DYNAMIC LIMIT PRICING*

Flavio Toxvaerd†

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ABSTRACT. I study a multi-period model of limit pricing under one-sided incomplete information. I characterize pooling and separating equilibria and their existence, and determine when these involve limit pricing. For some parameter constellations, the unique equilibrium surviving a D1 type refinement involves immediate separation on monopoly prices. For others, there are limit price equilibria surviving the refinement in which different types may initially pool and then (possibly) separate. Separation involves setting prices such that the inefficient incumbent's profits from mimicking are negative. As the horizon increases or as firms become more patient, limit pricing becomes increasingly difficult to sustain in equilibrium.

Keywords: Dynamic limit pricing, entry deterrence, dynamic signaling, equilibrium selection.

JEL Classification: D43, D82, L11, L41.

1. INTRODUCTION

Since Bain's (1949) pioneering work, limit pricing has been a staple of industrial economics. In a nutshell, limit pricing is the practice by which an incumbent firm (or cartel) deters potential entry to an industry by pricing below the profit maximizing price level. Early work on the subject took its cue from the casual observation that in some industries, firms price below the myopic profit maximizing price level *on a persistent basis*. This observation lead to the notion that by doing so, incumbent firms could somehow discourage potential entry which would otherwise have occurred, in effect by sacrificing profits in the short run in return for a maintenance of the monopoly position in the long run.

Bain (1949) noted that "[...] established sellers persistently or "in the long run" forego prices high enough to maximize the industry profit for fear of thereby attracting new entry to the industry and thus reducing the demands for their outputs and their own profits".

The present work revisits received wisdom on equilibrium limit pricing in dynamic contexts, by way of a dynamic extension of a simple static model of one-sided incomplete information in the spirit of Milgrom and Roberts (1982). I demonstrate that whereas some aspects of the standard (static) analysis may be preserved qualitatively when moving to dynamic contexts, the quantitative results may radically differ. The analysis shows that when the horizon is sufficiently long and the players sufficiently patient, limit pricing becomes infeasible altogether.

In this article, I analyze a model of limit pricing with one-sided incomplete information in which a simple entry game is repeated as long as entry has not occurred. In this model,

†Faculty of Economics, University of Cambridge. Email: fmot2@cam.ac.uk.

I identify two distinct regimes, a *monopoly price regime* and a *limit price regime*. In the monopoly price regime, limit price equilibria may exist but all such equilibria are ruled out by using a combination of equilibrium dominance and Cho and Kreps' (1987) criterion D1 at all information sets off the equilibrium path, as compared to a natural benchmark equilibrium in which the uninformed player makes use of all available information (in a sense that will be made precise). The unique equilibrium, using this notion, is one of immediate separation on monopoly prices. In the limit price regime, both pooling and separating equilibria may exist and all these involve limit pricing. I find that in the limit price regime, the basic logic of separating equilibria of a static single-round setting carries over to the separating equilibria of the dynamic setting. In particular, I find that by sacrificing enough at some (single) stage of the game, the efficient incumbent may credibly convey his identity to the entrant. Whether this signaling effectively precludes entry, and is thus worthwhile from the perspective of the incumbent firm, in turn depends on entrant's incentives to enter. In the dynamic setting, as the future becomes more important, the relevant conditions needed to deter entry are increasingly unlikely to be satisfied. Specifically, I show that as the horizon becomes longer, it becomes more difficult to deter entry simply because the entrant's one-off cost of entry may not outweigh a long sequence of post-entry profits, even if discounted. Similarly, I show that for a sufficiently patient entrant firm, an infinite sequence of discounted future profits will outweigh any bounded entry fee and thus make entry inevitable. In both cases, adding dynamics to a static limit pricing model makes entry deterrence through limit pricing more difficult (or impossible) to sustain as an equilibrium outcome. Thus immediate entry is likely to result, with each firm settings its respective monopoly price (regardless of the prevailing regime).

Although these results cast serious doubt on the viability of limit pricing, it should be mentioned that the basic tradeoff found in the static analysis can be recovered in the dynamic setting, if one disregards the caveats above and assume all incentive constraints to be satisfied. Even in this case, the dynamics of the model make somewhat unrealistic predictions. Specifically, one important difference with a static setting is that in the static setting, the benefits from deterring entry are bounded, whereas this is no longer the case in the dynamic setting, if the players are sufficiently patient. For a large enough discount factor and a sufficiently long horizon, the efficient incumbent needs to press the inefficient incumbent to make strictly negative profits from mimicking (e.g. by pricing below marginal cost). When the players are very patient, the short-term losses necessary to credibly signal to be a low-cost incumbent are unbounded.

Assuming that all the relevant feasibility constraints are satisfied, in the limit price regime all equilibria satisfying the D1 type refinement (*anchored* D1) belong to a single class, consisting of (i) a (possibly non-zero and possibly infinite) number of periods during which the two types of incumbent pool, (ii) a period in which the efficient type engages in costly signaling whereas the inefficient type reveals himself and invites entry and (iii) continuation play in which the efficient type charges monopoly prices in all subsequent periods and deters entry whereas the inefficient type competes against the entrant.

The welfare properties of these equilibria are not straightforward. It is true, as is the case in the static benchmark model, that in the period where separation takes place, welfare is unambiguously higher than it would be under symmetric information. This is because entry occurs under the same states of nature as under symmetric information, but the efficient type sets lower prices than would a monopolist. But if separation is preceded by periods with pooling, the conclusion is less clear cut. This is because pooling deters entry, which counterweights the benefits of lower prices set by the incumbent.[1] In the special case where the static pooling equilibrium yields lower welfare than under complete information, social welfare is unambiguously higher the earlier separation occurs. This is because immediate separation

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[1] For a nice discussion of the welfare properties of such equilibria, see Tirole (1988).

(which has good welfare properties) precludes periods with pooling (which have bad welfare properties). But if instead welfare under the static pooling equilibrium is higher than under complete information, then the welfare comparison of equilibria that differ in their timing of separation becomes impossible, without making explicit assumptions about parameters and the nature of post-entry competition. This is because one must then compare magnitudes of positive payoffs rather than comparing signs of payoffs.[2]

Empirical Evidence of Limit Pricing. Direct evidence of limit pricing is difficult to come by, for several reasons. First, limit pricing is a response to the *threat* of entry rather than to *actual* entry, as emphasized by Goolsbee and Syverson (2008). The threat of entry is typically not observed, but must instead be inferred from context, as the few existing empirical articles on limit pricing do. Second, if successfully carried out, limit pricing causes potential entrants to stay out of the industry. Again, it is not straightforward to identify the "absence of entry", whereas in instances of predatory pricing, an active firm is easily identified as leaving the industry. Third, even pricing patterns that are broadly consistent with the basic idea of limit pricing may be the result of other types of dynamic demand linkages, such as the building up of a loyal customer base (see e.g. Bain, 1949 and Gedge et al., 2013).[3]

More recently, the literature has identified two industries in which the threat of entry can be inferred from context, namely the airline industry and the cable TV industry. Entry into the airline industry has been analyzed by Goolsbee and Syverson (2008), Morrison (2001) and Gedge et al. (2013). In this literature, the threat of entry on a route between two airports A and B is identified with the presence of an entrant airline in airports A and B separately. That is, if an entrant airline is already serving the route A-C and now starts operating on route B-D, then the presence of the entrant on both endpoints of the route A-B is taken as a threat that it will start operating on this route as well. The results of Goolsbee and Syverson (2008), although suggestive (or at least consistent with) limit pricing, are inconclusive. Gedge et al. (2013) also study the airline industry to conclude that strategic considerations are in play.

In the cable TV industry, Seamans (2013) uses a similar approach, by exploiting the fact that entering (and offering cable TV services to) a geographical area, is significantly more appealing for a company that already serves an adjacent area (because fixed costs can be shared). Thus the threat of entry of an entrant can be identified as the physical proximity of the entrant's existing foothold to the market in question. The analysis of Seamans (2013) concludes that the evidence from the cable TV industry is consistent with limit pricing.

Although the analysis of these industries seems to lend some credence to the practice of limit pricing, one has to be cautious in interpreting this evidence. The reason is that in both cases, both entrant and incumbent firms are engaged in multi-market contact. In other words, each entry scenario is but a small part of a larger game played by the firms across different geographical locations and markets. This type of competitive environment does not fit neatly into the standard entry deterrence framework, as exemplified by the Milgrom and Roberts (1982) model. For example, if an incumbent signals strength via limit pricing on a given route, what does that imply for the entrant's entry decisions on other routes? Similarly, if simultaneous entry into multiple markets is attempted, should signaling on these markets by a multi-market incumbent be coordinated? If so, how? It quickly becomes clear that the full analysis of such markets goes well beyond the simple single incumbent/single entrant model.

A third industry in which the threat of entry can be identified clearly is the pharmaceuticals

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[2]Saloner (1984) also finds that the welfare properties of equilibria in an alternative dynamic model of limit pricing are ambiguous.

[3]Several articles, such as Harrington (1986) and Jun and Park (2010), construct models in which the weak incumbent wants to price higher than the strong incumbent, either to signal to the entrant that it may have high costs or to encourage weak entrants to enter at the expense of stronger entrants.

industry.[4] Producers of patented medicines are, by definition, not exposed to potential entry, but when patents expire, the possibility of entry and competition by producers of generic alternatives materializes. Because the expiry dates of patents is known, one may fruitfully analyze the pricing behavior of incumbent firms close to the expiration date of their patents and try to ascertain whether limit pricing is taking place. Interestingly, both advocacy groups and regulators have taken an active interest in this industry and its pricing behavior, but the academic literature on pre-expiry pricing behavior is sparse. The Rx Price Watch Report (2011) from the AARP Public Policy Institute finds evidence that for a number of blockbuster drugs, firms significantly *increase* their prices prior to patent expiry. This type of pricing behavior is noteworthy, because an ostensibly profit maximizing incumbent monopolist would *not* wish to increase its price in the face of entry. In fact, an increase in price is prima facie evidence that the firm was not charging its monopoly price till that point. This type of pricing turns out to be consistent with a type of equilibrium described in this article, namely an equilibrium in which the informed firm sets an uninformative price for a while (i.e. pools) and then separates by setting a high monopoly price that subsequently prompts entry into the industry. Although suggestive, this phenomenon should be contrasted with another common practice which casts doubt on the limit pricing story, namely the adoption of so-called *pay-for-delay* agreements. As noted by the Federal Trade Commission (2012), there is an increasing trend of incumbent firms engaging in this type of arrangement with potential generic competitors. Under a pay-for-delay agreement, the entrant delays its introduction of a competing generic drug, in return for the incumbent's promise not to introduce an *authorized generic* (i.e. a non-branded version of the incumbent's drug that competes with the entrant's generic drug). To appreciate that this type of arrangement is incompatible with limit pricing, note that the incumbent firm is essentially forgoing future profits from the introduction of an authorized generic, in return for a short term gain (i.e. that it will remain a monopolist in the short term). But the basic mechanics of limit pricing are the exact opposite, namely that the firm makes a short term loss in order to secure a long term gain.

To conclude, the extent to which limit pricing is helpful in explaining pre-expiry pricing behavior in the pharmaceuticals industry is unclear.

Overall, the empirical literature does not allow one to confidently assert that limit pricing is a successful strategy for deterring entry (to the extent that the strategy is used at all, something that is also not clear). Being careful not to overstate the conclusions based on a theoretical analysis, the present article may suggest additional reasons why clear actual examples of limit pricing are hard to come by.

The Theoretical Literature on Limit Pricing. In his analysis, Bain (1949) identified two possible channels through which current prices may deter entry: (i) a low current price may signal to potential entrants that existing and future market conditions are unfavorable to entry and that (ii) a low current price may signal to potential entrants something about the incumbent's response to entry. The first generation of contributions focused almost exclusively on explanation (ii) and featured models that were fully dynamic in nature, an approach which is suitable for the study of ongoing relationships between competing firms. This literature expounded a number of interesting characterizations of equilibrium limit price paths that could in principle be confronted with the data (see Carlton and Perloff, 2004). Nonetheless, most contributions had the unsatisfactory feature that entrants' decisions were not the outcome of rational deliberations, but rather mechanistic (assumed) responses to the incumbent's pricing behavior.[5] Furthermore, incumbents in these models were endowed with perfect ability to

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[4]I thank an anonymous referee for directing my attention to this industry.

[5]Gaskins (1971) assumes that entry is a deterministic function of pre-entry price and Berck and Perloff (1988) that it is proportional to future profitability. Stochastic entry is assumed by Kamien and Schwartz

commit to future (i.e. post-entry) pricing behavior. This critique, first articulated by Friedman (1979), cast serious doubt on the received wisdom from Bain's insights. Milgrom and Roberts (1982) confronted this challenge by reformulating the situation as one of incomplete information and drew on both explanations (i) and (ii). In doing so, they succeeded in validating Bain's insights. Unlike the early theory on limit pricing, the Milgrom-Roberts analysis is essentially static in nature (because signaling through prices occurs once and for all and the potential entrant only has one opportunity to enter). This raises the question as to how robust their findings are to dynamic extensions and to what extent their predictions are reconcilable with those of the previous literature.

The second generation of models (distinguished by featuring incomplete information as initiated by Milgrom and Roberts), has greatly shaped the way economists think about limit pricing. As such, it is important to determine to what extent the lessons are robust to variations in the modeling approach. Matthews and Mirman (1983) consider the possibility that the incumbent's price only provide noisy information to the entrant about the profitability of entry. Under certain conditions, they find that limit pricing can be successfully employed by the incumbent to limit entry. Harrington (1986) considers a variation of the basic model in which the entrant is uncertain of his own costs, which are in turn correlated with those of the incumbent. This modeling approach means that a high pre-entry price may signal that the entrant's costs are likely to be high, thereby making entry less appealing. In turn, this may imply that in equilibrium, the incumbent charges a price higher than the monopoly price and also deters entry. Jun and Park (2010) consider a dynamic setup where the incumbent faces a sequence of entrants that can be either weak or strong, of which only the former can be deterred. Rather than having a strong opponent enter, the incumbent may wish to appear weak by charging a price higher than the monopoly price, thereby encouraging entry by weak entrants. This conclusion should be contrasted to that gained from the Milgrom-Roberts analysis.

The third generation of work on limit pricing seeks to come full circle by integrating the dynamic nature of first generation models with a careful treatment of informational issues as emphasized in second generation models. The present article is a contribution to this branch of the literature. Closest to my analysis is the work of Kaya (2009), who studies repeated signaling in a reduced form setup. She assumes one-sided asymmetric information and focuses on separating equilibria. Her work complements the current analysis, focusing on somewhat different issues. In particular, she does not select between equilibria and focuses on the least cost separating equilibrium which allows the informed party to smooth costly signaling intertemporally.[6] In unpublished work, Saloner (1984) extends the Matthews and Mirman (1983) setup of noisy signaling to multi-period settings. He assumes that not only is signaling noisy, but that market conditions evolve randomly over time. This introduces a real-options dimension to the entrant's entry problem which may give it a strategic motive to delay entry. The evolving market approach is also taken by Roddie (2010), who treats signaling games with a particular monotone structure. A recent article by Gryglewicz (2009) treats a continuous-time signaling model in which the informed party's type is constant across time. His analysis focuses on pooling equilibria in which the incumbent's type is never revealed. Sorenson (2004) treats a dynamic model of limit pricing similar to the present one, but implicitly assumes that the informed party is unable to credibly signal his type in a single period. This gives rise to repeated signaling over time.

In contrast to static models and to the existing dynamic models in the literature, my modeling approach allows me to study dynamic limit pricing behavior in which there is delayed revelation of information (i.e. separation may occur immediately, with a delay or not at all).

(1971) and others.

[6]This approach to repeated signaling was first adopted by Cho (1990).

Furthermore, my analysis shows that once dynamic considerations are introduced, limit pricing may no longer be viable as an equilibrium phenomenon.

The remainder of the article is structured as follows. In Section 2, I introduce the benchmark static model that will constitute the building block of the dynamic analysis and then extend it to a fully dynamic model. A detailed analysis of the benchmark model is available as an Online Appendix. I then analyze the dynamic setting and compare the outcomes of this analysis to the static setting. Furthermore, in Section 3, I perform comparative statics analysis and discuss sensitivity of equilibrium existence with respect to the length of the horizon and the discount factor. Section 4 concludes. Most proofs are relegated to the Appendix whereas additional analysis and worked examples can be found in the Online Appendix.

2.   The Model

In this section, I set out a dynamic model of limit pricing played between an incumbent firm and a potential entrant. The overall game consists of a number of rounds of pricing and entry decisions, with each round sub-divided into three stages. For the sake of clarity, I will first describe a single round of the overall game.

The Benchmark Setting.   Consider an incumbent monopolist I and a potential entrant E. The monopolist serves a market with demand $Q(p)$ and the entrant can enter the market at cost $F > 0$ to compete with the incumbent. The monopolist can be one of two types, high cost (H) or low cost (L), with probability μ and $(1 - \mu)$ respectively. The incumbent knows his type, but his type is unknown to the entrant (who only knows the probability μ). Let $C_H(q)$ and $C_L(q)$ be the cost functions of H and L respectively. Denote by $\pi_i(p)$ the profit function of the incumbent of type $i = H, L$ when he sets price p. These profits are given by

$$\pi_i(p) = pQ(p) - C_i(Q(p)), \quad i = H, L \tag{1}$$

Let D_i be the duopoly profit of the incumbent of type $i = H, L$ when competing against E and let $D_E(i)$ be the duopoly profits of E when competing against the incumbent of type $i = H, L$. Denote by p_H^M and p_L^M the monopoly prices under the technologies $C_H(\cdot)$ and $C_L(\cdot)$ respectively.

