STOCK PRICES AND MONETARY POLICY SHOCKS: 
A GENERAL EQUILIBRIUM APPROACH

EDOUARD CHALLE†    CHRYSSI GIANNITSAROU‡

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Abstract. Empirical literature documents that unexpected changes in the nominal 
interest rates have a significant effect on real stock prices: a 100-basis point increase in the 
nominal interest rate is associated with an immediate decrease in broad real stock indices that 
may range from 2.2 to 9 percent, followed by a gradual decay as real stock prices revert towards 
their long-run expected value. In this paper, we assess the ability of a general equilibrium New 
Keynesian asset-pricing model to account for these facts. The model we consider is a production 
economy with elastic labor supply, staggered price and wage setting, as well as time-varying 
risk aversion through habit formation. We find that the model predicts a stock market response 
to policy shocks that matches empirical estimates, both qualitatively and quantitatively. Our 
findings are robust to a range of variations and parametrizations of the model.

Keywords: Monetary policy; Asset prices; New Keynesian general equilibrium model

JEL Classification: E31, E52, G12

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1. Introduction

The reaction of the stock market to monetary policy shocks has been the subject of much empirical research in the last ten-fifteen years. In particular, this literature documents that an unexpected change in the nominal interest rates has significant and persistent effects on real stock prices. Papers focusing on the instant stock market response to such a shock report that a normalized 100-basis points increase in the Fed funds rate is associated with an immediate decrease in broad US stock indices that ranges from 2.2 to 9 percent, depending on the sample and estimation method being used (e.g., Craine and Martin, 2004; Rigobon and Sack, 2004, Bernanke and Kuttner, 2005; Bjørnland and Leitemo, 2009). Moreover, various authors document the dynamic effects of policy shocks and report a gradual mean reversion of real stock prices and returns following the shock (e.g., Lastrapes, 1998; Rapach, 2001; Neri, 2004). Such estimated reactions of the stock market to policy shocks are of potential interest for researchers in macro-finance for two reasons. First, they convey important information on the transmission channels of monetary policy, since policy shocks affect financial variables immediately, while they only have delayed effects on macroeconomic variables. Second, these estimates provide raw stylized facts against which the quantitative predictions of alternative theoretical frameworks can be evaluated.

The present paper has two goals. First, we assess the ability of a standard New Keynesian asset-pricing model to account for the impact and dynamic adjustment of the stock market following a nominal interest rate shock. Second, we use the model to disentangle the economic channels that may jointly contribute to the observed stock-price multiplier. The decomposition that we propose is motivated by the following simple observation. Stock prices are the expected discounted value of future dividends, hence they respond to nominal interest rate shocks because the latter affect the course of dividends and/or that of discount rates, i.e. the entire term structure. Since the discount rate can in turn be decomposed into a riskless rate and a risk premium, we ultimately end up with three potential channels, namely dividend, riskless rate and risk premium, via which policy shocks may affect stock prices. Our general-equilibrium framework thus makes the necessary assumptions for all these three channels to be operative and potentially affect stock prices.

First and foremost, a surprise increase in the nominal interest rate directly affects the real risk-free rates under the assumption that nominal prices are sticky. This directly raises the rate at which future dividends are discounted, so the risk-free rate channel contributes to take stock prices down after the policy shock.

Another implication of rising real interest rates is that they affect intertemporal substitution, as summarized by the consumption Euler equation, so that current aggregate demand falls. In our New Keynesian framework, firms are in a monopolistically competitive environment and the fall in aggregate demand affects profits and ultimately paid out dividends, in two conflicting ways. First, it directly impact sales, which consequently reduces profits. Second, firms respond to the shock by producing less, which exerts a downward pressure on the equilibrium wage and thereby on the marginal cost that all firms face. This indirect, general-equilibrium effect contributes to an increase in profits. As we show, when wages are fully flexible and for standard parameter values, the general equilibrium effect dominates the direct effect, implying that dividends counterfactually rise after an increase in the policy rate. However, we show that a plausible level of nominal wage stickiness mutes down the general-equilibrium effect sufficiently to make the direct effect on sales dominate. Hence, profits and dividends fall after an increase in the policy rate, so that the dividend
channel also contributes to a reduction in stock prices after the shock.\footnote{Christiano et al. (2005) and Bernanke and Kuttner (2005) have shown that the responses of profits and dividends conditional on a monetary policy shock are procyclical. Of course, this procyclicality need not hold conditionally on other shocks or unconditionally.}

Finally, in addition to changes in the risk-free rate, changes in risk premia or expected excess returns may also affect the discount rate and thus contribute to variations in stock prices and ex post returns (see, e.g. Campbell and Shiller, 1988; Campbell, 2003; Bernanke and Kuttner, 2005). We therefore introduce an active role for time-varying risk premia in the stock market reaction to policy shocks, by assuming that households form consumption habits, with a specification for habit formation that generates time-variations in households’ risk aversion. Our utility specification implies that risk aversion and the implied risk premia are countercyclical. Hence, the risk aversion channel concurs with the risk-free rate and dividend channels in taking stock prices down after the shock.

We establish our basic results in two steps. First, we solve the full model with a third-order perturbation method that preserves time-variations in risk premia (as in, e.g., Rudebusch and Swanson, 2008).\footnote{As is well known, standard log-linearizations of asset pricing models around the deterministic steady state eliminate higher order terms that are important when analyzing equity premia and asset returns; second order approximations or the usual log-linear log-normal approach bring back second order terms but imply constant risk premia.} Using a standard parameterization, we find that the predicted stock price and ex post return multipliers are well inside the range of available empirical estimates. Moreover these numbers are broadly robust to a variety of parametrizations, and variations of the model including one that allows for capital accumulation. Our results thus suggest that the baseline New Keynesian model provides a plausible general equilibrium explanation for the observed stock market reaction to monetary policy shocks.

In a second step, we use the model to quantitatively measure the contributions of the three channels discussed above to the overall stock-price multiplier. This cannot be done with the third-order approximation used to compute the overall multiplier, because the non-linearities involved make it hard to disentangle and isolate the contribution of each of the potential channels for the transmission of shocks. We thus propose a hybrid of the log-linear log-normal approach that allows us to express real stock prices as a linear function of future dividends, real interest rates and time-varying risk aversion, in the spirit of Campbell and Shiller (1988). The method is based on a log-linear approximation of the stochastic discount factor that makes it possible to track time-variations in risk aversion along the business cycle. This method yields stock price and return multipliers that are almost identical to those produced using a third-order approximation to the model, whilst allowing for an analytical breakdown of the three channels that contribute to the overall multiplier.

Our work relates to various strands of the literature. We have already mentioned the empirical papers on which our quantitative investigation is based (more details are provided in section 2). Of course, there is also a long tradition in assessing the asset pricing implications of dynamic macro-economic models, particularly within the Real Business Cycle tradition (see, for example, Jermann, 1998; Boldrin et al., 2001; Lettau, 2003). Within the New Keynesian tradition, Blanchard (1981) and Svensson (1986) provide early theoretical analyses of the stock market response to a monetary shock using rational expectations models with sticky goods prices and flexible asset prices. Some papers have studied the implications of sticky prices and non-neutral monetary policy for the shape...
and business cycle properties of the yield curve (e.g. Rudebusch and Swanson, 2008; Rudebusch and Wu, 2008; Doh, 2009; Bekaert, Cho and Moreno, 2010; Amisano and Tristani, 2011). Some more recent theoretical contributions that broadly analyze positive questions regarding asset prices in New Keynesian settings include Milani (2008), Li and Palomino (2009), Wei (2009), De Paoli, Scott and Weecken (2010), Castelnuovo and Nisticò (2010), Nisticò (2012). The major difference between our paper and these is that we provide an analytical decomposition of the effects of monetary shocks on real stock prices into three distinct channels of transmission. Moreover, Bhamra, Fisher and Kuehn (2011) study the implications of nominal rigidities in the value of firms debt for the way corporate bond spreads respond to monetary policy shocks. Finally, to the extent in which nominal interest rate shocks can be broadly viewed as generating uncertainty about monetary policy, our paper contributes to the literature of the effects of uncertainty about government policies on the stock market; for example see Sialm (2006) and Pastor and Veronesi (2012).

The rest of the paper is organized as follows. Section 2 summarizes the available evidence on the stock market reaction to monetary policy shocks. Section 3 introduces the basic New Keynesian model with a stock market. Section 4 presents the parametrization and results. In section 5 we explain and then implement the solution procedure we use to compute and decompose the stock-price multiplier. Section 6 evaluates the robustness of our baseline results as we alter, one by one, its key underlying assumptions. In section 7 we summarize our findings and provide some concluding remarks.

2. EMPIRICAL EVIDENCE

Table 1 reports the main pieces of recent evidence relating to the impact effects of unanticipated monetary policy shocks, in the U.S. For each study we refer to, we only report the baseline estimates of the reaction of broad stock market indices, mostly leaving aside results based on robustness checks and less representative indices (e.g., the NASDAQ). The figures reported in the last column give the reaction of the stock market value or index return following a one percentage point surprise increase in the short term nominal interest rate (the two measures are nearly identical since price changes govern ex post returns changes at high frequency). The exact value of the multiplier may vary across specifications, depending on the particular empirical methodology being implemented or the underlying data being used (e.g. the exact stock market index whose variation is measured, or the specific futures rate used to extract markets expectations and isolate the surprise component of policy shocks). However, despite these variations the overall picture that emerges from these numbers is consistent across papers, with a monetary policy shock having a significant impact on the stock market and estimated multipliers ranging from -2.20% to -9.00%.

Apart from their immediate impact on real stock market indices, monetary policy shocks are also shown to have different and persistent effects on financial asset prices. For example, Patelis (1997) shows that monetary policy indicators such as the Fed funds rate or the term spread help forecast future excess returns. Other papers have used identified VARs to recover the dynamic adjustment of real stock prices to policy shocks. For example, Lastrapes (1998) documents that the reversion of real stock prices following a money supply shock is of comparable speed as that of macroeconomic variables in a number of OECD countries. In related work, Rapach (2001) and Bjørnland and Leitemo (2009) extend and confirm this observation of a gradual decay of real

\[3\] For European countries estimates vary between -1.2% to -9.40%. See Bohl et al. (2008) and Kholodilin et al. (2009).
Table 1: Stock prices or ex post returns responses to surprise increases in the policy interest rate. The multipliers are normalized semi-elasticities summarizing the proportional change in prices or returns following a 100 basis point increase in the level of the nominal interest rate.

stock prices following a monetary policy shock. Such impulse-response patterns suggest that stock-price variables share much of the dynamic properties of other economic aggregates (at least at the quarterly frequency that we are considering here) and that they can consequently be modelled using similar macroeconomic models.

A Basic New Keynesian Model with a Stock Market

We now introduce our baseline macroeconomic model with a stock market. The macroeconomic block of the model is essentially a version of the New Keynesian framework along the lines of Amato and Laubach (2003), Woodford (2003), Smets and Wouters (2003, 2007) and Christiano et al. (2005), augmented with a stock market and accordingly an asset pricing block.

