Minimum Variance Beamformers for Coherent Plane-Wave Compounding

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ABSTRACT

In this paper we present and analyse a technique for applying minimum variance distortionless response (MVDR) beamforming to a coherent plane-wave compounding (CPWC) acquisition system. In the past, this has been done using a spatial smoothing approach that reduces the effective size of the receive aperture and degrades the image resolution. In this paper, we apply the MVDR algorithms in a novel way to the acquired data from the individual transducer elements, before any summation or other compounding. This enables us to propose a new approach for estimation of the covariance matrix that decorrelates the coherence among the components at all the different acquisition angles. This results in a new approach to receive beamforming for CPWC acquisition. The new beamformer is demonstrated on imaging data acquired with a research scanner. We find the new beamformer offers substantial improvements over the DAS method. It also significantly outperforms the previously published MVDR/CPWC beamformer on phantom studies where the signal from the main target is dominated by noise and interference. These improvements motivate further study in this new approach for enhancing image quality.

Keywords: Coherent plane-wave compounding, minimum variance, adaptive beamforming, acoustic reciprocity, spatial correlation, image quality

1. INTRODUCTION

In ultrasound imaging, a broad or unfocused beam has been used to accelerate image acquisition at the expense of resolution. One of the first systems employing this approach is the explososcan, developed by Shattuck et al. In that scanner, the transmit beam was slightly defocused so that four scanlines were generated within its boundaries. Consequently, the data acquisition rate was increased by a factor of four. The implementation was demonstrated on an in vivo study and extended to achieve volumetric imaging. In another approach for achieving a high frame-rate, Lu and Greenleaf proposed the use of a non-diffracting beam. Based on this work, Lu later developed a theory in which plane-wave transmission could be used with limited-diffraction array beam weightings applied on receive. This was developed to produce a spatial Fourier transform of the scanned object which also could be used for 3-D reconstructions.

With the developments of hardware and computational technology, imaging methods have been extended to improve the resolution of plane wave approaches. Cheng and Lu upgraded their previous work using multiple plane-waves tilted in different angles. Images generated from individual transmissions were then compounded incoherently to reduce the speckle size. Later, Montaldo et al. used the same data acquisition but compounded the echo signals coherently in the radio-frequency (RF) domain. The image resolution generated was improved and uniform over the entire imaging region. The coherent plane-wave compounding (CPWC) part of the algorithm, however, was still performed using a classical delay-and-sum (DAS) algorithm that compromised the image quality.

Being a prominent method in array signal processing, the minimum variance distortionless response (MVDR) beamformer has been investigated frequently, as an alternative to DAS, as a way of enhancing image resolution. The method was first developed by Capon to detect the origin of seismic signals. It combines data collected from the sensor array in a manner that maintains unity gain in a desired direction while minimizing noise and interference from other directions. When applied to ultrasound imaging, MVDR was found to narrow the
mainlobe and reduce the sidelobes of the pulse-echo beam. It results in enhancements to both spatial and contrast resolution compared to the DAS beamformer.

An advantage of the MVDR is that its weighting coefficients can be calculated directly by inverting the data covariance matrix. There is, however, a practical issue involving the estimation of this matrix when only one sample of the data vector, or snapshot, is available. Beamformer performance is limited by errors in the estimation of the covariance matrix. If these errors are too large, MVDR-based algorithms can quickly degrade the image quality compared to that obtained with the DAS. To enhance the robustness of this process, the data covariance matrix is usually estimated using the spatial smoothing approximation. This method divides the data vector into smaller overlapped subarrays so that several snapshots can be formed across the aperture. To avoid ill-conditioning, the number of snapshots should be greater than or equal to the matrix size. The method reduces the effective aperture which degrades the image resolution. This prevents MVDR from being used extensively despite its potential benefits.

