The Output Cost of Gender Discrimination: A Model-Based Macroeconomics Estimate*

Tiago Cavalcanti and José Tavares

June 3, 2015

Abstract

We use a growth model in which saving, fertility and labour market participation are endogenous, to quantify the cost that barriers to female labour force participation impose in terms of an economy's output. The model is calibrated to mimic the U.S. economy's behavior in the long-run. We find that a 50 percent increase in the gender wage gap leads to a 35 percent decrease in income per capita in steady-state. Using independent estimates of the female to male earnings ratio for a wide cross-section of countries, we construct an economy with parameters similar to those calibrated for the U.S. economy, except for the degree of gender barriers. Higher discrimination decreases steady-state output per capita for two distinct reasons: a direct effect due to the decrease in female labour market participation, and an indirect effect working through an increase in fertility. For several countries, a large fraction of the difference between the country's output and U.S. output can be ascribed to differences in gender discrimination. In addition, we find that close to half of the overall decrease in output per capita is due to the effect of gender discrimination in fertility.

Keywords: Development, Gender Barriers, Female Labour Force Participation, Fertility.

J.E.L. codes: E0; J1; O1.

*This paper has benefited from the financial support from Égide and Nova Forum at Universidade Nova de Lisboa, from the Fundação para a Ciência e Tecnologia (FCT) and POCTI through FEDER, and from the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil). We thank two anonymous referees, the editor Wouter den Haan, Stefania Albanesi, Alberto Alesina, Emanuela Cardia, Christian Conrad, Partha Dasgupta, Mathias Doepke, Allan Drazen, Francesco Franco, Nuno Garoupa, Martin Gassebner, Moshe Hazan, Michael Lamla, Andrea Lassmann, Fernanda Lllusá, Marina Costa Lobo, Jorge Braga de Macedo, Heiner F. Mikosch, Sarah Rupprecht, Jan-Egbert Sturm, David Weil and Richard Zeckhauser, as well as participants at the SED Conference and the Meeting of the Brazilian Econometric Society, seminars at Universidade Nova de Lisboa, the Instituto de Ciências Sociais, the Instituto Superior de Economia e Gestão and the KOF-ETH Zurich for useful comments. Corresponding Author: Tiago Cavalcanti, University of Cambridge, Faculty of Economics, Sidgwick Avenue, CB3 9DD, Cambridge, United Kingdom and Sao Paulo School of Economics - FGV. Email: `tvdvc2@cam.ac.uk`.

Everywhere females find it more difficulty than males to access market activities, political power, or health and education inputs.[1] Hausmann, Tyson, and Zahidi (2006) suggest that "*no country in the world has yet reached equality between women and men in critical areas such as economic participation, education, health, and political empowerment.*"[2] Gender discrimination has many guises, probably interrelated in their causes and consequences, as they are part of a complex system of social, cultural, and economic determinants. The economics literature has studied the microeconomics of job and wage discrimination in some detail, thus far focusing on the individual cost of discrimination.[3] We believe it is time to estimate the aggregate cost of gender discrimination, and for that purpose we use a long-run macro model where discrimination affects investment, fertility, and the steady-state features of the economy

Providing an estimate of the cost of discrimination to aggregate output is important for several reasons. First, gender inequality remains high in many countries (e.g., Doepke, Tertilt, and Voena, 2012; Hausmann, Tyson, and Zahidi, 2006). Second, although gender discrimination is largely determined by social and cultural characteristics which hardly change in the short run,[4] there are several gender-related policies which could foster gender equality and therefore reduce discrimination.[5] Finally, to our knowledge, this is the first paper to use a structural model to quantify the long-run output effects of barriers to female labour force participation. A model-based estimate is key as it takes into consideration how agents and prices respond to different levels of gender discrimination. Such a model also captures indirect effects of gender discrimination, including endogenous changes in the fertility rate, while allowing us to run counterfactual experiments that take into account general equilibrium effects.

From the seminal paper by Galor and Weil (1996), a strand of the literature has examined

[1]As an example, Alesina, Lotti, and Mistrulli (2013) show that Italian women have to pay higher interest rates than men, even when exhibiting a better credit history, and after controlling for a host of personal and firm characteristics.

[2]See also the 2012 World Development Report (cf., World Bank, 2011) on gender equality and development.

[3]Some authors have argued that the tax rates on second earners (usually the woman) are much higher than those on the first earner, de facto further discouraging female labour force participation (cf., Bick and Fuchs-Schündeln, 2012). Alesina, Ichino, and Karabarbounis (2011)have suggested going further and imposing differential gender-based tax rates, especially given the higher tax elasticity of women's labour supply.

[4]See Fernandez (2007) and Algan and Cahuc (2007) for the importance of culture and family characteristics on FLFP. For the role of religion on FLFP, see Guiso, Sapienza, and Zingales (2003), Psacharopoulos and Tzannatos (1989), and Siaroff (1994). Fernández, Fogli, and Olivetti (2004) show that men whose mothers worked while they were growing up tend to marry working women. See also Acemoglu, Autor, and Lyle (2004).

[5]For instance, improvement in women's legal rights, and female empowerment through gender-related education and training programs. See Doepke, Tertilt, and Voena (2012) for a documentation of how women's rights vary across countries.

gender-related issues within a growth framework.[6] An example is Lagerlöf (2003) who focuses on the relationship between gender discrimination and long-run growth. His model of a secular economy inspired on the European historical experience, relates gender discrimination with the industrial revolution and the demographic transition.[7] Doepke and Tertilt (2009) develop a model to explain how women's economic and legal rights are endogenously extended over the process of economic development.[8] We build our model based on this literature but we study a different question, which is how gender discrimination quantitatively impacts the economy over the development process. In our model, gender discrimination drives a wedge between women's labour productivity and female wages. We calibrate the model economy so that the long-run equilibrium matches key statistics of the United States economy, including the gender wage gap. We then explore how the equilibrium properties of the model change with the level of gender discrimination. This counterfactual exercise provides an estimate of U.S. output per capita and fertility when the gender wage gap is the same as in, for instance, Egypt. The effect of gender discrimination on the economy depends on two forces: less discrimination increases market activity by women, which directly increases output per capita; it also reduces fertility and increases capital accumulation and thus long-run equilibrium per capita output. We show how, for some countries, a very large fraction of the difference between the country's output and U.S. output is explained by differences in gender discrimination. We also provide an extension of the model in which the wedge between women's productivity and their wage decreases with capital accumulation. Our results are robust to this extension.

As to the nature of the exercise, the study most directly related to ours is Hsieh, Hurst, Jones, and Klenow (2012), who investigate the aggregate productivity gains in the United States, between 1960 and 2008, which can be attributed to decreases in labour market discrimination toward African-Americans and women. While their paper focuses on the aggregate costs of labour market

[6] There is a related literature which studies empirically the effects of gender inequality on human capital accumulation and economic development and economic growth (e.g., Daly, 2007; Dollar and Gatti, 1999; Hausmann, Tyson, and Zahidi, 2006; Seguino, 2000). Using growth accounting exercises Young (1995) found that the rise in female labour force participation accounted for between 0.6 and 1.6 percent of annual per capita growth in the four East Asian tiger economies. Our research is complementary to this literature since we provide a structural estimation instead of a reduced form estimation.

[7] See also Greenwood, Seshadri, and Yorukoglu (2005) who investigate the role of innovation in home appliances on fertility and female labour force participation.

[8] See also Hazan and Maoz (2002), who show how social norms regarding women's labour force participation change over time. Bertocchi (2011) offers a rationale for the decision to extend the franchise to females, emerging from within a politico-economic model. The driving force in her analysis is the increase in the return for intellectual labour relative to physical labour.

discrimination on occupational allocations in the United States, we take the US economy as our benchmark and then compute the costs of gender discrimination in the labour market for a cross section of countries, given endogenous savings and fertility rates.

The paper proceeds as follows. Section 1 presents the model, describes the behaviour of each agent in the economy, and defines the equilibrium. Section 2 derives some analytical results using a simplified version of the model, which is useful to provide the intuition behind our quantitative exercises. Section 3 describes the model calibration and contains simulations designed to evaluate the effects of gender discrimination on the benchmark economy. It also considers a model extension in which the wedge between women's productivity and their wage changes with capital accumulation. Finally, Section 4 contains concluding remarks.[9]

1. The Model

In this section we develop a model to study the cost of gender discrimination to output. Our strategy is to use a simple growth model with endogenous fertility and female labour market participation to assess the costs of gender discrimination.

1.1. Women and Men

Our economy is made up of men and women who live for three periods. In the first period, as children, women and men are indistinguishable, do not make any specific decision, and "consume" a fraction of their parents' time endowment, our proxy for parental care. In their second period of life, agents become adult men and women, organized as couples, and differ in their productivity in raising children. We assume that women are more productive than men in raising children, but they are equally productive in market activities. Both men and women can use one unit of time, divided between time at work and time raising children. During this second period of life, couples decide how many children to have and allocate their time between the labour market and the task of raising children. In the third period, each couple consumes the life savings.

The novelty relative to macroeconomic models of fertility and labour market participation is the introduction of gender discrimination. We assume that there are barriers to female labour

[9] There is also an appendix with some robustness exercises.

market participation in the form of wage discrimination. If we take w_t to be the labour wage rate, women receive the fraction $\phi < 1$ of this wage rate, so a lower ϕ represents a more discriminatory society.[10] Therefore, ϕ corresponds to a wedge faced by women between their labour productivity and their wage.[11] Therefore, it acts as a distortionary tax and therefore generates some type of inefficiency (misallocation) in the economy. It can be due to direct discrimination (preference-based discrimination as in Becker (1957)) and restricted job's opportunity, such as glass ceiling. We do not explicitly model how gender discrimination maps into different values for ϕ, a task conducted in Fernández, Fogli, and Olivetti (2004), Doepke and Tertilt (2011), and Rahim and Tavares (2012), who take gender discrimination in the labour market as endogenous. Discrimination costs are redistributed back to households in a non-distortionary, lump-sum manner such that, in our model, the cost that discrimination generates is solely the wedge between women's wage and their marginal productivity.[12] Giving micro-foundations for ϕ is clearly an important extension, but it goes beyond the goal of this paper, which is to measure whether or not barriers to female labour force participation might have sizeable quantitative effects on output per capita and what the channels are. A similar approach is taken by Hall and Jones (1999) and Parente and Prescott (1994), who quantify how differences in Total Factor Productivity (TFP) explain differences in output per worker in the long run; or by Hsieh and Klenow (2009), Restuccia and Rogerson (2008) and Guner, Ventura, and Yi (2008), who quantify the effects of input misallocation on productivity in different countries.

[10] A similar approach is used by Hsieh, Hurst, Jones, and Klenow (2012) and Jones, Manuelli, and McGrattan (2003), who argue that the narrowing wage gap alone explains a large part of the recent increase in female labour force participation in the United States. Lagerlöf (2003), instead, sets up a growth model in which gender differences arise endogenously in equilibrium through a coordination process. Related to this article is the model presented by Soares and Falcão (2008) in which increases in female labour force participation and reductions in the gender wage gap are the output of reductions in fertility and in mortality rates.

[11] Here we assume that ϕ is fixed. In Subsection 3.4 we let ϕ change with capital accumulation. This is indeed a more realistic assumption. However, the approach of an exogenous ϕ simplifies the analysis and the proof of existence of a balanced growth path equilibrium.

[12] This is immaterial for the qualitative nature of the results in our model. In addition, our quantitative analysis will provide conservative measures of the output costs of discrimination.

1.2. *Technology*

The production technology uses capital, K_t, and labour, L_t, to produce output, Y_t, according to a constant returns to scale production function. More specifically,

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \tag{1}$$

where $A_t = (1+\mu)^t$, and $\alpha \in (0,1)$. Parameter $\mu \geq 0$ corresponds to the rate of technical progress. Given the technology and input prices, the representative firm chooses inputs so that profits are maximized.[13] Let w_t be the wage rate and r_t^K denote the rental rate of capital. The first-order conditions associated with the representative firm's problem are:

$$w_t = (1-\alpha)K_t^{\alpha}(A_t L_t)^{-\alpha} A_t, \tag{2}$$

$$r_t^K = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha}. \tag{3}$$

1.3. *Preferences*

As suggested above, couples draw utility from consumption in their second and third period of life, and from the number of children. Let n_t be the number of children born at period t,[14] and c_t and d_{t+1} be the consumption of a couple in their second and third period of life, respectively. Preferences are represented by

$$U_t = \ln(c_t + \bar{\mathbf{c}}_t) + \beta \ln d_{t+1} + \gamma \ln n_t, \;\; \beta, \gamma \in (0,1), \tag{4}$$

where β is the subjective discount factor and γ represents the relative weight of children in the couple's utility function. Quantity $\bar{\mathbf{c}}_t$ corresponds to home produced consumption goods. As in Greenwood, Seshadri, and Vandenbroucke (2005), $\bar{\mathbf{c}}_t$ generates a decline in fertility as the economy develops. See Hazan and Berdugo (2002) for a similar approach. We assume that home production increases with technical change, such that $\bar{\mathbf{c}}_t = A_t \bar{\mathbf{c}}$, where $\bar{\mathbf{c}} > 0$.[15]

[13] Output is taken as the numeraire.