In the benchmark single-round setting, I make the following assumptions:

Assumptions

1 $C_i(q)$, $i = H, L$ and $Q(p)$ are differentiable, for $q > 0$ and $p > 0$ respectively.

2 $C_H'(q) > C_L'(q), \forall q \in \mathbb{R}_+$, with $C_H(0) \geq C_L(0)$.

3 $Q'(p) < 0, \forall p \geq 0$.

4 $D_E(L) - F < 0$.

5 $D_E(H) - F > 0$.

6 $\pi_i(p)$ is strictly increasing for $p < p_i^M$ and strictly decreasing for $p > p_i^M$, $i = H, L$.

7 $\pi_i(p_i^M) > D_i$, $i = H, L$.

8 $\mu D_E(H) + (1 - \mu)D_E(L) - F < 0$.

Assumption 2 makes precise the sense in which type L is more efficient than type H. Assumption 3 simply states that demand is downward sloping. Assumptions (4)-(5) imply that E will not enter in the benchmark setting if he knows that I is of type L, whereas he will

enter if he knows that I is of type H. Thus these conditions are necessary for a separating limit price equilibrium to exist. Assumption 6 means that the incumbent's profit function is single peaked, whereas Assumption 7 ensures that entry deterrence is desirable for the incumbent, ceteris paribus. Under Assumption 8, the entrant expects to make negative profits against the incumbent in the benchmark setting if he cannot distinguish between the two types and thus stays out. This is a necessary condition for a pooling limit price equilibrium to exist. Once the game is extended to a multiple round interaction, Assumptions 4, 5 and 8 will have to be suitably modified, whereas the remaining assumptions will remain in place throughout.

Each round of the game between I and E is played in three stages.[7] At the first stage, I sets a price that will serve as a signal for E of I's type. After observing the price set by I, E decides at the second stage whether or not to enter (incurring the entry fee F). Denote E's entry decision by $s_E \in \{0, 1\}$, where $s_E = 0$ stands for *stay out* and $s_E = 1$ stands for *enter*. At the third stage, if E enters he will learn I's type and compete against him in complete information fashion. Both incumbent and entrant discount the future by a factor $\delta \in [0, 1]$.

A strategy for I is a price for each of his two types, p_H or p_L, at the first stage, a price at the second stage if the entrant stays out and a quantity or price to set at the third stage if the entrant enters (depending on the mode of competition), both as functions of his type and the decisions made at the first stage. A strategy for E is a decision rule to enter or not as a function of the price set by I at the first stage and a quantity or price to set at the third stage in case he enters (again, depending on the mode of competition).

In a single round, strategies are defined as follows. Let $\sigma \equiv (p_L, p_H, \bar{p})$ denote a triple of pure strategies of the game, i.e. a price charged by each type of I and a threshold price governing E's entry decision. Throughout this article, attention will be restricted to pure strategy perfect Bayesian equilibria. Denote by p_H^* and p_L^* the equilibrium prices charged by the H type and the L type respectively.

Definition 1 The triple σ is a separating equilibrium if $p_H^* \neq p_L^*$ and a pooling equilibrium if $p_H^* = p_L^*$. Furthermore, σ is a limit price equilibrium if $p_H^* < p_H^M$ or $p_L^* < p_L^M$ or both.

Note that under the maintained assumptions, ceteris paribus, the high cost incumbent will wish to set a higher monopoly price than the low cost incumbent. An implication of this fact is that an inefficient incumbent would only set lower prices than an inefficient incumbent's monopoly price, in order to convince the entrant that it is in fact an efficient incumbent.

Summary of the Benchmark Setting.   The analysis of the single-round model is well understood and the details are therefore omitted (see Online Appendix for a complete analysis with the present notation). If the game admits equilibria, there are typically a continuum of such equilibria within each class, i.e. a continuum of separating equilibria and a continuum of pooling equilibria. The multiplicity relies on choosing different beliefs off the equilibrium path. Using standard equilibrium selection techniques, such as equilibrium dominance, a unique equilibrium can be selected within each class.

In addition, after performing equilibrium selection, the set of equilibria can, if non-empty, be divided into two distinct regimes, namely a *limit price regime* and a *monopoly price regime*. These regimes will reappear in an important way in the dynamic game. In the monopoly price regime, the unique equilibrium satisfying equilibrium dominance is characterized by firms separating by setting their respective monopoly prices, whereas in the limit price regime, both pooling and separating limit price equilibria coexist, both satisfying equilibrium dominance. Which regime obtains, depends on the parameter constellation and on the specifics of the mode of competition.

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[7]In the dynamic version of the model, stages one and two will together constitute a period and stage three will be a separate period.

For later reference, the monopoly price regime obtains if and only if

$$\pi_H(p_H^M) + \frac{\delta D_H}{1-\delta} \geq \frac{\pi_H(p_L^M)}{1-\delta} \tag{2}$$

This inequality has an interesting interpretation. The left-hand side is the profit for the H type of revealing his type by earning monopoly profits in the first period and then earning discounted duopoly profits in perpetuity thereafter. The right-hand side is the discounted profit stream for the H type from mimicking the L type's monopoly price in perpetuity.

The Dynamic Setting. To introduce proper dynamics to the model, the benchmark model is repeated $T-1$ times as long as entry has not occurred (so that period $T < \infty$ is the last period and period $T-1$ is the last period in which signaling and/or entry may occur).[8] Note that this is *not* a repeated game, as entry can only occur once and thus the stage game is not unvarying across periods; hence the use of the term *rounds*.

Next, I formally define what is meant by a separating and a pooling equilibrium in this dynamic setting. Let $\sigma^T \equiv \{p_{t,L}, p_{t,H}, \overline{p}_t\}_{t=1}^{T-1}$ denote a triple of pure strategies of the game. Denote by $\{p_{t,L}^*\}_{t=1}^{T-1}$ and $\{p_{t,H}^*\}_{t=1}^{T-1}$ the equilibrium price sequences for the two types of incumbent.

Definition 2 The triple σ^T is a separating equilibrium if $\{p_{t,H}^*\}_{t=1}^{T-1} \neq \{p_{t,L}^*\}_{t=1}^{T-1}$ and a pooling equilibrium if $\{p_{t,H}^*\}_{t=1}^{T-1} = \{p_{t,L}^*\}_{t=1}^{T-1}$.

These definitions are the natural generalizations of their static counterparts. In essence, they extend the notion that upon observing the incumbent's equilibrium strategy, the entrant can infer the incumbent's type. Importantly though, it is quite possible that such an inference can only be made upon observing the *entire* strategy. One reason for adopting this definition is that if the incumbent's type has to be recognizable after all partial (i.e. non-terminal) histories, as is the case in Kaya (2009) and Noldeke and van Damme (1990), then there cannot *by assumption* be any delay in separation. I shall not impose such a restriction as it may rule out interesting equilibria with delayed information revelation. In what follows, it is useful to distinguish between *immediate separation equilibria* and *delayed separation equilibria* (of which a special case is the pooling equilibrium).

As in any signaling game, out of equilibrium beliefs must be assigned. An optimal decision rule for the entrant will prescribe entry if the incumbent is believed to be of the H type and no entry otherwise, that is if either the incumbent is believed to be of the L type or the two types cannot be distinguished.[9] I will assume for simplicity that the incumbent will be interpreted to be of the H type for any observed price above the L type's equilibrium price (either separating or pooling) and to be the L type otherwise. These beliefs amount to the following (optimal) monotone decision rule as a function of the observed price p_t for $t = 1, ..., T-1$, if entry has not occurred by time t:

$$s_E(p_t) = \begin{cases} 1 \text{ if } & p_t > \overline{p}_t \\ 0 \text{ if } & p_t \leq \overline{p}_t \end{cases} \tag{3}$$

for an appropriately chosen sequence $\{\overline{p}_t\}_{t=1}^{T-1}$ (determined by E).[10] Note that although the dependence of the decision rule on past price observations has been suppressed in the notation, the entrant is allowed to condition his entry decision on all available information, including the exact history of prices set by the incumbent.

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[8] Although the benchmark model has two periods, it is static in the sense that signaling and entry can take place only once.

[9] Assumptions on the primitives of the model that ensure the optimality of this decision rule will be introduced below.

[10] This type of monotone strategy is similar to the trigger strategies considered by Saloner (1984).

Equilibrium Selection in the Dynamic Model. Rather than characterizing the entire set of equilibria of the dynamic game, I will argue that only a subset of the equilibria are reasonable in a sense to be made precise.[11] Note that ex post separation, the sequence $(p_L^M, p_L^M, ..., p_L^M)$ yields the *highest possible* payoff to the L type incumbent (as long as the entrant's beliefs that he is facing the L type are not disturbed). In other words, as long as the L type sticks to this price sequence, there is no possible deviation that can yield a higher payoff to him for *any* beliefs that a deviation could feasibly induce. On the other hand, there are deviations that would make the H type strictly better off. For example, consider the sequence $(p_H^M, p_L^M, ..., p_L^M)$. If the entrant gives the incumbent the benefit of the doubt and ignores the out of equilibrium price p_H^M (which he is entitled to do because there are no restrictions on beliefs off the equilibrium path), then the H type is strictly better off under this price sequence than under the sequence $(p_L^M, p_L^M, ..., p_L^M)$ in which he mimics the strategy of the L type incumbent. But this means that the set of best responses of the entrant that makes the H type want to deviate is strictly larger than the set of best responses that would induce the L type to deviate (because this latter set is empty). This line of reasoning implies that the entrant should actually conclude that it was the high-cost incumbent who deviated, thus justifying the removal of this potential Bayesian Nash equilibrium. This is very similar to the heuristic embodied in criterion D1. The reason that this is not simply an application of the standard D1 refinement, is that the deviation is compared to the equilibrium under complete information rather than to an arbitrary equilibrium. In what follows, the analysis will be confined to equilibria that are selected using this approach. This anchored D1 criterion is used in all post-separation periods. In the separation period itself, where beliefs are not degenerate, I will apply standard equilibrium dominance to select equilibria.[12]

This approach to equilibrium selection may have applicability to a larger class of dynamic signaling models. For that reason, I will now formally define the anchored D1 criterion. The D1 criterion is usually defined for static signaling games, typically by making use of reduced-form payoff functions for the sender and the receiver (i.e. the informed and the uninformed party, respectively). In what follows, the reduced-form payoff functions will be composed of two separate parts, namely the payoff in the current period and the discounted expected payoffs from future equilibrium play. Using continuation equilibrium play in this way, allows one to make use of the D1 criterion period by period, like in Roddie (2010) and Gedge et al. (2013).

First, define the reduced-form payoff to the type $i = H, L$ incumbent as

$$\Pi_i^t(p, s_E) \equiv \pi_{t,i}(p) + \delta V_i^t(p, s_E, \mu') \tag{4}$$

In this definition, $V_i^t(p, s_E, \mu')$ is the expected equilibrium continuation value (for some given equilibrium) for the incumbent when setting price p in the current period, the entrant's entry decision is s_E and the entrant's beliefs upon observing p are given by μ'.

For any period $t = 1, ..., T - 1$ for which beliefs are non-degenerate, the usual equilibrium dominance criterion is applied. To define D1, let S be a non-empty subset of the type space $\{H, L\}$ and define the entrant's best response

$$BR(S, p) \equiv \cup_{\mu':\mu(S|p)=1} BR(\mu', p) \tag{5}$$

where

$$BR(\mu', p) \equiv \arg \max_{s_E \in \{0,1\}} \sum_{i \in \{H,L\}} \mu(i|p)\Pi_E(p, s_E, i) \tag{6}$$

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[11] A more complete discussion of this concept and its relation to the existing literature can be found in the Online Appendix.

[12] An alternative to this approach would be to employ the standard D1 criterion to pre-separation periods. This would effectively rule out delayed-entry equilibria.

and $\mu(i|p)$ is the entrant's belief assigned to type i upon observing price p set by the incumbent. The term $\Pi_E(p, s_E, i)$ is simply the entrant's discounted, expected payoff in the given equilibrium. Pick an equilibrium in which the type i incumbent's payoff is Π_i^* and define the sets

$$\mathcal{D}(i, S, p) \equiv \cup_{\mu:\mu(S|p)=1} \left\{ p_E \in BR(\mu, p) : \Pi_i^* < \Pi_i^t(p, s_E) \right\} \tag{7}$$

$$\mathcal{D}^0(i, S, p) \equiv \cup_{\mu:\mu(S|p)=1} \left\{ p_E \in BR(\mu, p) : \Pi_i^* = \Pi_i^t(p, s_E) \right\} \tag{8}$$

The set $\mathcal{D}(i, S, p)$ is simply the best responses for the entrant that make the incumbent want to deviate from the equilibrium action, whereas $\mathcal{D}^0(i, S, p)$ is the set of best responses for which the incumbent is indifferent between deviating and taking the equilibrium action.

The standard definition of D1 (in the formulation of Fudenberg and Tirole, 1991) is then as follows:

Prune the type-strategy pair (i, p) under criterion D1 if there exists some type $j \in \{H, L\}$ such that

$$\left\{ \mathcal{D}(i, \{H, L\}, p) \cup \mathcal{D}^0(i, \{H, L\}, p) \right\} \subset \mathcal{D}(j, \{H, L\}, p) \tag{9}$$

This definition means that if the set of entry decisions for the entrant that makes type $i = H, L$ willing to deviate to some price p is strictly smaller than the set of of entry decisions that makes type j willing to deviate, then the entrant should believe it to be infinitely more likely that the deviation to price p came from type j rather than from type i.

Next, for any period $t = 1, ..., T - 1$ for which beliefs are degenerate, apply criterion D1 as above but replacing the equilibrium payoff Π_i^* to the incumbent by $\overline{\Pi}_i^*$, which is the discounted equilibrium payoff to the incumbent under complete information (given the degenerate beliefs).

In what follows, I will make use of the following definition:

Definition 3 An equilibrium price sequence $\{p_{t,i}^*\}_{t=1}^{T-1}$, $i = H, L$ satisfies the anchored D1 criterion, if the anchored D1 criterion is applied separately to each pre-entry period in which beliefs are degenerate.

In the present model, once beliefs are centered on the H type, entry occurs and no further learning can take place (because the incumbent's type is then perfectly revealed). In more general settings, further rounds of equilibrium selection may be necessary and thus somewhat more delicate arguments would be needed to select between equilibria.

Separating Limit Price Equilibria.　To make limit pricing with separation feasible, it must be the case that the entrant would find it optimal to enter against the H type incumbent but to stay out against the L type incumbent. A necessary condition for a separating limit price equilibrium with separation in any period $t = 1, ..., T - 1$ to exist is that

$$D_E(L) < \left(\frac{1 - \delta}{1 - \delta^{T-t+1}} \right) F < D_E(H), \quad t = 1, ..., T \tag{10}$$

As the coefficient on the entry fee F in this condition is decreasing in the number of remaining periods, the condition may fail to hold for some t.[13] In order to avoid time varying necessary conditions at this stage of the analysis, I instead impose the following restrictions, which ensure that separation is feasible in an arbitrary period $t = 1, ..., T$:

Assumptions

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[13]In particular, it may be the case that the necessary condition for a separating limit price equilibrium to be feasible is that the remaining number of periods be small. This case will be considered in the next section.

4′ $D_E(L) < \left(\frac{1-\delta}{1-\delta^T}\right) F.$

5′ $D_E(H) > F.$

It should be pointed out that although Assumption A5' coincides with Assumption A5, Assumption A4' is stronger than Assumption 4. In Section 3, I will explicitly analyze the dependence of these conditions on the discount factor and the length of the interaction.

Characterization. The characterization of equilibria of the dynamic model follows similar steps as that of the single-round model, although the analysis is complicated by the dynamic nature of the problem. Based on the discussion above, a separating equilibrium price sequence $\{p_{t,L}^*\}_{t=1}^{T-1}$ for the L type satisfying the anchored D1 refinement is of the general form $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$, with separation occurring in period $t = 1, ..., T-1 \leq \infty$.[14]

Next, the entrant's strategy can be characterized as follows:

Lemma 1. *(entrant's optimal decisions)*
Consider the equilibrium price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ *in which separation occurs in period* $t = 1, ..., T-1$. *Then (i)* $\bar{p}_s = p_s^*$ *and* $\bar{p}_s \leq p_L^M$, $s = 1, ..., t-1$, *(ii)* $\bar{p}_t = p_{t,L}^*$ *and* $\bar{p}_t < p_L^M$ *and (iii)* $\bar{p}_s = p_{s,L}^* = p_L^M$, $s = t+1, ..., T-1$.