The MVDR beamformer was first applied to the CPWC by Austeng et al.13 In that work, they used the MVDR to combine low-resolution images, generated from individual plane-wave transmissions. The MVDR beamformer was calculated using the spatial smoothing approximation and demonstrated on data simulated using the Field II program.14,15 They found improvements from MVDR over results from CPWC alone. In this paper, we observe that in previous algorithms, the incoming data to the MVDR covariance calculation has already been subject to a delay-and-sum process. By basing our algorithm on the raw acquired data, we show that it is possible to develop a new approach to estimate the covariance matrix without using the spatial smoothing approximation in the compounding process. Instead of varying the size of the vector, we generate the snapshots for matrix estimation by using different combinations of received signals. This results in a new beamformer that preserves the effective size of the receive aperture. Through demonstrations on a series of imaging datasets,16 we show the new MVDR beamformer outperforms the DAS method in terms of spatial and contrast resolution. It also shows improvements over the previously proposed MVDR/CPWC beamformer on phantom studies.

The rest of the paper is organized as follows. In Section 2, we summarize the CPWC and show how it can be combined with a MVDR beamformer using spatial smoothing.13 We then propose a new method of estimating the data covariance matrix. In Section 3, we demonstrate the new beamformer on imaging data. The results are discussed and compared to the other methods based on the spatial and contrast resolutions. Finally, we summarize the work in Section 4.

2. METHODS

2.1 Coherent Plane-Wave Compounding

Similar to the synthetic aperture imaging and pixel-based beamforming,17,18 the CPWC collects data for superposition using time delay calculations. Data at an imaging point \( P(x, z) \) receive contributions from multiple transmits. While the receive time delay is based on the geometrical distance from \( P \) to the corresponding element, which is the same in all transmissions, the transmit time delay depends on the tilted angle of the transmit beam. With steering angle \( \alpha \) (see Fig. 1(a)), the transmit time delay is given by

\[
\tau^t_p = \frac{z \cos \alpha + x \sin \alpha}{c},
\]

where \( c \) is the sound-speed \((c = 1540 \text{ m/s})\). Without any steering (see Fig. 1(b)), the calculation is simplified to \( \tau^t_p = z/c \). By combining this with the receive time-delay, we are able to extract the echo signal from the received waveform for data superposition. We generate the image from \( M \) firing angles and use an \( N \)-element array to receive the data. The collected signals at time instant \( n \) for each pixel can be arranged in a 2-D matrix \( \mathbf{X}(n) \), given by

\[
\mathbf{X}(n) = \begin{bmatrix}
x_{1,1}(n) & x_{1,2}(n) & \cdots & x_{1,N}(n) \\
x_{2,1}(n) & x_{2,2}(n) & \cdots & x_{2,N}(n) \\
\vdots & \vdots & \ddots & \vdots \\
x_{M,1}(n) & x_{M,2}(n) & \cdots & x_{M,N}(n)
\end{bmatrix},
\]

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Figure 1. Diagrams for time-delay calculations with plane-wave transmission: (a) with steered angle $\alpha$ and (b) without any steering.

where $x_{ij}(n)$ is the signal acquired with firing angle $i$ and received on element $j$. By averaging all elements of the matrix, we obtain the compounded data generated from CPWC. In this paper, we refer to this as the delay-and-sum (DAS) beamformer to differentiate it from other methods described in the following sections.

2.2 Transmit MVDR Beamformer

The first MVDR beamformer applied to CPWC was proposed by Austeng et. al.\textsuperscript{13} It was used to weight the coherent compounding among low-resolution images generated within individual transmits. Since each low-resolution image is generated by using the DAS, this strategy is equivalent to applying the MVDR to vector data $z(n)$ that contains the summed signals over all rows of $X(n)$. In particular, $z(n) = [z_1(n), z_2(n), \ldots, z_M(n)]^T$, where $z_i(n)$ is given by

$$z_i(n) = \sum_{j=1}^{N} x_{i,j}(n) \text{ for } i = 1, M.$$  \hspace{1cm} (3)

The beamforming weight vector (for narrow-band signals) is calculated by

$$w = \frac{R_{zz}^{-1}a}{a^HR_{zz}^{-1}a},$$  \hspace{1cm} (4)

where $a$ is the vector of ones, $R_{zz}$ is the covariance matrix of $z(n)$, and $(\cdot)^H$ stands for the Hermitian transpose.