Time is discrete. The economy is populated by a continuum of households of measure 1, indexed by $\iota \in [0, 1]$, a continuum of intermediate good firms of measure 1, indexed by $h \in [0, 1]$, a large number of final goods firms and a monetary authority. Our baseline model has no capital, so that the dividends being priced are pure monopolistic rents. In Section 6.1 we analyze an extended version of the model with capital accumulation.

3.1. Households. There is a continuum of households of measure one, indexed by $\iota \in [0, 1]$. Each household is the monopolistic supplier of a specific variety of labour service and enjoys consumption of the final good, which is the numeraire in the economy. At date $t$, household $\iota$ maximizes lifetime expected utility

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} [u(C_{t+s}(\iota), H_{t+s}) - v(N_{t+s}(\iota))],$$

with the instantaneous utility function being given by:

$$u(C_t(\iota), H_t) - v(N_t(\iota)) = \frac{(C_t(\iota) - H_t)^{1-\sigma}}{1 - \sigma} - \frac{N_t(\iota)^{1+\eta}}{1 + \eta}, \quad \sigma > 0, \eta > 0,$$

where $C_t(\iota)$ and $N_t(\iota)$ are consumption and labor supply of household $\iota$, and $H_t$ is an external habit term that only depends on past aggregate consumption, i.e. $H_t = bC_{t-1}$, where $b \in (0, 1)$ is

---

4We sketch the main elements of the model here, and leave its complete derivation in a technical appendix available upon request.
is aggregate consumption at date \( t - 1 \). Note that the joint assumption of full insurance and separability between consumption and leisure implies that consumption is identical across households so that \( C_t(t) = C_t \) for all \( t \).

The type of habit formation posited here is similar to that in Jermann (1998) and Boldrin et al. (2001), with the difference that the habit stock affects households’ utility externally rather than internally. We adopt the habit formation assumption for two reasons. First, habits typically introduce sluggishness in the endogenous response of aggregate consumption to aggregate shocks. In particular, Fuhrer (2000) and Christiano et al. (2005) have shown that habit formation is required to generate a hump shape response of consumption to a monetary policy shock. In as much as habits alter the way aggregate demand responds to a monetary policy shock, they also affect the paths of profits, dividends and equilibrium stock prices. Second, specifying that habits enter as a difference (rather than as a ratio) in the households’ utility function generates time-varying risk aversion, which will naturally affect asset prices through changes in the expected excess returns at which dividends are discounted.

In every period, household \( h \) chooses consumption, labour supply and asset holdings, taking goods and asset prices as given, so as to maximize expected lifetime utility. Households can transfer wealth across periods using both one-period nominal bonds and infinitely-lived shares, which are claims to the dividend flow paid out by firms. Nominal bonds are in zero net supply and the number of shares of each intermediate goods firm \( h \) is normalized to one. The budget constraint for household \( h \) is given by

\[
C_t + \frac{B_t(t)}{P_t} + \int_0^1 V_t(t, h) Q_t(h) dh = \frac{W_t(t) N_t(t)}{P_t} + I_{t-1} B_{t-1}(t) + \int_0^1 V_{t-1}(t, h) (Q_t(h) + D_t(h)) dh.
\]

In expression (1), \( P_t \) is the nominal price of final goods and \( W_t(t) \) is the nominal wage of type \( t \) labor, both at date \( t \). \( B_t(t) \) and \( V_t(t, h) \) denote the holdings of nominal bonds and shares of firm \( h \) by household \( t \) at the end of period \( t \), respectively. \( I_{t-1} \) is the gross interest rate on nominal bonds from date \( t - 1 \) to date \( t \), and \( Q_t(h) \) and \( D_t(h) \) are the real price of a share of firm \( h \) and the dividend paid out by firm \( h \), respectively, both expressed in terms of the final good.

**Asset holdings.** The Euler equations for bonds and shares \( h \in [0, 1] \) are given by

\[
\mathbb{E}_t [M_{t,t+1} I_t / \Pi_{t+1}] = 1, \quad (2)
\]

\[
\mathbb{E}_t [M_{t,t+1} R_{t+1}^e(h)] = 1, \quad (3)
\]

where \( \Pi_t \equiv P_t/P_{t-1} \) is the gross inflation rate, \( M_{t,t+i} \equiv \beta \Lambda_{t+i}/\Lambda_t \) is the stochastic discount factor (SDF) for a payoff paid at date \( t + i \), with \( \Lambda_t \equiv (C_t - H_t)^{-\sigma} \) the date \( t \) marginal utility of consumption, and \( R_{t+1}^e(h) \) is the ex post return on holding a share of firm \( h \) from date \( t \) to date \( t + 1 \). It is given by

\[
R_{t+1}^e(h) = (Q_{t+1}(h) + D_{t+1}(h))/Q_t(h), \quad (4)
\]

where \( D_t(h) \) and \( Q_t(h) \) are the stock dividend and trading price of firm \( h \) at date \( t \), respectively.
Wage setting. Household $i$ has monopolistic market power over the raw labor type it supplies, with this power depending on the substitutability between labor types. More specifically, households’ differentiated raw labor types are combined into homogenous final labor by a competitive intermediary sector with production function

$$N^d_t = \left( \int_0^1 N_t(\iota) \phi(\iota) \, d\iota \right)^{\theta_w - 1} \phi(\iota), \quad \theta_w > 1,$$

where $N_t(\iota)$ is the supply of type $\iota$ raw labor and $N^d_t$ is the economy wide demand for final labor by intermediate goods firms equal to $\int_0^1 N_t(h) \, dh$, where $N_t(h)$ is labor demand by firm $h$. Cost minimization by the labour intermediary and the zero-profit condition

$$\int_0^1 W_t(\iota) N_t(\iota) \, d\iota = W_t N^d_t$$

give the labour demand schedules for each labour type

$$N_t(\iota) = (W_t(\iota) / W_t)^{-\theta_w} N^d_t,$$  (5)

where

$$W_t = \left( \int_0^1 W_t(\iota)^{1-\theta_w} \, d\iota \right)^{1/(1-\theta_w)}$$  (6)

is the nominal price of a unit of final labor. Note that from (5) we have that aggregate labour supply (or employment) is

$$N_t = \int_0^1 N_t(\iota) \, d\iota = \Delta_{w,t} N^d_t,$$  (7)

where

$$\Delta_{w,t} \equiv \int_0^1 (W_t(\iota) / W_t)^{-\theta_p} \, d\iota$$  (8)

is an index of cross-household wage dispersion.

Household $i$ sets the wage charged so as to maximize intertemporal utility taking as given his budget set, the general price and wage levels $P_t$ and $W_t$, the demand curve for labour type $\iota$ and the exogenous constraints on nominal wage adjustment. There are rigidities in the nominal wage-setting process. Following Erceg et al. (2000) and Christiano et al. (2005), we assume that every household resets its nominal wage optimally with probability $1 - \psi_w \in [0, 1]$ in every period, and lets its past nominal wages $W_{t-1}(\iota)$ grow at rate $\Pi_{w,t-1} \equiv W_{t-1}/W_{t-2}$, i.e. last period’s gross wage inflation, with probability $\psi_w$. The optimal wage for a household that can reset its wage is identical across households. Denoting it by $W^*_t$, we can show that satisfies the following optimality condition:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi^i_w M_{t+1} \left[ \frac{W^*_t J_{t+1}^i}{P_{t+1}} - \frac{\theta_w}{\theta_w - 1} \left( \frac{W^*_t J_{t+1}^i}{W_{t+1}} \right)^{-\eta w} S_{t+1}^i \right] \left( \frac{W_{t+1}}{W_t} \right)^{\theta_w} \frac{J_{t+1}^i N_{t+1}^d}{\Delta_{w,t+1}} = 0,$$  (9)

where $J_{t+1}^i \equiv W_{t+1-i} / W_{t-1}$ reflects the role of indexation for non reset wages and $S_t \equiv v' (N_t) / \Lambda_t$ is the average marginal rate of substitution (MRS) between leisure and consumption. With monopolistically competitive labour markets, optimizing households wish to keep their wage markup
intact and thus raise the wage charged in response to an increase in the consumption-leisure MRS relative to the current real wage (see Erceg et al., 2000). From the constraint on nominal wage adjustment described above, it can be shown that the wage rate for final labor and wage dispersion index for raw labor evolve according to the following laws of motion:

\[
W_t^{1-\theta_w} = \left[ (1 - \psi_w) \left( W_t^* \right)^{1-\theta_w} + \psi_w \left( \Pi_{w,t-1} W_{t-1} \right)^{1-\theta_w} \right],
\]

\[
\Delta_{w,t} = (1 - \psi_w) \left( \frac{W_t^*}{W_t} \right)^{-\theta_w} + \psi_w \left( \frac{\Pi_{w,t-1}}{\Pi_{w,t}} \right)^{-\theta_w} \Delta_{w,t-1}.
\]

### 3.2. Firms.

There are two production layers. Intermediate goods firms use the final labor supplied by labor intermediaries as inputs to produce specialized intermediate goods, which are then combined into an homogenous final good by final goods firms.

**Final goods firms.** Final goods firms produce with technology

\[
Y_t = \left( \int_0^1 Y_t (h)^{\theta_p-1} \left( \frac{\theta_p}{\theta_p-1} \right) dh \right)^{\frac{\theta_p}{\theta_p-1}}, \theta_p > 1,
\]

where \( Y_t (h) \) is the use of intermediate goods of type \( h \). Minimizing the cost of producing \( Y_t \) gives the demand schedules:

\[
Y_t (h) = (P_t (h) / P_t)^{-\theta_p} Y_t,
\]

where \( P_t (h) \) is the nominal price of intermediate good \( h \) and

\[
P_t = \left( \int_0^1 P_t (h)^{1-\theta_p} \left( \frac{1}{\theta_p} \right) dh \right)^{\frac{1}{1-\theta_p}}
\]

is the nominal price of final goods. For later use, we also define the following index of cross-firm nominal price dispersion:

\[
\Delta_{p,t} = \int_0^1 (P_t (h) / P_t)^{-\theta_p} dh.
\]

**Intermediate goods firms.** Intermediate good firm \( h \) produces with technology \( Y_t (h) = Z_t N_t (h) \), where \( Y_t (h) \) is the output of firm \( h \), \( N_t (h) \) the amount of final labor used by firm \( h \) and \( Z_t \) an aggregate productivity shock obeying the following log-AR(1) process:

\[
\dot{z}_t = \kappa \dot{z}_{t-1} + u_t, \ u_t \sim N \left( 0, \sigma^2_w \right).
\]

where \( \dot{z}_t = \ln Z_t \).

