6.00 Å\(^3\)/atom) for other materials. The densities for LiBC\(_2\) and Li\(_2\)B\(_2\)C\(_{10}\) are 2.805 and 2.787 g/cm\(^3\), respectively. These values are much lower than the densities of other well-known superhard materials such as 3.510 g/cm\(^3\) for diamond, 3.483 g/cm\(^3\) for c-BN and 3.265 g/cm\(^3\) for B\(_4\). It is worth noting that even though Li\(_2\)B\(_2\)C\(_{10}\) has a smaller volume per atom than LiBC\(_2\), because the formula unit contains more light atoms, it is less dense than LiBC\(_2\), making it the lightest superhard material reported. This property may be related to the unexpected and so-far unidentified hard and transparent carbon phase found in a rock sample with an estimated density of 2.5 g/cm\(^3\) from the Popigai impact crater in Russia\(^{35}\).

The thermodynamic stability of the two Li-B-carbides with respect to the decomposition into the respective elements, can be quantified by the formation enthalpies of two different reaction routes. The positive reaction enthalpies of reactions (1) and (2) below indicate that LiBC\(_2\) and Li\(_2\)B\(_2\)C\(_{10}\) are thermodynamically metastable. Li-substitution, as expected, is highly endothermic, as seen in reactions (3) and (4). B-substitution helps to stabilise Li-insertion into the carbon cages. The successful synthesis of several isolated closo-carboranes, like 1,5-C\(_2\)B\(_2\)H\(_7\), 1,6-C\(_2\)B\(_2\)H\(_7\) and 2,4-C\(_2\)B\(_2\)H\(_7\), indicate that there is a good possibility that the bulk solid closo-carboranes can be also synthesised although there are experimental challenges to overcome the activation barriers. Nonetheless, the example of the existence of metastable diamond demonstrates that routes to the synthesis of these compounds may indeed be tractable.

\begin{align}
\text{Li}+\text{B}+\text{C}_2 &= \text{LiBC}_2 + \text{C}; \Delta H = 4.937 \text{ eV/f. u.} \\
2\text{Li}+2\text{B}+2\text{C}_2 &= 2\text{LiBC}_2 + 2\text{C}; \Delta H = 4.233 \text{ eV/f. u.} \\
\text{B}+\text{LiBC}_2 &= \text{LiBC}_2 + \text{C}; \Delta H = -2.406 \text{ eV/f. u.} \\
2\text{B}+2\text{LiBC}_2 &= 2\text{LiBC}_2 + 2\text{C}; \Delta H = -5.642 \text{ eV/f. u.}
\end{align}

The structural stability of both LiBC\(_11\) and Li\(_2\)B\(_2\)C\(_{10}\) has been investigated by calculations of their phonon band structures and elastic constants. As can be seen in Fig. 3, there are no imaginary phonons in the Brillouin zone for both LiBC\(_11\) and Li\(_2\)B\(_2\)C\(_{10}\), confirming their dynamical stability. Based on the mechanical stability criteria\(^{36}\) of orthorhombic or tetragonal crystals (where combinations of elastic constants have to exceed or equal zero), the calculated elastic constants also indicate that both LiBC\(_11\) and Li\(_2\)B\(_2\)C\(_{10}\) are mechanically stable under ambient pressure.

![Fig. 3](image)

Fig. 3 (colour online). Calculated phonon dispersion curves for (a) LiBC\(_11\) and (b) Li\(_2\)B\(_2\)C\(_{10}\) at 0 GPa.

The mechanical hardness of a material is the ability to resist plastic deformation from hydrostatic compression, tensile load, and shear. Therefore, a superhard material usually requires a high bulk modulus (B\(_0\)) to resist volume decrease created by compression and also high shear modulus (G\(_0\)) to limit the creation and mobility of dislocations. In Table 3 we list the bulk and shear moduli of LiBC\(_11\) and Li\(_2\)B\(_2\)C\(_{10}\) derived from the calculated elastic constants. The calculated values for B\(_0\) of LiBC\(_11\) and Li\(_2\)B\(_2\)C\(_{10}\) are 302 GPa and 208 GPa, and G\(_0\) are calculated to be 271 GPa and 232 GPa, respectively, indicating the highly incompressible nature of these structures. We further estimated the Vickers hardness (H\(_v\)) using a empirical model\(^{37}\) based on the correlation of the shear modulus with hardness. A comparison with the empty C\(_{12}\) cage structure is listed in Table 3. High hardness values of 48.8 GPa and 37.7 GPa are estimated for LiBC\(_11\) and Li\(_2\)B\(_2\)C\(_{10}\) respectively. These values are slightly lower than that of the bare C\(_{12}\) cage (51.5 GPa). To elucidate the microscopic mechanism of bond-deformation and breaking, we present below a first-principle strain-stress calculation, to further probe the mechanical properties of LiBC\(_11\) and Li\(_2\)B\(_2\)C\(_{10}\) under large structural deformations.