[14] Since the household is organized as a couple, we could interpret n_t as the number of couples generated by each household.

[15] Therefore, there is technical change in home production, as suggested by Cavalcanti and Tavares (2008) and Greenwood, Seshadri, and Yorukoglu (2005). In addition, this generates a long-run balanced growth path equilibrium

Let h_t^w and h_t^h denote the time of the wife and the husband spent in raising children. In the spirit of Greenwood, Seshadri, and Vandenbroucke (2005), we assume that children consume time resources according to the equation

$$n_t = D(h_t^w + \eta h_t^h)^\theta, \;\; D > 0, \;\; \theta \in (0,1). \tag{5}$$

Parameter $D > 0$ and $\theta > 0$ determine the level and curvature, respectively, of the production function to raise children. We assume that $\eta \in (0,1)$, which implies that women are more productive than men in raising children, as emphasized by Albanesi and Olivetti (2009) in relation to women's ability to breastfeed.[16]

1.4. Budget Constraints

The couple's budget constraints are:

$$c_t + s_t \leq w_t(1 - h_t^h) + \phi w_t(1 - h_t^w) + \varpi_t, \tag{6}$$

$$d_{t+1} \leq (1 + r_{t+1})s_t, \tag{7}$$

where s_t represents investment and the right-hand side shows net income of the couple. Variable ϖ_t stands for transfers to households. We assume that discrimination costs are redistributed lump-sum back to households. This implies that the cost of discrimination we calculate is due only to a wedge between women's marginal productivity of labor and their wage. In other words, the effect is due to misallocation and distortionary discrimination, not to any waste of resources.

Couples choose the level of consumption when young, c_t, and when old, d_{t+1}, the number of children n_t, the fraction of time allocated to household chores h_t^w and h_t^h, and savings, s_t, all this so as to maximize (4) subject to (5), (6), and (7). It can be shown that when $h_t^w < 1$ then $h_t^h = 0$,[17] and

$$h_t^w = \frac{\gamma\theta}{(1 + \beta + \gamma\theta)}[\frac{1+\phi}{\phi} + \frac{\bar{\mathbf{c}}_t}{\phi w_t} + \frac{\varpi_t}{\phi w_t}], \tag{8}$$

in which per worker capital and output grow at the rate of technical change.

[16] In Appendix 5 we present a model based on Galor and Weil (1996) in which there are two types of labour, mental and physical labour, and for each unit of time men have a higher endowment of physical labour than women but an equal endowment of mental labour. Quantitative results are similar using both approaches.

[17] Observe that, for $h_t^w < 1$ and $h_t^h > 0$ we would need $\eta \geq 1/\phi$, which cannot ever be the case since $\phi < 1$ and $\eta < 1$.

which is increasing in gender inequality (lower ϕ). When $h_t^w < 1$ then $h_t^h = 0$, then consumption and investment are:

$$c_t = \frac{1}{1+\beta+\gamma\theta}[(1+\phi)w_t - (\gamma\theta+\beta)\bar{\mathbf{c}}_t + \omega_t], \tag{9}$$

$$s_t = \frac{\beta}{1+\beta+\gamma\theta}[(1+\phi)w_t + \bar{\mathbf{c}}_t + \omega_t], \tag{10}$$

$$d_{t+1} = (1+r_{t+1})s_t. \tag{11}$$

But it can be the case that $h_t^w = 1$ and $h_t^h \in (0,1)$. Then, we have that

$$h_t^h = \frac{\gamma\theta}{(1+\beta+\gamma\theta)}[\frac{1+\eta}{\eta} + \frac{\bar{\mathbf{c}}_t}{w_t} + \frac{\varpi_t}{w_t}] - \frac{1}{\eta}. \tag{12}$$

If η is sufficiently small ($\eta \to 0$), then $h_t^h = 0$ for any finite wage. We make this assumption here and investigate the model in which $h_t^h = 0$ and $h_t^w \in (0,1]$.

1.5. Equilibrium

Let P_t denote the number of adult households in period t. In equilibrium, demand equals supply in all markets. In the labour market this means that $L_t = P_t(2 - h_t^w)$. Let $\hat{k}_t$ be the capital level per unit of efficiency couple, i.e., $\hat{k}_t = \frac{K_t}{A_t P_t}$. We also have $\varpi_t = (1-\phi)(1-h_t^w)w_t$, which corresponds to the resources firms lose because they discriminate. Transfers are equal to the cost of discrimination. Then, using equation (2) into (8), yields

$$h_t^w = \min\{1, \frac{\gamma\theta}{(1+\beta)\phi+\gamma\theta}[2 + \frac{\bar{\mathbf{c}}}{(1-\alpha)\hat{k}_t^\alpha(2-h_t^w)^{-\alpha}}]\}, \tag{13}$$

From the expression above, a necessary condition for women to participate in the labour market is that

Assumption 1: $\frac{2\gamma\theta}{(1+\beta)\phi+\gamma\theta} \leq 1 \Rightarrow \phi \geq \frac{\gamma\theta}{1+\beta}$.

Assumption 1 implies a limit for the size of gender discrimination in the labour market. We have that

$$n_t = D[\min\{1, \frac{\gamma\theta}{(1+\beta)\phi+\gamma\theta}[2 + \frac{\bar{\mathbf{c}}}{(1-\alpha)\hat{k}_t^\alpha(2-h_t^w)^{-\alpha}}]\}]^\theta, \tag{14}$$

and the following proposition holds:

PROPOSITION 1. *Let Assumption 1 be satisfied so that women participate in the labour market. Then we have:*

(i) *Female hours of work in the market increase with capital accumulation, $\hat{k}_t$, and decrease with labour market discrimination (low ϕ);*

(ii) *Fertility decreases with capital accumulation, $\hat{k}_t$, and increases with labour market discrimination (low ϕ).*

Proof. Equation (13) determines h_t^w as an implicit function of $\hat{k}_t$, $\psi(\hat{k}_t, \phi)$, and a critical value $\hat{k}^*(\phi)$ such that

$$h_t^w = \begin{cases} 1 & \text{for } \hat{k}_t \leq \hat{k}^*(\phi), \\ \psi(\hat{k}_t, \phi) & \text{for } \hat{k}_t \geq \hat{k}^*(\phi), \end{cases} \tag{15}$$

and $\psi(\hat{k}_t, \phi) \in (0, 1]$ $\forall$ $\hat{k}_t \geq \hat{k}^*(\phi)$, where:

$$\hat{k}^*(\phi) = [\frac{\bar{\mathbf{c}}\gamma\theta}{(1-\alpha)(\phi(1+\beta)-\gamma\theta)}]^{\frac{1}{\alpha}}. \tag{16}$$

Using the implicit function theorem we can show that $\psi_1(\hat{k}_t, \phi) < 0$ and $\psi_2(\hat{k}_t, \phi) < 0$, as long as $\hat{k}_t \geq \hat{k}^*(\phi)$. The second part of the proof is trivial since $n_t = D(h_t^w)^\theta$ and $D > 0$ and $\theta > 0$. □

Proposition 1 implies that time devoted to home activities decreases with capital accumulation. Observe that when barriers to female labour force participation are high (ϕ is low), women work fewer hours in the market. Since fertility is an increasing function of hours at home, the number of children decreases with capital accumulation and increases with gender discrimination in the form of barriers to female labour force participation.

In equilibrium we have that savings are given by:

$$s_t = \begin{cases} \frac{\beta}{1+\beta}[w_t + \bar{\mathbf{c}}_t] & \text{for } h_t^w = 1, \\ \frac{\phi\beta}{(1+\beta)\phi+\gamma\theta}[2w_t + \bar{\mathbf{c}}_t] & \text{for } h_t^w < 1. \end{cases} \tag{17}$$

The condition that equilibrates the capital market is

$$K_{t+1} = P_t s_t. \tag{18}$$

Since $P_{t+1} = n_t P_t$, we have that capital per effective unit of workers is

$$\hat{k}_{t+1} = \frac{s_t}{(1+\mu)A_t n_t}. \tag{19}$$

Using equations (14) and (17) into (19), we have that:

$$\hat{k}_{t+1} = \begin{cases} \frac{\beta[(1-\alpha)\hat{k}_t^{\alpha}+\bar{\mathbf{c}}]}{D(1+\beta)(1+\mu)} & \text{for } \hat{k}_t \leq \hat{k}^*, \\ \\ \Delta(\phi)[(1-\alpha)\hat{k}_t^{\alpha}(2-h_t^w)^{-\alpha}]^{\theta}[\bar{\mathbf{c}} + 2(1-\alpha)\hat{k}_t^{\alpha}(2-h_t^w)^{-\alpha}]^{1-\theta} & \text{for } \hat{k}_t \geq \hat{k}^*, \end{cases} \tag{20}$$

where $\Delta(\phi) = \frac{\phi\beta}{D(1+\mu)[(1+\beta)\phi+\gamma\theta]^{1-\theta}(\gamma\theta)^{\theta}}$ is a positive constant. Using (15) into (20) defines a non-linear difference equation $\hat{k}_{t+1} = \xi(\hat{k}_t)$.

Proposition 2. *Let Assumption 1 be satisfied. Then:*

(i) *If* $\bar{\mathbf{c}} = 0$, *then there exists a unique globally stable steady-state equilibrium for capital per effective unit of labour,* $\hat{k}_{t+1} = \hat{k}_t = \hat{k}^{ss}$, *such that female hours of work in the market are constant;*

(ii) *If* $\bar{\mathbf{c}}$ *is sufficiently small, then there exists at least one locally stable steady-state equilibrium for capital per effective unit of labour,* $\hat{k}_{t+1} = \hat{k}_t = \hat{k}^{ss}$, *such that in the neighbourhood of* $\hat{k}^{ss}$ *female hours of work in the market increase with capital accumulation.*

Proof. When $\bar{\mathbf{c}} = 0$, then

$$h_t^w = h^w(\phi) = \frac{2\gamma\theta}{(1+\beta)\phi + \gamma\theta},$$

which is less than one by assumption 1. In this case, we have that

$$\hat{k}_{t+1} = 2^{1-\theta}\Delta(\phi)(1-\alpha)\hat{k}_t^{\alpha}\left(\frac{2(1+\beta)\phi}{(1+\beta)\phi+\gamma\theta}\right)^{-\alpha}.$$

Therefore, we can easily show that the difference equation $\hat{k}_{t+1} = \xi(\hat{k}_t)$ is strictly concave in $\hat{k}_t$ and $\lim_{\hat{k}_t \to 0} \xi'(\hat{k}_t) = \infty$ and $\lim_{\hat{k}_t \to \infty} \xi'(\hat{k}_t) = 0$. This implies that there exists a unique globally stable steady-state equilibrium for $\hat{k}$. When $\bar{\mathbf{c}} > 0$, then the difference equation $\hat{k}_{t+1} = \xi(\hat{k}_t)$ is represented by (20). There are several possible cases. However, when $\bar{\mathbf{c}}$ is sufficiently small such that $\hat{k}^*(\phi)$ is smaller than the capital stock per effective unit of labour in which the capital accumulation equation for $h_t^w = 1$

$$\hat{k}_{t+1} = \frac{\beta[(1-\alpha)\hat{k}_t^{\alpha} + \bar{\mathbf{c}}]}{D(1+\beta)(1+\mu)}$$

crosses the 45 degree line, then we can ensure that the difference equation $\hat{k}_{t+1} = \xi(\hat{k}_t)$ has at least one locally steady-state equilibrium, $\hat{k}_{t+1} = \hat{k}_t = \hat{k}^{ss}$, such that in the neighbourhood of $\hat{k}^{ss}$ female hours of work in the market increase with capital accumulation. This follows from the fact that $\hat{k}_{t+1} = \xi(\hat{k}^*(\phi)) > \hat{k}^*(\phi)$ and we can show that $\lim_{k_t\to\infty} \xi'(\hat{k}_t) = 0$, which completes the proof since $\xi(\hat{k}_t)$ is a continuous function of $\hat{k}_t$. □

When $\bar{\mathbf{c}} = 0$ preferences are homothetic and the substitution effect implies that fertility should decrease with capital accumulation and the wage rate, while the income effect implies that fertility increases with capital accumulation and the wage rate. The log-utility implies that the substitution and income effects exactly cancel each other out, such that fertility and therefore female hours of work in the market are constant and do not vary with capital accumulation. When $\bar{\mathbf{c}} > 0$, then preferences are not homothetic and households spend a larger fraction of their income on consumption in the first period of life as their labour income rises so that capital accumulation leads to a decrease in fertility.

