Title: Using an experimentally-determined model of the evolution of pore structure for the calcination of cycled limestones

Article Type: Research Paper

Keywords: Calcium looping; Modelling; Calcination; Pore evolution

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Corresponding Author's Institution: University of Cambridge

First Author: Peng Dai
Order of Authors: Peng Dai; Belén González; John S Dennis

Abstract: A pseudo-steady state model of reaction and diffusion has been constructed to model the non-isothermal calcination of limestone particles which have been subjected to a history of cycling between the calcined and carbonated states. This typically occurs when using Ca-based materials for removing CO2 from the flue gas of plants such as a power station, cement plant and steel factory in certain schemes for carbon capture and storage. The model uses a Cylindrical Pore Interpolation Model to describe the intraparticle mass transfer of CO2 through the pores of the material coupled with an experimentally-determined function, \( f(X) \), describing the pore evolution as a function of the conversion of the CaCO3 present to CaO. The intrinsic rate of calcination was taken to be first order in concentration driving force. External to the limestone particle, the Stefan-Maxwell equations were used to describe the diffusion of CO2 away from the particle and into the particulate phase of the fluidised bed. The equation of energy was used to allow for the enthalpy of the reaction. In order to validate the use of the \( f(X) \) function, the theoretical predictions were compared with experiments conducted to measure the rates and extent of conversion, at various temperature and different particle sizes, of Purbeck and Compostilla limestones that had been previously cycled between the carbonated and fully-calcined state. Excellent agreement between experiment and theory was obtained, and the model using the \( f(X) \) approach predicted the conversion of particles of various sizes well at temperatures different to that at which the function was derived, thus indicating that the \( f(X) \) solely depended on the evolution of the morphology of the particle.

Response to Reviewers: All related changes are highlighted in the text when it is possible.

Reviewer 1
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In answering issue 1 of referee 1, the authors state "However, in his model the small pore trees will be kinetically limited while the larger pore trees are diffusion limited. This is the opposite of what one would normally expect."
I disagree with this statement. If the small trees have significant diffusion resistance, the larger trees having much larger length scales will have much stronger diffusion limitations, and the particle scale Thiele modulus will be very large. The authors may read the text of Bischoff and Froment for a discussion of this aspect with regard to micropore and macropore diffusion limitations.

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The authors next state "The existing mathematical pore models such as the random pore model, the pore tree model and the grain model have various degrees of mathematical complexity, but, more importantly, contain one or more unknown parameters which have to be fitted to experimental results." I cannot agree completely with this. For example the structural parameter of the Random Pore Model can be independently estimated from the pore size distribution.

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While I do not want to insist on fitting of their rate curve to a mechanism, it remains a weakness if they do not do so. While such fitting does have parameters, in most cases they can be reconciled with existing literature information obtained in a different context or experiment. By providing such mechanistic interpretation the authors could strengthen the paper and its appeal.

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The referee’s point is taken and will be thoroughly considered in the subsequent study where we will apply the model to simulating the sulphation of limestones where there are two different mechanisms for the reaction to terminate depending on temperature. This is where our model could be very useful in determining which mechanism the reaction would follow and when the transition would occur. However, the purpose of this study was to verify experimentally the continuous reaction model using the f(X) concept, originally developed and verified for gasification of chars, to describe the evolution of pores. Therefore, the primary objective of this paper is to communicate fully the features associated with the theoretical model and to demonstrate the applicability to a distinctively different gas-solid reaction other than the gasification of chars reported previously.
Although some new references have been included in the introduction, more references are required to support some statements in the introduction. Examples are:

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1. The following text is added below Eq. 4:
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"Here, yn is the mole fraction of species n"
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Page 23: "Thus, the intrinsic rate of carbonation changes little with temperature". This is true in the temperature range analysed in the paper.
Response

The text on page 23 now read:

"Thus, the intrinsic rate of carbonation changes little with temperature in the temperature range studied in the paper."
11 February 2016

Editorial office
Chemical Engineering Journal

Dear Editor,

**Original Research Paper for Consideration by Chemical Engineering Journal**

Please find attached an original research paper, which we would like to be considered for publication in *Chemical Engineering Journal*. It presents a new method of modelling the rate of calcination of particles of limestone, in which the key improvement is to determine the parameter affecting pore morphology experimentally, rather than using artificial models of pore structure to infer how the morphology evolves with reaction. The model has been compared with experiments at various conditions and appears to be a robust predictor of rate of reaction and conversion versus time.

We have prepared this manuscript according to Guide for Authors and in compliance with the Ethics in Publishing Policy as described in the Guide for Authors. We also confirm that:

- This is an original submission by the authors and has not been submitted for publication elsewhere.
- The list of suggested reviewers with their contact details are attached in a separate document.
- Keywords are provided.
- References are in the correct format.
- The PDF of the manuscript is in correct order upon submission.
- The length of the manuscript is in compliance with the Guide for Authors.

We thank you for considering this manuscript for publication.

Yours sincerely,

Peng Dai

(On behalf of all the co-authors)
<table>
<thead>
<tr>
<th>First name</th>
<th>Maria</th>
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<tbody>
<tr>
<td>Last name</td>
<td>Diego</td>
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<tr>
<td>Degree</td>
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<tr>
<td>Institution</td>
<td>Instituto Nacional del Carbón</td>
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<tr>
<td>Email</td>
<td><a href="mailto:marlen@incar.csic.es">marlen@incar.csic.es</a></td>
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**Reasons:**
Dr Diego has extensive research contributions on calcium looping process for CO2 capture, where her work spans from fundamental studies of the reaction kinetics and circulating fluidised bed to investigations of the large scale calcium looping pilot plant.

<table>
<thead>
<tr>
<th>First name</th>
<th>Marcin</th>
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<tr>
<td>Last name</td>
<td>Broda</td>
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<td>Degree</td>
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<td>Department</td>
<td>Institute of Energy Technology</td>
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<tr>
<td>Institution</td>
<td>Eidgenossische Technische Hochschule Zurich</td>
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<tr>
<td>Email</td>
<td><a href="mailto:mbroda@student.ethz.ch">mbroda@student.ethz.ch</a></td>
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**Reasons:**
Dr Broda has a long list of publications on calcium looping process where he worked extensively on the studies of the synthetic Ca-based sorbents and their performance in the looping process.

<table>
<thead>
<tr>
<th>First name</th>
<th>Vasilije</th>
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<tr>
<td>Last name</td>
<td>Manovic</td>
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<tr>
<td>Degree</td>
<td>PhD</td>
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<tr>
<td>Position</td>
<td>Professor</td>
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<td>Department</td>
<td>Energy and Power Engineering Division, School of Engineering</td>
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<tr>
<td>Institution</td>
<td>Cranfield University</td>
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<tr>
<td>Email</td>
<td><a href="mailto:v.manovic@cranfield.ac.uk">v.manovic@cranfield.ac.uk</a></td>
</tr>
</tbody>
</table>

**Reasons:**
Professor Manovic is a leading authority in the area of calcium looping – a new technology for CO2 capture, and he is an internationally recognised authority in several other CO2 capture areas including chemical looping combustion, oxy-fuel combustion, and CO2 capture from low-CO2-concentration sources such as industrial gases, natural gas, and air. His research interests also include the areas of SO2 capture, NOx reduction from fossil fuel, biomass, and waste combustion. He has also studied pyrolysis, and gasification, and been responsible for the modelling of these processes. In addition, his expertise includes chemistry, geochemistry, and analytical chemistry of fossil fuels.

<table>
<thead>
<tr>
<th>First name</th>
<th>Ben</th>
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<tr>
<td>Last name</td>
<td>Anthony</td>
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<tr>
<td>Degree</td>
<td>PhD</td>
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<tr>
<td>Position</td>
<td>Professor</td>
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<tr>
<td>Department</td>
<td>Centre for Combustion, Carbon Capture &amp; Storage</td>
</tr>
<tr>
<td>Institution</td>
<td>Cranfield University</td>
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<tr>
<td>Email</td>
<td><a href="mailto:b.j.anthony@cranfield.ac.uk">b.j.anthony@cranfield.ac.uk</a></td>
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<td>Reasons:</td>
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<td></td>
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Response to the comments

All related changes are highlighted in the text when it is possible.

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Page 23: "Thus, the intrinsic rate of carbonation changes little with temperature”. This is true in the temperature range analysed in the paper.

**Response**

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“Thus, the intrinsic rate of carbonation changes little with temperature in the temperature range studied in the paper.”
Highlights

- A pseudo time-stepping model with spatial variation for calcination of cycled limestones.
- An experimentally determined function to describe the evolution of pore structure.
- Good fit between the model and the experimental measurements.
Using an experimentally-determined model of the evolution of pore structure for the calcination of cycled limestones

Peng Dai*, Belén González, John S. Dennis

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Pembroke Street, Cambridge, CB2 3RA, United Kingdom

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Abstract

A pseudo-steady state model of reaction and diffusion has been constructed to model the non-isothermal calcination of limestone particles which have been subjected to a history of cycling between the calcined and carbonated states. This typically occurs when using Ca-based materials for removing CO$_2$ from the flue gas of plants such as a power station, cement plant and steel factory in certain schemes for carbon capture and storage. The model uses a Cylindrical Pore Interpolation Model to describe the intraparticle mass transfer of CO$_2$ through the pores of the material coupled with an experimentally-determined function, $f(X)$, describing the pore evolution as a function of the conversion of the CaCO$_3$ present to CaO. The intrinsic rate of calcination was taken to be first order in concentration driving force. External to the limestone particle, the Stefan-Maxwell equations were used to describe the diffusion of CO$_2$ away from the particle and into the particulate phase of the fluidised bed. The equation of energy was used to allow for the enthalpy of the reaction. In order to validate the use of the $f(X)$ function, the theoretical predictions were compared with experiments conducted to measure the rates and extent of conversion, at various temperature and different particle sizes, of Purbeck and Compostilla limestones that had been previously cycled between the carbonated and fully-calcined state. Excellent agreement between experiment and theory was obtained, and the model using the $f(X)$ approach predicted the conversion of particles of various sizes well at temperatures different to that at which the function was derived, thus indicating that the $f(X)$ solely dependent on the evolution of the morphology of the particle.

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### List of symbols

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<tr>
<td>$A_A$</td>
<td>Coefficients in pressure gradient equation</td>
<td>Pa m s g</td>
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<td>$A_{30}$</td>
<td>Initial pore area per unit mass</td>
<td>m$^2$ g$^{-1}$</td>
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<td>Molar heat capacity of species $n$</td>
<td>J mol$^{-1}$ K$^{-1}$</td>
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<td>Diffusivity at arbitrary Knudsen number, for species $n$ and $m$</td>
<td>m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$D_{B,nm}$</td>
<td>Molecular diffusivity, involving species $n$ in $m$</td>
<td>m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$D_{K,n}$</td>
<td>Knudsen diffusivity of species $n$</td>
<td>m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$D_{ref}$</td>
<td>Molecular diffusivity $D_{B,12}$ at bulk condition</td>
<td>m $s^{-1}$</td>
</tr>
<tr>
<td>$d_B$</td>
<td>Mean bubble diameter</td>
<td>m</td>
</tr>
<tr>
<td>$E_a$</td>
<td>Activation energy</td>
<td>J mol$^{-1}$</td>
</tr>
<tr>
<td>$H_n$</td>
<td>Partial molar enthalpy of species $n$</td>
<td>J mol$^{-1}$</td>
</tr>
<tr>
<td>$H_f$</td>
<td>Partial molar enthalpy of formation</td>
<td>J mol$^{-1}$</td>
</tr>
<tr>
<td>$h$</td>
<td>Bed height</td>
<td>m</td>
</tr>
<tr>
<td>$h_{mf}$</td>
<td>Bed height at minimum fluidisation</td>
<td>m</td>
</tr>
<tr>
<td>$J_n$</td>
<td>Total molar flux of species $n$</td>
<td>mol m$^{-2}$ s$^{-1}$</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Rate constants of calcination reaction</td>
<td>mol m$^{-2}$ s$^{-1}$</td>
</tr>
<tr>
<td>$k'_{c}$</td>
<td>Modified rate constants of calcination reaction</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Arrhenius coefficient of rate constant $k'_{c}$</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$\bar{k}$</td>
<td>Rate constant of carbonation reaction</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$M_n$</td>
<td>Molecular mass of species $n$</td>
<td>g mol$^{-1}$</td>
</tr>
<tr>
<td>$P$</td>
<td>Total pressure</td>
<td>bar</td>
</tr>
<tr>
<td>$P_{\text{bulk}}$</td>
<td>Bulk pressure</td>
<td>bar</td>
</tr>
<tr>
<td>$p_{CO_2,\text{r}}$</td>
<td>Local partial pressure of CO$_2$</td>
<td>bar</td>
</tr>
<tr>
<td>$p_{CO_2,\text{bulk}}$, $p_{CO_2,\text{eq}}$</td>
<td>Bulk and equilibrium partial pressure of CO$_2$</td>
<td>bar</td>
</tr>
<tr>
<td>$q_c$</td>
<td>Intrinsic rate of calcination per unit of surface area</td>
<td>mol m$^{-2}$ s$^{-1}$</td>
</tr>
<tr>
<td>$Q_n$</td>
<td>Net rate of change of species $n$ inside the particle</td>
<td>mol m$^{-3}$ s$^{-1}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial distance from the particle centre</td>
<td>m</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Radius of a limestone particle</td>
<td>m</td>
</tr>
<tr>
<td>$r_{\text{pore}}$</td>
<td>Mean radius of the pore</td>
<td>nm</td>
</tr>
<tr>
<td>$R$</td>
<td>Universal gas constant</td>
<td>kJ mol$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$T$</td>
<td>Absolute temperature</td>
<td>K</td>
</tr>
<tr>
<td>$T_{\text{bulk}}$</td>
<td>Bulk temperature</td>
<td>K</td>
</tr>
<tr>
<td>$U_b$</td>
<td>Bubble velocity</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$U_{mf}$</td>
<td>Flow velocity at minimum fluidisation</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$u_M$</td>
<td>Mass-averaged velocity</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$V_{M,\text{CaO}}$</td>
<td>Molar volume of CaO</td>
<td>m$^3$ mol$^{-1}$</td>
</tr>
<tr>
<td>$V_{M,\text{CaCO}_3}$</td>
<td>Molar volume of CaCO$_3$</td>
<td>m$^3$ mol$^{-1}$</td>
</tr>
<tr>
<td>$X$</td>
<td>Solid conversion</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Mole fraction of species $n$</td>
<td>-</td>
</tr>
</tbody>
</table>

### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_{k_c}$</td>
<td>Pre-exponential coefficient of the rate constant $k_c$</td>
<td>mol m$^{-2}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{\text{eff}}$</td>
<td>Effective thermal conductivity of the particle</td>
<td>W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Initial porosity of the particle</td>
<td>-</td>
</tr>
<tr>
<td>$\varepsilon_b$</td>
<td>Bubble fraction</td>
<td>-</td>
</tr>
<tr>
<td>$\varepsilon_{\text{bed}}$</td>
<td>Porosity of the fluidised bed around the particle</td>
<td>-</td>
</tr>
<tr>
<td>$\varepsilon(X)$</td>
<td>Porosity of the particle as a function of conversion</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>Bulk density of a particle</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Skeletal density of the particle</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\tau^2$</td>
<td>Tortuosity factor of the particle</td>
<td>-</td>
</tr>
</tbody>
</table>
\( \tau_{\text{bed}} \) \hspace{1cm} \text{Tortuosity factor of the fluidised bed} \\
\( \delta \) \hspace{1cm} \text{External diffusion boundary layer thickness} \\
\( \eta_r \) \hspace{1cm} \text{Dimensionless radius} \\
\( \nu_{\text{mix}} \) \hspace{1cm} \text{Kinematic viscosity of gas mixture} \\
\( \nu_n \) \hspace{1cm} \text{Stoichiometric coefficient of species } n \\

\textbf{Subscripts} \\
\( n = 1, 2 \) \hspace{1cm} \text{CO}_2 \text{ and N}_2 \\

\textbf{Abbreviation} \\
BET \hspace{1cm} \text{Brunauer–Emmett–Teller} \\
CPIM \hspace{1cm} \text{cylindrical pore interpolation model} \\
DGM \hspace{1cm} \text{dusty gas model} \\
MPTM \hspace{1cm} \text{mean pore transport model} \\
OCFE \hspace{1cm} \text{orthogonal collocation on finite element} \\
RPM \hspace{1cm} \text{random pore model}
1 Introduction

Broadly-speaking, two classes of model exist to describe the non-catalytic reaction between a gas and a solid, namely the Shrinking Core Model (SCM) and the Continuous Reaction Model (CRM). The SCM has been commonly used to describe, for example, the calcination of non-porous, virgin particles of limestone (generally close to pure CaCO₃), where the reaction occurs at a sharp front which recedes towards the centre of the particle [1]. Generally, the rate of reaction might be limited variously by (i) chemical kinetics, (ii) diffusion through the porous product layer, (iii) transport of heat to or from a reaction interface, or (iv) diffusion through the external gas film. On the other hand, the CRM is a better description where there is slow reaction of a gas, and, or, transfer of heat, within a porous solid across a broad front or the entire particle [1]. Fig. 1 shows the difference in local conversion profiles between two models at fixed average conversion of particle. The calcination of cycled, as opposed to virgin, limestone, possesses the characteristics of the CRM, where the initially-porous particles, containing a mixture of CaCO₃ and unreacted CaO, become more porous as the CaCO₃ is calcined to CaO during thermal decomposition. Such particles are created when raw limestone particles have been subjected to a history of cycling between the calcined and carbonated states. This typically would occur when using such Ca-based materials for removing CO₂ from the flue gas of plants such as a power station, cement plant and steel factory in certain schemes for carbon capture and storage. The present work is concerned with these cycled particles.