<table>
<thead>
<tr>
<th>Structure</th>
<th>C(_{11})</th>
<th>C(_{22})</th>
<th>C(_{33})</th>
<th>C(_{44})</th>
<th>C(_{55})</th>
<th>C(_{66})</th>
<th>C(_{12})</th>
<th>C(_{13})</th>
<th>C(_{23})</th>
<th>B(_0)</th>
<th>G(_0)</th>
<th>G(_0)/B(_0)</th>
<th>H(_v)</th>
</tr>
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<tbody>
<tr>
<td>C(_{12})</td>
<td>797</td>
<td>300</td>
<td>102</td>
<td>334</td>
<td>319</td>
<td>0.955</td>
<td>51.5</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>LiBC(_{11})</td>
<td>727</td>
<td>732</td>
<td>631</td>
<td>253</td>
<td>246</td>
<td>268</td>
<td>94</td>
<td>115</td>
<td>106</td>
<td>302</td>
<td>271</td>
<td>0.895</td>
<td>48.8</td>
</tr>
<tr>
<td>Li(_2)B(<em>2)C(</em>{10})</td>
<td>676</td>
<td>676</td>
<td>209</td>
<td>209</td>
<td>156</td>
<td>87</td>
<td>282</td>
<td>232</td>
<td>0.824</td>
<td>37.7</td>
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</tr>
</tbody>
</table>

Table 3 The calculated elastic constants C\(_{ij}\) (GPa), bulk modulus B\(_0\) (GPa), shear modulus G\(_0\) (GPa), G\(_0\)/B\(_0\) and Vickers hardness H\(_v\) of C\(_{12}\), LiBC\(_{11}\) and Li\(_2\)B\(_2\)C\(_{10}\).
The ideal strength, the maximum stress that a material can sustain, is the upper bound to the critical stress for crack formation and dislocation nucleation in the material. When the applied stress exceeds the ideal strength, the crystal structure will collapse even at zero temperature. Therefore, a strain-stress calculation also serves to describe the upper bound limit of the hardness of a material.

We have examined the stress-strain relations of LiBC$_2$ and Li$_2$B$_4$C$_{10}$ under tensile loadings. The results are shown in Fig. 4 (upper panels). LiBC$_2$ shows remarkably strong stress response in the <011> directions with the peak tensile stress reaching 110 GPa. The peak tensile stresses along the <011>, <110>, <100>, <111>, and <101> directions are also high, ranging from 62 GPa to 86 GPa. The lowest tensile strength, corresponding to the weakest direction, lies along the <011> direction and peaks at 49 GPa. Our results show that when the tensile stress exceeds 49 GPa, the <011> type planes of the crystal first become unstable against cleavage fracture. For Li$_2$B$_4$C$_{10}$, the highest tensile strength is 99 GPa along the <100> directions, followed by 91 GPa along the <001> directions. The weakest tensile strength is along <101> type directions with a value of 49 GPa, indicating that Li$_2$B$_4$C$_{10}$ would fail by cleavage in the <101> direction at 49 GPa. Similar to the estimations from the empirical model, the conclusion that LiBC$_2$ and Li$_2$B$_4$C$_{10}$ are both superhard is valid.

We now turn to ideal shear strength in the tensile-weakest (easiest-cleavage) plane. Various non-equivalent directions along the shear-sliding planes have been systematically studied under shear deformations, as shown in Fig. 4 (lower panels). It can be seen that the shear strengths of LiBC$_2$ range from 52 GPa to 63 GPa, with the weakest one being the <011>[011] shear system with the value of 49 GPa. Therefore, the resulting hardness of LiBC$_2$ compound is calculated to be 49 GPa, which is in good agreement with the estimated value from the empirical microscopic hardness model (0.41%). For Li$_2$B$_4$C$_{10}$ the shear strengths fall over a narrow range from 53 GPa to 56 GPa for most of the shear directions, apart from (011)[010] direction, along which the Li$_2$B$_4$C$_{10}$ crystal is unstable against slip on crystallographic planes when the shear stress exceeds 41 GPa. The lowest shear strength of 41 GPa is smaller than the ideal tensile strength, suggesting that the hardness of Li$_2$B$_4$C$_{10}$ is slightly lower. Nevertheless, since the lowest shear strength surpasses the threshold (40 GPa), it still can be classified as superhard materials.

Conclusion

Using first-principle calculations, we have investigated the structural, electronic, dynamical, and mechanical properties of two Li-doped B-substituted carbon cages: LiBC$_2$ and Li$_2$B$_4$C$_{10}$. The electronic structures suggest that both compounds are semiconductors. Phonon dispersion and elastic constant calculations demonstrate that both are dynamically and mechanically stable at ambient condition. First-principles strain-stress relations at large strains were also computed to examine the structural and mechanical properties. The established ideal tensile strength of 49 GPa in the <011> direction and ideal shear strength of 41 GPa along (101)[010] direction both suggest that LiBC$_2$ and Li$_2$B$_4$C$_{10}$ may be regarded as superhard materials.

Acknowledgements

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