Firms maximize the present value of the monopolistic profits that are paid out to their owners (i.e. the households) in the form of dividends. The real dividend paid out by firm \( h \) is given by

\[
D_t (h) = Y_t (h) \left( \frac{P_t (h)}{P_t} - \Phi_t \right),
\]

where \( P_t (h) / P_t \) is the real price of good \( h \) and

\[
\Phi_t = \frac{1}{Z_t} \frac{W_t}{P_t}
\]
is the marginal cost faced by all firms. A firm $h$ sets the selling price of its variety, $P_t(h)$, taking as given the demand it faces, $Y_t(h)$, the general price and wage levels $P_t$ and $W_t$, and the exogenous constraints on price setting. The price adjustment mechanism assumed here is similar to that in Christiano et al. (2005). In every period a firm is allowed to reset its price optimally with probability $1 - \psi_p \in [0, 1]$. In case it does not, it applies the indexation rule $P_t(h) = P_{t-1}(h) \Pi_{t-1}$. It can then be shown that the optimal nominal reset price $P_t^*$, common to all firms resetting their price, is given by:

$$
\sum_{j=0}^{\infty} \psi_p^j M_{t+j} P_{t+j} = \left( \frac{\Pi_t}{\Pi_{t+j}} \right)^{1-\theta_p} \left( \frac{P_t^*}{P_t} \right) - \frac{\theta_p}{\theta_p - 1} \Phi_{t+j} \left( \frac{\Pi_t}{\Pi_{t+j}} \right)^{-\theta_p} = 0.
$$

The description of the dynamics of intermediate and final goods’ prices is complete with the laws of motion for the price of final goods, $P_t$, and price dispersion, $\Delta_{p,t}$. From the Calvo resetting process described above, those two quantities evolve as follows:

$$
P_t^{1-\theta_p} = (1 - \psi_p) \left( P_t^* \right)^{1-\theta_p} + \psi_p \left( \Pi_{t-1} P_{t-1} \right)^{1-\theta_p},
$$

$$
\Delta_{p,t} = (1 - \psi_p) \left( P_t^* \right)^{-\theta_p} + \psi_p \left( \Pi_{t-1} \right)^{-\theta_p} \Delta_{p,t-1}.
$$

### 3.3. Monetary policy.

The model is closed by specifying the way the central bank provides nominal anchor. In our baseline specification, we assume that the central bank reacts to current inflation and current output according to the following Taylor rule:

$$
I_t = \left( \frac{I_{t-1}}{\bar{I}} \right)^\gamma \left[ \left( \frac{P_t}{\bar{P}_{t-1}} \right)^{\rho_p} \left( \frac{Y_t}{\bar{Y}} \right)^{\rho_y} \right]^{1-\gamma} \tilde{E}_t,
$$

where $Y_t$ is aggregate output and $\bar{I}$ and $\bar{Y}$ are the steady state values of the nominal interest rate and output respectively. The constants $\rho_p$ and $\rho_y$ are positive reaction coefficients, $\gamma \in (0, 1)$ reflects the degree of interest-rate smoothing by the central bank and $\tilde{E}_t$ is a nominal interest rate shock, with $\tilde{v}_t = \ln \tilde{E}_t$ and

$$
\tilde{v}_t = \nu \tilde{v}_{t-1} + \varepsilon_t.
$$

We generally allow for a persistent monetary policy shock, i.e. we consider cases such that $0 \leq \nu < 1$.\(^5\)

### 3.4. Market clearing.

**Goods market.** In the absence of capital accumulation, the market-clearing condition for final goods is $C_t = Y_t$. Market clearing for intermediate goods requires the total demand for such goods by the final goods sector to be equal to the total supply by intermediate goods firms. From (7) and (12) and the definition of the wage and price dispersion indices above, this equality simplifies to $\Delta_{w,t} \Delta_{p,t} Y_t = Z_t N_t$. Hence, clearing of the final and intermediate goods markets requires:

$$
C_t = Y_t = \frac{Z_t N_t}{\Delta_{p,t} \Delta_{w,t}}.
$$

\(^5\)Evidence supporting persistence of monetary policy shocks can be found in Rudebusch (2002) and more recently in Coibion and Gorodnichenko (2012).
Table 2: Baseline parameterisation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIS parameter</td>
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<td>Elast. of demand for goods</td>
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</tr>
<tr>
<td>Labour supply coef.</td>
<td>0.0000</td>
<td>Elast. of demand for labour</td>
<td>4.0000</td>
</tr>
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<td>Habit persistence</td>
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<td>Interest rate persistence</td>
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</tr>
<tr>
<td>Discounting</td>
<td>0.9900</td>
<td>Persistence of mon. shock</td>
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</tr>
<tr>
<td>Fraction of unchanged prices</td>
<td>0.6000</td>
<td>Standard deviation of mon. shock</td>
<td>0.0028</td>
</tr>
<tr>
<td>Fraction of unchanged wages</td>
<td>0.9000</td>
<td>Persistence of tech. shock</td>
<td>0.9900</td>
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<tr>
<td>Responsiveness to inflation</td>
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<td>Standard deviation of tech. shock</td>
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<td>Covariance of tech. and mon. shock</td>
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<tr>
<td>Discounting</td>
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</table>

Asset markets. Since nominal bonds are in zero net supply we must have:

\[ \int_0^1 B_t(t) \, dt = 0. \] (24)

The market clearing equations for stocks are given by:

\[ \int_0^1 V_t(t, h) \, dt = 1 \text{ for all } h \in [0, 1]. \] (25)

Equations (3), (4) and (25) determine the trading price of a share of firm \( h \), \( Q_t(h) \). Individual dividends and stock prices can then be exactly aggregated into aggregate dividend and stock price indices as follows. From (12)–(13) and (16), the aggregate dividend index \( D_t \equiv \int_0^1 D_t(h) \, dh \) is:

\[ D_t = Y_t - \Omega_t \frac{N_t}{\Delta_{w,t}}, \] (26)

where \( \Omega_t = W_t/P_t \) is the real wage. From (3)–(4), the corresponding stock price index \( Q_t \equiv \int_0^1 Q_t(h) \, dh \) is given by

\[ Q_t = \mathbb{E}_t \left( M_{t+1} (Q_{t+1} + D_{t+1}) \right). \] (27)

Appendix A gathers all the equations characterizing the equilibrium of our economy.\(^6\)

4. MODEL SOLUTION AND RESULTS

We assume a quarterly specification for the parameters of the model. Our baseline parameterization is presented in Table 2. We discuss each of these parameters in turn. The parameter \( \sigma \) is typically assumed to vary between 1 and 5 in most of the macroeconomics literature, while the asset pricing literature allows this to be up to 12. We choose \( \sigma = 5 \), somewhere in the middle of such large range, and both acceptable for business cycle and asset pricing characteristics of the model. Next, we set the parameter \( \eta \) to be 0, which is a common assumption. For the discounting factor we choose \( \beta = 0.99 \) which is typical for quarterly calibrations. The habit parameter is set to \( b = 0.8 \) following existing literature such as Jermann (1998). Turning to the parameters of the Taylor rule, for the Volker-Greenspan era, a robust estimate for the US is around \( \gamma = 0.85 \). For example, Clarida, Gali and Gertler (2000) calculate \( \gamma \in [0.73, 0.88] \) depending on which sample/measure is used. Judd

\(^6\)Appendix A summarizes the full model with capital accumulation (as in Section 6 below), which nests the baseline model without capital as a special case.
and Rudebusch (1998) suggest $\gamma \in [0.56, 0.73]$, Amato and Laubach (1999) give $\gamma \in [0.78, 0.92]$ and Kozicki (1999) gives $\gamma \in [0.75, 0.82]$.

Conventional estimates for the response parameters in the Taylor rule are $\rho_n \approx 1.5$ and $\rho_y < 1.0$, but estimates may vary substantially from one paper to the other. For example Judd and Rudebusch (1998) estimate $\rho_n \in [1.46, 1.69]$ and $\rho_y \in [0.36, 0.99]$, Clarida et al. (2000) give $\rho_n \in [1.97, 2.15]$ and $\rho_y \in [0.55, 1.49]$ and Kozicki (1999) gives $\rho_n \in [1.05, 1.66]$ and $\rho_y \in [0.42, 0.52]$. We choose $\rho_n = 1.5$ and $\rho_y = 0.6$ in our benchmark experiment.\footnote{As discussed by Clarida, Gali and Gertler (2000), the pre-Volcker period was probably characterized by a value of $\rho_n$ lower than one, leading to the violation of the Taylor principle and, in the context of the New Keynesian model, to the appearance of multiple self-fulfilling equilibria. In these cases, learnability criteria may be used to select the appropriate equilibrium (see, e.g., McCallum, 2003). The evidence that motivates our analysis is based on data collected after the Volcker shock, during which the Taylor principle is commonly agreed to have been satisfied.}

The elasticities of the demands for good and labour varieties are set to $\theta_p = \theta_w = 4$. In the literature, these parameters vary between 3 and 10, although the estimates of Christiano et al. (2005) have a larger variation. Finally, we set the degree of price rigidity $\psi_p$, to 0.6 and the degree of wage rigidity $\psi_w$ to 0.9. Highly rigid wages ensure that firm profits and thus dividends are procyclical.

There are five more parameters to be determined, namely $\kappa$, $\sigma_u$, $\nu$, $\sigma_e$ and $\sigma_{ue}$. First, we set $\kappa = 0.99$ and $\sigma_u = 0.025$. This is within reasonable limits and captures volatility of output growth from US data. The literature reports numbers for $\sigma_u$ between 0.008 and 0.04 (see Wouters and Smets, 2003, Danthine and Kurman, 2004, Collard and Dellas, 2006 and Rabanal and Rubio-Ramirez, 2005). We set $\nu = 0.65$, following Coibion and Gorodnichenko (2012). A moderate persistence for the policy shock is necessary for generating the hump-shaped response of the nominal interest rate to its own innovation, such as documented by, e.g., Björnland and Leitemo (2009).

Next, we set $\sigma_e = 0.0028$. This number is set so that it replicates monetary policy shocks that generate 100 basis-points surprise increases in the annualized nominal interest rate. More details on how this number is generated can be found in Section 5.3. Last, we set $\sigma_{ue} = 0$, since the underlying assumption behind our thought experiment is that $\varepsilon_t$ represents monetary policy surprises and should thus be treated as a non-systematic reaction to changes in aggregate supply. Any correlation between monetary policy and technology shocks is by construction internalized in the Taylor rule.

We solve the model with a third order approximation of log-variables around their deterministic steady state, using Dynare. Figure 1 provides the impulse response functions (IRFs) of all variables of interest, following a surprise 100 basis point increase in the nominal interest rate. The impulse response functions are based on averaging 1,000 simulated series each variable.\footnote{Dynare generates IRFs using the approximate policy functions as follows: (i) It draws a series of the all the exogenous shocks for a number of $100 + T$ periods, where $T$ is the number of periods shown in the IRF graphs, here $T = 15$. (ii) It performs a simulation ‘S1’ of all the variables of interest using this realization of the shocks. (iii) It changes the sequence of exogenous shocks, by adding a one-standard deviation of the shock we are interested in, here $\varepsilon$, to the realization of period 101, (iv) It performs a new simulation ‘S2’ of all variables based on the new sequence of shocks. (v) It calculates the IRF from S2-S1. These steps are then repeated a number of times (we choose 1,000 replications) and the produced IRFs are averages of the series from these 1,000 experiments. This procedure ensures that the entire distribution of shocks to the economy is taken into account, which is important since the approximate policy function (at the third order) depends on the distributional characteristics of the shocks.}

The dynamic adjustment of macroeconomic variables to a nominal interest rate shock is broadly consistent with empirical impulse-responses (e.g. Christiano et al., 2005). The nominal interest rate rise is contractionary, which lowers both price and wage inflation, the overall implication of both being a mildly procyclical real wage adjustment. The indexation of not re-optimized prices and wages
on their past respective inflation rates produces inertia in those variables and hence hump-shaped responses to the initial shock. Similarly, the presence of consumption habits generates output inertia and a hump-shaped response of this variable to the shock. In contrast, real stock prices are purely forward-looking and hence display no inertia; it follows that their maximal departure from their steady state value takes place at the very time of the shock. Finally, staggered wage adjustment generates procyclical profits and dividends, as is consistent with the data (we show in Section 6.2 below how the model behaves when nominal wages are fully flexible). Our baseline calibration generates a stock market impact multiplier of $-3.0691$, which is broadly consistent with the range of plausible point estimates reported in empirical studies and summarized in Table 1. This result suggests that our baseline New Keynesian model provides a potential general equilibrium explanation for the observed stock market reaction to monetary policy shocks.