Proof: The proofs of (i) and (ii) parallel those in the single-round setting (see Online Appendix) and are omitted, whereas that of (iii) follows from the equilibrium selection approach discussed above ∎

It should be emphasized that I do not make any use of support restrictions in the post separation game. With a support restriction and assuming that prices have revealed that the incumbent is of type L, the two different sequences of post-separation prices $(p_L^M, p_L^M, p_L^M, p_H^M, ...)$ and $(p_L^M, p_L^M, p_L^M, p_L^M, ...)$ would be treated equivalently in terms of beliefs and entry decisions, whereas with the equilibrium selection procedure I use, the former sequence would prompt the entrant to update his beliefs and subsequently enter.

I now proceed by first analyzing the incentive compatibility constraints of each type of incumbent and then move on to the issues of equilibrium existence and selection. Although matters are complicated somewhat by the dynamic nature of the model (there are in each case two regimes to consider, which depend on parameter values), the basic progression of the analysis is straightforward.

The Incentive Compatibility Constraints. As is the case in the single-round setting, the best alternative for the L type to setting the separating equilibrium price, is to set his monopoly price. In contrast, the best alternative for the H type to setting the separating equilibrium price, i.e. his monopoly price, is to mimic the L type's equilibrium price. With this in mind, the following partial characterization of the separating equilibrium price can be given, using the L type's incentive compatibility constraints:

Lemma 2. *(efficient incumbent's incentive constraints)*
For the price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ *to constitute a separating limit price equilibrium, it must satisfy*

$$\pi_L(p_s^*) \geq (1-\delta)\pi_L(p_L^M) + \delta D_L, \quad p_s^* < p_L^M, \quad s = 1, ..., t-1 \tag{11}$$

$$\pi_L(p_{t,L}^*) \geq \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_L \tag{12}$$

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[14]In fact, in a separating equilibrium satisfying the anchored D1 criterion, it must also be the case that $p_s^* = p_L^M$ for $s = 1, ..., t-1$ as will be shown below.

Proof: See Appendix ∎

To write the incentive compatibility constraint for the separation period in terms of prices, define the following set:

$$A_L(T,t) \equiv \left\{ p : \pi_L(p) = \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right)D_L \right\} \quad (13)$$

As $\pi_L(p) = D_L$ for some p, then by Assumptions 6 and 7 it follows that the set $A_L(T,t)$ is non-empty and contains at most two points. Let

$$\alpha_0(T,t) \equiv \min A_L(T,t), \quad \beta_0(T,t) \equiv \max A_L(T,t) \quad (14)$$

where $\alpha_0(T,t) < \infty$ and $\beta_0(T,t) \leq \infty$. Let the single-round cutoffs be denoted by α_0 and β_0, i.e. $\alpha_0 \equiv \alpha_0(2,1)$ and $\beta_0 \equiv \beta_0(2,1)$.

In terms of prices, the L type's incentive compatibility constraint for the separation period can then be written as

$$p_{t,L}^* \in [\alpha_0(T,t), \beta_0(T,t)] \quad (15)$$

For later use, note that by definition it is the case that

$$\pi_L(\alpha_0(T,t)) = \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right)D_L = \pi_L(\beta_0(T,t)) \quad (16)$$

I next consider the incentive compatibility constraints for the H type. These are slightly more complicated than those of the L type, due to the fact that the H type may in general wish to mimic the behavior of the L type for an arbitrary number of periods after the L type has chosen to separate. To see this more clearly, consider an equilibrium price sequence for the L type given by $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ for $t = 1, ..., T-1$. In equilibrium, the H type's strategy is given by a sequence $(p_1^*, ..., p_{t-1}^*, p_H^M, x_{t+1}, ..., x_T)$ where x_s is shorthand for H's post entry equilibrium strategy in period $s = t+1, ..., T$.

Consider possible deviations for the H type. First, H may wish to deviate during periods with pooling and so these pooling prices must respect appropriate incentive compatibility constraints. Next, the H type incumbent may deviate in the period where separation is prescribed, by mimicking the L type's strategy. Last, H may deviate by not only mimicking the L type's separating price, but also by mimicking L's post-separation strategy p_L^M for an arbitrary number of periods. It turns out that the optimal amount of mimicking undertaken by the H type out of equilibrium depends in a simple way on parameter values, as the following results show:

Lemma 3. *(inefficient incumbent's incentive constraints)*

(i) In the monopoly price regime, mimicking only once is the optimal off-equilibrium path strategy. Furthermore, for the price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ for $t = 1, ..., T-1$ to constitute a separating limit price equilibrium, it must satisfy

$$\pi_H(p_s^*) \geq (1-\delta)\pi_H(p_H^M) + \delta D_H, \quad p_s^* < p_L^M, \quad s = 1, ..., t-1 \quad (17)$$
$$\pi_H(p_{t,L}^*) \leq (1-\delta)\pi_H(p_H^M) + \delta D_H \quad (18)$$

(ii) In the limit price regime, mimicking perpetually is the optimal off-equilibrium path strategy. Furthermore, for the price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ for $t = 1, ..., T-1$ to

constitute a separating limit price equilibrium, it must satisfy

$$\pi_H(p_s^*) \geq (1-\delta)\pi_H(p_H^M) + \delta D_H, \quad p_s^* < p_L^M, \quad s = 1, ..., t-1 \tag{19}$$

$$\pi_H(p_{t,L}^*) \leq (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right) \pi_H(p_L^M) \tag{20}$$

Proof: See Appendix ∎

To express these incentive compatibility constraints in terms of prices, define the following set:

$$A_H(T,t) \equiv \left\{ p : \pi_H(p) = (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right) \pi_H(p_L^M) \right\} \tag{21}$$

Note that the coefficients on $\pi_H(p_H^M)$, D_H and $\pi_H(p_L^M)$ in the definition of $A_H(T,t)$ sum to one. It then follows from Assumptions 6 and 7 and the fact that $\pi_H(p_H^M) > \pi_H(p_L^M)$ that the set $A_H(T,t)$ contains at most two points. Let

$$\widehat{\alpha}(T,t) \equiv \min A_H(T,t), \quad \widehat{\beta}(T,t) \equiv \max A_H(T,t) \tag{22}$$

where $\widehat{\alpha}(T,t) < \infty$ and $\widehat{\beta}(T,t) \leq \infty$. Let the single-round cutoffs be denoted by $\widehat{\alpha}$ and $\widehat{\beta}$, i.e. $\widehat{\alpha} \equiv \widehat{\alpha}(2,1)$ and $\widehat{\beta} \equiv \widehat{\beta}(2,1)$.

For periods with pooling, the price sequence must thus satisfy

$$\max\{\alpha_0, \widehat{\alpha}\} \leq p_s^* \leq p_L^M < p_H^M, \quad s = 1, ..., t-1 \tag{23}$$

For the period in which separation is prescribed, the H type's incentive compatibility constraint when condition (2) is satisfied is that

$$p_{t,L}^* \notin [\widehat{\alpha}, \widehat{\beta}] \tag{24}$$

which is as in the single-round setting. In this case, only the inequality $p_{t,L}^* \leq \widehat{\alpha}$ is relevant, because $p_{t,L}^* < p_L^M < p_H^M < \widehat{\beta}$.

For the period in which separation is prescribed, the H type's incentive compatibility constraint when condition (2) is violated is that

$$p_{t,L}^* \notin [\widehat{\alpha}(T,t), \widehat{\beta}(T,t)] \tag{25}$$

In this case, only the inequality $p_{t,L}^* \leq \widehat{\alpha}(T,t)$ is relevant, because $p_{t,L}^* < p_L^M < p_H^M < \widehat{\beta}(T,t)$. For later use, note that by definition it is the case that

$$\pi_H(\widehat{\alpha}(T,t)) = (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right) \pi_H(p_L^M) = \pi_H(\widehat{\beta}(T,t)) \tag{26}$$

Before summarizing the analysis of the dynamic limit price equilibria, I will briefly discuss the issue of equilibrium existence.

Existence of Separating Limit Price Equilibria. When (2) is satisfied, the existence of separating limit price equilibria is ensured if $\widehat{\alpha} > \alpha_0(T,t)$ whereas if (2) is violated, then existence is ensured if $\widehat{\alpha}(T,t) > \alpha_0(T,t)$.

The relevant sufficient conditions for the existence of separating limit price equilibria are as follows:

Proposition 4. *(existence of separating limit price equilibria)*
  (i) In the monopoly price regime, if

$$\pi_L(p_L^M) - D_L > \left(\frac{1-\delta}{1-\delta^{T-t}}\right)\left[\pi_H(p_H^M) - D_H\right], \quad t = 1, ..., T-1 \qquad (27)$$

then $\widehat{\alpha} > \alpha_0(T, t)$ *and the set of separating limit price equilibria is non-empty.*
  (ii) In the limit price regime, if

$$\pi_L(p_L^M) - D_L > \left[\frac{(1-\delta)\delta^{T-t-1}}{1-\delta^{T-t}}\right]\pi_H(p_H^M) - D_H + \left[\frac{\delta - \delta^{T-2}}{1-\delta^{T-t}}\right]\pi_H(p_L^M), \quad t = 1, ..., T-1 \quad (28)$$

then $\widehat{\alpha}(T, t) > \alpha_0(T, t)$ *and the set of separating limit price equilibria is non-empty.*

Proof. See Appendix ■
  Note that in both regimes, the relevant sufficient condition for existence becomes easier to satisfy as the horizon recedes, i.e. equilibrium may exist in the dynamic setting even if none exist in the static setting.

  Equilibrium Selection.   I now determine which of the equilibria in the dynamic game satisfy the anchored D1 criterion after separation has taken place. I do this explicitly for the case where (2) is violated. The case where (2) is satisfied follows similar steps, with $\widehat{\alpha}(T, t)$ replaced by $\widehat{\alpha}$.

Proposition 5. *(uniqueness of separating limit price equilibrium satisfying anchored D1)*
  (i) In the limit price regime, only $p_{t,L}^* = \widehat{\alpha}(T, t)$ *satisfies the anchored D1 criterion.*
  (ii) In the monopoly price regime, only $p_{t,L}^* = p_L^M$ *satisfies the anchored D1 criterion.*

Proof: See Appendix ■
  Before turning to the comparative statics analysis, the following result is shown:

Proposition 6. *(characterization of separating limit price equilibrium satisfying anchored D1)*
  (i) In the monopoly price regime, all equilibria satisfying the anchored D1 refinement are immediate separation equilibria.
  (ii) In the limit price regime, all equilibria satisfying the anchored D1 refinement are of a form where, for $t = 1, ..., T-1$, *the L type's strategy is given by a sequence* $(p_L^M, ..., p_L^M, p_{t,L}^*, p_L^M, ..., p_L^M)$ *and the H type's strategy is given by a sequence* $(p_L^M, ..., p_L^M, p_H^M, x_{t+1}, ..., x_T)$ *where* x_s *is the H type's post entry strategy at time* $s = t+1, ..., T$.

Proof: (i) It can be shown in the single-round setting that in the monopoly price regime, i.e. when (2) is satisfied, no pooling equilibria surviving equilibrium dominance exist (see Online Appendix). The result then follows immediately from observing that the incentive compatibility constraint of the H type in periods of pre-separation pooling are identical to the incentive compatibility constraint in the single-round setting. (ii) The proof follows directly from the lemmas proved above ■
  This result means that in the monopoly price regime, the unique prediction is that each type of incumbent will set its corresponding monopoly price in the first period and deter entry in case of type L and invite entry in case of type H. That is, the equilibrium is necessarily an immediate separation equilibrium. In the limit price regime, equilibria are possibly of the delayed separation variety.

Pooling Limit Price Equilibria. In the dynamic setting, a pooling equilibrium consists of a price sequence $\sigma^T = \{p_t^*\}_{t=1}^{T-1}$ set by both types of incumbent. This means that in every period, the entrant cannot distinguish the two types. For pooling to be feasible in period $t = 1, ..., T-1$, the following conditions need to be imposed:

$$\mu D_E(H) + (1-\mu) D_E(L) < \left(\frac{1-\delta}{1-\delta^{T-t+1}}\right) F, \quad t = 1, ..., T \tag{29}$$

These constraints are more difficult to satisfy the farther away the final period is. In order to avoid time varying necessary conditions at this stage of the analysis, I instead impose the following condition that ensures that pooling is feasible in any arbitrary period $t = 1, ..., T-1$:

Assumption 8' $\mu D_E(H) + (1-\mu) D_E(L) - (1-\delta)F < 0.$

Interestingly, this condition is more difficult to satisfy than that in Assumption 8, pertaining to the benchmark setting. In other words, once dynamics are introduced, the necessary condition for a pooling equilibrium to be feasible is more difficult to satisfy. Furthermore, it becomes increasingly difficult the more patient the entrant becomes, i.e. the larger the discount factor δ becomes. This feature will be further explored in the next section.

The following characterization of the entrant's decision rule holds:

Lemma 7. *(entrant's optimal decisions)*
$\bar{p}_t = p_t^*$ and $p_t^* \leq p_L^M, \quad t = 1, ... T-1.$

Proof: Omitted ∎

The Incentive Compatibility Constraints. As is the case in the single-round setting, the best alternative for each type to setting the pooling price, is to set the monopoly price and thus invite entry. With this in mind, the following can be shown to hold:

Lemma 8. *(incumbent's incentive constraints)*
For the price sequence $\{p_t^*\}_{t=1}^{T-1}$ to constitute a pooling limit price equilibrium, it must satisfy

$$\pi_L(p_t^*) \geq (1-\delta)\pi(p_L^M) + \delta D_L, \quad p_t^* < p_L^M, \quad t = 1, ..., T-1 \tag{30}$$
$$\pi_H(p_t^*) \geq (1-\delta)\pi(p_H^M) + \delta D_H, \quad t = 1, ..., T-1 \tag{31}$$

Proof: See Appendix ∎

These results can be collected as follows:

Proposition 9. *(characterization of pooling limit price equilibria)*
In any pooling limit price equilibrium, it must be the case that

$$\max\{\alpha_0, \widehat{\alpha}\} \leq p_t^* \leq p_L^M < p_H^M, \quad t = 1, ..., T-1 \tag{32}$$

Equilibrium Selection. The incentive compatibility constraints in the dynamic pooling equilibrium are in fact equivalent to their static counterparts. It then follows from the same arguments as in the static analysis that only $p_t^* = p_L^M$ satisfies equilibrium dominance (see Online Appendix for details).

Existence of Limit Price Equilibria Satisfying Anchored D1 Criterion. As is the case in the analysis of the single-round setting, one may characterize two distinct regimes, namely a monopoly price regime and a limit price regime. In the monopoly price regime, the only outcome consistent with the anchored D1 refinement is separation on monopoly prices in the first period, whereas in the limit price regime, both pooling and separating equilibria coexist, both satisfying the anchored D1 refinement. In the pooling equilibrium, both types of incumbent set the efficient type's monopoly price and thus the equilibrium involves limit pricing. In the separating limit price equilibrium however, because the benefits from entry deterrence increase with the horizon and the patience of the players, credibly signaling to be of the efficient type may involve incurring arbitrarily large losses in the period in which separation is prescribed. Depending on the model specification and mode of competition in the market game, this may actually involve setting negative prices.[15]

The equilibrium price paths of the dynamic model should be contrasted to those of the early limit pricing literature. As Carlton and Perloff (2004) nicely show, some models predict that equilibrium prices will increase over time, others that they will decrease and yet others that price paths are not necessarily monotone. Because of the relatively weak restrictions on equilibrium behavior imposed by the incentive compatibility constraints, many different price profiles can be sustained in equilibrium. But not all such profiles are consistent with the refinements used in the present analysis.

In the *monopoly price regime*, the analysis predicts immediate separation on monopoly prices, with resulting entry against the H type incumbent and no entry against the L type incumbent (who will subsequently charge monopoly prices indefinitely). In the *limit price regime*, all equilibria share the same overall structure. Namely, they are characterized by a non-negative and possibly infinite number $N = 0, 1, ...$ of periods in which the two types of incumbent pool on the efficient type's monopoly price p_L^M, followed by a period $N + 1$ in which the firms separate. In case the incumbent is of type L, prices will dip in order to signal strength, after which prices will return to the pre-separation level p_L^M. In case the incumbent is of type H, prices will jump to p_H^M and then fall to some level $p < p_H^M$ (because of the ensuing entry and competition that will drive down prices).

Interestingly, in this model the timing of separation is indeterminate in the sense that in equilibrium, signaling can happen in any period, if ever. In other words, equilibrium does not pin down if and when signaling will take place. Note that this result is entirely unrelated to the equilibrium multiplicity created by choosing different off-equilibrium path beliefs in usual signaling games. Instead, the multiplicity is related to the coexistence of different classes of equilibria, i.e. pooling and separating equilibria. In the static benchmark setting, if both types of equilibria exist, there is no way to determine which of such different equilibria will be played. A similar situation arises in the dynamic setting, where separation may be preceded by multiple rounds of pooling. This indeterminacy effectively means that there are multiple equilibria (among which we cannot select) which differ in their predictions on the timing of separation and possible entry. It should be emphasized that timing indeterminacy is not inherently because of the dynamics of the model. Even in a static framework in which pooling and separating equilibria coexist, there is indeterminacy in this sense.[16]

Note that both types of incumbent are better off the later separation occurs. The efficient type earns monopoly profits as long as entry does not occur and is not called upon to engage in costly signaling. In turn, the inefficient type effectively deters entry as long as pooling

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[15] This will be the case, e.g., in a model with constant marginal costs and linear demand as that considered by Tirole (1988). In fact, the efficient incumbent would have to give its customers infinitely large subsidies to credibly convey his identity.