Matrix $R_{zz}$ can be estimated using the spatial smoothing and diagonal loading approximations.\textsuperscript{11} First, the spatial smoothing divides vector $z(n)$ into $N - L + 1$ overlapping subarrays of dimension $L$. By using the subarray averaging, $R_{zz}$ is calculated from

$$\hat{R}_{zz} = \frac{1}{N - L + 1} \sum_{l=1}^{N-L+1} z_l(n) z_l^H(n) + \epsilon I,$$  \hspace{1cm} (5)

where $z_l(n) = [z_l(n), z_{l+1}(n), \ldots, z_{l+L-1}(n)]^T$, and $\epsilon$ is the diagonal loading parameter.

The spatial smoothing method, however, reduces the dimension of the weight vector $w$ from $N$ to $L$. The data vector $z(n)$, therefore, needs to be modified to have the same dimensions as the averaged subarray $z(n)$ for calculating beamformed output. Vector $\tilde{z}(n)$ is given as

$$\tilde{z}(n) = \frac{1}{N - L + 1} \sum_{l=1}^{N-L+1} z_l(n).$$  \hspace{1cm} (6)
The beamformed signal is calculated as
\begin{equation}
    y(n) = w^H z(n) .
\end{equation}

For \( \hat{R}_{zz} \) to be nonsingular, the number of snapshots \( N - L + 1 \) should be greater than or equal to the matrix size \( L \), or \( L \leq (N + 1) / 2 \). Decreasing the subarray length increases the non-singularity of the estimated matrix but degrades the image resolution. Since the MVDR beamformer is applied to data from individual transmits, we name it the transmit MVDR (Tx-MVDR). In the next section, we propose a new beamformer without using the spatial smoothing approximation in the compounding process.

### 2.3 Subarray-Snapshot-based MVDR Beamformer

The MVDR beamformer is usually applied directly to echo signals received on transducer elements. In our study, however, each component of \( z(n) \) is a combination of the received signals \( x_{ij}(n) \). Thus, we propose a new method to generate the snapshots using different combinations of this data. Following the spatial smoothing approximation, we first divide the receive aperture in into \( M \) overlapping segments, each has a dimension of \( N - M + 1 \). Note that this \( M \) is the same as the number of different firing angles. Signals received on one segment are superposed to form a snapshot for \( z(n) \). In particular, we have a set of \( M \) snapshots \( u_k(n) \) for \( k = \frac{1}{M} \), where \( u_k(n) = [u_{1,k}(n), u_{2,k}(n), ..., u_{M,k}(n)]^T \) and
\begin{equation}
    u_{i,k}(n) = \sum_{j=k}^{N-M+k} x_{i,j}(n) .
\end{equation}

The covariance matrix estimation is thus given by
\begin{equation}
    \hat{R}_{zz,\text{SAS}} = \frac{1}{M} \sum_{k=1}^{M} u_k(n) u_k^H(n) .
\end{equation}

This is combined with diagonal loading to increase robustness. That is, the covariance matrix is calculated using \( \hat{R}_{zz,\text{SAS}} + \epsilon I \). If this is used to replace \( \hat{R}_{zz} \) in Eq. (4), we can calculate the weight coefficients for the new beamformer.

Since the matrix is calculated with the full size of the data vector, there is no need to modify \( z(n) \) to form the subarray average vector \( \bar{z}(n) \), which could provide some advantage in image resolution. Since the method uses superposed data on subarrays to generate snapshots, we name it the Sub-Array-Snapshot-based MVDR beamformer (SAS-MVDR). In the next section, we evaluate it on imaging data along with the DAS and Tx-MVDR beamformers.