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Figure 1: Impulse responses to monetary policy shock. IRFs to an unexpected (annualized) monetary policy shock of 100-basis points. Based on averaging one thousand simulations of a third-order approximation of the model around the deterministic steady state, using the baseline parametrization. The reported numbers are raw, that is an impact increase of 0.0025 of the nominal interest rate and an impact drop of -0.030691 of the stock price are interpreted as a 100-basis points suprise increase of the annualized interest rate and a -3.0691% decrease of the stock price index respectively.

---

9See Woodford (2003, chap. 3 and 5) for further discussion of inflation and output inertia.
5. Transmission channels

In this section, we propose an alternative method for approximating the proposed model, which allows us to decompose the overall stock price multiplier into its underlying determinants, namely the risk-free rate channel, the dividend channel and the risk aversion channel. While analysis based on a log-linearized dynamic system remains valid for small fluctuations around the deterministic steady state, it has a major drawback: by simply log-linearizing asset pricing Euler equations, we lose second-order information that enters expected returns and may affect the reaction of real stock prices to policy shocks.

The approach we propose combines log-linear and nonlinear elements in the following way. First, we log-linearize the set of relevant equilibrium conditions, and solve for its VAR representation using standard methods. Second, we derive an approximation to the stochastic discount factor that allows us to relate equilibrium expected excess returns to the degree of relative risk aversion, under the assumption that all variables are log-normally distributed. Third, we infer from our expression for expected stock returns one for equilibrium stock prices, which explicitly relates stock prices to the expected course of future riskless rates, dividends and risk premia. Finally, we use the VAR dynamics of the state vector to compute rational forecasts of dividends, real interest rates and expected excess returns, and hence to solve for equilibrium stock prices as a function of those three determinants.

5.1. Dividends, risk-free rate and expected excess returns. In what follows, lower case letters with a hat denote log-deviations of the corresponding variables from their steady state value. Log-linearizing equations (23) and (26), we obtain the following expressions for log-output and log-dividends:

\[
\begin{align*}
\hat{y}_t &= \hat{z}_t + \hat{n}_t - \hat{\delta}_{w,t} - \hat{\delta}_{p,t}, \\
\hat{d}_t &= \theta_p \hat{y}_t - (\theta_p - 1) (\hat{\omega}_t + \hat{n}_t - \hat{\delta}_{w,t}).
\end{align*}
\]

The log-linear dispersion terms \(\hat{\delta}_{w,t}\) and \(\hat{\delta}_{p,t}\) asymptotically vanish at the first order, therefore they can be ignored when generating IRFs and calculating multipliers. Then, substituting the first expression into the second we obtain

\[
\hat{d}_t = \hat{y}_t + (1 - \theta_p) (\hat{\omega}_t - \hat{z}_t).
\]

Next, we turn to the determination of the real interest rate of this economy. In principle,
the real interest rate on a risk-free one-period bond that pays out one unit of the consumption good can be decomposed into the contributions of the nominal bond rate, expected inflation and a correction term reflecting the negative compensation for not bearing the inflation risk associated with holding nominal one-period bonds. Here, however, we take a first-order approximation to this risk-free real interest rate and thus write its log-deviation from steady state as:

\[ \hat{r}_t^{f} = r_t^{f} + \ln \beta \approx i_t - \mathbb{E}_t (\pi_{t+1}), \]  

(29)

where \( r_t^{f} \) is the log risk-free rate and \( r_t^{f} = \ln \) is its value at the deterministic steady state.

We now analyze the equilibrium real stock returns and prices implied by our model. To this end, we start by characterizing equilibrium log-excess returns taking the stochastic discount factor as given, and then propose a approximate expression for the stochastic discount factor that allows us to explicitly solve for stock returns and prices as a function of the underlying macroeconomic variables. Let \( R_{t+1}^{c} \) denote the return on the stock market index, i.e.,

\[ R_{t+1}^{c} = (Q_{t+1} + D_{t+1}) / Q_t, \]  

(30)

where \( D_t \) is the aggregate dividend given by (26) and \( Q_t \) the price of a claim to the stream of aggregate dividends, i.e. the stock market index. From (27), \( R_{t+1}^{c} \) satisfies

\[ \mathbb{E}_t (M_{t+1} R_{t+1}^{c}) = 1, \]  

(31)

We now apply the usual log-normal framework to derive our approximate asset pricing equations. More specifically, we conjecture that the stochastic discount factor and ex post returns are jointly conditionally log-normally distributed, and then verify later on that this conjecture is true in equilibrium, under our approximated Euler equation for stocks.\(^{13}\) Under the joint log-normality assumption, (31) may be written as follows:

\[ \mathbb{E}_t (m_{t+1}) + \mathbb{E}_t (r_{t+1}) + \frac{1}{2} \left( \sigma_h^2 + \sigma_m^2 + 2 \sigma_{hm,t} \right) = 0, \]  

(32)

where \( m_{t+1} \equiv \ln M_{t+1} \) is the log stochastic discount factor, \( r_{t+1} \equiv \ln R_{t+1}^{c} \) the log-stock return, \( \sigma_m^2 \equiv \text{var}_t (m_{t+1}) \) the conditional variance of the log stochastic discount factor, \( \sigma_h^2 \equiv \text{var}_t (r_{t+1}) \) the conditional variance of log stock returns and \( \sigma_{hm,t} \equiv \text{cov}_t (r_{t+1}^{c}, m_{t+1}) \) the conditional covariance between log returns and the log stochastic discount factor. Since our equilibrium will feature conditionally homoskedastic ex post returns, we drop the time index in \( \sigma_h^2 \) from the outset. By contrast, as we shall see shortly, the log stochastic discount factor will be endogenously heteroskedastic (despite the homoskedasticity of its component, i.e., aggregate consumption), thereby generating a time-varying price of risk that will affect equilibrium real stock prices and excess returns. From (32), the expected log-excess return is then given by (see Campbell, 2003):

\[ \mathbb{E}_t (r_{t+1} - r_{t+1}) = -\sigma_{hm,t} - \frac{\sigma_h^2}{2}. \]  

(33)

Apart from the role of precautionary savings, which foster aggregate savings and thus lower excess

\(^{13}\)See, e.g., Campbell (1993), and more recently Restoy and Weil (2011) for a similar derivation of approximate asset pricing expressions (in the context of Epstein-Zin preferences, rather than habit formation).
returns (captured by the term $\sigma_h^2/2$), expression (33) reflects the usual pricing of systematic payoff risk in complete markets general equilibrium economies. For example, an asset payoff that is highly correlated with aggregate consumption provides a poor hedge against consumption fluctuations and thus commands high expected excess returns; this effect is reflected by the negative correlation between future marginal utility of consumption and the asset return and thus a high value of $-\sigma_{hm,t}$ in (33).

Let $\hat{r}_{t+1}^e$ be the deviation of the log expected return from the deterministic steady state where all shocks are set to zero at all times. Along this steady state, there is no risk premium and we have $r_{t+1}^e = r^f = -\ln \beta$; we may then rewrite (33) in terms of deviations from steady state as follows:

$$E_t(\hat{r}_{t+1}^e - \hat{r}_{t+1}^f) = -\sigma_{hm,t} - \frac{\sigma_h^2}{2}. \quad (34)$$

Excess equity returns in (34) affect asset prices through the discounting of dividend streams. Thus we need to determine the two components of the right-hand-side of (34) in order to analyze their effects on real stock prices. We defer the derivation of $\sigma_h^2$ to a later point, where we explain how to retrieve $\sigma_{hc}$ and $\sigma_h^2$ jointly.

Next, taking $\sigma_h^2$ as known, we need to derive an expression for $\sigma_{hm,t}$, which requires an explicit expression for the equilibrium SDF. We thus aim at expressing the time-varying covariance term in (34) as a function of variables that can be forecasted from the macroeconomic block of the model, while at the same time capturing the role played by time-varying risk aversion. The procedure described below delivers both these features. First let $\Lambda_{t+1} \equiv \Lambda_t (C_{t+1}, C_t)$ and

$$\Theta_t \equiv -\frac{C_t u_{11} (C_t, C_{t-1})}{u_1 (C_t, C_{t-1})} = \frac{\sigma}{1 - be^{-\Delta c_t}} \quad (35)$$

be the households’ (local) relative risk aversion coefficient at date $t$. Taking a first-order Taylor expansion of $\Lambda (C_{t+1}, C_t)$ around any point $(X, Y)$ that is sufficiently close to $(C_{t+1}, C_t)$ we obtain

$$\Lambda (C_{t+1}, C_t) \approx \Lambda (X, Y) + \Lambda_1 (X, Y) (C_{t+1} - X) + \Lambda_2 (X, Y) (C_t - Y). \quad (36)$$

Provided that consumption is sufficiently smooth, so that $C_t$ is sufficiently close to $C_{t-1}$, we may take $(X, Y) = (C_t, C_{t-1})$ as the point around which we expand. Then, we can rearrange this to get:

$$\frac{\Lambda (C_{t+1}, C_t) - \Lambda (C_t, C_{t-1})}{\Lambda (C_t, C_{t-1})} \approx \frac{\Lambda_1 (C_t, C_{t-1}) C_t}{\Lambda (C_t, C_{t-1})} \left( \frac{C_{t+1} - C_t}{C_t} \right) + \frac{\Lambda_2 (C_t, C_{t-1}) C_{t-1}}{\Lambda (C_t, C_{t-1})} \left( \frac{C_t - C_{t-1}}{C_{t-1}} \right). \quad (37)$$

This expression essentially approximates marginal utility growth (left hand side) with an appropriate weighted sum of current and past consumption growth (right hand side). We can now rewrite marginal utility growth as:

$$\Delta \ln \Lambda (C_{t+1}, C_t) \equiv \Delta \lambda_{t+1} \approx -\Theta_t (\Delta \hat{c}_{t+1} - \Delta \hat{c}_t) - \sigma \Delta \hat{c}_t. \quad (38)$$

The effect of consumption growth on risk aversion follows from our assumed utility function. For

\footnote{This approximation is in fact more accurate than linearizing $\Lambda (C_{t+1}, C_t)$ around steady state, since consumption persistence implies that $C_t$ is at least as close to $C_{t-1}$ as it is to its steady state value.}
example, when consumption falls relative to past consumption, so that \( \Delta \hat{c}_t < 0 \), then the local curvature of the utility function increases, thereby making households more risk averse. Under the approximation in (38), innovations to the log stochastic discount factor are given by:

\[
m_{t+1} - \mathbb{E}_t m_{t+1} = \Delta \ln \Lambda (C_{t+1}, C_t) - \mathbb{E}_t (\Delta \ln \Lambda (C_{t+1}, C_t)) = -\Theta_t (\hat{c}_{t+1} - \mathbb{E}_t \hat{c}_{t+1}). \tag{39}
\]

We can therefore approximately express the conditional covariance between the log stochastic discount factor and the log stock return as:

\[
\sigma_{hm,t} \approx \mathbb{E}_t \left[ -\Theta_t (\hat{c}_{t+1} - \mathbb{E}_t \hat{c}_{t+1}) (r^{e}_{t+1} - \mathbb{E}_t r^{e}_{t+1}) \right] = -\sigma_{hc} \Theta_t, \tag{40}
\]

where \( \sigma_{hc} \) has no time index since log-consumption and log-asset returns will be conditionally homoskedastic in our approximate equilibrium (see Section 5).