[16] One can then think of the separating equilibrium as an immediate separation equilibrium and of the pooling equilibrium as a delayed separation equilibrium.

takes place. Although pooling is indeed costly for the inefficient type, it still dominates entry. Therefore, it is not possible to use separation date as a screening device.

3. Comparative Analysis

To fully explore the differences between the static and dynamic settings, I will now consider the effects of changing the main distinguishing features of the dynamic setting, namely the length of the interaction T and the discount factor δ. I will in turn analyze the effects on the necessary conditions for the entrant and the incumbent respectively. For the former, the relevant conditions are A4'-A5', which ensure that E wishes to enter against H and to stay out against L and (29) (or A8'), which ensures that an uninformed entrant wishes to stay out. For the latter, the relevant conditions are the incentive compatibility constraints (12) and (20). Last, I will also analyze the effects on the necessary condition for a separating equilibrium to exist.

It is immediately clear that the constraints in pre-separation periods are unaffected by the length of the horizon. In periods where separation is prescribed however, the constraints do explicitly depend on the remaining number of periods (if $T < \infty$). First, consider the L type's incentive compatibility constraint $p_{t,L}^* \geq \alpha_0(T, t)$. The cutoff $\alpha_0(T, t)$ is decreasing in T as $\alpha_0(T, t) \leq p_L^M$ and the right-hand side of the equality defining the set $A_L(T, t)$ is decreasing in T. In the limit $T \to \infty$, $\alpha_0(T, t)$ is implicitly given by

$$\lim_{T \to \infty} \pi_L(\alpha_0(T, t)) = \left(1 - \frac{\delta}{1 - \delta}\right) \pi_L(p_L^M) + \left(\frac{\delta}{1 - \delta}\right) D_L \tag{33}$$

This means that as the horizon recedes, the L type's incentive compatibility constraint becomes easier to satisfy.

Now turn to the H type. In the monopoly price regime, the H type's incentive compatibility constraints are unaffected by changes in T. In the limit price regime however, the appropriate constraint is $p_{t,L}^* \leq \widehat{\alpha}(T, t)$. The cutoff $\widehat{\alpha}(T, t)$ is decreasing in T as $\widehat{\alpha}(T, t) \leq p_H^M$ and the right-hand side of the equality defining the set $A_H(T, t)$ is decreasing in T. In the limit $T \to \infty$, $\widehat{\alpha}(T, t)$ is implicitly given by

$$\lim_{T \to \infty} \pi_H(\widehat{\alpha}(T, t)) = \pi_H(p_H^M) + \left(\frac{\delta}{1 - \delta}\right) \left(D_H - \pi_H(p_L^M)\right) \tag{34}$$

As the horizon recedes, the H type's incentive compatibility constraint becomes more difficult to satisfy.

These observations mean that, in the monopoly price regime, the set of separating limit price equilibria expands with the length of the horizon, but the only equilibrium satisfying the anchored D1 refinement remains unchanged, namely immediate separation on monopoly prices. In the limit price regime, both critical cutoffs $\widehat{\alpha}(T, t)$ and $\alpha_0(T, t)$ decrease in the length of the horizon T and so both the largest and the smallest separating (and limiting) equilibrium prices decrease. Although the effect of an increase in T on the set of equilibrium prices is ambiguous, the unique equilibrium limit price satisfying the anchored D1 refinement is unambiguously decreasing. I gather these results in the following proposition (the following results implicitly assume that the relevant entry constraints A4' and A5' are satisfied for the entrant):

Proposition 10. *(dependence of equilibrium on length of interaction)*
(i) In the monopoly price regime, the unique equilibrium price satisfying the anchored D1 refinement is invariant in the length of the interaction.
(ii) In the limit price regime, the unique separating equilibrium limit price satisfying the anchored D1 refinement is decreasing in the length of the interaction.

Effects on the Cost of Signaling. In this subsection, I consider how the incumbent's cost of signaling changes when dynamics are introduced. In the single-round setting, the H type's profits from mimicking the L type's separating equilibrium strategy may be positive. Interestingly, this is no longer necessarily the case in the dynamic version of the game. In particular, I have the following result:

Proposition 11. *(cost of signaling with infinite horizon)*
   In the limit price regime,
$$\lim_{T \to \infty} \pi_H(\widehat{\alpha}(T,t)) < 0 \tag{35}$$

Proof: See Appendix ∎
   In fact, the result becomes even stronger as the future becomes increasingly important, as the next result demonstrates:

Corollary 12. *(cost of signaling with high discount factor)*
   In the limit price regime,
$$\lim_{\delta \to 1} \lim_{T \to \infty} \pi_H(\widehat{\alpha}(T,t)) = -\infty$$

Proof: The result follows from taking the limit $\delta \to 1$ of (34) and again noting that $\pi_H(p_L^M) > D_H$ when (2) is violated ∎
   The consequences of these results are worth emphasizing. They are that in the infinite horizon limit of the limit price regime, as the discount factor approaches one, the efficient type must force the inefficient type to make arbitrarily large losses in order to credibly signal that he is indeed efficient. This is because in this scenario, the benefits to H of perpetual incumbency approach infinity. This gives a very lopsided intertemporal profile of costs and benefits. The costs of signaling are all borne in a single period, whereas the benefits of effectively deterring entry accrue over an infinite number of periods.
   For T and δ sufficiently large, it may well be that the set

$$A_H^+(T,t) \equiv \{p \in A_H(T,t) \cap \mathbb{R}_+\}$$

is empty. In other words, depending on the details of the product market, it may be that there are no positive prices that satisfy the H type's incentive compatibility constraint. As $T \to \infty$ and $\delta \to 1$, positive prices can only be secured if demand has a vertical asymptote at $p = 0$, i.e. if $\lim_{p \to 0} Q(p) = \infty$. Even in this case, the equilibrium separating price may run afoul of the Areeda and Turner (1975) rule, requiring pricing above marginal cost.
   Pooling: As is the case in pre-separation periods in the separating equilibria, the constraints for the pooling equilibria do not depend explicitly on the remaining number of periods. It follows that the pooling equilibria are in fact not affected by the dynamic extension of the model (if they exist).

Effects on the Entry Decisions. In this subsection, I consider how the entrant's incentive to enter or stay our change when dynamics are introduced. I consider the necessary conditions for separation and pooling in turn.

   Separating Equilibrium. Recall that in the single-round setting, the necessary conditions for a separating limit price equilibrium are that $D_E(L) < F < D_E(H)$. The equivalent conditions in the T-period problem are that for $t = 1, ..., T-1$, it is the case that

$$D_E(L) < F\left(\frac{1-\delta}{1-\delta^{T-t+1}}\right) < D_E(H) \tag{36}$$

As the expression in parentheses is decreasing in the horizon T, the right-hand side inequality in (36) is trivially satisfied for any $T > 1$, if it is satisfied in the single-round setting. I therefore concentrate on the left-hand side inequality. There are two cases to consider. For $D_E(L) > (1 - \delta)F$, there exists some period T_S^* such that for $t > T - T_S^*$, the necessary condition is violated and thus there can be no separating limit price equilibrium. Note that an implication of this finding is that if one imposes conditions for a separating limit price equilibrium to be feasible for a long time horizon, a separating limit price equilibrium may not be feasible in the static setting.

For the case $D_E(L) < (1 - \delta)F$, a long time horizon is not enough to rule out the possibility of separation with limit pricing. But for sufficiently patient players, entry cannot be deterred through separation because the necessary condition is violated. Define the the following critical value of the discount factor:

$$\delta_S^* \equiv 1 - \frac{D_E(L)}{F} \tag{37}$$

The following result then follows:

Proposition 13. *For $\delta > \delta_S^*$, there can be no separation with limit pricing in the infinite horizon game.*

In conclusion, when the future is sufficiently important (either because the horizon is very long or because the players are very patient), limit pricing may become infeasible altogether because the discounted post-entry payoffs to the entrant are large enough to offset the entry fee F, even when competing against the efficient incumbent L. In this case, the only possible outcome is that of an immediate separation equilibrium with each type of incumbent setting its monopoly price and the entrant entering against the inefficient incumbent (and staying out against the efficient incumbent).

Pooling Equilibrium. For simplicity, define

$$R \equiv \mu D_E(H) + (1 - \mu)D_E(L) \tag{38}$$

and recall that in the single-round setting, a necessary condition for the existence of a pooling limit price equilibrium is that $R < F$. In contrast, in the T-period setting, the equivalent necessary condition for a pooling equilibrium to be feasible in an arbitrary period $t = 1, ..., T-1$ is that

$$R\left(\frac{1 - \delta^{T-t+1}}{1 - \delta}\right) < F \tag{39}$$

There are two cases to consider, depending on the magnitude of the left-hand side in the limit as the horizon becomes very distant. First, consider the case in which $R > (1 - \delta)F$. In this case, there exists some period T_P^* such that for $t > T - T_P^*$, the necessary condition (29) for pooling is violated. In other words, if the remaining game is sufficiently long, then there can be no pooling equilibria. The reason for this result is simply that if the remaining number of periods is very large, then the prospect of earning R per period upon entry (even if discounted) is sufficient to offset the entry fee F. It is therefore not possible to discourage entry, even if pooling is feasible in the benchmark setting.

Next, consider the case in which $R < (1-\delta)F$. In this case, pooling may be feasible even for very long horizons, for some values of the discount factor δ. But for sufficiently high patience, pooling can be ruled out even in this case. This is because the expected discounted post-entry profits are so large that entry is attractive even for an entrant who cannot distinguish the two types of incumbent. Define the the following critical value of the discount factor:

$$\delta_P^* \equiv 1 - \frac{R}{F} \tag{40}$$

The following result can then be established:

Proposition 14. *For $\delta > \delta_P^*$, there can be no pooling in equilibrium in the infinite horizon game.*

For completeness, note that $\delta_P^* < \delta_S^*$. This implies that for some intermediate values of the discount factor, it may be possible to rule out pooling (and thus delayed separation equilibria) and therefore conclude that the outcome will be that of an immediate separation equilibrium with limit pricing (if feasible).

4. Discussion

In this article, I analyzed a dynamic model of limit pricing and compared it with the outcome of a static single-round model. I showed that there are two regimes of interest. In one, the *monopoly price regime*, the only equilibrium satisfying the anchored D1 refinement involves separation in the first period on monopoly prices, i.e. it is an immediate separation equilibrium. In the other, the *limit price regime*, pooling limit price equilibria and separating limit price equilibria (both satisfying the anchored D1 refinement) coexist, which leads to the possibility of delayed separation equilibria.[17] Although the dynamic pooling equilibrium is essentially a repetition of the static outcome, with both types of incumbent pooling on the efficient type's monopoly price, the latter may differ quantitatively from the separating limit price equilibrium in the static setting.

The dynamic nature of the game changes the incentives of the incumbent and the entrant in important ways. First, a long time horizon and patient players may significantly increase the cost of signaling, to the point that the firms must set negative prices (and incur arbitrarily large losses). Second, the prospect of large discounted sums of post-entry profits (which are relevant when the horizon is long and the players are very patient) may make entry deterrence impossible to achieve. Last, in the infinite horizon version of the game, the incumbent and the entrant may choose to collude upon entry. Such collusion in turn makes entry deterrence less attractive ex ante (for the incumbent) and less deterring (for the potential entrant).[18] In summary, the analysis shows that for a number of different reasons, when moving from a static setting to a dynamic setting, the practice of entry deterrence through limit pricing seems to become less viable. This suggests that the issue of potential limit pricing should perhaps not be a main concern for competition authorities. Also, it should be noted that only some of these results depend on the equilibrium selection approach adopted in this article and would remain valid across a number of different environments.[19]

The basic model I have considered consisted of a single incumbent firm and a single potential entrant. As is true in most models of limit pricing, the incumbent can be interpreted as a profit maximizing cartel rather than as a single firm. As regards the entrant, the assumption that there is only a single such firm makes the problem tractable. As surveyed by Carlton and Perloff (2004), the early literature on limit pricing did indeed consider a number (and often a continuum) of potential entrants. Two assumptions made this tractable. First, it was typically assumed that there was no coordination problem between entrants (i.e. they would enter in an orderly and continuous fashion if price was sufficiently high). Second, it was assumed that there was no signaling taking place. In the signaling based limit pricing literature, the assumption

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[17]Note that in Saloner (1984), delayed entry is the outcome of a strategic choice by the entrant, whereas in the present article it is the outcome of the parties coordinating on a particular equilibrium amongst many.

[18]For further discussion of the effects of post-entry collusion, see the Online Appendix.

[19]In related work (available upon request), I show that in an infinite-horizon version of the model in which the incumbent's type evolves stochastically over time, when the players become sufficiently patient, entry cannot be deterred. Similar results are also likely to hold in the setting considered by Saloner (1984).

of a single entrant is ubiquitous. I have chosen to keep with this modeling assumption in order to make my contribution directly comparable to this strand of the literature.[20]

Last, the model has been solved under the important assumption that upon entry, the firms compete under complete information, i.e. there is no residual uncertainty about the incumbent's type. This is an assumption that it would be interesting to relax, as it would open up the possibility of post-entry predatory pricing, in the spirit of Benoit (1984). Nonetheless, a full analysis of such a setting seems very difficult to achieve, as one would have to consider repeated entry and exit both on and off the equilibrium path.

Appendix

A.   Main Proofs

This appendix contains the main proofs that have been omitted in the text.

Proof of Lemma 2 (efficient incumbent's incentive constraints).   I first derive the condition for the separating equilibrium price. The incentive compatibility constraints for the L type are given by

$$\pi_L(p_{1,L}^*) + \sum_{i=2}^{T} \delta^{i-1} \pi_L(p_L^M) \geq \pi_L(p_L^M) + \sum_{i=2}^{T} \delta^{i-1} D_L \tag{41}$$

$$\sum_{i=1}^{K+1} \delta^{i-1} \pi_L(p_i^*) + \delta^{K+1} \pi_L(p_{K+2,L}^*) + \sum_{i=K+3}^{T} \delta^{i-1} \pi_L(p_L^M) \tag{42}$$

$$\geq \sum_{i=1}^{M+1} \delta^{i-1} \pi_L(p_i^*) + \delta^{M+1} \pi_L(p_L^M) + \sum_{i=M+3}^{T} \delta^{i-1} D_L$$

for $0 \leq M \leq K = 0, 1, ..., T - 3$. The first constraint (41) reduces to

$$\pi_L(p_{1,L}^*) \geq \left(1 - \frac{\delta - \delta^T}{1 - \delta}\right) \pi_L(p_L^M) + \left(\frac{\delta - \delta^T}{1 - \delta}\right) D_L \tag{43}$$

Next, evaluate (42) at $M = K$ and rearrange to get

$$\pi_L(p_{K+2,L}^*) \geq \left(1 - \frac{\delta - \delta^{T-K-1}}{1 - \delta}\right) \pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-K-1}}{1 - \delta}\right) D_L \tag{44}$$

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[20]Kalish et al. (1978) and references therein study some of the complications of the presence of multiple (mutually aware) entrants.

which determines the separating prices. Next, evaluate (42) at two arbitrary consecutive periods $M = K - j$ and $M = K - j - 1$ respectively, with $j = 1, ..., K - 1$. These yield

$$\delta^{K-j+1}\pi_L(p^*_{K-j+2}) - \left(\delta^{K-j+1} - \sum_{i=K+3}^{T}\delta^{i-1}\right)\pi_L(p^M_L) \geq \tag{45}$$

$$\sum_{i=K-j+3}^{T}\delta^{i-1}D_L - \delta^{K+1}\pi_L(p^*_{K+2,L}) - \sum_{i=K-j+3}^{K+1}\delta^{i-1}\pi_L(p^*_i)$$

$$\delta^{K-j}\pi_L(p^*_{K-j+1}) - \left(\delta^{K-j} - \sum_{i=K+3}^{T}\delta^{i-1}\right)\pi_L(p^M_L) \geq \tag{46}$$

$$\sum_{i=K-j+2}^{T}\delta^{i-1}D_L - \delta^{K+1}\pi_L(p^*_{K+2,L}) - \sum_{i=K-j+3}^{K+1}\delta^{i-1}\pi_L(p^*_i) - \delta^{K-j+1}\pi_L(p^*_{K-ji+2})$$

Substituting (45) in (46), rearranging and reducing yields

$$\pi_L(p^*_{K-j+1}) \geq (1 - \delta)\pi_L(p^M_L) + \delta D_L \tag{47}$$

Last, if the equilibrium requires pooling in only the first period, then it must be that

$$\pi_L(p^*_1) \geq \left(1 - \sum_{i=3}^{T}\delta^{i-1}\right)\pi_L(p^M_L) + \sum_{i=2}^{T}\delta^{i-1}D_L - \delta\pi_L(p^*_{K+2,L}) \tag{48}$$