### 3. RESULTS AND DISCUSSIONS

#### 3.1 Imaging Data and Evaluation

In this study, we demonstrate the beamformers on imaging data provided by the Plane-wave Imaging Challenge in Medical Ultrasound (PICMUS). The datasets were recorded by using a Verasonics Vantage 256 research scanner and L11 probe (Verasonics Inc., Redmond, WA). The probe is 128-element linear array \((N = 128)\), used to received backscattered signals from 75 plane-wave transmissions \((M = 75)\). The plane-wave is steered in firing angles evenly distributed from \(-16^\circ\) to \(16^\circ\). All beamformers are calculated from radio-frequency (RF) data to generate envelope images with a pixel size of 0.0739 mm x 0.0986 mm.

Beamformed images are generated using all the data from 75 firing angles. For the Tx-MVDR beamformer, we set the subarray length \( L \) equal to a half of the data vector dimension. The diagonal loading parameter is selected using \( \epsilon = 5\% \Delta \) where \( \Delta = \text{Tr}(\hat{R})/L \) and \( \text{Tr}(\hat{R}) \) is the trace of the estimated covariance matrix. In the SAS-MVDR beamformer, we set \( \epsilon = 0.5\Delta \) for the point-target simulation and \( \epsilon = 5\Delta \) for other datasets. The DAS images are generated with an F-number of 1.75, while those obtained with the Tx-MVDR and SAS-MVDR are generated with an F-number of 1.0.
Each beamformer is evaluated based on the spatial and contrast resolutions of the generated images. The spatial resolution is quantified using the responses to individual scatterers. Because all MVDR beamformers are developed based on the assumption of narrow-band signals, they mainly improve the lateral resolution. Thus, we are interested in the full width at half maximum (FWHM) of these responses in this direction only. The narrower the lateral FWHM the better the beamformer performance.

The contrast resolution is measured using the contrast ratio (CR) between a lesion and the background, given by

$$CR = \frac{I_{\text{out}} - I_{\text{in}}}{\sqrt{I_{\text{out}}^2 + I_{\text{in}}^2}},$$

where $I_{\text{in}}$ and $I_{\text{out}}$ are the mean intensities (in decibels) measured inside and outside the lesion, respectively. CR has a value of 1 for perfect contrast, and a value of 0 for no contrast between the lesion and background. The background kernel is selected as a circular ring enclosing the lesion, with an area that is the same as that of the lesion. This helps minimise the effects of variations in the attenuation and diffraction of the ultrasound.

### 3.2 Simulation

![Simulated images of 12 point-targets generated with different beamformers: (a) DAS, (b) Tx-MVDR, and (c) SAS-MVDR. All images are log-compressed and displayed with a dynamic range of 60 dB.](http://proceedings.spiedigitallibrary.org/)

We first evaluate the beamformers on data from the point-target simulations. The data are generated by the Field II program\textsuperscript{14, 15} with simulation parameters based on the Verasonics Vantage system described above. The imaging regions contain 12 point-targets, eight of which are located on the centreline. They are ranged from 10 mm to 45 mm with a 5 mm separation. There are also two sets of 3 targets at depths of 20 mm and 40 mm. At each depth, the points are evenly distributed from $-5$ mm to 5 mm in the lateral direction.