2. Gender Barriers and Development: Analytical Results

In this section we derive some analytical results. For the model with $\bar{\mathbf{c}} = 0$ we can analytically find the steady-state equilibrium of the capital stock per effective worker.

$$\hat{k}^{ss}(\phi) = \begin{cases} \left(\frac{\beta(1-\alpha)}{D(1+\beta)(1+\mu)}\right)^{\frac{1}{1-\alpha}}, & \text{if } \phi \leq \frac{\gamma\theta}{1+\beta}, \\ \frac{\phi}{((1+\beta)\phi+\gamma\theta)^{\frac{1-\theta-\alpha}{1-\alpha}}} \left[\frac{2^{1-\theta-\alpha}\beta(1-\alpha)(1+\beta)^{-\alpha}}{D(1+\mu)(\gamma\theta)^{\theta}}\right]^{\frac{1}{1-\alpha}}, & \text{if } \phi > \frac{\gamma\theta}{1+\beta}. \end{cases} \tag{21}$$

For any $\phi > \frac{\gamma\theta}{1+\beta}$, it is straightforward to see that an increase in gender discrimination, i.e. a lower ϕ, implies a decrease in the long-run capital stock per effective worker. Moreover, output per effective unit of labour is given by

$$\hat{y}^{ss}(\phi) = \begin{cases} \left(\frac{\beta(1-\alpha)}{D(1+\beta)(1+\mu)}\right)^{\frac{\alpha}{1-\alpha}}, & \text{if } \phi \leq \frac{\gamma\theta}{1+\beta}, \\ \frac{\phi(2(1+\beta))^{1-\alpha}}{((1+\beta)\phi+\gamma\theta)^{\frac{1-\alpha(1+\theta)}{1-\alpha}}} \left[\frac{2^{1-\theta-\alpha}\beta(1-\alpha)(1+\beta)^{-\alpha}}{D(1+\mu)(\gamma\theta)^{\theta}}\right]^{\frac{\alpha}{1-\alpha}}, & \text{if } \phi > \frac{\gamma\theta}{1+\beta}. \end{cases} \tag{22}$$

When $\phi > \frac{\gamma\theta}{1+\beta}$ output per effective unit of labour is decreasing with gender discrimination in the labour market. If $\phi \leq \frac{\gamma\theta}{1+\beta}$, then women will not participate in the labour market and discrimination will not have any effect on output.

PROPOSITION 3. *Let Assumption 1 be satisfied such that $\phi > \frac{\gamma\theta}{1+\beta}$ and $\bar{\mathbf{c}} = 0$. Consider two economies (economy A and economy B) in which the only difference between these two economies is the level of gender barriers to female labour force participation. Then:*

(i) *the ratio of output per effective worker of the two economies is given by:*

$$\frac{\hat{y}^{ss}(\phi^A)}{\hat{y}^{ss}(\phi^B)} = \frac{\phi^A}{\phi^B}\frac{((1+\beta)\phi^B + \gamma\theta)^{\frac{1-\alpha(1+\theta)}{1-\alpha}}}{((1+\beta)\phi^A + \gamma\theta)^{\frac{1-\alpha(1+\theta)}{1-\alpha}}}. \tag{23}$$

(ii) *Moreover, in the limiting case in which $\alpha(1+\theta) \to 1$ (e.g., $\alpha \to \frac{1}{2}$ and $\theta \to 1$), then the ratio of output per worker of the two economies is given by the ratio in the level of gender barriers to female labour force participation, i.e.,*

$$\lim_{\alpha(1+\theta)\to 1} \frac{\hat{y}^{ss}(\phi^A)}{\hat{y}^{ss}(\phi^B)} = \frac{\phi^A}{\phi^B}. \tag{24}$$

Proof. This follows directly from equation (22). □

In the particular case of the second item of Proposition 3, differences in output per worker amongst similar economies will be proportional to differences in gender barriers to female labour force participation as long as $\phi > \frac{\gamma\theta}{1+\beta}$.[18] Since in this model gender barriers are reflected in the gender wage gap, we could map differences in output per effective worker with reference to differences in the gender wage gap. For instance, if we consider a counterfactual economy similar to the United States but with the gender wage gap of Egypt, then output per effective worker in the hypothetical economy would be 37 percent of the one in the United States, as long as $\phi > \frac{\gamma\theta}{1+\beta}$.[19]

[18] There is a limit of how gender discrimination in the labour market affects output per effective labour, since $\hat{y}^{ss}(\phi) \geq \left(\frac{\beta(1-\alpha)}{D(1+\beta)(1+\mu)}\right)^{\frac{\alpha}{1-\alpha}}$.

[19] Otherwise, the output per effective worker of this counterfactual economy will be higher than 37 percent of the United States economy.

Output per capita is given by

$$y_t = \frac{Y_t}{n_t P_t + P_t + \frac{P_t}{n_{t-1}}} = \frac{Y_t}{P_t} \frac{1}{(n_t + 1 + \frac{1}{n_{t-1}})} = A_t \hat{y}_t \frac{1}{(n_t + 1 + \frac{1}{n_{t-1}})}.$$

In the long run, the ratio in per capita output for economies A and B in the case in which $\bar{\mathbf{c}} = 0$, $\phi > \frac{\gamma\theta}{1+\beta}$, and $\alpha(1+\theta) \to 1$ would be given by

$$\frac{y_t^{ss}(\phi^A)}{y_t^{ss}(\phi^B)} = \frac{\phi^A}{\phi^B} \frac{(n(\phi^B) + 1 + \frac{1}{n(\phi^B)})}{(n(\phi^A) + 1 + \frac{1}{n(\phi^A)})}. \tag{25}$$

When population is still growing over time, i.e., $n(\phi^i) > 1$, for $i \in \{A, B\}$, then the difference in output per capita between economies A and B will be larger than the difference in output per effective worker of these two economies.[20]The reason is that since fertility is higher when gender barriers to female labour force participation are higher, then the total dependency ratio, the ratio of those out of the labour force (children and old adults) to those in the labour force, is larger for economies with smaller ϕ.

What are the effects of gender discrimination in the labour market on long-run welfare? On the one hand, gender discrimination decreases capital accumulation and therefore output, consumption, and consequently welfare. On the other hand, it increases fertility, which has a positive effect on the utility of households. The utility costs of gender wage gaps might therefore be lower than the output costs. For the case in which $\bar{\mathbf{c}} = 0$ and $\phi > \frac{\gamma\theta}{1+\beta}$, it can be shown that in the long run the indirect utility of the representative household is given by

$$V(\phi) = (1+\beta)\left[\ln(\phi) - \left(\frac{(1-\alpha(1+\theta))}{1-\alpha} + \theta(\gamma+\beta)\right)\right] \ln[(1+\beta)\phi + \gamma\theta] + \vartheta_t, \tag{26}$$

where ϑ_t is a term which depends on the time trend t and all constants except ϕ. We can see that when the gender wage gap increases (ϕ decreases), then there are two effects on welfare: (i) the first term on the right-hand side of (26) implies that welfare decreases; while the second term on the right-hand side of (26) suggests that welfare increases. Taking the derivative of $V(\phi)$ with respect

[20]Quantity $(n(\phi) + 1 + \frac{1}{n(\phi)})$ is negative related to ϕ when $n(\phi) < 1$.

to ϕ implies that:

$$V'(\phi) = (1+\beta)^2\theta \frac{[\frac{\gamma(1-\phi)}{1+\beta} + \phi(\frac{\alpha}{1-\alpha} - \frac{\beta}{1+\beta})]}{\phi[(1+\beta)\phi + \gamma\theta]}. \tag{27}$$

This derivative is positive if and only if the following condition holds:

$$\frac{\gamma(1-\phi)}{\phi} + \frac{\alpha(1+\beta)}{1-\alpha} > \beta. \tag{28}$$

Observe that a sufficient condition for this inequality to hold is that $\frac{\alpha(1+\beta)}{1-\alpha} > \beta$ and this is satisfied for the common estimate of capital share of income ($\alpha = \frac{1}{3}$), as documented by Gollin (2002).[21] In addition, we expect condition (28) to hold for small values of ϕ. Therefore, a decrease in gender barrier to female labour force participation (an increase in ϕ) should, in general, increase welfare in discriminatory societies.

It is important to highlight that the analytical results were derived for a particular model in which fertility and female hours worked in the labour market are constant over time. Results were also derived for the long-run balanced growth path equilibrium and transitional dynamics were not considered. In order to solve for the transitional dynamics and to quantify the effects of gender barriers to female labour market participation in a more general framework in which fertility is not necessarily constant, we need to solve the model numerically. This is what we do next.

3. Gender Barriers and Development: Numerical Results

In this section we quantify the cost of gender discrimination on output per capita. Our strategy is to choose parameter values consistent with empirical observations in the United States and then perform counter-factual analysis by investigating the effects of gender barriers to female labour force participation on output per capita and other statistics.

3.1. Measurement: Replicating a Baseline Economy

Table 1, Part I, provides all parameter values as well as a note on how each parameter was obtained. Below, we describe our calibration in detail. The model period in our economy is taken to be 25 years. Therefore, each agent lives for 75 years. We start the model period in 1900 and the

[21] If $\alpha = \frac{1}{3}$, then condition (28) implies that $\beta < 1$, which is true by assumption.

corresponding final year of analysis is after four periods, or in 2000.[22] The capital share α is set to 0.40, which is consistent with Gollin (2002). We set the value of μ such that total factor productivity (TFP) growth in the United States is equal to 1.5 percent per year, which is consistent with the TFP growth rate in the U.S. in the second half of the 20th Century (e.g., Greenwood, Seshadri, and Vandenbroucke, 2005).[23] We set the discount factor β such that the real rate of return of a risk free bond is roughly 2.2 percent, which is the average historical real rate of return on T-Bills from 1975 to 2000.[24] The altruism factor, γ, is calibrated so that the population is constant in the long-run equilibrium.[25] In the model, the fertility rate is 2.02 in 2000, which is close to the observed United States level - fertility rate adjusted for survival to age 10 in the United States is roughly 2 in 2000.[26] We set the values of the remaining five parameters - $\hat{k}_0$, $\bar{\mathbf{c}}$, ϕ, D, and θ - so that we approach five empirical observations for the U.S. economy:

i. The ratio of per capita income in 2000 relative to its level in 1900.[27]

ii. The fertility rate adjusted for survival to age 10 in 1900.[28]

iii. The female to male earnings in 2000;[29]

iv. The ratio of female to male hours of work in 2000;[30]

[22] After four periods the capital stock per unit of efficiency couple will be 95 percent of its steady-state level.

[23] According to Greenwood, Seshadri, and Vandenbroucke (2005), the annual growth rate of total factor productivity (TFP) in the United States was 1.41 percent between 1900 and 1948 and jumped to about 1.68 percent between 1948 and 1974. After 1974 there was a productivity slowdown as the TFP growth rate decreased by about 0.57 percent. From 1995 to 2000 the TFP growth rate increased to about 1.2 percent per year. We normalize the initial TFP value to $A_0 = 1$.

[24] This is the T-Bills 12 months nominal interest rate minus the consumer price index. For the T-Bills we use the H15 table from the Board of Governors of the Federal Reserve System and the inflation of the consumer price index we took from the international finance statistics of the IMF.

[25] In the model, a household means a couple and their children. Therefore, $n = 1$ implies a fertility rate of 2 in the data.

[26] See Goss (2010).

[27] According to Maddison (2006), the 2000 real per capita income in the United States was about 7 times higher than its level in 1900.

[28] According to Goss (2010) the fertility rate in 1900 adjusted by survival to age 10 was about 2.6. In our calibration, $\bar{\mathbf{c}} = 0.4$, which is a number close to the number used by Greenwood, Seshadri, and Vandenbroucke (2005) in a similar environment.

[29] Data from the United Nations (2005) show that the female to male earnings ratio is equal to 63 percent in the United States. This dataset uses National Accounts information and estimates the female to male earnings ratio using the non-agricultural wage, the female and male participation rates, and the female and male total populations. Using the Panel Study Income Dynamics (PSID) Olivetti and Petrangolo (2008) show that the gender wage gap is about 67.5 percent. We will make cross-country comparisons and the United Nations (2005) provide estimate of the female to male earnings ratio for a large sample of countries, including developing countries. Therefore, for comparison reasons we let the female to male earnings ratio to be equal to 63 percent in the baseline economy.