![Fig. 1. Local conversion profiles of shrinking core model (SCM) and continuous reaction model (CRM) for fixed average conversion of particle.](image)

A feature of most models of non-catalytic reactions between gases and solids generally is that the intrinsic rate of reaction, \( r \), at a local point within a solid particle is of the form \( r = g(C_i, T, P) \times f(X) \) [2,3]. Here, \( g \) describes the intrinsic reaction kinetics as a function of the temperature, \( T \), the total pressure, \( P \), and the concentration, \( C_i \), of the reaction gases. The term \( f(X) \) is a direct function of the conversion of the particle and is correlated with the internal morphology of the particle, e.g. surface area, pore size, pore size distribution etc. at a particular conversion. In addition, \( f(X) \) is not a function of \( C_i \), \( T \), or \( P \).
Many researchers have sought to model the development of internal pore structure with conversion during non-catalytic gas-solid reaction. For example, Szekely & Evans [4] assumed that solid particles consisted of an array of spherical grains with the space between them making up the voids. They made the first attempt to incorporate the structural parameters such as grain size, porosity and pore size into the reaction scheme in their grain model. Their original model assumed that the pore structure was unaffected by the progress of reaction, although later variants [5] were able to account for change in grain size with reaction. More recently, Liu et al. [6] developed an overlapping grain model using a fitted size distribution of grains to account for the evolution of pore structures during reaction. On the other hand, random pore models have been developed, e.g. Bhatia & Perlmutter [3] and Gavalas [7], using different approaches based on cylindrical pore assumptions, to model the total surface area at any conversion as a function of the initial morphological parameters, e.g. the initial porosity. In a somewhat different approach, Simons & Finson [8] built a mass transport model using statistical methods to specify the pore structure as a continuously branching tree. However, in his model the small pore trees will be kinetically limited while the larger pore trees are diffusion limited. This is the opposite of one might expect for pores with uniform length. The existing mathematical pore models such as the random pore model, statistical pore tree model and grain model contain parameters that are difficult to measure and thus become fitting parameters, e.g. the diffusivity of SO\textsubscript{2} through a layer of CaSO\textsubscript{4}, which are complicated and equally arbitrary. In many cases, the use of mathematical pore models leads to the need to modify the original models in order to fit experimental measurements [9–11].

On the other hand, the experimentally-determined \( f(X) \) function from the common measurements of reaction rate and conversion offers a straightforward method to describe the change of internal morphology at a local point within the particle being reacted under conditions affected by intraparticle mass transfer [12]. This is exemplified by recent studies of char gasification in a fluidised bed [12,13], where it was found that a simple, arbitrary function, \( f(X) \), could be determined from the plot of measurements of rate vs. conversion of the solid char in the kinetically-controlled regime. It was proposed that the ratio between the rate of reaction at any conversion and the initial rate of reaction reflects, generally, the variation in the pore structure as the reaction proceeds in the absence of intraparticle mass transfer limitation (e.g. at low temperature or using small particles or with particles of low reactivity) [13], thus giving \( f(X) = r(X)/r(X = 0) \).

Dai et al. [12] concluded that the application of the \( f(X) \) concept to the gasification of chars by CO\textsubscript{2} suffered the complication of there being multiple types of active sites for adsorption on the surface of char so that a single \( f(X) \) determined from experimental measurements at a low temperature was unable to fit satisfactorily all the measurements made at a substantially higher temperature. Accordingly, to investigate the basic hypothesis that
a gas-solid reaction can be characterised by \( r = g(C, T, P) \times f(\lambda) \), it is important to identify a solid which is unlikely to contain sites which vary in relative activity with temperature. The conversion of calcium carbonate to calcium oxide does not involve gas adsorption, thus limestones is a potential suitable candidate. Of course, as noted above, virgin limestone (CaCO\(_3\)) is almost non-porous, and the calcination reaction usually follows a shrinking core mechanism [14,15], unsuitable for the application of the \( f(\lambda) \) concept. However, limestones which have been successively calcined to CaO and carbonated in CO\(_2\) back to CaCO\(_3\) many times, present a different type of porous solid, which can be described by a CRM, as noted above. This is because the recarbonation is never complete and so after many cycles, the starting, carbonated material is, in fact, quite porous and so provides an appropriate candidate for verifying the \( f(\lambda) \) hypothesis. The purpose of this paper is therefore to examine if the \( f(\lambda) \) concept can be used for modelling non-catalytic gas – solid reactions, other than those involving gasification, using calcination as an example for different particle sizes and over a range of temperatures.

2 Experimental

2.1 Materials

Table 1. Composition of the fresh limestones in wt%.

<table>
<thead>
<tr>
<th>Component</th>
<th>Ca</th>
<th>Fe</th>
<th>Mg</th>
<th>Ni</th>
<th>Al</th>
<th>K</th>
<th>Mn</th>
<th>Si</th>
<th>S</th>
<th>Zr</th>
<th>Sr</th>
<th>Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compostilla</td>
<td>89.70</td>
<td>2.50</td>
<td>0.76</td>
<td>0</td>
<td>0.16</td>
<td>0.46</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0.46</td>
<td>0.16</td>
</tr>
<tr>
<td>Purbeck</td>
<td>97.67</td>
<td>0.49</td>
<td>0.61</td>
<td>0</td>
<td>0.21</td>
<td>0.09</td>
<td>0.14</td>
<td>0.65</td>
<td>0.11</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The gases used in the experiments were N\(_2\) (\( \geq 99.9 \) vol\%, oxygen \( \leq 2 \) ppmv) and CO\(_2\) (\( \geq 99.8 \) vol\%). All gas cylinders were supplied by BOC or Air Liquide. Natural, uncrushed silica sand (fraction C, David Ball Group plc., dry), sieved to 355 – 425 \( \mu \)m, was used as a fluidised bed material. The density of the non-porous sand particles was \( \sim 2690 \) kg m\(^{-3}\). Two types of limestone particles were used: (i) a Spanish limestone (Compostilla) after 8 cycles of calcination and carbonation, and (ii) a British limestone (Purbeck) after 6 such cycles. The number of cycles was chosen so that the particles become porous and the ultimate CO\(_2\) uptake by the limestones of the current cycle was close to that of the previous cycle. The compositions of the fresh limestones are shown in Table 1. After cycling, the internal pores of the limestone, even at the start in the fully-carbonated state, were macro-pores (> 50nm) so that the calcination would likely occur continuously throughout the entire particle, as shown in Table 2. The cycling of the limestone particles was conducted in a bed of sand fluidised by 15 vol\% CO\(_2\) balance N\(_2\) at 1 atm. The limestone particles were calcined at 1173 K for 10 minutes and then carbonated at 923 K for 10 minutes. Here, the temperature at which a partial pressure of CO\(_2\) of 0.15 atm is in thermodynamic
equilibrium with a mixture of CaO and CaCO$_3$ was calculated to be 1053 K [16]. The resulting carbonated particles were cooled in a desiccator and then were sieved from the sand. Two sieve size fractions were used in the experiments for each type of limestone: 710 – 850 µm and 1400 – 1700 µm. These sizes were selected in order to recover the cycled particles effectively from the sand and to compare the theoretical predications across different particle size.

2.2 Fluidised bed experiments

![Diagram of fluidised bed experiment setup](image)

**Fig. 2.** Schematic diagram of the arrangement for batch experiments in a quartz reactor (i.d. 30 mm).

Batch experiments were performed in a fluidised bed contained in a quartz reactor, internal diameter 30 mm and length 460 mm, provided with a porous frit (4 mm thick, pore size 100 – 160 µm) as the distributor, situated 110 mm from the base of the reactor. By using pressure taps at the inlet and the outlet, the pressure drop across the distributor and a 20 ml sand bed was measured to be 13 – 15 mbar at experimental conditions. The reactor was externally heated by an electric furnace. The temperature of the bed was measured by a K-type thermocouple (2 mm dia.) inserted into the top, with its tip 20 mm above the distributor. Flowrates of N$_2$ were controlled by a mass flow meter calibrated at 293 K and 1 bar. The off-gas leaving the fluidising bed was sampled at 16.7 mL s$^{-1}$ (STP) through a quartz tube. To prevent elutriated particles and water vapour in the sampled gas entering the analysers, the gas was passed through a glass wool filter and a drying tube filled with CaCl$_2$ in series. The mole fractions of CO$_2$ were measured by a non-dispersive infra-red gas analyser (ABB EL3020). Fig. 1
shows the arrangement of the apparatus. In an experiment, the reactor was filled with 20 ml of silica sand and heated to the desired temperature, viz. 1023 – 1173 K. For calcination, the fluidising gas was 100 mol% N₂. The total volumetric flowrate was 80 mL s⁻¹ (STP), giving \( U/U_{mf} \approx 6.3 – 7.9 \), with \( U \) being the superficial velocity at the temperature of the bed and \( U_{mf} \) being the value at incipient fluidisation predicted from the correlation of Wen and Yu [17]. From experimental measurements using different sample masses ranging from 0.1 – 0.5 g at 1173 K, the measurement of reaction rate starts decreased when the sample mass was bigger than 0.3 g due to complications arising from mass transfer between the bubble and the particulate phases. Hence a sample mass of 0.3 g was chose for all experiments. Each experiment was repeated at least 3 times. To ensure complete calcination of the limestones, the calcination experiment is ended 10 seconds after the measured concentration of \( \text{CO}_2 \) of the off-gas returns to zero.

### 2.3 Characterisation of the limestone particles

**Table 2.** Particle characterisation of fully carbonated limestones.

<table>
<thead>
<tr>
<th>Limestone</th>
<th>BET analysis</th>
<th>Mercury intrusion porosimetry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BET area / m² g⁻¹</td>
<td>BJH volume / cm³ g⁻¹</td>
</tr>
<tr>
<td>Compostilla, 8 cycles</td>
<td>7.2 × 10⁻³</td>
<td>72</td>
</tr>
<tr>
<td>Purbeck, 6 cycles</td>
<td>9.9 × 10⁻³</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 2 shows the measurements from the Brunauer–Emmett–Teller (BET) analysis (TriStar 3000) and mercury intrusion porosimetry (AutoPore IV 9500), both of which produce a pore size distribution of the particles. It is clear that these cycled particles have substantial pore volume, even in their fully-carbonated state, in contrast to virgin limestone, which has negligible porosity. In the subsequent modelling, the mean pore diameter \( d_{pore} = 4V/A \) from mercury porosimetry was used as the initial pore diameter of the particles, where \( V \) is the total intrusion volume and \( A \) is the total pore area. It should be noted that the limestone particles may have considerable unmeasured surface area and pore volume in the micro-porous range \( (d_{pore} < 2 \text{ nm}) \), as the BET and mercury intrusion analyser were unable to measure pore diameters smaller than 1.7 nm and 3 nm respectively.

Dai et al. [12] measured the particle size distribution of 600 – 1000 µm dia. char particles with the same sub-angular shape to the limestone particles by optical microscopy and showed that the effective particle diameter
\( D(3,2) \times \psi \), where \( D(3,2) \) was the Sauter mean diameter and \( \psi \) is a shape factor, is very close to the geometric mean of the mesh sizes. Therefore in the model, the mean, external particle diameter \( d_p \) was calculated from the geometric mean of the sieves \( d_p = (\text{lower mesh} \times \text{upper mesh})^{0.5} \).

3 Theoretical

The model described in this work assumed that the limestone particle is spherical and is calcined in a bed of silica sand fluidised by a stream of \( \text{N}_2 \). The only reaction occurring is:

\[
\text{CaCO}_3(\text{s}) \rightarrow \text{CaO}(\text{s}) + \text{CO}_2(\text{g}) \quad \Delta H^\circ_{298K} = +178 \text{ kJ mol}^{-1} \quad (I)
\]

The two principal assumptions were:

i) The material and energy balances inside and outside the particle are in pseudo-steady state, so that the gas concentrations, total fluxes, total pressure and temperature have no time dependence. By using the pseudo-steady state assumption here, it means that the time needed to establish an initial steady concentration profile is very small. Without this assumption, the initial concentration would have been zero everywhere. Nevertheless, as justified by Bischoff [18], the profiles generated using this assumption will be achieved very quickly in a gas-solid system. In fact, the thermal diffusivity of the solid at 800 °C is about \( 9 \times 10^{-6} \) m\(^2\)/s, even for a 2 mm dia. particle, the time constant estimated from \( \text{Radius}^2 / (2 \times \text{thermal diffusivity}) \) is very small \( \sim 0.2 \) s. Wen [19] also concluded that the pseudo-steady state solution was a good approximation for most of the solid-gas reaction systems except for systems with extremely high pressure and very low solid reactant concentration. However, the balances are affected indirectly by the conversion \( X \) which does have time dependence and affects the physical properties of the particle (e.g. porosity, pore diameter and particle size).

ii) The evolution of the internal morphology of a limestone particle during calcination can be described by an arbitrary \( f(X) \), a function of \( X \) – the local conversion of the maximum available \( \text{CaCO}_3 \) content within the particle after cycling. The reason for such a definition of conversion is that the limestone particle is only partially carbonated and the maximum \( \text{CO}_2 \) uptake decays gradually with the number of cycles. Hence the reaction rate at some point within the solid can be expressed in the form of \( r = g(C_i, T, P) \times f(X) \). It was assumed that \( f(X) \) applies everywhere within a particle and is independent of temperature. The value of \( f(X) \) changes with the local conversion, which will vary with distance from the centre of the particle. The \( f(X) \) can be obtained from a plot of the experimental rate of calcination against conversion obtained from experimental measurements in which the rate is controlled solely by intrinsic chemical kinetics [12,13].
3.1 The kinetics of the calcination reaction

The intrinsic rate of reaction per unit of surface area for reaction (I) was given by [14]

\[-q_c = k_c - kC_{CO_2} = k_c - \frac{\bar{k} \cdot p_{CO_2}}{RT}\]  
(1)

where \(k_c\) is the rate constant of the calcination reaction (here, in mol m\(^{-2}\) s\(^{-1}\)), \(\bar{k}\) is the rate constant of the reverse, carbonation reaction (m s\(^{-1}\)) and \(p_{CO_2}\) is the partial pressure of CO\(_2\). At equilibrium, the rate \(q_c = 0\), so that

\(k_c - \bar{k} \cdot \frac{p_{CO_2}^{eq}}{RT} = 0\). Therefore, the ratio of rate constants is:

\(\frac{k_c}{\bar{k}} = \frac{p_{CO_2}^{eq}}{RT}\)  
(2)

Substituting Eq. (2) into Eq. (1),

\[-q_c = k_c(1 - \frac{p_{CO_2}}{p_{CO_2}^{eq}})\]  
(3)

Here \(p_{CO_2}^{eq}\) is the equilibrium partial pressure of CO\(_2\) at local conditions. Barin and Platzki [16] gave the following expression for \(p_{CO_2}^{eq}\):

\(p_{CO_2}^{eq} = 4.083 \times 10^7 \exp(-20474/T)\)  
(4)

where the unit of \(T\) is in K. The rate constant \(k_c\) was assumed to be an activated quantity, thus

\(k_c = A_{k_c} \exp\left(-\frac{E_a}{RT}\right)\).