Substituting for the value of $\pi_L(p^*_{K+2,L})$ given by (44) and rearranging, yields

$$\pi_L(p^*_1) \geq (1 - \delta)\pi_L(p^M_L) + \delta D_L \tag{49}$$

This completes the proof ∎

Proof of Lemma 3 (inefficient incumbent's incentive constraints). The constraints are as follows:

$$\sum_{i=1}^{K+1} \delta^{i-1} \pi_H(p_i^*) + \delta^{K+1} \pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq$$

$$\sum_{i=1}^{M+1} \delta^{i-1} \pi_H(p_i^*) + \delta^{M+1} \pi_H(p_H^M) + \sum_{i=M+3}^{T} \delta^{i-1} D_H, \quad 0 \leq M < K = 0, ..., T-3 \quad (50)$$

$$\pi_H(p_H^M) + \sum_{i=2}^{T} \delta^{i-1} D_H \geq \pi_H(p_{1,L}^*) + \delta \pi_H(p_H^M) + \sum_{i=3}^{T} \delta^{i-1} D_H \quad (51)$$

$$\sum_{i=1}^{K+1} \delta^{i-1} \pi_H(p_i^*) + \delta^{K+1} \pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq$$

$$\sum_{i=1}^{K+1} \delta^{i-1} \pi_H(p_i^*) + \delta^{K+1} \pi_H(p_{K+2,L}^*) + \delta^{K+2} \pi_H(p_H^M) + \sum_{i=K+4}^{T} \delta^{i-1} D_H, \quad K = 0, ..., T-4 \quad (52)$$

$$\pi_H(p_H^M) + \sum_{i=K+2}^{T} \delta^{i-1} D_H \geq \pi_H(p_{1,L}^*) + \sum_{i=2}^{K+2} \delta^{i-1} \pi_H(p_L^M) + \delta^{K+2} \pi_H(p_H^M) + \sum_{i=K+4}^{T} \delta^{i-1} D_H,$$

$$K = 0, ..., T-4 \quad (53)$$

$$\sum_{i=1}^{K+1} \delta^{i-1} \pi_H(p_i^*) + \delta^{K+1} \pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq$$

$$\sum_{i=1}^{K+1} \delta^{i-1} \pi_H(p_i^*) + \delta^{K+1} \pi_H(p_{K+2,L}^*) + \sum_{i=K+3}^{M+2} \delta^{i-1} \pi_H(p_L^M) + \delta^{M+2} \pi_H(p_H^M) + \sum_{i=M+4}^{T} \delta^{i-1} D_H,$$

$$0 \leq K < M = 0, ..., T-4 \quad (54)$$

$$\pi_H(p_H^M) + \sum_{i=2}^{T} \delta^{i-1} D_H \geq \pi_H(p_{1,L}^*) + \sum_{i=2}^{T-1} \delta^{i-1} \pi_H(p_L^M) + \delta^{T-1} \pi_H(p_H^M) \quad (55)$$

$$\sum_{i=1}^{K+1} \delta^{j-1} \pi_H(p_i^*) + \delta^{K+1} \pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq$$

$$\sum_{i=1}^{K+1} \delta^{i-1} \pi_H(p_i^*) + \delta^{K+1} \pi_H(p_{K+2,L}^*) + \sum_{i=K+3}^{T-1} \delta^{i-1} \pi_H(p_L^M) + \delta^{T-1} \pi_H(p_H^M),$$

$$K = 0, ..., T-4 \quad (56)$$

These sets of constraints will be explained in turn. Roughly, the H type's off equilibrium behavior can be described by the sequence *pool-mimic-reveal*. That is, first H pools whenever the L type pools, then the H type mimics L's behavior for some number of periods and then he reveal his type, subsequently earning duopoly profits following entry by E. The first set (50) considers the possibility of the H type revealing his type, by setting the monopoly price earlier than the period in which the L type separates. These constraints will determine the pooling constraints for the H type. Next, the constraints (51) and (52) consider the H type mimicking the L type for a single period, in the cases of no prior pooling and an arbitrary number of prior periods with pooling, respectively. Constraints (53) and (54) consider the H type mimicking the L type for a number of periods, in the cases of no prior pooling and an arbitrary number of prior periods with pooling, respectively. Last, constraints (55) and (56)

consider the possibility of the H type perpetually mimicking the L type, again in the cases of no prior pooling and an arbitrary number of prior periods with pooling, respectively.

The first equations in parts (i) and (ii) of the Lemma follow from the constraints (50) and the same steps as those leading to the incentive compatibility constraints for the L type.

The next step is to order the magnitudes of the right-hand sides of constraints (51)-(56). Straightforward comparison shows that the order depends on whether or not

$$(1 - \delta)\pi_H(p_H^M) + \delta D_H \geq \pi_H(p_L^M) \tag{57}$$

If (57) is satisfied, then (51)-(52) imply (53)-(56), whereas if (57) is violated, then (51)-(54) are implied by (55)-(56). Note that condition (57) is in fact just a restatement of condition (2), i.e. the condition that delineates the monopoly price regime and the limit price regime, respectively.

The incentive compatibility constraints if (57) is satisfied are thus (52), which reduce to

$$\pi_H(p_{K+2,L}^*) \leq (1 - \delta)\pi_H(p_H^M) + \delta D_H \tag{58}$$

for $K = 0, ..., T - 4$, whereas the equivalent constraint for the first period follows from (51). If (57) is violated, then the relevant incentive compatibility constraints are (56), which reduce to

$$\pi_H(p_{K+2,L}^*) \leq \left(1 - \delta^{T-K-2}\right)\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-K-1}}{1 - \delta}\right)D_H - \left(\frac{\delta - \delta^{T-K-2}}{1 - \delta}\right)\pi_H(p_L^M) \tag{59}$$

for $K = 0, ..., T - 4$, whereas the equivalent constraint for the first period follows from (55) ∎

Proof of Proposition 4 (existence of separating limit price equilibrium). To prove the proposition, some preliminary notation and results are needed. As $D_H = \pi_H(p)$ for some p, then by Assumptions 6 and 7 the set A_H is non-empty and contains at most two points given by $\widehat{\alpha}$ and $\widehat{\beta}$, where $\widehat{\alpha} < \infty$ and $\widehat{\beta} \leq \infty$. For later use, note that by definition,

$$\pi_H(\widehat{\alpha}) = \delta D_H + (1 - \delta)\pi_H(p_H^M) = \pi_H(\widehat{\beta}) \tag{60}$$

Observe that $p_L^* < p_L^M < p_H^M < \widehat{\beta}$. In conclusion, for the H type's incentive compatibility constraint to hold, it must be that

$$p_L^* \leq \widehat{\alpha} \tag{61}$$

In other words, in order for the high cost incumbent to be willing to tell the truth, the low cost incumbent's strategy must be sufficiently low.

Next, as $D_L = \pi_L(p)$ for some p, then by Assumptions 6 and 7 the set A_L is non-empty and contains at most two points, given by α_0 and β_0, where $\alpha_0 < \infty$ and $\beta_0 \leq \infty$. By definition, it is the case that

$$\pi_L(\alpha_0) = \delta D_L + (1 - \delta)\pi_L(p_L^M) = \pi_L(\beta_0) \tag{62}$$

This means that for the low cost incumbent to be willing to engage in costly signaling, the separating equilibrium price must be high enough.

Last, the following result is needed:

Lemma 15. (relative efficiency of types)
   (i) $\pi_L(p) - \pi_H(p)$ is strictly decreasing in p and (ii) $p_H^M > p_L^M$.

Proof: See Online Appendix ∎

Next, I turn to the proof of the proposition:

(i) Solving (16) and (60) for D_L and D_H respectively, substituting into (27) and rearranging, yields

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}) \tag{63}$$

Adding and subtracting $\pi_H(p_L^M)$ yields

$$\pi_L(p_L^M) + \pi_H(p_L^M) + \left[\pi_H(p_L^M) - \pi_H(p_H^M)\right] > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}) \tag{64}$$

By the definition of p_H^M, it follows that $\pi_H(p_L^M) - \pi_H(p_H^M) \leq 0$. It thus follows from (64) that

$$\pi_L(p_L^M) + \pi_H(p_L^M) > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}) \tag{65}$$

As $\alpha_0(T,t) \leq p_L^M$, it follows by Lemma 15 that

$$\pi_L(\alpha_0(T,t)) - \pi_H(\alpha_0(T,t)) \geq \pi_L(p_L^M) - \pi_H(p_L^M) \tag{66}$$

Combined with (65), this implies that $\pi_H(\alpha_0(T,t)) < \pi_H(\widehat{\alpha})$. Finally, $\alpha_0(T,t) \leq p_H^M$ and $\widehat{\alpha} \leq p_H^M$ and therefore it follows by Assumption 6 that $\alpha_0(T,t) < \widehat{\alpha}$.

(ii) Solving (16) and (26) for D_L and D_H respectively, substituting into (28) and rearranging, yields

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}(T,t)) \tag{67}$$

Similar steps as in (i) then complete the proof ∎

Proof of Proposition 5 (uniqueness of separating limit price equilibrium). First, condition (2), which delineates the two regimes, holds if and only if $\widehat{\alpha}(T,t) \geq p_L^M$. To see this, note that from (26), it follows that

$$\delta D_H = \left(\frac{1-\delta}{1-\delta^{T-t}}\right)\left[\pi_H(\widehat{\alpha}(T,t)) - (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right)\pi_H(p_L^M)\right] \tag{68}$$

Substituting this in (2) yields

$$\pi_H(\widehat{\alpha}(T,t)) \leq \pi_H(p_L^M) \tag{69}$$

As $\widehat{\alpha}(T,t) < p_H^M$ and $p_L^M < p_H^M$, the result follows from the inequality and Assumption 6. Next, I make use of this result to prove the proposition. (i) Suppose that $\alpha_0(T,t) < \widehat{\alpha}(T,t) \leq p_L^M$ and let p' satisfy $p_L^* < p' < \widehat{\alpha}(T,t)$. Whichever strategy E picks, it is a strictly dominated strategy for H to choose p'. If $s_E(p') = 1$, then because $p' < \widehat{\alpha}(T,t) \leq p_L^M \leq p_H^M$, Assumption 6 implies that the H type can benefit from switching to p_H^M, thereby earning $\pi_H(p_H^M) - \pi_H(p') > 0$. Next, suppose that $s_E(p') = 0$. In equilibrium, the H type should set the price p_H^M and can never earn more out of equilibrium than by playing his optimal off equilibrium strategy. But the first element of this strategy is precisely given by $\widehat{\alpha}(T,t)$. It follows that the H type is better off by switching from p' to $\widehat{\alpha}(T,t)$. After deleting the price p' from the H type's strategy set, E must set $s_E(p') = 0$, as p' could only have been set by the L type. But because $p' < \widehat{\alpha}(T,t) \leq p_L^M$, it follows from Assumption 6 that the L type is better off by increasing his price to $\widehat{\alpha}(T,t)$. The proof of (ii) follows similar steps as that of (i) ∎

Proof of Lemma 8 (incumbents' incentive constraints). The incentive compatibility constraints for the L type are given by[21]

$$\sum_{i=1}^{T-1} \delta^{i-1} \pi_L(p_i^*) + \delta^{T-1} \pi_L(p_L^M) \geq \pi_L(p_L^M) + \sum_{i=2}^{T} \delta^{i-1} D_L \tag{70}$$

$$\sum_{i=1}^{T-1} \delta^{i-1} \pi_L(p_i^*) + \delta^{T-1} \pi_L(p_L^M) \geq \sum_{i=1}^{K+1} \delta^{i-1} \pi_L(p_i^*) + \delta^{K+1} \pi_L(p_L^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_L \tag{71}$$

for $K = 0, 1, ..., T-3$. The set of constraints (71), (one for each K) compares the equilibrium strategy with a strategy that pools until (and including) period $K+1$ and deviates in period $K+2$. Solving (70) for $\pi_L(p_1^*)$, yields

$$\pi_L(p_1^*) \geq (1 - \delta^{T-1}) \pi_L(p_L^M) + \sum_{i=2}^{T} \delta^{i-1} D_L - \sum_{i=3}^{T-1} \delta^{i-1} \pi_L(p_i^*) - \delta \pi_L(p_2^*) \tag{72}$$

Evaluating (71) at $K = 0$ and rearranging, yields

$$\delta \pi_L(p_2^*) \geq \sum_{i=K+3}^{T} \delta^{i-1} D_L + \delta \pi_L(p_L^M) - \sum_{i=3}^{T-1} \delta^{i-1} \pi_L(p_i^*) \tag{73}$$

Substituting this in (72) and rearranging, gives

$$\pi_L(p_1^*) \geq (1 - \delta) \pi_L(p_L^M) + \delta D_L$$

For arbitrary K, (71) reduces to

$$\sum_{i=K+2}^{T-1} \delta^{i-1} \pi_L(p_i^*) + \delta^{T-1} \pi_L(p_L^M) \geq \delta^{K+1} \pi_L(p_L^M) + \sum_{i=K+3}^{T} \delta^{i-t} D_L \tag{74}$$

Straightforward manipulation yields that this inequality can be rewritten as

$$\sum_{i=0}^{T-K-3} \delta^{i} \pi_L(p_{i+K+2}^*) + \delta^{T-K-2} \pi_L(p_L^M) \geq \pi_L(p_L^M) + \sum_{i=0}^{T-K-3} \delta^{i+1} D_L \tag{75}$$

In particular, this implies that

$$\pi_L(p_{K+2}^*) \geq \left(1 - \delta^{T-K-2}\right) \pi_L(p_L^M) + \sum_{i=0}^{T-K-3} \delta^{i+1} D_L - \sum_{i=2}^{T-K-3} \delta^{i} \pi_L(p_{i+K+2}^*) - \delta \pi_L(p_{K+3}^*) \tag{76}$$

But the constraint on $\pi_L(p_{K+3}^*)$ is in turn given by

$$\delta \pi_L(p_{K+3}^*) \geq \left(\delta - \delta^{T-K-2}\right) \pi_L(p_L^M) + \sum_{i=1}^{T-K-3} \delta^{i+1} D_L - \sum_{i=2}^{T-K-3} \delta^{i} \pi_L(p_{i+K+2}^*) \tag{77}$$

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[21]It is without loss of generality to consider a deviation in period 1, because if there is pooling in periods $s = 1, ..., t-1$, then the period t problem is essentially the same as that faced in period 1.

Substituting this back in (76) and rearranging, yields the following constraints:

$$\pi_L(p^*_{K+2}) \geq (1-\delta)\pi_L(p^M_L) + \delta D_L \tag{78}$$

for $K = 0, 1, ..., T-3$. Similar steps yield the equivalent constraints for the H type ∎

Proof of Proposition 11 (cost of signaling with infinite horizon). For $\lim_{T\to\infty} \pi_H(\widehat{\alpha}(T,t)) \leq 0$ to hold, it follows from (34) that the inequality

$$\pi_H(p^M_L) - D_H \geq \left(\frac{1-\delta}{\delta}\right)\pi_H(p^M_H) \tag{79}$$

must hold. This inequality can be rewritten as

$$\delta \geq \frac{\pi_H(p^M_H)}{\pi_H(p^M_H) - D_H + \pi_H(p^M_L)} \equiv \delta^{**} \tag{80}$$

Next, note that if $\pi_H(p^M_L) > D_H$, then (2) can be rewritten as

$$\delta \leq \frac{\pi_H(p^M_H) - \pi_H(p^M_L)}{\pi_H(p^M_H) - D_H} \equiv \delta^* \tag{81}$$

From Assumption 7 it follows that the violation of (2) is a sufficient (but not necessary) condition for $\pi_H(p^M_L) > D_H$ to hold. Last, note that $\delta^* \geq \delta^{**}$ if and only if $\pi_H(p^M_L) > D_H$, which is implied by the assumption that (2) is violated ∎

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DYNAMIC LIMIT PRICING: ONLINE APPENDIX

Flavio Toxvaerd[*]

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Abstract.   This appendix offers a detailed and self-contained analysis of the benchmark single-round version of the dynamic model presented in DYNAMIC LIMIT PRICING. In addition, the appendix offers a worked example and detailed discussion of different aspects of the dynamic extension omitted from the main article, such as equilibrium selection and post-entry collusion. Note that there is some overlap between this text and that in the article and that the equation numbering is independent from that of the main manuscript.

1. The Static Model

The following model is a simple version of the model of Milgrom and Roberts (1982). Consider an incumbent monopolist I and a potential entrant E. The monopolist serves a market with demand $Q(p)$ and the entrant can enter the market at cost $F > 0$ to compete with the incumbent. The monopolist can be one of two types, high cost (H) or low cost (L), with probability μ and $(1 - \mu)$ respectively. The incumbent knows his type, but his type is unknown to the entrant (who only knows the probability μ). Let $C_H(q)$ and $C_L(q)$ be the cost functions of H and L respectively. Denote by $\pi_i(p)$ the profit function of the incumbent of type $i = H, L$ when he sets price p. These profits are given by

$$\pi_i(p) = pQ(p) - C_i(Q(p)), \quad i = H, L \tag{1}$$

Let D_i be the duopoly profit of the incumbent of type $i = H, L$ when competing against E and let $D_E(i)$ be the duopoly profits of E when competing against the incumbent of type $i = H, L$. Denote by p_H^M and p_L^M the monopoly prices under the technologies $C_H(\cdot)$ and $C_L(\cdot)$ respectively.

Throughout, I make the following assumptions:

Assumptions

1 $C_i(q)$, $i = H, L$ and $Q(p)$ are differentiable, for $q > 0$ and $p > 0$ respectively.