[30] According to Erosa, Fuster, and Restuccia (2005), women worked 40 percent fewer hours than men. Men worked on average 37.6 hours per week while women worked 26.7 hours per week. About half of this difference in hours of

Table 1: *Parameter values, basic statistics, baseline economy*

Part I: Parameter Values		
Parameters	*Values*	*Comment/Observations*
α	0.4	Capital share based on Gollin (2002)
μ	0.4509	Rate of TFP growth based on Greenwood et al (2005)
β	0.3747	Calibrated to match the T-Bill annual real rate of return, 2.2%
γ	0.1220	Constant population level along the steady-state
D	2.184	Calibrated to match the average private cost of children/GDP
θ	0.965	Calibrated to match hours worked by women relative to hours worked by men in 2000
ϕ	0.63	Calibrated to match the U.S. female to male earnings ratio in 2000
$\bar{c}$	0.4	Calibrated to match the U.S. fertility rate adjusted for survival in 1900, Goss (2010)
$\hat{k}_0$	0.0505	Calibrated to match the U.S. $y^{US}_{2000}/y^{US}_{1900}$, (Maddison (2006))
Part II: Basic Statistics		
	U.S. economy	*Baseline economy*
ϕ	63%	63%
y_{2000}/y_{1900}	7.0	7.0
$1 - h^w_{2000}/1 - h^h_{2000}$	60%	55%
$\phi w_{2000} h^w_{2000}/y_{2000}$	40%	33%
Real interest rate, 1975-2000	2.2%	2.4%
Fertility rate, 1900	2.6	2.43

v. the average private cost of children (ie., the opportunity cost of staying at home) as a share of GDP per capita.[31]

Observe that the calibrated model matches most of the target values (see Table 1, Part II), except the ratio of female to male hours of work in 2000 and the private cost of children as a share of the GDP per capita.[32] Our model implies that women spend about 18 percent less hours in home activities in 2000 than in 1900. Estimates from Ramey and Francis (2009) suggest that the number

work is accounted for by the gender difference in hours per worker (intensive margin) while the remaining part is accounted for by the gender difference in participation (extensive margin). We target the overall difference in hours worked since there is no extensive margin decision in our model.

[31] According to Haveman and Wolfe (1995) this ratio is equal to 40 percent in the United States. See also Doepke, Hazan, and Maoz (2007).

[32] The difference between the data value and the model value were 8 percent and 17.5 percent for the ratio of female to male hours of work in 2000 and the private cost of children as a share of the GDP per capita, respectively. We define $G(\Omega)$ as the vector containing the absolute percent deviation between model moments and data moments at a vector of parameters $\Omega = (\hat{k}_0, \bar{c}, \theta, D, \beta)$. We chose $\hat{\Omega}$ to minimise the mean of $G(\Omega)$. In our calibration the mean of $G(\hat{\Omega})$ is less than 5 percent. The median is less than 1 percent.

of hours per woman in home production decreased by 40 percent from 1900 to 2000.[33] Our model thus underestimates the reduction in the number of hours spent by women in home activities over the development process. However, we highlight that in our model the driving force in the reduction of time spent in home activities is capital accumulation. As argued by Greenwood, Seshadri, and Yorukoglu (2005), there are other factors, such as technical progress in the home sector, that are important in accounting for the reduction in hours of housework.[34] Following Greenwood, Seshadri, and Vandenbroucke (2005), we could have increased parameter D in 1950 to mimic the technical progress that occurred in the home sector. This, however, would not have added any new insight to our analysis. In our model the capital-to-output ratio in 2000 is 5.77, which is higher than the one observed in the United States economy, which is roughly 3.5 (e.g., Chen, İmrohoroğlu, and İmrohoroğlu, 2009). However, in our model we do not consider intangible capital. McGrattan and Prescott (2011) show that if one considers investment in R&D and innovation as intangible capital, then the average capital-to-output ratio in the United States is 5.77.

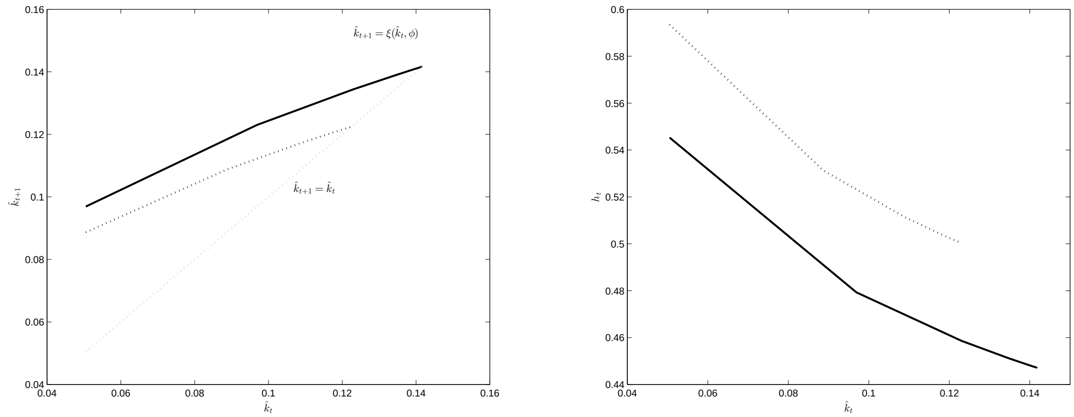
Figure 1 shows the evolution of the baseline economy, represented by the solid line. Figure 1(a) describes the evolution of the capital stock, with $\hat{k}_{t+1}$ on the y axis and $\hat{k}_t$ on the x axis, and the steady-state is found where this line is crossed by the 45 degree line. Simulations with the baseline parameter values show that there is a unique steady-state equilibrium for $\hat{k}_t > 0$.[35] Figure 1(b) shows the mechanics of the increase in women's hours worked: as capital is accumulated, then the opportunity cost of staying at home decreases fertility, and increases female labour market participation. The dotted black line in both graphs describes an economy with a female-to-male earnings ratio in 2000 of 56.7 percent instead of 63 percent, as in the baseline economy. Observe that, in this case, the capital per unit of efficiency couple is lower and women work fewer hours in the market. In the following section we exploit these "cross-section" changes further.

[33] According to Ramey and Francis (2009), women spent on average about 50 hours per week in home activities in 1900, compared with about 30 hours per week in 2000.

[34] In fact, Cavalcanti and Tavares (2008) show that a decrease in the relative price of home appliances has a first-order effect in female labour force participation. In the current paper we abstract from technical progress in the home sector, so we underestimate the reduction of hours in home production. Finally, Cavalcanti and Tavares (2011) show that the process of development is accompanied in virtually all countries by two changes in economic structure: the increase in the share of government spending in GDP, and the increase in female labour force participation. They find evidence that these two changes are causally related.

[35] We consider a fine grid for the capital stock, $\hat{\mathcal{K}} = [0 < \hat{k}^1 < ... < \hat{k}^n]$, such that $\hat{k}^n = 100 \times \hat{k}$, in which $\hat{k}$ is the steady-state value for the capital stock. In this grid we show that there is a unique-steady-state equilibrium for $\hat{k} > 0$.

Figure 1: Baseline economy.



(a) Evolution of capital per unit of efficiency couple. (b) Hours worked versus capital per unit of efficiency couple.

Notes: Black solid line: Baseline economy; dotted black line: economy with 10 percent more gender inequality.

3.2. Measurement: The Output Cost of Gender Discrimination

We now explore how the equilibrium properties of the model calibrated in the previous section change with gender discrimination, measured by the female to male earnings ratio. We vary parameter ϕ and examine the model's predictions along three dimensions: output per capita as a fraction of U.S. output per capita; female to male earnings ratio; and women's hours worked in the market. All statistics correspond to what would be observed in 2000.

Table 2 shows that as gender discrimination in labour market activities increases, the level of per capita output decreases, and hours spent by women in home activities increase. The effect of ϕ on output per capita is sizeable: a decrease in ϕ by a factor of two decreases output per capita by approximately 50 percent,[36] while hours at home increases by approximately 98 percent.[37] Welfare changes in the same direction of output, however, the welfare effects of gender barriers are smaller. Welfare effects are measured by the decrease/increase in permanent consumption such that the utility of the representative household in the baseline economy is similar to the economy with different gender barriers.[38] A decrease in ϕ by a factor of two decreases welfare by roughly 10

[36] Output per worker decreases by 42 percent.

[37] $1 - h_t$ can be interpreted as the fraction of the female population that participates in labour market activities in a homogeneous couple setup.

[38] Let the welfare measure be denoted by φ, then we have that $\varphi = \left(\frac{(c_{b,t} + \bar{\mathbf{c}}_t)}{(c_{a,t} + \bar{\mathbf{c}}_t)} (\frac{d_{b,t+1}}{d_{a,t+1}})^{\beta} (\frac{n_{b,t}}{n_{a,t}})^{\gamma} \right)^{\frac{1}{1+\beta}}$, where subscript '$b$' denotes the baseline economy and 'a' the economy with a different level of gender barriers (ϕ). The welfare reported

Table 2: *Gender inequality and development: Quantitative properties of the model*

	Output per capita, % baseline	Female to male earnings ratio	Hours at home, % baseline	Welfare % of consumption	Output per capita, % baseline (constant fertility)
U.S. (baseline)	100	63	100	0	100
$\phi = \frac{1}{1.5} \times \phi_{\text{base}}$	69	42	152	-5.64	85
$\phi = \frac{1}{2} \times \phi_{\text{base}}$	50	32	198	-9.87	72
$\phi = \frac{1}{3} \times \phi_{\text{base}}$	42	21	222	-13.67	51

percent. This is a large effect on welfare but substantially smaller than the output costs.

As barriers to female labour market participation increase (that is, ϕ decreases), there are two channels through which per capita output decreases.[39] First, output per capita decreases because women work fewer hours in the market (h_t decreases), and so output decreases for the same population. Second, output per capita also decreases because discrimination discourages women to work more hours in the market and decreases the couple's total income, leading couples to choose to have more children, that is, increase n_t.[40] What is the relative quantitative importance of the two effects in the overall impact of discrimination?

In the last column of Table 2 we present results for output per capita in the baseline economy when fertility is kept constant. We have solved a standard overlapping generations economy without fertility in which we feed exogenous values of h_t into the model as observed in each previous experiment. In this case, we are isolating the first channel through which gender discrimination affects output per capita, that is, the effect working solely through number of hours worked by women.[41] When the female to male earnings ratio decreases by a factor of two, output per capita, in the constant fertility case, decreases by 28 percentage points, compared to 50 percentage points in the first column. The effect of discrimination through women's hours at work accounts for about 46 percent of the total reduction in output observed in the model with endogenous fertility.

is the one of an economy with a different ϕ at 2000. When ϕ is reduced welfare is also reduced during the transition and along the balanced growth path equilibrium.

[39] Per capita output in this model is given by: $y_t = \frac{Y_t}{n_t L_t^p + L_t^p + \frac{L_t^p}{n_{t-1}}}$. The first term in the denominator corresponds to the number of existing children, the second term is the number of young couples, and the third term is the number of elderly couples.

[40] In our model, as discrimination limits utility gains through female participation and higher consumption, couples opt for increases in utility through fertility. This effect also accounts, in a larger model, for the lower opportunity cost of time spent at home, which is reflected in the decision to have more children.

[41] We can infer the role of fertility in the output decrease as the difference between the first and the last column.

There is a limit on how gender barriers to female labour force participation can affect aggregate output and welfare. As ϕ decreases, then h_t^w increases until $h_t^w = 1$. Further decreases in ϕ would not change the amount of hours devoted to household chores once $h_t^w = 1$.[42] The last row of Table 2 provides the case in which gender barriers are large and women do not participate in the labour market. Output would decrease by a 2.38 factor.

3.3. Measurement: Counterfactual Analysis

The exercises in the previous section describe the quantitative properties of the model for systematic variations in gender discrimination through wage inequality. We now feed the model with independent estimates of the female to male earnings ratio for several economies, keeping the other parameters exactly as in the baseline U.S. economy. The purpose of this counterfactual exercise is to assess how much the level of U.S. output per capita would decrease if gender discrimination were the same as in, say, Egypt. This will provide us with a first-ever macroeconomic estimate of how much of the existing difference in output per capita between Egypt and the United States can be accounted for by differences in gender inequality in pay. In effect, we conduct this exercise for a large sample of countries. For each country, we feed in an independent estimate of gender wage inequality and compare the model's predictions with the relevant country data. We keep all parameters at their baseline values, except parameter ϕ, which we adjust until the female to male earnings ratio is similar to what is observed in the data.[43] Table 3 reports results for selected economies.