Substituting our expression for \( \sigma_{hm,t} \) into (34), we find that expected excess returns, in terms of log-deviations from the deterministic steady state, are approximately given by:

\[
\mathbb{E}_t (\hat{r}^{e}_{t+1} - \mathbb{E}_t \hat{r}^{e}_{t+1}) = \sigma_{hc} \Theta_t - \frac{\sigma_h^2}{2}, \tag{41}
\]

which from (35), is only a function of \( \Delta \hat{c}_t \). In short, (41) states that rising current risk aversion, \( \Theta_t \), raises expected excess returns and therefore it increases the premium required for holding risky shares. This effect is scaled by the consumption risk associated with holding a share, i.e. the covariance of ex post returns with next period’s consumption \( \sigma_{hc} \). Loosely speaking, while the consumption risk of the stock market \( \sigma_{hc} \) is constant, the price of risk \( \Theta_t \) is time-varying because households become more risk-averse in recessions, due to the habit formation specification.

The key advantage of our way of linearizing the marginal utility of current consumption is that it allows us to arrive at a tractable expression for expected excess returns that preserves the key source of changes in risk aversion in the model (i.e., the changes in current consumption relative to past consumption), that would otherwise be lost with a standard log-linearization around the steady state of consumption.\(^{15}\)

5.2. Stock prices. Having derived expressions for all the underlying determinants of real stock prices (i.e., dividends, risk-free rates and expected excess returns), we may now turn to the implied equilibrium real stock prices. This may be done by using the log-linear present value model of Campbell and Shiller (1988). More specifically, linearizing (30) around the deterministic steady state and using (28), we may write ex post log-stock returns as follows:

\[
\hat{r}^{e}_{t+1} = \beta \hat{q}_{t+1} + (1 - \beta) \hat{d}_{t+1} - \hat{q}_t, \tag{42}
\]

where \( \hat{q}_t \) denotes the log-deviation of the stock market index from the deterministic steady state. Note that the unconditional means of \( \hat{r}^{e}_{t} \) and \( \hat{q}_t \) are different from zero here, since holding risky shares requires a positive average returns premium (i.e. \( \mathbb{E} (\hat{r}^{e}_{t+1}) > 0 \)) that depresses average real stock prices (i.e. \( \hat{q}_t < 0 \)), provided that the portfolio risk effect in (41) dominates the precautionary

\(^{15}\) This way of linearizing the consumption Euler equation (i.e., around current consumption or consumption growth, rather than their steady state counterparts) has proven useful elsewhere. One example is the literature on precautionary savings behavior, where this technique also allows to preserve important properties of the nonlinear Euler equation that would be lost otherwise (e.g., Dynan, 1993; Gourinchas and Parker, 2001).
savings effect (i.e. $\sigma_{hc}\Theta_t - \sigma_h^2/2 > 0$). However, the approximation in (42) will remain valid as long as fluctuations are sufficiently small, that is as long as $\mathbb{E}(\hat{r}^e_{t+1})$ is sufficiently close to $\mathbb{E}(\hat{r}^f_{t+1}) = 0$. On average, we have $\mathbb{E}(\hat{r}^e_{t+1}) = -(1-\beta)\mathbb{E}(\hat{q}_t)$ since $\mathbb{E}(\hat{d}_{t+1}) = 0$ in (42).

Solving (42) for $\hat{q}_t$, substituting it into (34) and applying the expectation operator on both sides, we get

$$
\hat{q}_t = \beta \mathbb{E}_t(\hat{q}_{t+1}) + (1 - \beta) \mathbb{E}_t \hat{d}_{t+1} - \hat{r}^f_{t+1} - \sigma_{hc}\Theta_t + \frac{\sigma_h^2}{2}.
$$

(43)

Finally, iterating (43) and rearranging under the condition that no rational bubble occurs (i.e., $\lim_{n \to \infty} \beta^n \hat{q}_{t+n} (h) < \infty$), the share price of firm $h$ may be written as

$$
\hat{q}_t = -\frac{\mu}{1 - \beta} + (1 - \beta) \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t(\hat{d}_{t+1+j}) - \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t(\hat{r}^f_{t+1+j}) - \sigma_{hc} \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t(\hat{\Theta}_{t+j}),
$$

(44)

where $\mu = \sigma_{hc}\bar{\Theta} - \sigma_h^2/2$ is the mean equity premium, $\bar{\Theta} = \sigma/(1-b)$ is the mean risk aversion coefficient and $\bar{\Theta}_t = \Theta_t - \bar{\Theta}$ its level-deviation from the mean.

Equation (44) is intuitive: real stock prices increase with future dividends (second term), but decrease with current and future risk-free rates (third term) and risk aversion (fourth term). The constant (first term) just reflects the difference between the average stock price along the stochastic equilibrium and its value at the deterministic steady state, around which the linearization was done. For example, a greater covariance between consumption and returns, $\sigma_{hc}$, makes asset $h$ more risky and thus lowers its average value, relative to the deterministic steady state; but higher return risk fosters precautionary savings, which tends to raise asset demand and prices, relative to the deterministic steady state. All summation terms are centered around their unconditional mean. The corresponding centered asset-price variable is simply $\tilde{\hat{q}}_t \equiv \hat{q}_t + \mu/(1-\beta)$.

Note that expression (44) is not quite yet operative, because real stock prices actually appear on both sides of it: the covariance term $\sigma_{hc}$ determines how time-variations in risk aversion affect prices, but $\sigma_{hc}$ is not a deep parameter of the model. It is an endogenous quantity that depends on equilibrium asset prices. Similarly, both $\sigma_h^2$ and $\sigma_{hc}$ enter the constant term while they are endogenously determined in equilibrium. In perfectly competitive economies, the ex post return on stocks would be given by the marginal product of capital and its first and second moments could be directly extracted from the macroeconomic block of the model (as, e.g., Jermann, 1998). This cannot be done in our imperfectly competitive model, so we must recover ex post returns from dividends and prices using (28), (42) and (44). However, we show next that under certain assumptions, there is only one possible combination of $\sigma_h^2$ and $\sigma_{hc}$ that is consistent with (44). This can be recovered from (44) and the VAR representation of the macro dynamics of the model.

5.3. Model solution. Our goal is to compute the reaction of real stock prices to an unexpected policy shock, where the three channels emphasized above (dividends, real interest rates, excess returns) play an active role in generating this reaction. We thus proceed as follows.

The first step is to solve for the joint dynamics of all variables that are log-linearized around the steady state. These variables are collected into a vector $\chi_t = [\hat{y}_t, i_t, \pi_t, \pi^v_t, \hat{s}_t, \hat{\omega}_t, \hat{d}_t, \hat{r}^f_{t+1}, \hat{z}_t, \hat{v}_t]^T$. 


which jointly solve (15), (22), (28), (29) and the following macro block:

\[
\begin{align*}
\dot{y}_t &= \left( \frac{b}{1+b} \right) \dot{y}_{t-1} + \left( \frac{1}{1+b} \right) \mathbb{E}_t (\ddot{y}_{t+1}) - \sigma \left( \frac{1-b}{1+b} \right) \left( i_t - \mathbb{E}_t (\pi_{t+1}) \right), \\
\pi_t &= \left( \frac{1}{1+\beta} \right) \pi_{t-1} + \left( \frac{\beta}{1+\beta} \right) \mathbb{E}_t (\pi_{t+1}) + \frac{(1-\beta \psi_p) (1-\psi_p)}{(1+\beta) \psi_p} (\hat{\omega} - \hat{z}_t), \\
\pi_t^w &= \left( \frac{1}{1+\beta} \right) \pi_{t-1}^w + \left( \frac{\beta}{1+\beta} \right) \mathbb{E}_t (\pi_{t+1}^w) + \frac{(1-\beta \psi_w) (1-\psi_w)}{(1+\eta \theta_w) (1+\beta) \psi_w} (\hat{s}_t - \hat{\omega}_t), \\
\dot{s}_t &= \left( \frac{\sigma}{1-b} + \eta \right) \dot{y}_t - \frac{b \sigma}{1-b} \dot{y}_{t-1} - \eta z_t, \\
\hat{\omega}_t &= \hat{\omega}_{t-1} + \pi_{t}^w - \pi_t, \\
i_t &= \gamma i_{t-1} + (1-\gamma) \left( \rho_x \pi_t + \rho_y \dot{y}_t \right) + \hat{\nu}_t.
\end{align*}
\]

The first equation of the block is the log linearized bond Euler equation, i.e. the dynamic IS curve of the model. The second and third equations are the price and wage Phillips curves under full indexation of non re-optimized prices and wages respectively. The fourth equation gives the consumption/leisure marginal rate of substitution, which enters the wage Phillips curve. The fifth equation is the real wage dynamics, while the sixth is the log-linear Taylor rule.

We then rewrite the full system in matrix form:

\[
\mathbb{E}_t \left[ \Psi_0 \chi_{1,t+1} + \Psi_1 \chi_{1,t} + \Psi_2 \chi_{1,t-1} + \Phi_0 \chi_{2,t+1} + \Phi_1 \chi_{2,t} \right] = 0,
\]

where

\[
\chi'_{1t} = \left[ \dot{y}_t, i_t, \pi_t, \pi_{t}^w, \dot{s}_t, \hat{\omega}_t, \hat{d}_t, \hat{\epsilon}_{t+1}^f \right]' \quad \text{and} \quad \chi'_{2t} = \left[ \hat{z}_t, \hat{\nu}_t \right]',
\]

and \( \Psi_i, i = 0,1,2 \) and \( \Phi_j, j = 0,1 \) are conformable matrices that are defined via equations governing the dynamics of \( \chi_t \).