2 $C_H'(q) > C_L'(q), \forall q \in \mathbb{R}_+$, with $C_H(0) \geq C_L(0)$.

3 $Q'(p) < 0, \forall p \geq 0$.

4 $D_E(L) - F < 0$.

5 $D_E(H) - F > 0$.

6 $\pi_i(p)$ is strictly increasing for $p < p_i^M$ and strictly decreasing for $p > p_i^M$, $i = H, L$.

7 $\pi_i(p_i^M) > D_i$, $i = H, L$.

8 $\mu D_E(H) + (1 - \mu)D_E(L) - F < 0$.

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[*]Faculty of Economics, University of Cambridge.

Assumption 2 makes precise the sense in which type L is more efficient than type H. Assumption 3 simply states that demand is downward sloping. Assumptions (4)-(5) imply that E will not enter if he knows that I is of type L, whereas he will enter if he knows that I is of type H. Thus these conditions are necessary for a separating limit price equilibrium to exist. Assumption 6 means that the incumbent's profit function is single peaked, whereas Assumption 7 ensures that entry deterrence is desirable for the incumbent, ceteris paribus. Under Assumption 8, the entrant expects to make negative profits against the incumbent if he cannot distinguish between the two types and thus stays out. This is a necessary condition for a pooling limit price equilibrium to exist.

The game between I and E is played in three stages. At the first stage, I sets a price that will serve as a signal for E of I's type. After observing the price set by I, E decides at the second stage whether or not to enter (incurring the entry fee F). Denote E's entry decision by $s_E \in \{0, 1\}$, where $s_E = 0$ stands for *stay out* and $s_E = 1$ stands for *enter*. At the third stage, if E enters he will learn I's type and compete against him in complete information fashion. Both incumbent and entrant discount the future by a factor $\delta \in [0, 1]$. The payoff to E is given by

$$\Pi_E(p) \equiv \begin{cases} 0 & \text{if } s_E = 0 \\ D_E(H) - F & \text{if } s_E = 1, \quad i = H \\ D_E(L) - F & \text{if } s_E = 1, \quad i = L \end{cases} \tag{2}$$

A strategy for I is a price for each of his two types, p_H or p_L, at the first stage, a price at the second stage if the entrant stays out and a quantity or price to set at the third stage if the entrant enters (depending on the mode of competition), both as functions of his type and the decisions made at the first stage. A strategy for E is a decision rule to enter or not as a function of the price set by I at the first stage and a quantity or price to set at the third stage in case he enters (again, depending on the mode of competition).

If E enters at the second stage, then at the third stage I and E play a duopoly game of complete information. Hence in any subgame perfect equilibria of the game after E's entry, I's equilibrium payoffs in the third stage are D_H or D_L. If E stays out, then I's equilibrium payoffs at the third stage are $\pi_H(p_H^M)$ or $\pi_L(p_L^M)$, depending on his type.[1] That is, the payoffs to the incumbent of type $i = H, L$ are given by

$$\Pi_i(p) \equiv \begin{cases} \pi_H(p) + \delta\pi_H(p_H^M) & \text{if } s_E = 0, \quad i = H \\ \pi_H(p) + \delta D_H & \text{if } s_E = 1, \quad i = H \\ \pi_L(p) + \delta\pi_L(p_L^M) & \text{if } s_E = 0, \quad i = L \\ \pi_L(p) + \delta D_L & \text{if } s_E = 1, \quad i = L \end{cases} \tag{3}$$

Next, I state some key definitions that will be used throughout this section. Let $\sigma \equiv (p_L, p_H, \overline{p})$ denote a triple of pure strategies of the game, i.e. a price charged by each type of I and a threshold price governing E's entry decision (details are given below). Throughout this article, attention will be restricted to pure strategy perfect Bayesian equilibria. Denote by p_H^* and p_L^* the equilibrium prices charged by the H type and the L type respectively.

Definition 1. σ *is a separating equilibrium if* $p_H^* \neq p_L^*$ *and a pooling equilibrium if* $p_H^* = p_L^*$. σ *is a limit price equilibrium if* $p_H^* < p_H^M$ *or* $p_L^* < p_L^M$ *or both.*

The aim of the analysis that follows is to characterize separating and pooling limit price equilibria of the game. Note that under the maintained assumptions, ceteris paribus, the high cost incumbent will wish to set a higher monopoly price than the low cost incumbent. Formally, the following result obtains:

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[1] In the dynamic version of the model, stages one and two will together constitute a period and stage three will be a separate period.

Lemma 2. *(relative efficiency of types)*
(i) $\pi_L(p) - \pi_H(p)$ *is strictly decreasing in p and (ii) $p_H^M > p_L^M$.*

Proof. (i) First, note that $\pi_L(p) - \pi_H(p) = C_H(Q(p)) - C_L(Q(p))$ and thus

$$\frac{\partial}{\partial p}[\pi_L(p) - \pi_H(p)] = Q'(p)[C'_H(Q(p)) - C'_L(Q(p))] \tag{4}$$

By Assumption 3, $Q'(p) < 0$. Thus, by Assumption 2 it follows that $\frac{\partial}{\partial p}[\pi_L(p) - \pi_H(p)] < 0$.
(ii) By the definition of monopoly prices and Assumption 6, it follows that

$$p_L^M Q_L^M - C_L(Q_L^M) > p_H^M Q_H^M - C_L(Q_H^M) \tag{5}$$
$$p_H^M Q_H^M - C_H(Q_H^M) > p_L^M Q_L^M - C_H(Q_L^M) \tag{6}$$

Adding these inequalities, I obtain

$$C_H(Q_L^M) - C_L(Q_L^M) > C_H(Q_H^M) - C_L(Q_H^M) \tag{7}$$

Hence, by Assumption 2, $Q_L^M > Q_H^M$ and by Assumption 3, $p_L^M < p_H^M$ ■

An implication of this fact is that an inefficient incumbent would only set lower prices than an inefficient incumbent's monopoly price, in order to convince the entrant that it is in fact an efficient incumbent.

Perfect Bayesian equilibrium requires that beliefs be derived from Bayes' rule whenever possible. This means that one must assign beliefs after out of equilibrium (i.e. probability zero) events have been observed. For simplicity, the out of equilibrium beliefs of E will be assumed to have the following monotone structure:

$$\mu'(p) = \begin{cases} 1 \text{ if } & p \leq p' \\ 0 \text{ if } & p > p' \end{cases}$$

where μ' is the probability assigned to the incumbent being of type L and p' is the L type's equilibrium strategy (i.e. either the separating price in a separating equilibrium or the common price in a pooling equilibrium).[2] That is, for any observed price above the L type's equilibrium price, the entrant will assign probability one to the incumbent being of type H. For prices below the L type's equilibrium price, the entrant will assign probability one to the incumbent being of type L.

This structure on beliefs is equivalent to a monotone decision rule for the entrant of the form

$$s_E(p) = \begin{cases} 1 \text{ if } & p > \bar{p} \\ 0 \text{ if } & p \leq \bar{p} \end{cases} \tag{8}$$

for some appropriately chosen threshold price $\bar{p}$ (determined by the entrant). The equivalence is straightforward (it follows from Assumptions 3 and 4) and is shown below for each of the two types of equilibria respectively. The restriction to such monotone entry rules is routine in the literature.

1.1. Separating Limit Price Equilibria.

Characterization. In a separating equilibrium, the entrant can, by definition, infer the incumbent's type merely by observing its chosen equilibrium price. Hence assume that $p_H^* \neq p_L^*$. The best reply strategy of E in this case is to enter if $p = p_H^*$ and to stay out if $p = p_L^*$, i.e. $s_E(p_H^*) = 1$ and $s_E(p_L^*) = 0$. Therefore the H type incumbent is best off setting $p = p_H^M$,

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[2]This is for simplicity only. Any off equilibrium beliefs that favor entry would do.

knowing that entry will occur in the second period, so $s_E(p_H^M) = 1$. Hence the high cost incumbent's equilibrium price is given by

$$p_H^* = p_H^M \qquad (9)$$

To obtain a limit price equilibrium, it is thus required that

$$p_L^* \neq p_L^M \qquad (10)$$

Next, the entrant's cutoff price can be characterized as follows:

Lemma 3. *(entrant's optimal decision)*
$\bar{p} = p_L^*$ *and* $\bar{p} < p_L^M$.

Proof. Suppose to the contrary that $\bar{p} \geq p_L^M$. Then $s_E(p_L^M) = 0$ and L is therefore best off switching from p_L^* to p_L^M, contradicting (10). Next, observe that in a separating equilibrium, $s_E(p_L^*) = 0$ (E knows that L set p_L^*) and hence $p_L^* \leq \bar{p}$. Suppose that $p_L^* < \bar{p}$. Since $\bar{p} < p_L^M$, it follows by Assumption 6 that L is better off by increasing his price from p_L^* to $\bar{p}$, which is a contradiction. Thus, $\bar{p} = p_L^*$ ∎

The characterization so far of the separating equilibrium prices may be summarized in the following way:

Corollary 4. *(characterization of separating equilibria)*
   (i) In any separating limit price equilibrium, $p_L^* < p_L^M$ and (ii) in any separating equilibrium, either $p_L^* = p_L^M \leq \bar{p}$ or $p_L^* = \bar{p} < p_L^M$.

These results completely characterize the entrant's equilibrium behavior. I proceed by further analyzing the incumbent's equilibrium strategy.

The Incentive Compatibility Constraints. Since $p_H^* = p_H^M$, the following incentive compatibility constraint for H should hold:

$$\Pi_H(p_H^M) \geq \Pi_H(p), \quad \forall p \qquad (11)$$

This simply means that the H type's equilibrium strategy is globally optimal. Clearly, (11) holds for $p > \bar{p}$ because in this case, E enters and I can do no better than to set the monopoly price. Consider p such that $p \leq \bar{p}$. By Lemma 2, in a separating limit pricing equilibrium $\bar{p} = p_L^*$ and hence by Assumption 6 and Lemma 1 (ii) (which can be found in Appendix), it follows that $p \leq \bar{p} = p_L^* < p_L^M < p_H^M$ and thus it is sufficient to consider the following inequality:

$$\Pi_H(p_H^M) \geq \Pi_H(p_L^*) \qquad (12)$$

By the definition of Π_H given in (3), (12) is equivalent to

$$\pi_H(p_L^*) \leq (1 - \delta)\pi_H(p_H^M) + \delta D_H \qquad (13)$$

For later reference, note that the right-hand side of (13) is strictly positive. This means that for the incentive compatibility constraint (13) to be satisfied, it is not *necessarily* the case that the H type's profits from mimicking the L type are negative. As shall be shown in Section 3, this result does not carry over to the dynamic setting.

To write (13) in terms of prices, first define the set

$$A_H \equiv \left\{ p : \pi_H(p) = (1 - \delta)\pi_H(p_H^M) + \delta D_H \right\} \qquad (14)$$

This set is simply the set of prices for which the H type's incentive compatibility constraint is binding. Since $D_H = \pi_H(p)$ for some p, then by Assumptions 6 and 7 the set A_H is non-empty and contains at most two points. Next, define

$$\widehat{\alpha} \equiv \min A_H, \quad \widehat{\beta} \equiv \max A_H \tag{15}$$

where $\widehat{\alpha} < \infty$ and $\widehat{\beta} \leq \infty$. Hence, according to (13), p_L^* must satisfy

$$p_L^* \notin [\widehat{\alpha}, \widehat{\beta}] \tag{16}$$

For later use, note that by definition,

$$\pi_H(\widehat{\alpha}) = \delta D_H + (1 - \delta)\pi_H(p_H^M) = \pi_H(\widehat{\beta}) \tag{17}$$

Observe that $p_L^* < p_L^M < p_H^M < \widehat{\beta}$. In conclusion, for the H type's incentive compatibility constraint to hold, it must be that

$$p_L^* \leq \widehat{\alpha} \tag{18}$$

In other words, in order for the high cost incumbent to be willing to tell the truth, the low cost incumbent's strategy must be sufficiently low.

I now turn to the L type. The incentive compatibility constraint for L is given by

$$\Pi_L(p_L^*) \geq \Pi_L(p), \quad \forall p \tag{19}$$

Again, this inequality simply states that the L type's equilibrium strategy is globally optimal. But the relevant alternative strategy p is only $p = p_L^M$ (because deterring entry is only optimal if it yields higher payoffs than setting the monopoly price in the first period and accommodating entry). Hence (19) becomes

$$\Pi_L(p_L^*) \geq \Pi_L(p_L^M) \tag{20}$$

By the definition of Π_L given by (3), inequality (20) is equivalent to

$$\pi_L(p_L^*) + \delta\pi_L(p_L^M) \geq \pi_L(p_L^M) + \delta D_L \tag{21}$$

Consequently,

$$\pi_L(p_L^*) \geq (1 - \delta)\pi_L(p_L^M) + \delta D_L \tag{22}$$

is the relevant incentive compatibility constraint for L. Define the set

$$A_L \equiv \left\{ p : \pi_L(p) = (1 - \delta)\pi_L(p_L^M) + \delta D_L \right\} \tag{23}$$

Again, this set is the set of prices for which the L type's incentive compatibility constraint is binding. Since $D_L = \pi_L(p)$ for some p, then by Assumptions 6 and 7 the set A_L is non-empty and contains at most two points. Let

$$\alpha_0 \equiv \min A_L, \quad \beta_0 \equiv \max A_L \tag{24}$$

where $\alpha_0 < \infty$ and $\beta_0 \leq \infty$.

In terms of prices, the L type's incentive compatibility (22) can then be written as

$$p_L^* \in [\alpha_0, \beta_0] \tag{25}$$

where, by definition, it is the case that

$$\pi_L(\alpha_0) = \delta D_L + (1 - \delta)\pi_L(p_L^M) = \pi_L(\beta_0) \tag{26}$$

This means that for the low cost incumbent to be willing to engage in costly signaling, the separating equilibrium price must be high enough. The previous results can be summarized as follows:

Proposition 5. *(characterization of separating limit price equilibria)*
Any separating limit price equilibrium is a triple $(p_H^*, p_L^*, \overline{p})$ such that (i) $p_H^* = p_H^M$, (ii) $\overline{p} = p_L^*$, (iii) $\alpha_0 \leq p_L^* \leq \widehat{\alpha}$ and (iv) $p_L^* < p_L^M$.

Hence, to show existence of a separating limit price equilibrium, I need to show that $\alpha_0 < \widehat{\alpha}$.[3]

Existence of Separating Limit Price Equilibria. The existence of separating limit price equilibria is now considered. Fortuitously, existence is secured under very mild conditions on the primitives of the model, as the following result shows:

Proposition 6. *(existence of separating limit price equilibria)*
Suppose that

$$\pi_L(p_L^M) - D_L > \pi_H(p_H^M) - D_H \tag{27}$$

Then $\widehat{\alpha} > \alpha_0$ and the set of separating limit pricing equilibria is non-empty.

Proof. From (27), (17) and (26), it follows that

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(\alpha_0) - \pi_H(\widehat{\alpha}) \tag{28}$$

Adding and subtracting $\pi_H(p_L^M)$ yields

$$\pi_L(p_L^M) + \pi_H(p_L^M) + \left[\pi_H(p_L^M) - \pi_H(p_H^M)\right] > \pi_L(\alpha_0) - \pi_H(\widehat{\alpha}) \tag{29}$$

By the definition of p_H^M, it follows that $\pi_H(p_L^M) - \pi_H(p_H^M) \leq 0$. It thus follows from (29) that

$$\pi_L(p_L^M) + \pi_H(p_L^M) > \pi_L(\alpha_0) - \pi_H(\widehat{\alpha}) \tag{30}$$

Since $\alpha_0 \leq p_L^M$, it follows by Lemma 1 that

$$\pi_L(\alpha_0) - \pi_H(\alpha_0) \geq \pi_L(p_L^M) - \pi_H(p_L^M) \tag{31}$$

Combined with (30), this implies that $\pi_H(\alpha_0) < \pi_H(\widehat{\alpha})$. Finally, $\alpha_0 \leq p_H^M$ and $\widehat{\alpha} \leq p_H^M$ and therefore it follows by Assumption 6 that $\alpha_0 < \widehat{\alpha}$ ∎

Condition (27) holds for the cases of Cournot competition with linear demand and fixed marginal costs (for sufficiently high demand intercept) and Bertrand competition with or without product differentiation (see Tirole, 1988).