We find that when fertility is endogenous, gender wage discrimination explains a large fraction of the difference in output per capita between some countries (see Table 3, Part I) and the United States. In the case of Saudi Arabia, barriers to female labour force participation explain almost the entire gap in relative output per capita. On the other hand, for Ireland the model over predicts the output gap between Ireland and the United States. Notice that, were the United States to have the level of gender pay inequality observed in Egypt, output per capita would be 42 percent of its

[42] Similar results will hold also when η is not sufficiently small and $h_t^h \in (0,1)$.

[43] In Appendix 6 we discuss three issues related to this approach: First, we consider an alternative measure of the gender wage gap, which is based on the "unexplained" gender wage gap. We show that for the sample of countries in which we have measures of the "unexplained" gender wage gap, results are similar using both measures. Second, we discuss how selection bias in female labour force participation could change our quantitative results. Finally, we consider issues related to the fact that countries might be in different stages of growth.

Table 3: *Empirical data and model predictions for reference economies. Source: United Nations (2005)*

Countries	**Data**		**Model**		
	Output per capita, % of baseline	Female to male earnings ratio	Output per capita, % of baseline	Female to male earnings ratio	Output per capita, % of baseline (const fert)
U.S. (baseline)	100	63	100	63	100
Part I					
Ireland	91	53	87	53	94
Greece	56	55	90	55	96
Singapore	71	51	84	51	93
Saudi Arabia	37	16	42	16	65
Iran	19	39	64	39	82
Egypt	10	23	42	23	65
India	8	31	49	31	71
Part II					
Finland	77	71	109	71	103
Norway	99	77	114	77	105
Sweden	77	81	117	81	107

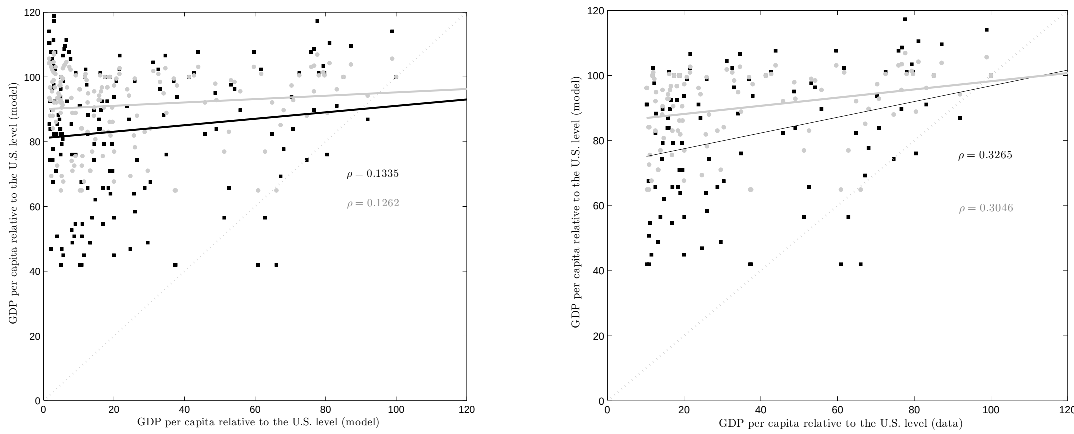
actual level. Since output per capita in Egypt is 10 percent of that of the United States, gender discrimination explains about 64 percent of the difference in output per capita between the two countries. When fertility is constant the model explains about 39 of the difference, still a sizeable fraction.[44]

Part II, shows what output per capita in the United States would be if gender barriers were similar to what is observed in Finland, Norway, and Sweden. Scandinavian countries are particularly interesting because gender inequality in earnings is lower in these countries than in the United States (see Table 3, part II), but output per capita is slightly lower than in the United States. Our computations show that the United States can increase output per capita significantly if gender inequality decreased to the level observed in Scandinavian countries. For instance, if gender inequality in the US were similar to the one in Sweden, output per capita would be 17 percent higher than the level observed in 2000.[45]

[44]Instead of using the female to male earnings ratio to estimate ϕ, we could have used the gender gap index (see Hausmann, Tyson, and Zahidi (2006)), which is the synthesis of gender discrimination indices in health, education, and political and economic empowerment. To determine the parameter estimate for ϕ for each country we could have multiplied the ratio of the gender gap index of a country to the U.S. value by the baseline $\phi = 0.63$. This is presented in Appendix 7. Notice that in this case, the experiments underestimate the gender wage gap and gender barriers explain a smaller fraction of international income differences (see Table 8 in Appendix 7). However, these experiments would require that the mapping of differences in the gender gap index would translate in the observed differences in ϕ.

[45]Welfare would be 3.73 percent higher relative to the baseline.

Figure 2: Empirical data and model predictions



(a) All countries

(b) Countries with income per capita greater than 10% of the U.S. level

Notes: Black squares represent model predictions with endogenous fertility and the black solid line is the best linear fit. Gray circles and the accompanying gray solid line correspond to the constant fertility model. Dotted gray line: 45 degree line.

Figure 2(a) summarizes the performance of our model for 118 countries, first for the baseline model and then for the model with constant fertility. The figure plots on the y axis the value of country output per capita relative to the U.S. level, as predicted by the model. On the x axis, we plot the value of the exact same variable, as observed in the data. If gender discrimination explained all of the difference in per capita output between a country and the U.S., the corresponding point would lie on the 45 degree line. The graphs reveal three extremely important features. First, the model tends to predict values of per capita output that are higher than those observed in the data. This is expected given that we focus only on barriers to female labour force participation and abstract from all other differences amongst countries, such as TFP differences, labour market institutions, and government policies. We also abstract from the effects of gender discrimination on human capital, working through a decrease in young girls'access to education, which is also expected to be considerable. Second, for some countries, gender discrimination explains all of the difference in relative output levels, as shown by the cases where the point lies very close to the 45 degree line. Third, the model with endogenous fertility shows a stronger positive correlation between predicted and actual values, when compared to the exogenous fertility model. The differences between these two correlations are statistically different from zero at the usual confidence levels.

Therefore, abstracting from fertility choice and considering only differences in female labour force participation might lead to misleading conclusions about the effects of barriers for women to work on development.

A final feature to notice is that for very poor countries gender barriers explain a low fraction of the difference in income level between these countries and the United States. Very poor countries have a low level of gender inequality in earnings. In fact, Goldin (1995) and Galor and Weil (1996) emphasize that female labour force participation has a U-shaped pattern. This is because female labour force participation is high and gender inequality is low in the traditional agriculture sector. Therefore, as in Galor and Weil (1996), our model is more appropriated to analyse economies that are consistent with the modern growth regime with a negative relationship between income and population growth (e.g., Galor and Weil (2000)). Figure 2(b) shows that the model has a better fit when we exclude from our simulations countries with output per capita that is below 10 percent of the U.S. level. The correlation between model predictions and data is almost three times larger than when the whole sample is included. The difference in correlations between data and model, with and without endogenous fertility, is also statistically different from zero at any usual confidence level.

3.4. Causality in gender barriers

In our model, the gender wage gap derives from the existence of barriers to female participation in the labour market and is constant over time. Although there is empirical evidence showing that gender inequality is largely determined by social and cultural norms at the national level that hardly change in the short run,[46] this does not imply that economic forces, including structural transformation, which raises women's relative wage, do not have an impact on those norms.[47] In this section, we alter our benchmark model and let capital accumulation affect the extent of barriers to female labour force participation. This is an important modification and is in fact more consistent with the empirical evidence than the model in which gender barriers are constant over time. The benchmark model is, however, simpler to present and its analytical results are

[46] See, for instance, Fernandez (2007).

[47] Hazan and Maoz (2002) provide an interesting model of the dynamics of female labour force participation based on endogenous changes of social norms. Doepke and Tertilt (2009) show how technical progress can lead men to choose the extension of women's rights.

more straightforwardly derived. Nevertheless, it is important to investigate whether or not the quantitative results presented in the previous section hold also in this framework with endogenous gender barriers. We view the results of this Subsection not as a robustness check of the model with exogenous gender barriers, but as the main quantitative analysis of the paper. We assume that $\phi_t = \phi(\hat{k}_t)$ with $\phi'(\cdot) > 0$ so that gender barriers decrease as the economy develops. In particular, we let

$$\phi_t = \phi(\hat{k}_t) = \begin{cases} \phi_1 \times (1 + \hat{k}_t), \quad \phi_1 > 0 & \text{if } \hat{k}_t \leq \frac{1}{\phi_1} - 1, \\ 1 & \text{if } \hat{k}_t \geq \frac{1}{\phi_1} - 1. \end{cases} \tag{29}$$

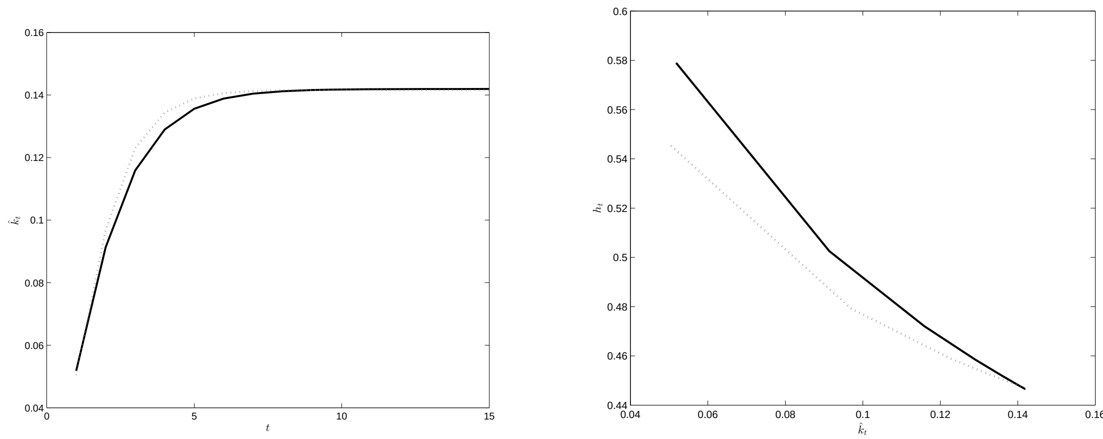
The rest of the framework is kept identical to that presented in Section 1. We calibrate the model to match the same statistics reported in Table 1.[48] In this case, the gender wage gap decreases as the economy accumulates capital and develops, for two reasons: differences in productivity across genders decrease, and gender discrimination decreases in response to the higher relative wage of women. Figure 3(a) shows that qualitatively the two economies - with ϕ exogenous and ϕ endogenous - behave similarly in terms of capital per effective unit of couple. The speed of convergence is faster when ϕ is exogenous than when ϕ depends on the capital stock.[49] Notice that the fall in hours worked at home is stronger with ϕ endogenous than when ϕ is exogenous. This is because not only the opportunity cost of staying at home increases with capital accumulation but also gender barriers decrease.

Table 4 provides some quantitative implications with the model presenting feedbacks from development to gender barriers (see Table 4). We vary the exogenous parameter ϕ_1 and examine the model's predictions along the same dimensions described in Table 2. Again, all statistics correspond to those observed for the year 2000. Results are very similar to those presented in Table 2. For instance, when ϕ_1 decreases by a factor of 2, output per capita decreases by 55

[48] The calibrated parameter values are: $\alpha = 0.4$, $\mu = 0.4509$, $\beta = 0.3747$, $\gamma = 0.123$, $D = 2.184$, $\theta = 0.965$, $\hat{k}_0 = 0.0518$, and $\phi_1 = 0.557$. Since barriers to female labour force participation decrease with capital accumulation, then $\phi_{1900} = 0.5859$ and $\phi_{2000} = 0.6325$. In addition, we have that $\frac{y_{2000}}{y_{1900}} = 7$, $\frac{1-h^w_{2000}}{1-h^h_{2000}} = 0.55$, $\frac{\phi w_{2000} h^w_{2000}}{y_{2000}} = 0.34$. Fertility in the calibrated economy is equal to 2.58 in 1900 and 2.03 in 2000. Therefore, the fall in fertility in this model is more consistent with the fall observed in the data than with what is generated in the benchmark model. In the data, fertility adjusted by survival to age 10 decreased from 2.6 in 1900 to about 2 in 2000. The real rate of return is equal to 2.2 percent between 1975 and 2000 and the capital-to-output ratio is 5.8 in 2000.