Figures 2(a)–(c) show the images obtained with DAS, Tx-MVDR, and SAS-MVDR beamformers, respectively. In the figure, the images generated by the two MVDR-based algorithms have all targets displayed at much finer resolution than the DAS image. In terms of point resolutions, the two MVDR beamformers are equal in performance. Their differences only can be observed on the lateral beam profiles, plotted in Figs. 3(a)–(d). In the figure, we show the responses of each beamformer to the central points at depths 15 mm, 25 mm, 35 mm, and 45 mm, respectively. The SAS-MVDR responses are shown to have narrowest main-lobes, however, they have side-lobes that are slightly higher than those generated with the Tx-MVDR beamformer. Meanwhile, the DAS has the widest main-lobes while its side-lobes are also much higher than the others. For each beamformers, we measure the FWHM of the responses to all 12 point-targets and take the average of them. The averaged FWHMs measured for the DAS, Tx-MVDR, and SAS-MVDR beamformer are of 0.53 mm, 0.25 mm, and 0.20 mm, respectively.
Figure 3. Beam profiles plotted at different depths: (a) 15 mm, (b) 25 mm, (c) 35 mm, and (d) 45 mm. They are generated from DAS, Tx-MVDR, and SAS-MVDR beamformers. The legend in plot (a) applies to all other plots.

To evaluate the imaging contrast obtained with each beamformer, we apply them to data simulated with nine anechoic lesions. The lesions are evenly distributed at depths from 17 mm to 43 mm. In the lateral direction, they are positioned at −12 mm, 0 mm, and 12 mm. Images obtained with the DAS, Tx-MVDR, and SAS-MVDR are respectively shown in Figs. 4(a)–(c). In the figure, the DAS image shows the lesions with the poorest contrast while the contrast in the SAS-MVDR image is the highest. The improvements are shown clearly in the lesions of the first row. While the lesions on the DAS image are generated with some blurring artefacts, there is almost uniformly and anechoic contrast inside the three lesions on the SAS-MVDR image. In this evaluation, the contrast of Tx-MVDR is higher than the DAS but a little lower than than achieved by the SAS-MVDR.

We use the CR defined in Eq. (10) to quantify the contrast generated on each image. For DAS, the ratio ranges from 0.75 to 0.92 for the nine lesions. It is lowest for the middle lesion of the first row and highest for the middle lesion of the third row. The ratio measured for Tx-MVDR ranges from 0.86 to 0.95. For the SAS-MVDR, it is from 0.89 to 0.95. On average, the contrast ratios of the DAS, Tx-MVDR, and SAS-MVDR beamformers are 0.84, 0.90, and 0.92, respectively.

3.3 Phantom Studies

We demonstrate the beamformers on experimental data recorded by scanning a multi-purpose tissue-mimicking phantom (model 040GSE, CIRS, Norfolk, VA, USA). The sound speed is reported as 1540 ± 10 m/s with a background attenuation coefficient slope of 0.5 dB cm⁻¹ MHz⁻¹. The first dataset is acquired by imaging seven nylon-monofilament wires, 100-microns in diameter, suspended against a diffuse scattering background. Five of the targets are located on the centreline and distributed from about 10 mm to 50 mm in depth. The other two targets are at depth of 40 mm, positioned at −10 mm and +10 mm in the lateral direction.
Figure 4. Simulated images of nine anechoic lesions generated with different beamformers: (a) DAS, (b) Tx-MVDR, and (c) SAS-MVDR. All images are log-compressed and displayed with a dynamic range of 60 dB.

Figure 5. Experimental images for resolution evaluation generated with different beamformers: (a) DAS, (b) Tx-MVDR, and (c) SAS-MVDR. All images are log-compressed and displayed with a dynamic range of 60 dB.

The images generated with the DAS, Tx-MVDR, and SAS-MVDR beamformers are shown in Figs. 5(a)-(c), respectively. Compared to the DAS image, all MVDR images demonstrate better image resolution. The improvements offered by the Tx-MVDR, however, are much reduced compared to its performance in the simulations. The reductions become clearer when one inspects the beam profiles, plotted in Figs. 4(a)-(c). These are the beamformer responses to three wire-targets at depths around 10 mm, 30 mm, and 50 mm, respectively. In these plots, the Tx-MVDR has slightly narrower main-lobes than those generated with the DAS beamformer. Significant improvements are only observed with the SAS-MVDR where the responses show the narrowest main-lobes at all depths. Similar to the simulation, we calculate the average of FWHMs measured with all the wire-targets. The averaged FWHM measured with the DAS is 0.57 mm while those with the Tx-MVDR and SAS-MVDR are 0.44 mm and 0.33 mm.