Equilibrium Selection. As seen above, the solution concept perfect Bayesian equilibrium fails to uniquely determine L's equilibrium price p_L^*. The reason for this lies in the arbitrariness of off-equilibrium path beliefs, which are not pinned down by Bayes' rule and on which the notion of perfectness imposes no restrictions. To get a sharp characterization of

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[3]If one eliminates the limit pricing requirement, then in addition to the set $\{(p_H^*, p_L^*, \overline{p}) : $ (i), (ii), (iii), (iv) satisfied$\}$ there are other separating equilibrium points if $\widehat{\alpha} \geq p_L^M$. In particular, any separating equilibrium is a triple $(p_H^*, p_L^*, \overline{p})$ such that (i) $p_H^* = p_H^M$, (ii) $\overline{p} = p_L^*$ and (iii) p_L^* satisfies the inequality $p_0 \leq p_L^* \leq \min\{\widehat{\alpha}, p_L^M\}$.

equilibrium behavior, I therefore make use of the notion of *equilibrium dominance*. This entails using notions of both backward and forward induction. Specifically, it requires that the incumbent's equilibrium strategy (at the signaling stage) form part of a perfect Bayesian equilibrium of the game obtained after deletion of strategies that are not a weak best response to any of the entrant's possible equilibrium strategies (at the entry stage). In other words, a deviation from an equilibrium price will be interpreted as coming from the L type whenever the H type cannot possibly benefit from such a deviation (for any best response of the entrant) whereas the L type incumbent would stand to benefit from such a deviation. The reasonableness of this criterion lies in the fact that it requires the entrant to assign probability zero to a type of incumbent who would find the observed action to be dominated by the equilibrium action, irrespective of the entrant's response to such a deviation from equilibrium play. In other words, if irrespective of the entrant's response to a non-equilibrium price, one type of incumbent could not possibly benefit from such a deviation and thus earn lower payoff than by setting its equilibrium price, the entrant will disregard the possibility that the incumbent is of that type. As the following proposition shows, this refinement yields a unique equilibrium:

Proposition 7. *(uniqueness of separating limit price equilibrium satisfying dominance)*
   (i) Suppose that $\alpha_0 < \widehat{\alpha} \leq p_L^M$. Then only $p_L^* = \widehat{\alpha}$ satisfies equilibrium dominance. (ii) Suppose that $\alpha_0 < p_L^M \leq \widehat{\alpha}$. Then only $p_L^* = p_L^M$ satisfies equilibrium dominance.

Proof. (i) Suppose that $\alpha_0 < \widehat{\alpha} \leq p_L^M$ and let p' satisfy $p_L^* < p' < \widehat{\alpha}$. Whichever strategy E picks, it is a strictly dominated strategy for H to choose p'. If $s_E(p') = 1$, then because $p' < \widehat{\alpha} \leq p_L^M \leq p_H^M$ it follows that

$$\pi_H(p') + \delta D_H < \pi_H(\widehat{\alpha}) + \delta D_H \tag{32}$$

If in turn $s_E(p') = 0$, then

$$\pi_H(p') + \delta \pi_H(p_H^M) < \pi_H(\widehat{\alpha}) + \delta \pi_H(p_H^M) = \pi_H(p_H^M) + \delta D_H \tag{33}$$

Hence, even if H fools E to believe that he is L, he will obtain less than $\pi_H(p_H^M) + \delta D_H$ which he would obtain under the equilibrium strategy $p_H^* = p_H^M$. In the game obtained after eliminating the strategy p' from H's strategy set, E must play $s_E(p') = 0$ because p' can have been set only by L and thus by backward induction staying out at the price p' is a best response for E. But in the new reduced game, L can profitably deviate from p_L^* to p' and obtain $\pi_L(p') - \pi_L(p_L^*) > 0$, which follows from Assumption 6 and the fact that $p' \leq p_L^M$. For completeness, note that no type of incumbent can benefit from deviations to prices such that $p' \in [\alpha_0, p_L^*]$. The proof of (ii) follows similar steps as that of (i) ∎
   The price selected by the equilibrium dominance approach is known as the least-cost separating equilibrium price, as it is the equilibrium price which involves the lowest possible cost for the L type in terms of foregone profits. In other words, it is the highest price (lower than the monopoly price) consistent with the incentive compatibility constraints. This outcome is known in the literature as the *Riley outcome*.

1.2. Pooling Limit Price Equilibria. In a pooling equilibrium, it is by definition the case that

$$p_L^* = p_H^* = p^* \tag{34}$$

This means that the entrant cannot infer the incumbent's type merely by observing his chosen equilibrium price. Observe first that if

$$\mu D_E(H) + (1 - \mu)D_E(L) - F > 0 \tag{35}$$

then pooling equilibria cannot exist, because the expected profit of E when he cannot distinguish between the incumbent's types is positive and he thus enters regardless of p^*. By backward induction, each type of incumbent is better off setting his monopoly price. Since $p_H^M > p_L^M$, I thus have that $p_L^* \neq p_H^*$, contradicting the supposition that the two types pool. Assumption 8 rules out this case, thus ensuring that a pooling equilibrium is feasible.

Characterization. Before characterizing the incumbent's equilibrium price, the entrant's cutoff rule can be characterized in the following way:

Lemma 8. *(entrant's optimal decision)*
   $\overline{p} = p^*$ and $p^* \leq p_L^M$.

Proof. Clearly $\overline{p} \geq p^*$. Otherwise, E's decision rule dictates entry if p^* is charged. That is, $s_E(p^*) = 1$ if $p^* > \overline{p}$ and thus each type of incumbent would benefit from deviating to their respective monopoly prices, contradicting (34). Next observe that if $p^* > p_L^M$, then L is best off setting the price p_L^M and entry will still be deterred (i.e. $s_E(p_L^M) = 0$). Consequently, $p^* \leq p_L^M$ as claimed. Finally, suppose to the contrary that $\overline{p} > p^*$. Since $p^* \leq p_L^M < p_H^M$, it follows by Assumption 6 that the H type is better off increasing his price slightly above p^* to increase profits while still deterring E's entry. Therefore $\overline{p} = p^*$ must hold as claimed ∎

The Incentive Compatibility Constraints. The incentive compatibility constraints for the H type and the L type are given by

$$\pi_H(p^*) + \delta\pi_H(p_H^M) \quad \geq \quad \pi_H(p_H^M) + \delta D_H \tag{36}$$
$$\pi_L(p^*) + \delta\pi_L(p_L^M) \quad \geq \quad \pi_L(p_L^M) + \delta D_L, \quad p^* < p_L^M \tag{37}$$

Note that for each type, the best alternative strategy to choosing the entry deterring pooling price is to set the monopoly price and inviting entry. Also note that if $p^* = p_L^M$, then there is no incentive compatibility constraint for the L type.[4] The two incentive compatibility constraints (36)-(37) can be rewritten as

$$\pi_H(p^*) \quad \geq \quad (1-\delta)\pi_H(p_H^M) + \delta D_H \tag{38}$$
$$\pi_L(p^*) \quad \geq \quad (1-\delta)\pi_L(p_L^M) + \delta D_L, \quad p^* < p_L^M \tag{39}$$

Using (15) and (24), inequality (38) holds if and only if $\widehat{\alpha} \leq p^* \leq \widehat{\beta}$ whereas (39) holds for $p^* \geq \alpha_0$ as long as $p^* < p_L^M$. Combining these constraints, I obtain:

Proposition 9. *(characterization of pooling limit price equilibria)*
   Any pooling equilibrium is a tuple $(p^*, \overline{p})$ such that (i) $\overline{p} = p^*$ and (ii) p^* satisfies

$$\max\{\alpha_0, \widehat{\alpha}\} \leq p^* \leq p_L^M < p_H^M \tag{40}$$

It should be noted that a pooling equilibrium necessarily involves limit pricing, because at least the H type (and potentially the L type) charges below his monopoly price.

Equilibrium Selection. As was the case with the set of separating limit price equilibria, there is a continuum of pooling limit price equilibria. Again, equilibrium dominance can be used to select a unique equilibrium satisfying equilibrium dominance as follows:

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[4]Throughout the article, the qualifier $p^* < p_L^M$ will reappear in connection with constraints on pooling prices. It will henceforth be implicit that if $p_t^* = p_L^M$ in some period t, then there is no incentive compatibility constraint for the L type in that period.

Proposition 10. *(uniqueness of pooling limit price equilibrium satisfying dominance)*
The only pooling equilibrium limit price satisfying equilibrium dominance is $p^* = p_L^M$.

Proof. The set of pooling equilibrium prices is the set

$$\left\{ p^* : \max\left\{\alpha_0, \widehat{\alpha}\right\} \leq p^* \leq p_L^M \right\} \tag{41}$$

Suppose that $p^* < p_L^M$. First note that $s_E(p_L^M) = 1$, for otherwise the L type is better off switching from p^* to p_L^M. Thus it is a strictly inferior strategy for H to select p_L^M or p_H^M. Indeed, by H's incentive compatibility constraint (36) I have

$$\pi_H(p^*) + \delta\pi_H(p_H^M) \geq \pi_H(p_H^M) + \delta D_H > \pi_H(p_L^M) + \delta D_H \tag{42}$$

Consider the new reduced game, which is obtained from the original game by eliminating p_L^M from H's strategy set. In the equilibrium of the new game, $s_E(p_L^M) = 0$, because this price can only have been set by the L type. Hence L, in the new game, is better off deviating from p^* to p_L^M ∎

As was the case with the selected separating limit price equilibrium, equilibrium dominance selects the least-cost pooling limit price equilibrium.

Last, note the following result, which further reduces the set of pooling limit price equilibria satisfying equilibrium dominance:

Proposition 11. *(possible non-existence of pooling limit price equilibrium satisfying dominance)*
If $\widehat{\alpha} > p_L^M$, then no pooling equilibrium satisfying equilibrium dominance exists.

Proof: First, note that $\widehat{\alpha} \geq p_L^M$ if and only if

$$(1 - \delta)\pi_H(p_H^M) + \delta D_H \geq \pi_H(p_L^M) \tag{43}$$

To see this, note that from (17) it follows that

$$\delta D_H = \pi_H(\widehat{\alpha}) - (1 - \delta)\pi_H(p_H^M) \tag{44}$$

Substituting this in (43) yields

$$\pi_H(\widehat{\alpha}) \leq \pi_H(p_L^M) \tag{45}$$

Since $\widehat{\alpha} < p_H^M$ and $p_L^M < p_H^M$, the result then follows from Assumption 6. Next, recall that for pooling on $p^* = p_L^M$ to be incentive compatible, inequality (38) must hold, i.e.

$$\pi_H(p_L^M) \geq (1 - \delta)\pi_H(p_H^M) + \delta D_H \tag{46}$$

The result then follows immediately. For the knife's edge case $\pi_H(p_L^M) = (1 - \delta)\pi_H(p_H^M) + \delta D_H$, pooling on $p^* = p_L^M$ is incentive compatible ∎

1.3. Existence of Limit Price Equilibria Satisfying Equilibrium Dominance. Before continuing the analysis, some comments on the existence of limit price equilibria satisfying equilibrium dominance are in order. Note that the above existence result concerns itself only with the existence of limit price equilibria and *not* with the existence of limit price equilibria satisfying equilibrium dominance. After performing equilibrium selection, the set of equilibria can, if non-empty, be divided into two distinct regimes, namely a *limit price regime* and a *monopoly price regime*. The former obtains if $\widehat{\alpha} < p_L^M$ and the latter if $\widehat{\alpha} \geq p_L^M$. These regimes will reappear in an important way in the dynamic game. In the monopoly price regime, the
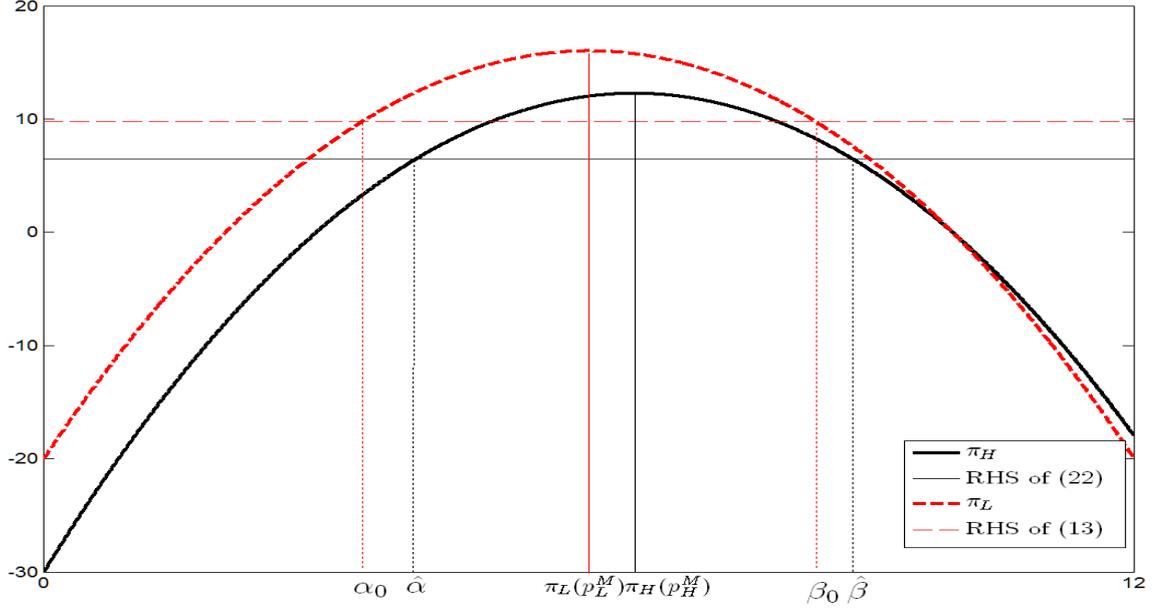
Figure 1: Profits and Incentive Compatibility Constraints in Static Setting.

unique selected equilibrium is characterized by firms separating by setting their respective monopoly prices, whereas in the limit price regime, both pooling and separating limit price equilibria satisfying equilibrium dominance coexist. Which regime obtains, depends on the parameter constellation and on the specifics of the mode of competition.

For later reference, it should be reiterated that the condition determining the regimes is given by (43). That is, the monopoly price regime obtains if and only if

$$\pi_H(p_H^M) + \frac{\delta D_H}{1-\delta} \geq \frac{\pi_H(p_L^M)}{1-\delta} \tag{47}$$

This inequality has an interesting interpretation. The left-hand side is the profit for the H type of revealing his type by earning monopoly profits in the first period and then earning discounted duopoly profits in perpetuity thereafter. The right-hand side is the discounted profit stream for the H type from mimicking the L type's monopoly price in perpetuity.

The profit functions and the incentive compatibility constraints are illustrated in Figure 1.[5]

1.4. Worked Example. In order to illustrate the analysis so far, I will next analyze a concrete functional form example. Consider the case in which the post-entry game takes the form of homogeneous goods Cournot competition. Production involves incurring fixed marginal costs but no fixed costs, so $C'_H(q) = c_H > c_L = C'_L(q)$ and $C_H(0) = C_L(0) = 0$. The entrant has fixed marginal costs $C'_E(q) = c_E$, but no fixed costs (other than the entry fee F). Market demand is given by the function $p = a - bQ$, where $a, b > 0$ and $Q = q_I + q_E$ is total quantity

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[5]The graphs are based on the worked example below.

produced. With this specification, it is straightforward to verify that

$$D_E(L) = \frac{(a + c_L - 2c_E)^2}{9b} \tag{48}$$

$$D_E(H) = \frac{(a + c_H - 2c_E)^2}{9b} \tag{49}$$

$$D_L = \frac{(a + c_E - 2c_L)^2}{9b} \tag{50}$$

$$D_H = \frac{(a + c_E - 2c_H)^2}{9b} \tag{51}$$

$$\pi_L(p_L^M) = \frac{(a - c_L)^2}{4b} \tag{52}$$

$$\pi_H(p_H^M) = \frac{(a - c_H)^2}{4b} \tag{53}$$

$$\pi_L(p_H^M) = \frac{(a - c_H)(a + c_H - 2c_L)}{4b} \tag{54}$$

$$\pi_H(p_L^M) = \frac{(a - c_L)(a + c_L - 2c_H)}{4b} \tag{55}$$

The sufficient condition for a separating limit price equilibrium to exist, i.e. inequality (23), reduces to the requirement that

$$2a + 7c_H + 7c_L - 16c_E \geq 0 \tag{56}$$

This condition holds provided that the demand intercept is sufficiently high or that the entrant is not too inefficient relative to the incumbent.

Parametric Example. Consider the following benchmark parameter constellation:

a	b	c_H	c_L	c_E	F	μ	δ
10	1	3	2	2	8	0.5	0.7

It is easily verified that for this choice of parameters, both separating and pooling limit price equilibria exist (i.e. assumptions A4 and A5 and conditions (13), (22) and (35) are satisfied). Furthermore, the relevant regime is the limit price regime (because condition (43) is violated). The monopoly prices are given by $p_L^M = 6$ and $p_H^M = 6.5$ respectively. The incentive compatibility constraints are given by

$$IC_H : \quad p_L^* \notin [4.1, 8.9] \tag{57}$$

$$IC_L : \quad p_L^* \in [3.5, 8.5] \tag{58}$$

It follows that the set of separating limit price equilibria are given by pairs of prices (p_L^*, p_H^*) with

$$p_H^* = 6.5 \tag{59}$$

$$p_L^* \in [3.5, 4.1] \tag{60}$$

The least cost separating equilibrium limit price is given by $p_L^* = 4.1$. Last, the least cost pooling limit price equilibrium is given by $p^* = 6$.