[49] The difference equation for capital per unit of efficiency couple $\hat{k}_{t+1} = \tilde{\xi}(\hat{k}_t)$ will be similar to that of equation (20), except that $\Delta(\phi(\hat{k}_t))$ will be a postive function of $\hat{k}_t$ and h^w_t will decrease faster with respect to $\hat{k}_t$. There are two effects on the speed of convergence. In the one hand, the speed of convergence increases because $\Delta(\phi)$ is positively related with ϕ and therefore with $\hat{k}_t$. In the other hand, h^w_t is decreasing in ϕ and this slows down convergence. In our numerical exercises, the second effect dominates the first one.

Figure 3: Model with endogenous gender barriers.



(a) Capital per unit of efficiency couple.

(b) Hours worked versus capital per unit of efficiency couple.

Notes: Black solid line: ϕ increases with $\hat{k}_t$; dotted black line: ϕ constant.

Table 4: *Gender inequality and development: Quantitative properties of the model*

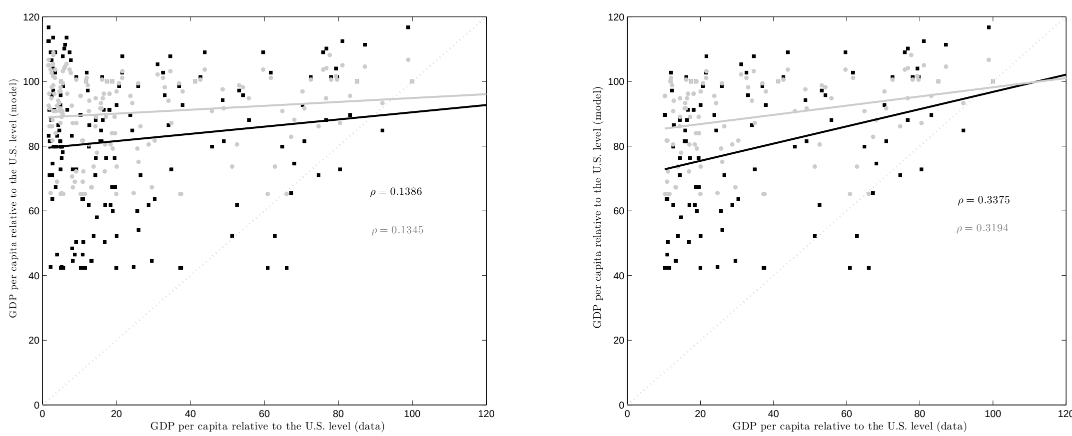
	Output per capita, % baseline	Female to male earnings ratio	Hours at home, % baseline	Welfare % of consumption	Output per capita, % baseline (constant fertility)
U.S. (baseline)	100	63	100	0	100
$\phi = \frac{1}{1.5} \times \phi_{\text{base}}$	65	40	159	-6.17	83
$\phi = \frac{1}{2} \times \phi_{\text{base}}$	45	29	211	-11	68
$\phi = \frac{1}{3} \times \phi_{\text{base}}$	42	19	218	-14	65

percent, while in the baseline case it decreased by 50 percent. Notice, however, that now the gender wage gap increases further, reflecting the additional assumption of endogenously decreasing discrimination. The female to male earnings ratio decreases from 63 percent to 29 percent, while in the baseline case it decreased to 32 percent. Therefore, to undertake a cross-country analysis, we would need a smaller variation in ϕ than in our benchmark model. For instance, a decrease in ϕ_1 so that the female to male earnings ratio is the same as in the baseline case (say 30 percent), delivers a decrease in output per capita that is roughly the same as when gender barriers were exogenous (and ϕ constant). This implies that a counterfactual analysis similar to the one implemented in Section 3.3 would yield similar results for this case when gender barriers depend on the level of capital per effective unit of couple.

In Figure 4 we implemented an exercise similar to that produced in Figure 2. We changed ϕ_1

such that the model reproduces the gender wage gap observed in 2000 for the same 118 economies considered in Figure 2. Firstly, notice the similarities of the two figures: In general, the model predicts values of per capita output that are higher than those observed in the data; for some countries, gender barriers explain most of the difference in relative output levels; and the model with endogenous fertility shows a stronger positive correlation between predicted and actual values than the exogenous fertility framework. The difference between these two correlations are statistically different from zero at the usual confidence levels. The correlation between the predicted relative income of the models presented in Figure 2(a) and 4(a) is 0.9981 when fertility is endogenous and 0.9972 when fertility is exogenous.[50] For instance, for the case of Ireland in which the gender wage gap in 2000 is 0.53, the output per capita will be 86 percent relative to the calibrated model. This is very similar to the number found in Table 3, which for Ireland was 87 percent relative to baseline. For the case of Egypt, output per capita will be 42.4 percent, which is roughly the same number generated in Table 3.

Figure 4: Empirical data and model predictions: Endogenous gender wage gap



(a) All countries

(b) Countries with income per capita greater than 10% of the U.S. level

Notes: Black squares represent model predictions with endogenous fertility and the black solid line is the best linear fit. Gray circles and the accompanying gray solid line correspond to the constant fertility model. Dotted gray line: 45 degree line.

[50]For the exercise in which we consider economies with per capita income larger than 10% of the United States level, then these correlations are 0.9978 and 0.9967, respectively.

4. Concluding Remarks

This paper develops a model of economic growth where fertility and female labour market participation are endogenous as a way to estimate the output costs of gender discrimination. In our model, gender discrimination drives a wedge between women's labour productivity and female wages. We calibrate the model economy so that the long-run equilibrium matches key statistics of the United States economy, including the gender wage gap. We then compare this benchmark economy with a counterfactual economy in which the value of all parameters are similar to those calibrated for the United States economy, except for the gender wage gap. Gender discrimination decreases output per capita in two ways: it discourages female labour market participation, thus decreasing output; and it increases fertility and thus population in steady state, thus decreasing output per capita. The two channels have similar quantitative relevance, with the decrease in labor market participation coming out as slightly more important.

A counterfactual exercise using 118 developing and developed countries shows that for some economies a large fraction of country differences in output per capita can be attributed to gender inequality. For Saudi Arabia, gender discrimination explains all of its output difference relative to the US. Were the U.S. to display the level of gender wage inequality present in, say, Egypt, its output per capita could drop by 58 percent relative to the initial level. This estimate is obtained by changing only the level of gender wage inequality in the U.S. benchmark economy so that it matches Egypt's value, while maintaining all other parameters as those calibrate for the United States economy. Our conclusion is that many countries can make substantially better use of their workforce and considerably increase output per capita by discouraging gender barriers in the labour market. This is also valid for the United States. Output per capita would increase by 17 percent if gender inequality were reduced to the level observed in, say, Sweden.

Further research should concentrate on two issues. The first is how distinct mechanisms of gender discrimination - bias against participation versus wage discrimination - affect output. The second, and most important, is the relationship between gender discrimination and the accumulation of human capital. In particular, the discouragement of girls's education might have a strong impact on human capital and output per capita, as suggested in (cf., Greenwood, Guner, Kocharkov, and Santos, 2012).

University of Cambridge and Sao Paulo School of Economics - FGV
Universidade Nova de Lisboa and CEPR

References

Acemoglu, D., D. Autor, and D. Lyle (2004): "Women, War and Wages: The Effect of Female Labor Supply on the Wage Structure of Mid-Century," *Journal of Political Economy*, 112(3), 497–551.

Albanesi, S., and C. Olivetti (2009): "Gender Roles and Medical Progress," *NBER Working Paper 14873*.

Alesina, A., A. Ichino, and L. Karabarbounis (2011): "Gender Based Taxation and the Division of Family Chores," *American Economic Journal: Economic Policy*, 3(2), 1–40.

Alesina, A., F. Lotti, and P. E. Mistrulli (2013): "Do Women Pay More for Credit? Evidence from Italy," *Journal of the European Economic Association*, 11(s1), 45–66.

Algan, Y., and P. Cahuc (2007): "The Roots of Low European Employment: Family Culture?," In: J. Frenkel and C. Pissarides (eds.), *NBER Macroeconomic Annual*, MIT Press.

Becker, G. (1957): *The Economics of Discrimination*. The University of Chicago Press.

Bertocchi, G. (2011): "The Enfranchisement of Women and the Welfare State," *European Economic Review*, 55(4), 535–553.

Bick, A., and N. Fuchs-Schündeln (2012): "Taxation and Labor Supply of Married Women across Countries: A Macroeconomic Analysis," *CPER Discussion Paper No 9115*.

Blau, F., and L. M. Kahn (2000): "Gender Differences in Pay," *Journal of Economic Perspectives*, pp. 75–99.

——— (2003): "Understanding International Differences in the Gender Wage Gap," *Journal of Labor Economics*, 21(1), 105–143.

Blinder, A. S. (1973): "Wage Discrimination: Reduced Form and Structural Estimates," *Journal of Human Resources*, 8(4), 436–455.

Cavalcanti, T. V., and J. Tavares (2007): "The Output Cost of Gender Discrimination: A Model-Based Macroeconomic Estimate," *CEPR Discussion Papers 6477.*

——— (2008): "Assessing the "Engines of Liberation": Home Appliances and Female Labor Force Participation," *Review of Economics and Statistics*, 90(1), 81–88.

——— (2011): "Women Prefer Larger Governments: Growth, Structural Transformation and Government Size," *Economic Inquiry*, 49(1), 155–171.

Chen, K., A. İmrohoroğlu, and S. İmrohoroğlu (2009): "A Quantitative Assessment of the Decline in the US Current Account," *Journal of Monetary Economics*, 56(8), 1135–1147.

Daly, K. (2007): "Gender Inequality, Growth and Global Ageing," *Society of Business Economist Journal*, 38(1).

Doepke, M., M. Hazan, and Y. D. Maoz (2007): "The Baby Boom and World War II: A Macroeconomic Analysis," *Mimeo, UCLA.*

Doepke, M., and M. Tertilt (2009): "Women's Liberation: What's in It for Men?," *Quarterly Journal of Economics*, 124(4), 1541–1591.

——— (2011): "Does Female Empowerment Promote Economic Development?," *World Bank Policy research Working Paper no. 5714.*

Doepke, M., M. Tertilt, and A. Voena (2012): "The Economics and Politics of Women's Rights," *Annual Review of Economics*, 4, 339–372.

Dollar, D., and R. Gatti (1999): "Gender Inequality, Income, and Growth: Are Good Times Good for Women?," *Policy Research Report on Gender and Development, Working Paper Series, No. 1.*

Erosa, A., L. Fuster, and D. Restuccia (2005): "A Quantitative Theory of the Wage Gender Gap," *Working Paper, University of Toronto.*

Fernandez, R. (2007): "Alfred Marshall Lecture: Women, Work, and Culture," *Journal of the European Economic Association*, 5(2-3), 305–332.

Fernández, R., A. Fogli, and C. Olivetti (2004): "Mothers and Sons: Preference Formation and Female Labor Force Dynamics," *Quarterly Journal of Economics*, 119(4), 1249 – 1299.

Galor, O., and D. N. Weil (1996): "The Gender Gap, Fertility, and Growth," *American Economic Review*, 85(3), 374–387.

——— (2000): "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *American Economic Review*, 90(4), 806–828.

Goldin, C. (1990): *Understanding the Gender Gap: An Economic History of American Women.* Oxford: Oxford University Press.

——— (1995): "The U-Shaped Female Labor Force Function in Economic Development and Economic History," In: T. Paul Schultz (ed.), *Investment in Women's Human Capital and Economic Development*, Chicago University Press.

——— (2006): "The Quiet Revolution that Transformed Women's Employment, Education, and Family," *American Economic Review*, 96(2), 1–20.

Gollin, D. (2002): "Getting Income Shares Right," *Journal of Political Economy*, 110(2), 458–474.

Goss, S. C. (2010): "The Future Financial Status of the Social Security Program," *Social Security Bulletin*, 70(3), 111.

Greenwood, J., N. Guner, G. Kocharkov, and C. Santos (2012): "Technology and the Changing Family: A Unified Model of Marriage, Divorce, Educational Attainment and Married Female Labor-Force Participation," *Working Paper, University of Pennsylvania.*

Greenwood, J., A. Seshadri, and G. Vandenbroucke (2005): "The Baby Boom and Baby Bust," *American Economic Review*, 5(1), 183–207.

Greenwood, J., A. Seshadri, and M. Yorukoglu (2005): "Engines of Liberalization," *Review of Economic Studies*, 72(1), 109–133.

Guiso, L., P. Sapienza, and L. Zingales (2003): "People's opium? Religion and economic attitudes," *Journal of Monetary Economics*, 50(1), 225–282.

Guner, N., G. Ventura, and X. Yi (2008): "Macroeconomic Implications of Size Dependent Policies," *Review of Economic Dynamics*, 11(4), 721–744.