Based on the assumption ii), the rate of reaction per unit volume of particle, \(Q_c\), is:

\(Q_c = q_c A_{2,0} \rho_{e,0} f(X) = -k_c(1 - \frac{p_{CO_2}}{p_{CO_2}^{eq}}) A_{2,0} \rho_{e,0} f(X)\)  
(5)

where the parameter \(A_{2,0}\) is the initial pore area per unit mass and \(\rho_{e,0}\) is the initial bulk density of the particles.

3.2 Equations of mass balance

A pseudo-steady mass balance over a spherical shell at radius \(r\) gives the flux equations for CO\(_2\) and N\(_2\):

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 J_n \right) = Q_n = -\nu_n Q_c = -\nu_n q_c A_{2,0} \rho_{e,0} f(X)\quad n = 1, 2
\]

where \(J_n\) is the total flux (i.e. diffusive flux + advective flux) of species \(n\). The parameter \(Q_n\) is the net rate of reaction of species \(n\), in mol m\(^{-3}\) s\(^{-1}\), which is positive for a net gain and negative for a net loss in species, and \(\nu_n\) is the stoichiometric coefficient of species \(n\) in reaction (I). The subscripts 1 and 2 are used to represent CO\(_2\) and N\(_2\).

A material balance on carbon across a differential element gives the variation of local conversion of CaCO\(_3\) with respect to time:

\[
\frac{dX}{dt} \bigg|_{r} = -M_{CaCO_3} \left( \frac{Q_c}{\rho_{e,0}} \right) = k_c A_{2,0} M_{CaCO_3} \left( 1 - \frac{p_{CO_2}}{p_{CO_2}^{eq}} \right) f(X)
\]

(7)

the initial condition of which is:
\[ t = 0: \quad X(r) = 0 \quad \text{for all } r \in [0, r_p] \]  

where the particle centre is \( r = 0 \) and the particle surface is \( r = r_p \). Since \( A_{x,0}M_{CaCO_3} \) is the initial pore area per unit volume of particle – a constant, the product of parameters \( k_c A_{x,0}M_{CaCO_3} \) can be replaced by a modified rate constant \( k_c': \)

\[
\frac{dX}{dt} \bigg|_r = k_c' \left( 1 - \frac{p_{CO_2}}{p_{CO_2}^{eq}} \right) f(X) \quad (9)
\]

\[
k_c' = \Lambda_k A_{x,0} M_{CaCO_3} \exp(-E_a/RT) = k_0 \exp(-E_a/RT)
\]

The activation energy of \( k_c' \) is the same as that of \( k_c \), but the pre-exponential becomes \( k_0 = \Lambda_k A_{x,0} M_{CaCO_3} \).

### 3.3 Equations of intraparticle mass transfer

A model of multi-component diffusion based on the Stefan-Maxwell equations within a porous medium was needed to describe the intraparticle diffusion rigorously. The two principal flux models for non-equimolar, multi-component mass transfer are the Dusty Gas Model (DGM) [20], and the Mean Pore Transport Model (MPTM) [21–23]. Given that both models are algebraically complicated, Young and Todd [24] developed a new MPTM called the Cylindrical Pore Interpolation Model (CPIM). Comparing all three models, the CPIM has a more rigorous treatment of continuum flow, a clearer interpolation procedure for transitional flow and a more compact form of the working equations which helps to clarify the roles of the governing parameters. Recent studies suggest that the CPIM is well suited to modelling multi-component diffusion in both catalyst pellets [25] and in gasifying char particles [12,13]. For this reason, the CPIM was selected to model intraparticle diffusion in the present work. The governing equations are:

\[
\frac{dy_n}{dr} = \frac{\tau^2 RT}{\varepsilon P} \sum_{m=1}^{2} \left( \frac{y_n \mu_m}{D_{A,mm}} \frac{y_m \mu_n}{D_{A,nm}} \right) \quad n = 1, 2 \quad (10)
\]

\[
\frac{dP}{dr} = -\frac{\tau^2 A_d}{\varepsilon} \sum_{n=1}^{2} \left( \sqrt{R_n} J_n \right) \quad (11)
\]

The boundary conditions for the above equations are given at the centre \( (r = 0) \) and the surface \( (r = r_p) \) of the particle:

\[
r = 0: \quad J_1 = 0 \quad J_2 = 0 \quad (12)
\]

\[
r = r_p: \quad y_n = y_n^{surface} \quad (n = 1, 2) \quad \text{&} \quad P = P^{surface} \quad (13)
\]

Here, \( y_n \) is the mole fraction of species \( n \), \( \tau^2 \) represents the tortuosity factor of the particle from mercury intrusion porosimetry measurements, \( \varepsilon \) is the porosity at the local point, which varies with conversion and is discussed later,
and $M_n$ is the molar mass of gas species $n$. The parameters $D_{A,n,m}$ and $A_A$ were found by interpolating between the extremes of continuum and Knudsen flow using the equations proposed by Young and Todd [24]:

$$\frac{1}{D_{A,n,m}} = \frac{1}{D_{K,n}} + \frac{1}{D_{B,n,m}} \quad \text{and} \quad \frac{1}{A_A} = \frac{1}{A_K} + \frac{1}{A_C}$$

(14)

where $D_{B,n,m}$ is the molecular diffusivity calculated from the Chapman-Engskog theory using the Lennard-Jones (6-12) potential [26]. The error in the predicted binary diffusivities by this method is ~ 7.3% [27]. $D_{K,n}$ is the Knudsen diffusivity and the parameters $A_K$ and $A_C$ are the coefficients in the pressure gradient equation in the continuum and Knudsen regime, given by [24,28]

$$D_{K,n} = \frac{2r_{pore}}{3} \frac{8RT}{\pi M_n} \quad A_K = \frac{3}{4r_{pore}} \frac{\pi RT}{2}$$

(15)

where the viscosity of the gas mixture, $\mu_{mix}$, was calculated using Chapman-Engskog theory.

The porosity, $\varepsilon$, changes with the local conversion of CaCO$_3$, $X$, during reaction and can be derived from the volume balance equation for a thin cylindrical shell inside the particle:

$$\varepsilon(X) = \varepsilon_0 + X(1 - \varepsilon_0) \left(1 - \frac{V_{m,CaO}}{V_{m,CaCO_3}}\right)$$

(16)

where $V_{m,CaO}$ and $V_{m,CaCO_3}$ are the molar volume of the non-porous CaO and CaCO$_3$ solids. In terms of the pore diameter, it was assumed that the particle has uniform cylindrical pores of initial diameter $d_{pore,0}$; the corresponding initial porosity of the particle was $\varepsilon_0$. Ignoring the small volume of crossing between pore channels, the local porosity can be estimated from $\varepsilon(r) \cdot \delta V = \pi d_{pore}^2 \sum_i L_i / 4$, where $\delta V$ is the volume of a differential element between $r$ and $r+dr$ and $\sum_i L_i$ is the sum of the length of the cylindrical pores within the element.

Assuming that the evolution of the pores during reaction occurs only in a radial direction so that the pore diameter changes while the length of the pore remains constant, then:

$$\varepsilon(r)/\varepsilon_0 = \left(\frac{d_{pore}}{d_{pore,0}}\right)^2$$

(17)

Substituting Eq. (16) into (17), the pore diameter at some time when the local conversion is $X$ is:

$$d_{pore} = d_{pore,0} \sqrt{\frac{\varepsilon_0 + X(1 - \varepsilon_0) \left(1 - \frac{V_{m,CaO}}{V_{m,CaCO_3}}\right)}{\varepsilon_0}}$$

(18)

### 3.4 Equations of external mass transfer

The particulate phase of the fluidised bed was considered to have a constant local tortuosity and porosity around the limestone particle. It was also assumed that there is no variation of pressure with radial distance
outside the limestone particle, since the interstitial velocity of fluidising gas – ~1.1 m/s (STP) is much larger than the mass average velocity of gas leaving the surface of a reaction particle – ~0.04 m/s (STP) calculated from the gas flux at the surface. This suggests that there is no tendency to form voids or bubbles around the reacting particle in the case under consideration and that pressure variations outside the particle can be neglected. The general Stefan-Maxwell equations [26] were used to model the external mass transfer within a diffusion boundary layer of thickness $\delta$ outside the particle:

$$\frac{d y_n}{dr} = \frac{\tau_{\text{bed}}^2 RT}{\varepsilon_{\text{bed}} P} \sum_{m=1}^{2} \left( \frac{y_n l_m - y_m l_n}{D_{nm}} \right) \quad n = 1, 2 \quad dP/dr = 0 \quad (19)$$

Here $\tau_{\text{bed}}^2$ is the tortuosity factor of the sand bed. It was experimentally measured to be $1.34^2 = 1.80$ for a packed bed with 200 µm dia. quartz sand by Zoia and Latrille [29]. Also, $\varepsilon_{\text{bed}}$ is the porosity of the bed, assumed to be 0.44, the same as the porosity at incipient fluidisation used by Hayhurst and Parmar [30] for a bubbling fluidised bed of silica sand. The parameter $D_{nm}$ refers to the binary molecular diffusivity of species $n$ and species $m$. The boundary conditions at the particle surface ($r = r_p$) and the edge of boundary layer ($r = r_p + \delta$) are:

$$r = r_p: \quad f_1 = f_1^{\text{bulk}}$$

$$r = r_p + \delta: \quad y_n = y_n^{\text{bulk}} \quad (n = 1, 2) \quad \& \quad P = P^{\text{bulk}} \quad (20)$$

3.5 Equations of energy balance

Outside the limestone particle, convective heat transfer in a bubbling fluidised bed involves packets of sand particles coming into contact with the limestone for a short time, then quickly moving away to be replaced by other packets. It was assumed that the heat transfer coefficient between the particulate phase of the fluidised bed and the surface of the particle was large, so that the particle surface is close to the temperature of the bulk. It was also assumed that the radiative contribution and transpiration contributions to heat transfer were small.

Inside the limestone particle, the energy flux, $E$, is given by [26]

$$E = \sum_{n=1}^{2} M_n j_n \left( \frac{u_n^2}{2} + \frac{H_n}{M_n} \right) - \lambda_{\text{eff}} \frac{dT}{dr} \quad (22)$$

Jackson [31] showed that the energy balance can be expressed as $\text{div}(E) = 0$: 14
\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = \frac{1}{\lambda_{\text{eff}}} \sum_{n=1}^{2} \left( H_n + \frac{u_M^2}{2} M_n \right) q_n + \frac{1}{\lambda_{\text{eff}}} \sum_{n=1}^{2} M_n j_n \frac{du_M}{dr} + \frac{1}{\lambda_{\text{eff}}} \sum_{n=1}^{2} M_n j_n \frac{du_M^2}{dr} \]

(23)

where \( C_{p,n} \) is the molar heat capacity of species \( n \) and \( \lambda_{\text{eff}} \) is the effective thermal conductivity. \( H_n \) is the partial molar enthalpy of species \( n \) at temperature \( T \), and is calculated from standard enthalpy of formation \( H^n_f \) by

\[
H_n = H^n_f + \int_{298}^{T} C_{p,n} dT.
\]

This equation makes specific allowance for the small change in momentum occurring as a result of the change in mass in the gas phase during the non-catalytic decomposition of the solid. The calculation of the thermal parameters \( C_p \) and \( \lambda \) is discussed in the next section. Finally, \( u_M \) is the mass-averaged velocity of the mixture which is given by

\[
u_M = \sum_{n=1}^{2} M_n j_n / \sum_{n=1}^{2} \rho_n.
\]

The boundary conditions for the internal energy balance were:

\[
\begin{align*}
  r = 0: & \quad \frac{dT}{dr} = 0 \\
  r = r_p: & \quad T = T_{\text{bulk}}
\end{align*}
\]

(24)

3.6 Calculation of parameters

The initial pore area per unit mass \( A_{p,0} \) was obtained from the BET area measurement of the limestone particles. The initial bulk density \( \rho_{e,0} \), the initial pore diameter \( d_{\text{pore},0} \), the tortuosity factor \( r^2 \) and the initial porosity of the particle \( \varepsilon_0 \) were determined from the mercury porosimetry measurements, as shown in Table 2. Alvarez et al. [32] did reported up to 50% increase in pore diameter of natural limestone particles after 100 cycles. In terms of the particle size, Wu et al. [33] reported only 2 – 7% reduction of particle diameter after 10 calcination – carbonation cycles. Hence in this study the overall particle size was assumed to be constant during calcination. Any change of the solid volume due to the difference in molar volumes of CaCO3 and CaO was taken only to affect the pore structure parameters e.g. porosity and pore diameter.

The boundary layer thickness \( \delta \) was given by Hayhurst and Parmar [30]:

\[
S_{\text{EMCD}} = 2\varepsilon_m (1 + r_p / \delta)
\]

(25)

\[
S_{\text{EMCD}} = 2\varepsilon_m + 0.61 \left( 2r_p U_p / \nu_{\text{mix}} \right)^{0.48} \left( \nu_{\text{mix}} / D_{B,12} \right)^{0.33}
\]

(26)

\[
U_p = U_{mf} (1 - \varepsilon_b) \left( 1 - 0.5\pi \ln(1 - 6\varepsilon_b / \pi) \right)
\]

(27)

where \( \varepsilon_{mf} = 0.44 \) was the voidage at incipient fluidisation for a bubbling fluidised bed with silica sand used by Hayhurst and Parmar [30]. Also, \( \nu_{\text{mix}} \) is the kinematic viscosity of the gas mixture calculated using Chapman-Engskog theory and \( D_{B,12} \) the binary molecular diffusivity for CO2 and N2. The bubble fraction \( \varepsilon_b \) was given by
\[ \varepsilon_b = \left( U - U_{mf} \right) / U_b = (h - h_{mf}) / h \text{, where } U_b = \left( U - U_{mf} \right) + 0.71 \left( g \sqrt{\bar{d}_b} \right)^{0.5} [34]. \] Here, \( h_{mf} \) was the bed height at incipient fluidisation, measured to be 0.029 m and \( \bar{d}_b \) was the mean bubble diameter estimated from the correlation of Darton et al. [35]: \( \bar{d}_b = 0.54 \left( U - U_{mf} \right)^{0.4} h^{0.8} / (2g^{0.2}) \) with \( h \) being the expanded height of the bed when fluidised at superficial velocity \( U \). Although the correlation was based on equimolar counter-diffusion (EMCD), it has been shown that it will yield the correct value of \( \delta \) from Eq. (25), even for non-EMCD [36].