2.   WORKED EXAMPLE IN DYNAMIC MODEL

To illustrate the results of the dynamic analysis, I now return to the example considered earlier. For ease of reference, the two relevant incentive compatibility constraints for the dynamic setting are reproduced below:

$$\pi_L(p_{t,L}^*) \geq \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right)D_L \tag{61}$$

$$\pi_H(p_{t,L}^*) \leq (1 - \delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right)D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right)\pi_H(p_L^M) \tag{62}$$

Now consider the case $T = 4$, i.e. where the static game is repeated once (as long as entry has not occurred). In this case, the incentive compatibility constraints (62) and (61) for separation at $t = 1$ are given by

$$IC_H : \quad p_L^* \notin [3, 10] \tag{63}$$
$$IC_L : \quad p_L^* \in [2.3, 9.7] \tag{64}$$

The set of separating equilibria (with separation in the first period) is in this case given by (p_L^*, p_H^*) with

$$p_H^* = 6.5 \tag{65}$$
$$p_L^* \in [2.3, 3] \tag{66}$$

It follows that when moving from $T = 2$ to $T = 4$, the range of possible limit price equilibria increases.[6] More importantly, the least cost separating equilibrium has decreased, from $p = 4.1$ to $p = 3$, in accordance with the results in the previous section.

For the purpose of illustration, I will also consider the case $T = 100$. For this long horizon, the set of separating equilibria is given by (p_L^*, p_H^*) with

$$p_H^* = 6.5 \tag{67}$$
$$p_L^* \in [-2.9, -2] \tag{68}$$

so even the least cost separating equilibrium (if it exists), will involve negative prices.

I next turn to the necessary conditions in Assumptions A4' and A8' for limit price equilibria to exist. As already noted above, these conditions hold at $t = 2$, at which point the remainder of the game coincides with the static game. But at $t = 1$ (when $T = 4$), the constraints in Assumptions A4' and A8' are in fact *violated*. This means that the entrant's expected, discounted post-entry profits are now so large, that *no amount of signaling can make entry unprofitable* and neither pooling nor separating limit price equilibria exist. Thus in this case, the unique equilibrium involves immediate separation on the monopoly prices $(p_L^*, p_H^*) = (p_L^M, p_H^M) = (6, 6.5)$ and it is in fact immaterial whether the prevailing regime is the limit price regime or the monopoly price regime. The important point to take from this example is that with this specification of post-entry competition and for this parameterization, limit pricing is feasible in the static setting, whereas under a minimal dynamic extension, it is not.

From the above analysis, it is clear that for a given set of product market parameters (in the parametric example, a, b, c_H, c_L, c_E) and parameters F, δ, μ, each of the incentive compatibility constraints and each of the many feasibility constraints deliver values of T for which the constraints hold (or do not hold, as the case may be). It is therefore tempting to try to rank such

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[6]To be precise, the measure of the interval of possible prices increases, but neither interval is contained in the other.
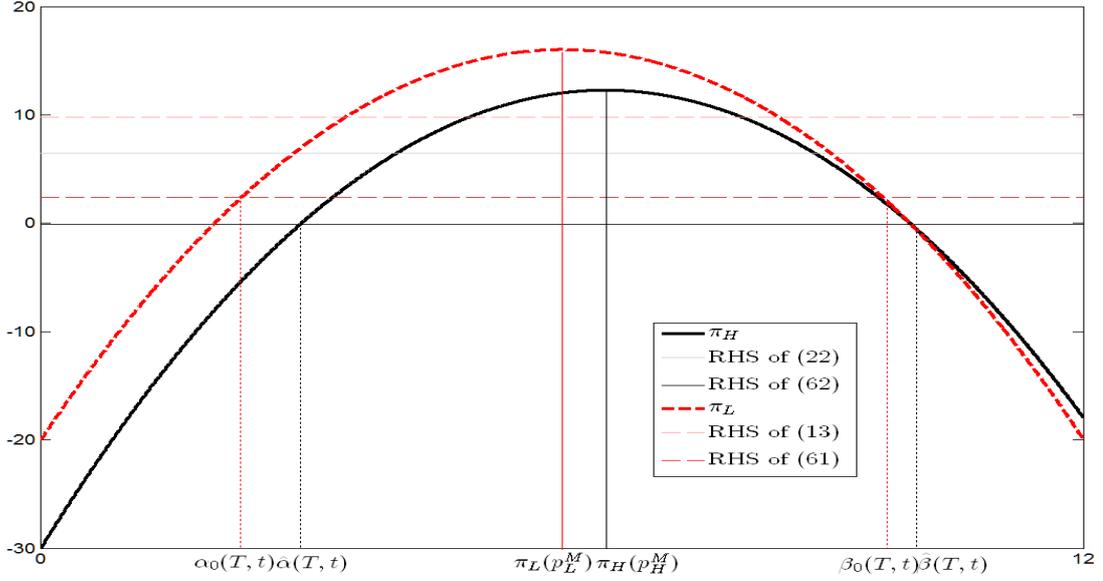
Figure 2: Profits and Incentive Compatibility Constraints in Dynamic Setting.

thresholds in a clean manner and draw general conclusions about existence of different types of equilibria for different horizons T. Unfortunately, doing this is an uphill battle, because (i) the ranking of thresholds may change depending on the choice of parameters $a, b, c_H, c_L, c_E, F, \delta, \mu$ and (ii) the constraints depend on these parameters in different and complicated ways. But as a practical matter, for a given choice of parameters, it is relatively straightforward to evaluate the different constraints and determine the existence and nature of the equilibria of the dynamic model.

3. DISCUSSION ON EQUILIBRIUM SELECTION IN DYNAMIC MODEL

Rather than characterizing the entire set of equilibria of the dynamic game, I will argue that only a subset of the equilibria are 'reasonable' in a specific sense to be developed in further detail below. Because the literature on signaling in dynamic settings is still in its infancy, I will start by giving a brief review of it and emphasize the distinct contribution of the present analysis to that literature. In the vast majority of signaling models, there is only one instance of signaling, even if the model is otherwise dynamic. When there are multiple opportunities for the informed party to engage in signaling, the details of how (and if) private information changes over time and its observability by the uninformed party become crucial. The most conventional analyses are those of models in which private information is regenerated each period or in which the uninformed party's observations are imperfect signals of the informed party's actions. In either case, updating on the equilibrium path can always be achieved by application of Bayes' rule. articles of this type include Mester (1992) and Vincent (1998) as well as Saloner (1984), Roddie (2010) and Gedge et al. (2013) in the context of limit pricing.

When private information is perfectly persistent over time and the informed party's actions are perfectly observable, then the modeler must confront the issue of assigning out of equilibrium beliefs. There have been two different approaches to deal with such beliefs in the existing literature, namely (i) *support restrictions* and (ii) *belief resetting*. Both approaches rely on the fact that the solution concept perfect Bayesian equilibrium does not impose any restrictions on beliefs after probability zero events. In the former approach, once posterior beliefs are concentrated entirely on some state of nature, no possible observation will prompt a shift of probability towards alternative states of nature. In other words, once posterior beliefs are

degenerate, the game is treated as one of perfect information regardless of how it subsequently unfolds. In the latter approach, posteriors are allowed to fluctuate over time. In particular, this approach allows beliefs that assign positive probability to events that previously were assigned zero probability.

Support restrictions have been used in different contexts by Rubinstein (1985), Grossman and Perry (1986) and LeBlanc (1992) and in a limit pricing context by Gryglewicz (2009). Although such restrictions may be perfectly appropriate for some analyses, there are instances in which they are clearly inappropriate. Madrigal et al. (1987), Noldeke and van Damme (1990a) and Vincent (1998) discuss the treatment of degenerate posteriors in depth and show that such support restricted equilibria may fail to exist.

As an alternative to support restrictions, some authors have resorted to repeated resetting of beliefs. In actual fact, beliefs are degenerate along the entire equilibrium path when employing this approach, but the equilibria are constructed *as if* beliefs are reset to their prior values.[7] This is the avenue taken by Cho (1990), Noldeke and van Damme (1990b), by Kaya (2009) in a limit pricing context and discussed by Vincent (1998). The equilibria studied in Kaya (2009) and Noldeke and van Damme (1990b) exploit the fact that the players may simply disregard the public information contained in past play and proceed "as if" they had not observed past play at all. The point here is not that the equilibria studied by these authors are not equilibria (which they clearly are). Rather, I argue that the reliance of such equilibria on the players ignoring past evidence can serve as a useful feature to help choose between different kinds of equilibria in this type of setting.

In terms of applications to economic modeling such as limit pricing, these two approaches differ radically in their predictions in that support restrictions effectively make repeated signaling impossible (by definition), whereas belief resetting allows for a potentially very rich set of equilibria, in which signaling occurs repeatedly. The precise assumptions adopted by the modeler therefore have profound implications for the analysis at hand and therefore merit scrutiny.

What support restrictions and belief resetting have in common, is that with neither approach does the observation of out of equilibrium play prompt the uninformed party to make sense of the deviation. This is at odds with the way that static signaling models are habitually analyzed. In such settings, out of equilibrium beliefs are not all treated equally, some being deemed more reasonable than others. In this way, equilibrium selection techniques are useful in that they reduce the equilibrium set significantly, sometimes even to a unique equilibrium. Hitherto, equilibrium selection techniques have not been widely applied to dynamic settings of signaling. This is unfortunate, because equilibrium selection obviates the need to choose between support restrictions and belief resetting. Furthermore, it is entirely consistent with the way that static signaling models are analyzed. The approach I adopt in the present article, is to make use of equilibrium selection reasoning in the dynamic game. Specifically, I make use of reasoning along the lines of criterion D1 against a natural benchmark equilibrium in which post separation beliefs are degenerate.

Last, it should be mentioned that some articles do feature repeated signaling without relying on the arbitrariness of out of equilibrium beliefs. These articles include Noldeke and van Damme (1990b), Bar-Isaac (2003) and Sorenson (2004). In these articles, the informed party is unable to effectively separate in a single period and is hence forced to distribute costly signaling over several periods.[8,9]

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[7]This distinction is immaterial since under this approach, beliefs are not fully incorporated into subsequent behavior.

[8]In the present article, all signaling takes place in (at most) a single period, for reasons mirroring those articulated by Weiss (1983) and Noldeke and Van Damme (1990b).

[9]Note that whereas many equilibrium selection approaches rely on the arbitrariness of off-equilibrium path

To be more specific about the differences between the approaches taken in the existing literature, suppose that the incumbent's past actions have convinced the entrant that its type is L and that the incumbent subsequently charges price p_H^M. This sequence of events should confound the entrant, because an L type could have set the preferred price p_L^M without suffering adverse consequences. There are different ways to interpret the situation. One is to insist on the informational content of past behavior and to simply ascribe p_H^M to a "mistake" by the L type incumbent (which echoes the approach prescribed by the notion of subgame perfection in complete information games). This type of obstinacy in updating is the essence of support restrictions. A second way to proceed is to suppose that past behavior was in fact "mistaken" and to infer from the observation of p_H^M that the incumbent is in fact not an L type after all. But if such past behavior is ignored, then the L type incumbent should set his price such as to (again) credibly convey the information that he is in fact an L type incumbent and thus deter entry, despite the fact that it is already "known" (or has already been inferred) that he is an L type. This is exactly the way in which belief resetting makes repeated signaling possible. In the former approach, the informational content of past actions is given all weight whereas in the latter, the informational content in the incumbent's current action is given all weight.

A third approach, is to consider the two pieces of conflicting evidence together and to make sense of the conflict by using heuristics familiar to the equilibrium refinement literature. This approach consists of asking which type of incumbent, *given the belief that he is type L*, would stand to gain from setting price p_H^M? It turns out that answering this question gives a very natural prediction in this game. The key is to observe that *given* that the entrant already assigns probability one to the incumbent being an L type, the L type *cannot possibly benefit from setting any price different from* p_L^M, as long as observing p_L^M does not prompt the entrant to revise his belief that the incumbent is of type L. On the other hand, an H type incumbent *would* benefit from this price if E disregards this piece of confounding evidence (which he is entitled to do as out of equilibrium beliefs are arbitrary in a perfect Bayesian equilibrium).

Extending this reasoning to the dynamic game, the natural benchmark equilibrium price sequence after separation has occurred is $(p_L^M, p_L^M, ..., p_L^M)$. Next, *given* this benchmark equilibrium price sequence, all deviations can be dealt with by using reasoning similar to that inherent in criterion D1 of Cho and Kreps (1987). The D1 criterion works as follows. Fix some perfect Bayesian equilibrium of the game under consideration and consider a deviation by the informed party from its equilibrium strategy. Criterion D1 then dictates that if the set of responses by the entrant that makes the type $i = H, L$ incumbent willing to deviate to the observed deviation price is strictly smaller than for type $j \neq i$, then the entrant should assign infinitely larger probability to incumbent j having deviated than to incumbent i. In the reasoning above, the benchmark equilibrium was simply that in which after separation, the L type sets its monopoly price p_L^M whereas the H type, off the equilibrium path, chooses to mimic the behavior of the L type by also setting the price p_L^M in periods after separation.[10]

Note that the procedure I make use of is not quite a direct application of D1, as I do not consider an arbitrary equilibrium. Rather, the present approach accords special significance to the equilibrium in which the uninformed party at each information set makes full use of all available information (i.e. acts without ignoring available evidence). The reason that this is sensible is that in the static setting, there is no sense in which prior information "favors" any equilibrium over the other. In the dynamic setting however, prior information "suggests" or "indicates" one particular equilibrium over all other equilibria. In contrast, belief resetting amounts to *actively disregarding* the most focal equilibrium, on which beliefs are naturally

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beliefs, some, such as those introduced in Fudenberg, Kreps and Levine (1988) are explicitly constructed to avoid this issue.

[10] That this is indeed the optimal deviation for the inefficient incumbent in the limit price regime, will be verified below.

anchored because of past play and application of Bayes' rule.

It should be noted that the equilibrium chosen by the anchored D1 criterion has a very nice property, namely that it is the equilibrium satisfying D1 which is preferred by both the incumbent and the entrant. That it satisfies D1 and is preferred by the incumbent follows from the arguments above. That it is also preferred by the entrant, can be seen by considering an alternative equilibrium which also satisfies D1. Assume that the game is in the limit price regime. In the benchmark equilibrium, the equilibrium strategies are given by (p_L^M, p_L^M) in the period after separation has occurred. If these prices do not form part of the highest payoff equilibrium satisfying D1, then there exists some other D1 equilibrium such that the H type would have strictly higher payoffs from setting an alternative price $p \neq p_L^M$. But by D1, in this alternative equilibrium it must be that $p_L^* = p_L^M$ (or else it would be better for the L type to switch to this price, as he would still be believed to be type L). But by D1, upon observing $p \neq p_L^* = p_L^M$, the entrant must assign probability one to the incumbent being type H, and thereby enter. As will be shown below, in the limit price regime, the optimal deviation has the H type incumbent set p_L^M and avoid entry (rather than setting the next best alternative p_H^M and inviting entry). In conclusion, there is no equilibrium satisfying D1 which is preferred by either the incumbent or the entrant, a feature which lends added support to the one selected by the anchored D1 criterion.

4. Infinite Horizon and the Effects of Post-Entry Collusion

In the infinite horizon setting, the possibility of post-entry cooperation (which is absent in the static setting) affects the desirability and feasibility of limit pricing in interesting ways. Before exploring these effects more systematically, it is worth recalling the tradeoffs involved for the incumbent and the entrant respectively. The reason that the incumbent may wish to engage in costly limit pricing in order to deter entry, is that post-entry competition reduces its profits. Similarly, the logic of limit pricing is that for a sufficiently low price, the incumbent convinces the entrant that its post-entry profits would be too low to offset the entry fee F. In short, the magnitude of post-entry profits for the two market participants directly affect the different incentive constraints necessary for limit pricing to be viable.

In any dynamic setting in which the final period T is finite, the post-entry payoffs $(D_i, D_E(i))$, $i = H, L$ to the incumbent and the entrant are given by the Nash equilibrium duopoly payoffs. This fact turns out to make entry deterrence through limit pricing much easier to achieve in the static setting than in the a setting with an infinite horizon. To see this, note that in the infinite horizon game, the set of payoffs that E and I can achieve in equilibrium is bounded below by the payoffs $(D_i, D_E(i))$, $i = H, L$ in the static setting. From the Folk Theorem, it is known that for a sufficiently high discount factor, any feasible and individually rational payoff vector can be sustained in equilibrium. In other words, the competing firms may be able to coordinate on payoffs that are above the levels achieved in the static setting. In turn, this directly influences the two firms' incentives to deter entry and to stay out respectively. To be specific, the higher the incumbent's (per-period) post-entry payoff D_i, the weaker is the incumbent's incentive to engage in costly signaling in the short term in order to maintain incumbency. This is seen most clearly in the L type's incentive compatibility constraint (61). As D_L increases, it becomes increasingly difficult to satisfy the constraint.[11]

Turning to the entrant, recall that the basic tradeoff influencing the entry decision is that of entry fees versus post-entry payoffs $D_E(i)$ from duopoly competition. But the higher these latter payoffs are, the more difficult is it to discourage entry, even under complete information. To see this, consider the constraint A4', which ensures that the entrant will choose to stay out against an efficient incumbent. As $D_E(L)$ increases, the constraint becomes increasingly difficult to satisfy.

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[11]When D_i reaches $\pi_L(p_L^M)$, the incentive compatibility constraint can only be satisfied with equality.

In conclusion, the prospect of less than cut-throat post-entry competition, makes the incumbent more reluctant to engage in costly entry deterrence. By the same token, the prospect of higher post-entry duopoly profits enjoyed by engaging in collusion with the incumbent makes it more difficult to discourage the potential entrant from entering the industry and competing with the incumbent.

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