Hall, R., and C. Jones (1999): "Why Do Some Countries Produce So Much More Output per Worker than Others?," *Quarterly Journal of Economics*, 114(1), 83–116.

Hausmann, R., L. D. Tyson, and S. Zahidi (2006): *The Global Gender Gap Report 2006.* World Economic Forum, Geneva, Switzerland.

Haveman, R., and B. Wolfe (1995): "The Determinants of Children's Attainments: A Review of Method and Findings," *Journal of Economic Literature*, 33(4), 1829–1878.

Hazan, M., and B. Berdugo (2002): "Child Labor, Fertility and Economic Growth," *Economic Journa*, 112(482), 810–828.

Hazan, M., and Y. Maoz (2002): "Women's Labor Force Participation and the Dynamics of Tradition," *Economics Letters*, 75, 193–198.

Hsieh, C.-T., E. Hurst, C. I. Jones, and P. J. Klenow (2012): "The Allocation of Talent and U.S. Economic Growth," *Mimeo, Stanford University.*

Hsieh, C.-T., and P. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124(4), 1403–1448.

Jones, L., R. E. Manuelli, and E. R. McGrattan (2003): "Why Are Married Women Working So Much?," *Federal Reserve Bank of Minneapolis, Staff Report.*

Lagerlöf, N.-P. (2003): "Gender Equality and Long Run Growth," *Journal of Economic Growth*, 8(4), 403–426.

Maddison, A. (2006): *The World Economy: Historical Statistics.* Paris: OECD Development Center.

McGrattan, E. R., and E. C. Prescott (2011): "On Efficiently Financing Retirement," *Minneapolis Fed, Working Paper 692.*

Mulligan, C., and Y. Rubinstein (2005): "Selection, Investment, and Women's Relative Wage Since 1975," *NBER Workin Paper 11159.*

Ngai, L. R. (2004): "Barriers and the Transition to Modern Growth," *Journal of Monetary Economics*, 51(7), 1353–1383.

Oaxaca, R. (1973): "Male-Female Wage Differentials in Urban Labor Markets," *International Economic Review*, 14(3), 693–709.

Olivetti, C., and B. Petrangolo (2008): "Unequal Pay or Unequal Employment? A Cross-Country Analysis of Gender Gaps," *Journal of Labor Economics*, 26(4), 621–654.

Parente, S. L., and E. C. Prescott (1994): "Barriers to technology adoption and development," *Journal of Political Economy*, 102(2), 298–321.

——— (2005): "A Unified Theory of the Evolution of International Income Levels," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. N. Durlauf, vol. 1, chap. 21, pp. 1371–1416. Elsevier.

Psacharopoulos, G., and Z. Tzannatos (1989): "Female Labor Force Participation: An International Perspective," *The World Bank Research Observer*, 4(2), 187–201.

Rahim, F., and J. Tavares (2012): "Gender Discrimination: The Role of Males and Per capita Income," *CEPR Working Paper No. 9045*.

Ramey, V. A., and N. Francis (2009): "A Century of Work and Leisure," *American Economic Journal: Macroeconomics*, 1(2), 189–224.

Restuccia, D., and R. Rogerson (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 11(4), 707–720.

Seguino, S. (2000): "Gender Inequality and Economic Growth: A Cross-Country Analysis," *World Development*, 28(7), 1211–1230.

Siaroff, A. (1994): "Work, Welfare and Gender Equality: A New Typology," In: Sainsbury, D. (ed.) *"Gendering Welfare States"*, pp. 82–100, Sage: London.

Soares, R. R., and B. L. S. Falcão (2008): "The Demographic Transition and the Sexual Division of Labor," *Journal of Political Economy*, 116(6), 1058–1104.

United Nations (2005): *Human Development Report 2005: Human Development Indicators.* United Nations.

Weichselbaumer, D., and R. Winter-Ebmer (2005): “A Meta-Analysis of the International Gender Wage-Gap,” *Jornal of Economic Survey*, 19(3), 479–511.

World Bank (2011): *World Development Report 2012: Gender Equality and Development.* World Bank, Washington D.C.

Young, A. (1995): “The Tyranny of Numbers: Confronting the Statistical Reality of the East Asian Growth Experience,” *Quarterly Journal of Economics*, 110(3), 641–680.

5. Model with gender productivity difference in the labour market

The model presented in Section 1 relies on the assumption that women and men differ as to their productivity in raising children, while they are equally productive in the labor market. Another, alternative possibility, is that there are two types of labour, physical and mental labor, as proposed in Galor and Weil (1996). While each man is endowed with one unit of physical labour and one unit of mental labour, each woman can offer one unit of mental labour only. As before, both men and women can dispose of one unit of time, which they can divide between time at work and time raising children.[51] Though the child rasing technology and production function have changed, the preferences of the couple are represented by the same utility function as before.

The production technology uses capital, mental labour, L_t^m, and physical labour, L_t^p, to produce output, according to a constant returns to scale production function:

$$Y_t = K_t^{\alpha}(A_t L_t^m)^{1-\alpha} + BA_t L_t^p, \tag{30}$$

where $A_t = (1+\mu)^t$, $B > 0$, and $\alpha \in (0, 1)$. While physical labour is a substitute for physical capital, mental labour is a complement to capital. Thus, physical labour loses importance as the economy accumulates physical capital and its compensation deteriorates in relative terms. The wage of

[51] A previous version of this paper was based on this model, which is available at the CEPR working paper series (see Cavalcanti and Tavares, 2007).

physical labour, w_t^p, does not depend on capital accumulation, while the wage of mental labour, w_t^m, increases with capital accumulation. Therefore, female labour force participation increases as the relative wage of mental labour increases and, concomitantly, the gender wage gap, $\frac{\phi w_t^m}{w_t^p + w_t^m}$ decreases.

Let h_t be the time that parents devote to raising children. The child raising production function is given by

$$n_t = Dh_t^\theta, \ D > 0, \ \theta > 0, \tag{31}$$

The opportunity cost of raising children is greater for a man, $(w_t^p + w_t^m)$, than for a woman, ϕw_t^m, $\phi \in (0, 1)$. Therefore, if $h_t \leq 1$, only the wife will spend time raising children. In the case where $h_t > 1$ both men and women will raise children, but the man will also work some time in the market

In equilibrium, demand equals supply in all markets. In the market for mental labour this means that $L_t^m = L_t^p(2 - h_t)$, or $m_t = \frac{L_t^m}{L_t^p} = 2 - h_t$. Let $\hat{k}_t$ be the capital level per unit of efficiency couple, i.e., $\hat{k}_t = \frac{K_t}{A_t L_t^p}$. We also have $\varpi_t = (1 - \phi)(1 - h_t)w_t^m$, such that transfers back to the individual equal to the cost of discrimination. Then:

$$h_t = \min\{1, \frac{\gamma\theta}{(1+\beta)\phi + \gamma\theta}[2 + \frac{B}{\phi(1-\alpha)\hat{k}_t^\alpha (2 - h_t)^{-\alpha}}]\}. \tag{32}$$

A necessary condition for women to participate in the labour market is still Assumption 1 and a proposition similar to Proposition 1 hold in this model.

Proposition 4. *Let assumption 1 be satisfied. Then female hours of work in the market increase with capital accumulation, $\hat{k}_t$, and decrease with labour market discrimination (low ϕ).*

Proof. Equation (32) determines h_t as an implicit function of $\hat{k}_t$, $\psi(\hat{k}_t, \phi)$, and a critical value $\hat{k}^*(\phi)$ such that

$$h_t = \begin{cases} 1 & \text{for } \hat{k}_t \leq \hat{k}^*(\phi), \\ \psi(\hat{k}_t, \phi) & \text{for } \hat{k}_t \geq \hat{k}^*(\phi), \end{cases} \tag{33}$$

and $\psi(\hat{k}_t, \phi) \in (0, 1] \ \forall \ \hat{k}_t \geq \hat{k}^*(\phi)$, where:

$$\hat{k}^*(\phi) = [\frac{B\gamma\theta}{(1-\alpha)(\phi(1+\beta) - \gamma\theta)}]^{\frac{1}{\alpha}}. \tag{34}$$

Using the Implicit Function Theorem we can show that $\psi_1(\hat{k}_t, \phi) < 0$ and $\psi_2(\hat{k}_t, \phi) < 0$, as long as $\hat{k}_t \geq \hat{k}^*(\phi)$. $\square$

In equilibrium it can be shown that

$$\hat{k}_{t+1} = \begin{cases} \frac{\beta[(1-\alpha)\hat{k}_t^\alpha + B]}{D(1+\beta)(1+\mu)} & \text{for } \hat{k}_t \leq \hat{k}^*, \\ \tilde{\Delta}(\phi)\frac{[(1-\alpha)\hat{k}_t^\alpha(2-h_t)^{-\alpha}]^\theta[B+(1-\alpha)\hat{k}_t^\alpha(2-h_t)^{-\alpha}(2-(1-\phi)h_t)]}{[B+2(1-\alpha)\hat{k}_t^\alpha(2-h_t)^{-\alpha}]^\theta} & \text{for } \hat{k}_t \geq \hat{k}^*, \end{cases} \tag{35}$$

where $\tilde{\Delta}(\phi) = \frac{\beta[(1+\beta)\phi+\gamma\theta]^\theta}{(1+\beta+\gamma\theta)(1+\mu)D(\gamma\theta)^\theta}$ is a constant. Using (33) into (35) defines a non-linear difference equation $\hat{k}_{t+1} = \xi(\hat{k}_t, \phi)$. It is algebraically demanding to show that a stable locally steady-state equilibrium exists. When $\phi = 1$, the model is similar to Galor and Weil (1996) and we can always guarantee the existence of such an equilibrium. For the calibrated parameters we always find numerically that there is a locally unique steady-state equilibrium for any $\hat{k}_0 > \hat{k}^*(\phi)$.

Table 5, Part I, provides all parameter values as well as a note on how each one was obtained. The calibration uses similar statistics to those used in the previous model. Now, we do not have to calibrate parameter $\bar{\mathbf{c}}$, but we have to calibrate parameter B, which is related to the gender wage gap. Then instead of targeting the fertility in 1900, we target the female to male wage earnings in 1900.[52]

We again explore how the equilibrium properties of the model change with gender barriers, measured by the female to male earnings ratio. Table 6 shows that results are qualitatively and quantitatively similar to those presented in Table 2. A decrease in ϕ by a factor of two decreases output per capita by approximately 42 percent when fertility is endogenous, and 17 percent when fertility is exogenous. Recall that a similar exercise using the model of Section 1 yielded 50 and 28 percent reductions in output per capita in the case of endogenous and exogenous fertility, respectively (see Table 2). Notice that the effects on welfare are greater in this case than in the model presented in Section 1. The reason is that parameter $\bar{\mathbf{c}}$ in the utility function of the model presented in Section 1 is independent of the level of female labour force participation.

[52] Goldin (1990) shows that in 1900 the average employed female earned about 46 percent of the average employed male across all occupations. In the manufacturing sector the female to male wage ratio was roughly 55 percent in 1900. As as result, we targeted a number for the female to male wage earnings in 1900 between 46 and 55 percent.

Table 5: *Parameter values and basic statistics*

Part I: Parameter Values		
Parameters	*Values*	*Comment/Observations*
α	0.4	Capital share based on Gollin (2002)
μ	0.4509	Rate of TFP growth based on Greenwood at al (2005)
β	0.3747	Calibrated to match the T-Bill annual real rate of return, 2.2%
γ	0.272	Constant population level along the steady-state
D	2.2	Calibrated to match the average private cost of children/GDP
θ	0.95	Calibrated to match hours worked by women relative to hours worked by men in 2000
B	0.025	Calibrated to match the U.S. female to male earnings ratio in 1900
ϕ	0.725	Calibrated to match the U.S. female to male earnings ratio in 2000
$\hat{k}_0$	0.0505	Calibrated to match the U.S. $y^{US}_{2000}/y^{US}_{1900}$, (Maddison (2006))
Part II: Basic Statistics		
	U.S. economy	*Baseline economy*
$\phi w^m_{1900}/(w^p_{1900}+w^m_{1900})$	46-55%	57%
$\phi w^m_{2000}/(w^p_{2000}+w^m_{2000})$	63%	62%
y_{2000}/y_{1900}	7.0	7.0
$1-h^w_{2000}/1-h^h_{2000}$	60%	56%
$\phi w^m_{2000}h^w_{2000}/y_{2000}$	40%	35%

Table 6: *Gender inequality and development: Quantitative properties of the model*

	Output per capita, % baseline	Female to male earnings ratio	Hours at home, % baseline	Welfare % of consumption	Output per capita, % baseline (constant fertility)
Baseline	100	63	100	0	100
$\phi = \frac{1}{1.5}\times\phi_{\text{base}}$	74	41	138	-13.05	91
$\phi = \frac{1}{2}\times\phi_{\text{base}}$	58	30	169	-22.01	83
$\phi = \frac{1}{3}\times\phi_{\text{base}}$	40	19	218	-34.19	71

6. Issues and Robustness

6.1. Measuring gender discrimination

Total wage inequality between men and women can be decomposed in two distinct parts: the first stems from differences in gender attributes - education, skills, among others, and the second from differences in the return to those attributes, including the effects due to gender discrimination.[53] In this paper we use the raw differences in gender pay for several reasons:

(i) Measures of gender wage discrimination are not readily available for a sufficiently high and diverse number of countries. An important source is Blau and Kahn (2003), who estimate the "unexplained" gender wage gap for the United States and OECD countries only, a total of 20 countries. Weichselbaumer and Winter-Ebmer (2005) provide a quantitative review of the vast empirical literature on the gender wage gap for a large sample of countries, but the period examined changes markedly across countries, which limits their use for cross-country comparisons as ours. In addition, most Middle-Eastern economies, generally characterized by high levels of gender discrimination, are absent from the sample.