The thermal conductivities of the gases were calculated from \( \lambda = C_1 T^{C_2} / \left( 1 + C_3 / T + C_4 / T^2 \right) \), where \( C_1 - C_4 \) are constants [27]. The effective thermal conductivity of the particle was calculated from \( \lambda_{eff} = (1 - \varepsilon) \lambda_{solid} + \varepsilon \lambda_{gases} \), where \( \lambda_{gases} = \sum_{a=1}^{n=3} y_a \lambda_a \). The overall thermal conductivity was largely influenced by that of the solid. The reported thermal conductivity of limestone (CaCO\(_3\)) and lime (CaO) is 2.25 and 0.84 W m\(^{-1}\) K\(^{-1}\) respectively [27]. The exact mole fraction of CaCO\(_3\) and CaO within the particles after cycling was unknown, hence, \( \lambda_{solid} \) was taken to be 1.5 W m\(^{-1}\) K\(^{-1}\). The specific heat capacity of each gas was estimated from \( C_p = E_1 + E_2 T + E_3 / T^2 \), where \( E_1 - E_3 \) are constants from Green and Perry [27].

### 3.7 Numerical solution

The system is described by Eq. (6) to (24). Both the intraparticle and external mass transfer models have five 1\(^{st}\) order ordinary differential equations (ODEs) in the space domain, and hence five boundary conditions are provided. The ODE for conversion is 1\(^{st}\) order in the time domain, hence only one initial condition is required. The energy equation is a 2\(^{nd}\) order ODE, hence two boundary conditions are required for both the internal and external cases.

The main difficulty in solving the system lies in efficient solution of the large system of equations. A numerical algorithm, Orthogonal Collocation on Finite Elements (OCFE) [37] was written in MATLAB to solve the model. Pseudo-steady state was assumed for all the other variables except for the conversion \( X \). Using the initial condition in Eq. (8), the 1\(^{st}\) order time-dependent ODE of local conversion \( X \), Eq. (7), was solved. The value of \( X(\eta) \) at time \( t + \Delta t \) could be calculated based on \( X(\eta) \) and \( dX / dt \) at time \( t \). The relevant model parameters (e.g. porosity and pore diameter) were updated with the new value of \( X(\eta) \), and then the internal and external models were solved for results at time \( t + \Delta t \). The iterative process was stopped when the overall conversion reached unity.

Since the model predicts the distributions of reaction rate and conversion across the radius of a particle, the overall values of rate and conversion need to be obtained from integration across the particle radius. For a
distribution of $\chi$ (e.g. $Q_c$ and $X$), its volume averaged value can be calculated from $\bar{\chi} = 3 \int_0^1 (\eta_r^2 \chi) d\eta_r$, where $\eta_r$ is the dimensionless radius used inside the particle. The integral was evaluated numerically.

4 Results

Fig. 3 shows the raw measurements of CO$_2$ mole fraction in the off-gas during the calcination of cycled Compostilla (plot a) and Purbeck limestone particles (plot b) in a bed of silica sand fluidised by pure N$_2$. The figure suggests that the calcination of Compostilla at 1173 K was completed after ~ 50 s while Purbeck at 1173 K finished calcining after 35 s. The peak concentration of CO$_2$ from Compostilla was about half that of Purbeck, hence the reactivity of Compostilla was significantly less than that of Purbeck. The equilibrium partial pressure of CO$_2$ at 1173 K is about 1.087 bar, so the concentration driving force $p_{\text{CO}_2}/p_{\text{CO}_2}^{\text{eq}} < 5\%$. Hence this confirms that the fluidised bed was close to a differential reactor, and it is reasonable to use 0% CO$_2$ as the bulk concentration in the model.

![Fig. 3. Measurements of CO$_2$ mole fraction during calcination of cycled limestones at atmospheric pressure: a) Compostilla 8 cycles 0.71 – 0.85 mm at 1173 K; b) Purbeck 6 cycles 0.71 – 0.85 mm at 1173 K.](image)

The overall rate of production of CO$_2$ from calcination in s$^{-1}$ is

$$dX/dt = \left(\dot{N}_{\text{out}Y_{\text{CO}_2,\text{out}}} - \dot{N}_{\text{in}Y_{\text{CO}_2,\text{in}}}\right)/\int\left(\dot{N}_{\text{out}Y_{\text{CO}_2,\text{out}}} - \dot{N}_{\text{in}Y_{\text{CO}_2,\text{in}}}\right)dt.$$ The parameters $\dot{N}_{\text{out}}$ and $\dot{N}_{\text{in}}$ are the total molar flows leaving and entering the reactor at the exit and entrance conditions, where $\dot{N}_{\text{out}}(1 - Y_{\text{CO}_2,\text{out}}) = \dot{N}_{\text{in}}(1 - Y_{\text{CO}_2,\text{in}})$ from the mass balance of nitrogen. The raw measurements were deconvoluted to account for the mixing and delay in the sampling line using the method described by Saucedo et al. [13].

Table 3. Kinetic parameters of the calcination of 0.71 – 0.85 mm dia. limestone particles.

<table>
<thead>
<tr>
<th>Limestone particles</th>
<th>$k_0$ / s$^{-1}$</th>
<th>$E_a$ / kJ mol$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compostilla</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Purbeck</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The Arrhenius coefficients and activation energies of the kinetic parameter $k'_c$ in Eq. (9), shown in Table 3, were determined from the initial rate extrapolated from the experimental measurements, as shown in Fig. 4. At the start of reaction, the particle conversion is 0 and $f(X) = 1$, and Eq. (9) can be rearranged

$$\ln\left(\frac{dX}{dt} |_{t=0}\right) = -\frac{E_a}{RT} + \ln(k_0) + \ln\left(1 - \frac{p_{CO_2}}{p_{CO_2}^{eq}}\right)$$

(28)

If the value of $p_{CO_2}/p_{CO_2}^{eq}$ is much smaller than 1, which is usually the case if the reaction is controlled by intrinsic kinetics, then

$$\ln\left(\frac{dX}{dt} |_{t=0}\right) = \ln(k'_c) = -\frac{E_a}{RT} + \ln(k_0)$$

(29)

A plot of $\ln(dX/dt |_{t=0})$ vs. $1/T$ should therefore yield the activation energy $E_a$ and the Arrhenius coefficient $k_0$. The linearity of Fig. 5 and Fig. 8 in section 4.1 and 4.2 respectively show that the calcination of the limestone was indeed controlled by chemical kinetics at low temperature, hence confirming the use of Eq. (29) for the determination of the kinetic parameters. The errors associated with the kinetic parameters mainly come from extrapolating the initial rates from measurements, and using a limited number of measurements for linear regression analysis. The 95% confidence intervals of the kinetic parameters are shown in Fig. 5 and Fig. 8.

![Fig. 4. Determining initial rate of reaction using linear extrapolation (--) on rate and conversion measurements (×) of Compostilla 0.71 – 0.85 mm particles (8 cycles) at 1073 K.](image)
4.1 Calcination of Compostilla limestone particles

![Graph showing the relationship between the logarithm of the rate constant and the inverse of the temperature. The graph includes a linear regression analysis with the following equations:

\[ k_0 = \exp(16.66) \times 10^7 \text{s}^{-1} \]

\[ E = 175 \text{kJ/mol (95% C.I.: 163, 186)} \]

\[ E_{\text{app}} = 166 \text{kJ/mol (95% C.I.: 152, 179)} \]

Fig. 5. Determining the kinetic parameters of Compostilla limestone particles (8 cycles). The measurements were obtained from the calcination of the limestone particles at 1023 K (0.71 – 0.85 mm only), 1073 K, 1123 K, 1148 K and 1173 K. The values of the kinetic parameters of the rate constant are shown with 95% confidence interval (C.I.).

Particles of Compostilla limestone with diameters of 0.71 – 0.85 mm and 1.40 – 1.70 mm were calcined at 1023 K (0.71 – 0.85 mm only), 1073 K, 1123 K, 1148 K and 1173 K. Using the initial rate extrapolated to zero conversion, Fig. 5 shows that the plot of \( \ln(dX/dt) \) vs. \( 1/T \) of each particle forms straight lines. The values of the kinetic parameters in Eq. (9) were determined from a linear regression analysis, yielding an activation energy of \( E_a = 175 \pm 12 \text{kJ/mol} \) and \( k_0 = 1.72 \times 10^7 \text{s}^{-1} \) for 0.71 – 0.85 mm dia. particle. For 1.40 – 1.70 mm dia. particle, the reaction rates were lower than those of 0.71 – 0.85 mm dia. particle and the apparent activation energy was 166 ± 14 kJ/mol, representing a 9 kJ/mol reduction that is within an error band of ±12 kJ/mol. It is expected that the gradient of a best fit line would approach a half of its intrinsic value if the reaction rate were significantly limited by intraparticle mass transfer [1]. However, this is not observed in Fig. 5. The values of the activation energy indicate that the reactions could not have been in mass transfer limited regime. Hence, it can be concluded that (i) the calcination of 0.71 – 0.85 mm dia. particles was controlled by intrinsic chemical kinetics; (ii) the reactions of 1.40 – 1.70 mm dia. particles were possibly affected by intraparticle mass transfer but not severely so.
Fig. 6. Determining the function $f(X)$ from the plot of normalised rate vs. conversion measurement of $0.71 - 0.85$ mm dia. Compostilla (8 cycles) at 1073 K.

The form of $f(X)$ needs to be determined from experimental measurements of calcination rate vs. conversion conducted under conditions where the reaction is controlled by intrinsic chemical kinetics. Owing to the low rate of reaction at 1023 K, the percentage fluctuation caused by random noise in the measurements of CO$_2$ concentration was very large. The resulting $f(X)$ was not a smooth function, as expected. However, since experiments at both 1023 K and 1073 K appear to be in the regime of chemical kinetic control, as shown in Fig. 5, the $f(X)$ function was determined from the measurements at 1073 K instead. Fig. 6 shows the plot of $f(X)$, a 6th order polynomial of $X$, determined from the normalised rate vs. conversion measurements of Compostilla limestone of sieve diameter 0.71 – 0.85 mm, calcined at 1073 K. The figure shows a peak higher than 1 at about 20% conversion. This is due to the fact that part of the surface area that is previously unreachable become accessible when the solid volume reduces during the initial stage of calcination. This increase in surface area only occurs at low conversion when the coalescence of pores is insignificant. Using this $f(X)$, the model was able to fit well the experimental results at 1023 K, as seen in Fig. 7(a). This strongly suggests that the $f(X)$ was not merely a fit valid for one particular experimental condition.
Fig. 7. Comparison of model results (lines) with experimental measurements (points) of the calcination of Compostilla limestone particles (8 cycles) by 100% N₂: a) 0.71 – 0.85 mm; b) 1.40 – 1.70 mm. The $f(X)$ was determined from the rate vs. conversion measurements of 0.71 – 0.85 mm particles at 1073 K, and was applied to all cases.

Further comparisons between model predictions and experimental measurements for Compostilla limestone of 0.71 – 0.85 mm dia. and 1.40 – 1.70 mm dia. are shown in Fig. 7, with generally good agreement being seen between experiment and theory. However, the experimental measurements for the 0.71 – 0.85 mm dia. particle at 1173 K were almost identical to those at 1148 K, which indicates either a severe limitation by external mass transfer or experimental error arising from the rapidity of the reaction and the problem in correcting for mixing in the sampling line. Fig. 5 shows that even for the larger particles at higher temperature, the rate of reaction was not limited by mass transfer as the gradients of the two measurements are almost the same. Therefore it can be concluded that the unexpected behaviour of the rate of 0.71 – 0.85 mm dia. at 1173 K is due to error.

4.2 Calcination of Purbeck limestone particles

Fig. 8. Determining the kinetic parameters of Purbeck limestone particles (6 cycles). The measurements are from calcination of 0.71 – 0.85 mm and 1.40 – 1.70 mm particles at 1023 K, 1073 K, 1098 K, 1123 K, 1148 K and
Experiments with Purbeck limestone were performed using 0.71 – 0.85 mm and 1.40 – 1.70 mm dia. particles at 1023 K, 1073 K, 1098 K, 1123 K, 1148 K and 1173 K. The same kinetic analysis was performed on the experimental measurements and the results are shown in Fig. 8. A linear regression line of the plot of \( \ln(dX/dt) \) vs. \( 1/T \) gives an activation energy \( E_a = 186 \pm 5 \text{ kJ/mol} \) and the rate constant \( k_0 = 6.50 \times 10^7 \text{ s}^{-1} \). At 1098 – 1173 K, the gradient of the regression lines of measurements, thus \( -E_a/R \), is reduced by about half at \( T > 1098 \text{ K} \). Hence, this figure suggests that the transition of the reaction regime from chemical kinetic control to mass transfer control starts at \( \sim 1098 \text{ K} \). Fig. 8 also shows that the rates of reaction of the 1.40 – 1.70 mm dia. particles are lower than those of the 0.71 – 0.85 mm dia. particles; the linear regression lines of the 1023 – 1098 K measurements show a 21 kJ/mol decline, larger than the \( \pm 5 \text{ kJ/mol} \) error, in the apparent activation energy, probably owing to a growing influence of the mass transfer limitation for larger particles. At \( T > 1098 \text{ K} \), the slope of the points of 1.40 – 1.70 mm dia. particle is almost the same as that of 0.71 – 0.85 mm particle, suggesting that the reaction becomes limited by mass transfer. Furthermore, for the 1.40 – 1.70 mm dia. particles the transition of the reaction regime occurs at a temperature lower than that of the 0.71 – 0.85 mm dia. particle, a consequence of the increased mass transfer limitation in larger particles.

**Fig. 9** compares the rates of conversion vs. time from experimental measurements (points) with the theory (line) for Purbeck limestone calcined in 100% \( \text{N}_2 \) at 1023 – 1173 K. Interestingly, the \( f(X) \) determined previously from the measurements on Compostilla limestone of 0.71 – 0.85 mm dia. at 1073 K, shown in **Fig. 6**, was successfully applied here for both size fractions of Purbeck limestone. The result shows that the model fits perfectly with the experimental measurements even for measurements at 1173 K.

![Fig. 9. Comparison of model results (lines) with experimental measurements (points) of the calcination of Purbeck limestone particles (6 cycles) by 100% \( \text{N}_2 \): a) 0.71 – 0.85 mm; b) 1.40 – 1.70 mm. The \( f(X) \) determined from Compostilla 710 – 850 \( \mu \text{m} \) particles at 1073 K was used here.](image-url)
Given the results, it can be concluded that using a constant $f(X)$ across different temperatures gives a satisfactory agreement between the model and the measurements for both Compostilla and Purbeck limestone. The fact that the $f(X)$ obtained from measurements of Compostilla could be successfully applied to the modelling of Purbeck suggests that the two limestone particles experienced similar changes of internal morphology during calcination. One reason that could explain this is that both particles had been periodically cycled several times before the final calcination reaction, which could have reduced the variations in pore structures thus making the two types of limestone particles more similar in terms of internal morphology. In addition, after a number of calcination – carbonation cycles, the reactivity of the particles approaches an asymptotic value. It might also be the case that the internal pore structure had developed into an “asymptotic” stage, where the original variations in pore structures between the two limestones had become slight on cycling.

5 Discussion

The above research is concerned with limestone which has been successively calcined and carbonated several times. Experimentally, the observed activation energies for the calcination of cycled, carbonated material were reasonably close to values in the literature, lying between 160 and 210 kJ/mol [14,15,38–40] for the calcination of virgin limestones, being 175 ± 12 kJ/mol and 186 ± 5 kJ/mol, respectively, for the Compostilla and Purbeck. These values, being close to the standard enthalpy of calcination, +178 kJ/mol, suggest that the activation energy for the reverse, carbonation reaction is small, being ~ -3 kJ/mol for Compostilla and ~ +8 kJ/mol for Purbeck. Thus, the intrinsic rate of carbonation changes little with temperature in the range of the temperature studied in the paper. Zawadzki and Bretsznajder [41] found that the rate of carbonation varies linearly with the difference between the partial pressure of CO$_2$ and its equilibrium value at 328 – 368°C, which suggested that the rate constant was the same for all temperature thus a zero activation energy of the carbonation rate. Nitsch [42] also concluded that the rate of carbonation has an activation energy close to zero as the rate versus partial pressure difference gave a single linear line for measurements at 800 – 850°C. The same conclusions were also reached by Bhatia and Perlmuter [43] and Dennis and Hayhurst [14] for carbonation experiments at 823 – 998 K and 1073 – 1248 K respectively.