To evaluate the imaging contrast, we apply the beamformers to another dataset acquired by scanning two anechoic cysts. The cysts are 3 mm in diameter and positioned at depths around 15 mm and 45 mm. Images
generated with the beamforming strategies are shown in Figs. 7(a)-(c) respectively for DAS, Tx-MVDR, and SAS-MVDR beamformers. In the figure, the DAS and Tx-MVDR have image contrasts on a par with each other. The average CR measured in the DAS and Tx-MVDR images are 0.70 and 0.71, respectively. Meanwhile, the SAS-MVDR has much higher contrast especially on the first cyst. The average CR measured in the SAS-MVDR image is 0.92.

3.4 Discussions

Similar to the earlier study, we find that the Tx-MVDR algorithm offers improvements compared to the DAS beamformer on simulated data. These improvements, however, are much reduced in the phantom studies. A major difference of the phantom studies from simulation is that the collected data is dominated by noise and interference. The experimental data is also generated by both incoherent and coherent scattering. Meanwhile, the benefits of the new SAS-MVDR beamformer are still preserved in the phantom studies. It offers a two-fold reduction in the FWHM of the blur measured on simulated point-targets and experimental wire-targets. The imaging contrast, quantified by CR, depends on the dynamic range. When both algorithms are working to a dynamic range of 60 dB, the SAS-MVDR also shows substantial enhancements over the DAS beamformer.

Usually, the MVDR beamformer is used to decorrelate the coherence of signals backscattered within a transmit beam. In this study, however, it is applied to an array of DAS beamformed signals generated from different transmit sequences. This can be described through the acoustic reciprocity theorem that exchanges the role
For example, the data vector $z(n)$ in Tx-MVDR beamforming can be considered as backscattered signals from a transmit beam generated by an 128-element linear array, and received in 75 directions. Thus, the Tx-MVDR beamformer is used to decorrelate the coherence of signals coming from these directions.

In the new matrix estimation of the SAS-MVDR beamformer, we vary the transmit beam to generate different realizations, or snapshots $\mathbf{u}_k(n)$ for $z(n)$. Signals in $\mathbf{u}_k(n)$ are also received in the same 75 directions. This allows the estimated covariance matrix to capture information about the relative position of the scatterers in relation to the receive elements. The new estimation approach allows the data vector to be decorrelated without reducing the effective aperture, which provides advantages for the image resolution. Signal coherence, however, also depends on the source aperture. In ultrasound imaging, the source is not self-radiating but reflected from the transmit beam. The snapshot generation, therefore, should take the transmit pressure profiles into account. This motivates further investigation of better approaches for estimating the covariance matrix.

4. SUMMARY AND FUTURE WORK

In this paper, we have developed a new MVDR beamformer for the CPWC without using the spatial smoothing approximation in the compounding process. By analyzing the first MVDR beamformer applied to this imaging modality, we develop a new approach for estimation of the covariance matrix. The new method decorrelates the data vector along its complete length which benefits the quality of the generated image. In demonstrations on imaging data, we show the new method significantly outperforms the DAS and a previously published MVDR beamformer in both contrast and spatial resolution.

The new MVDR beamformer is applied to the output signals of a DAS beamformer applied to data within individual transmits. These signals can also be generated using a MVDR beamformer. This could further improve the image quality at the cost of computation. Improvements in image resolution of the MVDR can lead to the need for fewer firing angles to achieve similar image quality to that obtained using DAS. Without compromising the hardware complexity, the new MVDR beamformer also can be applied to speed-up high quality image generation as low-cost high-performance computing advances.

REFERENCES