(ii) Weichselbaumer and Winter-Ebmer (2005) unveil a strong positive correlation between the gender wage gap and the unexplained residual, suggesting that the relative cost of discrimination across countries would remain substantially unaltered were we to use information on the gender wage residual across countries.[54]

(iii) Much of the difference in endowments between women and men is explained by gender barriers to the participation of women in the labour market (see Doepke and Tertilt, 2009). In long-run models such as ours, where education and work experience are not explicitly considered, it makes sense to estimate the effects of discrimination by using raw gender wage inequality rather than a measure of statistical discrimination since the incentives for females

[53] See Blinder (1973) and Oaxaca (1973).

[54] Figure 2 of Weichselbaumer and Winter-Ebmer (2005) plots the reported gender wage gap versus the reported wage residual. For countries above the 45^0 line (e.g., Cote d'Ivoire, Tanzania, and Korea) women have lower endowments than men. Part of the total wage gap, therefore, can be attributed to differences in human capital. In countries below the 45^0 line (e.g., Singapore, Guinea, and Costa Rica) the contrary is true and women have higher endowments than men, though still receiving less pay. The majority of countries, however, lies close to the 45^0 line.

to obtain education and the experience they accumulate in the labour market are likely to be, themselves, influenced by discrimination.

(iv) Finally, as reported by Blau and Kahn (2000) and Goldin (2006), there is a higher fraction of women than men in low-paying jobs and at lower levels of the managerial hierarchy. This can explain why returns on human capital characteristics, such as years of schooling and experience, are different for men and women. But these gender occupational differences might - are likely to - be driven also by discrimination.

The four reasons above suggest the use of a broad measure of discrimination rather than just the gender wage residual. We also calculate the effects of gender barriers on development for the estimates of the "unexplained" gender wage gap as provided by Blau and Kahn (2003). According to their estimates, the "unexplained" gender wage gap in the United States is such that $\phi = 0.6804$. We recalibrate our parameters so that we matched the statistics reported in Table 1.[55] Figure 5 reports output per capita relative to the United States, as generated by the model when the raw rather than the "unexplained" wage gap is used.[56] As we can observe, most of the points are very close to the 45 degree line. This feature implies that the counterfactual income per capita levels generated by the model using either the raw gender wage gap or the "unexplained" gender gap are very similar for this sample of countries.

6.2. Selection bias in female labour force participation

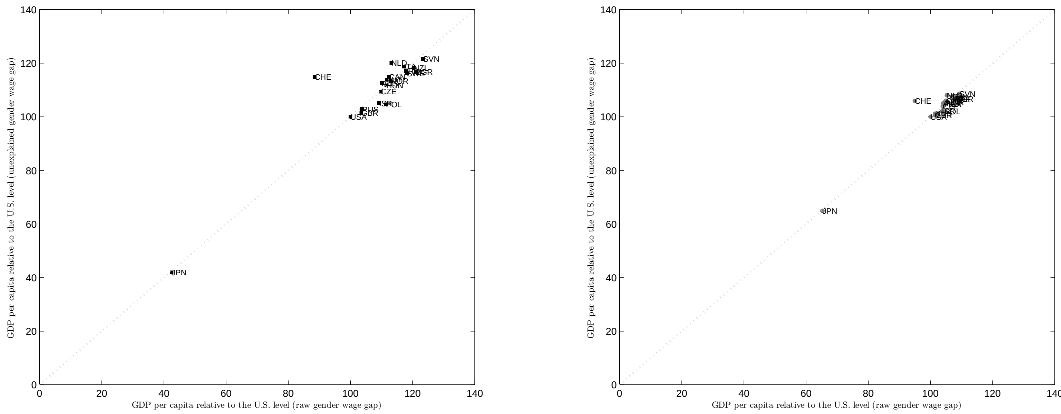
There is evidence that gender wage inequality might be affected by gender-based selection bias. Employed women tend to have relatively high levels of human capital and cognitive ability, a fact which impacts the raw gender wage gap.[57] Gender inequality in earnings would be even higher than observed were this selection bias to be taken into account. On the other hand, since highly productive women have a higher reservation wage than low skilled women, when gender barriers are large, highly productive women are less likely to work. Concomitantly, the observed gender

[55] The calibrated value of the parameters are the same for α, μ, β, D, and θ. Then we set $\gamma = 0.1315$, such that the population remains constant in the long run, and $\hat{k}_0 = 0.0509$. Then, we have $\frac{y_{2000}}{y_{1900}} = 7$, $\frac{1-h^w_{2000}}{1-h^h_{2000}} = 0.55$, $\frac{\phi w_{2000} h_{2000}}{y_{2000}} = 0.35$. The real rate of return is equal to 2.21 percent between 1975 and 2000 and the capital-to-output ratio is 5.9 in 2000. Fertility is equal to 2.42 in 1900 and 2 in 2000.

[56] Blau and Kahn (2003) also report the raw gender wage gap used in the analysis. In this case, for the United States we have that $\phi = 0.6737$.

[57] As in Olivetti and Petrangolo (2008).

Figure 5: Raw versus "unexplained" gender wage gap



(a) Model with endogenous fertility

(b) Model with exogenous fertility

Notes: Black squares represent model predictions with endogenous fertility. Gray circles represent model predictions with constant fertility model. Dotted gray line: 45 degree line.

wage gap would be higher than if we were to correct for selection bias. Using United States data, Mulligan and Rubinstein (2005) show that, in the 1980s and 1990s, working women typically had better backgrounds than women not working, but women not working in the 1970s had better backgrounds than women working. In sum, over time selection bias has changed signs, from negative to positive.

How would sample selection in female labour force participation affect our counterfactual estimates? In the case in which women not working have lower potential wages (lower productivity) than women working, an increase in gender barriers to female labour participation (decrease in ϕ) would decrease female labour participation, going from the left to the right of the productivity distribution. In this case, our estimates in Section 3.3 overestimate the true effect of barriers to female labour force participation on income levels since it is the low productivity women that are abandoning the market first. However, notice that the change in ϕ to match the new female to male earnings ratio would have to be higher, which suggests an underestimation of the true effect of gender barriers on development. If instead women not working have better skills than those in the labour market, the opposite is true. The overall quantitative implications of selection bias on our results would therefore depend on the specific type of sample selection bias in each country and the implied change in ϕ.

Table 7: *Empirical data and model predictions for reference economies. Source: United Nations (2005)*

Countries	**Data**		**Model**		
	Output per capita, % of baseline	Female to male earnings ratio	Output per capita, % of baseline	Female to male earnings ratio	Output per capita, % of baseline (const fert)
Baseline$_{\phi_1=0.557}$	100	63	100	63	100
Ireland$_{\phi_1=0.4835}$	91	53	88	53	95
Greece$_{\phi_1=0.499}$	56	55	91	55	96
Singapore$_{\phi_1=0.467}$	71	51	86	51	94
Saudi Arabia$_{\phi_1=0.1525}$	37	16	45	16	68
Iran$_{\phi_1=0.3904}$	19	39	66	39	84
Egypt$_{\phi_1=0.219}$	10	23	45	23	68
India$_{\phi_1=0.2935}$	8	31	51	31	73

Though we recognize that selection bias is an important issue in the study of gender barriers on economic development, it is unclear how it would change our results, qualitatively and quantitatively.

6.3. Transition versus balanced growth path

In the simulations so far, we varied the level of gender barriers to female labour force participation and calculated the effects of such barriers on output that would be observed in 2000. In such simulations, we implicitly assume that different countries are at the same development stage as the United States and that differences in the gender wage gap are due only to such gender barriers. This might be the case of developed countries, which went through the transition to modern growth at roughly the same time as the United States. It is unlikely to be true for developing countries. Differences in the gender wage gap might then be explained by differences in stages of development, as assumed in the subsection above. A similar point is made by Ngai (2004) and Parente and Prescott (2005), who compare the effects of barriers to technology adoption on differences in country incomes.

In Table 7 we change our gender barriers parameter in our U.S. calibrated economy of Subsection 3.4, such that we match the gender wage gap of a particular economy (e.g., Egypt) but, unlike our previous exercises, we consider the particular economy in the year 1950, not 2000. This choice of date implicitly suggests that the particular economy will take 50 years to reach its long-run equilibrium. We use the model with ϕ endogenous, but the benchmark model with ϕ constant yields similar results. Compared to Table 3, gender barriers explain a similar but smaller fraction of the difference in income levels between the U.S. and such selected economies. For countries with large gender wage gaps (e.g., Egypt, Iran, and Saudi Arabia), gender barriers remain a substantial factor in explaining differences in output per capita. For instance, gender barriers now explain 61 percent of the difference in relative income level between Egypt and the U.S (compared to 68 percent if we consider differences using steady-state levels). In fact, gender barriers explain a large fraction of the difference in income levels between Egypt and the U.S. even when we consider that the Egyptian economy will take 100 years to reach the actual U.S. stage of development. In this case, gender barriers *alone* still account for about 35 percent of the difference in output per capita between the two countries. This is a sizeable impact of discrimination on income per capita, given the parsimony of our model settings.

7. Gender inequality (ϕ) based on the gender gap index

In this Appendix we use the gender gap index (see Hausmann, Tyson, and Zahidi (2006)) to estimate ϕ, instead of the female to male earnings ratio. This index is a composition of gender discrimination indices in health, education, political and economic empowerment. To determine the parameter estimate for ϕ for each country we multiplied the ratio of the gender gap index of a country to the U.S. value by the baseline $\phi = 0.63$. For instance, the gender gap index in the United States and in Egypt are 70 and 58.1, respectively. The estimated value of ϕ for Egypt is therefore $\phi_{Egypt} = \frac{58.1}{70} \times \phi_{US} = \frac{58.1}{70} \times 0.63$=0.5229. Table 8 reports results using this approach. Notice that variations in gender barriers are smaller than what we observe in Table 3. For instance, in Table 3 we have that $\phi_{EGY} = 0.2736$, while in Table 8 the same parameter for the gender barriers in Egypt is $\phi_{EGY} = 0.5228$. As a result, in almost all cases the female to male earnings ratios are larger in the data than in model simulations presented in Table 8 and consequently the effects of gender barriers on output are smaller than using the raw gender wage gap. Notice that there are still important output losses. If gender barriers in the United States were similar to those observed in Egypt, then output per capita would be 14 percent lower.

Table 8: *Empirical data and model predictions for reference economies. Source: United Nations (2005)*

Countries	**Data**		**Model**		
	Output per capita, % of baseline	Female to male earnings ratio	Output per capita, % of baseline	Female to male earnings ratio	Output per capita, % of baseline (const fert)
Baseline$_{\phi_{US}=0.725}$	100	63	100	63	100
Part (a)					
Ireland$_{\phi_{IRL}=0.6713}$	91	53	105	67	102
Greece$_{\phi_{GRC}=0.5984}$	56	55	96	60	98
Singapore$_{\phi_{SGP}=0.5948}$	71	51	96	59	98
Saudi Arabia$_{\phi_{SAU}=0.5085}$	37	16	84	51	93
Iran$_{\phi_{IRN}=0.531}$	19	39	87	53	94
Egypt$_{\phi_{EGY}=0.5228}$	10	23	86	52	94
India$_{\phi_{IND}=0.5346}$	8	31	88	53	95
Part (b)					
Finland$_{\phi_{FIN}=0.8327}$	77	71	110	72	104
Norway$_{\phi_{NOR}=0.8348}$	99	77	110	73	104
Sweden$_{\phi_{SWD}=0.8441}$	77	81	111	73	104