Comparing the reaction rates of both limestones in Fig. 5 and Fig. 8, it can be seen that the reactivity of Compostilla is slightly lower than that of Purbeck for 0.71 – 0.85 mm dia. particles. In addition, Table 2 shows that the mean pore diameter of Purbeck limestones is only ~1/3 of that of Compostilla limestones. With higher
reactivity and smaller pore diameter, Purbeck limestone is indeed expected to experience more significant effects of intraparticle mass transfer on observed rate of reaction.

An interesting result from this study was that the $f(X)$ function determined from the measurements on the Compostilla limestone has been applied successfully in modelling the conversion of the Purbeck limestone. This implies that the evolution of the pore structure of both limestone particles are similar during calcination. One hypothetical reason for this observation is related to the cycling process of the limestones, where the change of pore structure become more stable as number of cycles increases. In fact, a study of the sulphation rate of cycled lime particles showed that different limestones followed a very similar conversion vs. time evolution after 50 cycles [44], which indicates that the cycling process does affect how the pore structure evolves with conversion.

6 Conclusions

It has been proposed that a simple arbitrary function $f(X)$, determined from experimental measurements of rate vs. conversion in the kinetically-controlled regime, could be used in place of mathematical pore models to describe the evolution of pore structure during a reaction that is influenced by intra-particle gas mass transfer. A model has been constructed using the Cylindrical Pore Interpolation Model for intraparticle mass transfer, first order rate equations of calcination, the Stefan-Maxwell equations for external mass transfer and the equations of energy. The model was solved numerically by orthogonal collocation on finite elements in MATLAB. The predicted results were compared with experimental measurements conducted using two size fractions of Compostilla (after 8 cycles) and Purbeck (after 6 cycles) limestones.

The results have shown that for the calcination of limestones, the empirically-determined $f(X)$ can be successfully applied to predicting the conversion of particles of various sizes across different temperature. In addition, it was found that the $f(X)$ determined from Compostilla limestones was successful in predicting the conversion of Purbeck limestones, which indicated that the two limestones had similar evolution of pore structure during calcination. This observation was attributed to the hypothesis that the calcination – carbonation cycling process might have significantly reduced the difference in the pore structures of the limestone particles and made them more homogenous.

The significance of this research is that the $f(X)$ concept presents a simple solution in modelling the evolution of pore structures during reactions of particles. Instead of using complicated mathematical pore models, one could determine the $f(X)$ from the experiments used for kinetic studies. This idea could be further applied to many other gas-solid reactions that involve change of pore structures during reactions. One needs to be aware of
the influence of multiple types of active sites which could lead to incorrect predictions. However, multiple sites are also not reflected in most published pore models.

Acknowledgements

The authors would like to thank Felix Donat, Wenting Hu and Zlatko Saracevic from the Department of Chemical Engineering and Biotechnology University of Cambridge for their assistance with the experimental work, and Peng Dai’s family for financial support. Belén González acknowledges the EU Research Fund for Coal and Steel (project number RFCR-CT-2012-00008).
References


Using an experimentally-determined model of the evolution of pore structure for the calcination of cycled limestones

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Abstract

A pseudo-steady state model of reaction and diffusion has been constructed to model the non-isothermal calcination of limestone particles which have been subjected to a history of cycling between the calcined and carbonated states. This typically occurs when using Ca-based materials for removing CO₂ from the flue gas of plants such as a power station, cement plant and steel factory in certain schemes for carbon capture and storage.

The model uses a Cylindrical Pore Interpolation Model to describe the intraparticle mass transfer of CO₂ through the pores of the material coupled with an experimentally-determined function, \( f(X) \), describing the pore evolution as a function of the conversion of the CaCO₃ present to CaO. The intrinsic rate of calcination was taken to be first order in concentration driving force. External to the limestone particle, the Stefan-Maxwell equations were used to describe the diffusion of CO₂ away from the particle and into the particulate phase of the fluidised bed. The equation of energy was used to allow for the enthalpy of the reaction. In order to validate the use of the \( f(X) \) function, the theoretical predictions were compared with experiments conducted to measure the rates and extent of conversion, at various temperature and different particle sizes, of Purbeck and Compostilla limestones that had been previously cycled between the carbonated and fully-calcined state. Excellent agreement between experiment and theory was obtained, and the model using the \( f(X) \) approach predicted the conversion of particles of various sizes well at temperatures different to that at which the function was derived, thus indicating that the \( f(X) \) solely dependent on the evolution of the morphology of the particle.

Keywords
Calcium looping; Modelling; Calcination; Pore evolution;
# List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit/Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda )</td>
<td>Coefficients in pressure gradient equation</td>
<td>Pa ( m ) g(^{0.5}) mol(^{0.5}) ( \cdot ) s ( \cdot ) m ( ^{-1})</td>
</tr>
<tr>
<td>( A_{m,0} )</td>
<td>Initial pore area per unit mass</td>
<td>m(^2) g(^{-1})</td>
</tr>
<tr>
<td>( C_{p,n} )</td>
<td>Molar heat capacity of species ( n )</td>
<td>J mol(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>( D_{n,m} )</td>
<td>Diffusivity at arbitrary Knudsen number, for species ( n ) and ( m )</td>
<td>m(^2) s(^{-1})</td>
</tr>
<tr>
<td>( D_{n,m} )</td>
<td>Molecular diffusivity, involving species ( n ) in ( m )</td>
<td>m(^2) s(^{-1})</td>
</tr>
<tr>
<td>( D_{K,n} )</td>
<td>Knudsen diffusivity of species ( n )</td>
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<tr>
<td>( D_{ref} )</td>
<td>Molecular diffusivity ( D_{ref} ) at bulk condition</td>
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<tr>
<td>( d_{b} )</td>
<td>Mean bubble diameter</td>
<td>m</td>
</tr>
<tr>
<td>( E_{a} )</td>
<td>Activation energy</td>
<td>kJ mol(^{-1})</td>
</tr>
<tr>
<td>( H_{n} )</td>
<td>Partial molar enthalpy of species ( n )</td>
<td>J mol(^{-1})</td>
</tr>
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<td>( H_{f} )</td>
<td>Partial molar enthalpy of formation</td>
<td>J mol(^{-1})</td>
</tr>
<tr>
<td>( h )</td>
<td>Bed height</td>
<td>m</td>
</tr>
<tr>
<td>( h_{mf} )</td>
<td>Bed height at minimum fluidisation</td>
<td>m</td>
</tr>
<tr>
<td>( J_{n} )</td>
<td>Total molar flux of species ( n )</td>
<td>mol m(^{-2}) s(^{-1})</td>
</tr>
<tr>
<td>( k_{c} )</td>
<td>Rate constants of calcination reaction</td>
<td>mol m(^{-2}) s(^{-1})</td>
</tr>
<tr>
<td>( k'_{c} )</td>
<td>Modified rate constants of calcination reaction</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>( k_{0} )</td>
<td>Arrhenius coefficient of rate constant ( k'_{c} )</td>
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<tr>
<td>( \bar{k} )</td>
<td>Rate constant of carbonation reaction</td>
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<tr>
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<td>Molecular mass of species ( n )</td>
<td>g mol(^{-1})</td>
</tr>
<tr>
<td>( P )</td>
<td>Total pressure</td>
<td>bar</td>
</tr>
<tr>
<td>( P_{bulk} )</td>
<td>Bulk pressure</td>
<td>bar</td>
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<tr>
<td>( p_{CO_{2}} )</td>
<td>Local partial pressure of CO(_2)</td>
<td>bar</td>
</tr>
<tr>
<td>( P_{eq} )</td>
<td>Bulk and equilibrium partial pressure of CO(_2)</td>
<td>bar</td>
</tr>
<tr>
<td>( q_{e} )</td>
<td>Intrinsic rate of calcination per unit of surface area</td>
<td>mol m(^{-2}) s(^{-1})</td>
</tr>
<tr>
<td>( Q_{e} )</td>
<td>Net rate of change of species ( n ) inside the particle</td>
<td>mol m(^{-3}) s(^{-1})</td>
</tr>
<tr>
<td>( r )</td>
<td>Radial distance from the particle centre</td>
<td>m</td>
</tr>
<tr>
<td>( r_{p} )</td>
<td>Radius of a limestone particle</td>
<td>m</td>
</tr>
<tr>
<td>( r_{pore} )</td>
<td>Mean radius of the pore</td>
<td>nm</td>
</tr>
<tr>
<td>( R )</td>
<td>Universal gas constant</td>
<td>K</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>( T )</td>
<td>Absolute temperature</td>
<td>K</td>
</tr>
<tr>
<td>( T_{bulk} )</td>
<td>Bulk temperature</td>
<td>K</td>
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<td>( U_{b} )</td>
<td>Bubble velocity</td>
<td>m s(^{-1})</td>
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<td>( U_{mf} )</td>
<td>Flow velocity at minimum fluidisation</td>
<td>m s(^{-1})</td>
</tr>
<tr>
<td>( \bar{u}_{m} )</td>
<td>Mass-averaged velocity</td>
<td>m s(^{-1})</td>
</tr>
<tr>
<td>( V_{M, CaO} )</td>
<td>Molar volume of CaO</td>
<td>m(^3) mol(^{-1})</td>
</tr>
<tr>
<td>( V_{M, CaCO_{3}} )</td>
<td>Molar volume of CaCO(_3)</td>
<td>m(^3) mol(^{-1})</td>
</tr>
<tr>
<td>( X )</td>
<td>Solid conversion</td>
<td>-</td>
</tr>
<tr>
<td>( y_{n} )</td>
<td>Mole fraction of species ( n )</td>
<td>-</td>
</tr>
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</table>

**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit/Dimension</th>
</tr>
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<tr>
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<td>Pre-exponential coefficient of the rate constant ( k_{c} )</td>
<td>mol m(^{-2}) s(^{-1})</td>
</tr>
<tr>
<td>( \lambda_{eff} )</td>
<td>Effective thermal conductivity of the particle</td>
<td>W m(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>( \varepsilon_{0} )</td>
<td>Initial porosity of the particle</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon_{b} )</td>
<td>Bubble fraction</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon_{bed} )</td>
<td>Porosity of the fluidised bed around the particle</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon(X) )</td>
<td>Porosity of the particle as a function of conversion</td>
<td>-</td>
</tr>
<tr>
<td>( \rho_{b} )</td>
<td>Bulk density of a particle</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>( \rho_{m} )</td>
<td>Skeletal density of the particle</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Tortuosity factor of the particle</td>
<td>-</td>
</tr>
</tbody>
</table>
\( \tau_{\text{bed}} \) Tortuosity factor of the fluidised bed

\( \delta \) External diffusion boundary layer thickness m

\( \eta_r \) Dimensionless radius -

\( \nu_{\text{mix}} \) Kinematic viscosity of gas mixture m² s⁻¹

\( \nu_n \) Stoichiometric coefficient of species \( n \) -

**Subscripts**

\( n = 1, 2 \) CO₂ and N₂

**Abbreviation**

BET Brunauer–Emmett–Teller

CPIM cylindrical pore interpolation model

DGM dusty gas model

MPTM mean pore transport model

OCFE orthogonal collocation on finite element

RPM random pore model
1 Introduction

Broadly-speaking, two classes of model exist to describe the non-catalytic reaction between a gas and a solid, namely the Shrinking Core Model (SCM) and the Continuous Reaction Model (CRM). The SCM has been commonly used to describe, for example, the calcination of non-porous, virgin particles of limestone (generally close to pure CaCO$_3$), where the reaction occurs at a sharp front which recedes towards the centre of the particle [1]. Generally, the rate of reaction might be limited variably by (i) chemical kinetics, (ii) diffusion through the porous product layer, (iii) transport of heat to or from a reaction interface, or (iv) diffusion through the external gas film. On the other hand, the CRM is a better description where there is slow reaction of a gas, and, or, transfer of heat, within a porous solid across a broad front or the entire particle [1]. Fig. 1 shows the difference in local conversion profiles between two models at fixed average conversion of particle. The calcination of cycled, as opposed to virgin, limestone, possesses the characteristics of the CRM, where the initially-porous particles, containing a mixture of CaCO$_3$ and unreacted CaO, become more porous as the CaCO$_3$ is calcined to CaO during thermal decomposition. Such particles are created when raw limestone particles have been subjected to a history of cycling between the calcined and carbonated states. This typically would occur when using such Ca-based materials for removing CO$_2$ from the flue gas of plants such as a power station, cement plant and steel factory in certain schemes for carbon capture and storage. The present work is concerned with these cycled particles.

![Fig. 1. Local conversion profiles of shrinking core model (SCM) and continuous reaction model (CRM) for fixed average conversion of particle.](image)

A feature of most models of non-catalytic reactions between gases and solids generally is that the intrinsic rate of reaction, $r$, at a local point within a solid particle is of the form $r = g(C_i, T, P) \times f(X)$ [2,3]. Here, $g$ describes the intrinsic reaction kinetics as a function of the temperature, $T$, the total pressure, $P$, and the concentration, $C_i$, of the reaction gases. The term $f(X)$ is a direct function of the conversion of the particle and is correlated with the internal morphology of the particle, *e.g.* surface area, pore size, pore size distribution *etc.* at a particular conversion. In addition, $f(X)$ is not a function of $C_i$, $T$, or $P$. 
Many researchers have sought to model the development of internal pore structure with conversion during non-catalytic gas-solid reaction. For example, Szekely & Evans [4] assumed that solid particles consisted of an array of spherical grains with the space between them making up the voids. They made the first attempt to incorporate the structural parameters such as grain size, porosity and pore size into the reaction scheme in their grain model. Their original model assumed that the pore structure was unaffected by the progress of reaction, although later variants [5] were able to account for change in grain size with reaction. More recently, Liu et al. [6] developed an overlapping grain model using a fitted size distribution of grains to account for the evolution of pore structures during reaction. On the other hand, random pore models have been developed, e.g. Bhatia & Perlmutter [3] and Gavalas [7], using different approaches based on cylindrical pore assumptions, to model the total surface area at any conversion as a function of the initial morphological parameters, e.g. the initial porosity. In a somewhat different approach, Simons & Finson [8] built a mass transport model using statistical methods to specify the pore structure as a continuously branching tree. However, in his model the small pore trees will be kinetically limited while the larger pore trees are diffusion limited. This is the opposite of one might expect for pores with uniform length. The existing mathematical pore models such as the random pore model, statistical pore tree model and grain model contain parameters that are difficult to measure and thus become fitting parameters, e.g. the diffusivity of SO$_2$ through a layer of CaSO$_4$, which are complicated and equally arbitrary. In many cases, the use of mathematical pore models leads to the need to modify the original models in order to fit experimental measurements [9–11].

On the other hand, the experimentally-determined $f(\lambda)$ function from the common measurements of reaction rate and conversion offers a straightforward method to describe the change of internal morphology at a local point within the particle being reacted under conditions affected by intraparticle mass transfer [12]. This is exemplified by recent studies of char gasification in a fluidised bed [12,13], where it was found that a simple, arbitrary function, $f(\lambda)$, could be determined from the plot of measurements of rate vs. conversion of the solid char in the kinetically-controlled regime. It was proposed that the ratio between the rate of reaction at any conversion and the initial rate of reaction reflects, generally, the variation in the pore structure as the reaction proceeds in the absence of intraparticle mass transfer limitation (e.g. at low temperature or using small particles or with particles of low reactivity) [13], thus giving $f(\lambda) = r(\lambda)/r(\lambda = 0)$.