We employ a standard undetermined coefficients method to solve for the dynamics of this system, and we may write the solution for the dynamics of \( \chi_t \), if it exists and is unique, in a compact form as

\[
\chi_t = F \chi_{t-1} + L e_t, \tag{45}
\]

where \( F \) and \( L \) are conformable matrices and where

\[
e_t = \left[ \begin{array}{c} u_t \\ \varepsilon_t \end{array} \right] \sim N \left( 0, \Sigma \right), \quad \Sigma = \begin{bmatrix} \sigma_u^2 & \sigma_{ue} \\ \sigma_{ue} & \sigma_e^2 \end{bmatrix}.
\]

The system can be solved with any of the known algorithms or toolboxes that are available for such problems. We use Christiano’s (2002) general approach.

The second step is to use (45) in order to derive an expression for the stock price as a function of present and past values of \( \chi \). At this stage, all sequences that enter the summation terms in (44) can be forecasted using (45), apart from \( \hat{\Theta}_t \) which is a nonlinear function of \( \Delta \hat{\epsilon}_t \) (see (35)). However, linearizing (35) and using the fact that \( \hat{\epsilon}_t = \hat{\epsilon}_{t-1} \), we can write the centered risk aversion coefficient as:

\[
\hat{\Theta}_t = \Theta_t - \bar{\Theta} \approx - \left( \frac{\Theta b}{1-b} \right) \Delta \hat{y}_t, \tag{47}
\]

which can now also be extracted from (45). We are also now in position to confirm the joint
log-normality of the stochastic discount factor and returns. Under the dynamics (45) and the maintained assumption that the underlying innovations are i.i.d. normal, all variables in $\chi_t$, are conditionally normally distributed and homoskedastic. It follows that $\hat{d}_t$ in (28), $\hat{r}_{t+1}^f$ in (29) and $\hat{\theta}_t$ in (47) are all conditionally normally distributed and homoskedastic, and so are the log-deviations from steady state of real stock prices, $\hat{q}_t(h)$ in (44), and stock returns, $\hat{r}_{t+1}^c(h)$ in (42). Finally, the conditional normality of $\hat{c}_{t+1}$ implies that marginal utility growth, $\Delta \lambda_{t+1}$, in (38), and hence the log stochastic discount factor, $m_{t+1} = \ln \beta + \Delta \lambda_{t+1}$, are also conditionally normally distributed. Thus, $R_{t+1}(h) = \exp \left[ - \ln \beta + \hat{r}_{t+1}^c(h) \right]$ and $M_{t+1} = \exp (m_{t+1})$ are confirmed to be conditionally lognormally distributed, as we assumed when going from (31) to (32).

Now let $e_k$ denote a column indicator vector that picks a generic variable $k$ from the vector $\chi_t$, i.e. a vector such that $k_t = e_k \chi_t$. Expectations of future dividends, risk-free rates and risk aversion coefficients are then given by

$$\mathbb{E}_t(\hat{d}_{t+1+j}) = e_d F^{j+1} \chi_t; \quad \mathbb{E}_t(\hat{r}_{t+1+j}^f) = e_{r,f} F^j \chi_t,$$

and

$$\mathbb{E}_t(\hat{\theta}_{t+1+j}) = e_{\theta}^f (F^{j+1} - F^j) \chi_t, \quad \text{for } j = 0, 1, \ldots$$

Then, substituting these sequences into (44), we can rewrite the value of the stock market index only as a function of constants and the current and last period’s value of the vector $\chi$:

$$\hat{q}_t = \mathbb{E}_t(\hat{q}_t) = -\frac{\mu}{1 - \beta} + (1 - \beta) e_d^f (I - \beta F)^{-1} F \chi_t - \frac{\beta \sigma_{hc}}{1 - \beta} e_d^y \left( \chi_{t-1} + \frac{1 - \beta}{1 - \beta} (I - \beta F)^{-1} \chi_t \right),$$

where $I$ is a $10 \times 10$ identity matrix.

The last step in computing equilibrium real stock prices is to $\sigma_h^2$ and $\sigma_{hc}$. Regarding $\sigma_h^2$, we first rewrite (50) as

$$\hat{q}_t = \xi_0 + \xi_1 \chi_t + \xi_2 \chi_{t-1},$$

where

$$\xi_0 = -\frac{\mu}{1 - \beta},$$

$$\xi_1 = (1 - \beta) \frac{\beta \sigma_{hc}}{1 - \beta} e_d^y (I - \beta F)^{-1} - e_{r,f}^f (I - \beta F)^{-1} + (1 - \beta) e_d^y (I - \beta F)^{-1} F,$$

$$\xi_2 = -\frac{\beta \sigma_{hc}}{1 - \beta} e_d^y.$$

Then, from (42), innovations to ex post returns are given by:

$$r_{t+1}^c(h) - \mathbb{E}_t(r_{t+1}^c(h)) = \beta (\hat{q}_{t+1} - E_t \hat{q}_{t+1}) + (1 - \beta) (\hat{d}_{t+1} - \mathbb{E}_t(\hat{d}_{t+1})) = (\beta \xi_1 + (1 - \beta) e_d^y) (\chi_{t+1} - E_t \chi_{t+1}).$$

Using the above expression, we can derive the conditional covariance of consumption and ex post
returns as follows:

\[
\sigma_{hc} = (\beta \xi_1 + (1 - \beta) e_d)' L \Sigma L' e_y. \tag{56}
\]

Since \(\xi_1\) is linear in \(\sigma_{hc}\), it is straightforward to retrieve it from the above expression once we have evaluated the matrices \(F\) and \(L\) from the rest of the parameter values. Similarly once we have \(\sigma_{hc}\), we can also get the conditional variance \(\sigma^2_h\), which is given by

\[
\sigma^2_h = (\beta \xi_1 + (1 - \beta) e_d)' L \Sigma L' (\beta \xi_1 + (1 - \beta) e_d). \tag{57}
\]

For a given \(\chi_t\), all terms and parameters in (50) are now pinned down by (35), the matrices \(F\) and \(L\) in (45) and the expressions for \(\sigma^2_h\) and \(\sigma_{hc}\) given by (57) and (56). The vector \(\chi_t\) is endogenously determined by the exogenous shock vector through (45). We thus have all the elements necessary for the computation of the impact and propagation of a nominal interest rate shock on the stock market, as well as for its decomposition into the relative contributions of the three underlying stock price determinants.

First, we use our approximate solution and the same parametrization as in the previous section to plot impulse response functions that are directly comparable to those in Figure 1. The dynamic adjustment of macroeconomic variables to a nominal interest rate shock is almost identical to the one generated by the third order approximation and therefore we do not include an additional figure for this.

Next, we focus on the decomposition of the effects of the monetary policy shock on the stock price index. The multiplier \(\Delta \tilde{q}_0/\Delta i_0\) can be decomposed into the three relevant components using (50). These are given by

\[
\mathcal{M}_q \equiv \Delta \tilde{q}_0 \quad \Delta i_0 = (1 - \beta) e_d'(I - \beta F)^{-1} FL \begin{bmatrix} 0 \\ \varepsilon_0 \end{bmatrix} - e_r'(I - \beta F)^{-1} L \begin{bmatrix} 0 \\ \varepsilon_0 \end{bmatrix} + \frac{\Theta \sigma_{hc}}{1 - \beta} e_y'(I - \beta F)^{-1} L \begin{bmatrix} 0 \\ \varepsilon_0 \end{bmatrix}. \tag{58}
\]

In order to simulate the reaction of real stock prices to 25 basis points increase in the central bank (quarterly) rate and generate impulse-response functions, we first need to calculate the size of the shock \(\varepsilon_0\) that does generate such a change. We have that

\[
0.25 = \Delta i_0 = e_f'L \varepsilon_0 = L_{22} \varepsilon_0 \implies \varepsilon_0 = 0.25/L_{22},
\]

where \(L_{22}\) is the elasticity of the nominal interest rate with respect to the monetary innovation. We thus impose a shock of size \(0.25/L_{22}\), and then recover model-generated semi-elasticities that are directly comparable to those in Table 1 by computing \(\Delta \tilde{q}_0/\Delta i\).

Table 3 gives the proportional change in real stock prices and ex post excess returns following this shock, as well as the breakdown of those in the three channels. The corresponding effect

\[16\] The impulse responses are generated in a way that makes them directly comparable to those generated by Dynare for the third order approximation, i.e. by taking into account both exogenous shocks and their distributions, in the sense that their standard deviations appear in expression that gives the multiplier (58).
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{M}_q$</td>
<td>-3.0694</td>
<td>-0.00883</td>
<td>-3.06039</td>
<td>-0.00006</td>
</tr>
<tr>
<td>$\mathcal{M}_{er}$</td>
<td>-3.0390</td>
<td>-0.00912</td>
<td>-3.02979</td>
<td>-0.00006</td>
</tr>
</tbody>
</table>

Table 3: Multipliers of stock prices and excess returns, with respect to 100 basis surprise increase in the nominal rate, for the baseline parametrization.

on ex post excess returns is obtained as a weighted average of price and dividend changes, i.e. 
$$\Delta \hat{r}_e = \beta \Delta \bar{y} + (1 - \beta) \Delta d_0.$$  
Our baseline calibration generates a stock market impact multiplier of -3.0694.

The decomposition of ex post excess returns following the policy shock that we obtain from the model (second line of Table 3) gives a surprisingly small role to variations in ex ante excess returns and a comparatively large one to changes in real interest rates. The relatively small contribution of changes in expected excess returns can be understood as follows. From the last summation term in (44), it is apparent that the excess returns contribution to the price multiplier is governed by (i) the conditional covariance between consumption and asset return, $\sigma_{hc}$, or consumption risk of the stock market and (ii) the variability of the (local) relative risk aversion coefficient, $\hat{\Theta}_t$, in response to aggregate shocks, i.e., the price of risk. Moreover, from equation (47), $\hat{\Theta}_t$ is the product of (minus) the change in log-output, $\Delta \bar{y}$ (which is also $\Delta c_t$ in the model), multiplied by the factor $\hat{\Theta}_b/ (1 - b) = \sigma b/(1 - b)^2$, which scales the impact on households’ risk aversion of the consumption fall that follows the shock. Now, the very nature of habit formation makes households reluctant to change current consumption relative to past consumption and thus limits the consumption response to exogenous shocks, as soon as consumption is optimally chosen by households (rather than being exogenously given as in pure exchange economies, see the discussion in Lettau and Uhlig, 2000). Formally, this shows up in the fact that in equilibrium, the value of $|\Delta \bar{y}|$ at the time of the shock is smaller when $b > 0$ than when $b = 0$, which in turn tends to limit the corresponding change in risk aversion. Second, our baseline value $\sigma = 5$ keeps the scaling factor $\sigma b/(1 - b)^2$ relatively low, thereby preventing small consumption changes to induce large changes in risk aversion. Finally, the limited conditional consumption variability after a shock due to habit formation implies a small conditional covariance between consumption and ex post stock returns, $\sigma_{hc}$. Hence, all factors determining the size of the excess returns contribution (i.e., $\sigma_{hc}$, $\hat{\Theta}_b/ (1 - b)$ and $-\Delta \bar{y}$) tend to be small.