Dai et al. [12] concluded that the application of the $f(\lambda)$ concept to the gasification of chars by CO$_2$ suffered the complication of there being multiple types of active sites for adsorption on the surface of char so that a single $f(\lambda)$ determined from experimental measurements at a low temperature was unable to fit satisfactorily all the measurements made at a substantially higher temperature. Accordingly, to investigate the basic hypothesis that
a gas-solid reaction can be characterised by \( r = g(C_i, T, P) \times f(X) \), it is important to identify a solid which is unlikely to contain sites which vary in relative activity with temperature. The conversion of calcium carbonate to calcium oxide does not involve gas adsorption, thus limestones is a potential suitable candidate. Of course, as noted above, virgin limestone (CaCO\(_3\)) is almost non-porous, and the calcination reaction usually follows a shrinking core mechanism [14,15], unsuitable for the application of the \( f(X) \) concept. However, limestones which have been successively calcined to CaO and carbonated in CO\(_2\) back to CaCO\(_3\) many times, present a different type of porous solid, which can be described by a CRM, as noted above. This is because the recarbonation is never complete and so after many cycles, the starting, carbonated material is, in fact, quite porous and so provides an appropriate candidate for verifying the \( f(X) \) hypothesis. The purpose of this paper is therefore to examine if the \( f(X) \) concept can be used for modelling non-catalytic gas – solid reactions, other than those involving gasification, using calcination as an example for different particle sizes and over a range of temperatures.

## 2 Experimental

### 2.1 Materials

<table>
<thead>
<tr>
<th>Component</th>
<th>Ca</th>
<th>Fe</th>
<th>Mg</th>
<th>Ni</th>
<th>Al</th>
<th>K</th>
<th>Mn</th>
<th>Si</th>
<th>S</th>
<th>Zr</th>
<th>Sr</th>
<th>Ti</th>
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<tbody>
<tr>
<td>Compostilla</td>
<td>89.70</td>
<td>2.50</td>
<td>0.76</td>
<td>0</td>
<td>0.16</td>
<td>0.46</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.46</td>
<td>0.07</td>
</tr>
<tr>
<td>Purbeck</td>
<td>97.67</td>
<td>0.49</td>
<td>0.61</td>
<td>0</td>
<td>0.21</td>
<td>0.09</td>
<td>0.14</td>
<td>0.65</td>
<td>0.11</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
</tr>
</tbody>
</table>

The gases used in the experiments were N\(_2\) (≥ 99.9 vol%, oxygen ≤ 2 ppmv) and CO\(_2\) (≥ 99.8 vol%). All gas cylinders were supplied by BOC or Air Liquide. Natural, uncrushed silica sand (fraction C, David Ball Group plc., dry), sieved to 355 – 425 μm, was used as a fluidised bed material. The density of the non-porous sand particles was ~ 2690 kg m\(^{-3}\). Two types of limestone particles were used: (i) a Spanish limestone (Compostilla) after 8 cycles of calcination and carbonation, and (ii) a British limestone (Purbeck) after 6 such cycles. The number of cycles was chosen so that the particles become porous and the ultimate CO\(_2\) uptake by the limestones of the current cycle was close to that of the previous cycle. The compositions of the fresh limestones are shown in Table 1. After cycling, the internal pores of the limestone, even at the start in the fully-carbonated state, were macro-pores (> 50nm) so that the calcination would likely occur continuously throughout the entire particle, as shown in Table 2. The cycling of the limestone particles was conducted in a bed of sand fluidised by 15 vol% CO\(_2\) balance N\(_2\) at 1 atm. The limestone particles were calcined at 1173 K for 10 minutes and then carbonated at 923 K for 10 minutes. Here, the temperature at which a partial pressure of CO\(_2\) of 0.15 atm is in thermodynamic
equilibrium with a mixture of CaO and CaCO$_3$ was calculated to be 1053 K [16]. The resulting carbonated particles were cooled in a desiccator and then were sieved from the sand. Two sieve size fractions were used in the experiments for each type of limestone: 710 – 850 µm and 1400 – 1700 µm. These sizes were selected in order to recover the cycled particles effectively from the sand and to compare the theoretical predications across different particle size.

2.2 Fluidised bed experiments

![Schematic diagram of the arrangement for batch experiments in a quartz reactor (i.d. 30 mm).]

Batch experiments were performed in a fluidised bed contained in a quartz reactor, internal diameter 30 mm and length 460 mm, provided with a porous frit (4 mm thick, pore size 100 – 160 µm) as the distributor, situated 110 mm from the base of the reactor. By using pressure taps at the inlet and the outlet, the pressure drop across the distributor and a 20 ml sand bed was measured to be 13 – 15 mbar at experimental conditions. The reactor was externally heated by an electric furnace. The temperature of the bed was measured by a K-type thermocouple (2 mm dia.) inserted into the top, with its tip 20 mm above the distributor. Flowrates of N$_2$ were controlled by a mass flow meter calibrated at 293 K and 1 bar. The off-gas leaving the fluidising bed was sampled at 16.7 mL s$^{-1}$ (STP) through a quartz tube. To prevent elutriated particles and water vapour in the sampled gas entering the analysers, the gas was passed through a glass wool filter and a drying tube filled with CaCl$_2$ in series. The mole fractions of CO$_2$ were measured by a non-dispersive infra-red gas analyser (ABB EL3020). Fig. 1
shows the arrangement of the apparatus. In an experiment, the reactor was filled with 20 ml of silica sand and heated to the desired temperature, \( \text{viz.} \ 1023 - 1173 \text{ K} \). For calcination, the fluidising gas was 100 mol\% \( \text{N}_2 \). The total volumetric flowrate was 80 mL s\(^{-1}\) (STP), giving \( U/U_{mf} \sim 6.3 - 7.9 \), with \( U \) being the superficial velocity at the temperature of the bed and \( U_{mf} \) being the value at incipient fluidisation predicted from the correlation of Wen and Yu [17]. From experimental measurements using different sample masses ranging from 0.1 – 0.5 g at 1173 K, the measurement of reaction rate starts decreased when the sample mass was bigger than 0.3 g due to complications arising from mass transfer between the bubble and the particulate phases. Hence a sample mass of 0.3 g was chose for all experiments. Each experiment was repeated at least 3 times. To ensure complete calcination of the limestones, the calcination experiment is ended 10 seconds after the measured concentration of \( \text{CO}_2 \) of the off-gas returns to zero.

### 2.3 Characterisation of the limestone particles

<table>
<thead>
<tr>
<th>Limestone</th>
<th>BET analysis</th>
<th>Mercury intrusion porosimetry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BET area ( / \text{m}^2 \text{g}^{-1} )</td>
<td>BJH volume ( / \text{cm}^3 \text{g}^{-1} )</td>
</tr>
<tr>
<td>Compostilla, 8 cycles</td>
<td>0.33 ( \times 10^{-3} )</td>
<td>7.2 \times 10^{-3}</td>
</tr>
<tr>
<td>Purbeck, 6 cycles</td>
<td>1.58 ( \times 10^{-3} )</td>
<td>9.9 \times 10^{-3}</td>
</tr>
</tbody>
</table>

*Table 2.* Particle characterisation of fully carbonated limestones.

Table 2 shows the measurements from the Brunauer–Emmett–Teller (BET) analysis (TriStar 3000) and mercury intrusion porosimetry (AutoPore IV 9500), both of which produce a pore size distribution of the particles. It is clear that these cycled particles have substantial pore volume, even in their fully-carbonated state, in contrast to virgin limestone, which has negligible porosity. In the subsequent modelling, the mean pore diameter \( d_{pore} = 4V/A \) from mercury porosimetry was used as the initial pore diameter of the particles, where \( V \) is the total intrusion volume and \( A \) is the total pore area. It should be noted that the limestone particles may have considerable unmeasured surface area and pore volume in the micro-porous range (\( d_{pore} < 2 \text{ nm} \)), as the BET and mercury intrusion analyser were unable to measure pore diameters smaller than 1.7 nm and 3 nm respectively.

Dai *et al.* [12] measured the particle size distribution of 600 – 1000 \( \mu \text{m} \) dia. char particles with the same sub-angular shape to the limestone particles by optical microscopy and showed that the effective particle diameter
\( D(3,2) \times \psi \), where \( D(3,2) \) was the Sauter mean diameter and \( \psi \) is a shape factor, is very close to the geometric mean of the mesh sizes. Therefore in the model, the mean, external particle diameter \( d_p \) was calculated from the geometric mean of the sieves \( d_p = (\text{lower mesh} \times \text{upper mesh})^{0.5} \).

3 Theoretical

The model described in this work assumed that the limestone particle is spherical and is calcined in a bed of silica sand fluidised by a stream of \( \text{N}_2 \). The only reaction occurring is:

\[
\text{CaCO}_3(s) \rightarrow \text{CaO}(s) + \text{CO}_2(g) \quad \Delta H_{298K}^\circ = +178 \text{ kJ mol}^{-1}
\]

The two principal assumptions were:

i) The material and energy balances inside and outside the particle are in pseudo-steady state, so that the gas concentrations, total fluxes, total pressure and temperature have no time dependence. By using the pseudo-steady state assumption here, it means that the time needed to establish an initial steady concentration profile is very small. Without this assumption, the initial concentration would have been zero everywhere. Nevertheless, as justified by Bischoff [18], the profiles generated using this assumption will be achieved very quickly in a gas-solid system. In fact, the thermal diffusivity of the solid at \( 800 \text{ °C} \) is about \( 9 \times 10^{-6} \text{ m}^2/\text{s} \), even for a 2 mm dia. particle, the time constant estimated from \( \text{Radius}^2 / (2 \times \text{thermal diffusivity}) \) is very small \( \sim 0.2 \text{ s} \). Wen [19] also concluded that the pseudo-steady state solution was a good approximation for most of the solid-gas reaction systems except for systems with extremely high pressure and very low solid reactant concentration. However, the balances are affected indirectly by the conversion \( X \) which does have time dependence and affects the physical properties of the particle (e.g. porosity, pore diameter and particle size).

ii) The evolution of the internal morphology of a limestone particle during calcination can be described by an arbitrary \( f(X) \), a function of \( X \) – the local conversion of the maximum available \( \text{CaCO}_3 \) content within the particle after cycling. The reason for such a definition of conversion is that the limestone particle is only partially carbonated and the maximum \( \text{CO}_2 \) uptake decays gradually with the number of cycles. Hence the reaction rate at some point within the solid can be expressed in the form of \( r = g(C_r, T, P) \times f(X) \). It was assumed that \( f(X) \) applies everywhere within a particle and is independent of temperature. The value of \( f(X) \) changes with the local conversion, which will vary with distance from the centre of the particle. The \( f(X) \) can be obtained from a plot of the experimental rate of calcination against conversion obtained from experimental measurements in which the rate is controlled solely by intrinsic chemical kinetics [12,13].
### 3.1 The kinetics of the calcination reaction

The intrinsic rate of reaction per unit of surface area for reaction (I) was given by [14]

$$-q_c = k_c - \bar{k}C_{CO_2} = k_c - \bar{k}p_{CO_2}/RT$$  \hspace{1cm} (1)

where $k_c$ is the rate constant of the calcination reaction (here, in mol m$^{-2}$ s$^{-1}$), $\bar{k}$ is the rate constant of the reverse, carbonation reaction (m s$^{-1}$) and $p_{CO_2}$ is the partial pressure of CO$_2$. At equilibrium, the rate $q_c = 0$, so that

$k_c - \bar{k}p_{CO_2}^{eq}/RT = 0$. Therefore, the ratio of rate constants is:

$$k_c/\bar{k} = p_{CO_2}^{eq}/RT$$  \hspace{1cm} (2)

Substituting Eq. (2) into Eq. (1),

$$-q_c = k_c(1 - p_{CO_2}/p_{CO_2}^{eq})$$  \hspace{1cm} (3)

Here $p_{CO_2}^{eq}$ is the equilibrium partial pressure of CO$_2$ at local conditions. Barin and Platzki [16] gave the following expression for $p_{CO_2}^{eq}$:

$$p_{CO_2}^{eq} = 4.083 \times 10^7 \exp(-20474/T)$$  \hspace{1cm} (4)

where the unit of $T$ is in K. The rate constant $k_c$ was assumed to be an activated quantity, thus

$$k_c = \Lambda k_c \exp(-E_a/RT).$$

Based on the assumption ii), the rate of reaction per unit volume of particle, $Q_c$, is:

$$Q_c = q_cA_{z,0}\rho_{e,0}f(X) = -k_c(1 - p_{CO_2}/p_{CO_2}^{eq})A_{z,0}\rho_{e,0}f(X)$$  \hspace{1cm} (5)

where the parameter $A_{z,0}$ is the initial pore area per unit mass and $\rho_{e,0}$ is the initial bulk density of the particles.

### 3.2 Equations of mass balance

A pseudo-steady mass balance over a spherical shell at radius $r$ gives the flux equations for CO$_2$ and N$_2$:

$$\frac{1}{r^2} \frac{d}{dr}(r^2J_n) = Q_n = -\nu_nQ_c = -\nu_nq_cA_{z,0}\rho_{e,0}f(X) \hspace{1cm} n = 1, 2$$  \hspace{1cm} (6)

where $J_n$ is the total flux (i.e. diffusive flux + advective flux) of species $n$. The parameter $Q_n$ is the net rate of reaction of species $n$, in mol m$^{-3}$ s$^{-1}$, which is positive for a net gain and negative for a net loss in species, and $\nu_n$ is the stoichiometric coefficient of species $n$ in reaction (I). The subscripts 1 and 2 are used to represent CO$_2$ and N$_2$.

A material balance on carbon across a differential element gives the variation of local conversion of CaCO$_3$ with respect to time:

$$\frac{dX}{dt} |_{r} = -M_{CaCO_3} \frac{Q_c}{\rho_{e,0}} = k_cA_{z,0}M_{CaCO_3}(1 - p_{CO_2}/p_{CO_2}^{eq})f(X)$$  \hspace{1cm} (7)

the initial condition of which is:
where the particle centre is $r = 0$ and the particle surface is $r = r_p$. Since $A_{x,0}M_{CaCO_3}$ is the initial pore area per unit volume of particle – a constant, the product of parameters $k_c A_{x,0}M_{CaCO_3}$ can be replaced by a modified rate constant $k'_c$:

$$\frac{dX}{dt}\bigg|_r = k'_c \left(1 - \frac{p_{CO_2}}{p_{CO_2}^{eq}}\right)f(X)$$

(9)

$$k'_c = \Lambda k_c A_{x,0}M_{CaCO_3} \exp(-E_a/RT) = k_0 \exp(-E_a/RT)$$

The activation energy of $k'_c$ is the same as that of $k_c$, but the pre-exponential becomes $k_0 = \Lambda k_c A_{x,0}M_{CaCO_3}$.