This prediction of a small excess returns contribution to the multipliers is in contrast to the VAR based decomposition of empirical returns proposed by Bernanke and Kuttner (2005), which suggests that ex post excess returns variations following a nominal interest rate shock work predominantly through variations in ex ante excess returns, with a small contribution of real interest rate changes. However, Bernanke and Kuttner’s result of a small real interest rate contribution naturally follows from their very quick estimated decay of the real interest rate following the policy shock: real rates deviations from the mean have a half-life of no more than two months and have completely died out after four. With such a rapid reversion of real rates, these are bound to have little effect on real stock prices since the latter ultimately depend on the infinite sequences of future real rates, dividends and excess returns. Although this speed of adjustment is not necessarily inconsistent with previous estimates based on monthly data (see Bernanke and Mihov, 1998), the quarterly macroeconomic evidence on which our model builds typically documents a much slower reversion.
of real interest rates following an exogenous policy shock and would thus imply a much larger role for such rates in explaining the stock market response to the shock (e.g., Amato and Laubach, 2003; Boivin and Giannoni, 2002; Christiano et al., 2005; Nakamura and Steinsson, 2013).

Finally, as expected, this model does not generate an equity premium even when non-linear solution methods are employed. In line with recent literature which recasts the notion of relative risk aversion in dynamic stochastic general equilibrium models (Swanson, 2012), this model does not allow for high enough relative risk aversion to generate a sizeable equity premium.

6. Model variations

In this section we explore how the model behaves as we move away from the baseline model specification described in Section 3. We consider four such variations, each analyzed in isolation: capital accumulation, flexible nominal prices, flexible nominal wage, and habit-free preferences. Table 4 presents the multipliers for these cases, and Figure 2 plots the impulse response functions against the baseline case. At the end of the section, we also comment on the sensitivity of our main result to other model variations and parameter changes.

6.1. Capital accumulation. Incorporating capital accumulation into the model may affect our baseline results for two reasons. First, with capital stock prices not only reflect the value of the stream of pure profits generated by monopolistic firms, but also the value of a claim to the income flow generated by the capital stock. Thus, after a monetary policy shock, we expect the economy with capital to generate a dividend path that differs from the economy without capital, and this will directly affect the response of stock prices in equilibrium. Second, capital accumulation offers an additional way for households to smooth consumption over time, which should translate into less consumption variability and thereby less variability in the pricing kernel determining stock prices and returns. We wish to assess whether these two effects matter quantitatively and affect the multiplier computed from the baseline model specification.

We introduce capital accumulation along the lines of Christiano et al. (2005) and Smets and Wouters (2003, 2007), with one important difference. In those papers, households own the capital stock and lend it to firms in every period in a competitive capital market after the distribution of Calvo shocks is revealed. Thus capital is efficiently reallocated across firms at the time firms make pricing decisions, which in turn ensures that capital-to-labor ratios and real marginal costs are identical across firms, even though any two firms could be choosing different levels of capital and labor inputs. In our model, households hold firm shares, but the firms own the capital stock and hence decide how much capital to accumulate. The problem of the firm is a priori complicated in this context, because firms face idiosyncratic (Calvo) shocks and thus firms having different

<table>
<thead>
<tr>
<th>Model</th>
<th>Stock Price Multiplier</th>
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<tbody>
<tr>
<td>Baseline</td>
<td>-3.0691</td>
</tr>
<tr>
<td>Flexible Prices</td>
<td>-3.0623</td>
</tr>
<tr>
<td>Flexible Wages</td>
<td>-6.5739</td>
</tr>
<tr>
<td>No Habit Formation</td>
<td>-3.4927</td>
</tr>
<tr>
<td>Capital Accumulation</td>
<td>-2.6547</td>
</tr>
</tbody>
</table>

Table 4: Multipliers for model variations.
histories of nominal prices will typically accumulate different levels of capital. We keep the model tractable by assuming that firms not only accumulate capital from one period to the next, but may also do within-period trade of capital units in a competitive market, once the distribution of Calvo shocks is realized. In detail, any firm \( h \) saves a quantity of capital \( K_t(h) \) at the end of date \( t-1 \) expecting that the competitive price of capital at date \( t \) will be \( R_t^h \). After the idiosyncratic and aggregate date \( t \) shocks are revealed, firm \( h \) may either sell or buy additional capital at price \( R_t^h \), which results in a quantity of capital in use \( \tilde{K}_t(h) \) (which will in general differ from \( K_t(h) \)). In every period, \( R_t^h \) adjusts so that total firms’ savings, \( \int_0^1 K_t(h) \, dh = K_t \) equal total capital in use, \( \int_0^1 \tilde{K}_t(h) \, dh \).

The production function for firm \( h \) is now

\[
Y_t(h) = Z_t \tilde{K}_t^\alpha(h) N_t^{1-\alpha}(h), \quad 0 \leq \alpha < 1, \tag{59}
\]

while its period budget constraint is:

\[
D_t(h) + \frac{W_t}{P_t} N_t(h) + R_t^h \tilde{K}_t(h) + X_t(h) = \frac{P_t(h)}{P_t} Y_t(h) + R_t^h K_t(h), \tag{60}
\]

i.e. a capital reallocation of size \( R_t^h (\tilde{K}_t(h) - K_t(h)) \) takes place for firm \( h \). Following Christiano et al. (2005), we assume that capital accumulation is partly impeded by adjustment costs, so that beginning-of-period capital is:

\[
K_{t+1}(h) = (1 - \delta) K_t(h) + \left( 1 - \tau \left( \frac{X_t(h)}{X_{t-1}(h)} \right) \right) X_t(h), \tag{61}
\]

where \( \delta \in (0,1) \) is the depreciation rate, \( X_t(h) \) is investment at firm \( h \), and \( \tau(.) \) the following function:

\[
\tau \left( \frac{X_t(h)}{X_{t-1}(h)} \right) = \frac{\vartheta}{2} \left( \frac{X_t(h)}{X_{t-1}(h)} - 1 \right)^2, \quad \vartheta > 0. \tag{62}
\]

Firms maximize value to their shareholders, i.e. they choose \( P_t(h), X_t(h), \tilde{K}_t(h) \) and \( N_t(h) \) to solve:

\[
V(K_t(h), P_{t-1}(h), X_{t-1}(h), C_t(h), S_t) = \max D_t(h) + \mathbb{E}_t [M_{t+1} V(K_{t+1}(h), P_t(h), X_t(h), C_{t+1}(h), S_{t+1})], \tag{63}
\]

where \( C_t(h) = 1 \) if firm \( h \) re-optimizes its selling price in period \( t \) and \( C_t(h) = 0 \) otherwise, and Note that past investment \( X_{t-1}(h) \) enters the current value function as a state variable, due to its effect on future adjustment costs.

Solving (63) subject to (59) and (62), the Calvo pricing process and the Markovian dynamics of the exogenous states vector \( S_t = (E_t, Z_t) \), and then aggregating across firms, we obtain
following four equilibrium conditions:

\[
\frac{K_t}{N_t} = \frac{\alpha}{1 - \alpha} \frac{W_t}{P_t} \frac{P_t}{R_t}^k,
\]

\[
K_{t+1} = (1 - \delta) K_t + \left[ 1 - \frac{\theta}{2} \left( \frac{X_t}{X_{t-1}} - 1 \right) \right]^2 X_t,
\]

\[
Q_t = \mathbb{E}_t \left[ M_{t+1} \left( R^k_{t+1} + (1 - \delta) Q_{t+1} \right) \right],
\]

\[
1 = Q_t \left[ 1 - \frac{\theta}{2} \left( \frac{X_t}{X_{t-1}} - 1 \right)^2 - \varphi \left( \frac{X_t}{X_{t-1}} - 1 \right) \frac{X_t}{X_{t-1}} \right] + \mathbb{E}_t \left[ M_{t+1} Q_{t+1} \varphi \left( \frac{X_{t+1}}{X_t} - 1 \right) \left( \frac{X_{t+1}}{X_t} \right)^2 \right].
\]

The first equation gives the optimal capital-to-labor ratio in any firm as a function of the relative price of inputs. The second equation is the capital accumulation equation, once aggregated across firms. In the third equation, \( Q_t \) denotes the present value of an additional unit of installed capital (i.e., Tobin’s marginal \( Q \)), which is a symmetric, forward-looking variable. The fourth equation gives aggregate investment, \( X_t \), as a function of past investment, future investment and the marginal value of capital. Finally, note that with capital the expression for the economy wide real marginal cost (17) becomes

\[
\Phi_t = \frac{1}{Z_t} \left( \frac{W_t}{P_t} \right)^{1-\alpha} \left( \frac{R^k_t}{\alpha} \right)^\alpha,
\]

i.e., a geometrically weighted average of unit labour and capital costs. Similarly, the expression for aggregate dividends (26) must now take into account investment and becomes:

\[
D_t = Y_t - \frac{W_t}{P_t} \frac{N_t}{\Delta x, t} - X_t.
\]

We then calibrate the three additional free parameters (\( \alpha, \delta \) and \( \varphi \)) and run the same computations as in Section 4 (i.e., using a third-order approximation of the full model). We set \( \alpha \) and \( \delta \) to the conventional quarterly values of 0.36 and 0.025, respectively. Smets and Wouters (2007) estimate the mean value of the posterior distribution of the capital adjustment cost parameter \( \varphi \) to be 5.74, and we adopt this value in our calibration. The implied stock price multiplier is \(-2.6547\), slightly below that in the model without capital \((-3.0691\)). Moreover, the responses of the endogenous variables to the shock in the full model with capital are very similar to those in the baseline model, except for a somewhat larger recessionary effect on output (see Figure 2). We conclude that our results are robust to the introduction of capital accumulation, for conventional assumptions about the specification and elasticity of investment adjustment costs.

### 6.2. Flexible prices, flexible wages, and habit-free preferences

We now return to the baseline model without capital, and relax one by one each of its three basic assumptions, namely, sticky prices, sticky wages, and habit formation. As it turns out, the economy with flexible nominal prices behaves almost identically to that with sticky prices, as long as the assumption of nominal
wage rigidities is maintained. This is true both for the stock price multiplier and for the IRFs.\footnote{The IRFs in the flexible price case are not reported in Figure 2, as they are almost identical to the IRFs in the baseline (sticky price) case.}

The economy without habit formation generates a \textit{larger}, not smaller, stock price reaction to a monetary policy shock, despite the fact that risk aversion is not time-varying in this case. However, this is only because lack of habits generates an implausibly large impact response of output to the shock, which is also counterfactually not hump-shaped (see the first panel of Figure 2). This illustrates the fact that habit formation is key in generating a realistic output response to a monetary policy shock, although it does not ultimately matter a great deal for the reaction of the stock market.