### 3.3 Equations of intraparticle mass transfer

A model of multi-component diffusion based on the Stefan-Maxwell equations within a porous medium was needed to describe the intraparticle diffusion rigorously. The two principal flux models for non-equimolar, multi-component mass transfer are the Dusty Gas Model (DGM) [20], and the Mean Pore Transport Model (MPTM) [21–23]. Given that both models are algebraically complicated, Young and Todd [24] developed a new MPTM called the Cylindrical Pore Interpolation Model (CPIM). Comparing all three models, the CPIM has a more rigorous treatment of continuum flow, a clearer interpolation procedure for transitional flow and a more compact form of the working equations which helps to clarify the roles of the governing parameters. Recent studies suggest that the CPIM is well suited to modelling multi-component diffusion in both catalyst pellets [25] and in gasifying char particles [12,13]. For this reason, the CPIM was selected to model intraparticle diffusion in the present work. The governing equations are:

$$\frac{dy_n}{dr} = \frac{\tau^2RT}{\varepsilon P} \sum_{m=1}^{2} \left( \frac{y_n J_m}{D_{A,mn}} - \frac{y_m J_n}{D_{A,nm}} \right) \quad n = 1, 2$$

(10)

$$\frac{dP}{dr} = -\frac{\tau^2 A_D}{\varepsilon} \sum_{n=1}^{2} (\sqrt{R_n} \cdot J_n)$$

(11)

The boundary conditions for the above equations are given at the centre ($r = 0$) and the surface ($r = r_p$) of the particle:

$$r = 0: \quad J_1 = 0 \quad J_2 = 0$$

(12)

$$r = r_p: \quad y_n = y_n^{surface} \quad (n = 1, 2) \quad \& \quad P = P_{surface}$$

(13)

Here, $y_n$ is the mole fraction of species $n$, $\tau^2$ represents the tortuosity factor of the particle from mercury intrusion porosimetry measurements, $\varepsilon$ is the porosity at the local point, which varies with conversion and is discussed later,
and \( M_n \) is the molar mass of gas species \( n \). The parameters \( D_{A,n} \) and \( A_A \) were found by interpolating between the extremes of continuum and Knudsen flow using the equations proposed by Young and Todd [24]:

\[
\frac{1}{D_A,n} = \frac{1}{D_{Kn,n}} + \frac{1}{D_{nm,n}} \quad \frac{1}{A_A} = \frac{1}{A_{Kn}} + \frac{1}{A_{C}}
\]

(14)

where \( D_{B,n} \) is the molecular diffusivity calculated from the Chapman-Enskog theory using the Lennard-Jones \((6-12)\) potential [26]. The error in the predicted binary diffusivities by this method is ~ 7.3% [27]. \( D_{Kn,n} \) is the Knudsen diffusivity and the parameters \( A_K \) and \( A_C \) are the coefficients in the pressure gradient equation in the continuum and Knudsen regime, given by [24,28]

\[
D_{Kn,n} = \frac{2r_{pore}^2}{3} \sqrt{\frac{8RT}{\pi M_n}} \quad A_K = \frac{3}{4r_{pore}^2} \sqrt{\frac{\pi RT}{2}}
\]

(15)

\[
A_C = 8 \mu_{mix} RT \left( \sum_{n=1}^{2} \left( \frac{y_n \sqrt{M_n}}{M_n} \right)^2 \right)
\]

where the viscosity of the gas mixture, \( \mu_{mix} \), was calculated using Chapman-Enskog theory.

The porosity, \( \varepsilon \), changes with the local conversion of \( \text{CaCO}_3 \), \( X \), during reaction and can be derived from the volume balance equation for a thin cylindrical shell inside the particle:

\[
\varepsilon(X) = \varepsilon_0 + X(1 - \varepsilon_0) \left( 1 - \frac{V_{m,\text{CaO}}}{V_{m,\text{CaCO}_3}} \right)
\]

(16)

where \( V_{m,\text{CaO}} \) and \( V_{m,\text{CaCO}_3} \) are the molar volume of the non-porous \( \text{CaO} \) and \( \text{CaCO}_3 \) solids. In terms of the pore diameter, it was assumed that the particle has uniform cylindrical pores of initial diameter \( d_{pore,0} \); the corresponding initial porosity of the particle was \( \varepsilon_0 \). Ignoring the small volume of crossing between pore channels, the local porosity can be estimated from \( \varepsilon(r) \cdot \delta V = \pi d_{pore}^2 \sum_{i=1}^{L} L_i / 4 \), where \( \delta V \) is the volume of a differential element between \( r \) and \( r+dr \) and \( \sum_{i=1}^{L} L_i \) is the sum of the length of the cylindrical pores within the element.

Assuming that the evolution of the pores during reaction occurs only in a radial direction so that the pore diameter changes while the length of the pore remains constant, then:

\[
\varepsilon(r)/\varepsilon_0 = \left( d_{pore}/d_{pore,0} \right)^2
\]

(17)

Substituting Eq. (16) into (17), the pore diameter at some time when the local conversion is \( X \) is:

\[
r_{pore} = r_{pore,0} \sqrt{\frac{\varepsilon_0 + X(1 - \varepsilon_0) \left( 1 - \frac{V_{m,\text{CaO}}}{V_{m,\text{CaCO}_3}} \right)}{\varepsilon_0}}
\]

(18)

### 3.4 Equations of external mass transfer

The particulate phase of the fluidised bed was considered to have a constant local tortuosity and porosity around the limestone particle. It was also assumed that there is no variation of pressure with radial distance.
outside the limestone particle, since the interstitial velocity of fluidising gas ~1.1 m/s (STP) is much larger than
the mass average velocity of gas leaving the surface of a reaction particle ~0.04 m/s (STP) calculated from the
gas flux at the surface. This suggests that there is no tendency to form voids or bubbles around the reacting
particle in the case under consideration and that pressure variations outside the particle can be neglected. The
general Stefan-Maxwell equations [26] were used to model the external mass transfer within a diffusion boundary
layer of thickness δ outside the particle:

\[
\frac{d y_n}{dr} = \frac{r^2_{\text{bed}} RT}{\varepsilon_{\text{bed}} P} \sum_{m=1}^{2} \left( \frac{y_n I_m - y_m I_n}{D_{B,nm}} \right) \quad n = 1, 2 \quad \frac{d P}{dr} = 0
\]  

(19)

Here \( r^2_{\text{bed}} \) is the tortuosity factor of the sand bed. It was experimentally measured to be \( 1.34^2 = 1.80 \) for a packed
bed with 200 µm dia. quartz sand by Zoia and Latrille [29]. Also, \( \varepsilon_{\text{bed}} \) is the porosity of the bed, assumed to be
0.44, the same as the porosity at incipient fluidisation used by Hayhurst and Parmar [30] for a bubbling fluidised
bed of silica sand. The parameter \( D_{B,nm} \) refers to the binary molecular diffusivity of species \( n \) and species \( m \). The
boundary conditions at the particle surface \( (r = r_p) \) and the edge of boundary layer \( (r = r_p + \delta) \) are:

\[
r = r_p: \quad f_1 = f_1^{\text{bulk}}
\]

\[
r = r_p + \delta: \quad y_n = y_n^{\text{bulk}} \quad (n = 1, 2) \quad \& \quad P = P^{\text{bulk}}
\]  

(20)

(21)

3.5 Equations of energy balance

Outside the limestone particle, convective heat transfer in a bubbling fluidised bed involves packets of sand particles coming into contact with the limestone for a short time, then quickly moving away to be replaced by other packets. It was assumed that the heat transfer coefficient between the particulate phase of the fluidised bed and the surface of the particle was large, so that the particle surface is close to the temperature of the bulk. It was also assumed that the radiative contribution and transpiration contributions to heat transfer were small.

Inside the limestone particle, the energy flux, \( E \), is given by [26]

\[
E = \sum_{n=1}^{2} M_n J_n \left( \frac{u_n^2}{2} + \frac{H_n}{M_n} \right) - \lambda_{\text{eff}} \frac{dT}{dr}
\]  

(22)

Jackson [31] showed that the energy balance can be expressed as \( \text{div}(E) = 0 \):
where \( C_{p,n} \) is the molar heat capacity of species \( n \) and \( \lambda_{\text{eff}} \) is the effective thermal conductivity. \( H_n \) is the partial molar enthalpy of species \( n \) at temperature \( T \), and is calculated from standard enthalpy of formation \( H_f^o,n \) by

\[
H_n = H_f^o,n + \int_{298}^{T} C_{p,n} dT.
\]

This equation makes specific allowance for the small change in momentum occurring as a result of the change in mass in the gas phase during the non-catalytic decomposition of the solid. The calculation of the thermal parameters \( C_p \) and \( \lambda \) is discussed in the next section. Finally, \( u_M \) is the mass-averaged velocity of the mixture which is given by \( u_M = \sum_{n=1}^{2} M_n J_n / \sum_{n=1}^{2} \rho_n \). The boundary conditions for the internal energy balance were:

\[
\begin{align*}
  r = 0: & \quad dT/dr = 0 \quad \quad r = r_p: & \quad T = T_{\text{bulk}}
\end{align*}
\]

### 3.6 Calculation of parameters

The initial pore area per unit mass \( A_{g,0} \) was obtained from the BET area measurement of the limestone particles. The initial bulk density \( \rho_{e,0} \), the initial pore diameter \( d_{\text{pore},0} \), the tortuosity factor \( r^2 \) and the initial porosity of the particle \( \varepsilon_0 \) were determined from the mercury porosimetry measurements, as shown in Table 2. Alvarez et al. [32] did reported up to 50% increase in pore diameter of natural limestone particles after 100 cycles. In terms of the particle size, Wu et al. [33] reported only 2 – 7% reduction of particle diameter after 10 calcination – carbonation cycles. Hence in this study the overall particle size was assumed to be constant during calcination. Any change of the solid volume due to the difference in molar volumes of CaCO\(_3\) and CaO was taken only to affect the pore structure parameters e.g. porosity and pore diameter.

The boundary layer thickness \( \delta \) was given by Hayhurst and Parmar [30]:

\[
S_{h_{\text{EMCD}}} = 2 \varepsilon_{mf} \left( 1 + r_p/\delta \right)
\]

\[
S_{h_{\text{EMCD}}} = 2 \varepsilon_{mf} + 0.61 \left( 2 r_p U_p / u_{\text{mix}} \right)^{0.48} \left( u_{\text{mix}} / D_{B,12} \right)^{0.33}
\]

\[
U_p = U_{mf} \left( 1 - \varepsilon_b \right) \left( 1 - 0.5 \pi \ln \left( 1 - 6 \varepsilon_b / \pi \right) \right)
\]

where \( \varepsilon_{mf} = 0.44 \) was the voidage at incipient fluidisation for a bubbling fluidised bed with silica sand used by Hayhurst and Parmar [30]. Also, \( u_{\text{mix}} \) is the kinematic viscosity of the gas mixture calculated using Chapman-Engskog theory and \( D_{B,12} \) the binary molecular diffusivity for CO\(_2\) and N\(_2\). The bubble fraction \( \varepsilon_b \) was given by
\[ \varepsilon_b = \left( U - U_{mf} \right) / U_b = \left( h - h_{mf} \right) / h, \] where \( U_b = \left( U - U_{mf} \right) + 0.71 \left( g d_b \right)^{0.5} \) [34]. Here, \( h_{mf} \) was the bed height at incipient fluidisation, measured to be 0.029 m and \( d_b \) was the mean bubble diameter estimated from the correlation of Darton et al. [35]: \( d_b = 0.54 \left( U - U_{mf} \right)^{0.4} h^{0.8} / (2g^{0.2}) \) with \( h \) being the expanded height of the bed when fluidised at superficial velocity \( U. \) Although the correlation was based on equimolar counter-diffusion (EMCD), it has been shown that it will yield the correct value of \( \delta \) from Eq. (25), even for non-EMCD [36].

The thermal conductivities of the gases were calculated from \( \lambda = C_1 T^{C_2} / (1 + C_3 / T + C_4 / T^2), \) where \( C_1 - C_4 \) are constants [27]. The effective thermal conductivity of the particle was calculated from \( \lambda_{eff} = (1 - \varepsilon) \lambda_{solid} + \varepsilon \lambda_{gases}, \) where \( \lambda_{gases} = \sum_{n=3}^{g} y_n \lambda_n. \) The overall thermal conductivity was largely influenced by that of the solid. The reported thermal conductivity of limestone (CaCO\(_3\)) and lime (CaO) is 2.25 and 0.84 W m\(^{-1}\) K\(^{-1}\) respectively [27]. The exact mole fraction of CaCO\(_3\) and CaO within the particles after cycling was unknown, hence, \( \lambda_{solid} \) was taken to be 1.5 W m\(^{-1}\) K\(^{-1}\). The specific heat capacity of each gas was estimated from \( C_p = E_1 + E_2 T + E_3 / T^2, \) where \( E_1 - E_3 \) are constants from Green and Perry [27].

### 3.7 Numerical solution

The system is described by Eq. (6) to (24). Both the intraparticle and external mass transfer models have five 1\(^{st}\) order ordinary differential equations (ODEs) in the space domain, and hence five boundary conditions are provided. The ODE for conversion is 1\(^{st}\) order in the time domain, hence only one initial condition is required. The energy equation is a 2\(^{nd}\) order ODE, hence two boundary conditions are required for both the internal and external cases.

The main difficulty in solving the system lies in efficient solution of the large system of equations. A numerical algorithm, Orthogonal Collocation on Finite Elements (OCFE) [37] was written in MATLAB to solve the model. Pseudo-steady state was assumed for all the other variables except for the conversion \( X. \) Using the initial condition in Eq. (8), the 1\(^{st}\) order time-dependent ODE of local conversion \( X, \) Eq. (7), was solved. The value of \( X(\eta_r) \) at time \( t + \Delta t \) could be calculated based on \( X(\eta_r) \) and \( dX / dt \) at time \( t. \) The relevant model parameters (e.g. porosity and pore diameter) were updated with the new value of \( X(\eta_r), \) and then the internal and external models were solved for results at time \( t + \Delta t. \) The iterative process was stopped when the overall conversion reached unity.

Since the model predicts the distributions of reaction rate and conversion across the radius of a particle, the overall values of rate and conversion need to be obtained from integration across the particle radius. For a
distribution of $\chi$ (e.g. $Q_c$ and $X$), its volume averaged value can be calculated from $\bar{\chi} = 3 \int_0^1 (\eta_r^2 \chi) d\eta_r$, where $\eta_r$ is the dimensionless radius used inside the particle. The integral was evaluated numerically.

4 Results

Fig. 3 shows the raw measurements of CO$_2$ mole fraction in the off-gas during the calcination of cycled Compostilla (plot a) and Purbeck limestone particles (plot b) in a bed of silica sand fluidised by pure N$_2$. The figure suggests that the calcination of Compostilla at 1173 K was completed after ~50 s while Purbeck at 1173 K finished calcining after 35 s. The peak concentration of CO$_2$ from Compostilla was about half that of Purbeck, hence the reactivity of Compostilla was significantly less than that of Purbeck. The equilibrium partial pressure of CO$_2$ at 1173 K is about 1.087 bar, so the concentration driving force $p_{\text{CO}_2}/p_{\text{CO}_2}^{\text{eq}} < 5\%$. Hence this confirms that the fluidised bed was close to a differential reactor, and it is reasonable to use 0% CO$_2$ as the bulk concentration in the model.

![Fig. 3](image)

**Fig. 3.** Measurements of CO$_2$ mole fraction during calcination of cycled limestones at atmospheric pressure: a) Compostilla 8 cycles 0.71 – 0.85 mm at 1173 K; b) Purbeck 6 cycles 0.71 – 0.85 mm at 1173 K.

The overall rate of production of CO$_2$ from calcination in s$^{-1}$ is

$$\frac{dX}{dt} = \left(\dot{N}_{\text{out}Y_{\text{CO}_2,\text{out}}} - \dot{N}_{\text{in}Y_{\text{CO}_2,\text{in}}}/\int(\dot{N}_{\text{out}Y_{\text{CO}_2,\text{out}}} - \dot{N}_{\text{in}Y_{\text{CO}_2,\text{in}}})dt\right).$$

The parameters $\dot{N}_{\text{out}}$ and $\dot{N}_{\text{in}}$ are the total molar flows leaving and entering the reactor at the exit and entrance conditions, where $\dot{N}_{\text{out}}(1 - y_{\text{CO}_2,\text{out}}) = \dot{N}_{\text{in}}(1 - y_{\text{CO}_2,\text{in}})$ from the mass balance of nitrogen. The raw measurements were deconvoluted to account for the mixing and delay in the sampling line using the method described by Saucedo et al. [13].

Table 3. Kinetic parameters of the calcination of 0.71 – 0.85 mm dia. limestone particles.

<table>
<thead>
<tr>
<th>Limestone particles</th>
<th>$k_0$ / s$^{-1}$</th>
<th>$E_a$ / kJ mol$^{-1}$</th>
</tr>
</thead>
</table>
The Arrhenius coefficients and activation energies of the kinetic parameter $k'_c$ in Eq. (9), shown in Table 3, were determined from the initial rate extrapolated from the experimental measurements, as shown in Fig. 4. At the start of reaction, the particle conversion is 0 and $f(X) = 1$, and Eq. (9) can be rearranged

$$\ln(dX/dt |_{t=0}) = -E_a/RT + \ln(k_0) + \ln\left(1 - p_{CO_2}/p_{CO_2}^{eq}\right)$$

If the value of $p_{CO_2}/p_{CO_2}^{eq}$ is much smaller than 1, which is usually the case if the reaction is controlled by intrinsic kinetics, then

$$\ln(dX/dt |_{t=0}) = \ln(k'_c) = -E_a/RT + \ln(k_0)$$

A plot of $\ln(dX/dt |_{t=0})$ vs. $1/T$ should therefore yield the activation energy $E_a$ and the Arrhenius coefficient $k_o$. The linearity of Fig. 5 and Fig. 8 in section 4.1 and 4.2 respectively show that the calcination of the limestone was indeed controlled by chemical kinetics at low temperature, hence confirming the use of Eq. (29) for the determination of the kinetic parameters. The errors associated with the kinetic parameters mainly come from extrapolating the initial rates from measurements, and using a limited number of measurements for linear regression analysis. The 95% confidence intervals of the kinetic parameters are shown in Fig. 5 and Fig. 8.
4.1 Calcination of Compostilla limestone particles

**Fig. 5.** Determining the kinetic parameters of Compostilla limestone particles (8 cycles). The measurements were obtained from the calcination of the limestone particles at 1023 K (0.71 – 0.85 mm only), 1073 K, 1123 K, 1148 K and 1173 K. The values of the kinetic parameters of the rate constant are shown with 95% confidence interval (C.I.).

Particles of Compostilla limestone with diameters of 0.71 – 0.85 mm and 1.40 – 1.70 mm were calcined at 1023 K (0.71 – 0.85 mm only), 1073 K, 1123 K, 1148 K and 1173 K. Using the initial rate extrapolated to zero conversion, **Fig. 5** shows that the plot of Ln(dX/dt) vs. 1/T of each particle forms straight lines. The values of the kinetic parameters in Eq. (9) were determined from a linear regression analysis, yielding an activation energy of $E_a = 175 \pm 12$ kJ/mol and $k_0 = 1.72 \times 10^7$ s$^{-1}$ for 0.71 – 0.85 mm dia. particle. For 1.40 – 1.70 mm dia. particle, the reaction rates were lower than those of 0.71 – 0.85 mm dia. particle and the apparent activation energy was 166 ± 14 kJ/mol, representing a 9 kJ/mol reduction that is within an error band of ±12 kJ/mol. It is expected that the gradient of a best fit line would approach a half of its intrinsic value if the reaction rate were significantly limited by intraparticle mass transfer [1]. However, this is not observed in **Fig. 5**. The values of the activation energy indicate that the reactions could not have been in mass transfer limited regime. Hence, it can be concluded that (i) the calcination of 0.71 – 0.85 mm dia. particles was controlled by intrinsic chemical kinetics; (ii) the reactions of 1.40 – 1.70 mm dia. particles were possibly affected by intraparticle mass transfer but not severely so.
Fig. 6. Determining the function $f(X)$ from the plot of normalised rate vs. conversion measurement of $0.71 - 0.85$ mm dia. Compostilla (8 cycles) at 1073 K.

The form of $f(X)$ needs to be determined from experimental measurements of calcination rate vs. conversion conducted under conditions where the reaction is controlled by intrinsic chemical kinetics. Owing to the low rate of reaction at 1023 K, the percentage fluctuation caused by random noise in the measurements of CO$_2$ concentration was very large. The resulting $f(X)$ was not a smooth function, as expected. However, since experiments at both 1023 K and 1073 K appear to be in the regime of chemical kinetic control, as shown in Fig. 5, the $f(X)$ function was determined from the measurements at 1073 K instead. Fig. 6 shows the plot of $f(X)$, a 6th order polynomial of $X$, determined from the normalised rate vs. conversion measurements of Compostilla limestone of sieve diameter $0.71 - 0.85$ mm, calcined at 1073 K. The figure shows a peak higher than 1 at about 20% conversion. This is due to the fact that part of the surface area that is previously unreachable become accessible when the solid volume reduces during the initial stage of calcination. This increase in surface area only occurs at low conversion when the coalescence of pores is insignificant. Using this $f(X)$, the model was able to fit well the experimental results at 1023 K, as seen in Fig. 7(a). This strongly suggests that the $f(X)$ was not merely a fit valid for one particular experimental condition.
Fig. 7. Comparison of model results (lines) with experimental measurements (points) of the calcination of Compostilla limestone particles (8 cycles) by 100% N₂: a) 0.71 – 0.85 mm; b) 1.40 – 1.70 mm. The \( f(X) \) was determined from the rate vs. conversion measurements of 0.71 – 0.85 mm particles at 1073 K, and was applied to all cases.

Further comparisons between model predictions and experimental measurements for Compostilla limestone of 0.71 – 0.85 mm dia. and 1.40 – 1.70 mm dia. are shown in Fig. 7, with generally good agreement being seen between experiment and theory. However, the experimental measurements for the 0.71 – 0.85 mm dia. particle at 1173 K were almost identical to those at 1148 K, which indicates either a severe limitation by external mass transfer or experimental error arising from the rapidity of the reaction and the problem in correcting for mixing in the sampling line. Fig. 5 shows that even for the larger particles at higher temperature, the rate of reaction was not limited by mass transfer as the gradients of the two measurements are almost the same. Therefore it can be concluded that the unexpected behaviour of the rate of 0.71 – 0.85 mm dia. at 1173 K is due to error.

4.2 Calcination of Purbeck limestone particles

Fig. 8. Determining the kinetic parameters of Purbeck limestone particles (6 cycles). The measurements are from calcination of 0.71 – 0.85 mm and 1.40 – 1.70 mm particles at 1023 K, 1073 K, 1098 K, 1123 K, 1148 K and
1173 K. The gradient of the linear regression line for both particles reduced by ~ 50% at 1098 – 1173 K. The values of the kinetic parameters are shown with 95% confidence interval (C.I.).

Experiments with Purbeck limestone were performed using 0.71 – 0.85 mm and 1.40 – 1.70 mm dia. particles at 1023 K, 1073 K, 1098 K, 1123 K, 1148 K and 1173 K. The same kinetic analysis was performed on the experimental measurements and the results are shown in Fig. 8. A linear regression line of the plot of 
\[ \ln(dx/dt) \] vs. \(1/T\) gives an activation energy \(E_a = 186 \pm 5 \text{ kJ/mol}\) and the rate constant \(k_0 = 6.50 \times 10^7 \text{ s}^{-1}\). At 1098 – 1173 K, the gradient of the regression lines of measurements, thus \(-E_a/R\), is reduced by about half at \(T > 1098\) K. Hence, this figure suggests that the transition of the reaction regime from chemical kinetic control to mass transfer control starts at ~ 1098 K. Fig. 8 also shows that the rates of reaction of the 1.40 – 1.70 mm dia. particles are lower than those of the 0.71 – 0.85 mm dia. particles; the linear regression lines of the 1023 – 1098 K measurements show a 21 kJ/mol decline, larger than the \(\pm 5 \text{ kJ/mol}\) error, in the apparent activation energy, probably owing to a growing influence of the mass transfer limitation for larger particles. At \(T > 1098\) K, the slope of the points of 1.40 – 1.70 mm dia. particle is almost the same as that of 0.71 – 0.85 mm particle, suggesting that the reaction becomes limited by mass transfer. Furthermore, for the 1.40 – 1.70 mm dia. particles the transition of the reaction regime occurs at a temperature lower than that of the 0.71 – 0.85 mm dia. particle, a consequence of the increased mass transfer limitation in larger particles.

Fig. 9 compares the rates of conversion vs. time from experimental measurements (points) with the theory (line) for Purbeck limestone calcined in 100% N\(_2\) at 1023 – 1173 K. Interestingly, the \(f(X)\) determined previously from the measurements on Compostilla limestone of 0.71 – 0.85 mm dia. at 1073 K, shown in Fig. 6, was successfully applied here for both size fractions of Purbeck limestone. The result shows that the model fits perfectly with the experimental measurements even for measurements at 1173 K.
Given the results, it can be concluded that using a constant $f(X)$ across different temperatures gives a satisfactory agreement between the model and the measurements for both Compostilla and Purbeck limestone. The fact that the $f(X)$ obtained from measurements of Compostilla could be successfully applied to the modelling of Purbeck suggests that the two limestone particles experienced similar changes of internal morphology during calcination. One reason that could explain this is that both particles had been periodically cycled several times before the final calcination reaction, which could have reduced the variations in pore structures thus making the two types of limestone particles more similar in terms of internal morphology. In addition, after a number of calcination – carbonation cycles, the reactivity of the particles approaches an asymptotic value. It might also be the case that the internal pore structure had developed into an “asymptotic” stage, where the original variations in pore structures between the two limestones had become slight on cycling.

5 Discussion

The above research is concerned with limestone which has been successively calcined and carbonated several times. Experimentally, the observed activation energies for the calcination of cycled, carbonated material were reasonably close to values in the literature, lying between 160 and 210 kJ/mol [14,15,38–40] for the calcination of virgin limestones, being $175 \pm 12$ kJ/mol and $186 \pm 5$ kJ/mol, respectively, for the Compostilla and Purbeck. These values, being close to the standard enthalpy of calcination, $+178$ kJ/mol, suggest that the activation energy for the reverse, carbonation reaction is small, being $\sim -3$ kJ/mol for Compostilla and $\sim +8$ kJ/mol for Purbeck. Thus, the intrinsic rate of carbonation changes little with temperature in the range of the temperature studied in the paper. Zawadzki and Bretsznajder [41] found that the rate of carbonation varies linearly with the difference between the partial pressure of CO$_2$ and its equilibrium value at 328 – 368°C, which suggested that the rate constant was the same for all temperature thus a zero activation energy of the carbonation rate. Nitsch [42] also concluded that the rate of carbonation has an activation energy close to zero as the rate versus partial pressure difference gave a single linear line for measurements at 800 – 850°C. The same conclusions were also reached by Bhatia and Perlmutter [43] and Dennis and Hayhurst [14] for carbonation experiments at 823 – 998 K and 1073 – 1248 K respectively.

Comparing the reaction rates of both limestones in Fig. 5 and Fig. 8, it can be seen that the reactivity of Compostilla is slightly lower than that of Purbeck for 0.71 – 0.85 mm dia. particles. In addition, Table 2 shows that the mean pore diameter of Purbeck limestones is only $\sim1/3$ of that of Compostilla limestones. With higher
reactivity and smaller pore diameter, Purbeck limestone is indeed expected to experience more significant effects of intraparticle mass transfer on observed rate of reaction.

An interesting result from this study was that the \( f(X) \) function determined from the measurements on the Compostilla limestone has been applied successfully in modelling the conversion of the Purbeck limestone. This implies that the evolution of the pore structure of both limestone particles are similar during calcination. One hypothetical reason for this observation is related to the cycling process of the limestones, where the change of pore structure become more stable as number of cycles increases. In fact, a study of the sulphation rate of cycled lime particles showed that different limestones followed a very similar conversion vs. time evolution after 50 cycles [44], which indicates that the cycling process does affect how the pore structure evolves with conversion.

6 Conclusions

It has been proposed that a simple arbitrary function \( f(X) \), determined from experimental measurements of rate vs. conversion in the kinetically-controlled regime, could be used in place of mathematical pore models to describe the evolution of pore structure during a reaction that is influenced by intra-particle gas mass transfer. A model has been constructed using the Cylindrical Pore Interpolation Model for intraparticle mass transfer, first order rate equations of calcination, the Stefan-Maxwell equations for external mass transfer and the equations of energy. The model was solved numerically by orthogonal collocation on finite elements in MATLAB. The predicted results were compared with experimental measurements conducted using two size fractions of Compostilla (after 8 cycles) and Purbeck (after 6 cycles) limestones.

The results have shown that for the calcination of limestones, the empirically-determined \( f(X) \) can be successfully applied to predicting the conversion of particles of various sizes across different temperature. In addition, it was found that the \( f(X) \) determined from Compostilla limestones was successful in predicting the conversion of Purbeck limestones, which indicated that the two limestones had similar evolution of pore structure during calcination. This observation was attributed to the hypothesis that the calcination – carbonation cycling process might have significantly reduced the difference in the pore structures of the limestone particles and made them more homogenous.

The significance of this research is that the \( f(X) \) concept presents a simple solution in modelling the evolution of pore structures during reactions of particles. Instead of using complicated mathematical pore models, one could determine the \( f(X) \) from the experiments used for kinetic studies. This idea could be further applied to many other gas-solid reactions that involve change of pore structures during reactions. One needs to be aware of
the influence of multiple types of active sites which could lead to incorrect predictions. However, multiple sites are also not reflected in most published pore models.

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References


Table 1. Composition of the fresh limestones in wt%.

<table>
<thead>
<tr>
<th>Component</th>
<th>Ca</th>
<th>Fe</th>
<th>Mg</th>
<th>Ni</th>
<th>Al</th>
<th>K</th>
<th>Mn</th>
<th>Si</th>
<th>S</th>
<th>Zr</th>
<th>Sr</th>
<th>Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compostilla</td>
<td>89.70</td>
<td>2.50</td>
<td>0.76</td>
<td>0.16</td>
<td>0.46</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.37</td>
</tr>
<tr>
<td>Purbeck</td>
<td>97.67</td>
<td>0.49</td>
<td>0.61</td>
<td>0.21</td>
<td>0.09</td>
<td>0.14</td>
<td>0.65</td>
<td>0.11</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Particle characterisation of fully carbonated limestones.

<table>
<thead>
<tr>
<th>Limestone</th>
<th>BET analysis</th>
<th>Mercury intrusion porosimetry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BET area / m² g⁻¹</td>
<td>BJH volume / cm³ g⁻¹</td>
</tr>
<tr>
<td>Compostilla, 8 cycles</td>
<td>0.33 7.2 × 10⁻³</td>
<td>72</td>
</tr>
<tr>
<td>Purbeck, 6 cycles</td>
<td>1.58 9.9 × 10⁻³</td>
<td>21</td>
</tr>
</tbody>
</table>
Figure 1
Figure 3

(a) 

(b)
Overall conversion $X$

Extrapolated initial rate

$dX/dt \text{s}^{-1}$

Overall conversion $X$
$E_a = 175$ kJ/mol (95% C.I.: 163, 186)

$k_0 = \exp(16.66)$ s$^{-1}$ (95% C.I.: $\exp(15.40)$, $\exp(17.93)$)

$E_{app} = 166$ kJ/mol (95% C.I.: 152, 179)
Figure 6
Figure 7

Overall Conversion $X$ from 1073 K

Time / s

1023K Model
1023K Expt.
1073K Model
1073K Expt.
1123K Model
1123K Expt.
1148K Model
1148K Expt.
1173K Model
1173K Expt.
$E_a = 186$ kJ/mol (95% C.I.: 181, 191)

$k_0 = \exp(17.99)$ s$^{-1}$ (95% C.I.: $\exp(17.44)$, $\exp(18.54)$)

$E_{app} = 165$ kJ/mol (95% C.I.: 140, 191)
Figure 9

(a) 1023K Model vs 1023K Expt.
(b) 1073K Model vs 1073K Expt.
(c) 1098K Model vs 1098K Expt.
(d) 1123K Model vs 1123K Expt.
(e) 1148K Model vs 1148K Expt.
(f) 1173K Model vs 1173K Expt.

Overall Conversion $\alpha$ vs Time / s
Figure 9 Supplementary MATLAB .fig files
Click here to download Supplementary MATLAB .fig files: FIG9.fig