Finally, the economy with fully flexible wages (and both sticky prices and habit formation) generates a substantially larger response of the stock market to the policy shock. Looking at the IRFs, we observe that this can be traced back to an implausibly strong reaction of the \textit{real} interest rate to the nominal interest rate shock, which is itself due to an excessively large inflation response. The reason is that we have an economy with a moderate degree of price stickiness,
in which a significant number of firms are able to maintain their desired markup over marginal cost. After a contractionary monetary policy shock, the fall in the demand for goods translates into a fall in the demand for labor, which under flexible nominal wage induces a large fall in the equilibrium real wage. This effect in turn exerts a strong downward pressure on the economy wide real marginal cost, and hence a large fall in many firms’ selling prices (due to the markup pricing rule), which are then reflected into the aggregate nominal price level.

To summarize, sticky prices have a marginal impact on our results (provided wages are sticky), sticky wages are needed to generate plausible responses of the real wage, inflation and the real interest rate to the policy shock, while habit formation is needed to obtain a mild, hump shaped response of output.

6.3. Other variations. Apart from these four variations of the model, we have also performed some sensitivity checks (not reported here) with respect to other structural assumptions of the model, but they turned out not to affect our baseline results significantly. For example, considering partial rather than full indexation of non-optimized prices and wages turns out to have very little impact, for plausible degrees of partial indexation. Similarly, considering a form of long-memory habit, leaves the results practically unaffected. Finally, the same applies to using several variations of the Taylor rule, including forward looking versions. We also found our measure $M_q$ to be very robust to changes in most of the model parameters. However, $M_q$ is somewhat sensitive to some of them, notably the utility parameter $\sigma$ and the Taylor rule parameters $\gamma, \rho_\pi, \rho_y$ and $\nu$. These are the parameters that have a direct effect on the behavior of consumption (utility parameters) and the real interest rates (Taylor rule parameters through their effects on nominal interest rates), i.e. the two variables that are relevant for understanding the breakdown of the impact of the shock on stock prices. Generally, when varying these parameters, almost all implied values of the multiplier stay within the interval consistent with the empirical studies, with the exception of somewhat extreme values of $\gamma$ and $\nu$. Similarly, such parameter changes do not alter the broad features of our impact decomposition, thus confirming the main conclusions drawn from the baseline specification. Finally, when calculating the relative contributions of each component to $M_q$, we find that these change very little, reinforcing our claim that our main result is robust to parameter changes.

7. Closing comments

The motivation behind our work comes from recent literature that documents the effects of unexpected monetary policy on the stock market. We ask and assess whether a basic DSGE model with New Keynesian features can account for the now well documented response of the stock market to changes in the nominal interest rate by the Central Bank, both qualitatively and quantitatively. The model we considered is the simplest possible version of a New Keynesian framework that may have the ability to explain such facts: Building on the basic New Keynesian model of Woodford (2003), we assumed that both prices and wages were sticky (with sticky wages ensuring that dividends are procyclical) and that households formed consumption habits (so that risk aversion was time-varying). The model was then augmented in a natural way with a financial market, which we analyzed in detail in order to address our asset pricing questions. The model was then parameterized in line with the business cycle literature, i.e. so that it generated commonly accepted dynamics for the main macroeconomic aggregates.

Our findings can be summarized as follows. On one hand, the model succeeds in matching the
main empirical fact that we wish to capture, i.e. the instantaneous response of the stock market to a surprise increase in the Fed funds rate and this result is robust to simple variations and parametrizations of the model. One the other hand, when attempting to break down the impact of unexpected monetary policy on the stock price to the three relevant channels (i.e. dividends, real interest rates and ex-ante excess returns), we find the relative contribution of real interest rates to the total impact on real stock prices to be larger than what some empirical studies have documented. We attribute this to (i) the slow mean reversion of real interest rates predicted by New Keynesian models and (ii) the smoothness of the endogenous consumption process under habit formation.

What can we learn from this analysis? First, we propose a mechanism for generating this interesting asset pricing fact in the context of a general equilibrium business cycle model. Given the general difficulty in reconciling the business cycle and asset pricing literatures, we believe that our paper goes a rather long way in understanding the links and interactions between monetary policy and the stock market. Our analysis thus provides a platform for further research that would seek to improve our understanding of how different factors may affect these links.

Second, an interesting by-product of our analysis is that the methodology for deriving present value expressions for the asset prices preserves some of the valuable second order information that is usually lost when linearizing dynamic systems. Although the methodology described here is particular to our New Keynesian framework, we conjecture that it can be easily applied to other settings.

APPENDIX

A. SYSTEM OF EQUILIBRIUM CONDITIONS

This appendix summarizes the dynamics of the complete nonlinear model with capital, which includes 25 endogenous variables. The full model with capital (25 variables) studied in Section 6 nests our baseline model without capital (21 variables) presented in Section 3. To go from the full model to the baseline model, we simply set $\alpha = 0$, remove equations (M22)–(M25) below from the full system, and ignore the following four endogenous variables: $X_t$, $K_t$, $Q_t$, $R_k^t$ (note that when $\alpha = 0$ $R_k^t$ disappears from (M6) while $X_t = 0$ in (M17)). The full model has two exogenous state variables, $Z_t$ and $E_t$, as well as the following 25 endogenous variables, where $\Xi_t$, $\Sigma_t$, $\Gamma_t$ and $\Upsilon_t$ are defined recursively by conditions (M6), (M7), (M12) and (M13).

$$C_t, X_t, Y_t, N_t, K_t, \Lambda_t, M_{t,t+1}, \Phi_t, \Delta_{p,t}, \Delta_{w,t}, \Omega_t, \Pi_t, \Pi^e_t, G_t, L_t, \Xi_t, \Sigma_t, \Gamma_t, \Upsilon_t, R_k^t, R_k^e, Q_t, D_t, Q_t, I_t.$$  

Those variables jointly solve the following 25 equations:

---

[^18]: A step-by-step derivation of equations (M1)–(M25) appears in a technical appendix available upon request.
Definitions:

\[ M_{t,t+1} = \beta \Lambda_{t+1}/\Lambda_t, \quad (M1) \]
\[ \Lambda_t = (C_t - bC_{t-1})^\alpha. \quad (M2) \]

Output and market clearing:

\[ Y_t = \frac{Z_t K_t^\alpha N_t^{1-\alpha}}{\Delta_{p,t} \Delta_{w,t}}, \quad (M3) \]
\[ C_t + X_t = Y_t. \quad (M4) \]

Wage dynamics:

\[ \Gamma_t = Y_t, \quad (M5) \]
\[ \Gamma_t \frac{G_t}{\Omega_t} = \frac{\Omega_t N_t}{\Delta_{w,t}} + \psi_w \mathbb{E}_t \left[ \frac{M_{t,t+1} \Gamma_{t+1}}{\Pi_{t+1}^w} \left( \frac{\Pi^w_{t+1}}{\Pi_t^w} \right)^{\theta_w-1} \right], \quad (M6) \]
\[ G_{t}^{\theta_w} Y_t = \frac{\theta_w \Lambda_{t-1}^{-1} \left( \frac{N_t}{\Delta_t^w} \right)^{1-\eta} + \psi_w \mathbb{E}_t \left[ M_{t,t+1} G_{t+1}^{\theta_w} Y_{t+1} \left( \frac{\Pi_{t+1}^w}{\Pi_t^w} \right)^{(1-\eta)\theta_w} \right]}{(1 - \psi_w) G_t^{1-\theta_w} + \psi_w \left( \frac{\Pi_{t-1}^w}{\Pi_t^w} \right)^{1-\theta_w}}, \quad (M7) \]
\[ \Delta_{w,t} = (1 - \psi_w) G_t^{1-\theta_w} + \psi_w \left( \frac{\Pi_{t-1}^w}{\Pi_t^w} \right)^{1-\theta_w} \Delta_{w,t}, \quad (M8) \]
\[ \Pi_t^w = \Omega_t \Pi_t^{w-1}. \quad (M9) \]

Price dynamics:

\[ L_t = \Xi_t / \Xi_t, \quad (M11) \]
\[ \Xi_t = \frac{\theta_p \Phi_t Y_t}{\theta_p - 1} + \psi_p \mathbb{E}_t \left[ M_{t,t+1} \Xi_{t+1} \left( \frac{\Pi_t}{\Pi_{t+1}} \right)^{-\theta_p} \right], \quad (M12) \]
\[ \Sigma_t = Y_t + \psi_p \mathbb{E}_t \left[ M_{t,t+1} \Sigma_{t+1} \left( \frac{\Pi_t}{\Pi_{t+1}} \right)^{1-\theta_p} \right], \quad (M13) \]
\[ 1 = (1 - \psi_p) L_t^{1-\theta_p} + \psi_p \left( \frac{\Pi_{t-1}}{\Pi_t} \right)^{1-\theta_p}, \quad (M14) \]
\[ \Delta_{p,t} = (1 - \psi_p) L_t^{1-\theta_p} + \psi_p \left( \frac{\Pi_{t-1}}{\Pi_t} \right)^{1-\theta_p} \Delta_{p,t-1}, \quad (M15) \]
\[ \Phi_t = \frac{1}{Z_t} \left( \frac{\Omega_t}{1 - \alpha} \right)^{1-\alpha} \left( \frac{R_t^\alpha}{\alpha} \right)^{\alpha}. \quad (M16) \]

Asset prices (with \( X_t = 0 \) if \( \alpha = 0 \)):

\[ D_t = Y_t - \frac{\Omega_t N_t}{\Delta_{w,t}} - X_t, \quad (M17) \]
\[ 1 = \mathbb{E}_t \left[ M_{t,t+1} I_{t+1} / \Pi_{t+1} \right], \quad (M18) \]
\[ 1 = \mathbb{E}_t \left[ M_{t+1} R_{t+1}^\beta \right], \quad (M19) \]
\[ R_t^\beta = (Q_t + D_t) / Q_{t-1}, \quad (M20) \]
\[ I_t = F^{1-\gamma} I_{t-1}^{\gamma} \left( \Pi_t^\alpha \left( Y_t / \bar{Y} \right)^{\alpha} \right)^{1-\gamma} \mathcal{E}_t. \quad (M21) \]
• Capital (for $\alpha > 0$ only):

\[
K_{t+1} = (1 - \delta) K_t + \left( 1 - \frac{\rho}{2} \left( \frac{X_t}{X_{t-1}} - 1 \right)^2 \right) X_t,
\]  
\text{(M22)}

\[
Q_t = \mathbb{E}_t \left[ M_{t+1} \left( R^k_{t+1} + (1 - \delta) Q_{t+1} \right) \right],
\]  
\text{(M23)}

\[
1 = Q_t \left[ 1 - \frac{\rho}{2} \left( \frac{X_t}{X_{t-1}} - 1 \right)^2 - \rho \left( \frac{X_t}{X_{t-1}} - 1 \right) \frac{X_t}{X_{t-1}} \right] + \mathbb{E}_t \left[ M_{t+1} Q_{t+1} \rho \left( \frac{X_{t+1}}{X_t} - 1 \right) \left( \frac{X_{t+1}}{X_t} \right)^2 \right],
\]  
\text{(M24)}

\[
\frac{K_t}{N_t} = \frac{\alpha}{1 - \alpha} \frac{\Omega_t}{\Delta_{w,t} R^k_t}.
\]  
\text{(M25)}

It is easily shown that combining (M5)–(M7) gives (9) in the body of the paper, while combining (M11)–(M13) gives (18).
References


